Physics Department, Bingham University

PHY 101 Lectures by Mrs. O. V. Oyelade

Part B: Introductory Classical Mechanics

Course content

- Work and Energy
- Rotational Motion
- Bodies in Equilibrium
- Newton’s Universal Gravitation
- Simple Harmonic Motion

Bodies in Equilibrium

- **Learning Objectives**
- To understand the conditions for static equilibrium
- To introduce the concept of the free-body diagram for rigid body
- To show how to solve rigid-body equilibrium problems using the equations of equilibrium
- To solve problems on center of gravity

Equilibrium

- Equilibrium implies the object is at rest (static) or its center of mass moves with a constant velocity (dynamic)
- **static equilibrium:** linear and angular velocities are equal to zero, and $v_{cm} = 0$ and $w = 0$
- Examples
  - *Book on table*
  - *Hanging mass*
  - *Ceiling fan – off/on*
  - *Ladder leaning against wall*

Conditions for Equilibrium

- The first condition of equilibrium is a statement of translational equilibrium:

  The net external force on the object must equal zero

$$\sum_k \vec{\mathbf{F}}_k = \vec{\mathbf{0}}$$

- This vector equation is equivalent to the following three scalar equations for the components of the net force:

$$\sum_k F_{kx} = 0, \quad \sum_k F_{ky} = 0, \quad \sum_k F_{kz} = 0.$$

- It states that the translational acceleration of the object's center of mass must be zero

Conditions for Equilibrium

- The second condition of equilibrium is a statement of rotational equilibrium
- The net external torque on the object must equal zero

$$\longrightarrow \quad \sum_k \vec{\tau}_k = \vec{0}$$

- This vector equation is equivalent to the following three scalar equations for the components of the net force:

$$\sum_k \tau_{kx} = 0, \quad \sum_k \tau_{ky} = 0, \quad \sum_k \tau_{kz} = 0.$$

- It states the angular acceleration of the object to be zero. This must be true for any axis of rotation

Conditions for Equilibrium

- If the object is modeled as a particle, then this is the only condition that must be satisfied

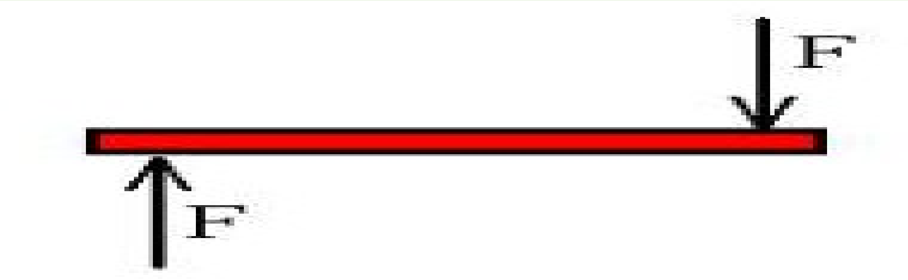
  The net force equals zero

- if the object cannot be modeled as a particle, this is needed

  The net torque equals zero

Static Equilibrium

- Consider a light rod subject to the two forces of equal magnitude as shown in figure. Choose the correct statement with regard to this situation:



(A) The object is in force equilibrium but not torque equilibrium.

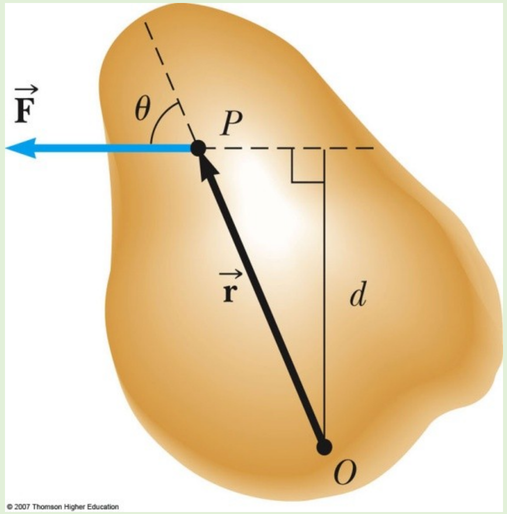
(B) The object is in torque equilibrium but not force equilibrium

(C) The object is in both force equilibrium and torque equilibrium

(D) The object is in neither force equilibrium nor torque equilibrium

Equilibrium Equations

- For simplicity, We will restrict the applications to situations in which all the forces lie in the xy plane.
- Equation 1:
- Equation 2:
- There are three resulting equations



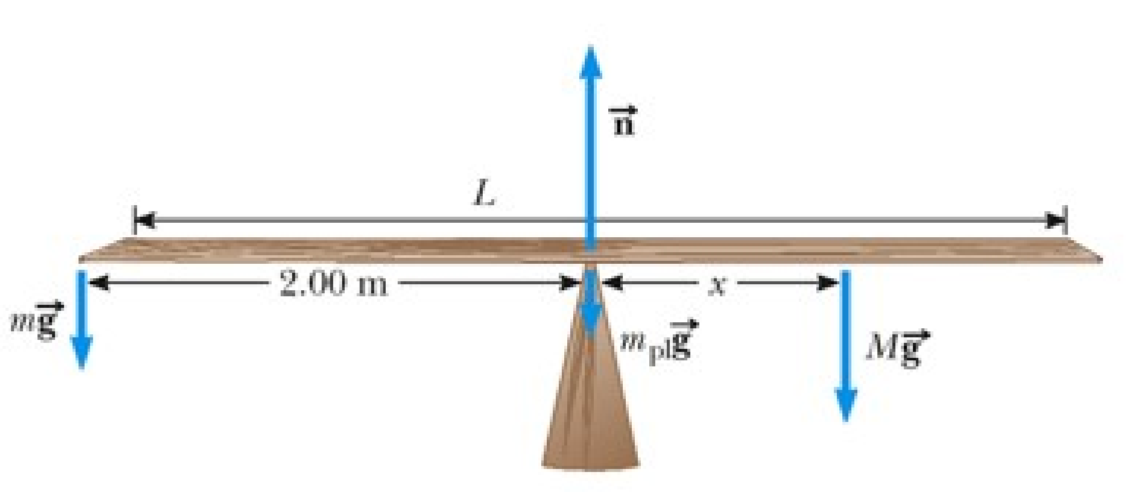
Free-body Diagram (FBD)

- FBD is the best method to represent all the known and unknown forces in a system
- FBD is a sketch of the outlined shape of the body, which represents it being isolated from its surroundings
- Necessary to show all the forces and couple moments that the surroundings exert on the body so that these effects can be accounted for when equations of equilibrium are applied

Example

- A uniform board of mass m_{pl} and length L supports two masses M and m. The support is under the center of gravity of the board, mass M is a distance d from the center, and the masse m is a distance 2.00 m from the center.
- A) Find the magnitude of the upward force **n** exerted by the support on the board.
- B) Find the position of M that will balance the system at rest.

Solution to Example



$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$

$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$= mgd - Mgx + 0 + 0 = 0$$

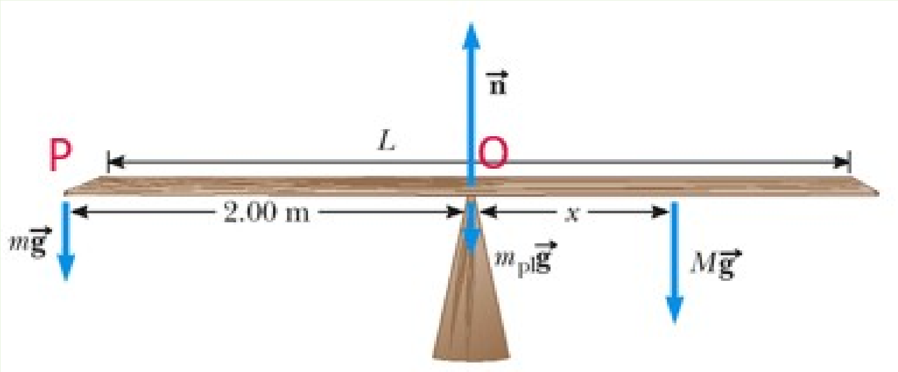
$$mgd = Mgx$$

$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\,\mathrm{m}$$

Axis of Rotation

- The net torque is about any axis in the xy plane
- The choice of an axis is arbitrary
- If an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis
- Choose a rotation axis to simplify problems

The position of M that will balance the system at rest



$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= mgd - Mgx + 0 + 0 = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= 0 - Mg(d + x) - m_{pl}gd + nd = 0$$
$$-Mgd - Mgx - m_{pl}gd + (Mg + mg + m_{pl}g)d = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$

Center of Gravity

- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- If g is uniform over the object, then the center of gravity of the object coincides with its center of mass
- If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Where is the Center of Mass?

Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

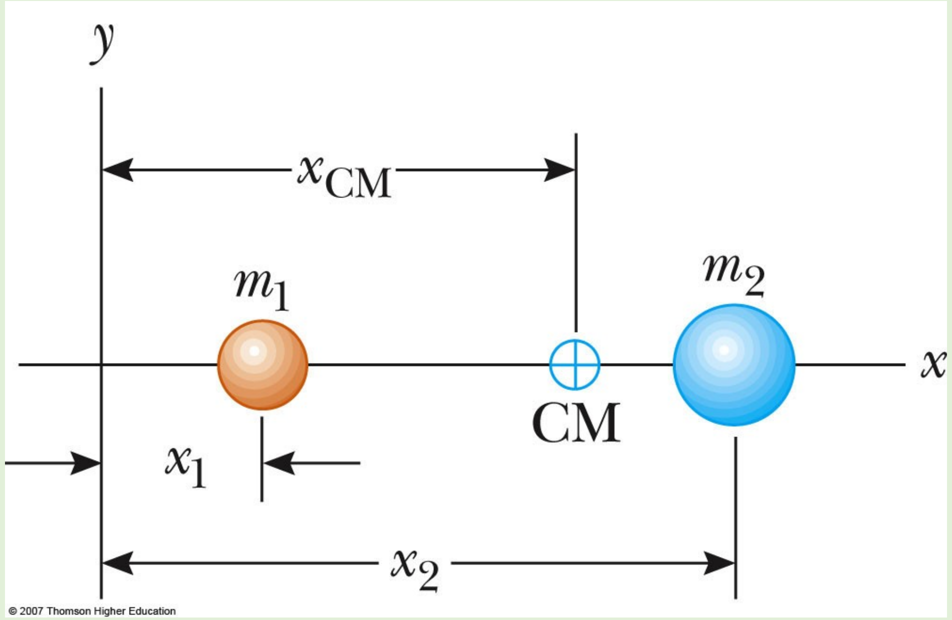
A) x_{CM} = 1 m

B) x_{CM} = 2 m

C) x_{CM} = 3 m

D) x_{CM} = 4 m

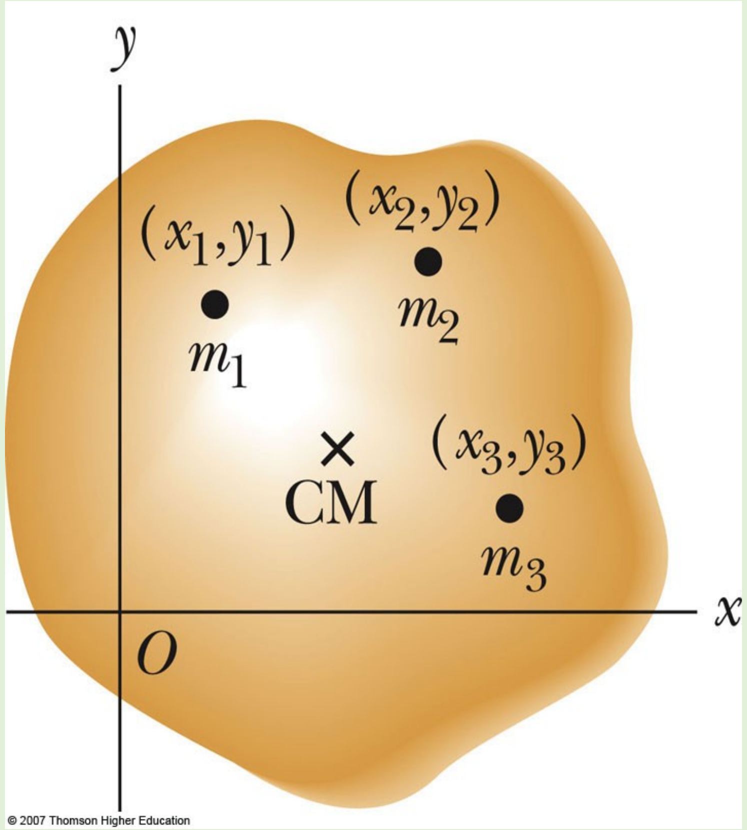
E) x_{CM} = 5 m



Center of Mass (CM)

- An object can be divided into many small particles
  - Each particle will have a specific mass and specific coordinates
- The x coordinate of the center of mass will be

- Similar expressions can be found for the y coordinates



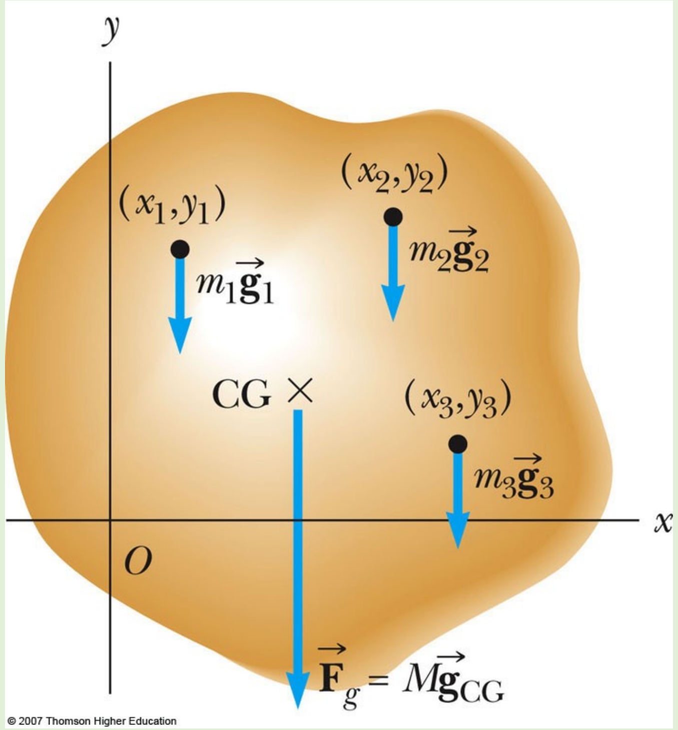
Center of Gravity (CG)

- All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG)

$$Mg_{CG}x_{CG} = (m_1 + m_2 + m_3 + \cdots)g_{CG}x_{CG}$$

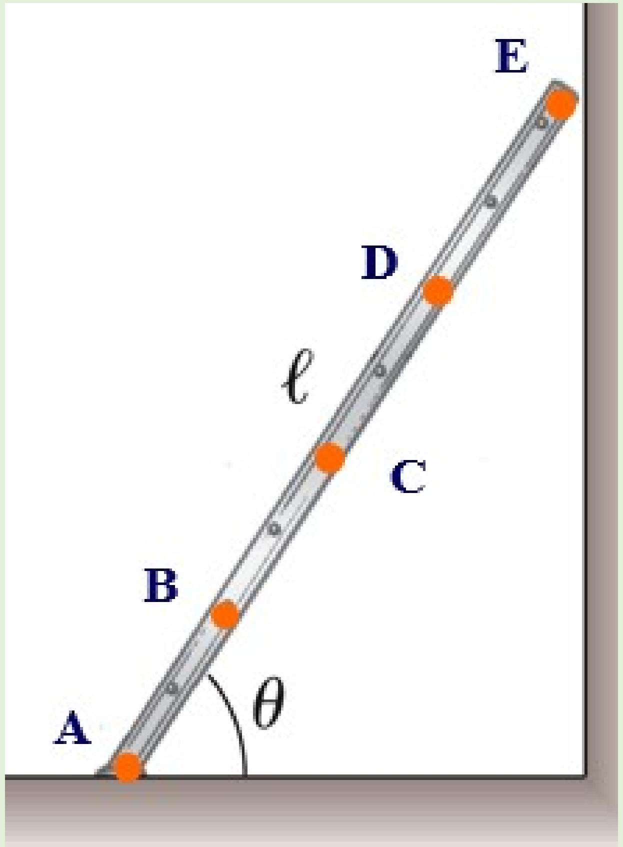
$$= m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots$$

- If
- then



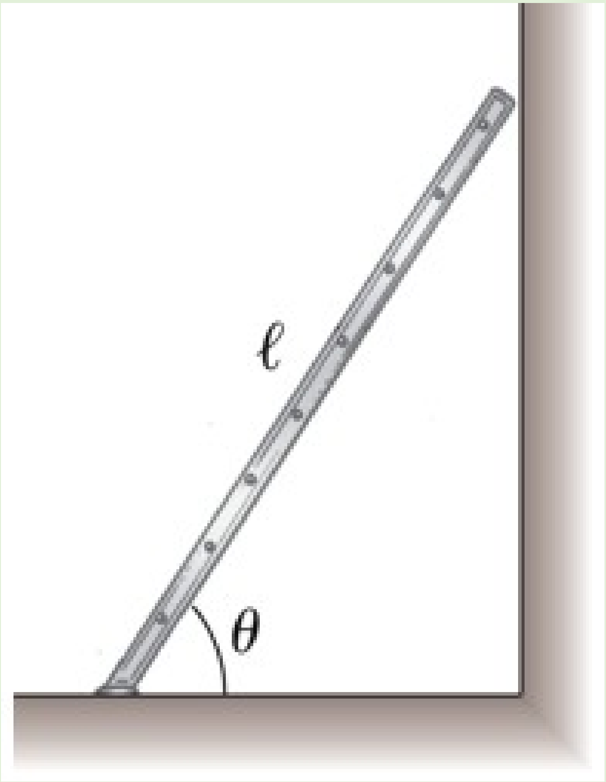
CG of a Ladder

- A uniform ladder of length l rests against a smooth, vertical wall. To calculate the torque due to the gravitational force, you have to find center of gravity of the ladder. The center of gravity should be located at which of these points?



Example

- A uniform ladder of length l rests against a smooth vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.

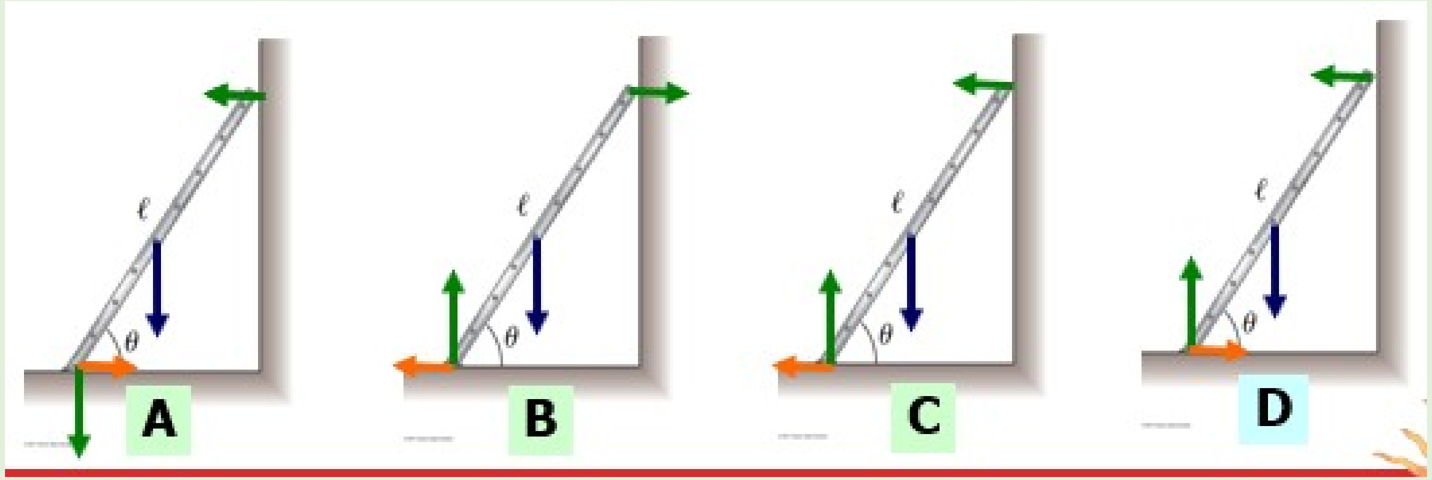


Problem-Solving Strategy 1

- Draw a sketch, decide what is in or out the system
- Draw a free body diagram (FBD)
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Note: use correct force directions (i.e. + or -)

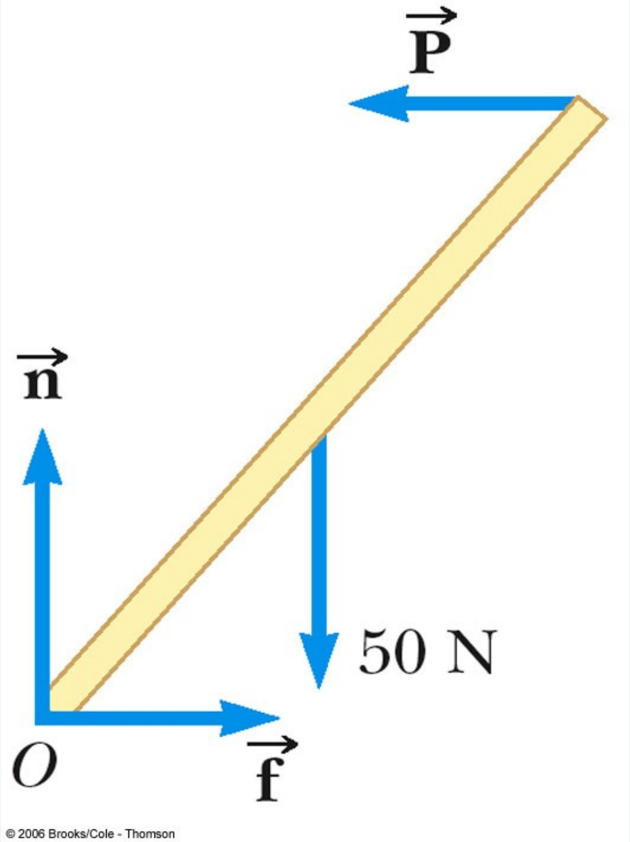
Which free-body diagram is correct?

- A uniform ladder of length l rests against a smooth vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$.
- Forces - gravity: blue, friction: orange, normal: green



Example

- A uniform ladder of length *l* rests against a smooth vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.

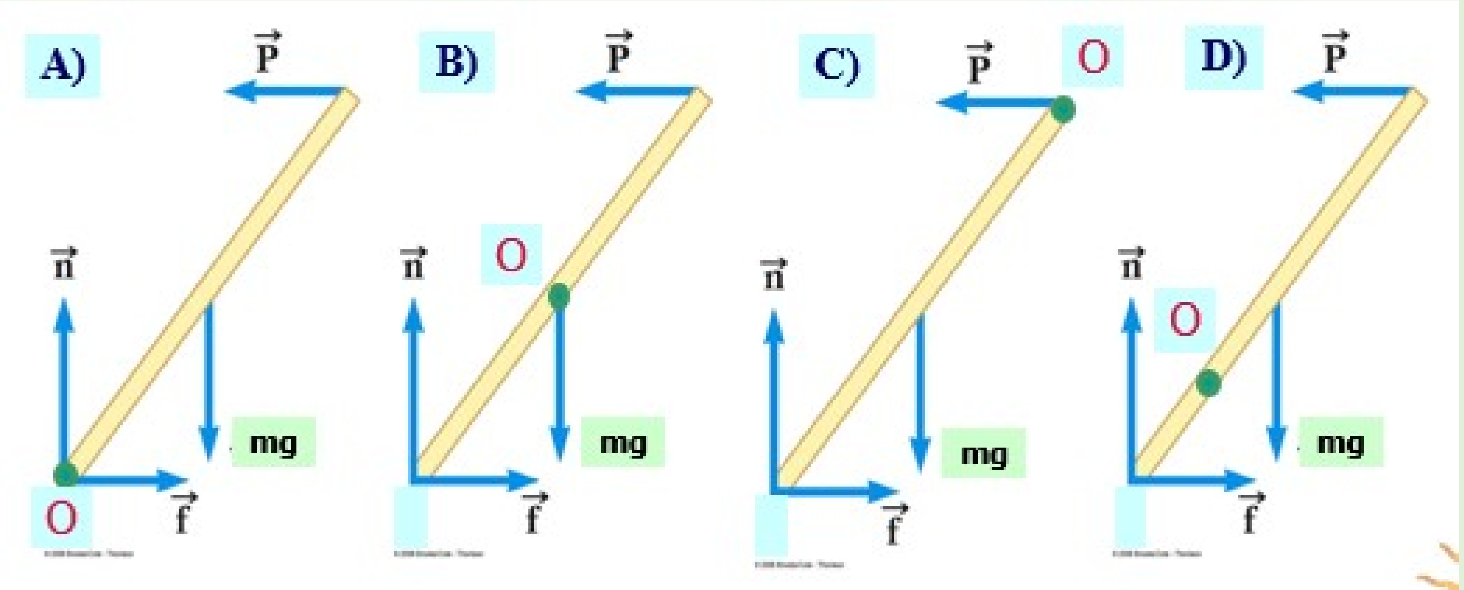


Problem-Solving Strategy 2

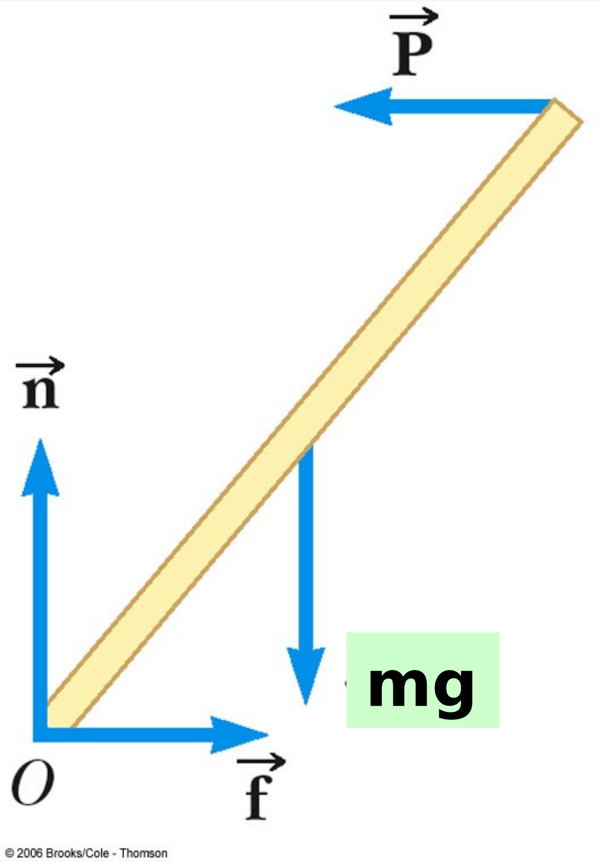
- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
  - positive if force tends to rotate object in counter clock wise direction (CCW)
  - negative if force tends to rotate object in clock wise direction (CW)
  - zero if force is on the rotational axis
- Apply the second condition for equilibrium

Choose an origin O that simplifies the calculations as much as possible ?

- A uniform ladder of length *l* rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle.



A uniform ladder of length *l* rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is μ_s = 0.40. Find the minimum angle θ at which the ladder does not slip.



Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

Exercises

- Ex. 12 nos.: 25, 31, 32 and 33

References

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