$$C = \begin{bmatrix} 2 & 4 & 3 \\ 2 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

$|C| = 1(2) - 1(4) + 0(3)$

$= 2 - 4 + 0 = -2$

$= -2$

$a_{11} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = (2 \times 1) - (0 \times 2)$

$2 - 0 = 2$

$C^* = \begin{bmatrix} 2 & -4 & 3 \\ -2 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

$a_{12} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = (2 \times 2) - (1 \times 0)$

$4 - 0 = 4$

$a_{13} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = (2 \times 2) - (1 \times 1)$

$4 - 1 = 3$

$C^T = \begin{bmatrix} 2 & -2 & 0 \\ -4 & 2 & 0 \\ 3 & -1 & -1 \end{bmatrix}$

$a_{21} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = (2 \times 1) - (0 - 2)$

$2 - 0 = 2$

$a_{22} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = (2 \times 1) - (0 \times 1)$

$2 - 0 = 2$

$\Sigma^{-1} = \begin{bmatrix} \frac{2}{-2} & \frac{-2}{-2} & \frac{0}{-2} \\ \frac{-4}{-2} & \frac{2}{-2} & 0 \\ \frac{3}{-2} & \frac{-1}{-2} & \frac{-1}{-2} \end{bmatrix}$

$a_{23} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = (2 \times 1) - (1 \times 1)$

$2 - 1 = 1$

$a_{31} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = (1 \times 0) - (1 \times 0)$

$0 - 0 = 0$

$= \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & 0 \\ 3/-2 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$a_{32} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} = (1 \times 0) - (0 \times 2)$

$0 - 0 = 0$

$a_{33} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = (1 \times 1) - (2 \times 1)$

$1 - 2 = -1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & 0 \\ -3/2 & 1/2 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1\times-1 + 1\times2 + 0\times\frac{3}{2} \\ 2\times1 + 1\times-1 + 0\times\frac{1}{2} \\ 1\times0 + 2\times0 + 2\times\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(7) $2x - y + 3z = 5$

$2x + 2y + 3z = 7$

$-2x + 3y + 0z = -3$

$$\begin{bmatrix} 2 & -1 & 3 \\ 2 & 2 & 3 \\ -2 & 3 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 5 \\ 7 \\ -3 \end{pmatrix}$$

$a_{11} = \begin{vmatrix} 2 & 3 \\ 3 & 0 \end{vmatrix} = (2\times0) - (3\times3) = 0 - 9 = -9$

$a_{33} \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} = (2\times2) - (-1\times2) = 4+2 = 6$ ✓

$a_{12} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = (2\times0) - (3\times-2) = 0+6 = 6$ ✓

$|A| = 2(-9) - (-1(6)) + 3(10)$

$a_{13} = \begin{vmatrix} 2 & 2 \\ -2 & 3 \end{vmatrix} = (2\times3) - (2\times-2) = 6+4 = 10$

$= -18 - (-6) + 30$

$= -18 + 6 + 30$

$a_{21} \begin{vmatrix} -1 & 3 \\ 3 & 0 \end{vmatrix} = (-1\times0) - (3\times3) = 0-9 = -9$

$= 24 \quad 18$

$a_{22} \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = (2\times0) - (3\times-2) = 0+6 = 6$ ✓

$C = \begin{bmatrix} -9 & 6 & 10 \\ -9 & 6 & 4 \\ -9 & 0 & 6 \end{bmatrix}$

$a_{23} \begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} = (2\times3) - (-1\times-2) = 6-2 = 4$ ✓

$C = \begin{bmatrix} -9 & -6 & 10 \\ 9 & 6 & -4 \\ -9 & 0 & 6 \end{bmatrix}$

$a_{31} \begin{vmatrix} -1 & 3 \\ 2 & 3 \end{vmatrix} = (-1\times3) - (3\times2) = -3-6 = -9$ ✓

$a_{32} \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = (2\times3) - (3\times2) = 6-6 = 0$ ✓

$$C^T = \begin{bmatrix} -9 & 9 & -9 \\ -6 & 6 & 0 \\ 10 & -4 & 6 \end{bmatrix} = A^{-1} \quad \begin{pmatrix} 5 \\ 7 \\ -3 \end{pmatrix}$$

$$\therefore = \begin{bmatrix} (-9 \times 5) + (9 \times 7) + (-9 \times -3) \\ (-6 \times 5) + (6 \times 7) + (0 \times -3) \\ (10 \times 5) + (-4 \times 7) + (6 \times -3) \end{bmatrix}$$

$$\begin{bmatrix} -45 + 63 + 27 \\ -30 + 42 + 0 \\ 50 - 28 - 18 \end{bmatrix} \quad 18/24 = 3/4 \quad x$$
$$12/24 = 1/2 \quad y$$
$$4/24 = 1/6 \quad z$$

$$2 \times 3/4 - 1/2 + 3 \times 1/6 =$$
$$3/2 - 1/2 + 1/2 =$$

$$45/18$$
$$3/18 = 1/6 \quad x \checkmark$$
$$12/18 = 2/3 \quad y \checkmark$$
$$4/18 = 2/9 \quad z \checkmark$$

$$1/3 - 2/3 + 7/3 = 5$$

$$2 \times 5/2 + 2^2/3 + 3 \cdot 2/9 =$$
$$5 + 1/3 + 2/3 = 7$$
$$-2 \times 5/2 + 3 \cdot 2/3 + 0 =$$
$$-5 + 2 = -3$$

$$\begin{bmatrix} 5x & + 2y & + 0 = 2 \\ 2x & + y & - z = 0 \\ 2x & + 3y & - z = 3 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 0 & 4 \\ -2 & -5 & 11 \\ -2 & -5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \qquad C = \begin{bmatrix} 2 & 0 & 4 \\ 2 & -5 & -11 \\ -2 & 5 & 1 \end{bmatrix}$$

$a_{11} \begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix} = \dfrac{(1 \times -1) - (3 \times -1)}{-1 + 3 = 2}$

$C^s = \begin{bmatrix} 2 & 2 & -2 \\ 0 & -5 & 5 \\ 4 & -11 & 1 \end{bmatrix} = Adj$

$a_{12} \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} = \dfrac{(2 \times -1) - (-1 \times 2)}{= -2 + 2 = 0}$

$a_{13} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = \dfrac{(2 \times 3) - (2 \times 1)}{6 - 2 = 4}$

$$\begin{bmatrix} (2 \times 2) + (2 \times 0) + (-2 \times 3) \\ (0 \times 2) + (-5 \times 0) + (5 \times 3) \\ (4 \times 2) + (-11 \times 0) + (1 \times 3) \end{bmatrix}$$

$a_{21} \begin{vmatrix} 2 & 0 \\ 3 & -1 \end{vmatrix} = \dfrac{(2 \times -1) - (0 \times 3)}{-2 - 0 = -2}$

$4 + (-6) = -2/10 = -1/5 \checkmark x$

$a_{22} \begin{vmatrix} 5 & 0 \\ 2 & -1 \end{vmatrix} = \dfrac{(5 \times -1) - (0 \times 2)}{-5 - 0 = -5}$

$15 = 15/10 = 3/2 \checkmark y$

$8 + 3 = 11/10 = 11/10 \checkmark z$

$a_{23} \begin{vmatrix} 5 & 2 \\ 2 & 3 \end{vmatrix} = \dfrac{(5 \times 3) - (2 \times 2)}{15 - 4 = 11}$

$5 \times -1/5 + 2 \times 3/2 + 0 = 2$

$a_{31} \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = \dfrac{(2 \times -1) - (0 \times 1)}{-2 - 0 = -2}$

$2 \times -1/5 + 3/2 - 1/10 = 0$

$-2/5 + 3/2 - 11/10 = 0$

$a_{32} \begin{vmatrix} 5 & 0 \\ 2 & -1 \end{vmatrix} = \dfrac{(5 \times -1) - (0 \times 2)}{-5 - 0 = -5}$

$-2/5 + 9/2 - 11/10 = 3$

$a_{33} \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = \dfrac{(5 \times 1) - (2 \times 2)}{5 - 4 = 1}$

$|A| = 5(2) - 2(0) + 0(-4)$

$= 10$

$$2x + y - 2z = 8$$
$$3x + 2y - 4z = 15$$
$$5x + 4y - 3z = 1$$

$$|A| = 2(10) - 1(11) + (-2(0))$$
$$= 20 - 11 - 4$$
$$= 5$$

$$\begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & -4 \\ 5 & 4 & -3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 8 \\ 15 \\ 1 \end{pmatrix}$$

$$C = \begin{bmatrix} 10 & 11 & 2 \\ 5 & 4 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

Soln

$$a_1 \begin{vmatrix} 2 & -4 \\ 4 & -3 \end{vmatrix} = \frac{(2 \times -3) - (4 \times -4)}{-6 + 16} = 10$$

$$C = \begin{bmatrix} 10 & -11 & 2 \\ -5 & 4 & -3 \\ 0 & 2 & 1 \end{bmatrix}$$

$$a_{12} \begin{vmatrix} 3 & -4 \\ 5 & -3 \end{vmatrix} = \frac{(3 \times -3) - (-4 \times 5)}{-9 + 20} = 11$$

$$a_{13} \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = \frac{(3 \times 4) - (5 \times 2)}{12 - 10} = 2$$

$$C^{T} = \begin{bmatrix} 10 & -5 & 0 \\ -11 & 4 & 2 \\ 2 & -3 & 1 \end{bmatrix} = Adj$$

$$a_{21} \begin{vmatrix} 1 & -2 \\ 4 & -3 \end{vmatrix} = \frac{(1 \times -3) - (-2 \times 4)}{-3 + 8} = 5$$

$$a_{22} \begin{vmatrix} 2 & -2 \\ 5 & -3 \end{vmatrix} = \frac{(2 \times -3) - (-2 \times 5)}{-6 + 10} = 4$$

$$(10 \times 8) + (-5 \times 15) + (0 \times 1)$$
$$(-11 \times 8) + (4 \times 15) + (2 \times 1)$$
$$(2 \times 8) + (-3 \times 15) + (1 \times 1)$$

$$a_{23} \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} = \frac{(2 \times 4) - (5 \times 1)}{8 - 5} = 3$$

$$a_{31} \begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix} = \frac{(-4 \times 1) - (-2 \times 2)}{-4 + 4} = 0$$

$$80 + (-75) + 0 = 5/5$$
$$-88 + 60 + 2 = -26/5$$

$$a_{32} \begin{vmatrix} 2 & -2 \\ 3 & -4 \end{vmatrix} = \frac{(2 \times -4) - (-2 \times 3)}{-8 + 6} = -2$$

$$16 - 45 + 1 = -28/5$$

$$a_{33} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = \frac{(2 \times 2) - (1 \times 3)}{4 - 3} = 1$$

$$x = 1$$
$$y = -26/5$$
$$z = -28/5$$

$2 \times 1 + \frac{-26}{5} + 2 \times \frac{28}{5}$

$2 - \frac{26}{5} + \frac{56}{5} = 8$

$3 - \frac{58}{5} + \frac{112}{5} = 15$

$5 - \frac{107}{5} + \frac{84}{5} = 1$