Modeling Momentum and Reversals

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August, 2018

Abstract

Stock prices are well known to exhibit behaviors that are difficult to model mathematically. Individual stocks are observed to exhibit short term price reversals and long term momentum, while their industries only exhibit momentum.

Here we show that individual stocks can be modeled by simple mean reverting processes in such a way that these behaviors are captured. Simulation shows that in such a market, strategies which use reversals would outperform long-only strategies, often by a factor of 10 in high mean reversion environments.

1 Introduction

There is a large array of literature that documents the mean reverting behavior in stock prices, otherwise known as reversals. For example, Hameed, Huang, and Mian [HHM10] observe reversals in the best and worst subset of monthly performers within industries, whereas the industries themselves only exhibit momentum.

This contrasts with longer term behavior, where stock prices exhibit momentum. Hameed, Huang, and Mian [HHM10] observe this, as does Figelman [Fig07] albeit on a different time scale.

Many articles try to explain these behaviors by appealing to investor behavior. Daniel, Hirshleifer, and Subrahmanyam [DHS98] try to explain these features via investor overconfidence and biased self-attribution. [Kah79] attribute such features to the certainty and isolation effects.

There is also a large array a literature about business cycles, including global cycles, country or currency cycles, and industry cycles. See for example Zarnowitz [Zar92].

Mean reversion as a general phenomenon is well known. Mean reversion is commonly posited for short rate models and mean reverting models are used in commodity option pricing. They are useful for modeling pairs trading, as well as analyzing the timing of entering and exiting positions [LL15].

Despite documentation of reversals and business cycles, these phenomena are rarely explained by equity factor models. Very little has been done to understand these phenomena within and across industry equity returns.

Here we show that mean reverting processes can be used to model reversals and momentum within and across industries. We analyze the model mathematically and through simulation. Simulation shows that in such a market, strategies which use reversals would outperform long-only strategies, often by a factor of 10 in high mean reversion environments.

2 Processes

We proceed to develop what might be considered a reduced form model for stock price movements.

Consider C_t, the value of an industry at time t. We could model the industry value in a number of ways. If we assume the industry value is deterministic with a time dependent growth rate g_t, then

$$dC_t = g_t C_t dt \tag{2.1}$$

In this case, the industry value at time t is:

$$C_{1,t} = e^{\int_0^t g_s ds} \tag{2.2}$$

Alternatively, we could assume the industry randomly deviates from a growth rate of g_t, in which case the industry value satisfies:

$$dC_t = g_t C_t dt + \sigma C_t dW_t \tag{2.3}$$

This is the geometric Brownian motion commonly assumed for stock processes when pricing options ("Black-Scholes" dynamics). In this case, the industry value at time t is:

$$C_{2,t} = e^{\int_0^t g_s ds - \sigma^2 t/2 + \sigma W_t} \tag{2.4}$$

Assuming the industry follows geometric Brownian motion implies that future perturbations are not impacted by past behavior. So, for example, if the underlying Brownian motion happened to trend upwards or downwards for some period of time, the industry would continue to grow from that higher level. This would correspond to an industry expanding and contracting randomly, rather than, for example, over-expansion being followed by contraction.

Instead of assuming the growth deviates randomly, we could assume the value itself deviates randomly. This can be modeled by a mean reverting process:

$$dC_{3,t} = a(C_{1,t} - C_{3,t})dt + \sigma C_{3,t} dW_t \tag{2.5}$$

This essentially says that the industry value process is still a log-normal process, but it's level returns to $C_{1,t}$ at a rate of a.

If firm values follow the industry except for random perturbations away from the industry (noise), it is reasonable to assume not that the stock returns themselves are randomly perturbed, but that the stock value itself deviates randomly from the industry and returns to it over a period of time. In this case, the firm value process S_t would be a mean reverting process:

$$dS_t = a(S_t - C_t)dt + \sigma S_t dW_t \tag{2.6}$$

Note that here we are transitioning from C_t being the overall value of the industry to expressing the optimal value of a company in this industry. For simplicity, we assume all firms revert to this overall level so as to capture behavior between the industry and the economy. The relationship between the sizes of different firms could be captured by instead having each firm revert to the appropriate percentage of the total industry level, and accounting for how these percentages change over time.

Suppose a given industry has a cyclical growth rate. Say its growth rate g_t is cyclical ranging from a low of l to a high of h with a period of p years:

$$g_t = \frac{l+h}{2} + \frac{h-l}{2} \sin(2\pi t/p) \tag{2.7}$$

If its value strictly follows this growth rate (case 1), then its value is

$$C_{1,t} = C_0 e^{\frac{l+h}{2}t + \frac{(h-l)p}{4\pi}(1-\cos(2\pi t/p))} \tag{2.8}$$

While it would be interesting to consider case 2 and case 3 for an industry following a periodic growth rate, we will instead consider $C_t = C_{1,t}$ with the above expression for g_t to be the normative industry value, and the actual industry value to be the sum of the values of the companies comprising said industry. This amounts to assuming that there is an underlying deterministic hidden variable driving the value of the industry and that it has cyclic growth.

To illustrate, we consider an industry with 30 companies. The industry follows C_1 with a 5 year cycle, a low rate of return of -0.01 and a high rate of return of 0.05. Each stock has a volatility of 20%. Figures 1 and 2 show the underlying dynamics with a mean reversion of 5 and of 1, respectively. We display the deterministic industry firm level, a sample path for the average stock value, and a sample path for one of the stocks in the industry. The average firm deviates around the average deterministic level, as do individual firms, albeit with much higher volatility. With lower mean reversion, the stock takes much longer to return to the industry level, and the industry does not follow the deterministic level as closely.

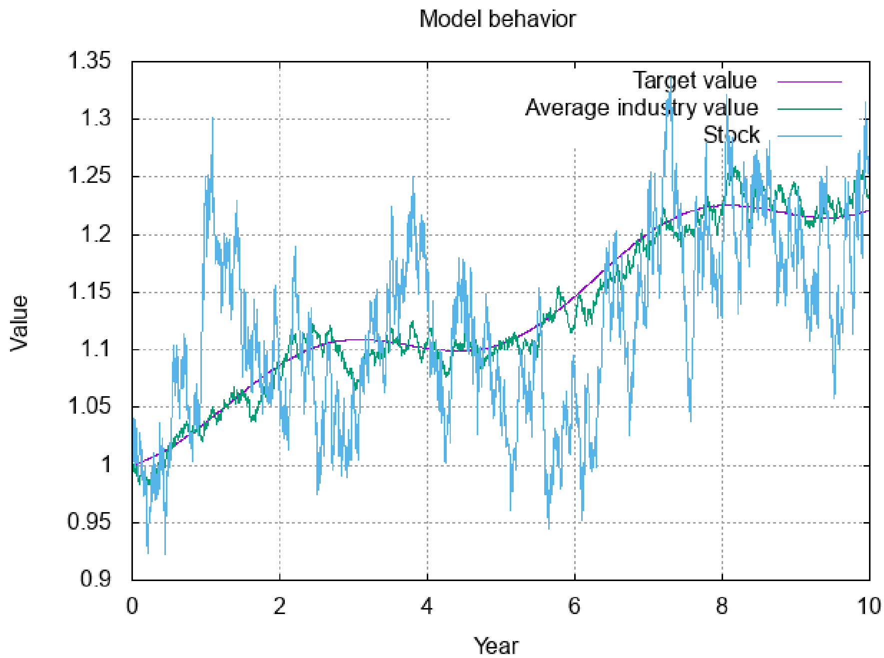

Figure 1: Market and one stock, mean reversion 5. The average firm value deviates around the underlying deterministic value, as do individual firms, but with higher volatility.

3 Properties

3.1 Bounded variance

Since the stock price process is mean reverting to the industry level, and the latter is deterministic, the stock prices do not exhibit GBM volatility (namely $\text{var}(\log S_t) = \sigma^2 t$). Instead, the variance tends to a limit.

While this goes against common assumptions, the fact is that we are attempting to estimate the variance of S_t from one sample path. As such, we attempt to estimate $\text{var}(S_t)$ from the sample variance of the historical $S_{t_i+\Delta t} - S_{t_i}$. For Δt greater than one year, we typically have very few samples, or overlapping (correlated) samples, or samples that span over substantial, world changing events. Not to mention the fact that changes in drift will impact sample variances. All of these issues reduce the confidence we have in such broad assumptions about the variance of S_t as t grows.

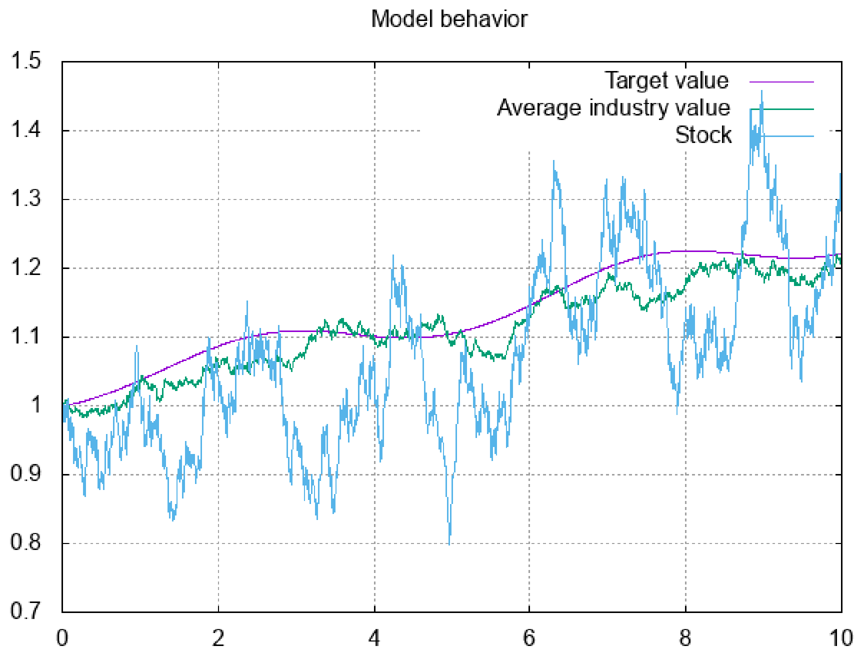

Figure 2: Market and one stock, mean reversion 1. Similar behavior is observed to the mean reversion 5 case, but the stock and market take longer to return to the deterministic level.

3.2 Reversals

Because the stock price process is mean reverting to C_t, the drift of S_t is lower than that of C_t when $S_t > C_t$, and is higher when $S_t < C_t$. This constitutes a reversal. Since the rate of reversion is a, the reversal roughly occurs on a time scale of $1/a$. So, strong reversals on a monthly basis would constitute a mean reversion of on the order of 6 to 12. Smaller mean reversions will exhibit reversals to a lesser degree.

On the other hand, the industry itself, being the sum of the values of the companies in the industry, will have greatly dampened reversal behavior. It will only exhibit reversal behavior when a large percentage of the companies have inflated values that are not compensated for by substantially deflated values of the remaining companies.

3.3 Momentum

On larger time scales, since the process mean reverts to C_t, when the business cycle is booming, all of the industry stocks will exhibit momentum in that they are mean reverting to C_t which itself exhibits momentum. Similarly, when the industry is in the bust part of the cycle, all of the industry stocks will exhibit poor long term performance.

Since reversals at the industry level are dampened, its momentum will be exhibited both on a small time scale as well as on long time scales.

3.4 Market efficiency

There has been much discussion of market efficiency since the ground breaking work of Fama [Fam70]. Discussion has largely been about the extent to which the markets exhibit various degrees of efficiency, and the properties exhibited by efficient markets.

More recently, there has been work on the relationship between market efficiency, martingale properties and the condition of a market being arbitrage free. This was first discovered by Samuelson [Sam65] and further elaborated on by Jensen [Jen78]. Most recently, Jarrow and Larsson [JL11] clarified and formalized the relationship between market efficiency, showing that a market is efficient if it satisfies the conditions of "No Free Lunch with Vanishing Risk" (NFLVR), and "No Dominance" (ND).

It is easy to see that the mean reverting market model that we propose satisfies both the NFLVR and ND conditions. One need only show that the change of measure factor that converts the mean reverting drifts to the risk free drift is in fact a martingale. This is an easy exercise. Then, it follows from Jarrow and Larsson [JL11] that the model proposed here satisfies the efficient market hypothesis. We may then conclude that momentum and reversals can exist in efficient markets, at least in the above sense.

4 Buy losers, sell winners

Due to the mean reversion, it is likely that good performance will be followed by bad performance and visa-versa. This will occur roughly on a time frame of $1/a$.

In practice, we will not know C_t, so we will not know for sure whether the drift of S_t exceeds that of the industry or lags that of the industry. However, it is likely that the best performing stocks at a given time are above C_t, while the worst performers are below C_t. As such, we expect that a strategy of buying the losers and selling the winners would outperform the industry.

This is borne out by simulating the processes in question. Figures 3 and 4 illustrate the results for the same stock and market parameters detailed above, where we buy the highest 30% of the industry, and short the lowest 30%. With a mean reversion of 5, we observe a monthly return rate on average of about 1.7% and an annualized continuous return of about 20% to 30%, substantially higher than the overall industry average with the above parameters, which is about 2%. With a mean reversion of 1, the improvements are much more modest, with 3 scenarios exhibiting returns less than 4%, the largest return being 14%, and most scenario returns between 4% and 8%. Improving on that under low mean reversion would require adjusting the strategy.

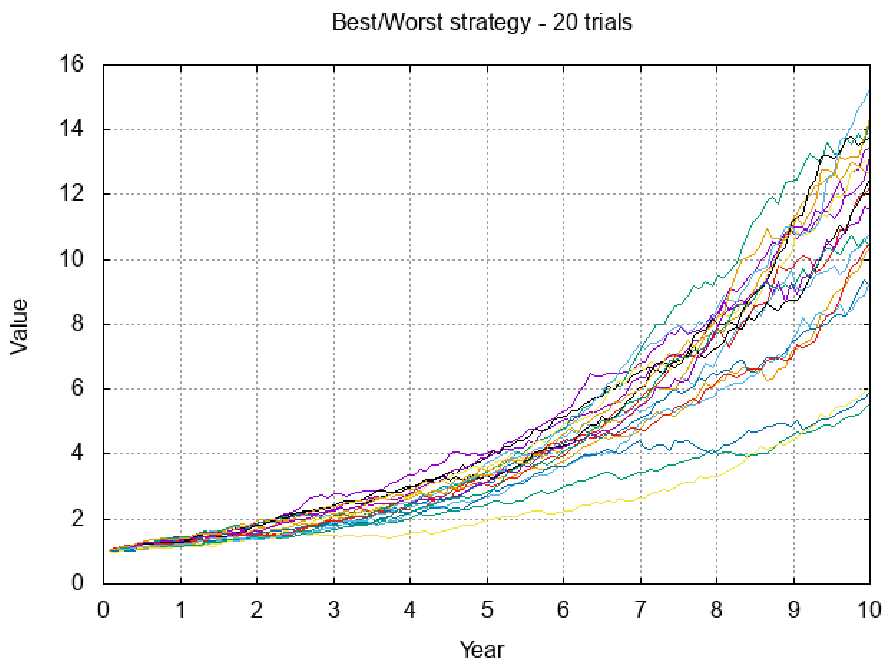

Figure 3: Best/worst strategy performance, 20 simulations of 30 stocks, mean reversion 5. The industry annualized return is around 2%, whereas the strategy yields returns from 20% to 30%.

5 Summary

Motivated by the observation of reversals and momentum in the stock market, we considered a stock process model where each stock in an industry reverts to an underlying deterministic cyclic level. We explored the properties of such a model. Mathematical considerations indicated that in such a model, one would observe short term reversals and long term momentum, while the industry would just exhibit momentum. This was borne out through simulation. We

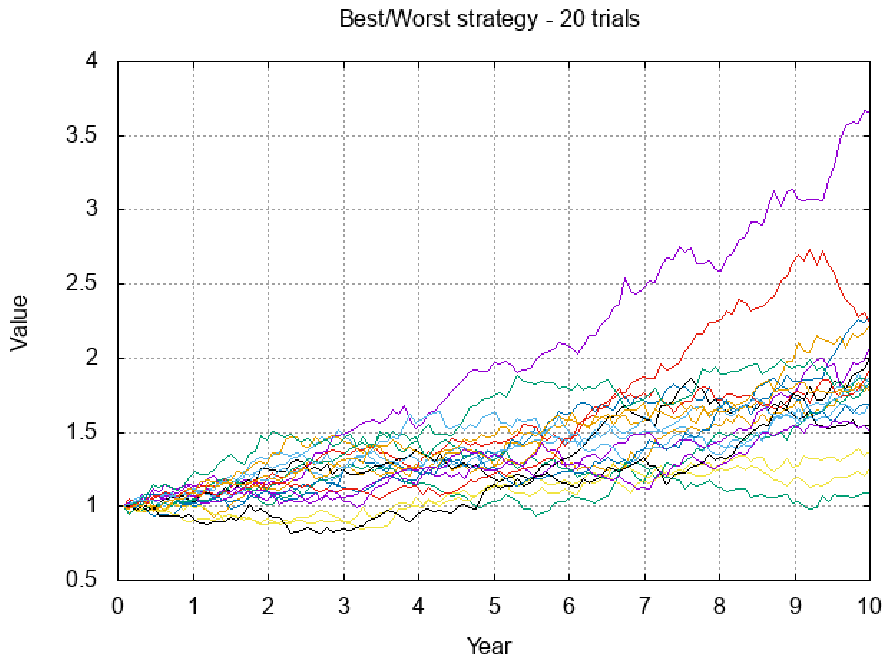

Figure 4: Best/worst strategy performance, 20 simulations of 30 stocks, mean reversion 1. With low mean reversion, strategy returns are more modest, and have a higher probability of underperforming the industry.

also demonstrated that in such a market, strategies which use reversals would outperform long-only strategies, often by a factor of 10 in high mean reversion environments.

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