CMP 418 (Slide 3 - Page 5)

Question

1 Find the Complexity class of the below Algorithm

Algorithm Selection Sort (A[0, ... n-1])

```
1  for i ← 0 to n-2 do
       min ← i
2      for j ← i+1 to n-1 do
           if A[j] < A[min]
               min ← j
       Swap A[j] and A[min]
```

NOTE

← means = (equal to)

for ... means for Loop

Σ means "Summation"

Solution

$T_{(n)} = ?$

Crıet summation of for Loop 1 (for i ← 0 to n-2 do)

$$\sum_{L}^{U}$$

U means upper

L means Lower

U = n - 2

L = i ← 0

$$= \sum_{L}^{U} = \sum_{i \leftarrow 0}^{n-2}$$

Note: ← means =
e.g $j \leftarrow 5$ same as $j = 5$

Get summation of for loop ②

for $j \leftarrow i + 1$ to $n - 1$ do

$$\sum_{L}^{U}$$

U means upper
L means lower

$U = n - 1$
$L = j \leftarrow i + 1$

$$= \sum_{L}^{U} = \sum_{j \leftarrow i+1}^{n-1}$$

Formula for finding Complexity class becomes

$$T_{(n)} = \text{for loop ① } \Sigma \times \text{for loop ② } \Sigma$$

$$T_{(n)} = \sum_{i \leftarrow 0}^{n-2} \times \sum_{j \leftarrow i+1}^{n-1}$$

Next Page: The main Calculation to find the Complexity class begins

No fear, e no hard

Finding the Complexity class

$$T(n) = \sum_{i \leftarrow 0}^{n-2} \cdot \sum_{j \leftarrow i+1}^{n-1}$$

multiplication

or $\left(\sum_{i \leftarrow 0}^{n-2} \times \sum_{j \leftarrow i+1}^{n-1} \right)$

Solve for loop ② $\sum$ first using [U - L + 1] formula

$$= \sum_{i \leftarrow 0}^{n-2} \cdot [U - L + 1]$$

$$= \sum_{i \leftarrow 0}^{n-2} \cdot [(n-1) - (i+1) + 1]$$

$$= \cdot [n - 1 - i - 1 + 1]$$

$$= \cdot [n - 1 - i]$$

$$= \sum_{i \leftarrow 0}^{n-2} \cdot (n - 1 - i)$$

Open bracket ($\sum$ multiplies $(n - 1 - i)$)

$$= \sum_{i \leftarrow 0}^{n-2} n - \sum_{i \leftarrow 0}^{n-2} 1 - \sum_{i \leftarrow 0}^{n-2} i$$

Whenever you get $\sum$ (summation) with 2 (i's) in the position $\sum_{i \ldots}^{\ldots} i$, you must replace it with $= \frac{1}{2} n^2$

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(Rosa Nwaelu)
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$$= \sum_{i \leftarrow 0}^{n-2} n - \sum_{i \leftarrow 0}^{n-2} 1 - \frac{1}{2}n^2$$

Solve for the remaining $\sum$ (summation) using the formula
[U - L + 1]

$$= (U - L + 1) - (U - L + 1) - \frac{1}{2}n^2$$

$$= (n - 2 - 0 + 1) - (n - 2 - 0 + 1) - \frac{1}{2}n^2$$

$$= n - 2 - 0 + 1 \quad - n + 2 + 0 - 1 \quad - \frac{1}{2}n^2$$

cancel out

$$= n - 2 - 0 \quad - n + 2 + 0 \quad - \frac{1}{2}n^2$$

remove zeros

$$= n - 2 \quad - n + 2 \quad - \frac{1}{2}n^2$$

cancel out

$$= n \quad - n \quad - \frac{1}{2}n^2$$

cancel out

$$\therefore T(n) = -\frac{1}{2}n^2$$ # Complexity class found ☑

Lastly, remove the Coefficient $(-\frac{1}{2})$ since "Complexity Class" solutions don't permit it

$T = n^2$ // # Final Answer ☑

∴ We can now say the Complexity Class of our above algorithm is

$\Theta(n^2)$ # Theta of n^2

<u>TIP</u>: Try solving it by yourself 3 to 5 times and you should get how it works