# Fast Volume Visualization with Ray Casting

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Figure 1: A simple grey scale image generated by volpack-1.0 library

## 1 Introduction

#### 1.1 The Need for Volume Visualisation

Three dimensional data has been employed in many fields. In medicine it is used to represent the information coming from a magnetic resonance (MR) or a computed tomography(CT) machine. Geologists use three dimensional data for many purposes varying from seismic explorations to locating petrol oil reservoirs. In Confocal Microscopy the detailed structures of biological specimens are stored in three dimensional arrays. In engineering, dynamics of the fluids in designed systems are explored with three dimensional data showing how the fluid behaves.

But bare three dimensional array of data, especially with an immense size, is hard to understand for human beings. We need to visualize the three dimensional information in order to grasp it, but the quantity of data in the fields of study mentioned above often exceeds our imagination.

At this point, volume visualisation establishes itself as a valuable tool for us. The volume visualizer projects a view of the three dimensional dataset onto a two dimensional image plane, aiding us in comprehension of the structure within the data. The figure ?? represents a three dimensional medical data by making use of the volume visualisation technique.

#### 1.2 Volume Rendering

Volume rendering is the computer implementation which simulates the physical the behavior of light, propagating in a colored semi-transparent gel.

A volume renderer takes a three dimensional array of samples, assigns opacity and color values to each member of the three dimensional array and then projects them on a image plane, by blending their colors according to their transparencies.

The array, which it takes as input is called *volume* and each sample on a cell of the three dimensional matrix is called a *voxel*. A voxel of a data set can either represent a small cube in space or a sample point of a continuous function, which we are going to use for our volume visualisation algorithm.

In real life the interaction of light with a three dimensional semi-transparent gel is too complicated. It involves many possibilities for a light ray such as being absorbed, scattered or emitted by the volume. Furthermore, there is in physical world the cases of fluorescence or phosphorescence reaction. But these complexities are superfluous for volume rendering.

#### 1.2.1 Volume Rendering Equation

For Volume Rendering the physical phenomenon can be approximated to finding the color value of each pixel x on two dimensional image plane by adding color contributions of all voxels on the viewing ray intersecting with the image plane in that particular pixel x as it is shown in figure ??.

The color value L(x) for the vertex x on image plane can be found with the simplified function ??. In this function, we assume that the number of voxels on the viewing ray is n and each voxel has the color value  $c_i$  and opacity value  $\alpha_i$ . The opacity value 0 corresponds to full transparency and 1 corresponds to total opacity.

$$L(x) = \sum_{i=0}^{n-1} c_i \prod_{j=0}^{i-1} 1 - \alpha_i$$
 (1)

The volume rendering function ?? , which calculates the color value for a pixel on the image plane, is called the *volumetric composing equation*. It adds the color values by multiplying them with the approximated energy they receive. The approximation for energy value is done by multiplying the opacities through the first voxel to the current voxel.

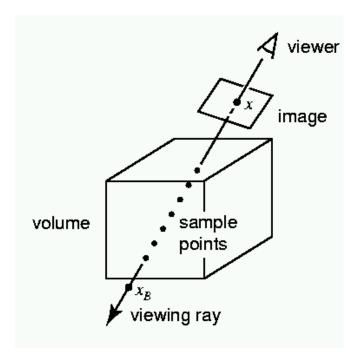


Figure 2: The value of pixel x is computed by summing color contributions of voxels standing between x and  $x_b$ .

#### 1.2.2 Data Representation

As it is mentioned above, a sample in a three dimensional matrix can be interpreted either as a box in space or a sample point of a continuous function.

If the latter is assumed, as we will do for our implementation, in equation ?? the attributes of the mentioned voxels do not have to come directly from the three dimensional array since the values on that array would be just sample values of a continuous function at arbitrary coordinates.

These arbitrary coordinates correspond to a grid in real life. As it is also shown in figure ??, the grid, to which the real life coordinates belong, may be in three types; either unstructured, curvilinear or regular. Regular sampling grids take samples from the real world so that each sample has a uniform distance to other samples. A curvilinear grid is warped form of a regular grid so that the edges are are no further linear. An unstructured sampling grid takes its samples from the real world arbitrarily.

Through the whole work we are going to work with regular sampling grids, since they are common and fast because of their simplicity. While using regular grids, the required values of the continuous function are interpolated by using some function on the three dimensional array. This process is called

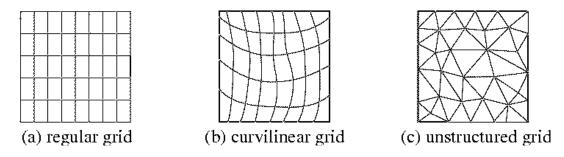


Figure 3: Examples of sampling grids.

Reconstruction or Resampling.

#### 1.2.3 Visualisation Process

As mentioned in the preceding text, the three dimensional array contains only sample values. Thus, the values  $c_i$  and  $\alpha_i$  used in function ?? should be computed from these sample values by two functions.

The function which maps samples to an  $\alpha_i$  for an arbitrary point in volume is called *classification function*. In our work classification functions are omitted. We assume that classification is done prior to rendering.

The other function which maps sample values into a color value for a point is called *shading function*. Differing from the classification function, shading function depends on other variables such as viewing direction and light source direction. In our implementation, shade function will be composed of a standard Phong shader and arbitrary functions to implement different illuminations.

Beside the two topics another important factor effecting the visualisation is view point and projection type. We will use parallel projection due to its simplicity so that it will facilitate a reasonable sample implementation.

Due to the complexity of the operations at this point, we don't consider a fully interactive rendering system.

# 2 Background

## 2.1 Volume Rendering Algorithms

There are actually four classes of volume rendering algorithms:ray casting, splatting, cell projection and multi-pass resampling. For our purposes, we

shall be examining ray casting and splatting algorithms. The hybrid system will be mentioned in next sections.

Cell projection is used with non-regular grids. Since we are going to work with regular grids cell projection algorithms remain out of our focus. Multipass algorithms are conceived for parallel systems. At this stage we are going to implement only serial programs, so this type of algorithms is not related to us.

#### 2.1.1 Ray Casting

Ray Casters are image order algorithms. They traverse the volume from image space to object space. This means the image space iterations stand in outer loops as it is in algorithm ??.

#### **Algorithm 1** Ray Casting

```
for y_i \leftarrow 1 to ImageHeight do

for x_i \leftarrow 1 to ImageWidth do

for z_i \leftarrow 1 to RayLength do

for all x_o in ResamplingFilter(x_i, y_i, z_i) do

for all y_o in ResamplingFilter(x_i, y_i, z_i) do

for all z_o in ResamplingFilter(x_i, y_i, z_i) do

ContributeVoxel(Voxel[x_o, y_o, z_o], ImagePixel[x_i, y_i])
end for
end for
end for
end for
end for
```

The disadvantage of these algorithms is that they do not access the volume in storage order and they require costly addressing arithmetic.

#### 2.1.2 Splatting

Contrasted to ray casting these types of algorithms operate in object order. They project the volume onto the image plane, meaning that they iterate over object coordinates in the outer loops as in algorithm ??.

The major drawback in Splatting is the expensive resampling, however the addressing arithmetic is much simpler.

#### Algorithm 2 Splatting

```
for z_o \leftarrow 1 to VolumeDepth do

for y_o \leftarrow 1 to VolumeHeight do

for x_o \leftarrow 1 to VolumeWidth do

for all z_i in ResamplingFilter(x_o, y_o, z_o) do

for all y_i in ResamplingFilter(x_o, y_o, z_o) do

for all x_i in ResamplingFilter(x_o, y_o, z_o) do

ContributeVoxel(Voxel[x_o, y_o, z_o], ImagePixel[x_i, y_i])
end for
end for
end for
end for
end for
```

### 2.2 An Efficacious Ray Casting Algorithm

First, we will implement an efficient ray caster which makes parallel projection. This will give us a platform on which we can build more complicated applications, test our objects for new applications, and serve as a sample implementation.

This effective ray caster will use the volume as a set of samples from a continuous three dimensional function. The ray will be traversing its way through these cubes according to a simple differential analysis algorithm and the color of the image plane pixel will be calculated by using ?? with n = len, where len is the number of points which should be resampled between the two points of the cast ray intersecting with the boundaries of the volume.

## 3 Shear-Warp Factorization

## 3.1 A Hybrid Algorithm

To summarize our findings about image order and object order algorithms, we can state that they offer the following advantages

- In image space algorithms high-quality resampling can be implemented
  efficiently, and the optimization by early ray termination is easily achievable.
- In object space algorithms, the addressing arithmetic is simpler and they offer an optimization due to spatial data structures since they

#### Algorithm 3 Efficient Ray Casting

```
for all (i, j) in ImagePlane do
  (i,j) \leftarrow BackgroundVoxel
end for
for all (i, j) in ImagePlane do
  [u, v, w] \leftarrow RayEnterVolume(i, j)
  [x, y, z] \leftarrow RayLeavesVolume(i, j)
  InitializeLineAlg([u, v, w], [x, y, z])
  len \leftarrow distance([u, v, w], [x, y, z])
  Voxel \leftarrow EmptyVoxel
  while (len \leftarrow len - 1) and not (opaque(Voxel)) do
    if not Transparent(Volume[u, v, w]) then
       Voxel \leftarrow ComposeTwoVoxels(Volume[u, v, w], Voxel)
    end if
     [u, v, w] \leftarrow NextVoxelAlongRay()
  end while
  if not opaque(Voxel) then
     Voxel \leftarrow ComposeTwoVoxels(BackgroundVoxel, Voxel)
  end if
  DrawPixel(i, j, Voxel)
end for
```

access the volume data in storage order.

Therefore, both classes of algorithms make distinct kind of improvements over the simplest possible implementation. An algorithm that would attain the advantages of both kinds would be desirable. However, such an algorithm can be neither strictly an image space algorithm, nor an object space one. Then, we should look for an algorithm that falls into both classes. Such an algorithm would assume both the early ray termination capability of ray casters, and the spanning advantages of splatting algorithms.

In this section, we describe a framework for obtaining an algorithm that is edible in both manners. The basic change is that we operate in an intermediate space that makes translation from the object space to image space and in the reverse direction a trivial task. We accomplish this by factoring the viewing transformation from the volume to the image. This process which is called shear-warp factorization involves first shearing the volume so that the volume is easier to process, secondly obtaining an intermediate image that is simpler to compute directly from that volume, and finally warping the intermediate image to the final image. The factorization can be accomplished both for affine viewing transformations and perspective viewing transformations, however we will examine only affine viewing transformations since they give rise to slightly simpler algorithms. We also describe the properties of the factorization that allow us to implement a volume rendering algorithm that is faster than both pure object space and pure image space algorithms.

## 3.2 Obtaining an Intermediate Space

Object space algorithms usually suffer from the complicated mapping from volume to image; this difficulty prevents efficient filtering and projection. Also in image space algorithms this difficulty causes costly arithmetic while projecting pixels to volume. That is why, we choose to work in an intermediate space that has a simple mapping from volume to image. Since the simplest case possible is that the cast rays are orthogonal to the object space, we transform the volume to a new volume which will make this possible and render an intermediate image which will be warped to the resulting image.

Consider the rays that are cast from the image plane in the case of parallel projection; an axis in the object space makes the least angle with all the rays. This axis is called the principal viewing axis and plays a big role in the factorization. We conceive the slices along the principal viewing direction. The kind of transformation that we should perform in order to align viewing rays with the principal viewing axis turns out to be very simple. If we place a secondary image plane attached to the closest slice, the viewing rays of it are

perpendicular to the object space. When we apply the regular deformation shear to the slices in the direction of the principal viewing axis so that viewing rays of the secondary image plane make the same angle with the slices that the viewing rays of the primary image plane makes with the original slices, we would have obtained a space with the desired property. The only remaining problem would be to compute the primary image from the secondary image.

We call the image on the secondary image plane the intermediate image. The intermediate image only requires a 2D warp in order to take us to the final image. <sup>1</sup>

The intermediate space with the orthogonality property gives us the simplest possible mapping from object space to image space since a projection of the slice along the principal viewing axis is equivalent to a translation of the slice.

The only addition for factorization of perspective viewing transformation is that each slice has to be scaled after sheared.

### 3.3 A Brute Force Algorithm

Using the shear warp algorithm it is possible to write a straightforward volume rendering algorithm (see alg. ??). The algorithm, without implementing any of the optimizations, is not much better then a brute force ray casting algorithm, except that the arithmetic overhead has been significantly reduced.

It is a hybrid algorithm because what it does is simply iterating the slices in front to back order, which have spaces identical with the image space. The iterated slices are immediately combined with the **over** operator to form the intermediate image.

In order to do the factorization for both parallel and perspective viewing transformation, one considers the equation

$$M_{view} = M_{warp} \cdot M_{shear} \tag{2}$$

To summarize, the parallel projection which we are more concerned with takes the viewing vector assuming that principal viewing axis is +z axis of object space  $v_i = (0,0,1)$  and derives  $v_0$ , the viewing direction vector

<sup>&</sup>lt;sup>1</sup>An analogy could perhaps better illustrate the solution. Assume that a block of highly detailed gel is being examined by a camera. In a dark room with a point light source, the picture will contain the kind of image that we wish to construct. Now also imagine that the block of gel stands on top of a table with a glass top and that the camera views the gel from below. A refractor is fixed under the table that can bend the light to a desired angle. The picture will be deformed. Now, if you deform the gel in a certain way, you may obtain the original picture of the gel. It might be quite impossible to do this experiment in real life, though.

#### Algorithm 4 Brute-Force-Shear-Warp

```
for z_0 \leftarrow 1 to VolumeDepth do

for y_i \leftarrow 1 to ImageHeight do

for x_i \leftarrow 1 to ImageWidth do

for all y_o in ResamplingFilter(x_i, y_i) do

for all x_o in ResamplingFilter(x_i, y_i) do

ContributeVoxel(Voxel[x_0, y_0, z_0], ImagePixel[x_i, y_i])
end for
end for
end for
end for
```

transformed to object space, where  $v_i = M_{view,3\times3}$ . The shear necessary in both x and y directions are plainly computed as  $s_x = -(v_{0,x})/v_{0,z}$  and  $s_y = -v_{0,y}/v_{0,z}$ .

#### 3.4 How Properties Permit Optimizations

In the preceding subsections, we had claimed that our hybrid volume rendering framework allows both of the significant optimizations, namely early ray termination and speedups from spatial data structures, to be incorporated. We now show how the orthogonality and equivalence of the intermediate image space and subspaces of the object spaces in slices may lead to better opportunities for optimization.

#### 3.4.1 Overview

The main advantage of the hybrid algorithm is that we can simultaneously traverse both object space and image space without high cost incurring. Since the intermediate space has the property that slice spaces are identical to intermediate image space and the property that viewing rays are parallel to principal viewing axis, only a simple translation suffices to project an entire slice onto the intermediate image plane. Thus, as we traverse a slice along the principal viewing axis, we can switch arbitrarily to the intermediate image as well.

This procedure is assistive in that it provides us to construct view independent spatial data structures in a preprocessing step prior to all rendering assuming that the classification remains constant.<sup>2</sup> All that is required will

<sup>&</sup>lt;sup>2</sup>A change in classification implies that all opacity values are going to be re evaluated.

be the encoding of the volume for each principal viewing axis possible, which amounts to three.

Among others, the particularly interesting spatial encoding is run length encoding, since it gives us the opportunity to implement both transparent voxel omission and early ray termination.

#### 3.4.2 Transparent Voxel Omission

We have indicated that it's possible to construct view independent spatial encoding of the volume. The encoding of interest to us is run length encoding since it can allow us to skip transparent voxels while processing a slice. Recalling that we can span a slice in both object space and image space efficiently and simultaneously, it will be clear that a run length encoding can give us the ability to distinguish consecutive voxels in a slice that should be omitted from evaluation. Since the run length encoding of the volume is going to be performed once while preprocessing the volume, the user will benefit from this optimization while interacting with the volume. As for the non transparent voxels, we try to make as least passes as possible. In practice, a very tight loop that functionally composes resampling, shading and composition is desirable since multiple passes would give rise to overheads due to decoding.

#### 3.4.3 Early Ray Termination

In ray casting algorithms, the backward projection iteration is terminated whenever the accumulated voxel is considered opaque. All remaining voxels are determined to be occluded, hence there is little need to shade them. For the early ray termination to work, a front to back order traversal of the object space is required. In the brute force algorithm ?? we already do that. However, a second requirement has to be fulfilled. An efficient memoization of the opacity of pixels must be supplied. In particular, we wish to skip consecutive pixels that are all opaque within a scanline. While the slice is being processed, the opacity data on the intermediate image is also considered so that the parts of the slice that would be projected onto the intermediate image are skipped at once.

Considering the order of our algorithm, what we do may be regarded as casting slices through an intermediate volume in front-to-back order. Then, if the memoization is handled efficiently, the early ray termination optimization will have been implemented as fast as on a traditional ray caster.

Even if we merely checked whether we needed to evaluate a pixel as a sort of *z-buffer*, this would give a valuable speed up. However, it is possible

to even further that with the employment of a disjoint set implementation. In disjoint sets, only a member of the set represent a set, and there exist efficient algorithms for disjoint set operations. If we take consecutive opaque pixels as disjoint sets, then whenever two such sets become neighbors, we will perform the Union operation and whenever we wish to skip over one we will perform the Find operation. In our Algorithms textbook, there is a complete elaboration of the subject. In order to implement the disjoint sets with forest of trees and path compression optimization, within the intermediate image, pixel indices for each pixel should be stored.<sup>3</sup>

#### 3.4.4 Resampling and the Opacity Correction

In the shear-warp factorization algorithms, we stick to 2D resampling since a 3D resampling would be considerably less efficient. However, a 2D bilinear interpolation filter that works by interpolating two scanlines of voxel data to produce one sampled voxel scanline is observed to produce high quality results. In addition to this, the resampling must be wary of the two main optimizations in the rendering algorithm since the optimizations would be corrupt otherwise. That is, it should those voxels that are non-transparent and non-occluded that are actually used for resampling.

Another issue is the opacity correction which should be addressed by any object space renderer. The correction may be applied just before resampling, and optimized by consulting a view dependent LUT.

# 4 Development Platform and Strategy

## 4.1 Language and Platform choice

Already, the public domain VolPack library implements the ideas presented in an OpenGL like C library. However, it is not our purpose to fit a user interface and build a complete visualisation application based on the library. In agreement with this, the software components we create must not be an exact replica of the VolPack software. Instead, we wish to implement a more object oriented approach in which we keep all the efficiency concerns. Since C++ has recently proven to be a very decisive tool for scientific applications, we have chosen C++ as our implementation platform. Although the study of algorithms has not been an easy task, it is our belief that the use of standard library and generic programming gives us quite a leverage. The platform for

<sup>&</sup>lt;sup>3</sup>The pixel pointers are basically the tree representation of disjoint sets.

development is set as GNU/Linux and the g++ compiler naturally.<sup>4</sup>

#### 4.2 Plan

The development will follow implementation of two algorithms first of which is the efficient ray caster and the second being a parallel projection shearwarp algorithm. They will be contained in the same program, and we want the user to be able to try both algorithms on the same data. Since we work with regular grids, we shall probably adapt Stanford CG Lab's file format.

The resampling, shading and composition of voxels are virtually identical in both cases, so we will achieve the implementation in an iterative manner. Where possible, we will give remarks for a comprehensive visualisation system.

## 5 Implementation

In this section, we report our implementation of the two algorithms we have undertaken. We shall explicate the consecutive phases of implementation in time order.

## 5.1 Establishing mathematical primitives

The usual array of mathematics should be at hand for volume rendering. We have been engaged in the development of vector and matrix mathematics from scratch by making use of generic programming. We have also established support for linear transformations, analytic geometry and rotation with Euler angles. The resulting code encapsulates all mathematics code within classes responsible for representing and operating on vectors, matrices, angles and geometric objects such as ray, interval and prism.

# 5.2 Object design and implementation for the volume visualisation domain

Volume visualisation requires the same approach for implementing polygon based systems. Not only the mathematics but the hierarchy and model which we use are quite similar. In particular, the idea of transformation proves to be very useful with which we can define many three dimensional objects that make a decent model. With transformation types we encapsulate typical linear transformations such as rotation, translation and scaling.

<sup>&</sup>lt;sup>4</sup>Considering the availability, standard C++ support and portability of the system.

We have implemented transformations and viewing transformations as classes from which a simple three dimensional component class, which also collects attributes such as shading parameters, is constructed. The basic three dimensional types are camera, light source, and volume, all of which are subtypes of a general component type. The volume is elaborated; it is a parametrized type which can contain user defined voxel types. We have defined two types of voxels and colors: gray scale and RGB. Another property of the volume is that it can be specified to an arbitrary size and subdivision easily. Since there can be distinct representations of the same regular grid of samples, we have taken that different classes may represent the same data so we have aimed a consistent interface which can be implemented in volume types utilizing different encodings. The interface has been defined with the raw volume type, which obviously encodes samples as a three dimensional array.

Surely, these basic components come together to form higher level objects just as voxels come together within a prism and a component type<sup>5</sup>. Our design rests on MVC<sup>6</sup> design pattern, that is why model and view classes are separate. The model class is exclusively streamlined to support volume visualisation for our purposes, though designed to be extensible in a scalable manner. We have not included support for multiplicity and part-of hierarchy in the model type, but the component type is evident and a more complicated structure is edible. The view class is deemed a visualizer. A visualizer can render any volume, and multiple visualisers can render the same volume. The visualizer has been designed to be generic, however we have not done so. It has only been given a clean interface that can be revised for a type parametrisation. It could also be thought that visualizers may be parametrized according to

- the type of volume they will be used for. Then, a raw volume would be rendered by one type of visualizer and a, for instance, run length encoded volume would be rendered by another type.
- the class of visualisation algorithm. It would be convenient to have different classes for ray casting and splatting.

Although we have not implemented controller classes, it would be an easy extension to do so. With controller classes, and a part-of hierarchy system, a decent modeling framework may be obtained. Another concern has been efficiency; we have tried to avoid copy constructors in most places, however the

<sup>&</sup>lt;sup>5</sup>We have used multiple inheritance to indicate proper is-a hierarchies.

<sup>&</sup>lt;sup>6</sup>Originally advocated by Smalltalk standard library.

vector library remains somewhat problematic. The vector template classes need to be specialized for faster vector operations. We have also tried to design the architecture allowing copy-on-write where possible.

Also at this stage, we have included support for rendering on a canvas with the GTK- - library. The visualizer classes use GTK- - drawing areas, so that GTK+ user interfaces can be quickly plugged into our volume rendering architecture.

The testing at this stage has proved to be most useful. By observing example vectors and transformations with simple animations, we have improved and corrected our implementation so that more advanced stages do not suffer from bugs.

### 5.3 Implementation of Efficient Ray Casting algorithm

For the ray casting algorithm, we have revised the types and methods for three dimensional objects. Most importantly, ray casting requires a working resampling method for obtaining values of the continuous color and transparency functions, and three dimensional arithmetic to detect intersections with the volume and interpolating rays through the volume.

We have also tried to keep the interface to visualizer and model as flexible as possible. The model can be easily modified since volume, light, and camera share the same component interface. The basic objects have also been made more configurable. The visualizer has suitable functions to serve as callbacks in a signaling system, so that timings and user actions can be accounted for.

After implementing the image order ray-caster, we have tested it with a test volume consisting of gradients which may easily be observed. Implementing the over operator, intersection and resampling correctly has proven to be most important at this stage. The next step has been implementing volpack's raw file format in order to read raw volumes from it. The file format is read through a coder class, different coder classes might be used for different file formats, and converters might facilitate going from one representation to another in an efficient way.

Once we have obtained the correct decoding, we have moved on to implement shading over the samples. This is achieved by a shader class. Implementing different shading functions in shader classes would be a suitable idea for C++, then those classes could be template parameters of visualizer classes. A simple Phong shading which does not produce specular highlights has been written.<sup>7</sup> Of course, for the Phong shader to operate, normals have

<sup>&</sup>lt;sup>7</sup>It seems that specular highlights are not extremely meaningful for volume visualisation, though it might be given meaningful interpretation.

to be known. This is accomplished in a preprocessing step, and is computed by an approximation of the gradient of density function.

# 5.4 Implementation of Shear-Warp Factorization algorithm

The shear-warp factorization requires a shear transformation and a general 2D warp. Although they seem to be clear, they do come at a price. The mathematics require differing procedures for ranges of viewing angles, and processing orders. We have implemented only one range of viewing angles, although the full range may be achieved by using permutation matrices which first transform the viewing volume to our assumed viewing angle range, and then permute back in the 2D warp phase.

In order to implement the two phases, an intermediate image is first composited from the sheared volume and then warped to the final image. Thus, there is the need for an intermediate image which consists of voxels. We have defined an image type following volume type and used it for that purpose. Also the projection from sheared slices to the intermediate image needs a bit tweaking. Since the slices depart from each other when the viewing angle changes the size of the intermediate image and changes, and the translation from the slice to the intermediate image is confusing. This translation has to be accounted for while projecting images, and then warping images, since the warp procedure expects the sheared coordinates to be available. Since the viewing angle from the principle viewing angle can vary  $\pi/2$  radiants, the intermediate image would be bigger than the slice size by the length of volume in z direction.

# 6 Code, Application, and Conclusion

We have accomplished a sample implementation which covers both a ray casting algorithm and shear warp factorization algorithm with parallel projection. The source code amounts to over 90 KBytes of C++ files, with 30 modules which define more than 25 primary classes some of which are parametrized types. The implementation is indeed a library which encompasses mathematics, three dimensional objects, rendering architecture, and volume rendering functionality.

The applications lets the user to try out both algorithms and compare their running time and results. We have taken simplicity, comprehensibility, modularity, and object oriented design to be prior to other goals since those goals may be realized by revision within a proper design. The algorithms which we have implemented demonstrate two of the software rendering techniques used for volume visualisation. The object order algorithms, in general, seem to be superior to image order algorithms because they can implement the two major optimizations at a less cost of processing. Our application also supports the view that object order algorithms tend to be faster at no serious loss of quality. The sample implementation is effective as it builds on modern software, and is extensible due to object oriented design.

# 7 Future Work and Program Licensing

This is only a sample implementation. Much better functionality would be required for a state-of-the-art visualisation library. Implementing all the improvements over the shear-warp factorization algorithm is a demanding task, however implementing those upon our framework would facilitate code reuse. As a first step, the two optimizations and perspective projection, which is not available in volpack, should be implemented.

Above that, a full rendering architecture might be implemented. Such an architecture could provide support for the parametrisations that we have mentioned and a hybrid volume and polygon renderer.

Another sensible area for future work is parallelizing the sequential shearwarp factorization algorithm. The intermediate space lends itself to a natural and efficient parallelisation, so it must not require a complete rewrite.

We wish to put the program under GPL, since it can be a basis for free medical visualisation programs utilizing the DICOM3.0 library by Eray Özkural, and would encourage other people to improve on it.

# References

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