Habitat

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Introduction

This homework centers around the predator-prey dynamic, which is one of the most fundamental ecological inter-action model in nature. This report presents the implementation of a two-dimensional particle-based simulation modelling the interaction between rabbits (prey) and wolves (predator) in a habitat with periodic boundary conditions. The simulation uses a agent based approach, where the agents follow a stochastic movement patterns and interact according to probabilistic rules for reproduction and predation.

Default Simulation Setup

The baseline simulation uses the following configuration:

- **Domain**: A square habitat of size $L = 10$ with periodic boundary conditions.

- **Initial populations**: 900 rabbits and 100 wolves distributed uniform randomly across the domain.

- **Movement**: Both species move with a Gaussian step size distribution with standard deviation $\sigma = 0.5$.

- **Predation**: Wolves can eat each rabbit within a radius of 0.5 with probability 0.02 per time step.

- **Reproduction**: Rabbits reproduce with probability 0.02 per time step.

- **Mortality**: Rabbits die after reaching a maximum age of $t_d^r = 100$.

- **Starvation**: Wolves die after 50 time steps without eating.

- **Spatial indexing**: The domain is divided into 20×20 grid cells,

Further information regarding the implementation can be found in the README.md file in the root folder of the project.

1 Scenario A: Standard Case

In Figure 1 we can observe the population dynamics between wolves and rabbits with parameter $\sigma = 0.5$ and $t_d^r = 100$ the system has the for predator prey models typical oscillatory behavior characteristic. For roughly the first 2500 time steps, the simulation has relatively stable oscillations with the rabbits fluctuating between approximately 300 to 1000 individuals, while the wolves population fluctuates roughly around 100. Furthermore you can observe that oscillation of the wolves is slightly delayed (phase difference). This behavior is consistent with the Lotka-Volterra differential equations, which are given as follows:

$$\begin{aligned} \frac{dr}{dt} &= \alpha r - \beta r w \\ \frac{dw}{dt} &= -\gamma w + \delta r w \end{aligned} \tag{1}$$

where r represents the number of rabbits, w the number of wolves, α the rabbits growth rate, β effect of the presence of wolves, γ describes the predators death rate and δ the effect of rabbits present on the growth rate of the wolves. However after time step 2500 the systems exhibits a more unstable state, showing significantly larger amplitudes. The population of rabbits has two major peaks one with roughly 2500 individuals and another with even more individuals (9000). These peaks are followed by a very steep decrease, similar to a crash. The wolves show a similar peak slightly delayed as reaction to the massive surge in rabbits.

We can observe several key similarities with Lotka-Volterra equations

- Cyclical nature of population changes

- Phase lag between predator and prey

- Dependency of predator growth on prey availability

However, the agent-based model reveals more complex dynamics than the standard Lotka-Volterra model:

- Non-uniform oscillations, amplitudes and phases

- Extreme population events

- Effects of spatial heterogeneity

  - Rabbits and Wolves can form clusters, therefore creating "hotspots", where interactions are more frequent. On the other hand there could also be "save zones", where rabbits can multiply rapidly without being eaten.

  - The agents have finite movement speeds and a wolf can only eat rabbits that are physically close.

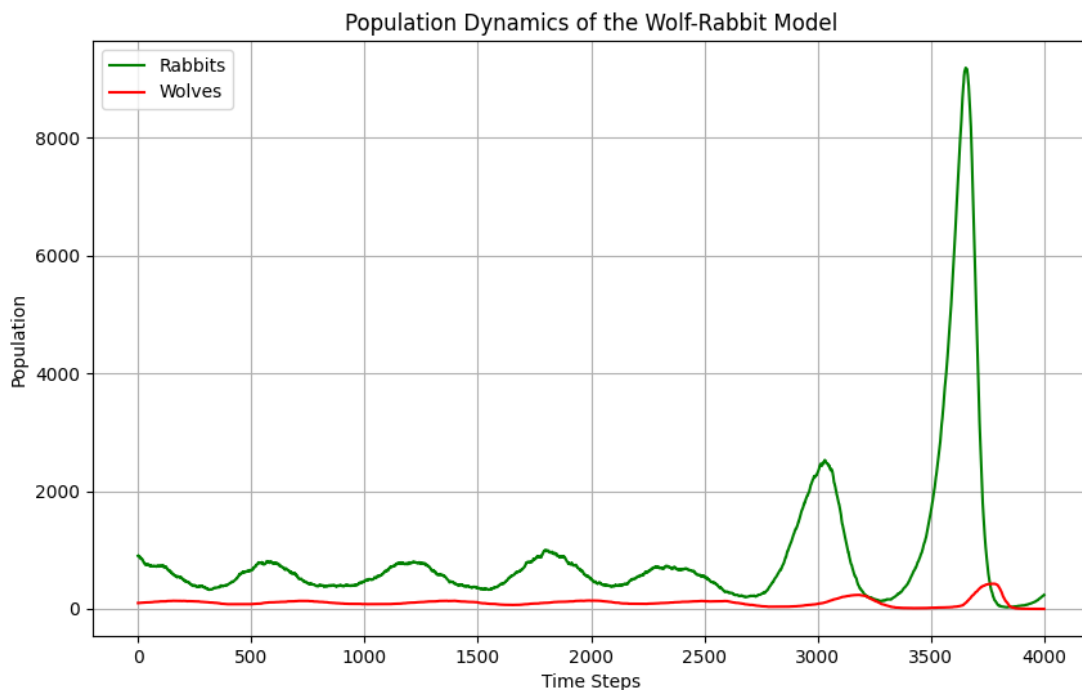 - Stochastic encounters, the interactions and the chance of meeting depends on probabilistic dynamics.

Figure 1: **Population Dynamics** $\sigma = 0.5$, $t_d^r = 100$

2 Scenario B: Reduced rabbit lifespan

In comparison to exercises A, we half the rabbit lifespan to $t_d^r = 50$, while maintaining the parameter σ. Figure 2 shows a dramatic shift in behaviour, unlike the oscillatory behaviour observed in the previous exercise, the systems shows a rapid collapse of both populations. The rabbit population shows a steep decline, then stabilizes around 100 rabbits, before continuing to decline until the rabbit population goes instinct. After feasting on the initial population of rabbits the wolves go instinct even earlier. This Scenario can be explained by several factor that interrelate with each other:

- The halved lifespan of rabbits increases the mortality rate, that the reproduction rate (i.e. 0.02) cannot compensate for.

- When the rabbit population falls under a critical threshold, the wolves cannot find enough prey in their radii to uphold their own populations, leading to extinction by starvation.

- Even though the wolves die off and none of the rabbits are eaten, the mortality is still too high for the rabbits to recover their population.

The result differs from the Lotka-Volterra models, where usually the extinction of the predator leads to the prey's population recovery. This simulation showed that parameters such as lifespan can override the oscillatory population dynamic. This scenario highlights the significance of balanced parameters in order to maintain stable predator-prey system and shows how slight changes can already alter the system in a significant way.
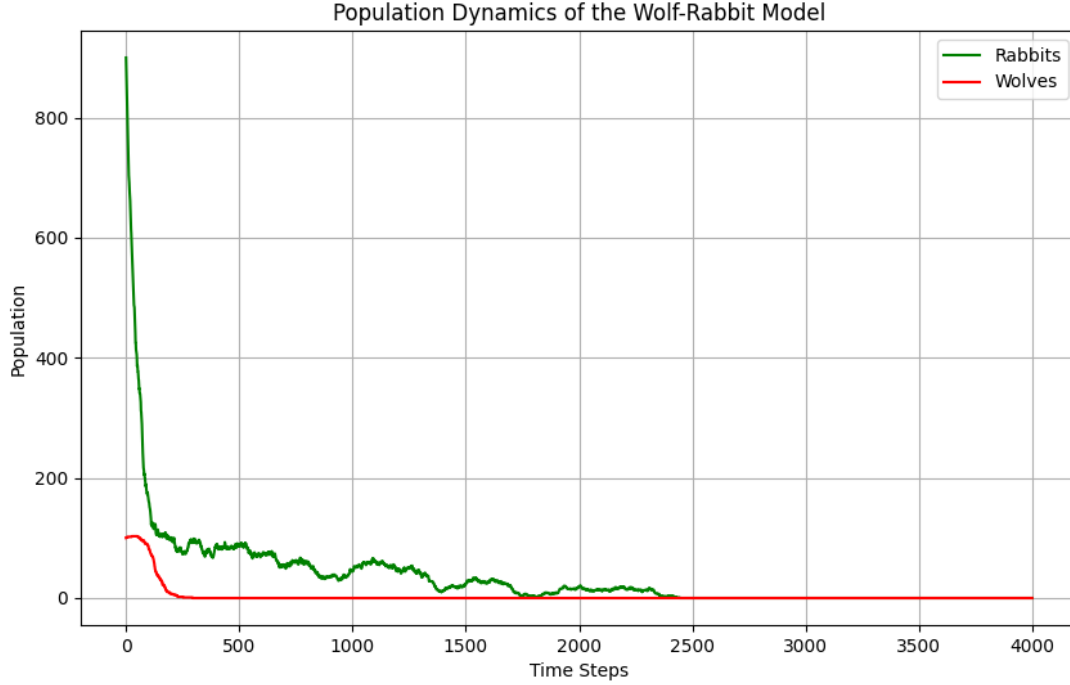

Figure 2: **Population Dynamics** $t_d^r = 50$

Scenario C: Reduced mobility

In this simulation we are interested in the result what happens if we reduce the movement parameter to $\sigma = 0.05$. When looking at Figure 3, the rabbits population exhibits multiple very high peaks, reaching even higher numbers than in exercise A, even though we have the same reproduction and mortality parameters. The following key observation can be made about the system

- The rabbit population has extreme booms and crashes (compared to exercise A) with peaks being roughly 10 times higher.

- As seen before the wolf population shows a delayed response, with way smaller peaks.

- The small movement parameter limits the mixing of population.

Looking at the species in more detail we can draw the following conclusions:

- Wolves are unable to effectively find rabbits across the domain, resulting in areas where the rabbit population can grow unchecked.

- Rabbits are able to form dense reproductive clusters, resulting in extremely high booms of rabbit population.

- The eventual crash is likely caused by a wolf being eventually able to reach the hotspot of rabbits, leading to a rapid crash of rabbits due to the abundance of rabbits for the wolf to eat and therefore being able to reproduce.

The classic Lotka-Volterra model is in this scenario a rather poor approximation of the system dynamics, as the assumption of well-mixed populations is severely violated. The extreme spikes in population demonstrate how spatial effects can dominate the systems behaviour when movement is constrained. We create a boom-bust cycles which is driven by the positions of the predators and prey instead of an interaction driven feedback mechanism captured in the differential equations (1).
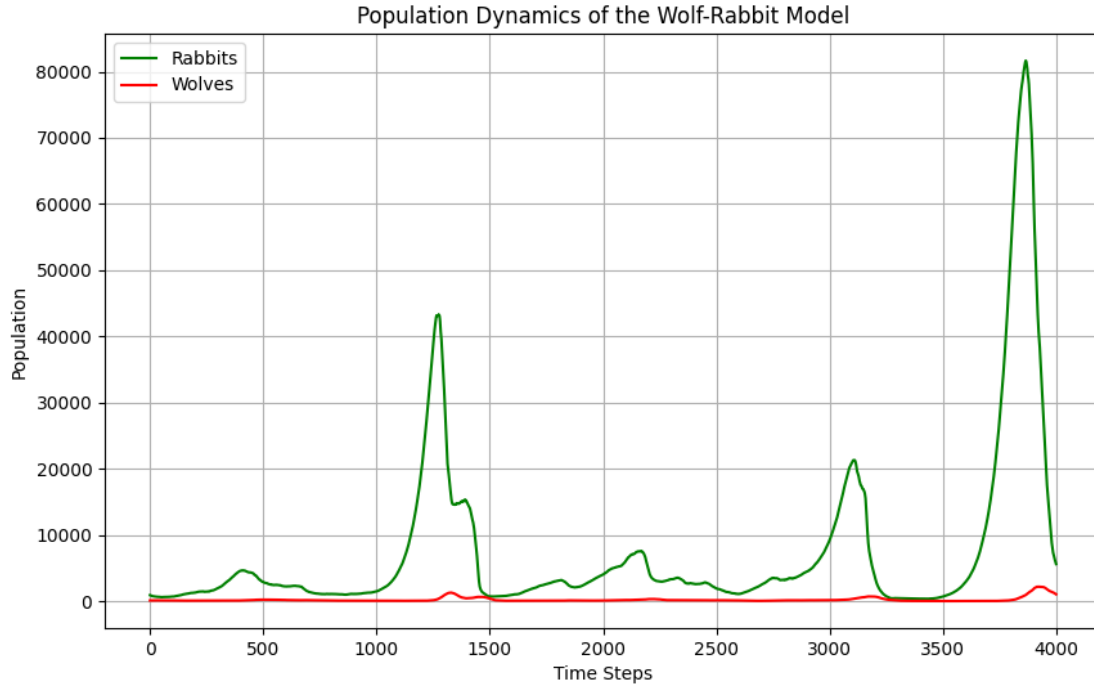
Figure 3: **Population Dynamics** $L = 8$, $\sigma = 0.05$

Conclusion

This agent-based simulation of the predator-prey dynamics showed several significant insights that extend beyond the classical the Lotka-Volterra models. The incorporation of spatial heterogeneity, stochastic movement, and interactions on an individual level, we observed wide variety of behaviors across different configurations.

In Scenario A, we observed the expected patterns with a phase difference between predator and prey populations. However, the spatial component of our model caused a more complex dynamics compared to what the differential equations would predict. This included non-uniform oscillations and extreme population counts after the 2500-time step mark.

By reducing the rabbit lifespan in Scenario B, we were able to observe the critical importance of prey longevity to ecosystem stability. Rather than oscillations, the system experienced a complete collapse. This highlights the importance of threshold effects and critical population densities.

Perhaps most interesting result were obtained in Scenario C, where we observed extreme peaks and crashes of rabbit populations, by reducing the mobility. By creating "safe zones", where prey can multiply unchecked until eventually intruded by predators, leading to boom-bust cycles on an order of magnitude more extreme than in Scenario A.

The most important findings can be boiled down to the following points:

- Spatial dynamics and the movement of individual agents highly influence population stability, probably even more than reproduction or mortality rates.

- There is a high parameter sensitivity in predator-prey, small changes can cause regime shifts from stable oscillations to extinction or extreme fluctuations.

- The addition of spatial heterogeneity reveals dynamics that which are not present in traditional Lotka-Volterra models, clearly points out the importance of agent-based approaches.

For future work this rather simple simulation could be extend by incorporating multiple ecological complexities. This could include food for rabbits that grows and depletes, terrain heterogeneity, introducing sexes for mating, adapting behaviour of agents among others. These extensions would further bridge the gap between theoretical models and the complex dynamics observed in the real world.