

CMP 418 (Slide 3 - Page 5)

Question

1 Find the Complexity class of the below Algorithm
Algorithm Selection Sort ($A[0, \dots, n-1]$)

① for $i \leftarrow 0$ to $n-2$ do
 $\text{min} \leftarrow i$
② for $j \leftarrow i+1$ to $n-1$ do
 if $A[j] < A[\text{min}]$
 $\text{min} \leftarrow j$
 swap $A[j]$ and $A[\text{min}]$

NOTE

\leftarrow means = (equal to)

for ... means for Loop

Σ means "Summation"

SOLUTION

$T(n) = ?$

Create summation of for loop ① (for $i \leftarrow 0$ to $n-2$ do)

$$\sum_L^U$$

U means upper

L means lower



$$U = n-2$$

$$L = i \leftarrow 0$$

$$= \sum_L^U = \sum_{i \leftarrow 0}^{n-2}$$

Note! \leftarrow means =
e.g $j \leftarrow 5$ same as $j = 5$

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Deaconess
ROSELINE EDEH
(Rosa Nwaelu)

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Get summation of for loop ②
for $j \leftarrow i+1$ to $n-1$ do

$$\sum$$

U means upper
L means lower

$$U = n - 1$$

$$L = j \leftarrow i + 1$$

$$= \sum_{j \leftarrow i+1}^{n-1} = \sum_{j \leftarrow i+1}^{n-1}$$

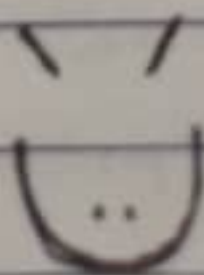
Formula for finding Complexity class becomes

$$T(n) = \text{for loop ① } \sum \times \text{for loop ② } \sum$$

$$T(n) = \sum_{i \leftarrow 0}^{n-1} \times \sum_{j \leftarrow i+1}^{n-1}$$

Next Page: The math Calculation to find the
Complexity class begins

No fear, e no hard



Finding the Complexity class

$$T(n) = \sum_{i=0}^{n-2} \cdot \sum_{j=i+1}^{n-1} \quad \# \text{ or } \left(\sum_{i=0}^{n-2} \times \sum_{j=i+1}^{n-1} \right)$$

multiplication

Solve for loop ② \sum ~~summation~~ ^{first} using $[U - L + 1]$ formula

$$= \sum_{i=0}^{n-2} \cdot [U - L + 1]$$

$$= \sum_{i=0}^{n-2} \cdot [(n-1) - (\cancel{i} + 1) + 1]$$

$$= \sum_{i=0}^{n-2} \cdot [n - 1 - \underbrace{i}_{\text{cancel}} - \underbrace{1}_{\text{cancel}} + 1]$$

$$= \sum_{i=0}^{n-2} \cdot [n - 1 - i]$$

$$= \sum_{i=0}^{n-2} \cdot (n - 1 - i)$$

Open bracket (\sum multiplies $(n - 1 - i)$)

$$= \sum_{i=0}^{n-2} n - \sum_{i=0}^{n-2} 1 - \sum_{i=0}^{n-2} i$$

Whenever you get \sum (summation) with 2 (i's) in the position $\sum_{i=0}^{\dots} (i)$, you must replace it with $= \frac{1}{2} n^2$

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$$= \sum_{i=0}^{n-2} n - \sum_{i=0}^{n-2} 1 = \frac{1}{2} n^2$$

Solve for the remaining \sum (summation) using the formula
[U - L + 1]

$$= (U - L + 1) - (U - L + 1) - \frac{1}{2} n^2$$

$$= (n - 2 - 0 + 1) - (n - 2 - 0 + 1) - \frac{1}{2} n^2$$

$$= n - 2 - 0 + 1 \quad - n + 2 + 0 - 1 \quad - \frac{1}{2} n^2$$

cancel out

$$= n - 2 - 0 \quad - n + 2 + 0 \quad - \frac{1}{2} n^2$$

remove zeros

$$= n - 2 \quad - n + 2 \quad - \frac{1}{2} n^2$$

cancel out

$$= n \quad - n \quad - \frac{1}{2} n^2$$

cancel out

$$T(n) = -\frac{1}{2} n^2 \quad \# \text{ Complexity class found}$$

Lastly, remove the coefficient $(-\frac{1}{2})$ since "Complexity Class" solutions don't permit it

$$T = n^2 //$$

Final Answer $\boxed{\checkmark}$

\therefore We can now say the Complexity Class of our above algorithm is

$\Theta(n^2)$ # Theta of n^2

TIP: Try solving it by yourself 3 to 5 times and you should get how it works