MTH 205 1ST TEST — SOLUTIONS QUESTIONS & ANSWERS

1a Evaluate the Determinant of the Matrix

$$A=\begin{bmatrix} t+2 & 0 & 1 \\ 5 & t+3 & 1 \\ 6 & -6 & t+4 \end{bmatrix}$$

SOLUTION

$$|A| = t+2\left((t+3)\cdot(t+4)-(1\cdot -6)\right)-0\left((5\cdot(t+4)-(1\cdot 6)\right)+1\left((5\cdot -6)-(t+3\cdot 6)\right)$$

$$= t+2\left((t^2+4t+3t+12)-(-6)\right)-0\left((5t+20)-(6)\right)+1\left((-30)-(6t+18)\right)$$

$$= t+2\left(t^2+4t+3t+12+6\right)-0(5t+20-6)+1(-30-6t-18)$$

$$= t+2\left(t^2+7t+18\right)-0(5t+14)+1(-6t-48)$$

$$= (t^3+7t^2+18t+2t^2+14t+36)-0-6t-48$$

$$= (t^3+9t^2+32t+36)-6t-48$$

$$= t^3+9t^2+32t+36-6t-48$$

$$= t^3+9t^2+26t-12 \;//$$

1b $A=\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ Suppose the matrix is symmetric. Find x.

SOLUTION

Symmetric says $A = A^T$

$$\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$$

~~$4=4$~~ Equal so cancel

$x+2 = 2x-3$

$2x-3 = x+2$

~~$x+1 = x+1$~~ Equal so Cancel

- Solve for x in $x+2 = 2x-3$

$x+2 = 2x-3$

$2+3 = 2x-x$

$x = 5$ // x found ✓

- To prove $x=5$, Solve for x in $2x-3 = x+2$

$2x-3 = x+2$

$2x-x = 2+3$

$x = 5$ // ✓ Therefore $x = 5$ //

1c ~~2c~~ Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$, Evaluate $f(A)$, where $f(x) = 2x^3 - 4x^2 + 5x + 3$

SOLUTION

$f(x) = 2x^3 - 4x^2 + 5x + 3$

Evaluate $f(A)$. Substitute A into polynom

$$f(A) = 2\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}^3 - 4\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}^2 + 5\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= 2\left[\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] - 4\left[\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] + 5\left[\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

multiply multiply

$$= 2\left[\begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] - 4\left[\begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}\right] + 5\left[\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

multiply

$$= 2\left[\begin{pmatrix} -7 & 30 \\ 60 & -67 \end{pmatrix}\right] - 4\left[\begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}\right] + 5\left[\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\right] + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} - 4\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + 5\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix} - \begin{bmatrix} 36 & -16 \\ -32 & 68 \end{bmatrix} + \begin{bmatrix} 5 & 10 \\ 20 & -15 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -50 & 76 \\ 152 & -202 \end{bmatrix} + \begin{bmatrix} 5 & 10 \\ 20 & -15 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -45 & 86 \\ 172 & -217 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -42 & 86 \\ 172 & -214 \end{bmatrix} //$$

2a Solve by Matrix Inverse Method the system of Equations

$x_1 + 2x_2 - x_3 = 3$

$2x_1 + 5x_2 - 4x_3 = 5$

$5x_1 + 4x_2 + 2x_3 = 12$

SOLUTION

$AX = B$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -4 \\ 5 & 4 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

To Solve for X, Multiply both sides of $AX = B$ by A^{-1}

$A^{-1} \cdot AX = A^{-1}B$ (NOTE: $A^{-1} \cdot A = I$ identity matrix which is basically 1)

$X = A^{-1}B$ where $A^{-1} = \frac{1}{|A|} AdjA$

• Find Determinant $|A|$

$|A| = 1\left((5\cdot2)-(-4\cdot4)\right) - 2\left((2\cdot2)-(-4\cdot5)\right) + (-1)\left((2\cdot4)-(5\cdot5)\right)$

$= 1(26) - 2(24) - 1(-17)$

$= -5$

• Adjoint / Adjugate (AdjA)

$a_{11} = \begin{vmatrix} 5 & -4 \\ 4 & 2 \end{vmatrix}$ $(5\cdot2)-(-4\cdot4)$ $= 26$

$a_{12} = \begin{vmatrix} 2 & -4 \\ 5 & 2 \end{vmatrix}$ $(2\cdot2)-(-4\cdot5)$ $= 24$

$a_{13} = \begin{vmatrix} 2 & 5 \\ 5 & 4 \end{vmatrix}$ $(2\cdot4)-(5\cdot5)$ $= -17$

$a_{21} = \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix}$ $(2\cdot2)-(-1\cdot4)$ $= 8$

$a_{22} = \begin{vmatrix} 1 & -1 \\ 5 & 2 \end{vmatrix}$ $(1\cdot2)-(-1\cdot5)$ $= 7$

$a_{23} = \begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix}$ $(1\cdot4)-(2\cdot5)$ $= -6$

$a_{31} = \begin{vmatrix} 2 & -1 \\ 5 & -4 \end{vmatrix}$ $(2\cdot-4)-(-1\cdot5)$ $= -3$

$a_{32} = \begin{vmatrix} 1 & -1 \\ 2 & -4 \end{vmatrix}$ $(1\cdot-4)-(-1\cdot2)$ $= -2$

$a_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix}$ $(1\cdot5)-(2\cdot2)$ $= 1$

$$\text{Minor} = \begin{bmatrix} 26 & 24 & -17 \\ 8 & 7 & -6 \\ -3 & -2 & 1 \end{bmatrix} \qquad \text{Place Sign} = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\text{Cofactor} = \begin{bmatrix} 26 & -24 & -17 \\ -8 & 7 & 6 \\ -3 & +2 & 1 \end{bmatrix} \qquad \text{Adj A} = \text{Cofactor}^T = \begin{bmatrix} 26 & -8 & -3 \\ -24 & 7 & +2 \\ -17 & 6 & 1 \end{bmatrix}$$

(Minor x place sign)

- Find $X = \frac{1}{|A|} \text{Adj A} \cdot B$

$$X = \frac{1}{-5} \begin{bmatrix} 26 & -8 & -3 \\ -24 & 7 & 2 \\ -17 & 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} (26 \cdot 3) + (-8 \cdot 5) + (-3 \cdot 12) \\ (-24 \cdot 3) + (7 \cdot 5) + (2 \cdot 12) \\ (-17 \cdot 3) + (6 \cdot 5) + (1 \cdot 12) \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} 2 \\ -13 \\ -9 \end{bmatrix}$$

$$= \begin{bmatrix} -2/5 \\ 13/5 \\ 9/5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2/5 \\ 13/5 \\ 9/5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2/5 \\ 13/5 \\ 9/5 \end{bmatrix}$$

$x_1 = -2/5$

$x_2 = 13/5$

$x_3 = 9/5$ //

2b Given the Below Matrices

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 0 & -3 \\ -1 & -2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Find (i) AB (ii) BC (iii) $B^T C$

SOLUTION

i) A · B = Not Possible Because:

= Matrix A Number of Columns is not equal to Matrix B Number of Rows

ii) $B \cdot C = \begin{pmatrix} 4 & 0 & -3 \\ -1 & -2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$$= \begin{pmatrix} (4 \cdot 2) + (0 \cdot -1) + (-3 \cdot 3) \\ (-1 \cdot 2) + (-2 \cdot -1) + (3 \cdot 3) \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 9 \end{pmatrix} //$$

iii) $B^T C = \begin{pmatrix} 4 & -1 \\ 0 & -2 \\ -3 & 3 \end{pmatrix}_{B^T} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}_{C}$

Not Possible Because:

= Matrix A Number of Columns is not equal to Matrix B Number of Rows

2c Solve the following System of Equations by row reducing the Augmented Matrix.

$$2x + y - 2z = 8$$
$$3x + 2y - 4z = 15$$
$$5x + 4y - z = 1$$

SOLUTION

Augmented Matrix

$$\text{Augmented Matrix} = \left|\begin{array}{ccc|c} 2 & 1 & -2 & 8 \\ 3 & 2 & -4 & 15 \\ 5 & 4 & -1 & 1 \end{array}\right|$$

$R_1 = \frac{1}{2} R_1$ to reduce $a_{11} = 1$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 3 & 2 & -4 & 15 \\ 5 & 4 & -1 & 1 \end{array}\right|$$

$R_2 = R_2 - 3R_1$ to reduce $a_{21} = 0$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 0 & \frac{1}{2} & -1 & 3 \\ 5 & 4 & -1 & 1 \end{array}\right|$$

$R_3 = R_3 - 5R_1$ to reduce $a_{31} = 0$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 0 & \frac{1}{2} & -1 & 3 \\ 0 & \frac{3}{2} & 4 & -19 \end{array}\right|$$

$R_2 = \frac{2}{1} R_2$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 0 & 1 & -\frac{2}{1} & 6 \\ 0 & \frac{3}{2} & 4 & -19 \end{array}\right|$$

$R_3 = R_3 - \frac{3}{2} R_2$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 0 & 1 & -2 & 6 \\ 0 & 0 & 7 & -28 \end{array}\right|$$

$R_3 = \frac{1}{7} R_3$

$$\left|\begin{array}{ccc|c} 1 & \frac{1}{2} & -1 & 4 \\ 0 & 1 & -2 & 6 \\ 0 & 0 & 1 & -4 \end{array}\right| \quad z = -4$$

Find y by substituting $z = -4$ into Row 2

$$0x + 1y - 2(-4) = 6$$
$$y + 8 = 6$$
$$y = 6 - 8$$
$$y = -2$$

Find x by substituting z & y into Row 1

$$1x + \frac{1}{2}(-2) - 1(-4) = 4$$
$$x + 3 = 4$$
$$x = 4 - 3$$
$$x = 1$$

$y = -2$

$z = -4$