

Properties of Gases and Gas Laws

ABDULLAHI Danjuma Kassim

Introduction - phases of matter

There are four major phases of matter:

- solids,
 - liquids,
 - gases and
 - plasmas.
- Starting from a solid at a temperature below its melting point, we can move through these phases by increasing the temperature.
- First, we overcome the bonds or intermolecular forces locking the atoms into the solid structure, and the solid melts.

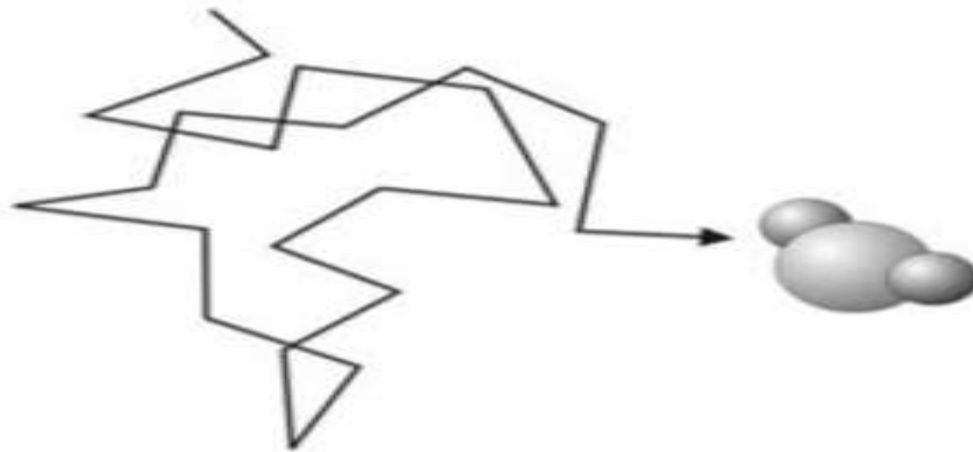
- phases of matter (Contn.)

- ▶ At higher temperatures we overcome virtually all of the intermolecular forces and the liquid vapourises to form a gas (depending on the ambient pressure and on the phase diagram of the substance, it is sometimes possible to go directly from the solid to the gas phase in a process known as sublimation).
- ▶ If we increase the temperature to extremely high levels, there is enough energy to ionise the substance and we form a plasma.
- ▶ This course is concerned solely with the properties and behaviour of gases.
- ▶ As we shall see, the fact that interactions between gas phase particles are only very weak allows us to use relatively simple models to gain

Characteristics of the gas phase

The gas phase of a substance has the following properties:

1. A gas is a collection of particles in constant, rapid, random motion (sometimes referred to as 'Brownian' motion). The particles in a gas are constantly undergoing collisions with each other and with the walls of the container, which change their direction – hence the 'random'. If we followed the trajectory of a single particle within a gas, it might look something like the figure on the right.
2. A gas fills any container it occupies. This is a result of the second law of thermodynamics i.e. gas expanding to fill a container is a spontaneous process due to the accompanying increase in entropy.



Characteristics of the gas phase

3. The effects of intermolecular forces in a gas are generally fairly small. For many gases over a fairly wide range of temperatures and pressures, it is a reasonable approximation to ignore them entirely. This is the basis of the 'ideal gas' approximation, of which more later.
4. The physical state of a pure gas (as opposed to a mixture) may be defined by four physical properties:

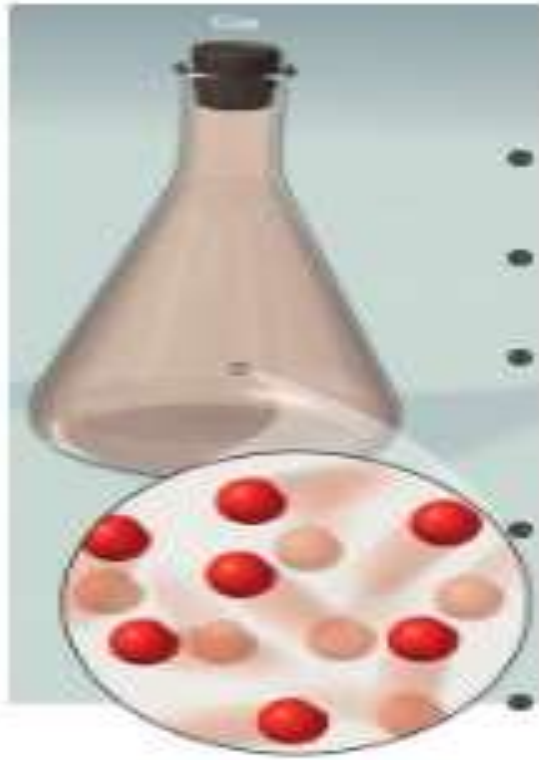
p – the pressure of the gas

T – the temperature of the gas

V – the volume of the gas

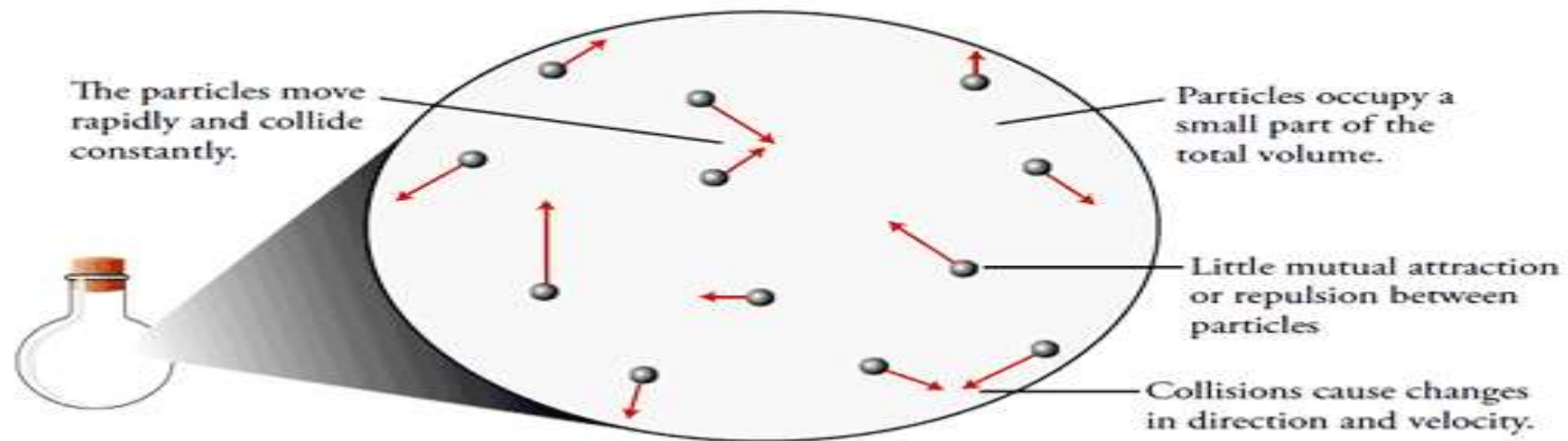
n – the number of moles of substance present

General Properties of Gases



- Gases do not have definite shape and volume
- Gases occupy the whole space available to them.
- Gases have unlimited expansibility and high compressibility.
- They have very low densities because of negligible intermolecular forces.
- Gases exert pressure on the walls of the container with perfectly elastic collisions.

- They diffuse rapidly through each other from homogeneous mixture against the electric and gravitational field.





The characteristics of gases are described in terms of following four parameters

- Mass
- Volume
- Pressure
- Temperature

1. Mass (M)

The mass of the gas is related to the number of moles as

$$n = w/M$$

Where n = number of moles

w = mass of gas in grams

M = molecular mass of the gas

2. Volume (V)

Since gases occupy the entire space available to them, therefore the gas volume

Units of Volume: Volume is generally expressed in litre (L), cm^3 & dm^3

$$1\text{m}^3 = 10^3 \text{ litre} = 10^3 \text{ dm}^3 = 10^6 \text{ cm}^3.$$

3. Pressure

Pressure of the gas is due to its collisions with walls of its container *i.e.* the force exerted by the gas per unit area on the walls of the container is equal to its pressure.

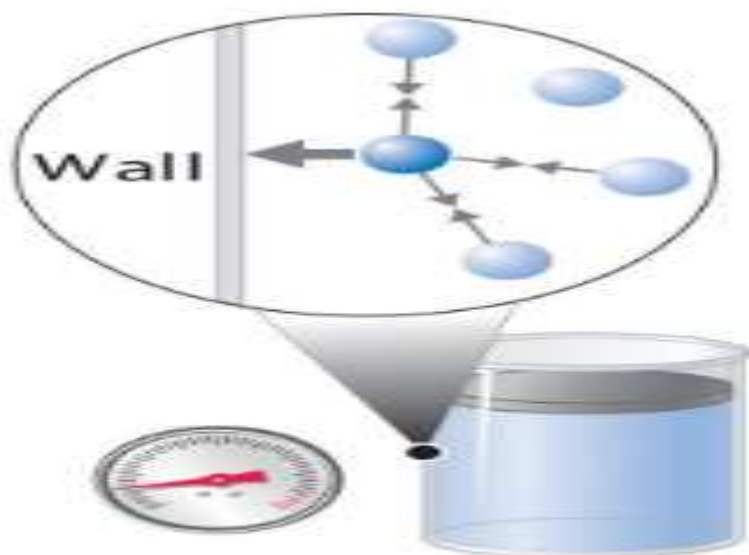
4. Temperature (T):

- Temperature is defined as the degree of hotness. The SI unit of temperature is Kelvin. °C and °F are the two other units used for measuring temperature. On the Celsius scale water freezes at 0°C and boils at 100°C where as in the Kelvin scale water freezes at 273 K and boils at 373 K.

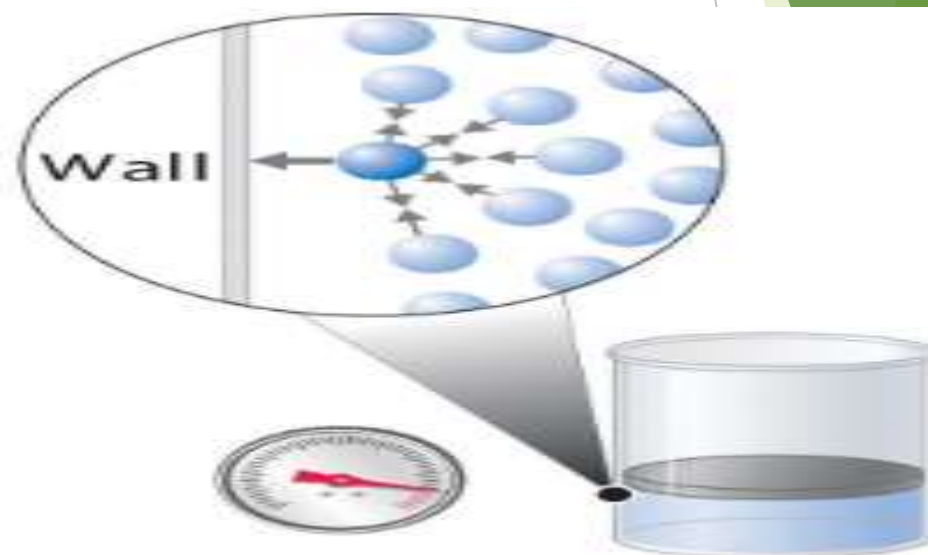
$$K = ^\circ C + 273.5$$

$$F = (9/5) ^\circ C + 32$$

- ❑ Pressure is exerted by a gas due to kinetic energy of its molecules.
- ❑ As temperature increases, the kinetic energy of molecules increases, which results in increase in pressure of the gas. So, pressure of any gas is directly proportional to its temperature.



(a) Low pressure



(b) High pressure

Units of Pressure:

The pressure of a gas is expressed in atm, Pa, Nm^{-2} , bar and lb/in^2 (psi).

$760 \text{ mm} = 1 \text{ atm} = 101325 \text{ KPa} = 101325 \text{ Pa} = 101325 \text{ Nm}^{-2}$

$760 \text{ mm of Hg} = 1.01325 \text{ bar} = 1013.25 \text{ milli bar} = 14.7 \text{ lb}/\text{in}^2$ (psi)

4. Temperature (T):

- Temperature is defined as the degree of hotness. The SI unit of temperature is Kelvin. °C and °F are the two other units used for measuring temperature. On the Celsius scale water freezes at 0°C and boils at 100°C whereas in the Kelvin scale water freezes at 273 K and boils at 373 K.

$$K = ^\circ C + 273.5$$

$$F = (9/5) ^\circ C + 32$$

Gas Laws

Boyle's Law

In 1662, Robert Boyle discovered that there existed a relation between the pressure and the volume of a fixed amount of gas at a fixed temperature. In his experiment, he discovered that the product of Pressure & Volume of a fixed amount of gas at a fixed temperature was approximately a constant. So, Boyle's law states that

"At constant temperature, the pressure of a fixed amount (i.e., number of moles n) of gas varies inversely with its volume".

Boyle's Law



Boyle's Law (Cont.)

It means that at constant temperature, product of pressure and volume of a fixed amount of gas is constant.

If a fixed amount of gas at constant temperature T occupying volume V_1 at pressure P_1 undergoes expansion, so that volume becomes V_2 and pressure becomes P_2 ,

Mathematically

$$P \propto \frac{1}{V}$$

$$P = k_1 \frac{1}{V} \Rightarrow PV = k_1 = \text{Constant}$$

then according to Boyle's law :

$$P_1 V_1 = P_2 V_2 = \text{Constant} \Rightarrow \frac{P_1}{P_2} = \frac{V_1}{V_2}$$

Using Boyle's Law we get,

$$d = \frac{m}{k} P$$

This indicates that at a constant temperature, pressure is directly proportional to the density of a fixed mass of the gas

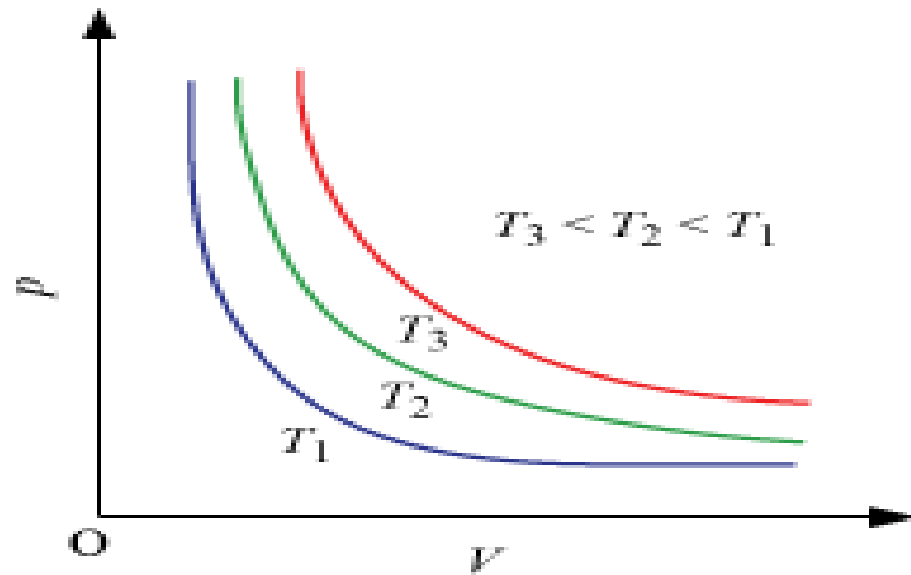
It is important to understand here the various units of pressure and their relationships.

$$1 \text{ atm} = 1.01325 \text{ bar}$$

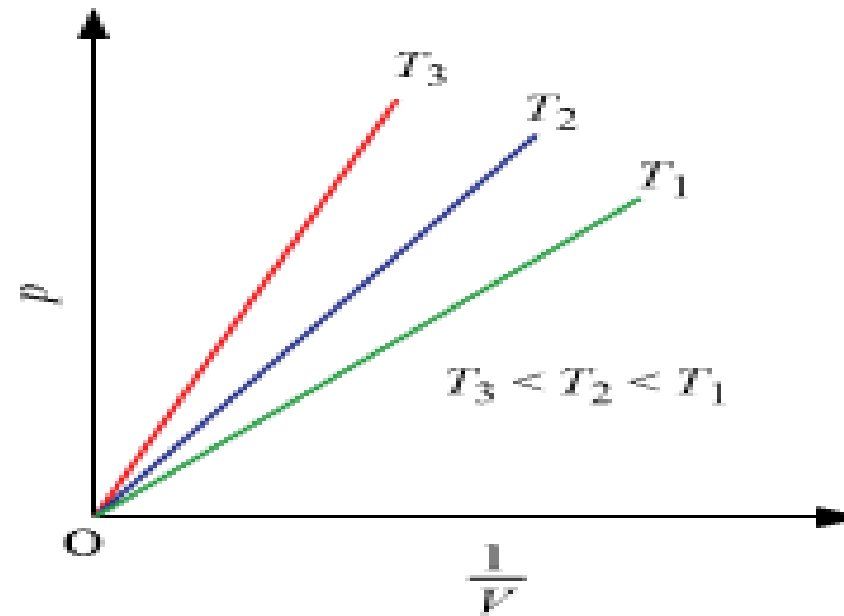
The SI unit of pressure is Nm^{-2} which is called 'Pascal' (Pa) The pressure at sea level due to the weight of the earth's atmosphere is approximately 10^5 Pa. Since pascal is a small unit, we express pressures in bar units, where $1 \text{ bar} = 10^5 \text{ Pa}$. Atmospheric pressure is about one bar. Atmospheric pressure can also be expressed in atmosphere units abbreviated as atm.

Graphical Representation of Boyle's Law :

- A plot of P versus $1/V$ at constant temperature for a fixed mass of gas would be a straight line passing through the origin.
- A plot of P versus V at constant temperature for a fixed mass of a gas would be a rectangular hyperbola.

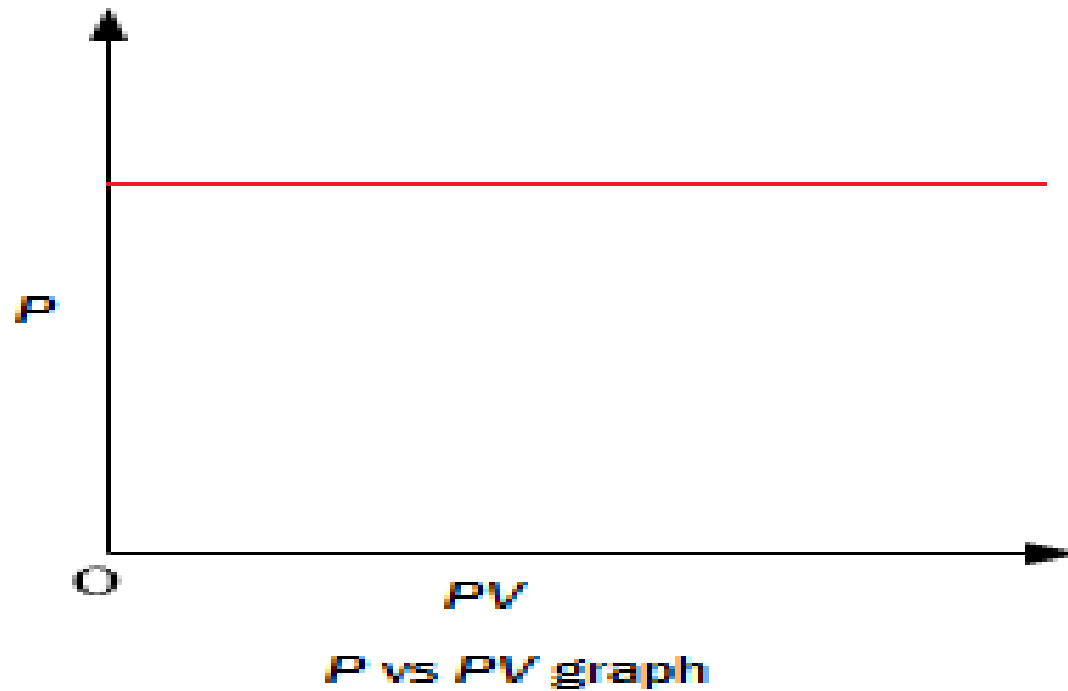


p vs V graph



p vs $\frac{1}{V}$ graph

- A plot of P (or V) versus PV at constant temperature for a fixed mass of a gas is a straight line parallel to the PV axis.



Example 1:

Question:

A gas is present at a pressure of 2 atm. What should be the increase in pressure so that the volume of the gas can be decreased to $\frac{1}{4}$ th of the initial value, If the temperature is maintained constant?

Solution:

PV = Constant for a given mass of gas at constant pressure

$$P_1V_1 = P_2V_2$$

$$P_1 = 2 \text{ atm}$$

$$V_1 = V$$

$$V_2 = V/4$$

$$2 \times V = P_2 \times V/4$$

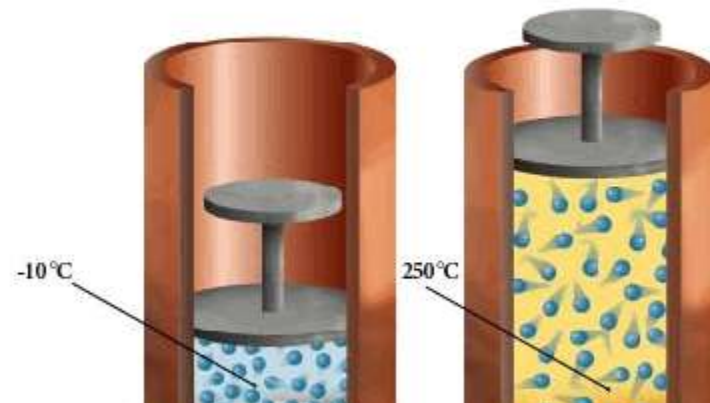
$$P_2 = 8 \text{ atm}$$

Pressure should be increased from 2 to 8 atm.

$$\text{Total increase} = 8 - 2 = 6 \text{ atm}$$

Charles's Law

In 1787, Jacques Charles discovered that if the pressure is kept constant, the volume of a gas sample increases linearly with the temperature for a fixed amount of gas. This law led to the idea of temperature. The unit of temperature used is Kelvin. Charles's law states that *"At constant pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature"*



Mathematically

$$V \propto T \Rightarrow V = kT$$

or

$$\frac{V}{T} = k = \text{constant}$$

Hence, if at constant pressure the volume of a gas V_1 at temperature T_1 change to V_2 at T_2 we have

Mathematically

$$V \propto T \Rightarrow V = kT$$

or

$$\frac{V}{T} = k = \text{constant}$$

This equation is known as **Charle's Law equation or formula.**

For each degree change in temperature, the volume of sample of a gas changes by the fraction of $1/273.5$ of its volume at 0°C .

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant}$$

or $\log V - \log T = \text{Constant}$

So,

$$V_t = V_0 \left[\frac{273.15 + t}{273.15} \right]$$

This equation is known as **Charles-Gay-Lussac equation.**

Where,

V_t = volume of gas at temperature $t^{\circ}\text{C}$

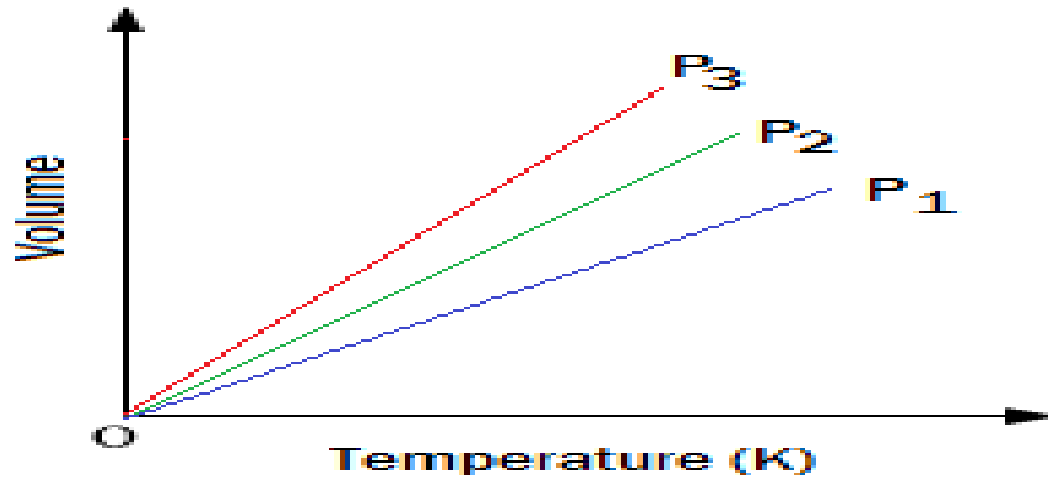
V_0 = volume of gas at 0°C

T = temperature in $^{\circ}\text{C}$

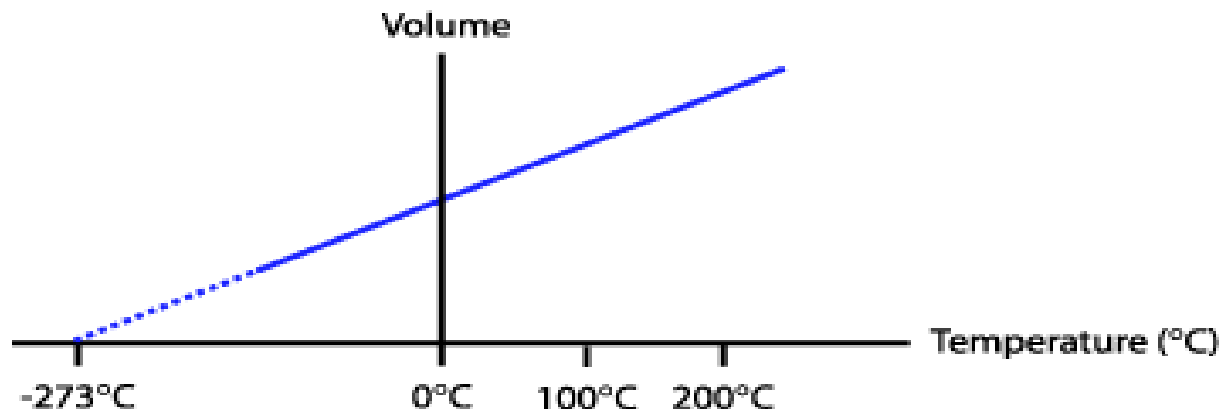
- ▶ The temperature of -273°C , at which the volume of a gas would theoretically be reduced to zero, is called **absolute zero**. At absolute zero temperature, the volume, pressure, kinetic energy and heat content of a gas is zero.

Graphical Representation of Charles's Law :

1. For a definite mass of the gas a plot of V vs T ($^{\circ}\text{K}$) at constant pressure is a straight line passing through the origin.



2. A plot of V vs t ($^{\circ}\text{C}$) at constant pressure is a straight line cutting the temperature axis at -273°C



Example 2:

Question:

Volume of given amount of a gas at 57°C and constant pressure is 425.8 cm^3 . If the temperature is decreased to 37°C at constant pressure, then what would be the volume of gas?

Solution:

According to Charle's Law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Here

$$V_1 = 425.8\text{ cm}^3$$

$$V_2 = ?$$

$$T_1 = 273 + 57 = 330\text{ K}$$

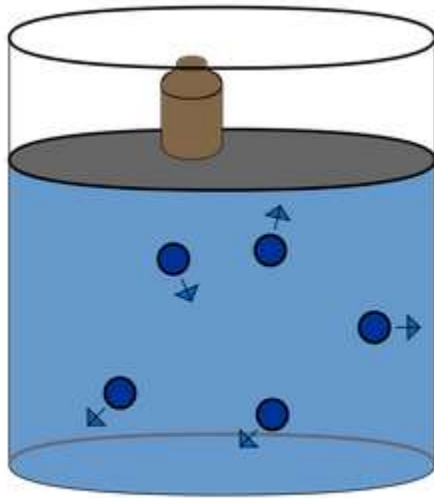
$$T_2 = 273 + 37 = 310\text{ K}$$

So,

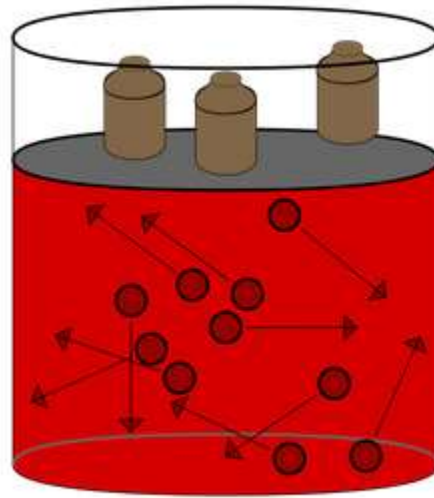
$$V_2 = \frac{425.8 \times 310}{330} = 400\text{ cm}^3$$

Gay-Lussac's Law or Amonation's Law

► This law states that ***“at constant volume, the pressure of a given mass of a gas is directly proportional to its absolute temperature”***.



Temperature T



Temperature 3T

Mathematically

$P \propto T$ (at constant volume)

$$\Rightarrow P = kT \Rightarrow \frac{P}{T} = k = \text{constant}$$

Where,

P = Pressure of Gas

T = Absolute Temperature

If the pressure and temperature of a gas changes from P_1 & T_1 to P_2 & T_2 , volume remaining constant, we have

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}$$

or $\log P - \log T = \text{constant}$

$$P_t = P_0 \left(1 + \frac{t}{273.15} \right)$$

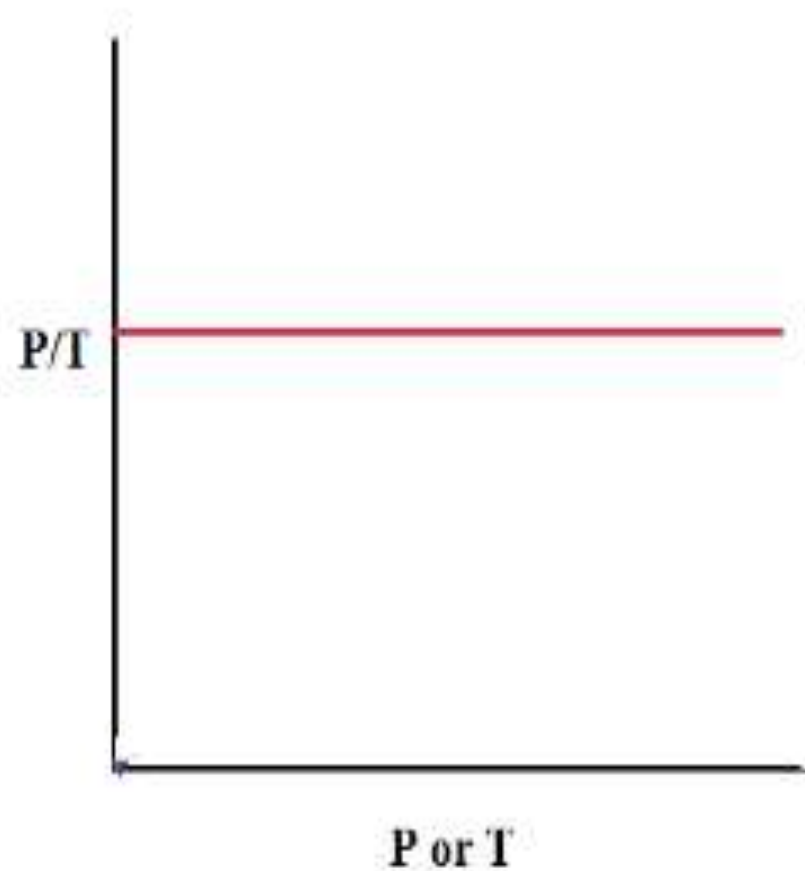
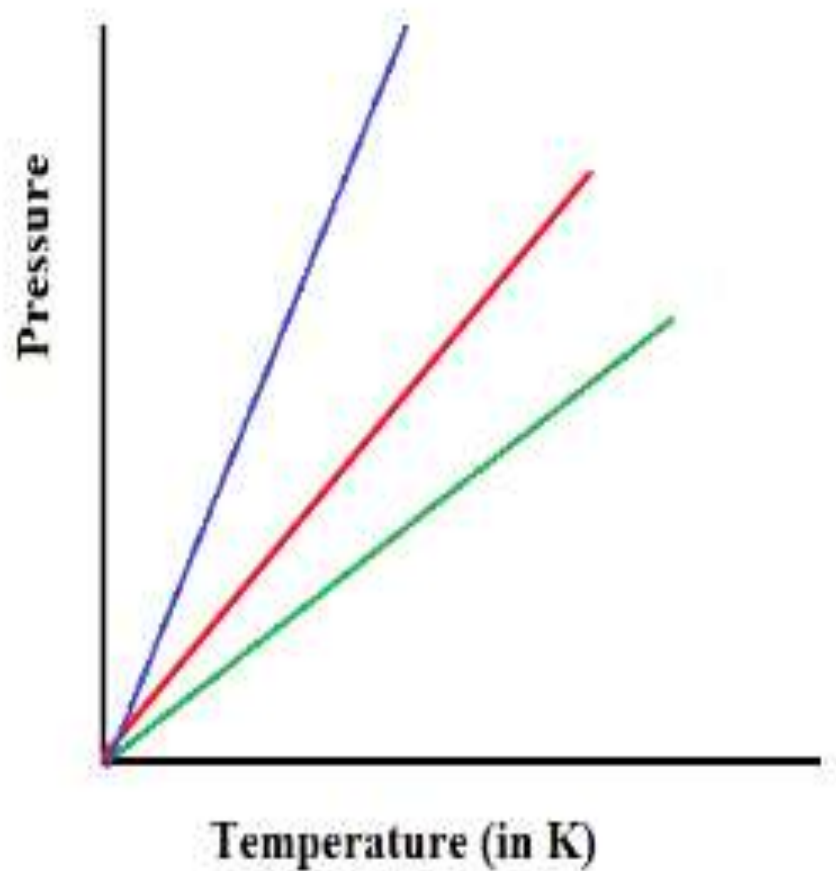
where,

P_t = Pressure of gas at t °C

P_0 = Pressure of gas at 0 °C

t = Temperature in °C.

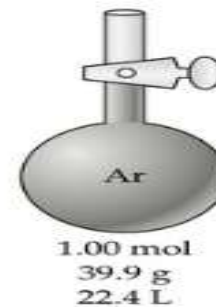
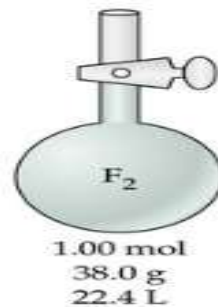
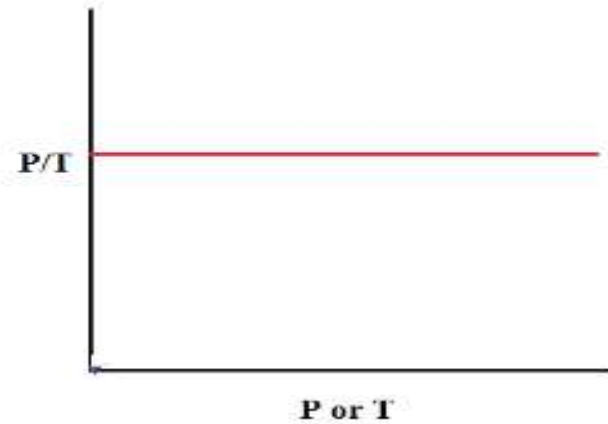
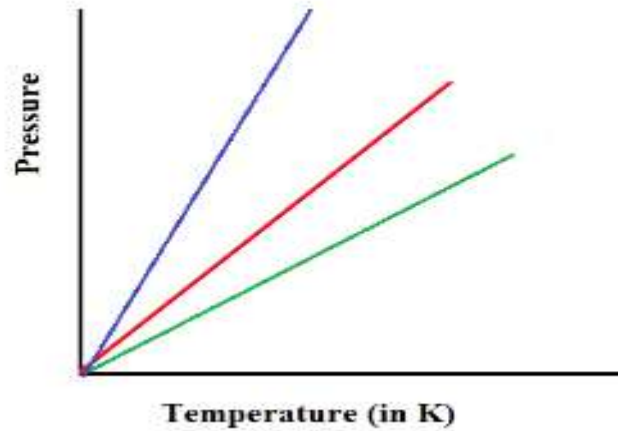
Graphical Representation of Gay-Lussac's Law



Avogadro's Law

In 1812, Amadeo Avogadro stated that

“Samples of different gases which contain the same number of molecules (any complexity, size, shape) occupy the same volume at the same temperature and pressure”.



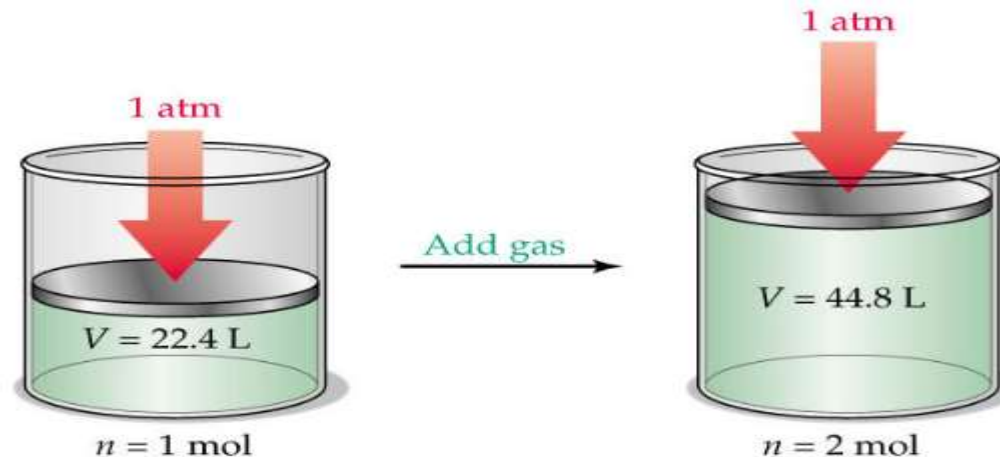
**It follows from Avogadro's hypothesis
that (when T and P are constant).**

$$\frac{V}{n} = k$$

Mathematically

$$V \propto n \Rightarrow V = kn$$

$$\Rightarrow \frac{V}{n} = k = \text{Constant}$$



Since volume of a gas is directly proportional to the number of mole of each gas at standard temperature and pressure (STP) **same**

Standard temperature and pressure means 273.15 K (0°C) temperature and 1 bar (i.e., exactly 10⁵ pascal) pressure. **At STP molar volume of any gas or a combination of ideal gases is 22.71098 L mol⁻¹**

We know that **number of moles of any gas (n) = m/M**

Where m = mass of the gas under investigation and M = molar mass

So, we can conclude from equation that the density of a gas is directly proportional to its molar mass.

A gas that follows Boyle's law, Charles' law and Avogadro law strictly is called an **ideal gas**. Such a gas is hypothetical. It is assumed that intermolecular forces are not present between the molecules of an ideal gas. Real gases follow these laws only under certain specific conditions when forces of intermolecular attraction are negligible.

Ideal Gas Equation

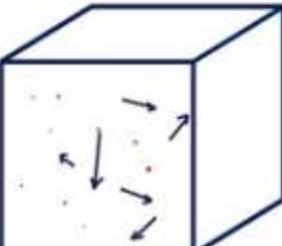
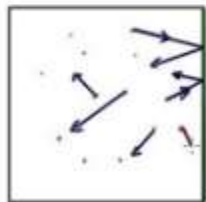
$P \propto \frac{1}{V}$
 Boyle's Law

$P \propto T$ $V \propto T$
 Charles' Law

$PV = nRT$
 Ideal Gas Equation

6.02×10^{23}

$PV = NkT$
 Ideal gas eqⁿ
 (alternate form)

The Ideal Gas Equation

PV = nRT

- P = pressure
- V = volume
- n = number of moles
- T = temperature
- R = gas constant

$$P = \frac{nRT}{V}$$

$$n = \frac{PV}{RT}$$

$$V = \frac{nRT}{P}$$

$$T = \frac{PV}{nR}$$

This equation give the relation between pressure P, volume V and absolute temperature T of a gas.

$$PV = nRT \quad 1$$

Derivation. According to Boyle's law

$$V \propto \frac{1}{P} \quad \dots(1) \quad 1$$

According to Charle's law

$$V \propto T \quad \dots(2) \quad 1$$

Comparing (1) and (2), we have

$$\frac{PV}{T} = \text{constant} \quad 1$$

As $PV = RT$

For n moles of gas $PV = nRT \quad 1$

This is perfect or ideal gas equation.

Ideal Gas Equation (IGE)

- Combines temperature, pressure, volume and number of molecules into one equation

$$PV = nRT$$

n = # of moles of gas

R = Ideal Gas Constant ($0.0821 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$)

Ideal Gas Law

$$\text{pressure } PV = nRT \text{ temperature (in K)}$$

volume # of gas gas constant
molecules

$$P = 749 \text{ torr} \times \frac{1 \text{ atm}}{760 \text{ torr}} = 0.9855 \text{ atm}$$

$$T = 125^\circ\text{C} + 273 = 398 \text{ K}$$

$$V = 0.421 \text{ L}$$

$$R = 0.0821 \text{ L atm/mol K}$$

$$n = \frac{PV}{RT} = \frac{(0.9855 \text{ atm})(0.421 \text{ L})}{(0.0821 \text{ L atm/mol K})(398 \text{ K})} = 0.01270 \text{ mol}$$

$$\text{molar mass} = \frac{1.67 \text{ grams}}{0.01270 \text{ moles}} = 132 \text{ g/mol}$$

$$pV = \frac{\text{mass (g)}}{\text{mass of 1 mole (g)}} \times RT$$

$$101325 \times 0.001 = \frac{1.264}{\text{mass of 1 mole (g)}} \times 8.31441 \times 293$$

$$\begin{aligned} \text{mass of 1 mole (g)} &= \frac{1.264 \times 8.31441 \times 293}{101325 \times 0.001} \\ &= 30.4 \text{ g} \end{aligned}$$

A 23.8 L balloon is filled with 88g CO_2 at 15°C , what is the pressure in kPa?

$$PV = nRT$$
~~$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$~~

$$PV = nRT$$

$$P(23.8 \text{ L}) = (n) \cdot$$

$$\begin{aligned} V &= 23.8 \text{ L} \\ T &= 15^\circ\text{C} + 273 = 288 \text{ K} \\ P &= ? \text{ kPa} \\ 88 \text{ g } \text{CO}_2 &\times \frac{1 \text{ mol } \text{CO}_2}{(12 + 16 + 16)} = \frac{2 \text{ mol}}{44 \text{ g}} \end{aligned}$$

The Gas Constant R

- Repeated experiments show that at standard temperature (273 K) and pressure (1 atm or 101325 N/m²), one mole ($n = 1$) of gas occupies 22.4 L volume. Using this experimental value, you can evaluate the gas constant R ,

$$\begin{aligned} R &= \frac{PV}{nT} = \frac{1 \text{ atm } 22.4 \text{ L}}{1 \text{ mol } 273 \text{ K}} \\ &= 0.08205 \frac{\text{L atm}}{\text{mol} \cdot \text{K}} \end{aligned}$$

When SI units are desirable, $P = 101325 \text{ N/m}^2$ (Pa for pascal) instead of 1 atm. The volume is 0.0224 m³. The numerical value and units for R are

$$\begin{aligned} R &= \frac{101325 \frac{\text{N}}{\text{m}^2} 0.0224 \text{ m}^3}{1 \text{ mol } 273 \text{ K}} \\ &= 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \end{aligned}$$

Suggested Reading

Physical Chemistry, P. W. Atkins

Foundations of Physics for Chemists, G. Ritchie and D. Sivia

Physical Chemistry, W. J. Moore

University Physics, H. Benson

Note that $1 \text{ L atm} = 0.001 \text{ m}^3 \times 101325 \frac{\text{N}}{\text{m}^2} = 101.325 \text{ J (or N m)}$.

Since energy can be expressed in many units, other numerical values and units for R are frequently in use.

For your information, the gas constant can be expressed in the following values and units.

$$R = 0.08205 \frac{\text{L atm}}{\text{mol} \cdot \text{K}}$$

$$= 8.3145 \frac{\text{L kPa}}{\text{mol} \cdot \text{K}}$$

$$= 8.3145 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$= 1.987 \frac{\text{cal}}{\text{mol} \cdot \text{K}}$$

$$= 62.364 \frac{\text{L torr}}{\text{mol} \cdot \text{K}}$$

Notes:

$$1 \text{ atm} = 101.32 \text{ kPa}$$

$$1 \text{ J} = 1 \text{ L kPa}$$

$$1 \text{ cal} = 4.182 \text{ J}$$

$$1 \text{ atm} = 760 \text{ torr}$$



$$\frac{PV}{T} = nR$$

For a fixed mass of gas, n is fixed, so:

$$\frac{PV}{T} = \text{a constant}$$

Or,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

initial
values

final
values

Boyles Law: $P = (nRT) \frac{1}{V}$, where nRT is a constant.

For a fixed mass of gas, n is fixed, so:

$$\frac{PV}{T} = \text{a constant}$$

Charles Law: $V = \left(\frac{nR}{P}\right)T$, where $\left(\frac{nR}{P}\right)$ is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$P = \left(\frac{nR}{V}\right)T$, where $\left(\frac{nR}{V}\right)$ is constant.

- Boyle's law $V \propto 1/P$
- Charles's law $V \propto T$
- Avogadro's law $V \propto n$

$$\left. \begin{array}{l} V \propto 1/P \\ V \propto T \\ V \propto n \end{array} \right\} V \propto \frac{nT}{P}$$

$$V = R \frac{nT}{P}$$

$$PV = nRT$$

Avogadro's Law: $V = \left(\frac{RT}{P}\right)n$, where $\left(\frac{RT}{P}\right)$ is constant.

Gay-Lusac's Law of Combining Gas Volumes

► Gay-Lusac's Law of Combining Gas Volumes states that:

The volume of gases taking part in a chemical reaction show simple whole number ratios to one another when those volumes are measured at the same temperature and pressure.

When gas A reacts with gas B to produce gas C at constant temperature and pressure, then the ratio of the gas volumes will be a simple whole number ratio:

For gaseous reaction at constant temperature and pressure

volume of Gas A : volume of Gas B : volume of Gas C
 x : y : z

where x , y and z are all whole numbers

