A formula summary for physics

Classical mechanics

Kinetics

velocity: $\mathbf{v} = \frac{d\mathbf{r}}{dt} = r'_x\mathbf{i} + r'_y\mathbf{j} + r'_z\mathbf{k}$
acceleration: $\mathbf{a} = \frac{d\mathbf{v}}{dt} = v'_x\mathbf{i} + v'_y\mathbf{j} + v'_z\mathbf{k}$
motion laws:
$v = v_0 + at$
$x - x_0 = v_0 t + \frac{1}{2}at^2$
$v^2 - v_0^2 = 2a(x - x_0)$
$x - x_0 = \frac{(v_0+v)t}{2}$
centripetal acceleration: $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2 = \omega^2 r$
velocity in two frames: $\mathbf{v}_{\mathrm{PA}} = \mathbf{v}_{\mathrm{PB}} + \mathbf{v}_{\mathrm{BA}}$
same acceleration measured in all frames: $\mathbf{a}_{\mathrm{PA}} = \mathbf{a}_{\mathrm{PB}}$

Kinetics 2

Newton's second law: $\mathbf{F}_{\mathrm{T}} = m\mathbf{a}$
Newton's third law: $\mathbf{F}_{\mathrm{A\ to\ B}} = -\mathbf{F}_{\mathrm{B\ to\ A}}$

acceleration of simple pulley: $a = \frac{m_1 - m_2}{m_1 + m_2} g$

drag force (body in fluid): $D = \frac{1}{2}C\rho A v^2$
terminal speed: $v = \sqrt{\frac{2mg}{C\rho A}}$ (1)
work: $W = \int \mathbf{F}\cdot d\mathbf{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$
Hooke's law: $\mathbf{F} = -k\mathbf{x}$
work by spring: $W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$
kinetic energy: $K = \frac{1}{2}mv^2$
work-kinetic energy theorem: $W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
power: $P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$

Kinetics 3

definition of change in potential energy: $\Delta U = -W$
change in mechanical energy: $\Delta K + \Delta U = 0$
total mechanical energy: $E = U + K$ (isolated)

U-x graph:
external force: $F(x) = -\frac{d}{dx}U(x)$
neutral equilibrium: $E = U$
unstable equilibrium: $U'' < 0, U' = 0$
stable equilibrium: $U'' > 0, U' = 0$

total m.energy of block-spring system: $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
total m.energy of particle-earth system: $E = mgy + \frac{1}{2}mv^2$
conservation of energy:
$\Delta K + \Delta U + \Delta E_{\mathrm{internal}}$ (+other forms) $= 0$ (isolated)
external work done: $W = \Delta K + \Delta U + \Delta E_{\mathrm{internal}}$
change in energy: $\Delta E = \Delta K + \Delta U$
loss in mechanical energy (friction): $\Delta E = -fd$
energy loss due to emitted light: $E_x - E_y = hf$

System kinetics

centre of mass: $\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i \mathbf{r}_i$
continuous: $\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\int \mathbf{r}\rho dV = \frac{1}{V}\int \mathbf{r} dV$
relation: $dm = \rho dV$

linear momentum: $\mathbf{p} = m\mathbf{v}$
relation: $K = \frac{p^2}{2m}$
net force: $\mathbf{F}_{\mathrm{T}} = \frac{d\mathbf{p}}{dt}$
Newton's second law: $\sum \mathbf{F}_{\mathrm{external}} = M\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{P}}{dt}$
conservation of linear momentum: $\mathbf{P} = $ constant

циолковский's rocket formula:
$\mathbf{F} = (\mathbf{v} - \mathbf{u})\frac{dm}{dt} + m\frac{d\mathbf{v}}{dt}$ or $\frac{d}{dt}m\mathbf{v} = \mathbf{F} + \mathbf{u}\frac{dm}{dt}$
v: rocket's velocity in earth, u: fuel's velocity in earth

change in translational k. energy: $\Delta K_{\mathrm{CM}} = F_{\mathrm{ext}} d_{\mathrm{CM}}$
König's theorem: $K = K_{\mathrm{related\ to\ CM}} + \frac{1}{2}mv_{\mathrm{CM}}^2$

Collisions

impulse: $\mathbf{J} = \Delta\mathbf{p} = \int \mathbf{F} dt$

elastic collision:
$v'_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$
$v'_2 = \frac{m_2 - m_1}{m_1 + m_2}v_2 + \frac{2m_1}{m_1 + m_2}v_1$
$v_{\mathrm{CM}} = \frac{P}{m_1 + m_2}$

complete inelastic collision: $m_1 v_1 + m_2 v_2 = (m_1 + m_2)V_{\mathrm{CM}}$
heat of reactions: $Q = -\Delta mc^2$ ($+Q$: exothermic, $-Q$: endothermic)

Rotation

angular position: $\theta = s/r$
angular velocity: $\omega = \frac{d\theta}{dt}$
angular acceleration: $\alpha = \frac{d\omega}{dt}$
motion laws:
$\omega = \omega_0 + \alpha t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega^2 - \omega_0^2 = 2\alpha\theta$
$\theta = \frac{(\omega_0 + \omega)t}{2}$
linear-angular relation: $s = \theta r$, $v = \omega r$
tangent acceleration: $a_t = \alpha r$
radial acceleration: $a_r = \frac{v^2}{r} = \omega^2 r$

rotational inertia: $I = \sum_i m_i r_i^2$
r.i. for continuous objects: $I = \int r^2 dm$
total rotational inertia: $I_{\mathrm{whole}} = \sum_i I_i$ (all to one axis)
parallel-axis theorem: $I = I_{\mathrm{CM}} + Mh^2$
perpendicular-axis theorem: $I_P = I_x + I_y$ (no thickness)
rotational kinetic energy: $K = \frac{1}{2}I\omega^2$
torque: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
Newton's second law: $\tau_{\mathrm{T}} = I\alpha$
work: $W = \int F_t r d\theta = \int \tau d\theta$
power: $P = \frac{dW}{dt} = \tau\omega$
work-kinetic energy theorem: $W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

Rotational inertia

hoop, central axis: $I = MR^2$
hoop, diameter: $I = \frac{1}{2}MR^2$
annular cylinder, central axis: $\frac{1}{2}M\left(R_1^2 + R_2^2\right)$
annular cylinder, central diameter, $\frac{1}{4}M\left(R_1^2 + R_2^2\right)$
solid cylinder/disk, central axis: $\frac{1}{2}MR^2$
solid cylinder/disk, central diameter: $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
rod, centre of length: $I = \frac{1}{12}ML^2$
rod, one end: $I = \frac{1}{3}ML^2$
triangle, parallel to base a (whose height is h, through CM): $I = \frac{1}{18}Mh^2$
solid sphere, diameter: $I = \frac{2}{5}MR^2$
spherical shell, diameter: $I = \frac{2}{3}MR^2$
slab, centre: $I = \frac{1}{12}M(a^2 + b^2)$
slab, along edge b: $I = \frac{1}{3}Ma^2$

Rolling

CM displacement/distance rolled: $x_{\mathrm{CM}} = \theta R$, $v_{\mathrm{CM}} = \omega R$
kinetic energy: $K = K_{\mathrm{rot}} + K_{\mathrm{tra}} = \frac{1}{2}I_{\mathrm{CM}}\omega^2 + \frac{1}{2}mv_{\mathrm{CM}}^2$
accleration of ideal yoyo: $a = -g(\frac{1}{1 + I/MR^2})$
angular momentum: $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$
a.m. for rigid, fixed axis: $L = I\omega$
angular impulse: $\Delta\boldsymbol{L} = \int \boldsymbol{\tau} dt$
Newton's second law: $\boldsymbol{\tau}_{\mathrm{T}} = \frac{d\boldsymbol{L}}{dt}$
conservation of angular momentum: $\boldsymbol{L} = $ constant

(1) C: drag coefficient, A: effective cross-sectional area, ρ: air density.

Elasticity

static equilibrium: $\mathbf{P} = 0, \boldsymbol{L} = 0$
requirements of equilibrium: $\sum \mathbf{F}_{\text{ext}} = 0, \sum \boldsymbol{\tau}_{\text{ext}} = 0$
tensile stress: $\frac{F}{A} = E\frac{\Delta L}{L}$, E: Young's modulus
sheering stress: $\frac{F}{A} = G\frac{\Delta x}{L}$, G: sheer modulus
hydraulic compression: $p = B\frac{\Delta V}{V}$, B: Bulk modulus

Oscillation

simple harmonic motion ($F = -m\omega^2 x$):
- $\omega = \frac{2\pi}{T} = 2\pi f$
- $x(t) = x_m \cos(\omega t + \phi)$
- $v(t) = -\omega x_m \sin(\omega t + \phi)$
- $a(t) = -\omega^2\, x(t)$

linear oscillator
- definition: $\frac{d^2x}{dt^2} + \omega^2 x = 0$
- angular frequency: $\omega = \sqrt{\frac{k}{m}}$
- period: $T = 2\pi\sqrt{\frac{m}{k}}$
- potential energy: $U(t) = \frac{1}{2}kx_m^2\cos^2(\omega t + \phi)$
- kinetic energy: $K(t) = \frac{1}{2}kx_m^2\sin^2(\omega t + \phi)$
- total energy: $E = \frac{1}{2}kx_m^2$
- series spring: $\frac{1}{K} = \sum_j \frac{1}{k_j}$, two: $K = \frac{k_1 k_2}{k_1 + k_2}$
- parallel spring: $K = \sum_j k_j$

simple pendulum
- period: $T = 2\pi\sqrt{\frac{L}{g}}$
- restoring force: $F \approx -(\frac{mg}{L})s$

torsion pendulum
- period: $T = 2\pi\sqrt{\frac{I}{\kappa}}$
- restoring torque: $\tau = -\kappa\theta$

physical pendulum
- period: $T = 2\pi\sqrt{\frac{I}{mgh}}$
- restoring torque: $\tau = -(mg\sin\theta)h$

damped simple harmonic motion
- damping force: $F_d = -bv$, b: damping constant
- definition: $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$
  - displacement: $x(t) = x_m e^{-bt/2m}\cos(\omega_d t + \phi)$
- angular frequency: $\omega_d = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
- total energy: $E(t) \approx \frac{1}{2}kx_m^2 e^{-bt/m}$

Gravitation

Newton's law of gravitation: $F = \frac{GMm}{r^2}$
gravitational constant: $G = 6.67 \cdot 10^{-11}\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$
- differential: $dF = \frac{Gm_1}{r^2}dm$

gravitational field: $g = \frac{GM}{r^2}$
gravitational potential energy: $U = \int_\infty^r \frac{GMm}{x^2}dx = -\frac{GMm}{r}$
- escape speed: $v = \sqrt{\frac{2GM}{r}}$

orbits:
- path of planet: $r = \frac{p}{1 + e\cos\theta}$
  - $p = \frac{L^2}{GMm^2}$, $e = \sqrt{1 + \frac{2EL^2}{G^2M^2m^3}}$
- net angular momentum: $L = mr^2\dot{\theta}$
- net mechanical energy:

$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{r} = \frac{1}{2}m\dot{r}^2 + (\frac{L^2}{2mr^2} - \frac{GMm}{r})$
total energy of satellite-earth ellipse system:
$E = -K = -\frac{GMm}{2r}$
- perihelion: $r_1 = a - c$
- aphelion: $r_2 = a + c$
  - $a = \frac{r_1 + r_2}{2} = -\frac{GMm}{2E}$, $c = \frac{r_2 - r_1}{2}$,
  - $b = \sqrt{a^2 - c^2} = \sqrt{r_1 r_2} = \frac{L}{\sqrt{-2mE}}$

total energy of satellite-earth hyperbola system: $E = \frac{GMm}{2r}$
total energy of satellite-earth parabola system: $E = 0$
law of periods: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

Fluids

pressure: $p = \frac{\Delta F}{\Delta A}$ (all direction)
pressure in liquid: $p = p_0 + \rho gh$
Pascal's principle: $\Delta p_{\text{int}} = \Delta p_{\text{ext}}$
Archimede's principle: $F_{\text{buoyancy}} = G_{\text{displaced water}}$
equation of continuity:
- volume flow rate $R = Av =$ constant
- mass flow rate $m = Av\rho =$ constant

Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho gy =$ constant

Transverse waves

transverse displacement: $y(x,t) = y_m\sin(kx - \omega t)$
- angular wave number: $k = \frac{2\pi}{\lambda}$
- waver number: $\kappa = \frac{1}{\lambda}$
- angular frequancy: $\omega = \frac{2\pi}{T}$
- frequency: $f = \frac{1}{T} = \frac{\omega}{2\pi}$
- wave speed: $v = \frac{\omega}{k} = \lambda f$
  - material expression: $v = \sqrt{\frac{\tau(\text{tension})}{\mu(\text{density of media})}}$
- transverse speed: $u = \frac{\partial y}{\partial t}$
- average power: $\overline{P} = \frac{1}{2}\mu v\omega^2 y_m^2$

adding waves
- superposition:

$y = y_1 + y_2 = y_m\sin(kx - \omega t + \phi) + y_m\sin(kx - \omega t)$
- new wave: $y = (2y_m\cos\frac{1}{2}\phi)\sin(kx - \omega t + \frac{1}{2}\phi)$
- new amplitude: $2y_m\cos\frac{1}{2}\phi$
- phase shift: $+\frac{1}{2}\phi$

standing waves
- superposition:

$y = y_1 + y_2 = y_m\sin(kx - \omega t) + y_m\sin(kx + \omega t)$
- new wave: $y = [(2y_m)\sin kx]\cos\omega t$
- new amplitude: $2y_m\sin kx$
- nodes: $x = n\frac{\lambda}{2}, n = 0, 1, 2, ...$
- antinodes: $x = (n + \frac{1}{2})\frac{\lambda}{2}, n = 0, 1, 2, ...$
- resonant frequency: $f_r = \frac{v}{\lambda} = \frac{v}{2l}n, n = 1, 2, 3, ...$

Longitudinal waves

speed of sound: $v = \sqrt{\frac{B}{\rho}}$
- bulk modulus: $B = -\frac{\Delta p}{\Delta V/V}(= \rho v^2)$

longitudinal displacement: $s = s_m\cos(kx - \omega t)$
air pressure: $\Delta p = \Delta p_m\sin(kx - \omega t)$
- relation: $\Delta p_m = (v\rho\omega)s_m$

interference
- phase shift: $\phi = \frac{\Delta d}{\lambda}2\pi$
- fully constructive: $\phi = m2\pi, m = 0, 1, 2, ...$
- fully destructive: $\phi = (m + \frac{1}{2})2\pi, m = 0, 1, 2, ...$

sound intensity: $I = \frac{1}{2}\rho v\omega^2 s_m^2$
sound level: $\beta = (10\text{ dB})\log(\frac{I}{I_0})$
- standard reference intensity: $I_0 = 10^{-12}\mathrm{W/m}^2$

resonant frequency
- pipe, two opens: $f_r = \frac{v}{\lambda} = \frac{v}{2L}n, n = 1, 2, 3$
- pipe, one open: $f_r = \frac{v}{\lambda} = \frac{v}{4L}n, n = 1, 3, 5, ...$

beat frequency: $f_{beat} = f_1 - f_2$

doppler effect: $f' = f\frac{v \pm v_L}{v \mp v_S}$
cone angle at supersonic speed: $\sin\theta = \frac{v}{v_s}$

Heat, Second law of thermodynamics

Heat

coefficient of linear expansion: $\alpha = \frac{\Delta L/L}{\Delta T}$

area expansion: $\beta = 2\alpha$

volume expansion: $\gamma = 3\alpha$

heat capacity: $Q = cm(T_f - T_i) = C(T_f - T_i)$

heat of transformation: $Q = Lm$

volume work: $W = \int_{V_i}^{V_f} p dV$

first law of thermodynamics: $\Delta E_{\text{int}} = E_{\text{int,f}} - E_{\text{int,i}} = Q - W$

rate of heat transfer: $H = \frac{Q}{t} = kA\frac{T_H - T_C}{L}$ (2)

multiple slabs: $H = A\frac{T_H - T_C}{\sum(L/k)}$

Kinetic theory of gases

ideal gas law: $pV = nRT$

gas constant $R = 8.31\text{J/mol}\cdot\text{K}$

volume work of expansion at constant pressure:
$W = \int \frac{nRT}{V} dV = nRT\ln(\frac{V_f}{V_i})$

gas pressure: $p = \frac{nMv_{\text{rms}}^2}{3V}$

translational kinetic energy: $\overline{K} = \frac{3}{2}kT$

Boltzman constant $k = R/N_A$

mean free path: $\lambda = \frac{1}{\sqrt{2}\pi dN/V}$ (3)

Maxwell's speed distribution:
$P(v) = 4\pi(\frac{M}{2\pi RT})^{3/2}v^2e^{-Mv^2/2RT}$

most propable speed: $v_p = \sqrt{\frac{2RT}{M}}$

average speed: $\overline{v} = \sqrt{\frac{8RT}{\pi M}}$

rms speed: $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

internal energy of monoatomic gas: $E_{\text{int}} = (nN_A)\overline{K} = \frac{3}{2}nRT$

monoatom: $3/2$ $(f = 1)$

diatom: $5/2$ $(f = 2)$

5-atom: 3 $(f = 5)$

molar specific heat of monoatomic gas at constant volume:
$C_v = \frac{3}{2}R = 12.5\,\text{J/molK}$

constant volume, change in internal energy:
$\Delta E_{\text{int}} = Q = nC_v(T_f - T_i)$

molar specific heat of monoatomic gas at constant pressure:
$C_p - C_v = R$

heat: $Q = nC_p(T_f - T_i)$

work: $W = nR(T_f - T_i)$

law of adiabatic expansion: $pV^\gamma =$ constant, or
$TV^{\gamma-1} =$ constant

$\gamma = C_p/C_v = 1 + 2/f$

Second law of thermodynamics

thermal effiency of engine: $e = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|}$

max: $e_{\text{Car}} = \frac{T_H - T_C}{T_H}$

coefficient of performance of refrigerator:
$e = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$

max: $e_{\text{Car}} = \frac{T_C}{T_H - T_C}$

first law of thermodynamics in closed system:
$|W| = |Q_H| - |Q_C|$

entropy: $dS = \frac{dQ}{T}$ and $\oint dS \le 0$

reversible process: $S_f - S_i = \int_i^f dS = \int_i^f \frac{dQ}{T}$

free expansion: $S_f - S_i = \frac{1}{T}\int_i^f dQ = nR\ln\frac{V_f}{V_i}$

irreversible heat transfer: $S_f - S_i = cm\ln\frac{T^2}{T^2 - \Delta T^2}$

Electricity and Magnetism

Electrostatic forces

Coulomb's law: $F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

permitivity constant in vacuum: $\epsilon_0 = 8.85\cdot 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$

charge is quantized: $q = ne$

elementary charge $e = 1.6\cdot 10^{-14}\,\text{C}$

Electric field

electric field: $\mathbf{E} = \frac{\mathbf{F}}{q_0}$

differential: $d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^3}\mathbf{r}$ (r from dq to point)

point charge: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

straight rod (perpendicular): $E = \frac{\lambda a}{2\pi\epsilon_0 r}\frac{1}{\sqrt{4r^2+a^2}}$

arc (to centre): $E = \frac{\lambda}{4\pi\epsilon_0 r}(2\sin\frac{\theta}{2})$

ring (perpendicular): $E = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$

round disk (perpendicular): $E = \frac{\sigma}{2\epsilon_0}(1 - \frac{z}{\sqrt{z^2+R^2}})$

electrostatic force in a field: $\mathbf{F} = q\mathbf{E}$ (signed)

Gauss' law

Gauss' law: $\Phi_E = \oiint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\epsilon_0}$

conducting surface: $E = \frac{\sigma}{\epsilon_0}$

nonconducting surface: $E = \frac{\sigma}{2\epsilon_0}$

straight rod: $\frac{\lambda}{2\pi r\epsilon_0}$

two conducting plates (+ greater):
$|E_L| = |E_R| = |E_{(+)} - E_{(-)}|$, $|E_{\text{in}}| = E_{(+)} + E_{(-)} = \frac{\sigma_1+\sigma_2}{\epsilon_0}$

shell: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ (outside), $E = 0$ (inside)

sphere: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ (outside), $E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right)r$ (inside)

cylinder: $E = \frac{R^2\rho}{2\epsilon_0 r}$ (outside), $E = \frac{\rho}{2\epsilon_0}r$ (inside)

Potential

work: $W = \int \mathbf{F}\cdot d\mathbf{s} = q_0\int\mathbf{E}\cdot d\mathbf{s}$

electric potential difference: $\Delta V = -\frac{W_{if}}{q_0} = \frac{\Delta U}{q_0}$

E-V relation: $\mathbf{E} = -\nabla V$, $V = -\int_{i_0}^f \mathbf{E}\cdot d\mathbf{s}$

electric field of parallel plates: $E = \frac{\Delta V}{\Delta d}$

point charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$(signed)

discrete points: $V = \frac{1}{4\pi\epsilon_0}\sum_i\frac{q_i}{r_i}$

continuous charge: $V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$

rod (perpendicular to one end): $V = \frac{\lambda}{4\pi\epsilon_0}\ln(\frac{L+(L^2+d^2)^{1/2}}{d})$

arc (to centre): $V = \frac{\lambda\theta}{4\pi\epsilon_0}$

ring (perpendicular): $V = \frac{q}{4\pi\epsilon_0\sqrt{z^2+R^2}}$

disk (perpendicular): $V = \frac{\sigma}{2\epsilon_0}(\sqrt{z^2+R^2} - z)$

Current and circuits

current: $i = \frac{dq}{dt}$

current density: $J = i/A$

relation: $i = \iint \mathbf{J}\cdot d\mathbf{A}$

draft speed: $\mathbf{v}_\text{d} = \mathbf{J}/(ne)$ (4)

resistance law: $R = \frac{V}{i}$

isotropic resistivity: $\rho = E/J$

relation: $\mathbf{E} = \rho\mathbf{J}$

conductivity: $\sigma = 1/\rho$

resistance: $R = \rho\frac{L}{A}$

variation with temperature: $\rho - \rho_0 = \rho_0\alpha(T - T_0)$ (5)

$T_0 = 293\,\text{K}$, $\rho_0 = 1.69\,\mu\Omega\cdot\text{cm}$

rate of electricity supply: $P = iV$

resistive dissipation: $P = i^2R = \frac{V^2}{R}$

electromotive force: $\varepsilon = \frac{dW}{dq}$

supplying current: $i = \frac{\varepsilon}{R}$

Kirchhoff's circuit laws

resistance rule: $\Delta V = -iR$ (current), $\Delta V = +iR$ (opposite)

emf rule: $\Delta V = +\varepsilon$ (current), $\Delta V = -\varepsilon$ (opposite)

series charge: $q = q_1 = q_2 = ...$

parallel charge: $q = \sum_j q_j$

series current: $i = i_1 = i_2 = ...$

parallel current: $i = \sum_j i_j$

series voltage: $V = \sum_j V_j$

parallel voltage: $V = V_1 = V_2 = ...$

series resistance: $R = \sum_j R_j$

parallel resistance: $\frac{1}{R} = \sum_j\frac{1}{R_j}$, two: $R = \frac{R_1R_2}{R_1+R_2}$

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(2) k: media's thermal conductivity. (3) d: diameter, N: number of molecules. (4) n: number of carriers per unit volume. (5) ρ: temperature coefficient of resistivity.

Capacitance

capacitance: $C = \frac{q}{V}$

parallel-plate: $C = \epsilon_0 \frac{A}{d}$

cylindrical: $C = 2\pi\epsilon_0 L \frac{1}{\ln(b/a)}$

spherical: $C = 4\pi\epsilon_0 \frac{ab}{b-a}$

isolated sphere: $C = 4\pi\epsilon_0 R$

series capacitor: $\frac{1}{C} = \sum_j \frac{1}{C_j}$

parallel capacitor: $C = \sum_j C_j$

potential energy: $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$

volume energy density: $u = \frac{1}{2}\epsilon_0 E^2$

q unchanged: $U_f = U_i/\kappa$

V unchanged: $U_f = \kappa U_i$

RC circuit

charging equation: $R\frac{dq}{dt} + \frac{q}{C} = \varepsilon$

charge function: $q = C\varepsilon(1 - e^{-t/\tau_C})$

current function: $i = (\frac{\varepsilon}{R})e^{-t/\tau_C}$

discharging equation: $R\frac{dq}{dt} + \frac{q}{C} = 0$

charge function: $q = q_0 e^{-t/\tau_C}$

current function: $i = -i_0 e^{-t/\tau_C}$

capacitive time constant $\tau_C = RC$

Magnetism

force due to moving charge: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

force due to current-carrying wire: $\mathbf{F}_B = i\mathbf{L} \times \mathbf{B}$

L along direction of conventional i

circular motion under F_B: $qvB = m\frac{v^2}{r}$

period: $T = \frac{2\pi m}{qB}$

Hall effect, density of carriers: $n = \frac{Bi}{Vle}$

$l = A/d$: thinkness of strip

Biot-Savart law: $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i d\mathbf{s} \times \mathbf{r}}{r^3}$

vacuum permeability: $\mu_0 = 4\pi \cdot 10^{-7}\mathrm{T \cdot m/A}$ (H/m)

arc (to centre): $B = \frac{\mu_0 i\theta}{4\pi R}$

Ampere's circuital law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

long straight wire: $B = \frac{\mu_0 i}{2\pi r}$

solid wire: $B = \frac{\mu_0 i}{2\pi r}$ (outside), $B = (\frac{\mu_0 i}{2\pi R^2})r$ (inside)

ideal solenoid: $B = \mu_0 i_0 n$, $n = N/L$: turns per unit length

ideal toroid: $B = \frac{\mu_0 i_0 N}{2\pi r}$

induced emf: $\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \mathbf{B} \cdot d\mathbf{S}$

for coils: $\varepsilon = -N\frac{d\Phi_B}{dt}$

Maxwell-Faraday equation: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\iint \mathbf{B} \cdot d\mathbf{S}$

induced electrodynamic field, circle:

$E = \frac{R^2}{2}\frac{dB}{dt}\frac{1}{r}$ (outside), $E = \frac{1}{2}\frac{dB}{dt}r$ (inside)

Inductance

inductance: $L = \frac{N\Phi_B}{i}$

solenoid: $L/l = \mu_0 n^2 A$

toroid: $L = \frac{\mu_0 N^2 h}{2\pi}\ln(\frac{b}{a})$

self-induced emf: $\varepsilon_L = -L\frac{di}{dt}$

potential energy: $U_B = \frac{1}{2}Li^2$

energy density: $u_B = \frac{B^2}{2\mu_0}$

LR circuit

rise in current: $iR + L\frac{di}{dt} = \varepsilon$

current function: $i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$

decay in current: $iR + L\frac{di}{dt} = 0$

current function: $i = i_0 e^{-t/\tau_L}$

inductive time constant: $\tau_L = L/R$

series inductance: $L = \sum_j L_j$

parallel inductance: $\frac{1}{L} = \sum_j \frac{1}{L_j}$

mutual induction, two coils:

$\varepsilon_2 = -M\frac{di_1}{dt}$, $\varepsilon_1 = -M\frac{di_2}{dt}$

LC oscillation

definition: $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

charge function: $q = Q\cos(\omega t + \phi)$

angular frequency: $\omega = \frac{1}{\sqrt{LC}}$

electric potential energy: $U_E = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$

magnetic potential energy: $U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$

total energy: $U = \frac{Q^2}{2C}$

series RLC oscillation

net energy dissipation: $\frac{dU}{dt} = -i^2 R$

definition: $\frac{d^2q}{dt^2} + \frac{1}{LR}\frac{dq}{dt} + \frac{1}{LC}q = 0$

charge function: $q = Qe^{Rt/2L}\cos(\omega' t + \phi)$

angular frequency: $\omega' = \sqrt{\omega^2 - (R/2L)^2}$

Electromagnetic waves

magnetic field induced by electric field:

$\oint \mathbf{B}_E \cdot d\mathbf{s} = +\mu_0\epsilon_0\frac{d\Phi_E}{dt} = +\mu_0\epsilon_0\frac{d}{dt}\iint \mathbf{E} \cdot d\mathbf{S}$

"displacement current"between parallel plates, circle:

$B = \frac{\mu_0\epsilon_0 R^2}{2}\frac{dE}{dt}\frac{1}{r}$ (outside), $\frac{\mu_0\epsilon_0}{2}\frac{dE}{dt}r$ (inside)

displacement current: $i_d = \epsilon_0\frac{d\Phi_E}{dt}$

Electromagnetic waves

B and E are in phase:

$E = E_m\sin(kx - \omega t)$

$B = B_m\sin(kx - \omega t)$

wave speed: $c = \frac{\omega}{k}$

magnitude ratio: $\frac{E_m}{B_m} = c$

speed of light: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

direction of wave/poynting vector: $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$

plane wave's instantaneous flow rate: $S = \frac{1}{c\mu_0}E^2$ ($S = P/A$)

wave intensity: $I = \overline{S} = \frac{1}{c\mu_0}E_{\mathrm{rms}}^2$

momentum of light:

$\Delta p = \frac{\Delta U}{c}$ (total absorption), $\Delta p = \frac{2\Delta U}{c}$ (total reflection)

radiation pressure:

$p_r = \frac{I}{c}$ (total absorption), $p_r = \frac{2I}{c}$ (total reflection)

law of Malus: $I = I_m\cos^2\theta$

AC

resistive circuit: $V_R = I_R R$

capacitive circuit: $V_C = I_C X_C$

capacitive reactance: $X_C = \frac{1}{\omega C}$

inductive circuit: $V_L = I_L X_L$

inductive reactance: $X_L = \omega L$

series RLC circuit

current: $i = I\sin(\omega t - \phi)$

voltage: $\varepsilon = v_R + v_C + v_L$

current amplitude: $I = \frac{\varepsilon_m}{Z}$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

phase constant: $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

average power: $\overline{P} = I_{\mathrm{rms}}^2 R = \varepsilon_{\mathrm{rms}} I_{\mathrm{rms}}\cos\phi$

I is in phase with v_R; leads v_C by 90°, lags hehind v_L by 90°

ideal transformer (rms)

voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ (AC supply at p end, sends to s end)

current: $\frac{I_s}{I_p} = \frac{N_p}{N_s}$

resistances: $R_{eq} = (\frac{N_p}{N_s})^2 R$ (R at s)

Dipoles

electric dipole

electric field produced: $E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$ (dipole axis)

electric potential: $V(\theta) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$

net torque: $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

potential energy: $U(\theta) = -\mathbf{p} \cdot \mathbf{E}$

dipole moment: $\mathbf{p} = q\mathbf{d}$ ($-$ to $+$)

magnetic dipole/current loop

magnetic field produced: $\mathbf{B} = \frac{\mu_0}{2\pi}\frac{\boldsymbol{\mu}}{z^3}$

net torque: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

potential energy: $U(\theta) = -\boldsymbol{\mu} \cdot \mathbf{B}$
magnetic dipole moment: $\boldsymbol{\mu} = Ni\mathbf{A}$, N: turns

Maxwell's equations

Gauss' law: $\oiint \mathbf{E} \cdot d\mathbf{S} = q/\epsilon_0$
Gauss' law: $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$
Gauss' law for magnetism: $\oiint \mathbf{B} \cdot d\mathbf{S} = 0$
Gauss' law for magnetism: $\nabla \cdot \mathbf{B} = 0$
Maxwell-Faraday equation: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S}$
Maxwell-Faraday equation: $\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$
Ampere's circuital law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{S}$
Ampere's circuital law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \mathbf{E}$
electric displacement: $\mathbf{D} = \epsilon \mathbf{E}$
magnetic field: $\mathbf{H} = \mathbf{B}/\mu$

Magnets

magnetism due to spinning electron
spin angular momentum: $S = \frac{h}{4\pi} = 5.2729 \times 10^{-35}\mathrm{J/s}$
spin magnetic moment: $\mu_S = \mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\mathrm{J/T}$
magnetism due to orbital motion
orbital angular momentum: $L_{\mathrm{orb}} = mvr$
orbital magnetic moment: $\mu_{\mathrm{orb}} = \frac{1}{2}evr$
negative charge: $\boldsymbol{\mu}_{\mathrm{orb}} = -\frac{e}{2m}\boldsymbol{L}_{\mathrm{orb}}$

Optics

Geometric optics

law of reflection: $\theta_1 = \theta_2$
law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$
total internal refraction, critical angle: $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$
(incident from greater n_1)
Brewster angle: $\theta = \tan^{-1}(\frac{n_2}{n_1})$ (incident from n_1)

spherical mirror (Real side is where reflected)
focus: $f = \frac{r}{2}$ (+: concave, −: convex)
relationship of object, image distance: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$
(+: Real side, upright; −: Virtual side, inverted) (p is +)
lateral magnification: $|m| = \frac{h_{\mathrm{image}}}{h_{\mathrm{obj}}}$, $m = -\frac{i}{p}$
(+: same orientation; −: opposite)
spherical refracting surface (Real side is where refracted)
relationship: $\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$ (p is +)

thin lens
relation 1: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$
relation 2: $\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$
($n = n_{\mathrm{lens}}/n_{\mathrm{medium}}$, r_1: first side light goes through)

angular magnification, simple magnifer: $m_\theta = \frac{15\ \mathrm{cm}}{f}$
angular magnification, refracting telescope: $m_\theta = -\frac{f_{\mathrm{ob}}}{f_{\mathrm{eye}}}$
magnification, compound microscope: $M = -\frac{|f'_{\mathrm{ob}} - f_{\mathrm{eye}}|}{f_{\mathrm{ob}}} \frac{15\ \mathrm{cm}}{f_{\mathrm{eye}}}$

Interference and diffraction

index of refraction: $n = \frac{c}{v}$
wavelength in medium: $\lambda_n = \frac{\lambda}{n}$
two mediums, same light, number of wavelength difference:
$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$

double-slit interference
fully constructive: $d\sin\theta = m\lambda, m = 0, 1, 2, ...$
fully destructive: $d\sin\theta = (m + \frac{1}{2})\lambda, m = 0, 1, 2, ...$
illumination intensity: $I = 4I_0 \cos^2(\frac{1}{2}\phi)$, $\phi = \frac{2\pi d}{\lambda}\sin\theta$
(I_0: intensity of one slit when the ohter covered, d: seperation of slits), $\overline{I} = 2I_0$

real double-slit
intensity: $I = I_m \underbrace{(\cos^2\beta)}_{\mathrm{intfr}} \underbrace{(\sin\alpha/\alpha)^2}_{\mathrm{diffr}}$
$\beta = (\frac{\pi d}{\lambda})\sin\theta$, $\alpha = (\frac{\pi a}{\lambda})\sin\theta$

multiple slits (N slits)
grating maxima: $d\sin\theta = m\lambda, m = 0, 1, 2, ...$
line width: $\Delta\theta = \frac{\lambda}{Nd\cos\theta}$
dispersion/seperation of lines: $D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$
resolving power: $R = Nm = \frac{\overline{\lambda}}{\Delta\lambda}$

thin film, $n_1, n_3 > n_2$ (incident at n_1)
(every larger n of refraction side causes phase change of $\lambda/2$)
fully constructive: $2n_2L = (m + \frac{1}{2})\lambda, m = 0, 1, 2, ...$
fully destructive: $2n_2L = m\lambda, m = 0, 1, 2, ...$

single-slit diffraction
intensity minima: $a\sin\theta = m\lambda, m = 1, 2, 3, ...$
intensity maximum: I_m at centre
intensity: $I = I_m(\frac{\sin\alpha}{\alpha})^2$, $\alpha = (\frac{\pi a}{\lambda})\sin\theta$ (a: width)

Rayleigh's criteria: $\theta_R = 1.22\frac{\lambda}{d}$ (d: lens' diameter)
seperation of two sources: $\Delta x \approx f\theta$ (f: may be viewing distance)

Modern physics

Special relativity

speed parameter: $\beta = v/c$
Lorentz factor: $\gamma = \frac{1}{\sqrt{1-\beta^2}}$
time dilation: $\Delta t = \gamma \Delta t_0$
length contraction: $L = \frac{L_0}{\gamma}$

Lorentz transformation (S, S'):
$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma(t - vx/c^2) \end{cases}, \begin{cases} x = \gamma(x' + vt') \\ t = \gamma(t' + vx'/c^2) \end{cases}$$
difference in x, t:
$$\begin{cases} \Delta x' = \gamma(\Delta x - v\Delta t) \\ \Delta t' = \gamma(\Delta t - v\Delta x/c^2) \end{cases}, \begin{cases} \Delta x = \gamma(\Delta x' + v\Delta t') \\ \Delta t = \gamma(\Delta t' + v\Delta x'/c^2) \end{cases}$$
relativistic velocity law: $V_{\mathrm{CA}} = \frac{V_{\mathrm{CB}} + V_{\mathrm{BA}}}{1 + V_{\mathrm{CB}}V_{\mathrm{BA}}/c^2}$
Doppler effect: $f = f_0\sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$

conservation of space-time interval:
$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = \mathrm{constant}$
relation of proper time: $\Delta s = c\Delta\tau$
4-displacement: $x^\mu(\tau) = [ct(\tau), x(\tau), y(\tau), z(\tau)]$
4-velocity: $u^\mu = \frac{d}{d\tau}x^\mu = \gamma(v)[c, v]$
magniture of velocity: $|u^\mu| = \sqrt{(u^0)^2 - (u^1)^2} = c$
accleration: $a^\mu = \frac{d}{d\tau}u^\mu \perp u^\mu$
4-momentum: $p^\mu = mu^\mu = m\gamma[c, v] = [\frac{E}{c}, \gamma mv]$
relativistic kinetic energy: $K = mc^2(\gamma - 1)$
total energy: $E = \gamma mc^2 = \underbrace{mc^2}_{\text{rest E}} + \underbrace{K}_{\text{kinetic E}}$
relations: $E^2 = (pc)^2 + (mc^2)^2$, $(pc)^2 = K^2 + 2Kmc^2$

Quantum

Photons, particles

single slit experiment:
distance between central and first max: $y = \frac{\lambda L}{d}$
width of central max: $w = 2y = \frac{2\lambda L}{d}$
De Braglie relation: $h = \lambda p = \lambda\sqrt{2mK}$
Planck's constant: $h = 6.626 \cdot 10^{-34}\ \mathrm{m}^2 \cdot \mathrm{kg/s}$
energy of photon: $E = cp = c\frac{h}{\lambda} = hf$
photoelectric effect: $K_{\max} = hf - \phi$
Compton scattering:
electron stationary: $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$
electron head-on: $\lambda' \approx \frac{hc}{E_e}\left[1 + \frac{m_e^2c^4\lambda}{4hcE^e}\right]$

blackbody radiation (low frequency)

distribution finding entities with energy E:

$p_E(E) = \frac{1}{KT} e^{-E/kT}$

average energy: $\overline{E} = kT$

distribution of number of standing waves per unit volume:

$p_N(f) = \frac{N(f)}{V} = \frac{8\pi}{c^2} f^2$

distribution of energy per unit volume

$p_{E/V}(f) = \frac{E(f)}{V} = \frac{8\pi}{c^3} f^2 kT$

number of photons in standing wave: $n \approx \frac{kT}{hf}$

ultraviolet catastrophe (high frequency)

energy of photons: $E = nhf$

average energy: $\overline{E}(f) = \frac{hf}{e^{hf/kT} - 1}$

distribution of number of standing waves per unit volume:

$p_N(f) = \frac{8\pi f^2}{c^3} \frac{hf}{e^{hf/kT} - 1}$

$p_N(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$

confining standing waves

probability distribution of finding particle: $p(x) = |\Psi(n)|^2$

wavelength, ground state: $\lambda_0 = 2L$

momentum, ground state: $p_0 = \frac{h}{2L}$

energy, ground state: $E_0 = \frac{h^2}{8L^2 m}$

wavelength, excited state: $\lambda_n = \frac{\lambda_0}{n}$, $n = 1, 2, ...$

momentum, excited state: $p_n = np_0$, $n = 1, 2, ...$

energy, excited state: $E_n = n^2 E_0$, $n = 1, 2, ...$

Heisenberg uncertainty principle: $\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$

corollary: $\Delta E \Delta t \propto \hbar$

particle diffraction, max angle: $\Delta\theta_{\max} = \frac{\lambda_0}{L}$

Waves 1

orbit of hydrogen atoms

constructive interference: $2\pi r = n\lambda, n = 1, 2, ...$

Bohr radius: $r_n = a_0 n^2$, $a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.529\,\text{Å}$

original quantization condition: $p_n = \frac{\hbar}{a_0} \frac{1}{n}$, $L_n = \hbar n$

total energy: $E_n = -K_n = -\frac{\hbar^2}{2ma_0} \frac{1}{n^2} = -\frac{E_1}{n^2}$

ground state energy: $E_1 = 13.6\,\text{eV}$

free particle, 2D complex wave: $\Psi(x, t) = Ae^{\frac{1}{\hbar}(px - Et)}$

general Schrödinger equation:

$i\hbar \frac{\partial}{\partial t} \Psi(x, y, z, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(x, y, z, t) + U(x, y, z)\Psi(x, y, z, t)$

time-independent Schrödinger equation:

$E\Psi(x, y, z) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(x, y, z) + U(x, y, z)\Psi(x, y, z)$

probability flux: $\mathbf{J} = \frac{p}{m} \nabla s$

normalisation: $\int |\Psi(x, y, z, t)|^2 dv = 1$

particle bounded by nodes 0, L

potential function: $V(x) = \begin{cases} 0, & 0 \leq x \leq L \\ \infty, & x < 0 \vee x > L \end{cases}$

momentum: $p_n = \frac{h}{2L} n, n = 1, 2, ...$

energy: $E_n = \frac{h^2}{8mL^2} n^2, n = 1, 2, ...$

wave function, time-independent:

$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}), n = 1, 2, ...$

wave function, time-dependent:

$\Psi_n(x, t) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}) \cdot e^{-\frac{i}{\hbar} E_n t}, n = 1, 2, ...$

wave function, time-independent, 3D:

$\psi_n(x, y, z) =$

$\sqrt{\frac{2}{L}} \sin(\frac{n_x \pi x}{L}) \cdot \sqrt{\frac{2}{L}} \sin(\frac{n_y \pi y}{L}) \cdot \sqrt{\frac{2}{L}} \sin(\frac{n_z \pi z}{L}), n_i = 1, 2, ...$

energy assuming constant time: $E = \frac{\hbar}{2m} \frac{\pi^2}{L^2} (n_x^2 + n_y^2 + n_z^2)$

finite potential well

momentum in regions:

$p_x = \begin{cases} \pm\sqrt{2mE}, & 0 \leq x \leq L \\ \pm\sqrt{2m(E - U_0)}, & x < 0 \vee x > L \end{cases}$

momentum amplitude: $|p_x| = \hbar k = \sqrt{2m(U_0 - E)}$

tunneling probability: $P_T \approx \alpha e^{-2kL}$, $\alpha = 16 \frac{E}{U_0} (1 - \frac{E}{U_0})$

harmonic oscillator

zero-point energy: $E_0 = \frac{1}{2} hf$

energy at nth level:

$E_n = (n + \frac{1}{2})hf = \underbrace{nhf}_{\text{number of photons}} + \underbrace{1/2hf}_{\text{zero-point energy}}$

energy in vacuum: $E_{\text{vac}} = -\frac{hc}{48} L$

quantum numbers

total energy: $E_n = -(\frac{1}{4\pi\epsilon_0})^2 \frac{me^4}{2\hbar} \frac{1}{n^2} = -\frac{13.6\,\text{eV}}{n^2}, n = 1, 2, ...$

magnitude of $\mathbf{L}$: $L = \sqrt{\ell(\ell + 1)}\hbar, \ell = 1, 2, ..., n - 1$

z-projection of $\mathbf{L}$: $L_z = m_\ell \hbar, m_\ell = 0, \pm1, \pm2, ..., \pm\ell$

End of formula sheet

Version 1.0.1

This summary was made possible by *cos*. It was coded to help learners recall their knowledge and understanding of general physics. As physics equations do not change according to textbooks thus remaining universal, please reply to the thread where this was published in case of any errors.