A formula summary for physics

Classical mechanics

Kinetics

velocity: $\mathbf{v} = \frac{d\mathbf{r}}{dt} = r'_x\mathbf{i} + r'_y\mathbf{j} + r'_z\mathbf{k}$
acceleration: $\mathbf{a} = \frac{d\mathbf{v}}{dt} = v'_x\mathbf{i} + v'_y\mathbf{j} + v'_z\mathbf{k}$
motion laws:

$v = v_0 + at$
$x - x_0 = v_0 t + \frac{1}{2}at^2$
$v^2 - v_0^2 = 2a(x - x_0)$
$x - x_0 = \frac{(v_0+v)t}{2}$

centripetal acceleration: $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2 = \omega^2 r$
velocity in two frames: $\mathbf{v}_{\mathrm{PA}} = \mathbf{v}_{\mathrm{PB}} + \mathbf{v}_{\mathrm{BA}}$
same acceleration measured in all frames: $\mathbf{a}_{\mathrm{PA}} = \mathbf{a}_{\mathrm{PB}}$

Kinetics 2

Newton's second law: $\mathbf{F}_{\mathrm{T}} = m\mathbf{a}$
Newton's third law: $\mathbf{F}_{\mathrm{A\ to\ B}} = -\mathbf{F}_{\mathrm{B\ to\ A}}$

acceleration of simple pulley: $a = \frac{m_1 - m_2}{m_1 + m_2}g$
drag force (body in fluid): $D = \frac{1}{2}C\rho A v^2$
terminal speed: $v = \sqrt{\frac{2mg}{C\rho A}}$ (1)
work: $W = \int \mathbf{F}\cdot d\mathbf{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$
Hooke's law: $\mathbf{F} = -k\mathbf{x}$
work by spring: $W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$
kinetic energy: $K = \frac{1}{2}mv^2$
work-kinetic energy theorem: $W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
power: $P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$

Kinetics 3

definition of change in potential energy: $\Delta U = -W$
change in mechanical energy: $\Delta K + \Delta U = 0$
total mechanical energy: $E = U + K$ (isolated)

U-x graph:
external force: $F(x) = -\frac{d}{dx}U(x)$
neutral equilibrium: $E = U$
unstable equilibrium: $U'' < 0, U' = 0$
stable equilibrium: $U'' > 0, U' = 0$

total m.energy of block-spring system: $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$
total m.energy of particle-earth system: $E = mgy + \frac{1}{2}mv^2$
conservation of energy:
$\Delta K + \Delta U + \Delta E_{\mathrm{internal}}$ (+other forms) = 0 (isolated)

(1) C: drag coefficient, A: effective cross-sectional area, ρ: air density.

external work done: $W = \Delta K + \Delta U + \Delta E_{\mathrm{internal}}$
change in energy: $\Delta E = \Delta K + \Delta U$
loss in mechanical energy (friction): $\Delta E = -fd$
energy loss due to emitted light: $E_x - E_y = hf$

System kinetics

centre of mass: $\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i\mathbf{r}_i$
continuous: $\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\int \mathbf{r}\rho dV = \frac{1}{V}\int \mathbf{r} dV$
relation: $dm = \rho dV$

linear momentum: $\mathbf{p} = m\mathbf{v}$
relation: $K = \frac{p^2}{2m}$
net force: $\mathbf{F}_{\mathrm{T}} = \frac{d\mathbf{p}}{dt}$
Newton's second law: $\sum \mathbf{F}_{\mathrm{external}} = M\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{P}}{dt}$
conservation of linear momentum: $\mathbf{P} =$ constant

циолковский's rocket formula:
$\mathbf{F} = (\mathbf{v} - \mathbf{u})\frac{dm}{dt} + m\frac{d\mathbf{v}}{dt}$ or $\frac{d}{dt}m\mathbf{v} = \mathbf{F} + \mathbf{u}\frac{dm}{dt}$
v: rocket's velocity in earth, u: fuel's velocity in earth

change in translational k. energy: $\Delta K_{\mathrm{CM}} = F_{\mathrm{ext}} d_{\mathrm{CM}}$
König's theorem: $K = K_{\mathrm{related\ to\ CM}} + \frac{1}{2}mv_{\mathrm{CM}}^2$

Collisions

impulse: $\mathbf{J} = \Delta\mathbf{p} = \int \mathbf{F} dt$

elastic collision:

$v'_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$
$v'_2 = \frac{m_2 - m_1}{m_1 + m_2}v_2 + \frac{2m_1}{m_1 + m_2}v_1$
$v_{\mathrm{CM}} = \frac{P}{m_1 + m_2}$

complete inelastic collision: $m_1 v_1 + m_2 v_2 = (m_1 + m_2)V_{\mathrm{CM}}$

Rotation

angular position: $\theta = s/r$
angular velocity: $\omega = \frac{d\theta}{dt}$
angular acceleration: $\alpha = \frac{d\omega}{dt}$
motion laws:

$\omega = \omega_0 + \alpha t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega^2 - \omega_0^2 = 2\alpha\theta$
$\theta = \frac{(\omega_0+\omega)t}{2}$

linear-angular relation: $s = \theta r$, $v = \omega r$
tangent acceleration: $a_t = \alpha r$
radial acceleration: $a_r = \frac{v^2}{r} = \omega^2 r$

rotational inertia: $I = \sum_i m_i r_i^2$
r.i. for continuous objects: $I = \int r^2 dm$
total rotational inertia: $I_{\mathrm{whole}} = \sum_i I_i$ (all to one axis)
parallel-axis theorem: $I = I_{\mathrm{CM}} + Mh^2$
perpendicular-axis theorem: $I_P = I_x + I_y$ (no thickness)
rotational kinetic energy: $K = \frac{1}{2}I\omega^2$
torque: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
Newton's second law: $\tau_{\mathrm{T}} = I\alpha$
work: $W = \int F_t r d\theta = \int \tau d\theta$
power: $P = \frac{dW}{dt} = \tau\omega$
work-kinetic energy theorem: $W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

Rotational inertia

hoop, central axis: $I = MR^2$
hoop, diameter: $I = \frac{1}{2}MR^2$
annular cylinder, central axis: $\frac{1}{2}M\left(R_1^2 + R_2^2\right)$
annular cylinder, central diameter,$\frac{1}{4}M\left(R_1^2 + R_2^2\right)$
solid cylinder/disk, central axis: $\frac{1}{2}MR^2$
solid cylinder/disk, central diameter: $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
rod, centre of length: $I = \frac{1}{12}ML^2$
rod, one end: $I = \frac{1}{3}ML^2$
triangle, parallel to base a (whose height is h, through CM): $I = \frac{1}{18}Mh^2$
solid sphere, diameter: $I = \frac{2}{5}MR^2$
spherical shell, diameter: $I = \frac{2}{3}MR^2$
slab, centre: $I = \frac{1}{12}M(a^2 + b^2)$
slab, along edge b:$I = \frac{1}{3}Ma^2$

Rolling

CM displacement/distance rolled: $x_{\mathrm{CM}} = \theta R$, $v_{\mathrm{CM}} = \omega R$
kinetic energy: $K = K_{\mathrm{rot}} + K_{\mathrm{tra}} = \frac{1}{2}I_{\mathrm{CM}}\omega^2 + \frac{1}{2}mv_{\mathrm{CM}}^2$
accleration of ideal yoyo: $a = -g(\frac{1}{1+I/MR^2})$
angular momentum: $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$
a.m. for rigid, fixed axis: $L = I\omega$
angular impulse: $\Delta\boldsymbol{L} = \int \boldsymbol{\tau} dt$
Newton's second law: $\boldsymbol{\tau}_{\mathrm{T}} = \frac{d\boldsymbol{L}}{dt}$
conservation of angular momentum: $\boldsymbol{L} =$ constant

Elasticity

static equilibrium: $\mathbf{P}=0, \boldsymbol{L}=0$
requirements of equilibrium: $\sum \mathbf{F}_{\text{ext}}=0, \sum \boldsymbol{\tau}_{\text{ext}}=0$
tensile stress: $\frac{F}{A}=E\frac{\Delta L}{L}$, E: Young's modulus
sheering stress: $\frac{F}{A}=G\frac{\Delta x}{L}$, G: sheer modulus
hydraulic compression: $p=B\frac{\Delta V}{V}$, B: Bulk modulus

Oscillation

simple harmonic motion ($F=-m\omega^2 x$):
$\omega=\frac{2\pi}{T}=2\pi f$
$x(t)=x_m\cos(\omega t+\phi)$
$v(t)=-\omega x_m\sin(\omega t+\phi)$
$a(t)=-\omega^2\, x(t)$

linear oscillator
definition: $\frac{d^2x}{dt^2}+\omega^2 x=0$
angular frequency: $\omega=\sqrt{\frac{k}{m}}$
period: $T=2\pi\sqrt{\frac{m}{k}}$
potential energy: $U(t)=\frac{1}{2}kx_m^2\cos^2(\omega t+\phi)$
kinetic energy: $K(t)=\frac{1}{2}kx_m^2\sin^2(\omega t+\phi)$
total energy: $E=\frac{1}{2}kx_m^2$
series spring: $\frac{1}{K}=\sum_j\frac{1}{k_j}$, two: $K=\frac{k_1k_2}{k_1+k_2}$
parallel spring: $K=\sum_j k_j$

simple pendulum
period: $T=2\pi\sqrt{\frac{L}{g}}$
restoring force: $F\approx-(\frac{mg}{L})s$

torsion pendulum
period: $T=2\pi\sqrt{\frac{I}{\kappa}}$
restoring torque: $\tau=-\kappa\theta$

physical pendulum
period: $T=2\pi\sqrt{\frac{I}{mgh}}$
restoring torque: $\tau=-(mg\sin\theta)h$

damped simple harmonic motion
damping force: $F_d=-bv$, b: damping constant
definition: $\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=0$
displacement: $x(t)=x_m e^{-bt/2m}\cos(\omega_d t+\phi)$
angular frequency: $\omega_d=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$
total energy: $E(t)\approx\frac{1}{2}kx_m^2e^{-bt/m}$

Gravitation

Newton's law of gravitation: $F=\frac{GMm}{r^2}$
gravitational constant: $G=6.67\cdot10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2$
differential: $dF=\frac{Gm_1}{r^2}dm$
gravitational field: $g=\frac{GM}{r^2}$
gravitational potential energy: $U=\int_\infty^r\frac{GMm}{x^2}dx=-\frac{GMm}{r}$
escape speed: $v=\sqrt{\frac{2GM}{r}}$

plane motion of point mass:
$a_r=\ddot{r}-r\dot{\theta}^2, a_\theta=2\dot{r}\dot{\theta}+r\ddot{\theta}$

orbits:
path of planet: $r=\frac{p}{1+e\cos\theta}$
$p=\frac{L^2}{GMm^2}$, $e=\sqrt{1+\frac{2EL^2}{G^2M^2m^3}}$
net angular momentum: $L=mr^2\dot{\theta}$
net mechanical energy:
$E=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-\frac{GMm}{r}=\frac{1}{2}m\dot{r}^2+(\frac{L^2}{2mr^2}-\frac{GMm}{r})$
total energy of satellite-earth ellipse system:
$E=-K=-\frac{GMm}{2r}$
perihelion: $r_1=a-c$
aphelion: $r_2=a+c$
$a=\frac{r_1+r_2}{2}=-\frac{GMm}{2E}$, $c=\frac{r_2-r_1}{2}$,
$b=\sqrt{a^2-c^2}=\sqrt{r_1r_2}=\frac{L}{\sqrt{-2mE}}$
total energy of satellite-earth hyperbola system: $E=\frac{GMm}{2r}$
total energy of satellite-earth parabola system: $E=0$
law of periods: $\frac{T^2}{r^3}=\frac{4\pi^2}{GM}$

Fluids

pressure: $p=\frac{\Delta F}{\Delta A}$ (all direction)
pressure in liquid: $p=p_0+\rho gh$
Pascal's principle: $\Delta p_{\text{int}}=\Delta p_{\text{ext}}$
Archimede's principle: $F_{\text{buoyancy}}=G_{\text{displaced water}}$
equation of continuity:
volume flow rate $R=Av=$ constant
mass flow rate $m=Av\rho=$ constant
Bernoulli's equation: $p+\frac{1}{2}\rho v^2+\rho gy=$ constant

Transverse waves

transverse displacement: $y(x,t)=y_m\sin(kx-\omega t)$
angular wave number: $k=\frac{2\pi}{\lambda}$
waver number: $\kappa=\frac{1}{\lambda}$
angular frequancy: $\omega=\frac{2\pi}{T}$
frequency: $f=\frac{1}{T}=\frac{\omega}{2\pi}$
wave speed: $v=\frac{\omega}{k}=\lambda f$
material expression: $v=\sqrt{\frac{\tau(\text{tension})}{\mu(\text{density of media})}}$
transverse speed: $u=\frac{\partial y}{\partial t}$
average power: $\overline{P}=\frac{1}{2}\mu v\omega^2y_m^2$

adding waves
superposition:
$y=y_1+y_2=y_m\sin(kx-\omega t+\phi)+y_m\sin(kx-\omega t)$
new wave: $y=(2y_m\cos\frac{1}{2}\phi)\sin(kx-\omega t+\frac{1}{2}\phi)$
new amplitude: $2y_m\cos\frac{1}{2}\phi$
phase shift: $+\frac{1}{2}\phi$

standing waves
superposition:
$y=y_1+y_2=y_m\sin(kx-\omega t)+y_m\sin(kx+\omega t)$
new wave: $y=[(2y_m)\sin kx]\cos\omega t$
new amplitude: $2y_m\sin kx$
nodes: $x=n\frac{\lambda}{2}, n=0,1,2,...$
antinodes: $x=(n+\frac{1}{2})\frac{\lambda}{2}, n=0,1,2,...$
resonant frequency: $f_r=\frac{v}{\lambda}=\frac{v}{2l}n, n=1,2,3,...$

Longitudinal waves

speed of sound: $v=\sqrt{\frac{B}{\rho}}$
bulk modulus: $B=-\frac{\Delta p}{\Delta V/V}(=\rho v^2)$
longitudinal displacement: $s=s_m\cos(kx-\omega t)$
air pressure: $\Delta p=\Delta p_m\sin(kx-\omega t)$
relation: $\Delta p_m=(v\rho\omega)s_m$

interference
phase shift: $\phi=\frac{\Delta d}{\lambda}2\pi$
fully constructive: $\phi=m2\pi, m=0,1,2,...$
fully destructive: $\phi=(m+\frac{1}{2})2\pi, m=0,1,2,...$

sound intensity: $I=\frac{1}{2}\rho v\omega^2s_m^2$
sound level: $\beta=(10\text{ dB})\log(\frac{I}{I_0})$
standard reference intensity: $I_0=10^{-12}\mathrm{W/m^2}$

resonant frequency
pipe, two opens: $f_r=\frac{v}{\lambda}=\frac{v}{2L}n, n=1,2,3$
pipe, one open: $f_r=\frac{v}{\lambda}=\frac{v}{4L}n, n=1,3,5,...$
beat frequency: $f_{beat}=f_1-f_2$

doppler effect: $f'=f\frac{v\pm v_L}{v\mp v_S}$
cone angle at supersonic speed: $\sin\theta=\frac{v}{v_s}$

Heat, Second law of thermodynamics

Heat

coefficient of linear expansion: $\alpha = \frac{\Delta L/L}{\Delta T}$

area expansion: $\beta = 2\alpha$

volume expansion: $\gamma = 3\alpha$

heat capacity: $Q = cm(T_f - T_i) = C(T_f - T_i)$

heat of transformation: $Q = Lm$

volume work: $W = \int_{V_i}^{V_f} p dV$

first law of thermodynamics: $\Delta E_{\text{int}} = E_{\text{int,f}} - E_{\text{int,i}} = Q - W$

rate of heat transfer: $H = \frac{Q}{t} = kA\frac{T_H - T_C}{L}$ (2)

multiple slabs: $H = A\frac{T_H - T_C}{\sum(L/k)}$

Kinetic theory of gases

ideal gas law: $pV = nRT$

gas constant $R = 8.31 \text{J/mol} \cdot \text{K}$

volume work of expansion at constant pressure:

$W = \int \frac{nRT}{V} dV = nRT \ln(\frac{V_f}{V_i})$

gas pressure: $p = \frac{nMv_{\text{rms}}^2}{3V}$

translational kinetic energy: $\overline{K} = \frac{3}{2}kT$

Boltzman constant $k = R/N_A$

mean free path: $\lambda = \frac{1}{\sqrt{2}\pi d N/V}$ (3)

Maxwell's speed distribution:

$P(v) = 4\pi(\frac{M}{2\pi RT})^{3/2} v^2 e^{-Mv^2/2RT}$

most propable speed: $v_p = \sqrt{\frac{2RT}{M}}$

average speed: $\overline{v} = \sqrt{\frac{8RT}{\pi M}}$

rms speed: $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

internal energy of monoatomic gas: $E_{\text{int}} = (nN_A)\overline{K} = \frac{3}{2}nRT$

monoatom: $3/2$ $(f = 1)$

diatom: $5/2$ $(f = 2)$

5-atom: 3 $(f = 5)$

molar specific heat of monoatomic gas at constant volume:

$C_v = \frac{3}{2}R = 12.5 \,\text{J/molK}$

constant volume, change in internal energy:

$\Delta E_{\text{int}} = Q = nC_v(T_f - T_i)$

molar specific heat of monoatomic gas at constant pressure:

$C_p - C_v = R$

heat: $Q = nC_p(T_f - T_i)$

work: $W = nR(T_f - T_i)$

law of adiabatic expansion: $pV^\gamma = \text{constant}$, or

$TV^{\gamma-1} = \text{constant}$

$\gamma = C_p/C_v = 1 + 2/f$

(2) k: media's thermal conductivity. (3) d: diameter, N: number of molecules. (4) n: number of carriers per unit volume. (5) ρ: temperature coefficient of resistivity.

Second law of thermodynamics

thermal effiency of engine: $e = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|}$

max: $e_{\text{Car}} = \frac{T_H - T_C}{T_H}$

coefficient of performance of refrigerator:

$e = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$

max: $e_{\text{Car}} = \frac{T_C}{T_H - T_C}$

first law of thermodynamics in closed system:

$|W| = |Q_H| - |Q_C|$

entropy: $dS = \frac{dQ}{T}$ and $\oint dS \leq 0$

reversible process: $S_f - S_i = \int_i^f dS = \int_i^f \frac{dQ}{T}$

free expansion: $S_f - S_i = \frac{1}{T}\int_i^f dQ = nR\ln\frac{V_f}{V_i}$

irreversible heat transfer: $S_f - S_i = cm\ln\frac{T^2}{T^2 - \Delta T^2}$

Electricity and Magnetism

Electrostatic forces

Coulomb's law: $F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

permitivity constant in vacuum: $\epsilon_0 = 8.85 \cdot 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2$

charge is quantized: $q = ne$

elementary charge $e = 1.6 \cdot 10^{-14}\,\text{C}$

Electric field

electric field: $\mathbf{E} = \frac{\mathbf{F}}{q_0}$

differential: $d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^3}\mathbf{r}$ (r from dq to point)

point charge: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

straight rod (perpendicular): $E = \frac{\lambda a}{2\pi\epsilon_0 r}\frac{1}{\sqrt{4r^2 + a^2}}$

arc (to centre): $E = \frac{\lambda}{4\pi\epsilon_0 r}(2\sin\frac{\theta}{2})$

ring (perpendicular): $E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}$

round disk (perpendicular): $E = \frac{\sigma}{2\epsilon_0}(1 - \frac{z}{\sqrt{z^2 + R^2}})$

electrostatic force in a field: $\mathbf{F} = q\mathbf{E}$ (signed)

Gauss' law

Gauss' law: $\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$

conducting surface: $E = \frac{\sigma}{\epsilon_0}$

nonconducting surface: $E = \frac{\sigma}{2\epsilon_0}$

straight rod: $\frac{\lambda}{2\pi r\epsilon_0}$

two conducting plates (+ greater):

$|E_L| = |E_R| = |E_{(+)} - E_{(-)}|$, $|E_{\text{in}}| = E_{(+)} + E_{(-)} = \frac{\sigma_1 + \sigma_2}{\epsilon_0}$

shell: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ (outside), $E = 0$ (inside)

sphere: $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ (outside), $E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r$ (inside)

cylinder: $E = \frac{R^2\rho}{2\epsilon_0 r}$ (outside), $E = \frac{\rho}{2\epsilon_0}r$ (inside)

Potential

work: $W = \int \mathbf{F} \cdot d\mathbf{s} = q_0 \int \mathbf{E} \cdot d\mathbf{s}$

electric potential difference: $\Delta V = -\frac{W_{if}}{q_0} = \frac{\Delta U}{q_0}$

E-V relation: $\mathbf{E} = -\nabla V$, $V = -\int_{i_0}^f \mathbf{E} \cdot d\mathbf{s}$

electric field of parallel plates: $E = \frac{\Delta V}{\Delta d}$

point charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$(signed)

discrete points: $V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

continuous charge: $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$

rod (perpendicular to one end): $V = \frac{\lambda}{4\pi\epsilon_0}\ln(\frac{L + (L^2 + d^2)^{1/2}}{d})$

arc (to centre): $V = \frac{\lambda\theta}{4\pi\epsilon_0}$

ring (perpendicular): $V = \frac{q}{4\pi\epsilon_0\sqrt{z^2 + R^2}}$

disk (perpendicular): $V = \frac{\sigma}{2\epsilon_0}(\sqrt{z^2 + R^2} - z)$

Current and circuits

current: $i = \frac{dq}{dt}$

current density: $J = i/A$

relation: $i = \iint \mathbf{J} \cdot d\mathbf{A}$

draft speed: $\mathbf{v}_{\text{d}} = \mathbf{J}/(ne)$ (4)

resistance law: $R = \frac{V}{i}$

isotropic resistivity: $\rho = E/J$

relation: $\mathbf{E} = \rho\mathbf{J}$

conductivity: $\sigma = 1/\rho$

resistance: $R = \rho\frac{L}{A}$

variation with temperature: $\rho - \rho_0 = \rho_0\alpha(T - T_0)$ (5)

$T_0 = 293\,\text{K}$, $\rho_0 = 1.69\,\mu\Omega \cdot \text{cm}$

rate of electricity supply: $P = iV$

resistive dissipation: $P = i^2 R = \frac{V^2}{R}$

electromotive force: $\mathscr{E} = \frac{dW}{dq}$

supplying current: $i = \frac{\mathscr{E}}{R}$

Kirchhoff's circuit laws

resistance rule: $\Delta V = -iR$ (current), $\Delta V = +iR$ (opposite)

emf rule: $\Delta V = +\mathscr{E}$ (current), $\Delta V = -\mathscr{E}$ (opposite)

series charge: $q = q_1 = q_2 = \ldots$

parallel charge: $q = \sum_j q_j$

series current: $i = i_1 = i_2 = \ldots$

parallel current: $i = \sum_j i_j$

series voltage: $V = \sum_j V_j$

parallel voltage: $V = V_1 = V_2 = \ldots$

series resistance: $R = \sum_j R_j$

parallel resistance: $\frac{1}{R} = \sum_j \frac{1}{R_j}$, two: $R = \frac{R_1 R_2}{R_1 + R_2}$

Capacitance

capacitance: $C = \frac{q}{V}$

parallel-plate: $C = \epsilon_0 \frac{A}{d}$

cylindrical: $C = 2\pi\epsilon_0 L \frac{1}{\ln(\text{b/a})}$

spherical: $C = 4\pi\epsilon_0 \frac{ab}{b-a}$

isolated sphere: $C = 4\pi\epsilon_0 R$

series capacitor: $\frac{1}{C} = \sum_j \frac{1}{C_j}$

parallel capacitor: $C = \sum_j C_j$

work to charge capacitor: $W = \int dW = \int V' dq'$

potential energy: $U = \frac{\epsilon_0 A V^2}{2d}$

potential energy(parallel-plate): $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$

volume energy density: $u = \frac{1}{2}\epsilon_0 E^2$

q unchanged: $U_f = U_i/\kappa$

V unchanged: $U_f = \kappa U_i$

RC circuit

charging equation: $R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}$

charge function: $q = C\mathscr{E}(1 - e^{-t/\tau_C})$

current function: $i = (\frac{\mathscr{E}}{R})e^{-t/\tau_C}$

discharging equation: $R\frac{dq}{dt} + \frac{q}{C} = 0$

charge function: $q = q_0 e^{-t/\tau_C}$

current function: $i = -i_0 e^{-t/\tau_C}$

capacitive time constant $\tau_C = RC$

Magnetism

force due to moving charge: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

force due to current-carrying wire: $\mathbf{F}_B = i\mathbf{L} \times \mathbf{B}$

L along direction of conventional i

circular motion under F_B: $qvB = m\frac{v^2}{r}$

period: $T = \frac{2\pi m}{qB}$

Hall effect, density of carriers: $n = \frac{Bi}{Vle}$

$l = A/d$: thinkness of strip

Biot-Savart law: $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{id\mathbf{s}\times\mathbf{r}}{r^3}$

vacuum permeability: $\mu_0 = 4\pi \cdot 10^{-7}\text{T} \cdot \text{m/A (H/m)}$

point charge: $B = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$

arc (to centre): $B = \frac{\mu_0 i\theta}{4\pi R}$

Ampere's circuital law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

long straight wire: $B = \frac{\mu_0 i}{2\pi r}$

solid wire: $B = \frac{\mu_0 i}{2\pi r}$ (outside), $B = (\frac{\mu_0 i}{2\pi R^2})r$ (inside)

ideal solenoid: $B = \mu_0 i_0 n$, $n = N/L$: turns per unit length

ideal toroid: $B = \frac{\mu_0 i_0 N}{2\pi r}$

induced emf: $\mathscr{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \mathbf{B} \cdot d\mathbf{S}$

for coils: $\mathscr{E} = -N\frac{d\Phi_B}{dt}$

Maxwell-Faraday equation: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\iint \mathbf{B} \cdot d\mathbf{S}$

induced electrodynamic field, circle:

$E = \frac{R^2}{2}\frac{dB}{dt}\frac{1}{r}$ (outside), $E = \frac{1}{2}\frac{dB}{dt}r$ (inside)

Inductance

inductance: $L = \frac{N\Phi_B}{i}$

solenoid: $L/l = \mu_0 n^2 A$

toroid: $L = \frac{\mu_0 N^2 h}{2\pi}\ln(\frac{b}{a})$

self-induced emf: $\mathscr{E}_L = -L\frac{di}{dt}$

potential energy: $U_B = \frac{1}{2}Li^2$

energy density: $u_B = \frac{B^2}{2\mu_0}$

LR circuit

rise in current: $iR + L\frac{di}{dt} = \mathscr{E}$

current function: $i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L})$

decay in current: $iR + L\frac{di}{dt} = 0$

current function: $i = i_0 e^{-t/\tau_L}$

inductive time constant: $\tau_L = L/R$

series inductance: $L = \sum_j L_j$

parallel inductance: $\frac{1}{L} = \sum_j \frac{1}{L_j}$

mutual induction, two coils:

$\mathscr{E}_2 = -M\frac{di_1}{dt}$, $\mathscr{E}_1 = -M\frac{di_2}{dt}$

LC oscillation

definition: $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

charge function: $q = Q\cos(\omega t + \phi)$

angular frequency: $\omega = \frac{1}{\sqrt{LC}}$

electric potential energy: $U_E = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$

magnetic potential energy: $U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$

total energy: $U = \frac{Q^2}{2C}$

series RLC oscillation

net energy dissipation: $\frac{dU}{dt} = -i^2 R$

definition: $\frac{d^2q}{dt^2} + \frac{1}{LR}\frac{dq}{dt} + \frac{1}{LC}q = 0$

charge function: $q = Qe^{Rt/2L}\cos(\omega' t + \phi)$

angular frequency: $\omega' = \sqrt{\omega^2 - (R/2L)^2}$

Electromagnetic waves

magnetic field induced by electric field:

$\oint \mathbf{B}_E \cdot d\mathbf{s} = +\mu_0\epsilon_0\frac{d\Phi_E}{dt} = +\mu_0\epsilon_0\frac{d}{dt}\iint \mathbf{E} \cdot d\mathbf{S}$

"displacement current"between parallel plates, circle:

$B = \frac{\mu_0\epsilon_0 R^2}{2}\frac{dE}{dt}\frac{1}{r}$ (outside), $\frac{\mu_0\epsilon_0}{2}\frac{dE}{dt}r$ (inside)

displacement current: $i_d = \epsilon_0\frac{d\Phi_E}{dt}$

Electromagnetic waves

B and E are in phase:

$E = E_m\sin(kx - \omega t)$

$B = B_m\sin(kx - \omega t)$

wave speed: $c = \frac{\omega}{k}$

magnitude ratio: $\frac{E_m}{B_m} = c$

speed of light: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

direction of wave/poynting vector: $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$

plane wave's instantaneous flow rate: $S = \frac{1}{c\mu_0}E^2$ $(S = P/A)$

wave intensity: $I = \overline{S} = \frac{1}{c\mu_0}E^2_{\text{rms}}$

momentum of light:

$\Delta p = \frac{\Delta U}{c}$ (total absorption), $\Delta p = \frac{2\Delta U}{c}$ (total reflection)

radiation pressure:

$p_r = \frac{I}{c}$ (total absorption), $p_r = \frac{2I}{c}$ (total reflection)

law of Malus: $I = I_m\cos^2\theta$

AC

resistive circuit: $V_R = I_R R$

capacitive circuit: $V_C = I_C X_C$

capacitive reactance: $X_C = \frac{1}{\omega C}$

inductive circuit: $V_L = I_L X_L$

inductive reactance: $X_L = \omega L$

series RLC circuit

current: $i = I\sin(\omega t - \phi)$

voltage: $\mathscr{E} = v_R + v_C + v_L$

current amplitude: $I = \frac{\mathscr{E}_m}{Z}$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

phase constant: $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

average power: $\overline{P} = I^2_{\text{rms}}R = \mathscr{E}_{\text{rms}}I_{\text{rms}}\cos\phi$

I is in phase with v_R; leads v_C by 90°, lags hehind v_L by 90°

ideal transformer (rms)

voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ (AC supply at p end, sends to s end)

current: $\frac{I_s}{I_p} = \frac{N_p}{N_s}$

resistances: $R_{eq} = (\frac{N_p}{N_s})^2 R$ (R at s)

Dipoles

electric dipole

electric field produced: $E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$ (dipole axis)

electric potential: $V(\theta) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$

net torque: $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

potential energy: $U(\theta) = -\mathbf{p} \cdot \mathbf{E}$

dipole moment: $\mathbf{p} = q\mathbf{d}$ ($-$ to $+$)

magnetic dipole/current loop

magnetic field produced: $\mathbf{B} = \frac{\mu_0}{2\pi} \frac{\boldsymbol{\mu}}{z^3}$

net torque: $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$

potential energy: $U(\theta) = -\boldsymbol{\mu} \cdot \mathbf{B}$

magnetic dipole moment: $\boldsymbol{\mu} = Ni\mathbf{A}$, N: turns

Maxwell's equations

Gauss' law: $\oiint \mathbf{E} \cdot d\mathbf{S} = q/\epsilon_0$

Gauss' law: $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$

Gauss' law for magnetism: $\oiint \mathbf{B} \cdot d\mathbf{S} = 0$

Gauss' law for magnetism: $\nabla \cdot \mathbf{B} = 0$

Maxwell-Faraday equation: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S}$

Maxwell-Faraday equation: $\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$

Ampere's circuital law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{S}$

Ampere's circuital law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \mathbf{E}$

electric displacement: $\mathbf{D} = \epsilon \mathbf{E}$

magnetic field: $\mathbf{H} = \mathbf{B}/\mu$

Classical optics

Geometric optics

law of reflection: $\theta_1 = \theta_2$

law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

total internal refraction, critical angle: $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$

(incident from greater n_1)

Brewster angle: $\theta = \tan^{-1}(\frac{n_2}{n_1})$ (incident from n_1)

spherical mirror (Real side is where reflected)

focus: $f = \frac{r}{2}$ (+: concave, −: convex)

relationship of object, image distance: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$

(+: Real side, upright; −: Virtual side, inverted) (p is +)

lateral magnification: $|m| = \frac{h_{\text{image}}}{h_{\text{obj}}}$, $m = -\frac{i}{p}$

(+: same orientation; −: opposite)

spherical refracting surface (Real side is where refracted)

relationship: $\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$ (p is +)

thin lens

relation 1: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$

relation 2: $\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$

($n = n_{\text{lens}}/n_{\text{medium}}$, r_1: first side light goes through)

angular magnification, simple magnifer: $m_\theta = \frac{15\text{ cm}}{f}$

angular magnification, refracting telescope: $m_\theta = -\frac{f_{\text{ob}}}{f_{\text{eye}}}$

magnification, compound microscope: $M = -\frac{|f'_{\text{ob}} - f\text{eye}|}{f_{\text{ob}}} \frac{15\text{ cm}}{f_{\text{eye}}}$

Interference and diffraction

index of refraction: $n = \frac{c}{v}$

wavelength in medium: $\lambda_n = \frac{\lambda}{n}$

two mediums, same light, number of wavelength difference:

$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$

double-slit interference

fully constructive: $d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$

fully destructive: $d\sin\theta = (m + \frac{1}{2})\lambda, m = 0, 1, 2, \ldots$

illumination intensity: $I = 4I_0 \cos^2(\frac{1}{2}\phi)$, $\phi = \frac{2\pi d}{\lambda} \sin\theta$

(I_0: intensity of one slit when the ohter covered, d: seperation of slits), $\overline{I} = 2I_0$

real double-slit

intensity: $I = I_m \underbrace{(\cos^2\beta)}_{\text{intfr}} \underbrace{(\frac{\sin\alpha}{\alpha})^2}_{\text{diffr}}$

$\beta = (\frac{\pi d}{\lambda})\sin\theta$, $\alpha = (\frac{\pi a}{\lambda})\sin\theta$

multiple slits (N slits)

grating maxima: $d\sin\theta = m\lambda, m = 0, 1, 2, \ldots$

line width: $\Delta\theta = \frac{\lambda}{Nd\cos\theta}$

dispersion/seperation of lines: $D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$

resolving power: $R = Nm = \frac{\overline{\lambda}}{\Delta\lambda}$

thin film, $n_1, n_3 > n_2$ (incident at n_1)

(every larger n of refraction side causes phase change of $\lambda/2$)

fully constructive: $2n_2L = (m + \frac{1}{2})\lambda, m = 0, 1, 2, \ldots$

fully destructive: $2n_2L = m\lambda, m = 0, 1, 2, \ldots$

single-slit diffraction

intensity minima: $a\sin\theta = m\lambda, m = 1, 2, 3, \ldots$

intensity maximum: I_m at centre

intensity: $I = I_m(\frac{\sin\alpha}{\alpha})^2$, $\alpha = (\frac{\pi a}{\lambda})\sin\theta$ (a: width)

Rayleigh's criteria: $\theta_R = 1.22\frac{\lambda}{d}$ (d: lens' diameter)

seperation of two sources: $\Delta x \approx f\theta$ (f: may be viewing distance)

Modern physics

Special relativity

speed parameter: $\beta = v/c$

Lorentz factor: $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

time dilation: $\Delta t = \gamma \Delta t_0$

length contraction: $L = \frac{L_0}{\gamma}$

Lorentz transformation (S, S'):

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma(t - vx/c^2) \end{cases}, \begin{cases} x = \gamma(x' + vt') \\ t = \gamma(t' + vx'/c^2) \end{cases}$$

difference in x, t:

$$\begin{cases} \Delta x' = \gamma(\Delta x - v\Delta t) \\ \Delta t' = \gamma(\Delta t - v\Delta x/c^2) \end{cases}, \begin{cases} \Delta x = \gamma(\Delta x' + v\Delta t') \\ \Delta t = \gamma(\Delta t' + v\Delta x'/c^2) \end{cases}$$

relativistic velocity law: $V_{\text{CA}} = \frac{V_{\text{CB}} + V_{\text{BA}}}{1 + V_{\text{CB}} V_{\text{BA}}/c^2}$

Doppler effect: $f = f_0\sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$

conservation of space-time interval:

$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = \text{constant}$

relation of proper time: $\Delta s = c\Delta\tau$

4-displacement: $x^\mu(\tau) = [ct(\tau), x(\tau), y(\tau), z(\tau)]$

4-velocity: $u^\mu = \frac{d}{d\tau}x^\mu = \gamma(v)[c, v]$

magnitude of velocity: $|u^\mu| = \sqrt{(u^0)^2 - (u^1)^2} = c$

acceleration: $a^\mu = \frac{d}{d\tau}u^\mu \perp u^\mu$

4-momentum: $p^\mu = mu^\mu = m\gamma[c, v] = [\frac{E}{c}, \gamma mv]$

magnitude of momentum: $|p^\mu| = mc$

relativistic kinetic energy: $K = mc^2(\gamma - 1)$

total energy: $E = \gamma mc^2 = \underbrace{mc^2}_{\text{rest E}} + \underbrace{K}_{\text{kinetic E}}$

relations: $E^2 = (pc)^2 + (mc^2)^2$, $(pc)^2 = K^2 + 2Kmc^2$

Photons, particles

single slit experiment:

distance between central and first max: $y = \frac{\lambda L}{d}$

width of central max: $w = 2y = \frac{2\lambda L}{d}$

De Braglie relation: $h = \lambda p = \lambda\sqrt{2mK}$

Plank's constant: $h = 6.626 \cdot 10^{-34}\,\text{m}^2 \cdot \text{kg/s}$

energy of photon: $E = cp = c\frac{h}{\lambda} = hf$

photoelectric effect: $K_{\max} = hf - \phi$

Compton scattering:

electron stationary: $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

electron head-on: $\lambda' \approx \frac{hc}{E_e}\left[1 + \frac{m_e^2 c^4 \lambda}{4hcE_e}\right]$

blackbody radiation (low frequency)

distribution finding entities with energy E:

$p_E(E) = \frac{1}{KT}e^{-E/kT}$

average energy: $\overline{E} = kT$

distribution of number of standing waves per unit volume:

$p_N(f) = \frac{N(f)}{V} = \frac{8\pi}{c^2}f^2$

distribution of energy per unit volume:

$p_{E/V}(f) = \frac{E(f)}{V} = \frac{8\pi}{c^3}f^2 kT$

number of photons in standing wave: $n \approx \frac{kT}{hf}$

ultraviolet catastrophe (high frequency)

energy of photons: $E = nhf$

average energy: $\overline{E}(f) = \frac{hf}{e^{hf/kT}-1}$

distribution of number of standing waves per unit volume:

$p_N(f) = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT}-1}$

$p_N(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$

confining standing waves

probability distribution of finding particle: $p(x) = |\Psi(n)|^2$

wavelength, ground state: $\lambda_1 = 2L$

momentum, ground state: $p_1 = \frac{h}{2L}$

energy, ground state: $E_1 = \frac{h^2}{8L^2m}$

wavelength, excited state: $\lambda_n = \frac{\lambda_1}{n}$, $n = 1, 2, ...$

momentum, excited state: $p_n = np_1$, $n = 1, 2, ...$

energy, excited state: $E_n = n^2E_1$, $n = 1, 2, ...$

Heisenberg uncertainty principle: $\Delta x\Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$

corollary: $\Delta E\Delta t \propto \hbar$

particle diffraction, max angle: $\Delta\theta_{\max} = \frac{\lambda_0}{L}$

Waves 1

orbit of hydrogen atoms

constructive interference: $2\pi r = n\lambda, n = 1, 2, ...$

Bohr radius: $r_n = a_0n^2$, $a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.529\,\text{Å}$

original quantization condition: $p_n = \frac{\hbar}{a_0}\frac{1}{n}$, $L_n = \hbar n$

total energy: $E_n = -K_n = -\frac{\hbar^2}{2ma_0}\frac{1}{n^2} = -\frac{E_1}{n^2}$

ground state energy: $E_1 = 13.6\,\text{eV}$

free particle, 2D complex wave: $\Psi(x,t) = Ae^{\frac{1}{\hbar}(px-Et)}$

general Schrödinger equation:

$i\hbar\frac{\partial}{\partial t}\Psi(x,y,z,t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(x,y,z,t) + U(x,y,z)\Psi(x,y,z,t)$

time-independent Schrödinger equation:

$E\Psi(x,y,z) = -\frac{\hbar^2}{2m}\nabla^2\Psi(x,y,z) + U(x,y,z)\Psi(x,y,z)$

probability flux: $\mathbf{J} = \frac{p}{m}\nabla s$

normalisation: $\int|\Psi(x,y,z,t)|^2dv = 1$

particle bounded by nodes 0, L

potential function: $V(x) = \begin{cases} 0, & 0 \leq x \leq L \\ \infty, & x < 0 \vee x > L \end{cases}$

momentum: $p_n = \frac{h}{2L}n, n = 1, 2, ...$

energy: $E_n = \frac{h^2}{8mL^2}n^2, n = 1, 2, ...$

wave function, time-independent:

$\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}), n = 1, 2, ...$

wave function, time-dependent:

$\Psi_n(x,t) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})\cdot e^{-\frac{i}{\hbar}E_nt}, n = 1, 2, ...$

wave function, time-independent, 3D:

$\psi_n(x,y,z) =$
$\sqrt{\frac{2}{L}}\sin(\frac{n_x\pi x}{L})\cdot\sqrt{\frac{2}{L}}\sin(\frac{n_y\pi y}{L})\cdot\sqrt{\frac{2}{L}}\sin(\frac{n_z\pi z}{L}), n_i = 1, 2, ...$

energy assuming constant time: $E = \frac{\hbar}{2m}\frac{\pi^2}{L^2}(n_x^2 + n_y^2 + n_z^2)$

finite potential well

momentum in regions:

$p_x = \begin{cases} \pm\sqrt{2mE}, & 0 \leq x \leq L \\ \pm\sqrt{2m(E-U_0)}, & x < 0 \vee x > L \end{cases}$

momentum amplitude: $|p_x| = \hbar k = \sqrt{2m(U_0-E)}$

tunneling probability: $P_T \approx \alpha e^{-2kL}$, $\alpha = 16\frac{E}{U_0}(1-\frac{E}{U_0})$

harmonic oscillator

zero-point energy: $E_0 = \frac{1}{2}hf$

energy at nth level:

$E_n = (n+\frac{1}{2})hf = \underbrace{nhf}_{\text{number of photons}} + \underbrace{\frac{1}{2}hf}_{\text{zero-point energy}}$

energy in vacuum: $E_{\text{vac}} = -\frac{hc}{48}L$

The electrons

total energy of e^-: $E_n = -(\frac{1}{4\pi\epsilon_0})^2\frac{me^4}{2\hbar}\frac{1}{n^2} = -\frac{13.6\,\text{eV}}{n^2}$

principle quantum #: $n = 1, 2, ...$

angular momentum of e^-

magnitude of $\mathbf{L}$: $L = \sqrt{\ell(\ell+1)}\hbar$

orbital quantum #: $\ell = 1, 2, ..., n-1$

z-projection of $\mathbf{L}$: $L_z = m_\ell\hbar$

magnetic quantum #: $m_\ell = 0, \pm1, \pm2, ..., \pm\ell$

spin angular momentum of e^-

magnitude of $\mathbf{S}$: $S = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\hbar = \sqrt{\frac{3}{4}}\hbar$

z-projection of $\mathbf{S}$: $S_z = m_s\hbar$

spin quantum #: $m_s = \pm\frac{1}{2}$

z-projection of $\boldsymbol{\mu}_\mathbf{S}$: $\mu_z = -2.00232\frac{e}{2m}S_z$

orbital magnetic dipole moment: $\boldsymbol{\mu} = -\frac{1}{2}e\mathbf{r}\times\mathbf{v} = -\frac{e}{2m}\mathbf{L}$

spin magnetic dipole moment: $\boldsymbol{\mu}_\mathbf{S} = -2.00232\frac{e}{2m}\mathbf{S}$

Bohr magneton: $\mu_B = \mu_{e^-} = \mu_z = \frac{e\hbar}{2m} = 9.274\cdot10^{-24}\,\text{J/T}$

interacting energy: $U = m_\ell\mu_zB$

shell/subshell level convention

shell	K	L	M	N	...
n	1	2	3	4	...
max e^-	2	8	18	32	$2n^2$

subshell	s	p	d	f	g	h	...
ℓ	0	1	2	3	4	5	...
max e^-	2	6	10	14	18	22	...

Solid matter

diatomic molecular spectra

rotational energy level: $E_\ell = \ell(\ell+1)\frac{\hbar^2}{2I}, \ell = 0, 1, 2, ...$

vibrational energy level: $E_n = (n+\frac{1}{2})\hbar\sqrt{\frac{k'}{m_r}}, n = 0, 1, 2, ...$

reduced mass: $m_r = \frac{m_1m_2}{m_1+m_2}$

net moment of inertia: $I = m_rd^2$

1D sodium lattice

Fermi momentum: $p_F = N\Delta p\cdot2\cdot2$

Fermi energy: $E_F = \frac{h^2}{32m}(\frac{N}{L})^2$

gap energy: $E_{\text{gap}} = \overline{U_{\text{odd}}} - \overline{U_{\text{even}}}$

e^- position: $P_{\text{even}}(x) \propto \cos^2(\frac{\pi x}{a})$, $P_{\text{odd}}(x) \propto \sin^2(\frac{\pi x}{a})$

3D lattice (free particle in box)

energy of states: $E = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}(n_x^2 + n_y^2 + n_z^2)$, $n_i = 1, 2, ...$

number of states: $n(E) = \frac{(2m)^{3/2}V}{3\pi^2\hbar^3}E^{3/2}$

state density: $g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}$

distribution of state with E is filled (Fermi-Dirac):

$f_{T=0}(E) = \begin{cases} 1, & E \leq E_F \\ 0, & E > E_F \end{cases}$

$f_{T>0}(E) = \frac{1}{e^{(E-E_F)/kT}+1}$

Fermi energy: $E_F = \frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}n^{2/3}, n = \frac{N}{V}$

current-voltage relation (semiconductor): $I = I_s(e^{eV/kT}-1)$

Nuclear reactions

${}^A_Z\text{M}: \underbrace{A}_{\text{nucleon \#}} = \underbrace{Z}_{\text{protons}} + \underbrace{N}_{\text{neutrons}}$

radius of nucleon (approx.): $R = R_0A^{1/3}$, $R_0 = 1.2\,\text{Fm}$

volume of nucleon (approx.): $V = \frac{4}{3}\pi R_0^3A$

common nucleon masses

mass of ${}^{12}_6\text{C}$: $M_{{}^{12}_6\text{C}} = 1\,\text{u} = 1.660538782\cdot10^{-27}\,\text{kg}$

mass of ${}^1_1\text{H}$: $M_{{}^1_1\text{H}} = 1.007826\,\text{u}$

mass of proton: $m_p = 1.007276\,\text{u}$

mass of neutron: $m_n = 1.008665\,\text{u}$

mass of electron: $m_{e^-} = 0.00054858\,\text{u}$

spin quantum momentum of p and n

magnitude of $\mathbf{S}$: $S = \sqrt{\frac{3}{4}}\hbar$

spin quantum #: $m_s = \pm\frac{1}{2}$

z-projection of $\mathbf{S}$: $S_z = m_s\hbar$

total angular momentum: $\mathbf{J} = \mathbf{L} + \mathbf{S}$

nuclear magneton: $\mu_{\text{nucl}} = \frac{e\hbar}{2m_p} = 5.05078 \cdot 10^{-27}\,\text{J/T}$

proton magneton: $|\mu_p| = 2.7928\mu_{\text{nucl}}$

neutron magneton: $|\mu_n| = 1.9130\mu_{\text{nucl}}$

nuclear binding

procedure: ${}^{A}_{Z}\text{M} + \text{K} \rightarrow (Z)\,{}^{1}_{1}\text{H} + (N)\,\text{n}$

mass defect: $\Delta m = Z(m_p + m_e) + Nm_n - M_{\text{original}}$

nuclear binding energy: $E_B = c^2\Delta m$

binding energy per nucleon: $\frac{E_B}{A}$

liquid drop model

binding energy:

$E_B = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(A-2Z)^2}{A} + C_5 A^{-4/3}$

$C_1 = 15.75\,\text{MeV}$, $C_2 = 17.8\,\text{MeV}$,

$C_3 = 0.71\,\text{MeV}$, $C_4 = 23.69\,\text{MeV}$,

$C_5 = 39\,\text{MeV}(Z, N \text{ even}), -39\,\text{MeV}(Z, N \text{ odd}), 0\,(\text{else})$

semiemperical mass formula: ${}^{A}_{Z}M = (Z)M_{{}^{1}_{1}\text{H}} + (N)m_n - \frac{E_B}{c^2}$

alpha decay: ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He} + \text{K}$

result: $Z \rightarrow Z-2, N \rightarrow N-2, A \rightarrow A-4$

energy required: $M_{\text{parent}} > M_{\text{daughter}} + M_{{}^{4}_{2}\text{He}}$

centre of momentum frame: $m_{\text{He}}v_{\text{He}} = M_{\text{Y}}V_{\text{Y}}$

shared kinetic energy: $K = K_{\text{He}} + K_{\text{Y}}$

β^- decay: $\text{n} \rightarrow \text{p} + \text{e}^- + \overline{\nu_e} + \text{K}$

result: $Z \rightarrow Z+1, N \rightarrow N-1$

energy required: $M_{\text{parent}} > M_{\text{daughter}}$

β^+ decay: $\text{p} \rightarrow \text{n} + \text{e}^+ + \nu_e + \text{K}$

result: $Z \rightarrow Z-1, N \rightarrow N+1$

energy required: $M_{\text{parent}} > M_{\text{daughter}} + 2m_{e^-}$

electron capture: $\text{p} + \text{e}^- + \text{K} \rightarrow \text{n} + \nu_e$

result: $Z \rightarrow Z-1, N \rightarrow N+1$

energy required: $M_{\text{parent}} > M_{\text{daughter}}$

gamma decay: ${}^{A}_{Z}\text{X}^* \rightarrow {}^{A}_{Z}\text{X} + \gamma$

nuclear activities

activity: $|\frac{dN}{dt}| \equiv \lambda N(t)$

Curie: $1\,\text{Ci} = 3.7 \cdot 10^{10}\,\frac{\text{decays}}{\text{s}}$ (Bq)

# of radioactive nuclei: $N(t) = N_0 e^{-\lambda t}$

half-life: $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

mean-life: $T_{\frac{1}{e}} = \frac{1}{\lambda}$

nuclear reaction: $\text{A} + \text{B} \rightarrow \text{C} + \text{D} + \ldots$

reaction energy: $Q = c^2\Delta m = c^2(m_{\text{reactant}} - m_{\text{product}})$

($Q > 0$: exothermic, $Q < 0$: endothermic)

Cosmology

collider energies

available energy: $E_a = \sum_i m_i c^2$

relation: $E_a^2 = m_B^2c^4 + m_T^2c^4 + 2(E_BE_T + c^2\mathbf{p}_B \cdot \mathbf{p}_T)$

fixed target: $E_a^2 = m_B^2c^4 + m_T^2c^4 + 2m_Tc^2E_B$

high energy: $E_a = \sqrt{2E_Bm_Tc^2}$

head-on, same mass, high energy: $E_a = 2E_B$

max living time of force mediator: $\Delta t < \frac{\hbar}{2\Delta E}$

Useful mathematical relations

Geometry

area of rectangle: $S = xy$

length of arc: $s = \theta r$

area of sector: $S = \frac{1}{2}sr = \frac{1}{2}r^2\theta$

area of ellipse: $S = \pi ab$

volume of pyramid: $V = \frac{1}{3}Sh$

infinitesimal-width ring: $dS = d(\pi r^2) = 2\pi r dr$

infinitesimal-width hollow sphere: $dV = d(\frac{4}{3}\pi r^3) = 4\pi r^2 dr$

conic section

polar form: $r = \frac{ed}{1+e\cos\theta}$

circle: $e = 0$

ellipse: $0 < e < 1$

parabola: $e = 1$

hyperbola: $e > 1$

semi-latus rectum: $l = ed$

trigonometric identities

$\sin^2\alpha + \cos^2\alpha = 1$

$\tan^2\alpha + 1 = \sec^2\alpha$

$\cot^2\alpha + 1 = \csc^2\alpha$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\cos\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha + \sin\beta = 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

$\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

Algebra

binomial theorem: $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$

binomial theorem:

$(1+x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \ldots, |x| < 1$

complex division: $\frac{a+bi}{x+yi} = \frac{(a+bi)(x-yi)}{x^2+y^2}$

De Moivre's formula: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

triangle inequality: $|x| + |y| \geq |x+y|$

Cauchy–Schwarz inequality: $|x||y| \geq |x \cdot y|$

cross product: $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{vmatrix} \mathbf{i} & x_1 & x_2 \\ \mathbf{j} & y_1 & y_2 \\ \mathbf{k} & z_1 & z_2 \end{vmatrix}$

angle between vectors: $\cos\angle\langle\mathbf{x}, \mathbf{y}\rangle = \frac{\mathbf{x}\cdot\mathbf{y}}{||\mathbf{x}||||\mathbf{y}||}$

rotation matrix: $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Calculus

taylor series:

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, |x| < 1$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

curve length: $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx$

mean value approximation:

$f(x + \Delta x, y + \Delta y) \approx f(x, y) + f_x(x, y)\Delta x + f_y(x, y)\Delta y$

derivative vector: $\mathbf{f}'(t) = x'(t)\,\mathbf{i} + y'(t)\,\mathbf{j} + z'(t)\,\mathbf{k}$

gradient: $\nabla f = (\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k})f$

flux density: $\operatorname{div}\mathbf{f} = \nabla \cdot \mathbf{f}$

circulation density: $\operatorname{curl}\mathbf{f} = \nabla \times \mathbf{f}$

line integral: $\int_L \mathbf{F} \cdot d\mathbf{r} = \int_{t=a}^{t=b} \mathbf{F} \cdot \mathbf{r}'(t)dt$

gradient theorem: $\int_{L[\mathbf{a},\mathbf{b}]} \nabla F(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = F(\mathbf{b}) - F(\mathbf{a})$

Gauss' theorem: $\iiint_V \nabla \cdot \mathbf{F}\,dV = \oiint_S \mathbf{F} \cdot \mathbf{n}\,dS$

Stokes' theorem: $\oint_\Gamma \mathbf{F} \cdot d\mathbf{s} = \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$

End of formula sheet

Version 1.1.1

This summary was made possible by *cos*. It was coded to help learners recall their knowledge and understanding of general physics. As physics equations do not change according to textbooks thus remaining universal, please reply to the thread where this was published in case of any errors.