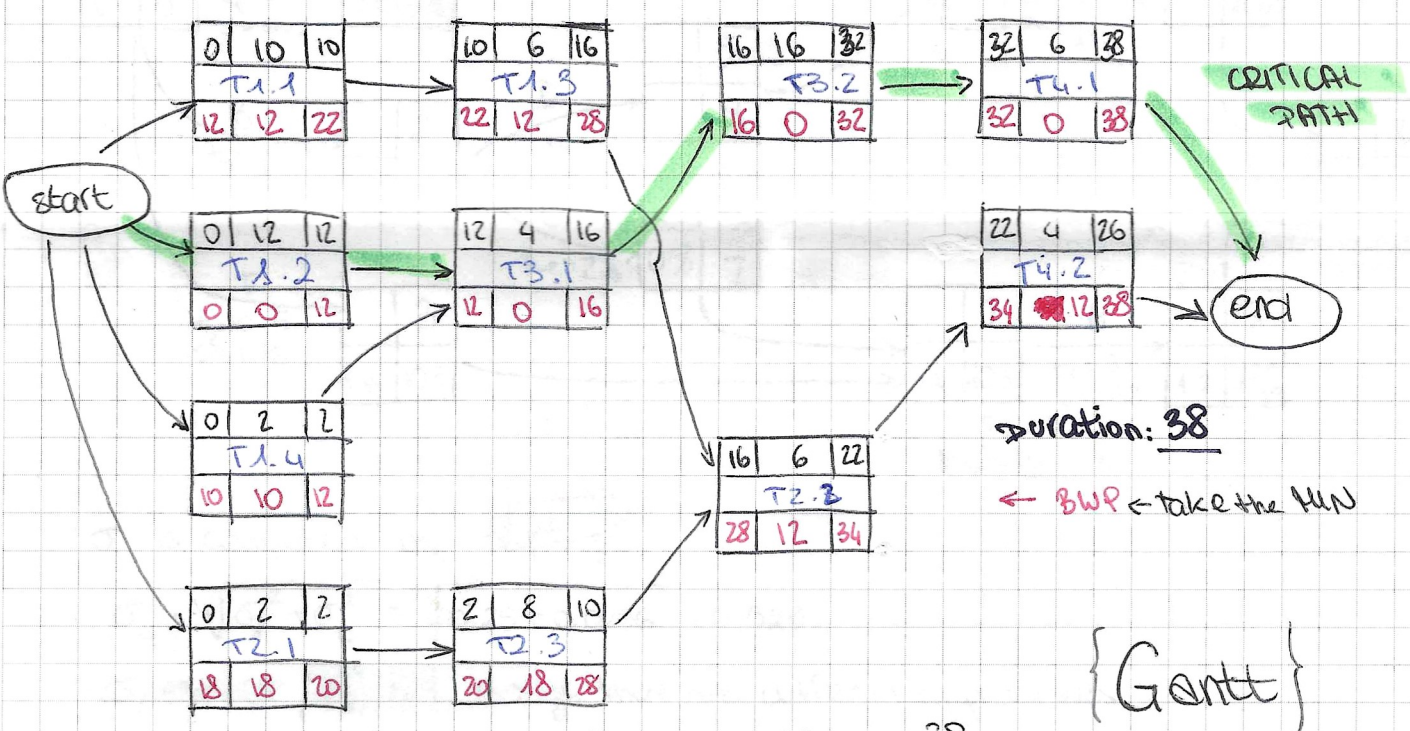
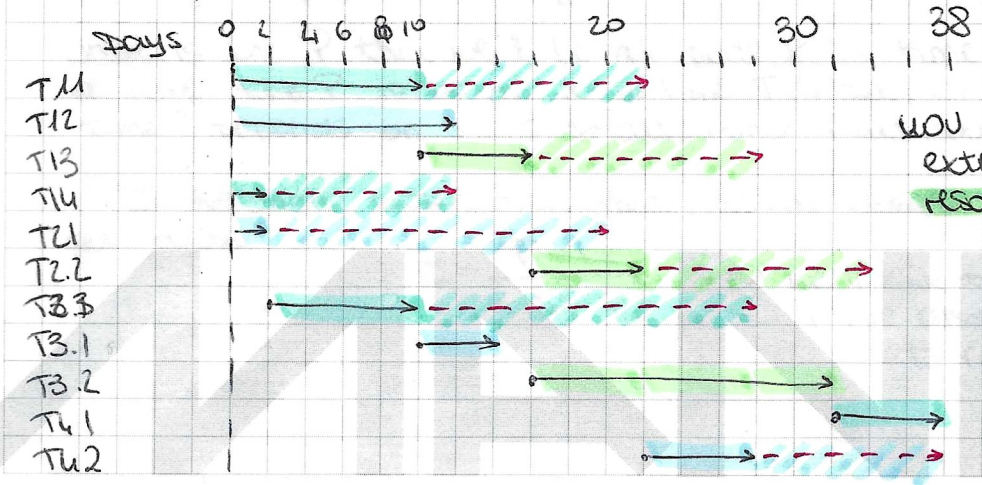
CPM

	ACT	DEP'S	DURATION	SD	JD
X	T1.1	/	10	20	10
X	T1.2	/	12	24	12
X	T1.3	T1.1	6	6	0
X	T1.4	/	2	2	2
X	T2.1	/	2	4	2
X	T2.2	T1.3, T2.3	6	12	6
X	T2.3	T2.1	8	0	8
X	T3.1	T1.2, T1.4	4	12	8
X	T3.2	T3.1	16	0	32
X	T4.1	T3.2	6	12	6
X	T4.2	T2.2	4	8	0

Network and prj duration:

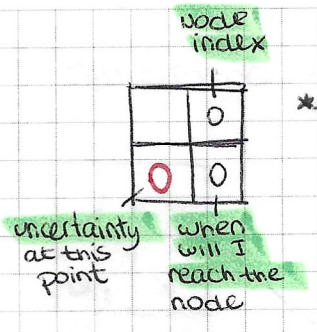


{Gantt}



YOU CAN MOVE activities with extra float to optimize the resources (MIND THE DEPENDENCIES THOUGH!)

PERT analysis



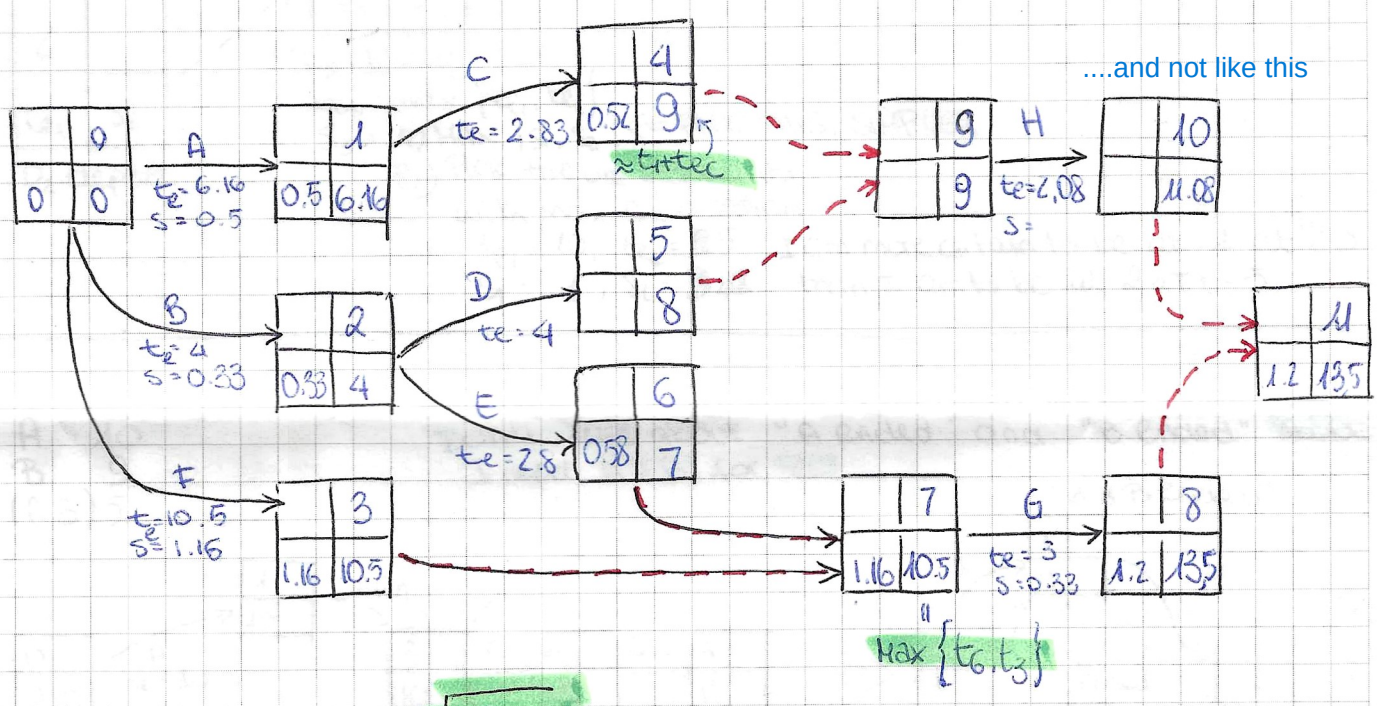
first node

i think you should MIRROR these?

this is the correct version indeed. I was watching a mirrored whiteboard, therefore I misunderstood the positions of these numbers at first. You should write them like this.

$$S = \frac{b-a}{6}$$

$$te = \frac{a+4m+b}{6}$$



σ is computed as $\sqrt{\sigma_1^2+\sigma_2^2}$

$$\sigma_4 = \sqrt{\sigma_1^2+\sigma_C^2} = \sqrt{0.25+0.0256} = 0,52$$

$\sigma_7 = \max\{\sigma_6, \sigma_3\}$ though there are different approaches

what is the P that we'll be at node X in time Y?
we have the P to be at X in time T, right, what about a DIFFERENT time? Depends on S: S small, minor fluctuation on our T.

Z-function: DISTRIBUTION that gives us probability of not meeting a target date

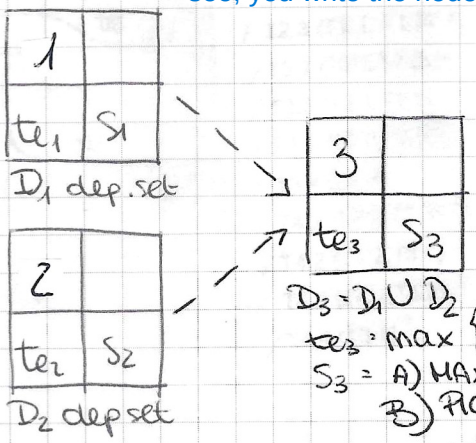
$$Z = \frac{T-te}{S}$$

=> we'll have to do 1-2 then! SEE TABLE

i hope and pray that you understand why to find P of REACHING

SIMPLIFYING the PERT Network

see, you write the nodes this way. i will do so from here on out.



$D_3 = D_1 \cup D_2$

$te_3 = \max\{te_1, te_2\}$

S_3 = A) MAX $|S_1, S_2|$ ⟸ PICK THIS, SIMPLER

B) Pick the one associated with max expectation

$te_1 = 10$, $te_2 = 5$ I am more certain to be at 1 at T=10 than I am to be at 2 at t=5.

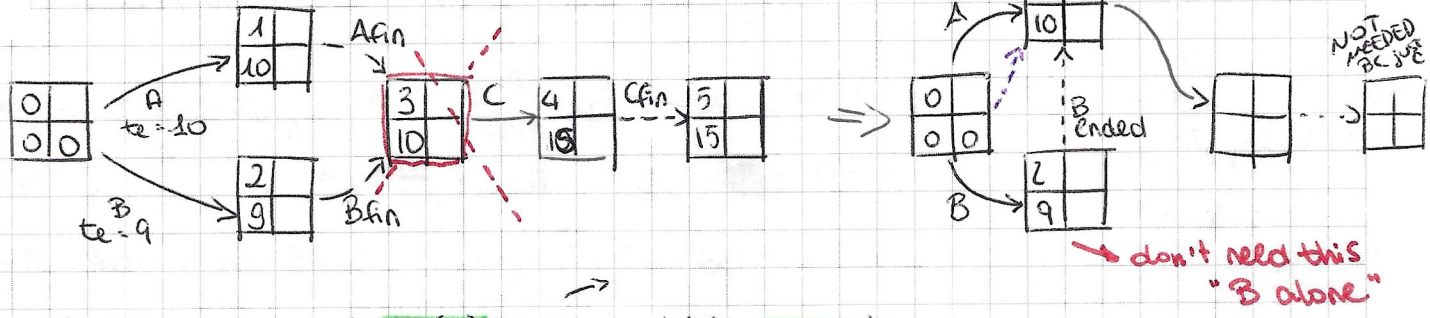
$S_1 = 1$, $S_2 = 3$

---

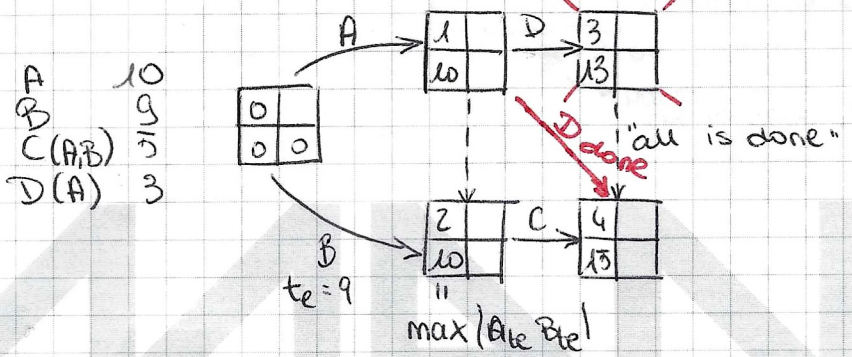
a. d.

A 10
B 9
C(A,B) 5

on dependencies: NODE ≡ event, some a's are finished, don't care about "A ended" and "B ended" ALONE, I need A∧B for C.

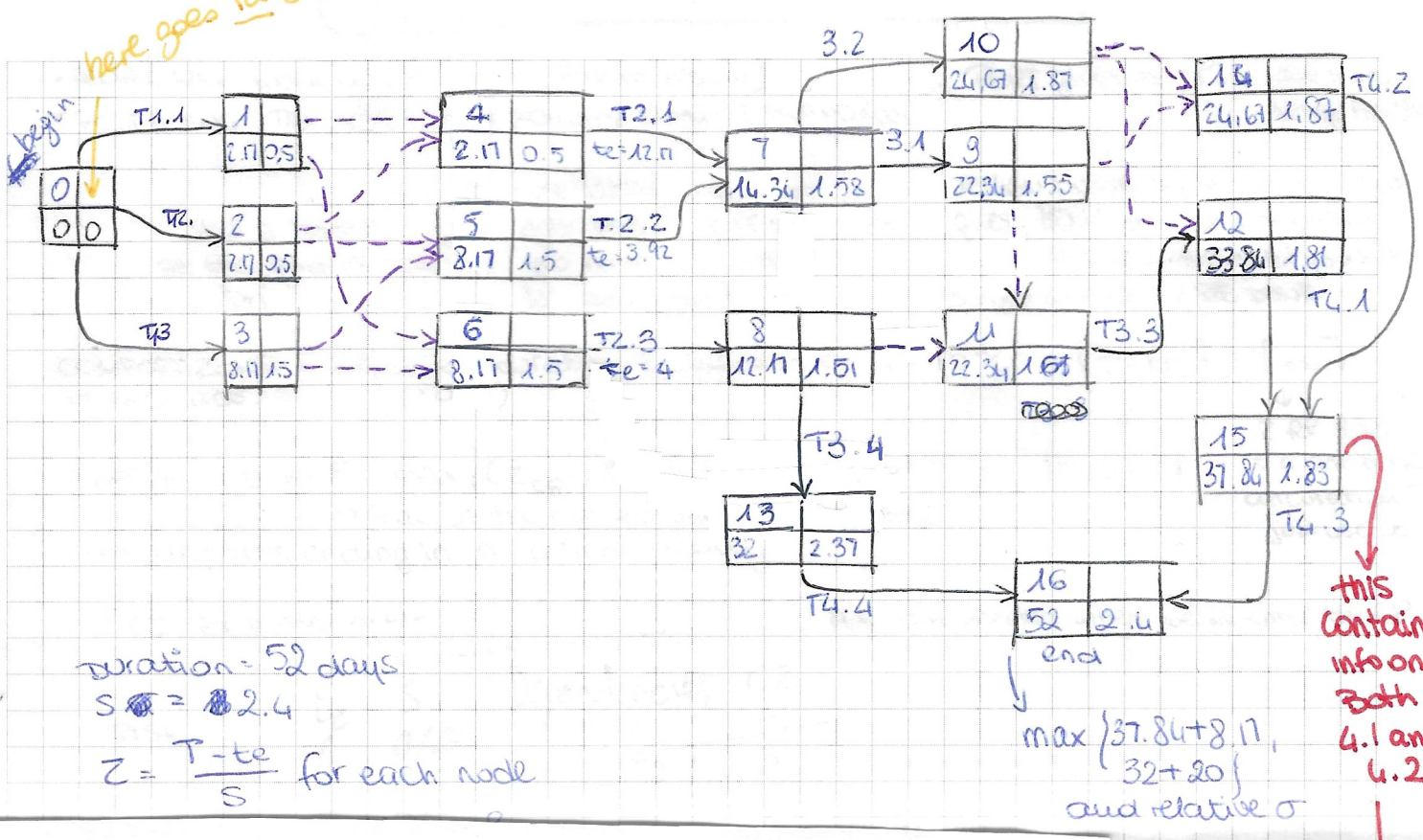


- IF I had a node D(A) this would be wrong! I would no longer have "A finished" alone, only "A∧B finished"



Basically you need one node for start, one for finish one for each "TYPE" of dependency.

here goes Target date



duration = 52 days
S = 2.4
$Z = \frac{T - te}{S}$ for each node

$\max\{37.84+8.17,\ 32+20\}$ and relative σ

this contains info on both 4.1 and 4.2

Activity (Precedents)	Activity Durations (weeks) Optimistic (a)	Most likely (m)	Pessimistic (b)	Expected te	Standard deviation (s)
T1.1	1	2	4	2,17	0,5
T1.2	1	2	4	2,17	0,5
T1.3	4	8	13	8,17	1,5
T2.1 (T1.1, T1.2)	8	12	17	12,17	1,5
T2.2 (T1.2, T1.3)	3	4	4,5	3,92	0,25
T2.3 (T1.1, T1.3)	2	4	6	4	0,66
T3.1 (T2.1,T2.2)	7	8	9	8	0,33
T3.2 (T2.1,T2.2)	4	12	10	10,33	1
T3.3 (T2.3,T3.1)	8	12	13	11,5	0,83
T3.4 (T2.3)	14	20	25	19,83	1,83
T4.1 (T3.2,T3.3)	3	4	5	4	0,33
T4.2 (T3.2,T3.1)	3	4	5	4	0,33
T4.3 (T4.1,T4.2)	6	8	11	8,17	0,83
T4.4 (T3.4)	16	20	24	20	1,33

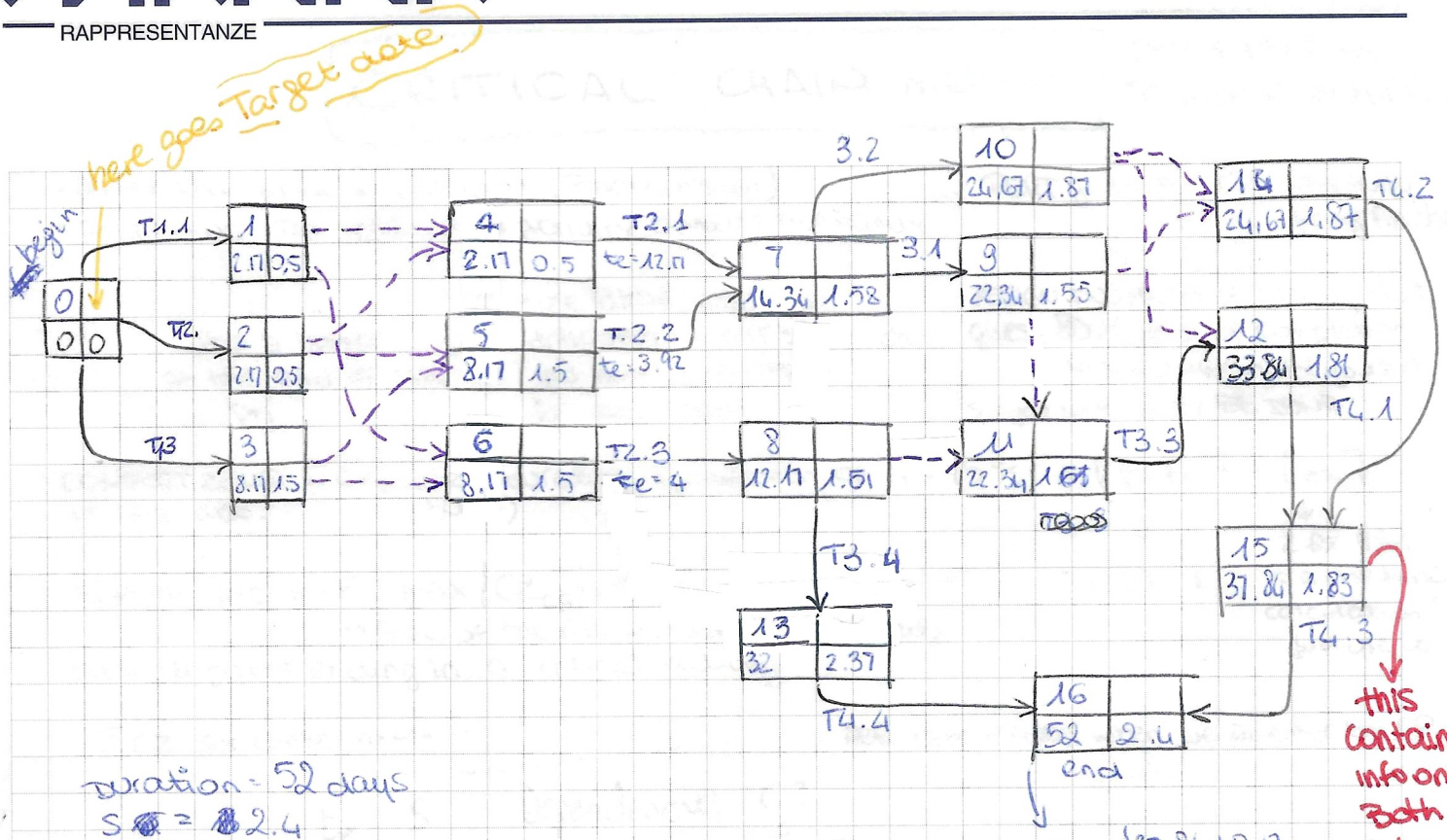
$S(N_{16}, T_{4.1}) = 1.9$ this we had not used fwd

for time $T_{16} + T_{4.1} = 28,67$ fwd

checks out :) BC we pick the MAX fwd

$26.2 + 4.6 \cong 46.53$

let T4.4 there!



duration = 52 days
S = 2.4

$Z = \frac{T - te}{S}$ for each node

what's T for activities?

this contains info on both 4.1 and 4.2

max {37.84+8.17, 32+20} and relative σ

BIGGER S, LOWER chance of meeting deadline
=> convert via table => chance of NOT meeting DL

- Prob of finishing in 52? $\frac{52-52}{2.4} = 0$? → Table → 50%
- P. " " " 56? $\frac{4}{2.4} = 1.67$ → Table → 95%
- At what date the P of having T4.3 is 60%

$S(N_{16}, T_{4.1}) = 1.9$ this we had not used fwd

↓

for time $T_{16} + T_{4.1} = 28,67$ fwd

checks out :)
BC we pick the MAX fwd

$T = Z \cdot S + te = 25 + (37.84 + 8.17) =$

$= Z \cdot (S(1.83, 0.23)) + 46,01 =$ (UN TABLE) $0.6 \cdot 2 + 46 = 0.26 \cdot 2 + 46 \cong 46.53$

$a = 2.66$ (closest value)

NOT THE ONE AT NODE 16, we had picked T4.4 there!

CRITICAL CHAIN method

CPM + PERT to Schedule activities

- PEOPLE WILL WORK ON DEADLINES (Parkinson's law)
- POSTPONING THE START OF AN ACTIVITY BENEFITS FINANCIALLY

=> PERT gives us: P% of activity being finished

⇓

ASSIGN DURATION of activity that gives 80% of P. of completion
↳ select threshold yourself
So you will LIKELY BE DONE

<= • use latest start but anticipate A BIT so you have a BUFFER for each Path

<= also a buffer at the end of the Prj.

COMFORT ZONE OF EACH act. = expected duration (50%) − duration of P=95% = $\lceil T - te \rceil = \lceil zs \rceil$

$\lceil 0.12 \rceil = 1$

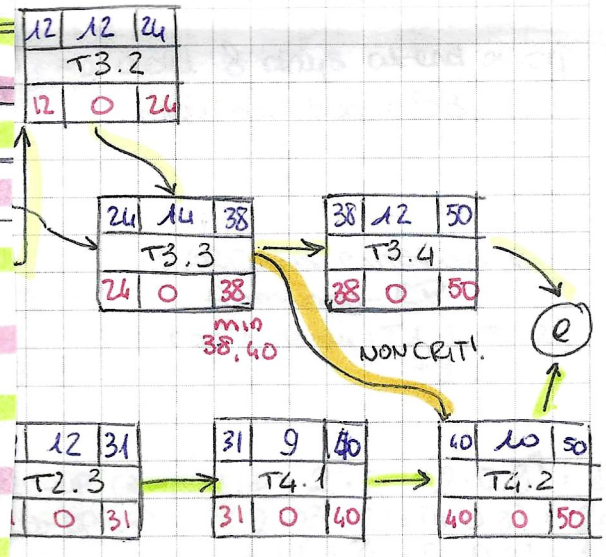
z for 95% is a one time calculation?? just use it

BUFFER SIZE = $F \cdot \max\{CZ_{CP_i}\}$; F a fraction you establish
↳ Frac. of the risk you want to acknowledge

consider paths ending in a critical activity

Σ CZ for these act's => compute FEEDING BUFFER to be inserted when we re-enter the CP.

Task	te	S	Dependencies	CZ
T1.1	6	0,66		2
T1.2	7	1,66	T1.1 T2.1	3
T1.3	12	1	T3.1	2
T2.1	12	1,33		3
T2.2	6	1	T1.1 T2.1 T3.1	2
T2.3	12	2	T2.2 T1.3	4
T3.1	7	1,66		3
T3.2	12	1,33	T1.1 T2.1	3
T3.3	14	1,33	T1.2 T3.2	3
T3.4	12	0,33	T3.3	1
T4.1	9	1,33	T2.2 T1.3 T2.3	3
T4.2	10	1,66	T3.3 T4.1	3



Nt quality as critical dependency in this path, 40 − 38 is not critical

CRITICAL CHAIN method

CPM + PERT to Schedule activities

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=> PERT gives us: P% of activity being finished

⇓

ASSIGN DURATION of activity that gives 80% of P. of completion
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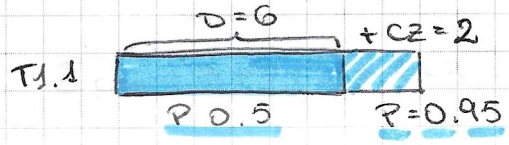
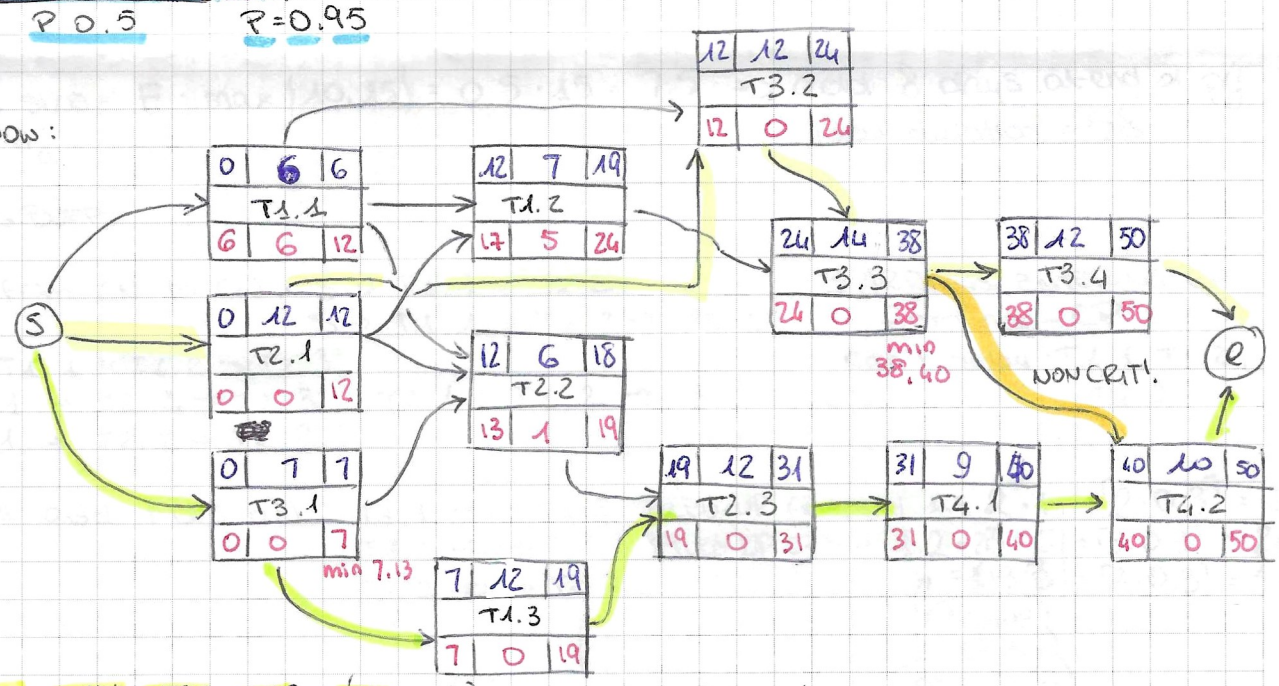
ΣCZ for these act's => compute FEEDING BUFFER to be inserted when we re-enter the CP.



CPM now:

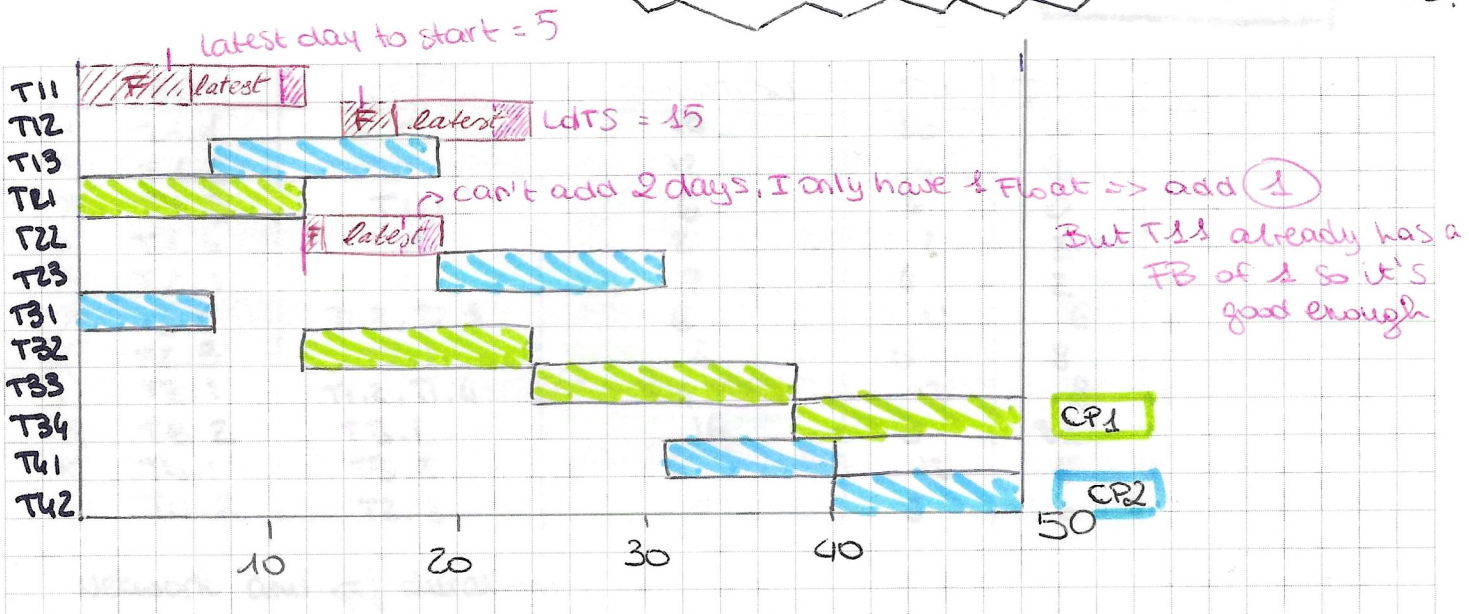


CP1: T21 - ~~T22~~ - T32 - T33 - (T34, T42) T42 doesn't qualify as critical dependency in this path, 40 - 38 is not critical

CP2: T31 - T13 - T23 - T41 - T42

C. Chain 2

MIND DEPENDENCIES WHEN MOVING T'S!



$\sum_{a \in CP1} cz_a = 3+3+3+1 = 10$ // if I had 50+10, all a's in CP1 would have 95% P. of completing

$\sum_{a \in CP_2} cz_a = 3+2+4+3+3 = 15$

BUFFER SIZE FOR Prj = $F \cdot \max\{10, 15\} = 0.5 \cdot 15 = 7.5$ => add 8 days at end of prj

New duration = 58

FEEDING BUFFS

Paths from CN to CN: S → T1.1 → T3.2
S → T1.1 → T1.2 → T3.3
S → T1.1 → T2.2 → T2.3
T2.1 → T2.2 → T2.3
T3.1 → T2.2 → T2.3
T2.1 → T1.2 → T3.3

3 ARROWS ENTERING CP from non-CP can't delay: T1.1, T1.2, T2.2

we will add FB's on: T1.1 → T3.2
T1.2 → T.3.3
T2.2 → T2.3

PILED-UP CZ'til 1.1 = $2 \cdot F = \lceil 2 \cdot 0.3 \rceil = 1$

PILED-UP CZ'til 1.2 = 2+3 = 5 $= \lceil 5 \cdot 0.3 \rceil = 2$
= ~~3~~+3-3 $= \lceil 3 \cdot 0.3 \rceil = 1$ } PICK MAX

Because 2 Paths can go to 1.2

PILEDUP CZ's til 2.2:
~~2+2~~ 2
~~3~~ 2
2+2
4 => $\lceil 4 \cdot 0.3 \rceil = 2$ Trouble above! But it's okay

NOTICE a 5-day float between T1.1 and T1.2
=> delaying T1.1 is not that bad, you might anticipate T1.2 of just (1)

$1.66^3 = 2.7566$
$1.33^2 = 1.7689$
$2.33 - 5.4289$

$0.83 = 0.6889$

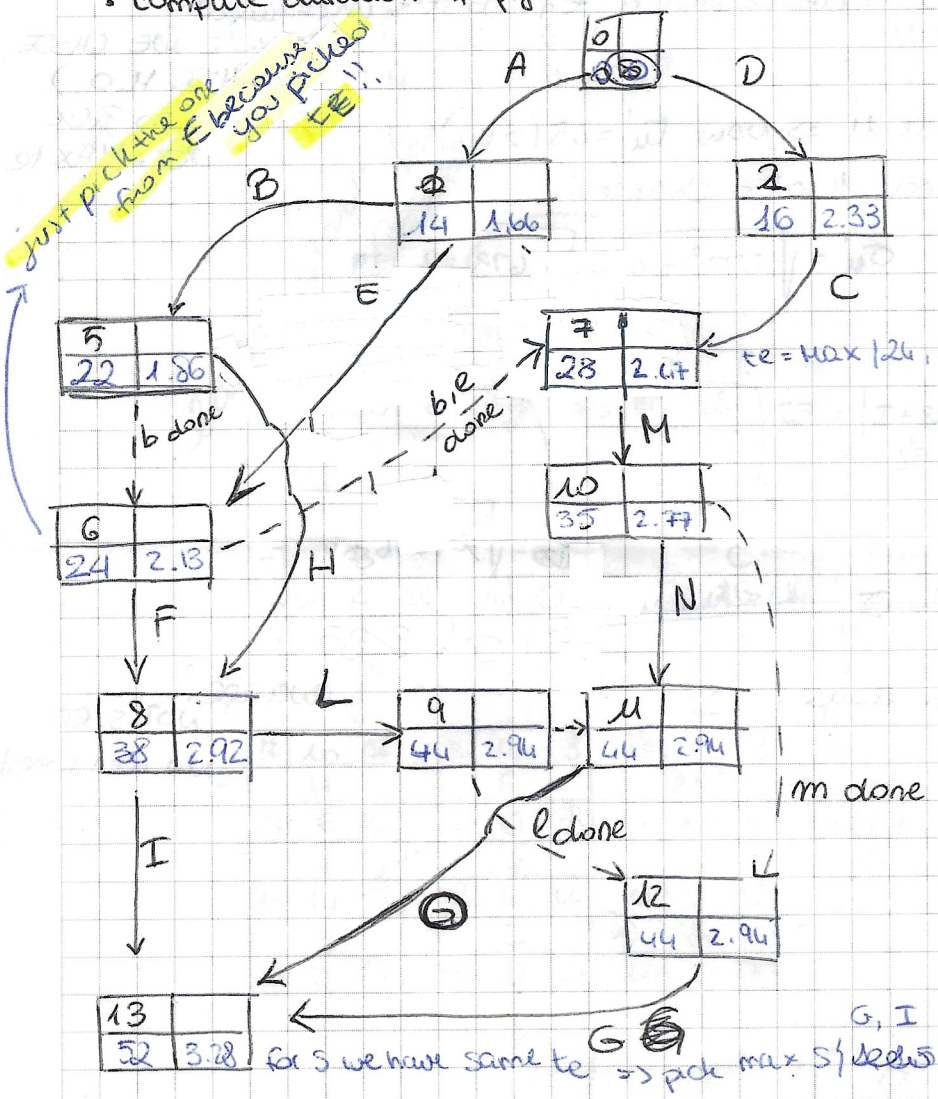
$2.7566 *$

{2.13, 1.86}

★ MOCK EXAM ★

E1 : PERT analysis SD cost × day / Analyst : 800 €
JD cost × day : 500 €
overhead 60% historically

- compute duration of prj:



just pick the one from E because you picked EE !!

Q: do I need this "ALONE" as a dependency? or does it always come with others?

for 3 we have same te => pick max {5, 3.28} = 5

b) P of completion at day 52:
50%

c) P of completion at day 60:
$z = 2.439 \Rightarrow \Phi 0.99 \Rightarrow 99\%$

d) P of compl. H at T = 38:
$z = \frac{38-32}{\sqrt{1.86^2+0.83^2}} = \frac{6}{2.04} = 2.941$
S_H
⇓
99%

44 + O = 51
44 + G = 52
38 + I = 52
} Dur. = 52 days (prob. 50%)

e) T of finishing O at P = 95%?

$T = te + zS = (44 + t_0) + zS = 51 + z_{95} S_0 = 51 + 1.65 \cdot 2.96 \cong 56$ days (4.88)

UN TABLE

f) P of finishing PRJ with P = 90%?

$T = te + zS = 52 + z_{90} \cdot 3.28 = 52 + 4.2 \cong 56$ or 57?
I'd say 57

g) imagine you have adopted "early start" - At day 35 you realize you should change values of N to $t_e' = 5$ and $\sigma' = 2$ - How does the modification impact duration and scheduling - Now assume it changed to $t_e'' = 10$ and $\sigma'' = 2$. Provide the day by which P of finishing all is 95%.

$t_e = 4 \Rightarrow 5$

$\sigma_e = 0.16 \Rightarrow 2$

N impacts node (11) $\Rightarrow$ t_{11}' = still 44
σ_{11}' = still 2.94 cuz we picked t_9 so it's unchanged (BECAUSE WE CHOSE THIS M.O.)

$t_e = 4 \Rightarrow 10$
$\sigma_e = 0.16 \Rightarrow 2$

N impacts node 11 $\Rightarrow$ now $t_{11}' = 45 (> 44)$
TO PICK σ = MAX t_e

so σ at node 11 also changes!

$$t_{11}' = 45 \quad \sigma_{11}' = \sqrt{2.77^2 + 4} = \sqrt{11.673} = 3.42$$

updated node 13

$$t_{e13} = \max \{ 44 + t_6 ; 45 + 0 ; 38 + I \} = 52$$
(52 ; 52 ; 52)

$\sigma_{13} = \max \sigma$? lol

$= \{1 ; 1.5 ; 2\}$! ~~Pick σ_N' it's the max~~

$\sigma_{13}' = \sqrt{11.696 + 4} =$

$\sigma' = 3.35$?

average? NOT SURE NO IDEA honestly

$P\ T_{P95} = 52 + 1.65 \cdot 3.35 = 60$ days

E2. Critical chain Method → USE LATEST START; YOU NEED CPM use te for duration

1) CZONES = 1/3 = $\lceil (te + z_{95} \cdot s) - te \rceil = \lceil z_{95} \cdot s \rceil = \lceil 1{,}65 \cdot s \rceil$

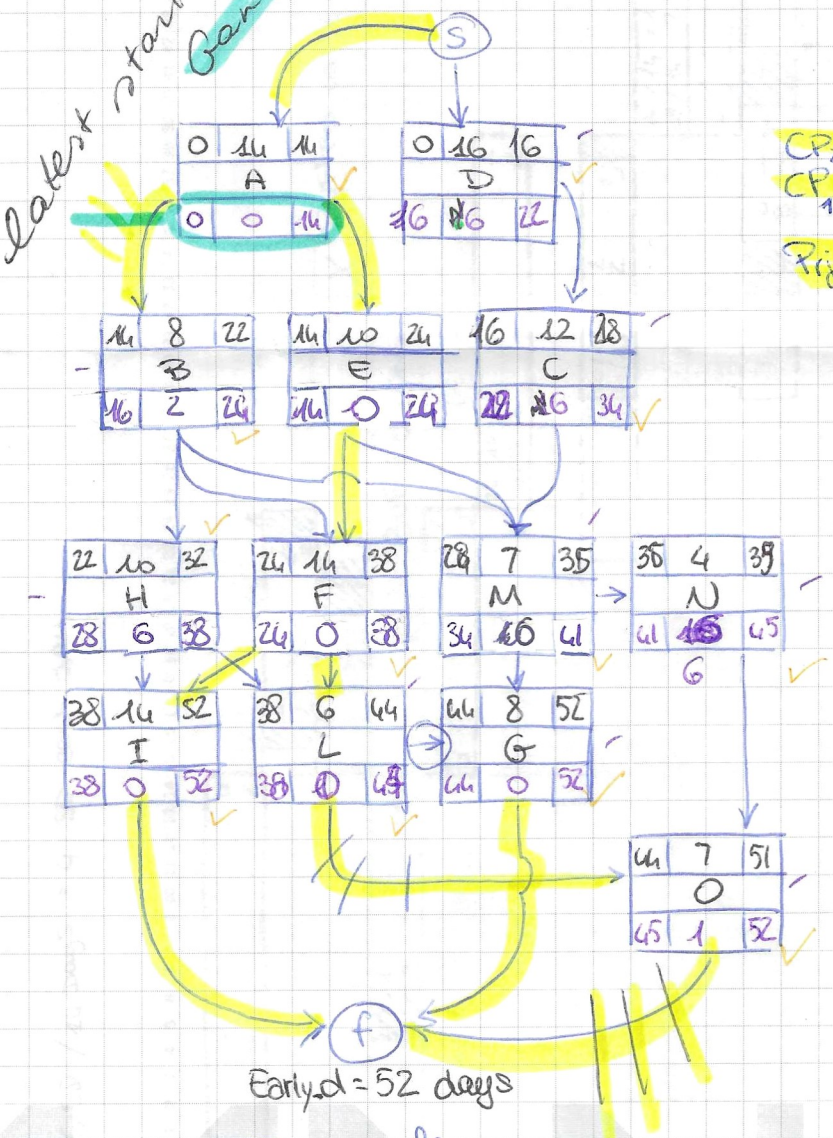
A = $\lceil z \cdot 1.66 \rceil = 3$
B = $\lceil z \cdot 0.83 \rceil = 2$
C = $\lceil z \cdot 0.83 \rceil = 2$
D = $\lceil z \cdot 2.33 \rceil = 4$

E = $\lceil z \cdot 1.33 \rceil = 3$
F = $\lceil z \cdot 2 \rceil = 4$
G = $\lceil z \cdot 1 \rceil = 2$
H = $\lceil z \cdot 0{,}83 \rceil = 2$

I = $\lceil z \cdot 1.5 \rceil = 3$
L = $\lceil z \cdot 0.33 \rceil = 1$
M = $\lceil z \cdot 1 \rceil = 2$
N = $\lceil z \cdot 0{,}16 \rceil = 1$
O = $\lceil z \cdot 0{,}33 \rceil = 1$

2) CPM

latest start for Gantt



CP2 = A-E-F-L-O, ΣCz = 13
CP1 = A-E-F-I-f ΣCz = 13
13 = 7 days

$Pr_{jBUF} = 0.5 \cdot$ (13) days

FEEDBUF H→I, H→L
B→F, M→G
O→end → USE PRJBUF

paths to count:
D-C-M
B-H

FOR M-G
↳ $Feed_{DCM} = \lceil 0.25 \cdot (2+2+4) \rceil = 2$
BIGGEST BUFF GOING INTO G-M

$Feed_{BF} = \lceil 0.25 \cdot 2 \rceil = 1$
only path for B→F

$Feed_{BH} = \lceil 0.25 \cdot 2 \rceil = 1$
↳ for both HI and HL

H B M are our SUB-CRITICAL activities

~~Paths from CD-CN with NCN: A-H-I~~
~~A-B-D~~
~~A-B-H-I~~

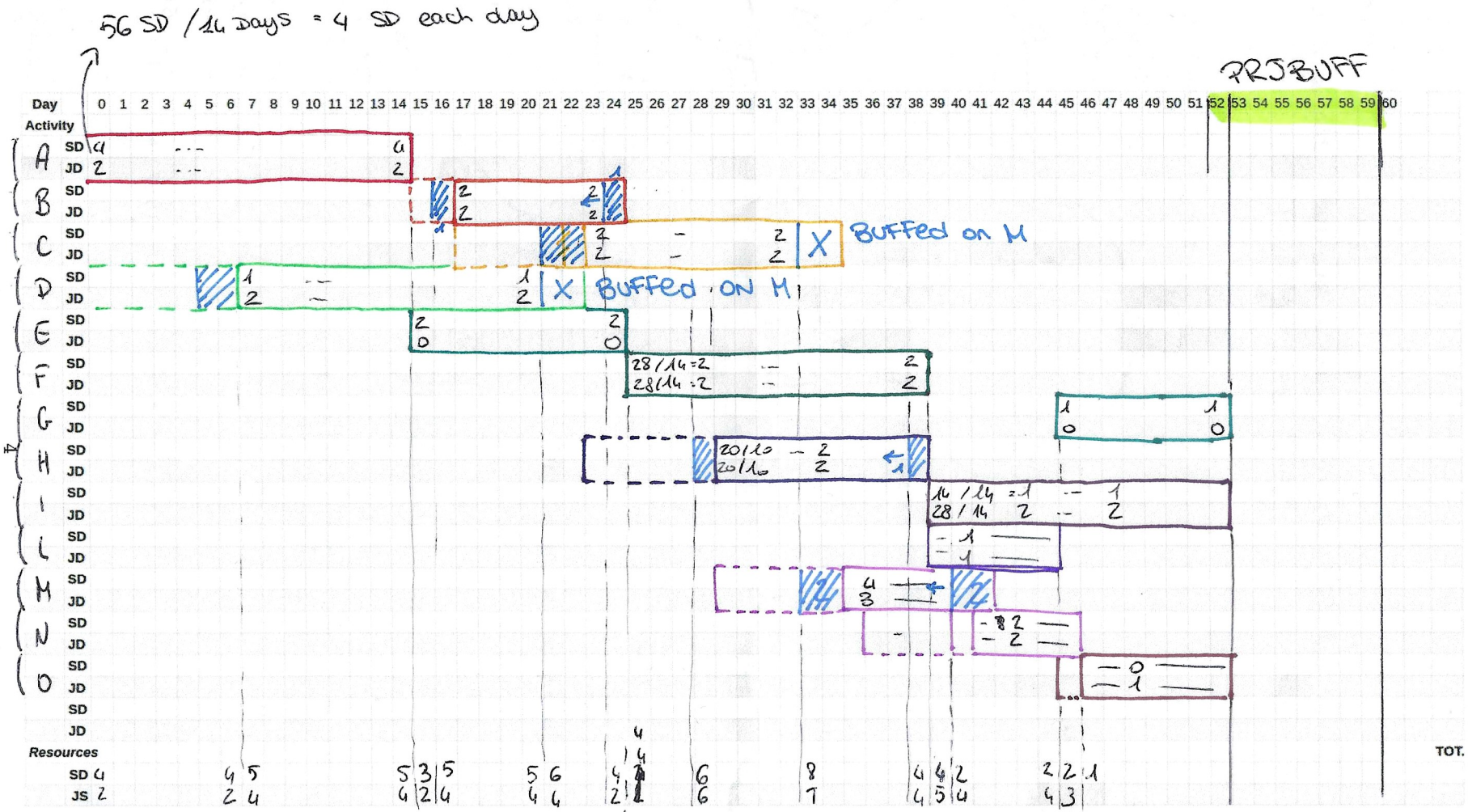


Table 3: Exercise 2 - Resource Allocation