

# Classification

Hendrik Santoso Sugiarto

IBDA2032 – *Artificial Intelligence*

# Capaian Pembelajaran

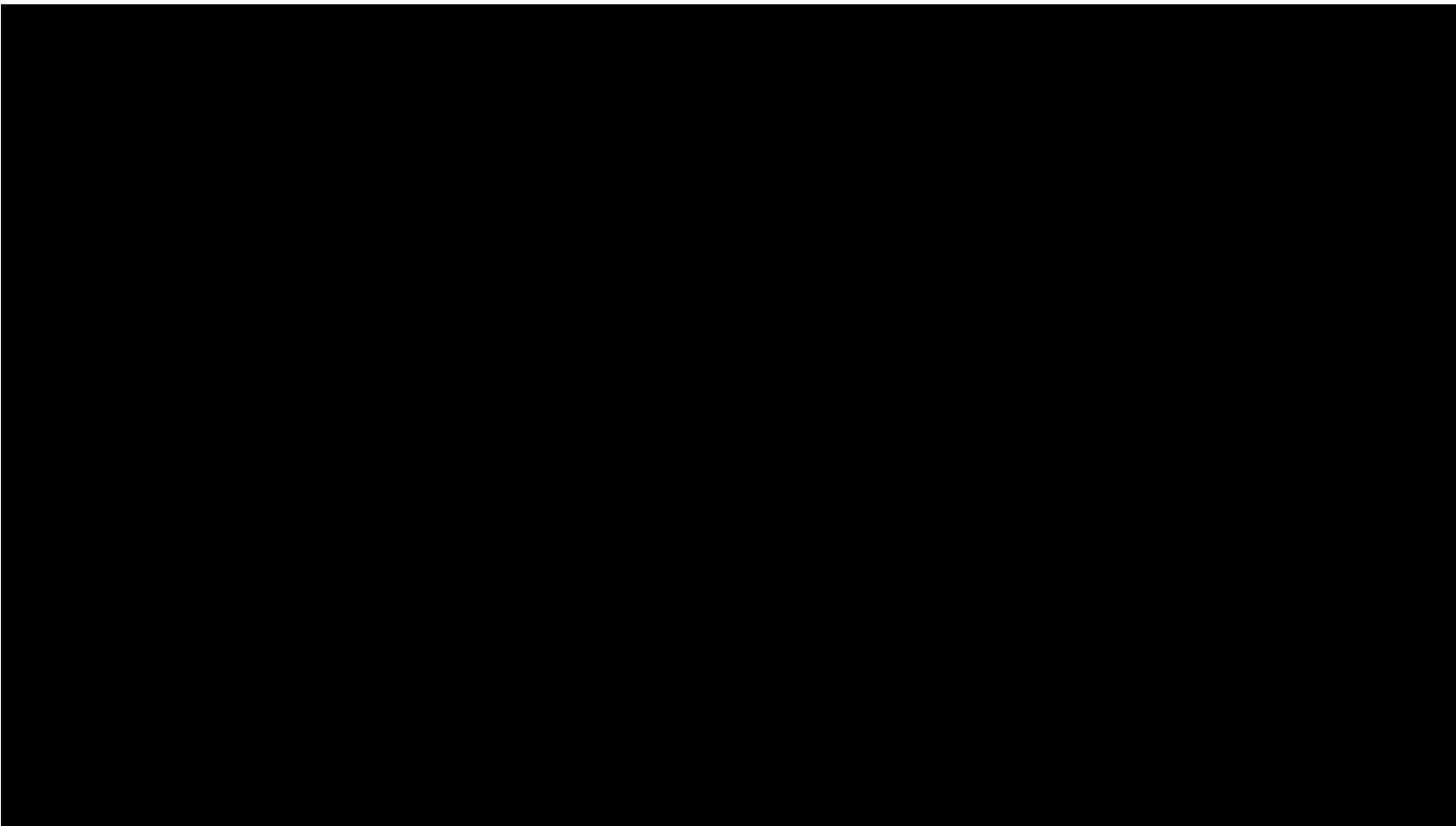
- Konsep Klasifikasi
- Klasifikasi Biner
- Klasifikasi Multi-Kelas
- Klasifikasi Multi-Label

# Klasifikasi

# Supervised Learning

- Bentuk formal:
  - Input:  $x \in \mathcal{X} \in \mathbb{R}^n$
  - Output:  $y \in \mathcal{Y} \in \begin{cases} \mathbb{R} \rightarrow \text{regresi} & \xrightarrow{\text{Real}} \\ \{+1, -1\} \rightarrow \text{klasifikasi biner} & (1, 0) \\ \{1, 2, \dots, K\} \rightarrow \text{klasifikasi multikelas} & \end{cases}$
  - Target function:  $f: \mathcal{X} \rightarrow \mathcal{Y}$  (*unknown*)
  - Training data:  $D = \{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$
  - Hypothesis:  $h: \mathcal{X} \rightarrow \mathcal{Y}$
  - Hypothesis space:  $h \in \mathcal{H}$

# Klasifikasi Satu Kelas



# Klasifikasi Multi Kelas



[https://www.youtube.com/watch?v=\\_pm7lrGmIQ](https://www.youtube.com/watch?v=_pm7lrGmIQ) Diakses pada tanggal 7 Maret 2021

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# Klasifikasi

- Proses mengelompokan sekumpulan data ke dalam kategori yang telah ditentukan sebelumnya
- Tipe klasifikasi:
  - Klasifikasi biner (ya/tidak, laki-laki/perempuan)
  - Klasifikasi multi-kelas (identifikasi suatu gambar angka)
  - Klasifikasi multi-label (identifikasi beberapa objek di satu gambar)
  - Klasifikasi multi-output (identifikasi objek dan lokasinya)

# Uji Pemahaman

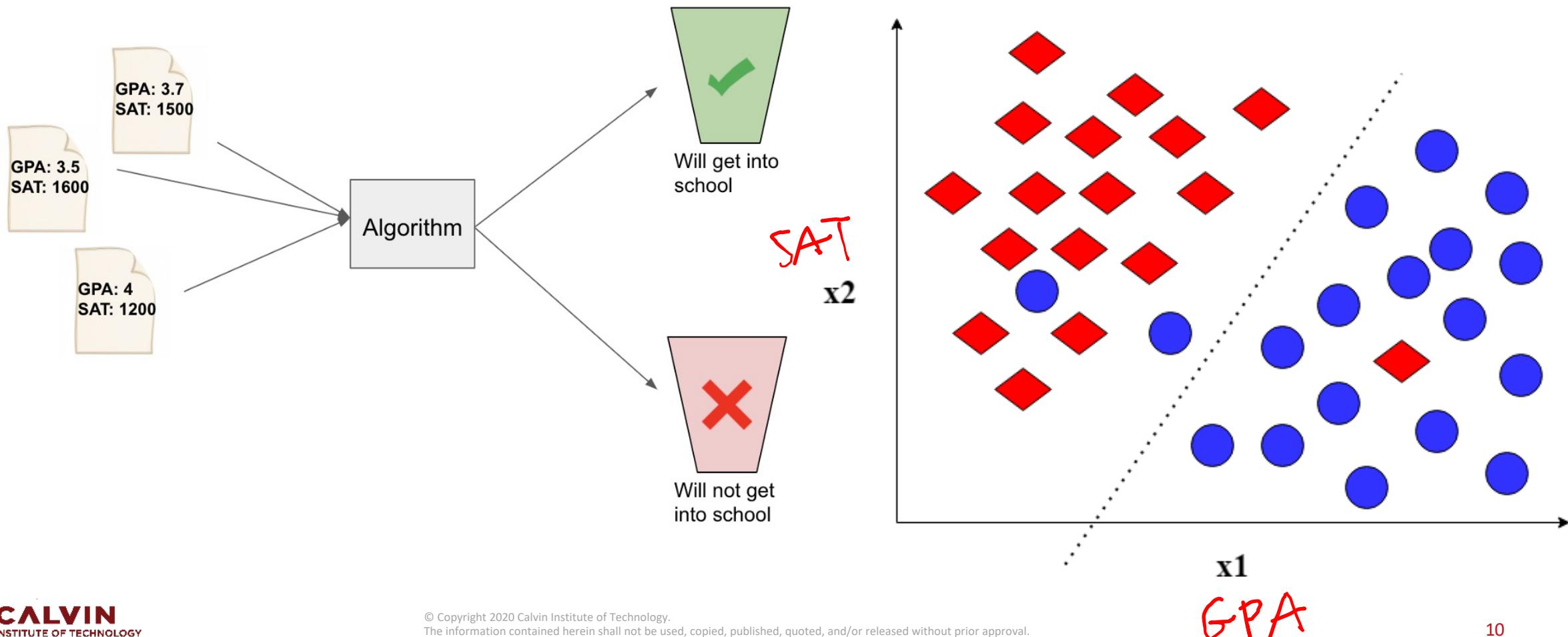
- Apakah jenis klasifikasi untuk memprediksi genre movie?

multilabel

# Klasifikasi Biner

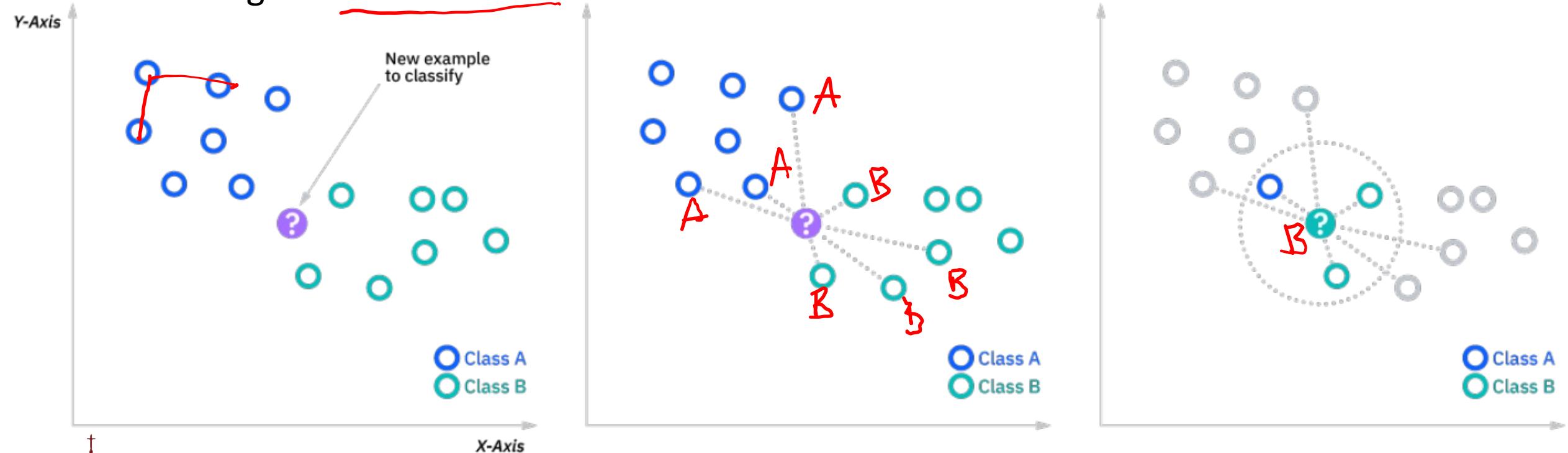
# Klasifikasi Biner

- Model yang dibuat dapat membagi data menjadi 2 kelas



# K-Nearest Neighbor

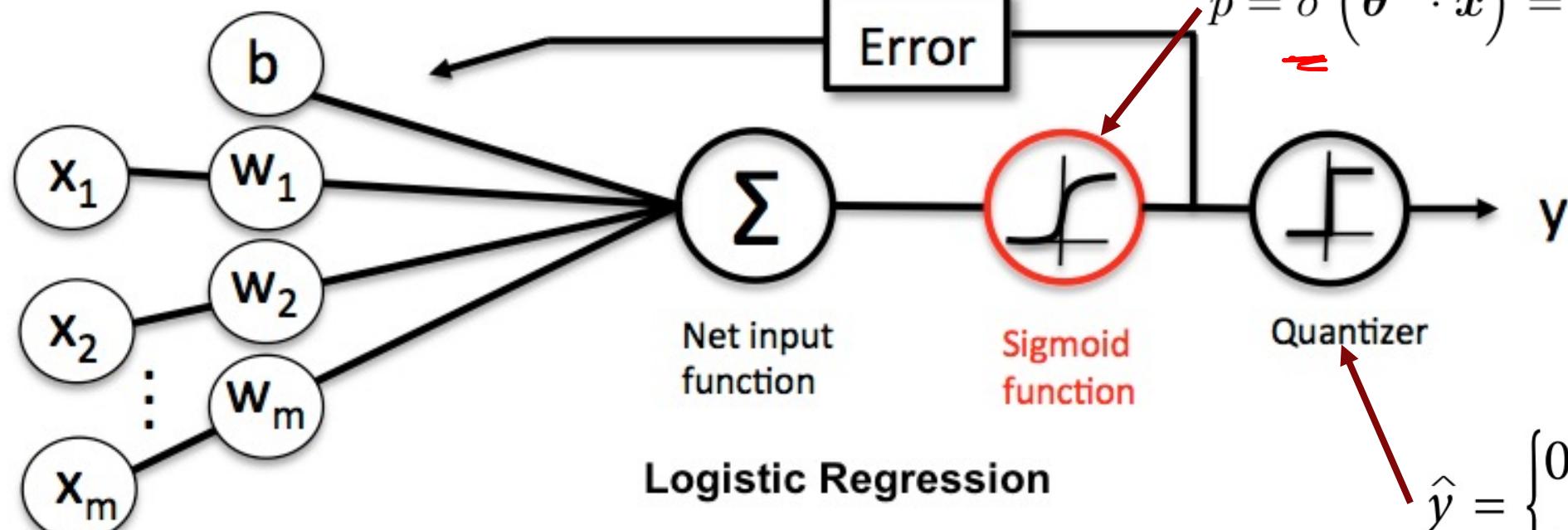
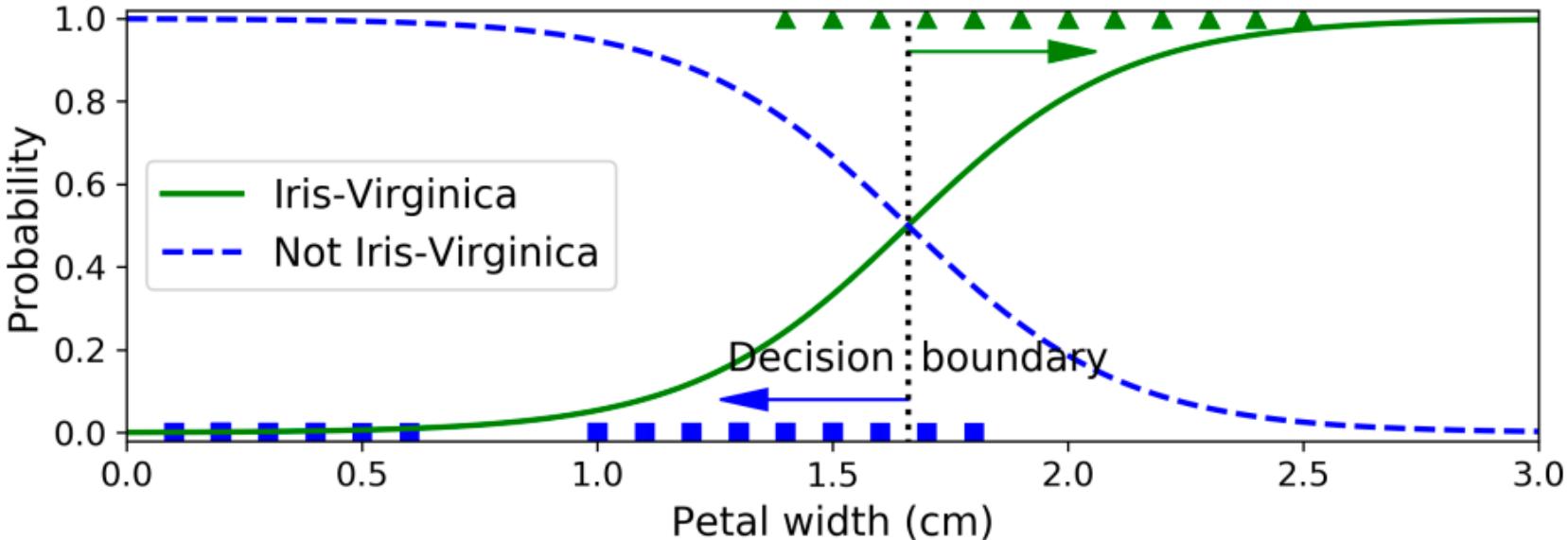
- ~~Lazy~~ learning *No learning*
- Hyperparameter:  $k$  ( $k$  terlalu besar  $\rightarrow$  underfitting,  $k$  terlalu kecil  $\rightarrow$  overfitting)
- Kelebihan: mudah diimplementasikan
- Kekurangan: tidak scalable



# Regresi Logistik

$$\theta_0 + \theta_1 x_1 + \theta_2 x_2 = \theta^T \cdot x$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$



$$\hat{y} = \begin{cases} 0 & \text{if } \hat{p} < 0.5 \\ 1 & \text{if } \hat{p} \geq 0.5 \end{cases}$$

# Gradient Descent

- Cost function menggunakan cross entropy:

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log(\hat{p}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{p}^{(i)})]$$

$\theta_j$

- Turunan parsial:

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^m (\sigma(\theta^T x^{(i)}) - y^{(i)}) x_j^{(i)}$$

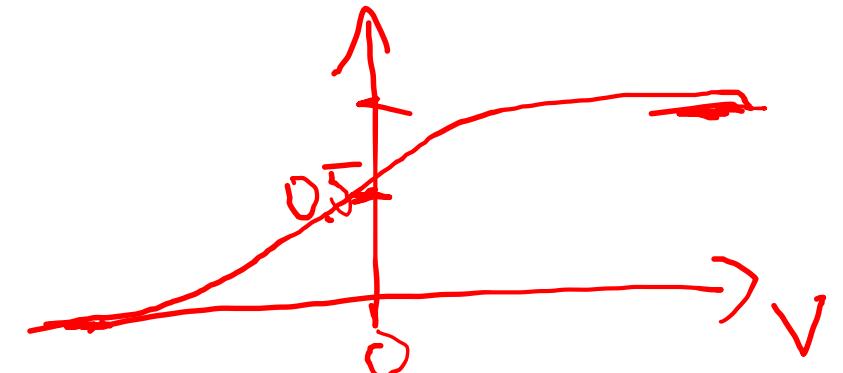
- Gradient Descent:

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_j) = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (\sigma(\theta^T x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$\frac{1}{1+e^{-c}}$$

# Uji Pemahaman

- Sebuah pembelajaran regresi logistik terhadap data barang rusak ( $y = 1 \rightarrow$  rusak,  $y = 0 \rightarrow$  tidak rusak) menghasilkan  $\theta_0 = 0$ ,  $\theta_1 = 1$ ,  $\theta_2 = 2$ . Apakah hasil prediksi dari:
  - $x_1 = 0, x_2 = 0$
  - $x_1 = 1, x_2 = 2$
  - $x_1 = 1, x_2 = -2$
- Apakah bentuk persamaan dari decision boundary nya?



$$h(x_1, x_2) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

$$\sigma(0 + 1 \cdot 0 + 2 \cdot 0) = 0.5$$

$$\sigma(0 + 1 \cdot 1 + 2 \cdot 2) = \sigma(5) \approx 1$$

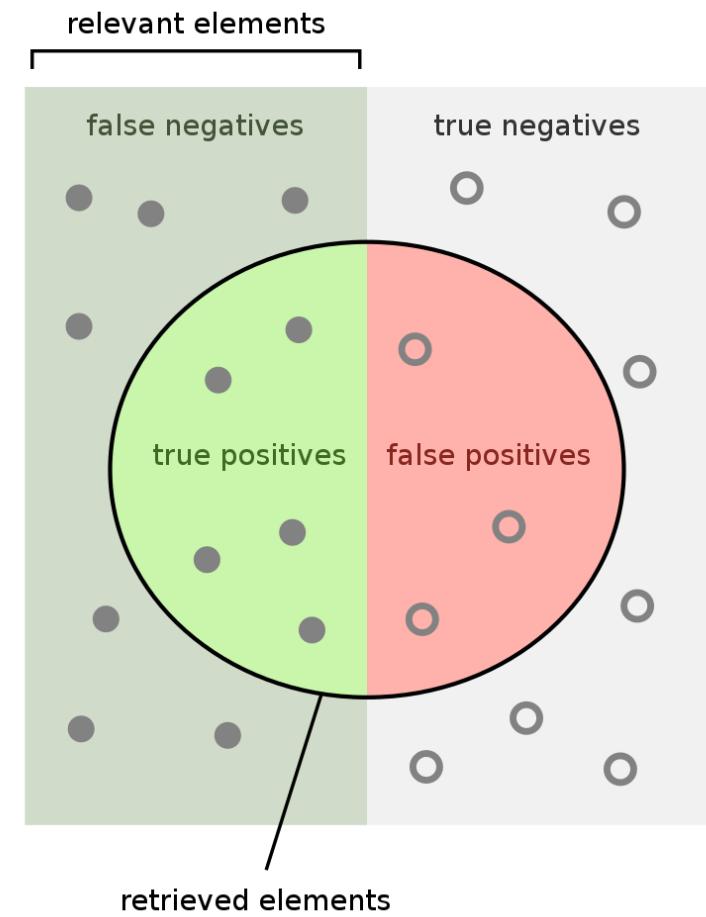
$$\sigma(0 + 1 \cdot 1 + 2 \cdot (-2)) = \sigma(-3) \approx 0$$

# Metrik Performa

- Confusion matrix:
  - True positives
  - False positives
  - False negatives
  - True negatives

$$\begin{aligned} \text{accuracy} &= \frac{TP+TN}{TP+FP+FN+TN} \\ \text{precision} &= \frac{TP}{TP+FP} \\ \text{recall} &= \frac{TP}{TP+FN} \\ F1 &= 2 \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \end{aligned}$$

|                 |   | Actual Class   |                |
|-----------------|---|----------------|----------------|
|                 |   | 1              | 0              |
| Predicted Class | 1 | True Positive  | False Positive |
|                 | 0 | False Negative | True Negative  |



# Uji Pemahaman

- Manakah yang lebih penting, menangkap false positive atau false negative?

tergantung situasinya

|                 |   | 1              | 0              |
|-----------------|---|----------------|----------------|
| Predicted Class | 1 | True Positive  | False Positive |
|                 | 0 | False Negative | True Negative  |

# Accuracy

$$\bullet \text{accuracy} = \frac{TP+TN}{TP+FP+FN+TN}$$

- Berapa % hasil yang diprediksi tepat

Well, here's the model: simply label every single person flying from a U.S. airport as "not a terrorist." Given the 800 million average passengers on U.S. flights per year and the 19 (confirmed) terrorists who boarded U.S. flights from 2000–2017, this model achieves an astounding accuracy of 99.9999999 percent! That might sound impressive, but I have a suspicion the Department of Homeland Security will not be calling anytime soon to buy this model. While this solution has nearly perfect accuracy, this problem is one in which accuracy is clearly not an adequate metric.

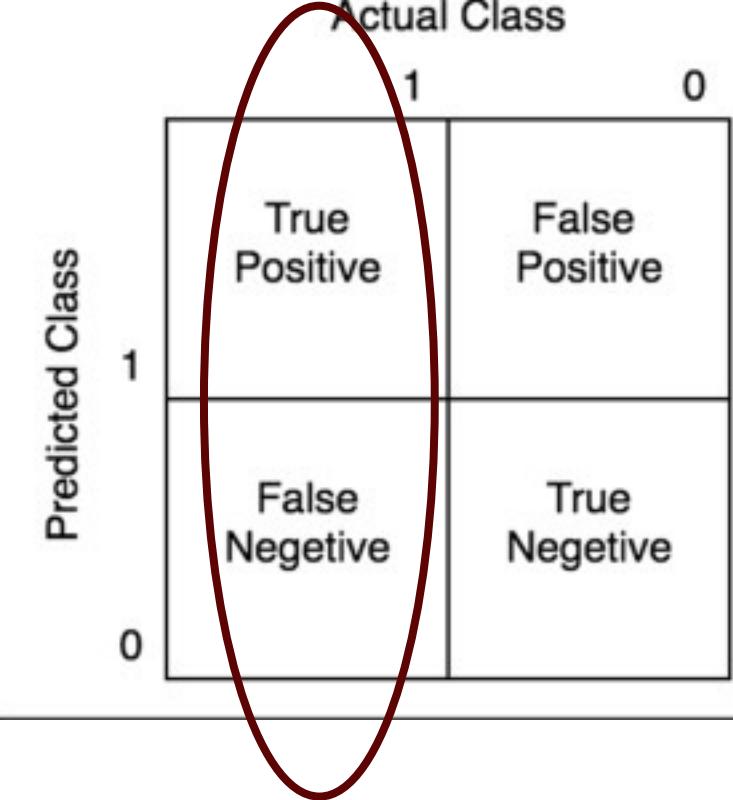
|                 |   | 1              | 0              |
|-----------------|---|----------------|----------------|
| Predicted Class | 1 | True Positive  | False Positive |
|                 | 0 | False Negative | True Negative  |

# Precision

- $precision = \frac{TP}{TP+FP}$
- Berapa % kasus yang diprediksi positif benar-benar positif
- High precision diperlukan ketika harga yang harus dibayar untuk kasus false positive tinggi
- Contoh false positive: delete email yang bukan spam → lebih baik terima spam daripada delete email penting

# Recall

- $recall = \frac{TP}{TP+FN}$
- Berapa % kasus yang benar-benar positif dapat teridentifikasi dengan tepat
- High recall diperlukan Ketika harga yang harus dibayar untuk false negative tinggi
- Contoh false negative: tidak diprediksi ada tsunami padahal akan terjadi tsunami → lebih baik berjaga-jaga untuk evakuasi daripada memakan korban jiwa



|                 |   | 1              | 0              |
|-----------------|---|----------------|----------------|
| Predicted Class | 1 | True Positive  | False Positive |
|                 | 0 | False Negative | True Negative  |

# F1

- $F1 = 2 \frac{precision \times recall}{precision + recall}$
- Harmonic mean of precision and recall
- Balance antara precision dna recall
- High F1 diperlukan jika harga yang harus dibayar untuk false positive dan flase negative sama tinggi
- Misalnya quality control → terlalu kaku banyak barang bagus yang terbuang, terlalu longgar banyak barang rusak yang lolos

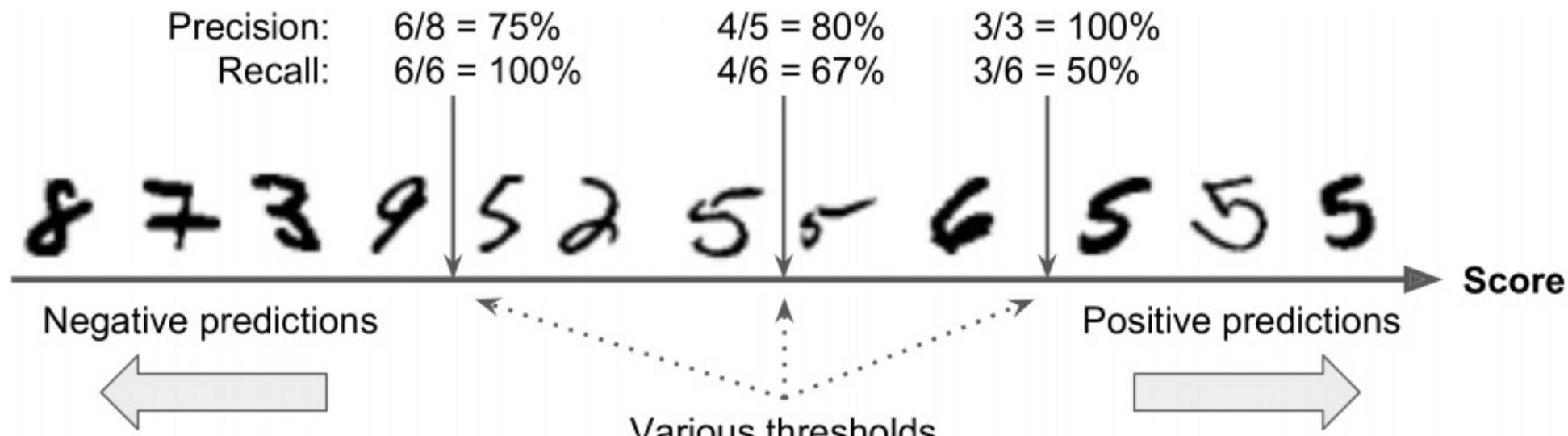
# Studi kasus

- Klasifikasi apakah suatu box pasta gigi kosong atau tidak?
- Menggunakan x-ray memakan biaya mahal



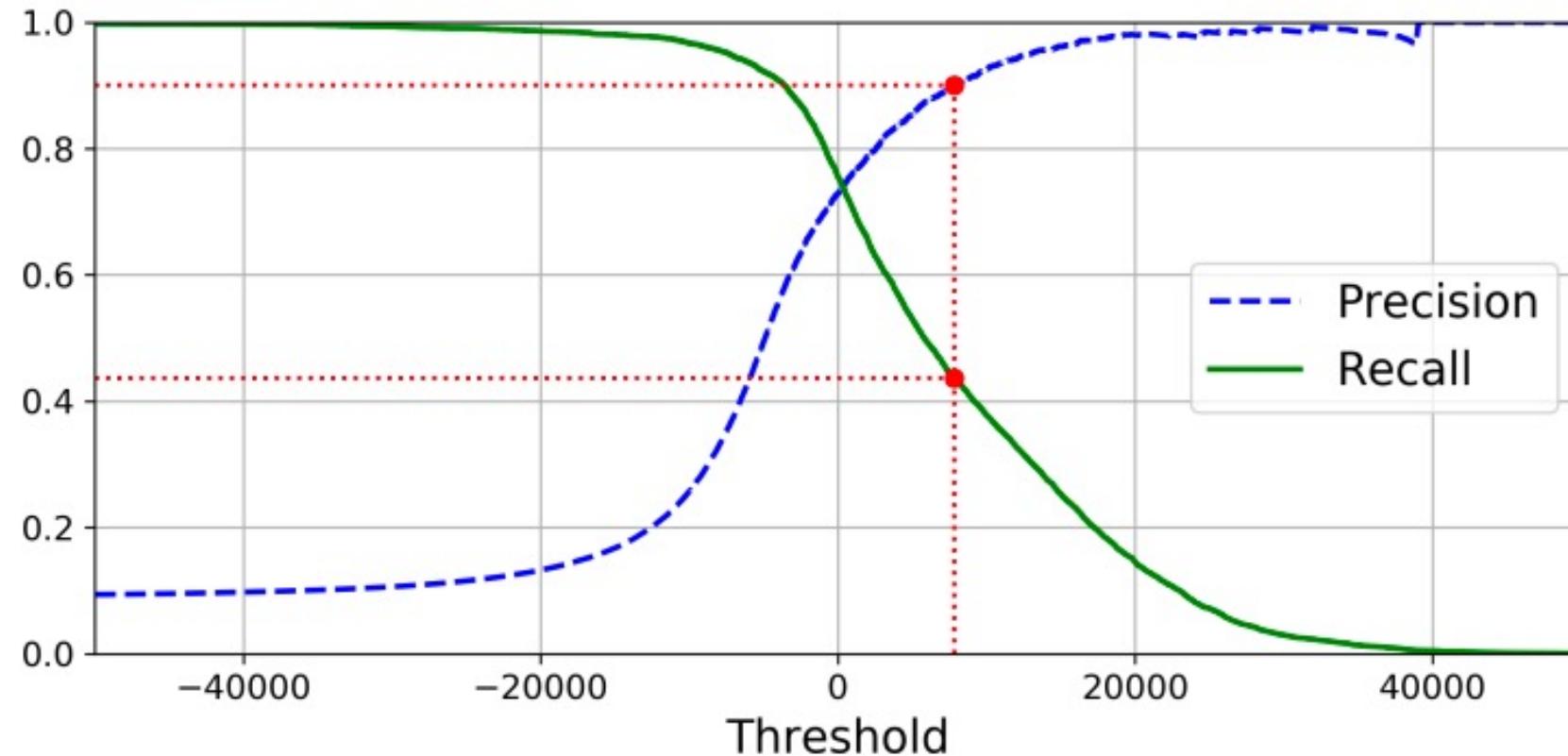
# Precision-Recall Tradeoff

- Meningkatkan precision akan menurunkan recall dan sebaliknya
- Manakah boundary decision yang tepat untuk menghasilkan klasifikasi yang baik?
- Misal kita pilih 50% threshold → 80% precision, 67% recall
- Misal kita pilih lebih tinggi → 100% precision, 50% recall
- Misal kita pilih lebih lemah → 75% precision, 100% recall



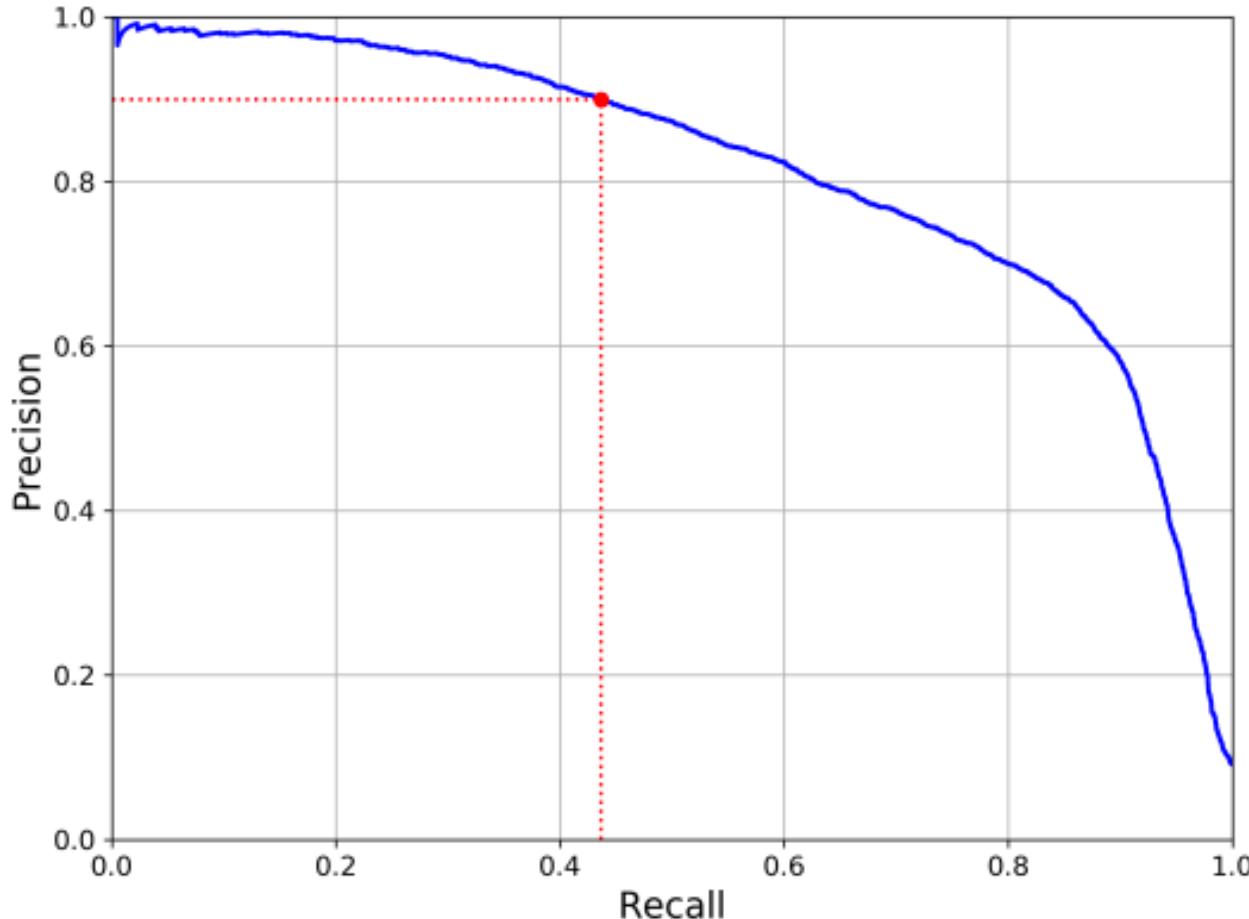
# Precision-Recall Curve

- Dapatkan skor untuk semua data dalam training set, hitung precision dan recall → plot precision-recall curve
- Titik merah menunjukan 90% precision, 43% recall



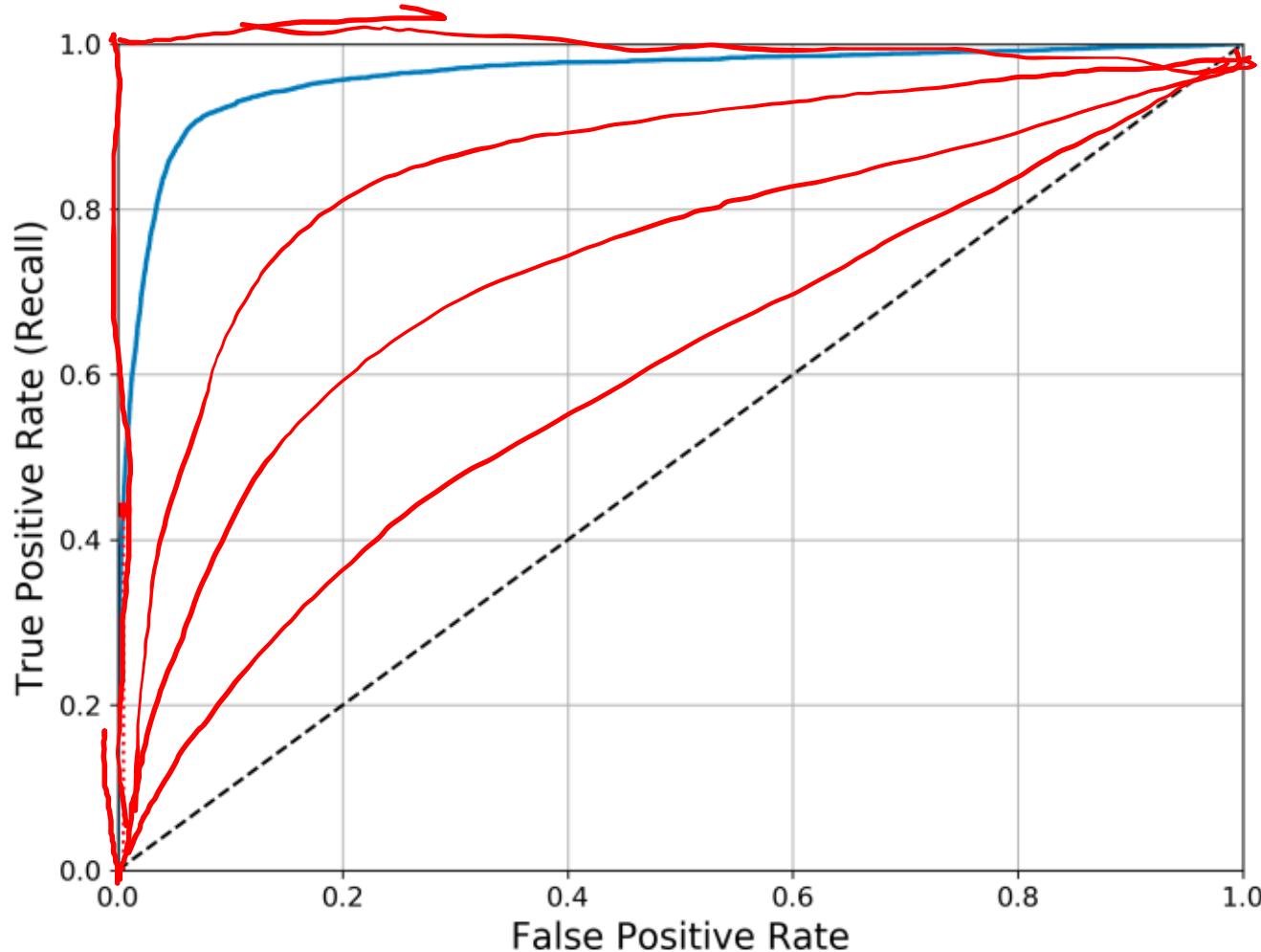
# Precision vs recall curve

- Plot precision vs recall untuk berbagai threshold (titik merah: 90% precision, 43% recall)
- Kurva menurun tajam >80% recall (disarankan memilih threshold sebelum titik curam)



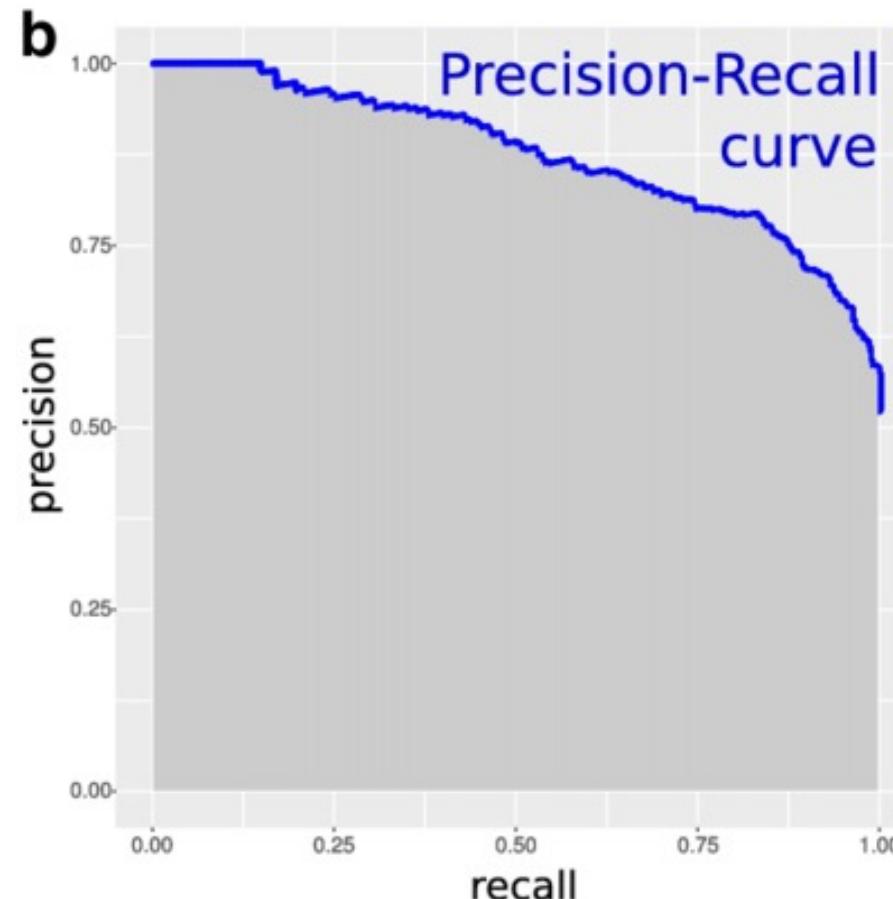
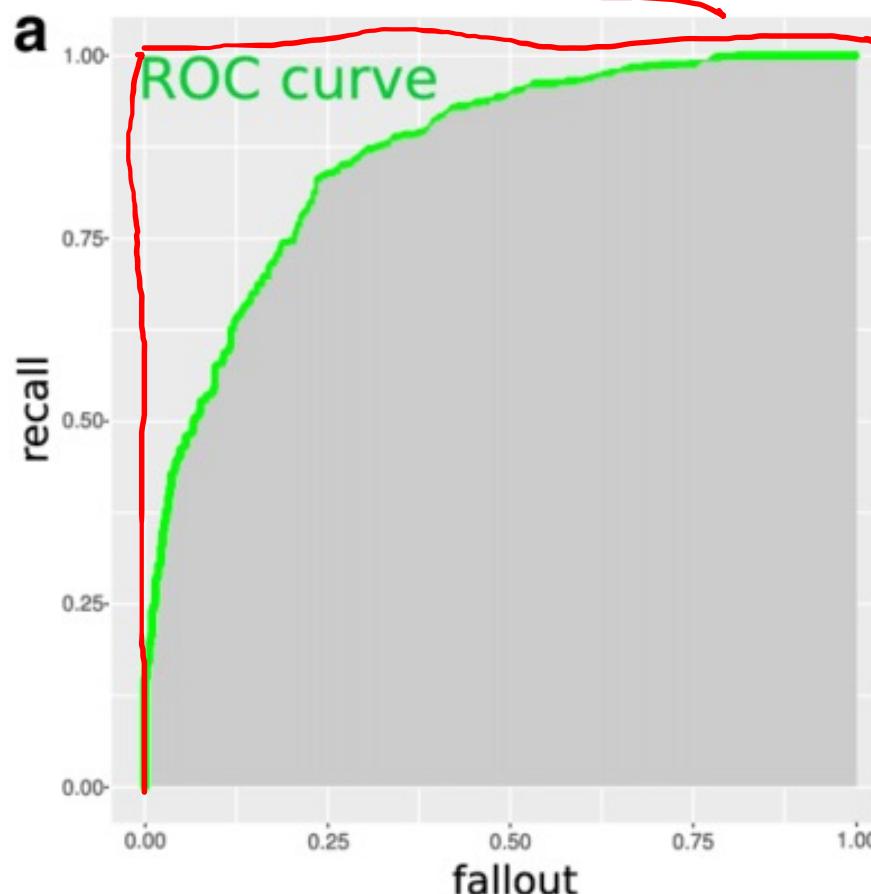
# Receiver operating characteristics (ROC)

- Plot TPR vs FPR untuk berbagai nilai thresholds
- Sumbu vertikal: true positive rate
- Sumbu horizontal: false positive rate
- Garis putus-putus: 50% prediksi benar



# Area Under Curve (AUC)

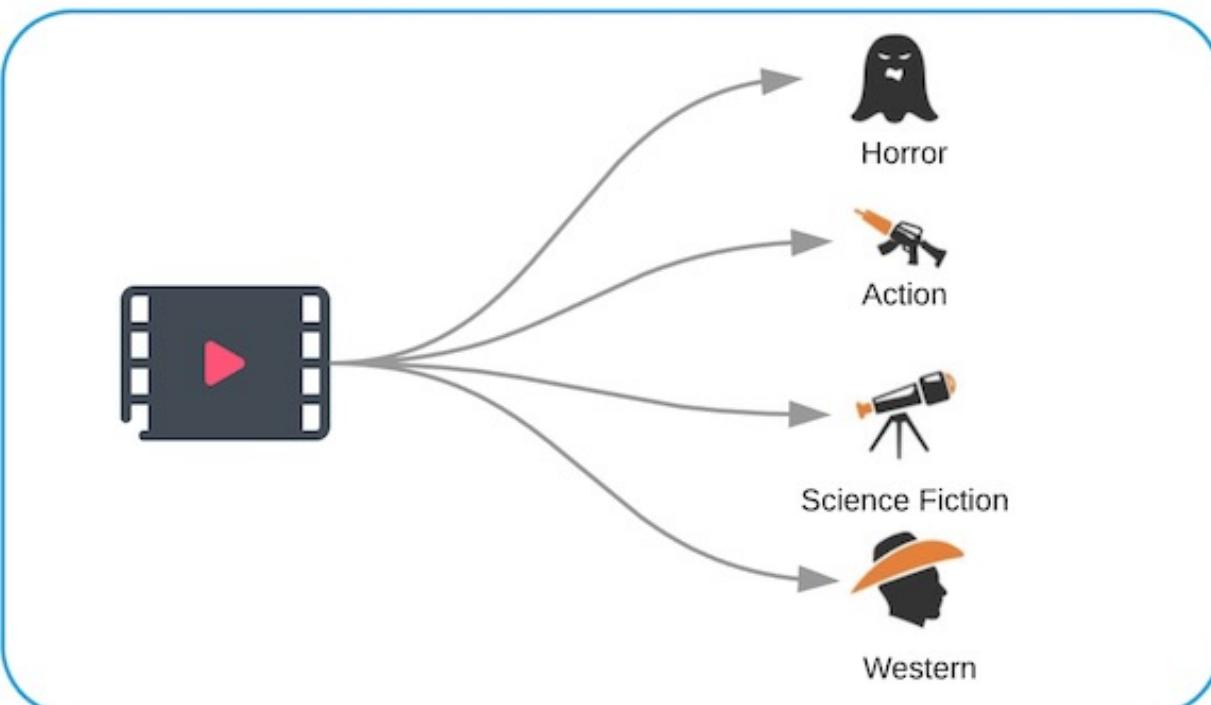
- Menghitung luas area dibawah suatu kurva
- Nilai AUC akan maksimal (AUC=1) jika kurva menentuh poiok



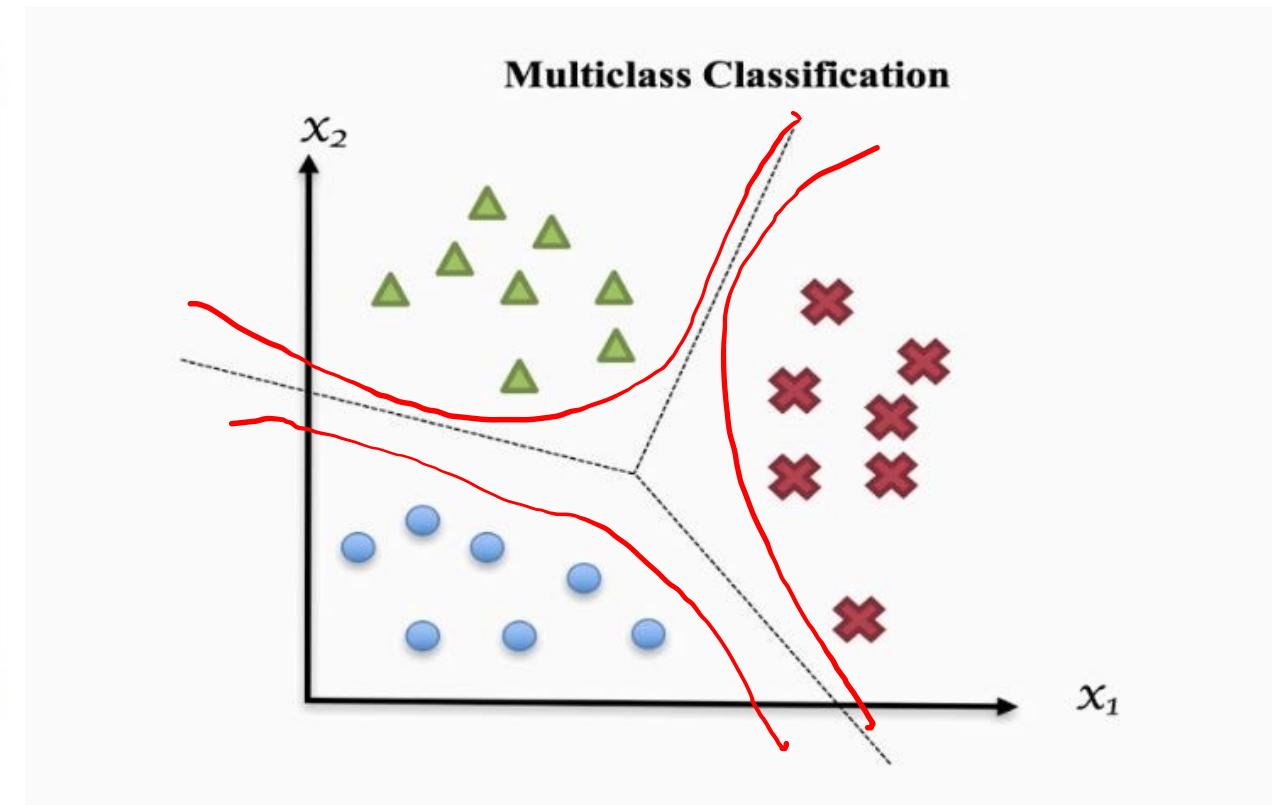
# Klasifikasi Multi-Kelas

# Klasifikasi Multi-Kelas

- Model yang dibuat dapat membagi data menjadi K kelas



Multi-class classification



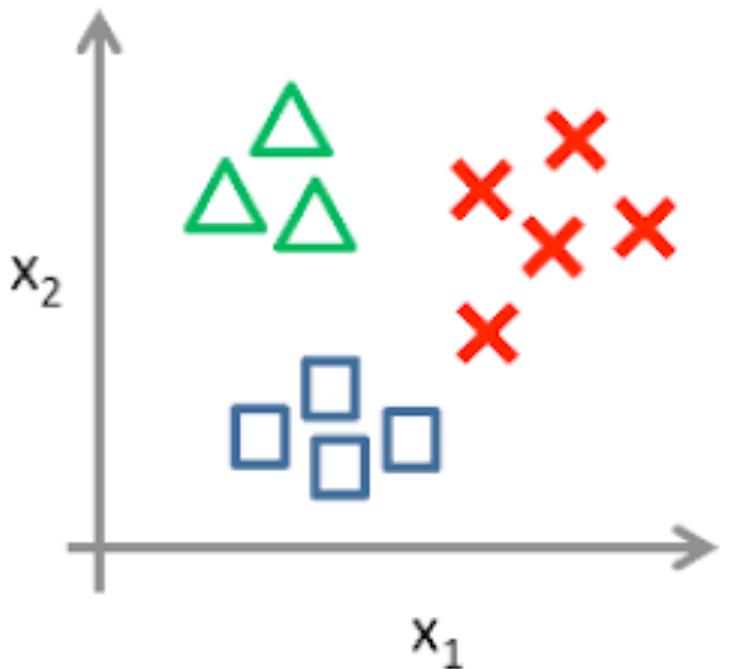
# Pendekatan

A  
B  
C

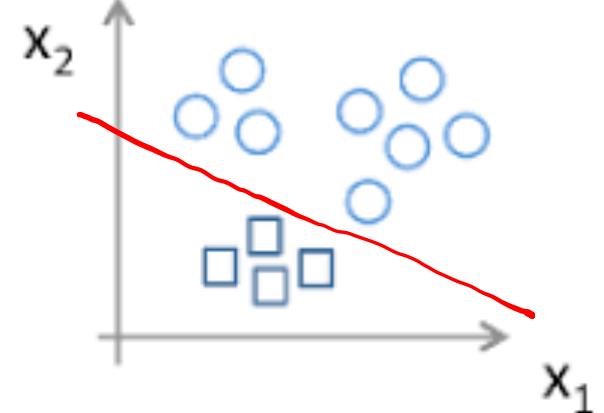
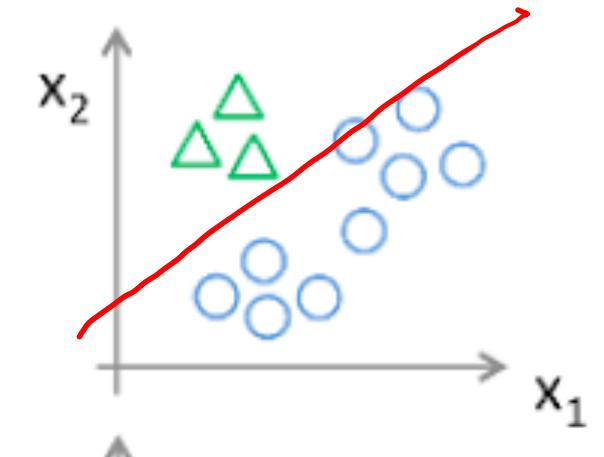
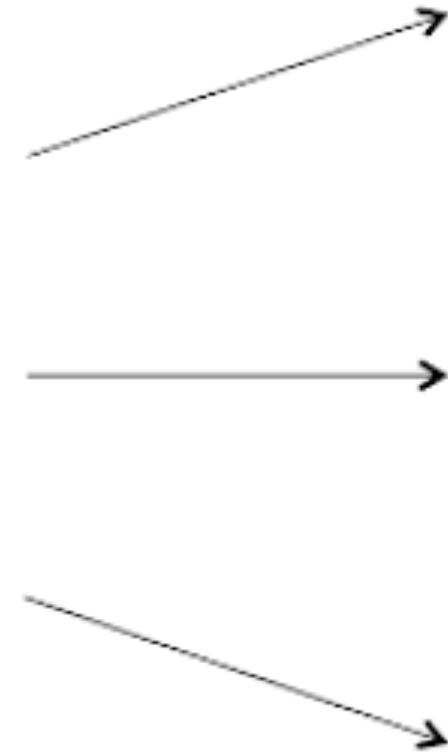
A VS BC  
B VS AC  
C VS AB

- Binary (one vs all): K-classifiers on large training sets (0-detectors, 1-detectors, ..., K-detectors) → hasil: skor dari setiap classifier → ambil nilai tertinggi
- Binary (one vs one):  $\frac{K(K-1)}{2}$ -classifiers on small training sets (0 vs 1, 0 vs 2, ..., K-1 vs K) → hasil: skor dari setiap duel → ambil kelas yang paling banyak menang duel
- Multiclass: 1-classifiers on entire training sets → hasil: skor dari setiap kelas → ambil nilai tertinggi

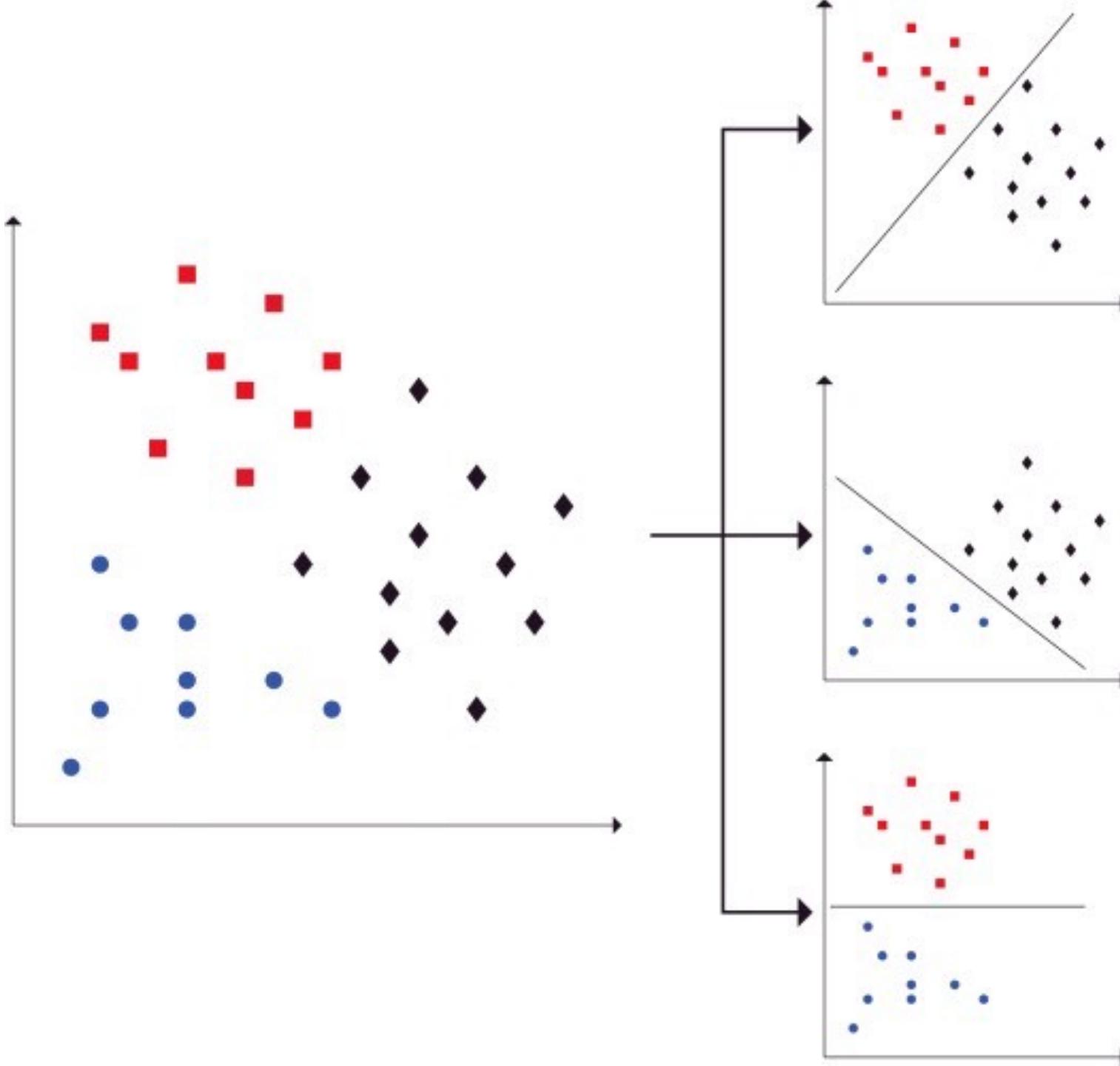
# 1 vs all One-vs-all (one-vs-rest):



Class 1: **Green**  
Class 2: **Blue**  
Class 3: **Red**

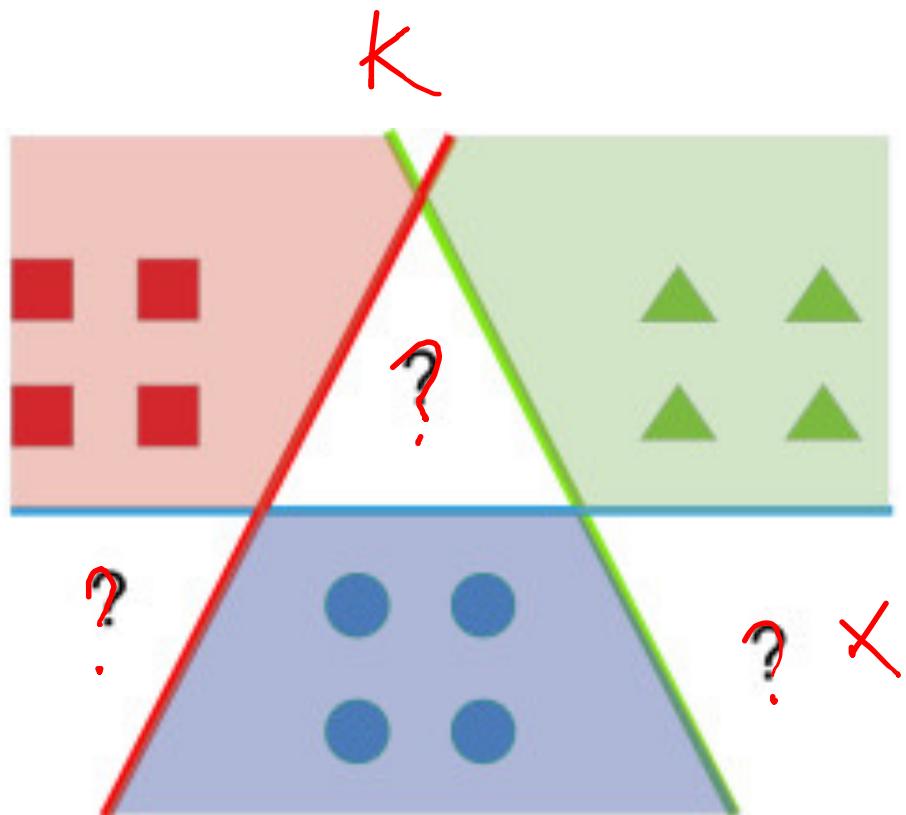


# 1 vs 1

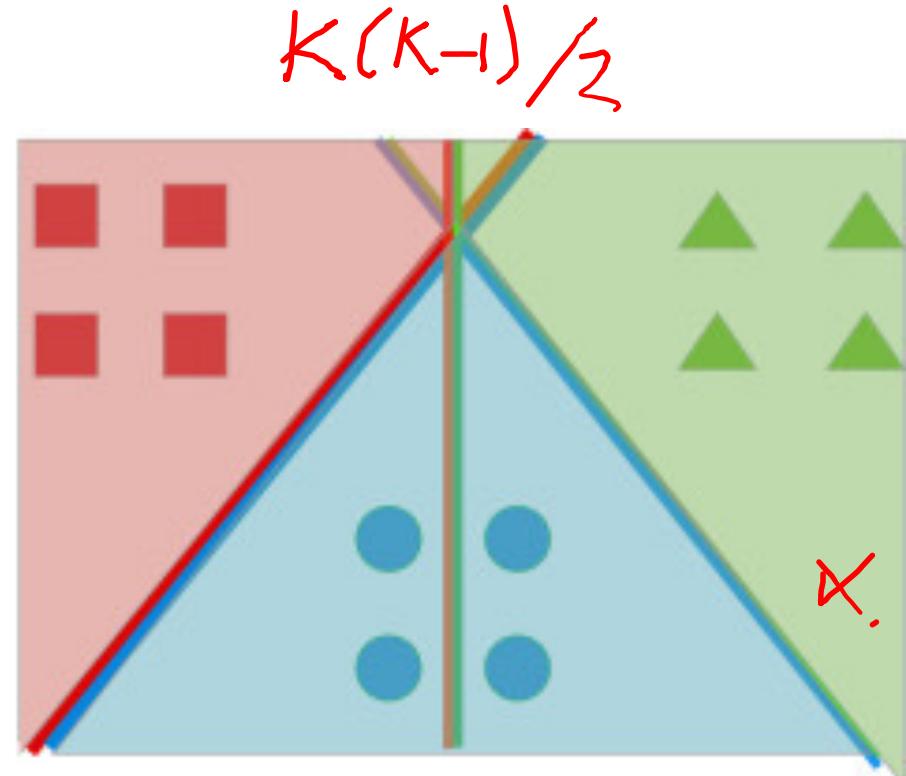


# OvA vs OvO

- OvA jauh lebih efisien, tapi dapat menghasilkan area ambigu yang sulit diprediksi



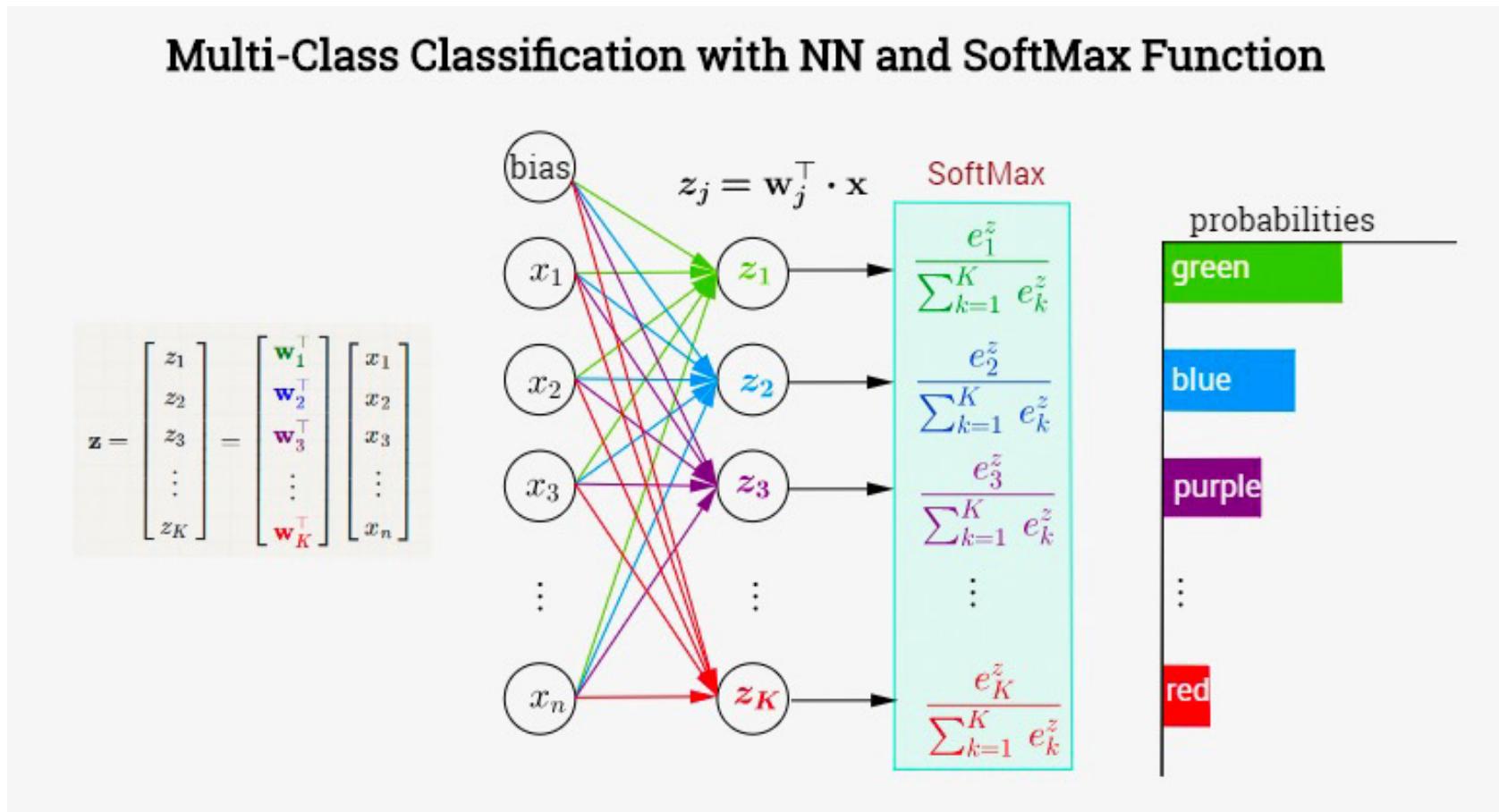
(a) Separation with OvA.



(b) Separation with OvO.

# Multiclass

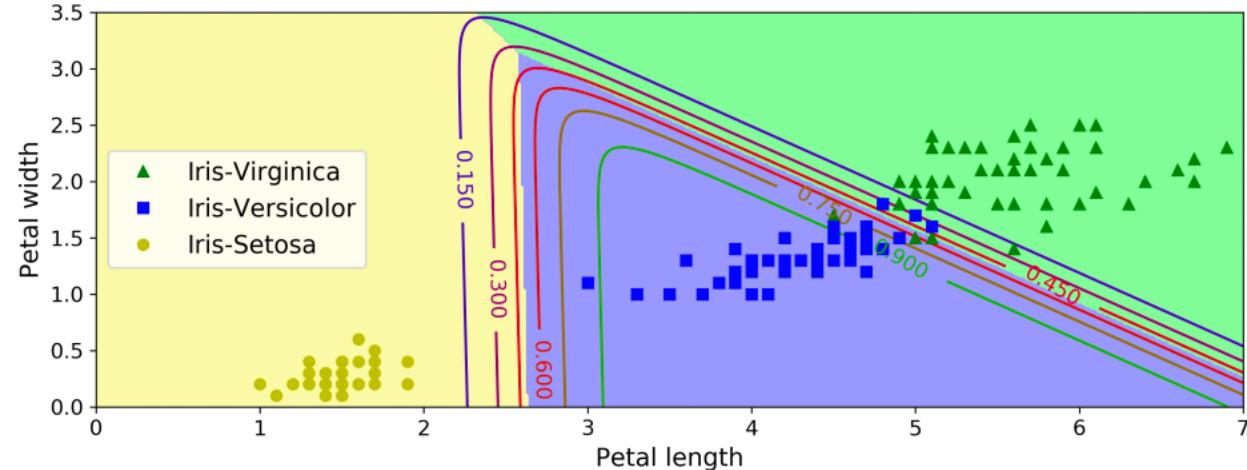
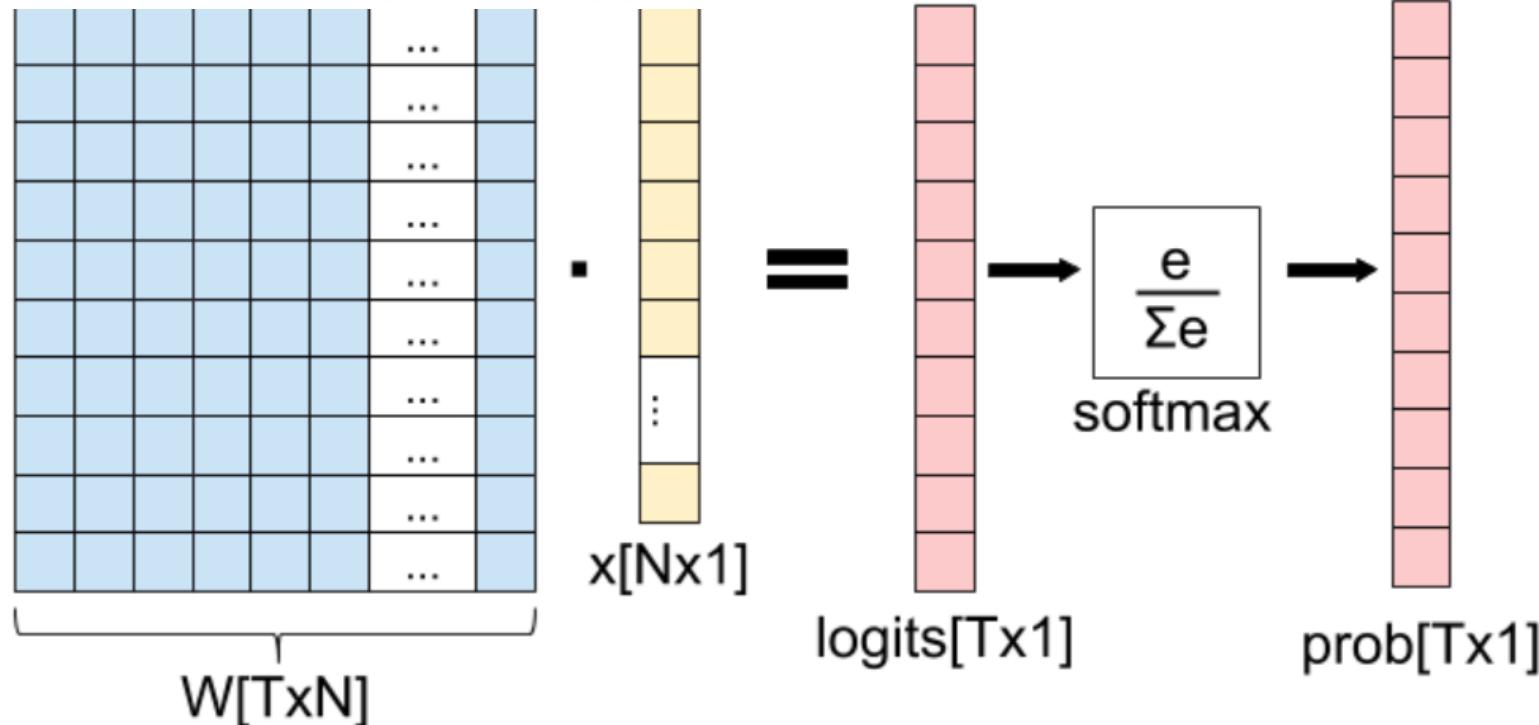
- Arsitektur model dapat secara otomatis menghasilkan K kelas (tidak perlu melakukan binary classification berkali-kali)



# Regresi Softmax

- Peluang tiap kelas = softmax skor tiap kelas

$$\hat{p}_k = \sigma(s(\mathbf{x}))_k = \frac{\exp(s_k(\mathbf{x}))}{\sum_{j=1}^K \exp(s_j(\mathbf{x}))}$$



# Gradient Descent

- Cost function menggunakan cross entropy:

$$J(\Theta) = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \log(\hat{p}_k^{(i)})$$

- Turunan parsial:

$$\frac{\partial}{\partial \theta_{jk}} J(\Theta) = \frac{1}{m} \sum_{i=1}^m (\hat{p}_k^{(i)} - y_k^{(i)}) x_j^{(i)}$$

- Gradient Descent:

$$\theta_{jk} := \theta_{jk} - \alpha \frac{\partial}{\partial \theta_{jk}} J(\theta_{jk}) = \theta_{jk} - \alpha \frac{1}{m} \sum_{i=1}^m (\hat{p}_k^{(i)} - y_k^{(i)}) x_j^{(i)}$$

# Uji Pemahaman

- Berapakah hasil penjumlahan dari semua prediksi kelas ( $\sum_k^K \hat{p}_k$ ) pada regresi softmax?

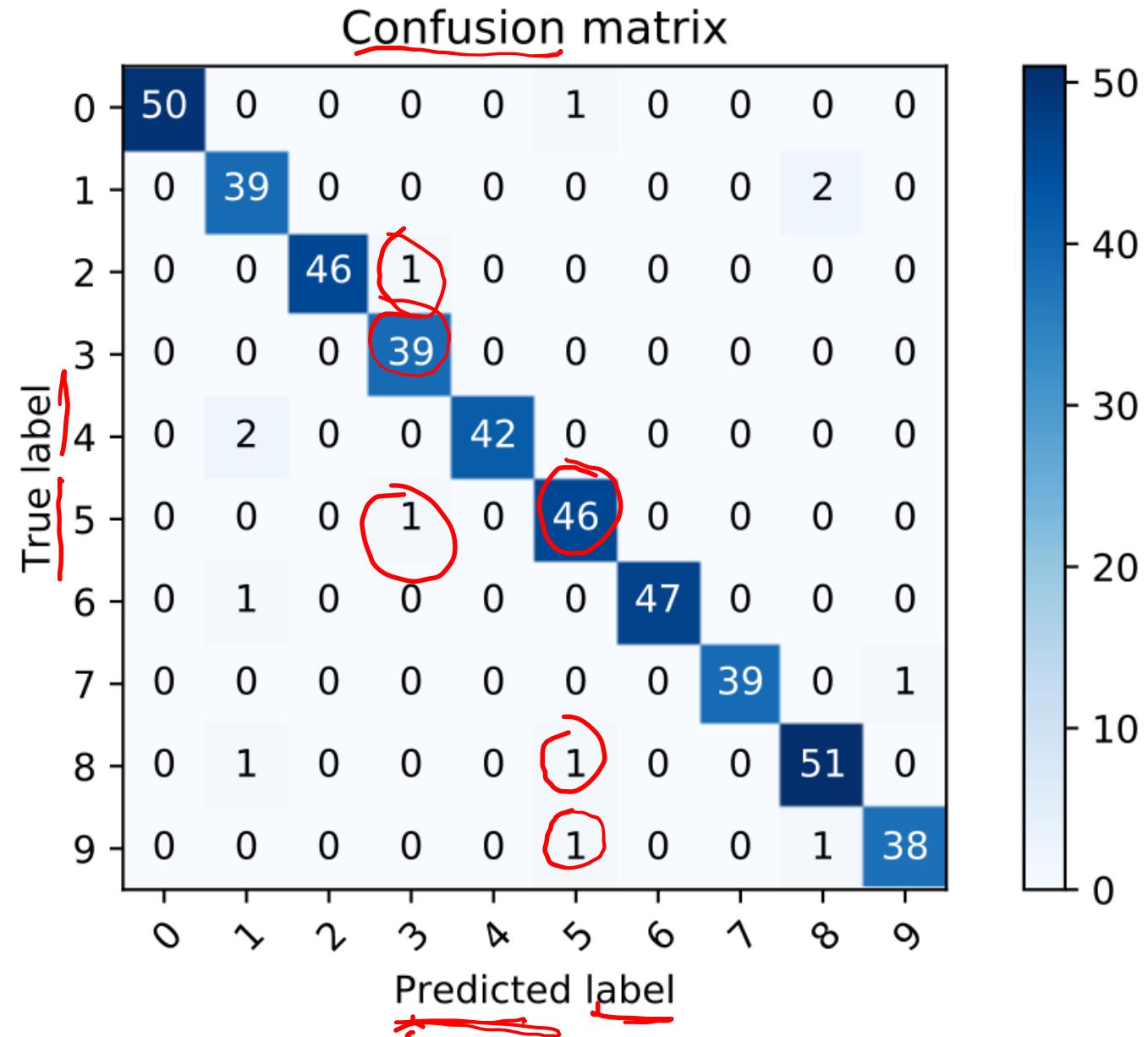
$$\hat{p}_k = \frac{e^{-\theta_k^T \cdot X}}{\sum_k e^{-\theta_k^T \cdot X}}$$

$$\sum_{k=1}^K \hat{p}_k = 1 = \sum_{k=1}^K \frac{e^{-\theta_k^T \cdot X}}{\sum_{k=1}^K e^{-\theta_k^T \cdot X}}$$

# Metrik Performa

- Confusion matrix:
  - True positives
  - False positives
  - False negatives
  - True negatives

|         |    | PREDICTED classification |    |    |    |
|---------|----|--------------------------|----|----|----|
|         |    | a                        | b  | c  | d  |
| Classes | a  | TN                       | FP | TN | TN |
|         | b  | FN                       | TP | FN | FN |
| c       | TN | FP                       | TN | TN |    |
| d       | TN | FP                       | TN | TN |    |



# Macro vs micro vs weighted

- <https://towardsdatascience.com/micro-macro-weighted-averages-of-f1-score-clearly-explained-b603420b292f>

|              | precision | recall | f1-score | support |
|--------------|-----------|--------|----------|---------|
| Aeroplane    | 0.67      | 0.67   | 0.67     | 3       |
| Boat         | 0.25      | 1.00   | 0.40     | 1       |
| Car          | 1.00      | 0.50   | 0.67     | 6       |
| accuracy     |           |        | 0.60     | 10      |
| macro avg    | 0.64      | 0.72   | 0.58     | 10      |
| weighted avg | 0.82      | 0.60   | 0.64     | 10      |

↓  
Average F1 scores

| No | Actual   | Predicted | Match |
|----|----------|-----------|-------|
| 1  | Airplane | Airplane  | ✓     |
| 2  | Car      | Boat      | ✗     |
| 3  | Car      | Car       | ✓     |
| 4  | Car      | Car       | ✓     |
| 5  | Car      | Boat      | ✗     |
| 6  | Airplane | Boat      | ✗     |
| 7  | Boat     | Boat      | ✓     |
| 8  | Car      | Airplane  | ✗     |
| 9  | Airplane | Airplane  | ✓     |
| 10 | Car      | Car       | ✓     |

# Metrik tiap kelas

| Label   | True Positive (TP) | False Positive (FP) | False Negative (FN) | Precision | Recall | F1 Score                                   |
|---|--------------------|---------------------|---------------------|-----------|--------|--|
|  Airplane | 2                  | 1                   | 1                   | 0.67      | 0.67   | $2 * (0.67 * 0.67) / (0.67 + 0.67) = 0.67$ |
|  Boat     | 1                  | 3                   | 0                   | 0.25      | 1.00   | $2 * (0.25 * 1.00) / (0.25 + 1.00) = 0.40$ |
|  Car     | 3                  | 0                   | 3                   | 1.00      | 0.50   | $2 * (1.00 * 0.50) / (1.00 + 0.50) = 0.67$ |

$$\frac{0.67 + 0.25 + 1}{3} = \frac{1.92}{3} = 0.64$$

$$\frac{6}{6+4} = 0.6$$

$$0.3 \times 0.67 + 0.1 \times 0.25 + 0.6 \times 1 = 0.826$$

# Micro-Averaged

| Label   | True Positive (TP) | False Positive (FP) | False Negative (FN) | Micro-Averaged Values                                     |
|---|--------------------|---------------------|---------------------|---|
|  Airplane | 2                  | 1                   | 1                   | $\text{Precision} = \frac{6}{6+4} = 0.60$                 |
|  Boat     | 1                  | 3                   | 0                   | $\text{Recall} = \frac{6}{6+4} = 0.60$                    |
|  Car     | 3                  | 0                   | 3                   |   |
| <b>TOTAL</b>  | <b>6</b>           | <b>4</b>            | <b>4</b>            | $\text{F1 Score} = \frac{6}{6 + \frac{1}{2}(4+4)} = 0.60$ |

# Macro vs micro vs weighted

- Contoh: F1

| Label  | True Positive (TP) | False Positive (FP) | False Negative (FN) | Micro-Averaged F1 Score  |
|--|--------------------|---------------------|---------------------|--|
|  Airplane | 2                  | 1                   | 1                   | $\frac{TP}{TP + \frac{1}{2}(FP+FN)} = \frac{6}{6 + \frac{1}{2}(4+4)} = 0.60$ |
|  Boat     | 1                  | 3                   | 0                   |  |
|  Car      | 3                  | 0                   | 3                   |  |
| <b>TOTAL</b>   | <b>6</b>           | <b>4</b>            | <b>4</b>            |  |

| Label   | Per-Class F1 Score | Macro-Averaged F1 Score               | Label  | Per-Class F1 Score | Support | Support Proportion | Weighted Average F1 Score                           |
|---|--------------------|---------------------------------------|--|--------------------|---------|--------------------|---|
|  Airplane | 0.67               | $\frac{0.67 + 0.40 + 0.67}{3} = 0.58$ |  Airplane | 0.67               | 3       | 0.3                | $(0.67 * 0.3) + (0.40 * 0.1) + (0.67 * 0.6) = 0.64$ |
|  Boat     | 0.40               |                                       |  Boat     | 0.40               | 1       | 0.1                |   |
|  Car      | 0.67               |                                       |  Car      | 0.67               | 6       | 0.6                |   |
|   |                    |                                       | Total  | -                  | 10      | 1.0                |   |

# Uji Pemahaman

- Bagaimana cara menghitung accuracy, precision, recall, F1 untuk kasus weighted, macro, dan micro averaged

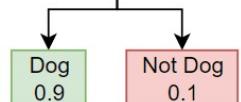
|          | Acc | Pres  | Rec | F1   |
|----------|-----|-------|-----|------|
| MACRO    | 0.6 | 0.64  |     | 0.6  |
| micro    | 0.6 | 0.6   |     | 0.58 |
| Weighted | 0.6 | 0.826 |     | 0.64 |

# Klasifikasi Multi-Label

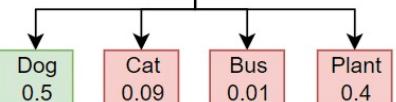
# Klasifikasi Multi-Label

- Model yang dibuat dapat melabeli data dengan beberapa K kelas

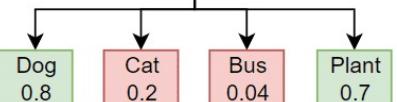
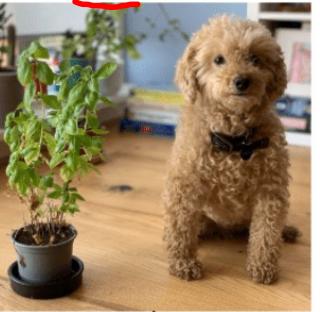
Binary Classification



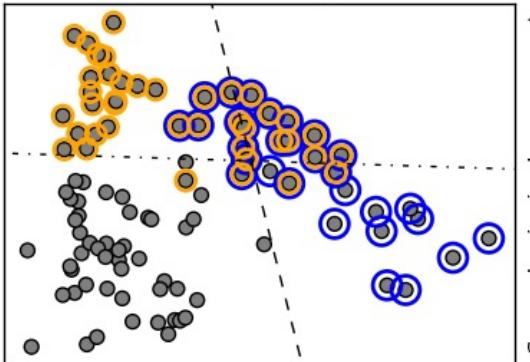
Multiclass Classification



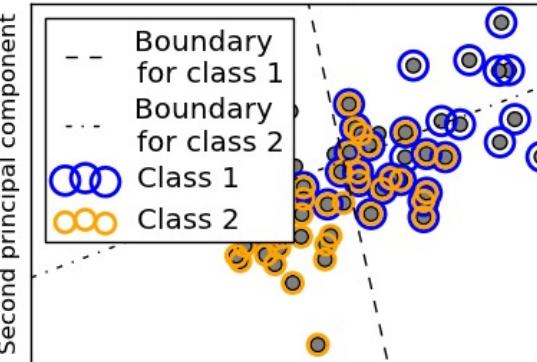
Multilabel Classification



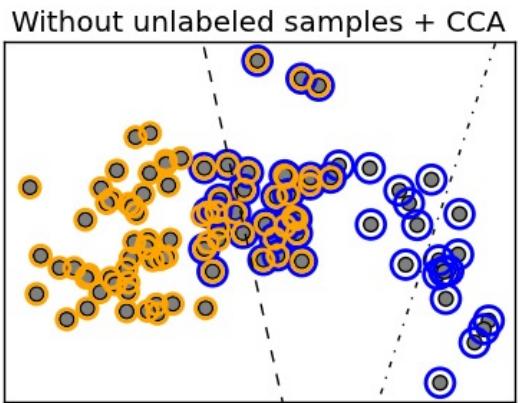
With unlabeled samples + CCA



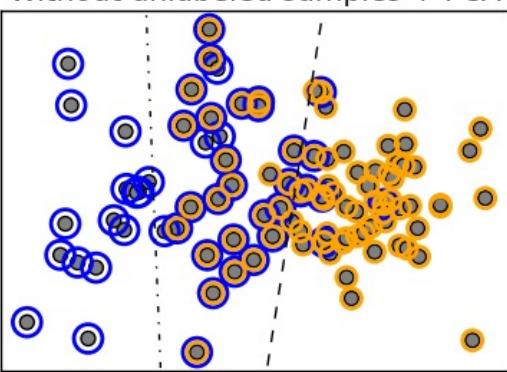
With unlabeled samples + PCA



First principal component



Without unlabeled samples + PCA



# Klasifikasi Multi-Label

- <https://www.geeksforgeeks.org/an-introduction-to-multilabel-classification/>

| TEXT   | SERVICE | FOOD | ANECDOTES | PRICE | AMBIENCE |
|--|---------|------|-----------|-------|----------|
| but the staff was so horrible to us                | 1       | 0    | 0         | 0     | 0        |
| to be completely fair the only redeeming facto...  | 0       | 1    | 1         | 0     | 0        |
| the food is uniformly exceptional with a very ...  | 0       | 1    | 0         | 0     | 0        |
| where gabriela personally greets you and recomm... | 1       | 0    | 0         | 0     | 0        |
| for those that go once and dont enjoy it all i...  | 0       | 0    | 1         | 0     | 0        |

# Accuracy

*data*

*predictive model*

True labels

Predicted labels

| TEXT   | SERVICE | FOOD | ANECDOTES | PRICE | AMBIENCE | SERVICE | FOOD | ANECDOTES | PRICE | AMBIENCE |   |
|--|---------|------|-----------|-------|----------|---------|------|-----------|-------|----------|---|
| but the staff was so horrible to us                | 1       | 0    | 0         | 0     | 0        | 0       | 1    | 0         | 0     | 0        | X |
| to be completely fair the only redeeming facto...  | 0       | 1    | 1         | 0     | 0        | 1       | 1    | 0         | 0     | 0        | X |
| the food is uniformly exceptional with a very ...  | 0       | 1    | 0         | 0     | 0        | 0       | 0    | 0         | 1     | 0        | X |
| where gabriela personally greets you and recomm... | 1       | 0    | 0         | 0     | 0        | 1       | 0    | 0         | 0     | 0        | ✓ |
| for those that go once and dont enjoy it all i...  | 0       | 0    | 1         | 0     | 0        | 1       | 0    | 0         | 0     | 0        | X |

Total number of predictions (TNP) = 5

Total number of correct predictions (TNCP) = 1

$$1/5 = 0.2$$

Accuracy = TNCP/TNP = 1/5 = 0.2

# Hamming loss

5

True labels

Predicted labels

| TEXT   |
|--|
| but the staff was so horrible to us                |
| to be completely fair the only redeeming facto...  |
| the food is uniformly exceptional with a very ...  |
| where gabriela personally greets you and recomm... |
| for those that go once and dont enjoy it all i...  |

|   | SERVICE | FOOD | ANECDOTES | PRICE | AMBIENCE |
|---|---------|------|-----------|-------|----------|
| 1 | 0       | 0    | 0         | 0     | 0        |
| 0 | 1       | 1    | 0         | 0     | 0        |
| 0 | 1       | 0    | 0         | 0     | 0        |
| 1 | 0       | 0    | 0         | 0     | 0        |
| 0 | 0       | 1    | 0         | 0     | 0        |

|   | SERVICE | FOOD | ANECDOTES | PRICE | AMBIENCE |
|---|---------|------|-----------|-------|----------|
| 0 | 0       | 1    | 0         | 0     | 0        |
| 1 | 1       | 1    | 0         | 0     | 0        |
| 0 | 0       | 0    | 0         | 1     | 0        |
| 1 | 0       | 0    | 0         | 0     | 0        |
| 1 | 1       | 0    | 0         | 0     | 0        |

5

Total number of predictions (TNP) = 25 =  $5 \times 5$

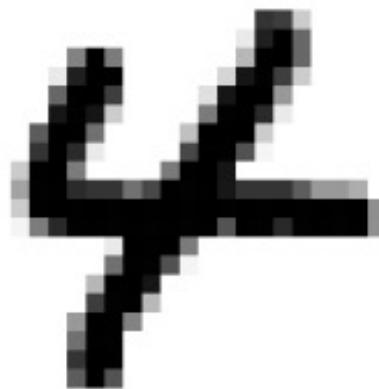
Total number of incorrect predictions (TNIP) = 8

Hamming loss

Accuracy = TNIP/TNP = 8/25 = 0.32

# Klasifikasi Multi-Output

- Model yang dibuat dapat melakukan beberapa klasifikasi sekaligus
- Contoh dibawah ini adalah angka 4 dengan dan tanpa noise
- Model dapat membedakan angka tanpa dan dengan noise maupun angka berapa dari 10 pilihan angka



multiclass

binary

# Regresi Multi-Output

X1 X2 X3 X4 X5 X6 X7 X8

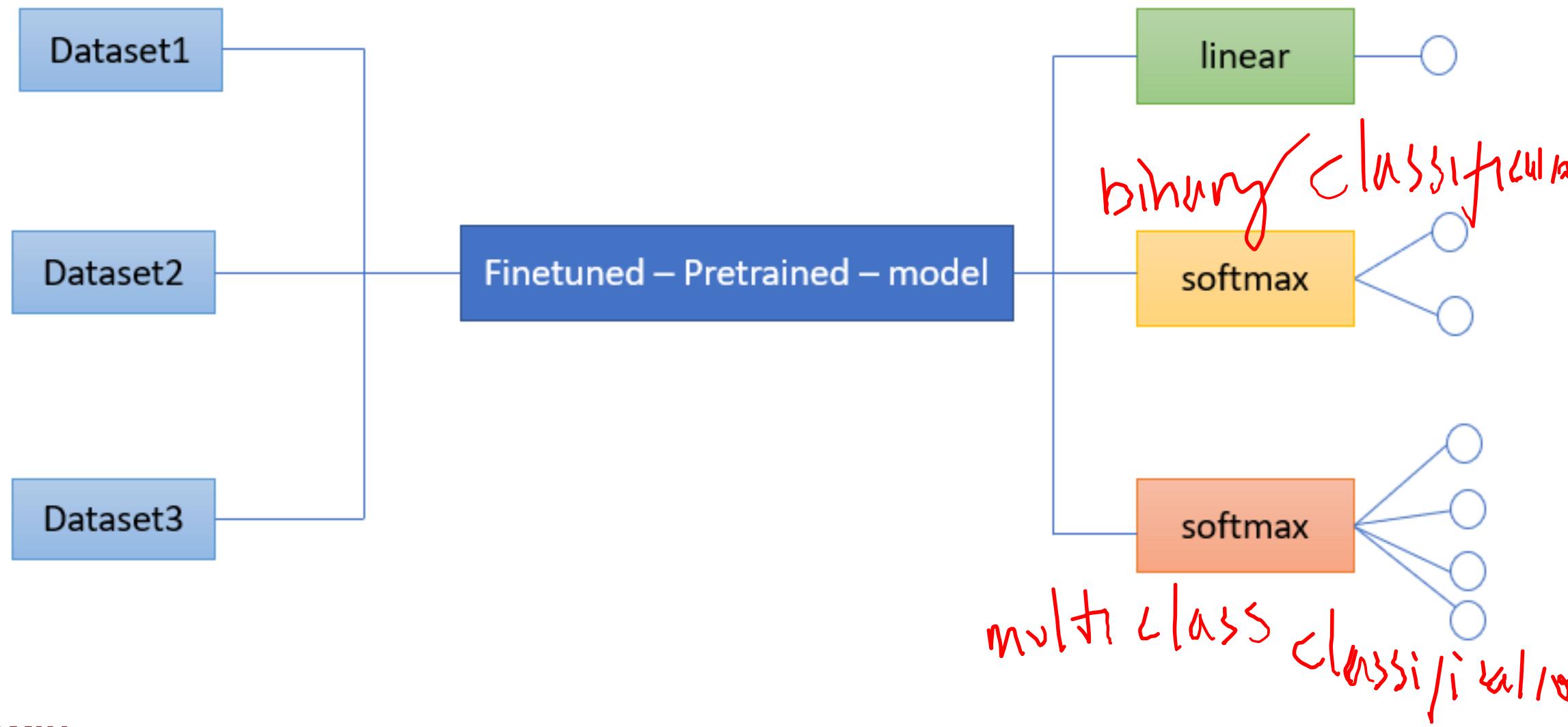
Y1

Y2

Y3

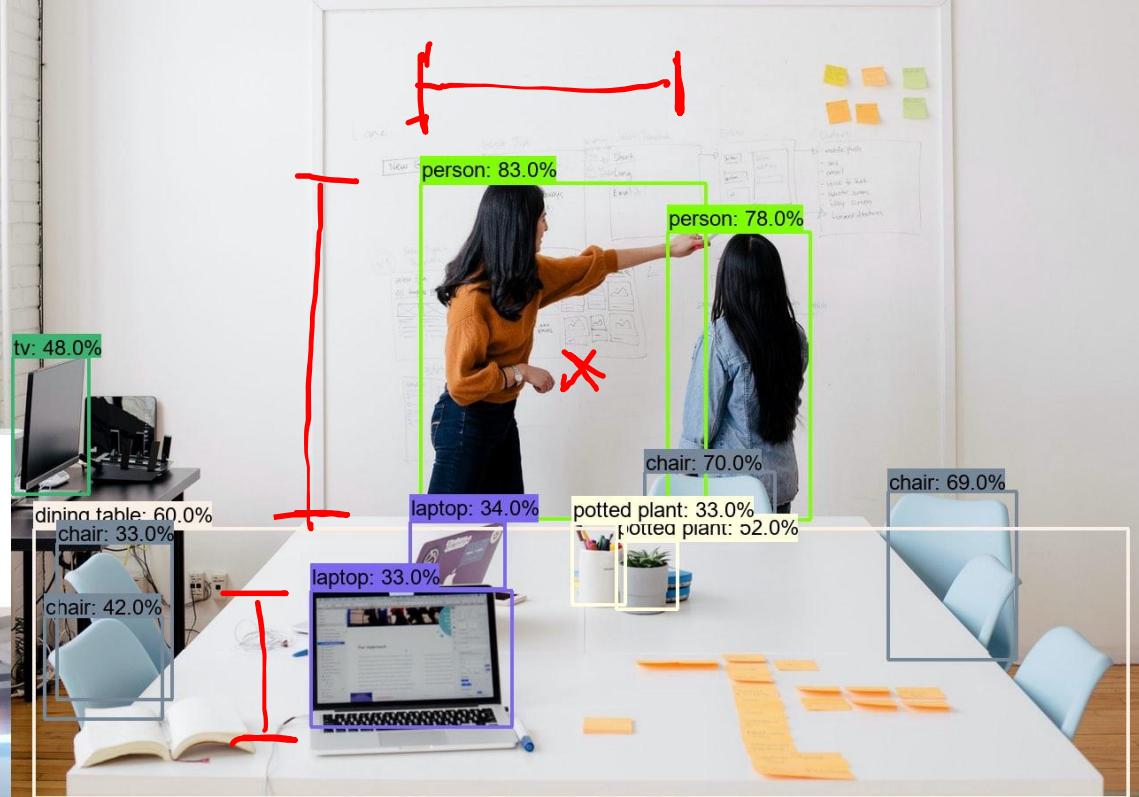
Sample Set

# Multi-Task



# Contoh Multi-Task

- Contoh: deteksi objek, segmentasi
- Terdapat klasifikasi dan regresi



1. Klasifikasi objek
2.  $x, y$  pusat objek
3.  $\Delta x, \Delta y$  kotak
4. Segmentasi Objek

# Contoh Cost Function YOLO

Regression loss

$$\lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 \right]$$

(kordinat objek) prediksi

$$+ \lambda_{\text{coord}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} \left[ (\sqrt{w_i} - \sqrt{\hat{w}_i})^2 + (\sqrt{h_i} - \sqrt{\hat{h}_i})^2 \right]$$

(ukuran objek) prediksi

Confidence loss

$$+ \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{obj}} (C_i - \hat{C}_i)^2$$

objek prediksi

$$+ \lambda_{\text{noobj}} \sum_{i=0}^{S^2} \sum_{j=0}^B \mathbb{1}_{ij}^{\text{noobj}} (C_i - \hat{C}_i)^2$$

prediksi

Classification loss

$$+ \sum_{i=0}^{S^2} \mathbb{1}_i^{\text{obj}} \sum_{c \in \text{classes}} (p_i(c) - \hat{p}_i(c))^2$$

kelas objek prediksi

# Tuhan Memberkati