Algoritma Kuantum

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IBDA4221 – Selected Topic in Computer Technology

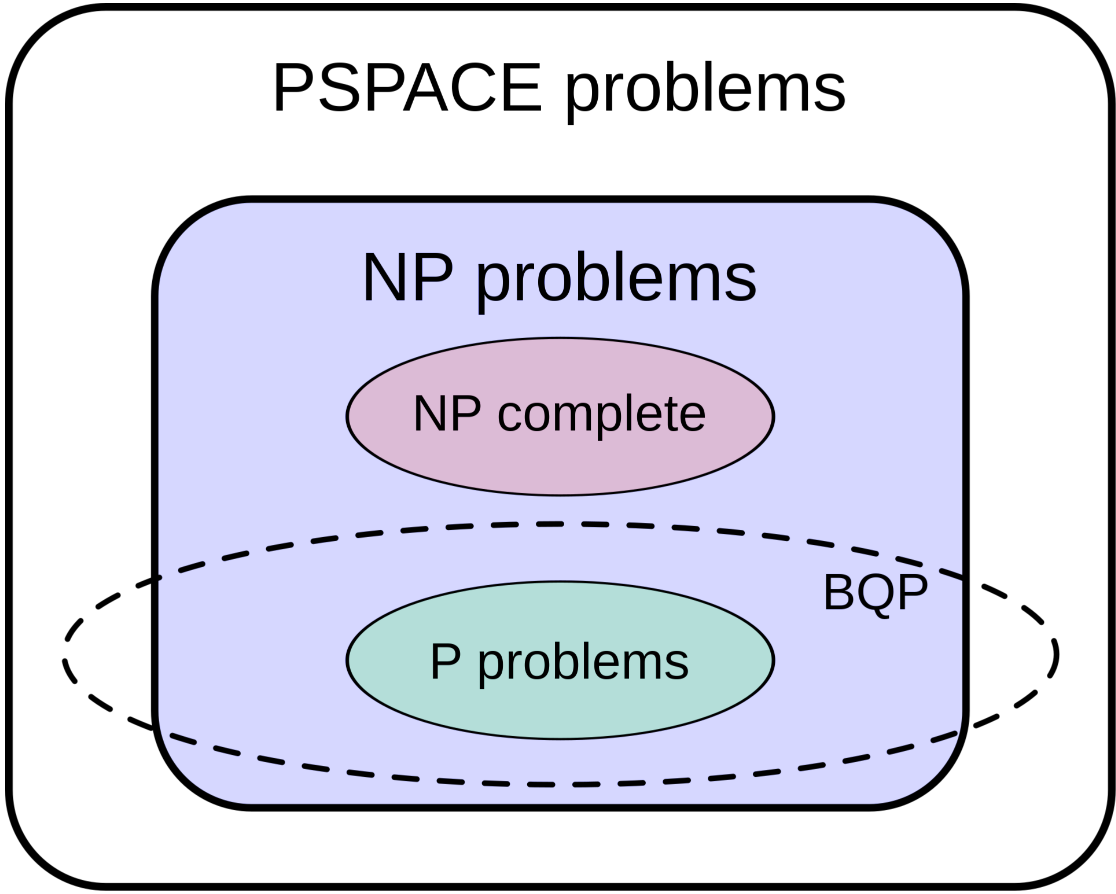
Quantum Computing

Capaian Pembelajaran

- Algoritma Deutsch-Josza
- Algoritma Berstein-Vazirani
- Algoritma Simon (Pengayaan)

Algoritma Kuantum

- Algoritma yang memanfaatkan komputasi kuantum untuk menyelesaikan permasalahaan dengan jauh lebih efisien dibanding komputasi klasik
- Kategori algoritma kuantum:
  - Quantum Fourier Transform: DJ, BV, Simon, QPE, Shor
  - Amplitude amplification: Grover
  - Quantum Walks
- Jenis problem algoritma kuantum:
  - Bounded error quantum polynomial time (BPQ)
  - Hybrid quantum/classical: QAOA, VQE, CQE



Deutsch Problem

God's People for God's Glory
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Deutsch-Jozsa Problem

$\{0,1\}^3$: 000
001
010
011
100
101
110
111

- Misalkan terdapat sebuah oracle (black box computer):

$$f: \{0,1\}^n \rightarrow \{0,1\}$$

- Dimana fungsi ini menggunakan n-bit binary input dan menghasilkan 0 atau 1 output.
- Fungsi ini menghasilkan antara constant (0 atau 1 untuk seluruh kemungkinan input) atau balanced (1 untuk setengah kemungkinan input dan 0 untuk sisanya)
- Tugas kita untuk memprediksi apakah fungsi ini constant ataukah balanced

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Contoh

x	f(x)
0	0
1	0

x	f(x)
0	1
1	1

x	f(x)
0	1
1	0

x	f(x)
0	0
1	1

- n=1
- Jika $f(0) = f(1) = 0 \rightarrow f$ constant
- Jika $f(0) = 0, f(1) = 1 \rightarrow f$ balanced

x	f(x)
00	1
01	0
10	0
11	1

- n=2
- Jika $f(00) = f(01) = f(10) = f(11) = 0 \rightarrow f$ constant
- Jika $f(00) = f(10) = 0, f(01) = f(11) = 1 \rightarrow f$ balanced

x	f(x)
00	0
01	0
10	0
11	0

x	f(x)
00	1
01	1
10	1
11	1

x	f(x)
00	0
01	0
10	1
11	1

x	f(x)
00	0
01	1
10	1
11	0

x	f(x)
00	0
01	1
10	0
11	1

x	f(x)
00	1
01	1
10	0
11	0

x	f(x)
00	1
01	0
10	1
11	0

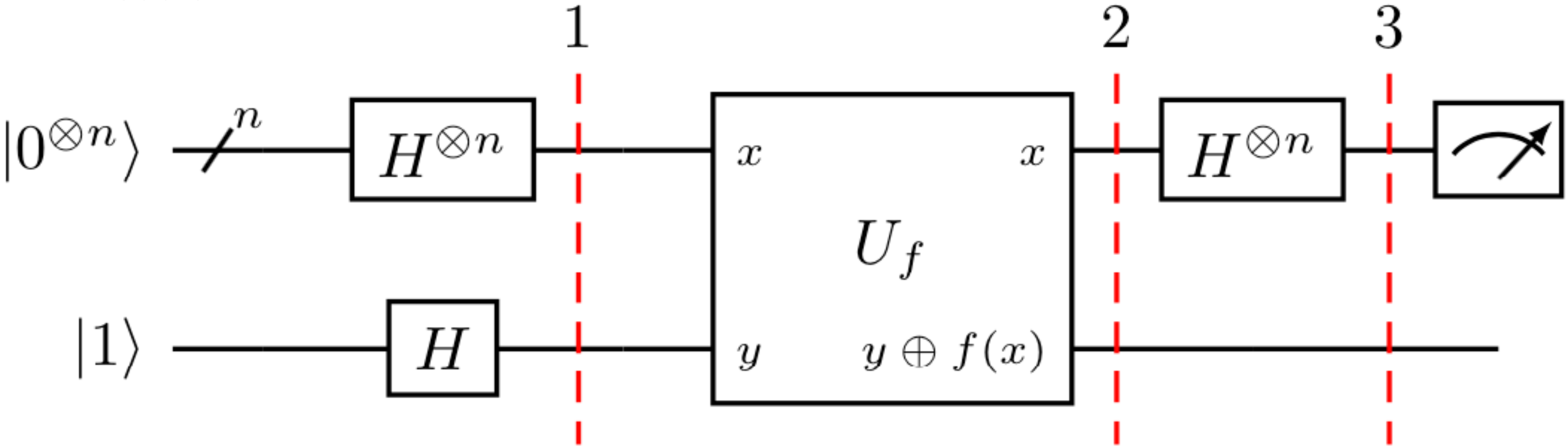
Solusi Klasik

- Pada best case, 2 queries dapat menentukan fungsi f balanced
- Misalnya kita memperoleh $f(0,0,0,\ldots) \rightarrow 0$ dan $f(0,0,0,\ldots) \rightarrow 1$, maka kita langsung mengetahui bahwa fungsi ini balanced karena menghasilkan 2 output yang berbeda
- Pada worst case, kita harus cek sebanyak $2^n + 1$ kali untuk menentukan f constant

$f(0000) = 1$
$f(0001) = 1$
$f(0010) = 1$
$f(0011) = 1$
$f(0100) = 1$

Solusi Kuantum

- Dengan komputer kuantum, kita dapat memecahkan masalah ini hanya sekali memanggil fungsi f, dimana f diimplementasikan sebagai oracle kuantum yang memetakan state $|y\rangle|x\rangle$menuju $|y \oplus f(x)\rangle|x\rangle$, dimana $\oplus$ adalah penjumlahan modulo 2



Deutsch Algorithm

$f(0) \oplus f(1)$: $0 \oplus 0 = 0$, $0 \oplus 1 = 1$, $1 \oplus 0 = 1$, $1 \oplus 1 = 0$

- Deutsch Algorithm adalah Deutsch-Jozsa Algorithm untuk kasus n=1→harus cek $f(0) = f(1)$
- Ini ekuivalen dengan cek $f(0) \oplus f(1)$, jika 0 →f constant, jika 1 → f balanced
- Kita mulai dari 2 qubit $|1\rangle|0\rangle$ lalu menggunakan H-gate terhadap kedua qubit:

$H|1\rangle \otimes H|0\rangle = |-\rangle \otimes |+\rangle$

$$|yx\rangle = \frac{1}{2}(|0\rangle - |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$(|0\rangle - |1\rangle)|0\rangle + (|0\rangle - |1\rangle)|1\rangle$

$(|0\rangle - |1\rangle)|0\rangle + (|1\rangle - |0\rangle)|1\rangle$

$(|1\rangle - |0\rangle)|0\rangle + (|0\rangle - |1\rangle)|1\rangle$

$(|1\rangle - |0\rangle)|0\rangle + (|1\rangle - |0\rangle)|1\rangle$

- Lalu menggunakan CU gate dimana qubit pertama adalah qubit kontrol:

$|0\rangle - |1\rangle = |-\rangle$

$$CU|yx\rangle = |(y \oplus f(x))x\rangle$$

$$CU|-+\rangle = \frac{1}{\sqrt{2}}(|0 \oplus f(0)\rangle - |1 \oplus f(0)\rangle)|0\rangle + \frac{1}{\sqrt{2}}(|0 \oplus f(1)\rangle - |1 \oplus f(1)\rangle)|1\rangle$$

$|1\rangle - |0\rangle = -|-\rangle$

$$CU|-+\rangle = |-\rangle \otimes (-1)^{f(0)} \frac{1}{\sqrt{2}}(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle)$$

$-1^0 = 1$ $\quad$ $-1^1 = -1$

$f(0) \oplus f(1) = 0 \to |+\rangle$, $1 \to |-\rangle$

$H|0\rangle$ $\quad$ $H|1\rangle$

- Jika kita menggunakan H-gate pada qubit pertama:

$$|\psi_1\rangle = \frac{1}{2}(|0\rangle + |1\rangle + (-1)^{f(0)\oplus f(1)}|0\rangle - (-1)^{f(0)\oplus f(1)}|1\rangle)$$
$$= \frac{1}{2}((1 + (-1)^{f(0)\oplus f(1)})|0\rangle + (1 - (-1)^{f(0)\oplus f(1)})|1\rangle)$$

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Pengukuran akhir

- Pengukuran pada state akhir dari qubit pertama akan mendapatkan beberapa kemungkinan:
- Jika $f(0) \oplus f(1) = 0$ maka $|\psi_1\rangle = \frac{1}{2}((1+1)|0\rangle + (\cancelto{0}{1-1})|1\rangle) = \underline{|0\rangle}$
- Jika $f(0) \oplus f(1) = 1$ maka $|\psi_1\rangle = \frac{1}{2}((\cancelto{0}{1-1})|0\rangle + (1+1)|1\rangle) = \underline{|1\rangle}$
- Jadi jika mengukur $|0\rangle$ maka $f(0) \oplus f(1) = 0$ → constant
- Jadi jika mengukur $|1\rangle$ maka $f(0) \oplus f(1) = 1$ → balanced

$O(2) \longrightarrow O(1)$

Classical

Quantum

Algoritma Deutsch-Jozsa

Deutsch-Jozsa Problem

$f(001011001) = 0$
$f(010010111) = 0$
$\vdots$

- Misalkan terdapat sebuah oracle (physical device yang tersembunyi):

$$f : \{0,1\}^n \to \{0,1\}$$

- Dimana fungsi ini menggunakan n-bit binary input dan menghasilkan 0 atau 1 output.
- Fungsi ini menghasilkan antara constant (0 atau 1 untuk seluruh kemungkinan input) atau balanced (1 untuk setengah kemungkinan input dan 0 untuk sisanya)
- Tugas kita untuk memprediksi apakah fungsi ini constant ataukah balanced
- Deutsch-Jozsa adalah generalisasi (n-bit extension) dari Deutsch problem (n=1)

Solusi Klasik

- Pada best case, 2 queries dapat menentukan fungsi f balanced
- Misalnya kita memperoleh $f(0,0,0,\ldots) \rightarrow 0$ dan $f(0,0,0,\ldots) \rightarrow 1$, maka kita langsung mengetahui bahwa fungsi ini balanced karena menghasilkan 2 output yang berbeda
- Pada worst case, kita harus cek sebanyak $2^{n-1} + 1$ kali untuk menentukan f constant

$n = 3$

$2^{3-1} + 1 = 2^2 + 1 = 5$

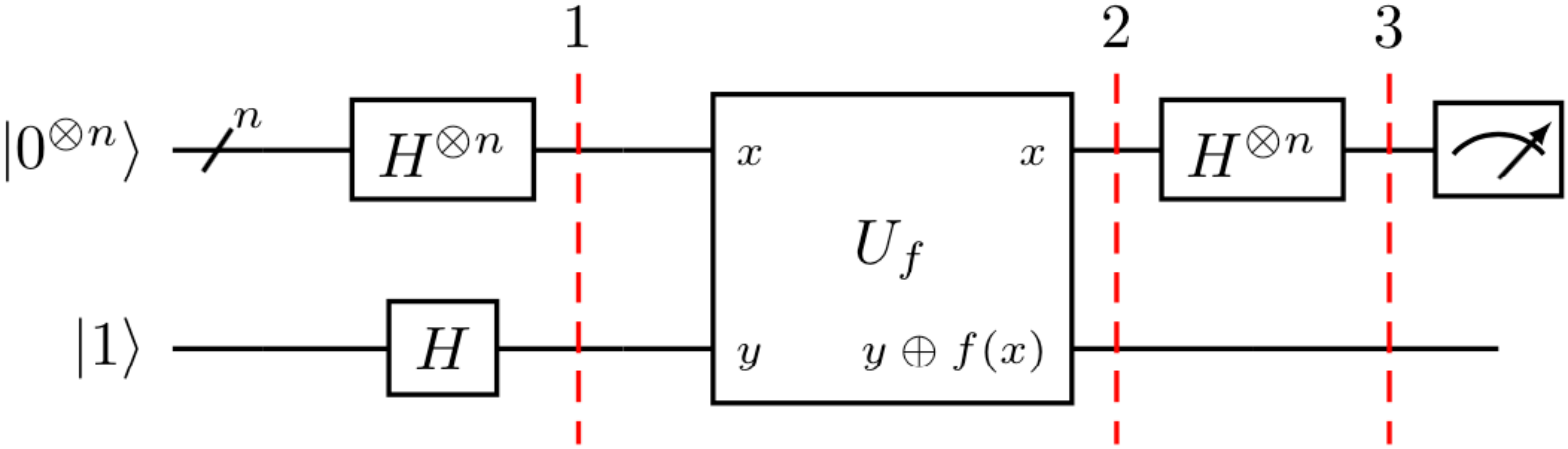
$f(000) = 0$
$f(001) = 0$
$f(010) = 0$
$f(011) = 0$
$f(100) = 1$
$f(101) = 1$
$\vdots$

$O(2^{n-1}) \longrightarrow O(1)$

time complexity

Solusi Kuantum

- Dengan komputer kuantum, kita dapat memecahkan masalah ini hanya sekali memanggil fungsi f, dimana f diimplementasikan sebagai oracle kuantum yang memetakan state $|y\rangle|x\rangle$ menuju $|y \oplus f(x)\rangle|x\rangle$, dimana $\oplus$ adalah penjumlahan modulo 2



Oracle

target control

- Oracle dapat dilihat sebagai unitary yang melakukan mapping $O_f|y\rangle|x\rangle = |y \oplus f(x)\rangle|x\rangle$

$|-\rangle|x\rangle$

$$O_f|y\rangle|x\rangle = \frac{1}{\sqrt{2}}(|0 \oplus f(x)\rangle|x\rangle - |1 \oplus f(x)\rangle|x\rangle)$$

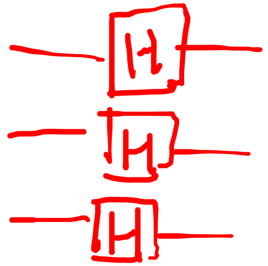
$$= \begin{cases} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|x\rangle, jika\ \underline{f(x) = 0} & \rightarrow |0 \oplus 0\rangle|x\rangle - |1 \oplus 0\rangle|x\rangle \\ & \quad |0\rangle|x\rangle - |1\rangle|x\rangle \\ \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)|x\rangle, jika\ \underline{f(x) = 1} & \rightarrow |0 \oplus 1\rangle|x\rangle - |1 \oplus 1\rangle|x\rangle \\ & \quad |1\rangle|x\rangle - |0\rangle|x\rangle \\ & \quad \rightarrow (-1)(|0\rangle - |1\rangle)|x\rangle \end{cases}$$

$$= (-1)^{f(x)}|y\rangle|x\rangle$$

- Oracle independent terhadap $|y\rangle$, dimana mirip sebuah phase oracle

$$P_f|x\rangle = \underline{(-1)^{f(x)}}|x\rangle$$

Hadamard pada n-qubits



$H^{\otimes n}$

$|+\rangle$ $\quad$ $|-\rangle$

- $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Untuk $x \in \{0,1\}$: $\quad x=0 \rightarrow |0\rangle + |1\rangle$, $x=1 \rightarrow |0\rangle - |1\rangle$

$$H|x\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^x|1\rangle) = \frac{1}{\sqrt{2}}((-1)^{0\cdot x}|0\rangle + (-1)^{1\cdot x}|1\rangle)$$

- Untuk $x \in \{0,1\}^n$:

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{k\in\{0,1\}^n} (-1)^{k\cdot x}|k\rangle$$

- Contoh $|x\rangle = |01\rangle$ $\quad \frac{1}{2}(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$

$$H|0\rangle \otimes H|1\rangle = |+\rangle \otimes |-\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$\frac{1}{\sqrt{2^n}} \sum_{k\in\{0,1\}^n} (-1)^{k\cdot x}|k\rangle = \frac{1}{2}(\overset{+1}{(-1)^0}|00\rangle + \overset{-1}{(-1)^1}|01\rangle + \overset{+1}{(-1)^0}|10\rangle + \overset{-1}{(-1)^1}|11\rangle)$$

$k \cdot x = \quad 00\cdot01 \quad 01\cdot01 \quad 10\cdot01 \quad 11\cdot01$

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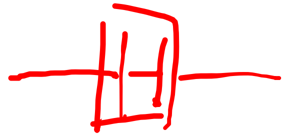
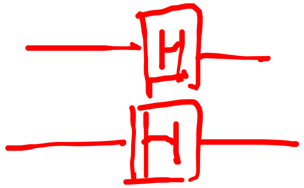
Uji Pemahaman

- Apa sajakah semua kemungkinan Hadamard pada 1-qubit?
- Apa sajakah semua kemungkinan Hadamard pada 2-qubit?





$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H^{\otimes 2}|00\rangle$$

$$H^{\otimes 2}|01\rangle$$

$$H^{\otimes 2}|10\rangle$$

$$H^{\otimes 2}|11\rangle$$

$$|x\rangle = |0\rangle$$

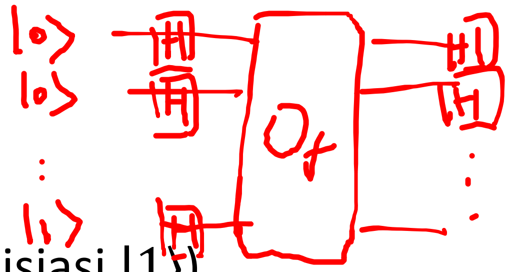
$$\frac{1}{\sqrt{2^n}}\sum_k (-1)^{k\cdot x}|k\rangle = \frac{1}{\sqrt{2}}\left[(-1)^{0\cdot 0}|0\rangle + (-1)^{1\cdot 0}|1\rangle\right] = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|x\rangle = |1\rangle \longrightarrow = \frac{1}{\sqrt{2}}\left[(-1)^{0\cdot 1}|0\rangle + (-1)^{1\cdot 1}|1\rangle\right] = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|x\rangle = |00\rangle \longrightarrow = \frac{1}{2}\left[(-1)^{00\cdot 00}|00\rangle + (-1)^{01\cdot 00}|01\rangle + (-1)^{10\cdot 00}|10\rangle + (-1)^{11\cdot 00}|11\rangle\right] = \frac{1}{2}\left[|00\rangle + |10\rangle + |01\rangle + |11\rangle\right]$$

$$|x\rangle = |11\rangle \longrightarrow = \frac{1}{2}\left[(-1)^{00\cdot 11}|00\rangle + (-1)^{01\cdot 11}|01\rangle + (-1)^{10\cdot 11}|10\rangle + (-1)^{11\cdot 11}|11\rangle\right] = \frac{1}{2}\left[|00\rangle - |01\rangle - |10\rangle + |11\rangle\right]$$

Deutsch-Josza Algorithm



- Siapkan 2 register (pertama: n-qubit diinisiasi $|0\rangle$, kedua: 1-qubit diinisiasi $|1\rangle$)

$$|\psi_0\rangle = |1\rangle|0\rangle^{\otimes n} = |1\rangle\,|0\rangle \otimes |0\rangle \otimes |0\rangle \cdots$$

- Gunakan Hadamard gate pada setiap qubit

$$|\psi_1\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}\underbrace{(|0\rangle - |1\rangle)}_{|-\rangle}|x\rangle$$

- Gunakan oracle $|y\rangle|x\rangle \to |y \oplus f(x)\rangle|x\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle)|x\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}(|0\rangle - |1\rangle)|x\rangle$$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$$|\psi_3\rangle = \frac{1}{2^{n-1}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}\left[\sum_{k=0}^{2^n-1}(-1)^{x\cdot k}|k\rangle\right] = \frac{1}{2^{n-1}}\sum_{k=0}^{2^n-1}\left[\sum_{x=0}^{2^n-1}(-1)^{f(x)}(-1)^{x\cdot k}\right]|k\rangle$$

$|000\ldots0\rangle$

- Ukur qubit pertama

$$\langle\psi_3|0\rangle^{\otimes n} = \left|\frac{1}{2^{n-1}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}\right|^2 = \begin{cases} 1 \; if \; f(x) \to constant \\ 0 \; if \; f(x) \to balanced \end{cases}$$

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Phase

- Constant Oracle: tidak mempengaruhi global phase input

$$H^{\otimes n}\begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix} = \frac{1}{\sqrt{2^n}}\begin{bmatrix}1\\1\\1\\\vdots\\1\end{bmatrix} \xrightarrow{\text{after } U_f} H^{\otimes n}\frac{1}{\sqrt{2^n}}\begin{bmatrix}1\\1\\1\\\vdots\\1\end{bmatrix} = \begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}$$

- Balanced Oracle: phase kickback menciptakan superposisi seimbang antara fase positif dan fase negatif

$$U_f\frac{1}{\sqrt{2^n}}\begin{bmatrix}1\\1\\1\\\vdots\\1\end{bmatrix} = \frac{1}{\sqrt{2^n}}\begin{bmatrix}-1\\1\\-1\\\vdots\\1\end{bmatrix}$$

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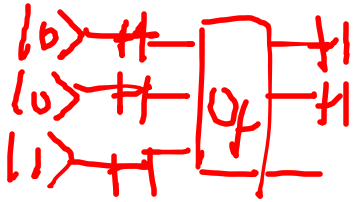
Contoh: 2-bit balanced

- Misalnya terdapat fungsi $f(x_0, x_1) = x_0 \oplus x_1$ dimana:
  - $f(0,0) = 0$ $\longrightarrow 0 \oplus 0 = 0$
  - $f(0,1) = 1$ $\longrightarrow 0 \oplus 1 = 1$
  - $f(1,0) = 1$ $\longrightarrow 1 \oplus 0 = 1$
  - $f(1,1) = 0$ $\longrightarrow 1 \oplus 1 = 0$
- Bentuk dari phase oraclenya adalah

$$U_f|x_1x_0\rangle = (-1)^{f(x_1,x_0)}|x\rangle$$

2-qubit DJ Algorithm



- Siapkan 2 register (pertama: 2-qubit diinisiasi $|0\rangle$, kedua: 1-qubit diinisiasi $|1\rangle$)

$$|\psi_0\rangle = |1\rangle|00\rangle$$

$$f(x) = x_0 \oplus x_1 = \begin{cases} 0 \oplus 0 = 0 \\ 0 \oplus 1 = 1 \\ 1 \oplus 0 = 1 \\ 1 \oplus 1 = 0 \end{cases}$$

- Gunakan Hadamard gate pada setiap qubit

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\underbrace{(|0\rangle - |1\rangle)}_{|-\rangle} \otimes \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|y + f(x)\rangle = \begin{cases} |-\rangle \\ -|-\rangle \end{cases}$$

- Gunakan oracle $|y\rangle|x\rangle \rightarrow |y \oplus f(x)\rangle|x\rangle = |y \oplus x_0 \oplus x_1\rangle|x\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) = \frac{1}{2\sqrt{2}}|-\rangle \otimes |-\rangle \otimes |-\rangle$$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$$H|-\rangle = |1\rangle$$

$$|\psi_3\rangle = (|0\rangle - |1\rangle)|1\rangle|1\rangle$$

- Ukur register pertama

$$P(11) = 100\%$$

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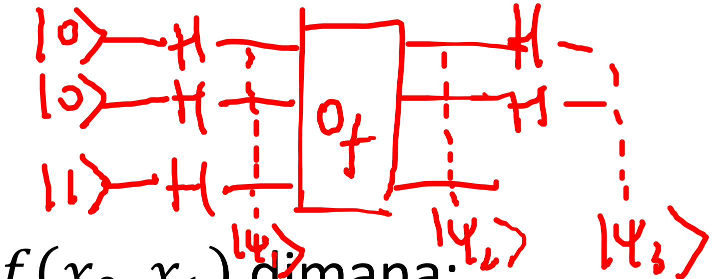
Uji Pemahaman



- Cobalah 2-qubit DJ untuk fungsi $f(x_0, x_1)$ dimana:
  - $f(0,0) = 0$
  - $f(0,1) = 0$
  - $f(1,0) = 0$
  - $f(1,1) = 0$

$O_f|y\rangle|x\rangle = |y \oplus f(x)\rangle|x\rangle$

$|++\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

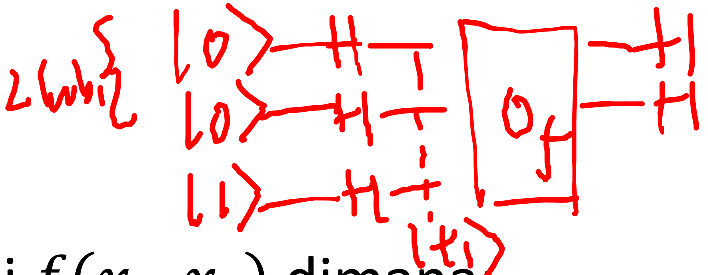
$|\psi_1\rangle = |-\rangle \otimes |+\rangle \otimes |+\rangle$

$|\psi_2\rangle = O_f|\psi_1\rangle = \frac{1}{2\sqrt{2}}\left(|0 \oplus \overset{0}{f(x)}\rangle - |1 \oplus \overset{0}{f(x)}\rangle\right)\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

$= \frac{1}{2\sqrt{2}}|-\rangle \underbrace{\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)}$

$|\psi_3\rangle = H|+\rangle \otimes H|+\rangle = |00\rangle \longrightarrow P(00) = 100\% \longrightarrow$ balance

Uji Pemahaman



- Cobalah 2-qubit DJ untuk fungsi $f(x_0, x_1)$ dimana:
  - $f(0,0) = 1$
  - $f(0,1) = 1$
  - $f(1,0) = 1$
  - $f(1,1) = 1$

$$|++\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$|\psi_1\rangle = H|1\rangle \otimes H|0\rangle \otimes H|0\rangle = |-\rangle \otimes |+\rangle \otimes |+\rangle$$

$$-(|0\rangle - |1\rangle)$$

$$|\psi_2\rangle = O_f|\psi_1\rangle = |0 \oplus f(x)\rangle|++\rangle - |1 \oplus f(x)\rangle|++\rangle = (|1\rangle - |0\rangle) \otimes |++\rangle$$

$$= -|-\rangle \otimes |+\rangle \otimes |+\rangle$$

$$|\psi_3\rangle = -H|+\rangle \otimes H|+\rangle = -|00\rangle \longrightarrow P(00) = |\langle 00|\psi_3\rangle|^2 = (-1)^2 = 100\%$$

constant

Aktivitas

- 2-qubit DJ
- 3-qubit DJ

Algoritma Berstein-Vazirani

Berstein-Vazirani Problem

- Algoritma Berstein-Vazirani adalah extension dari algoritma Deutsch-Jozsa
- Misalkan terdapat sebuah oracle dengan input string (x) dan output 0 atau 1

$$f(\{x_0, x_1, x_2, \ldots\}) \rightarrow 0 \; atau \; 1 \; dimana \; x_n \; adalah \; 0 \; atau \; 1$$

- Jika diberikan input x, $\boxed{f(x) = s \cdot x \; mod \; 2}$, kita harus menemukan s

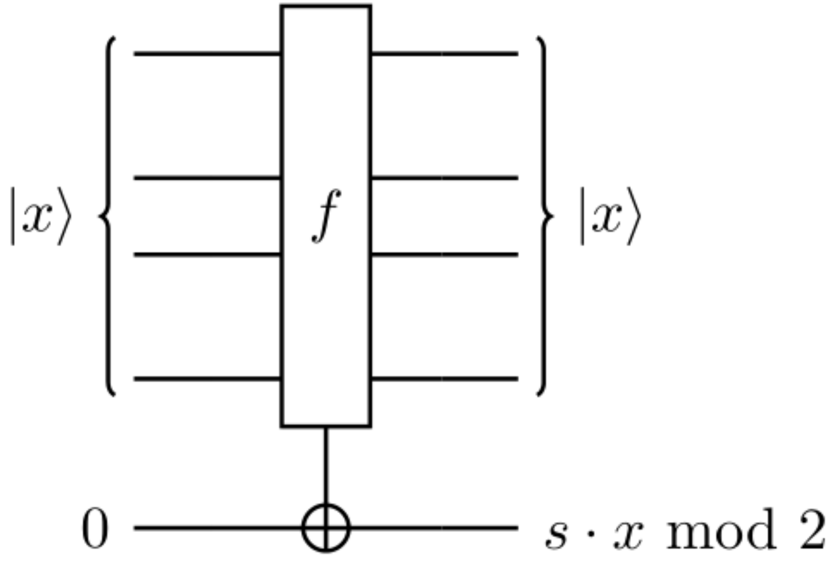
$f(0,0,1,0,0,0) = 0$

$f(1,0,0,0,0,0) = 1$

$s = 100111 \rightarrow$ password

$f(000001) = 100111 \cdot 000001 \; mod \; 2 = 1$

$f(010000) = 100111 \cdot 010000 \; mod \; 2 = 0$



Solusi Klasik

- Oracle akan menghasilkan:

$$\boxed{f_s(x) = s \cdot x \bmod 2}$$

$f(a) =$

- Untuk setiap input x, string tersembunyi akan muncul melalui query
- Dimana setiap query menampilkan bit s yang berbeda
- Jadi perlu memanggil fungsi $f_s(x)$ sebanyak n kali

$s = 100111$

$f(100000) = 100111 \cdot 100000 \bmod 2 = 1$

$f(010000) = 100111 \cdot 010000 \bmod 2 = 0$

$f(001000) = 100111 \cdot 001000 \bmod 2 = 0$

$f(000100) = \quad = 1$

$f(000010) = \quad = 1$

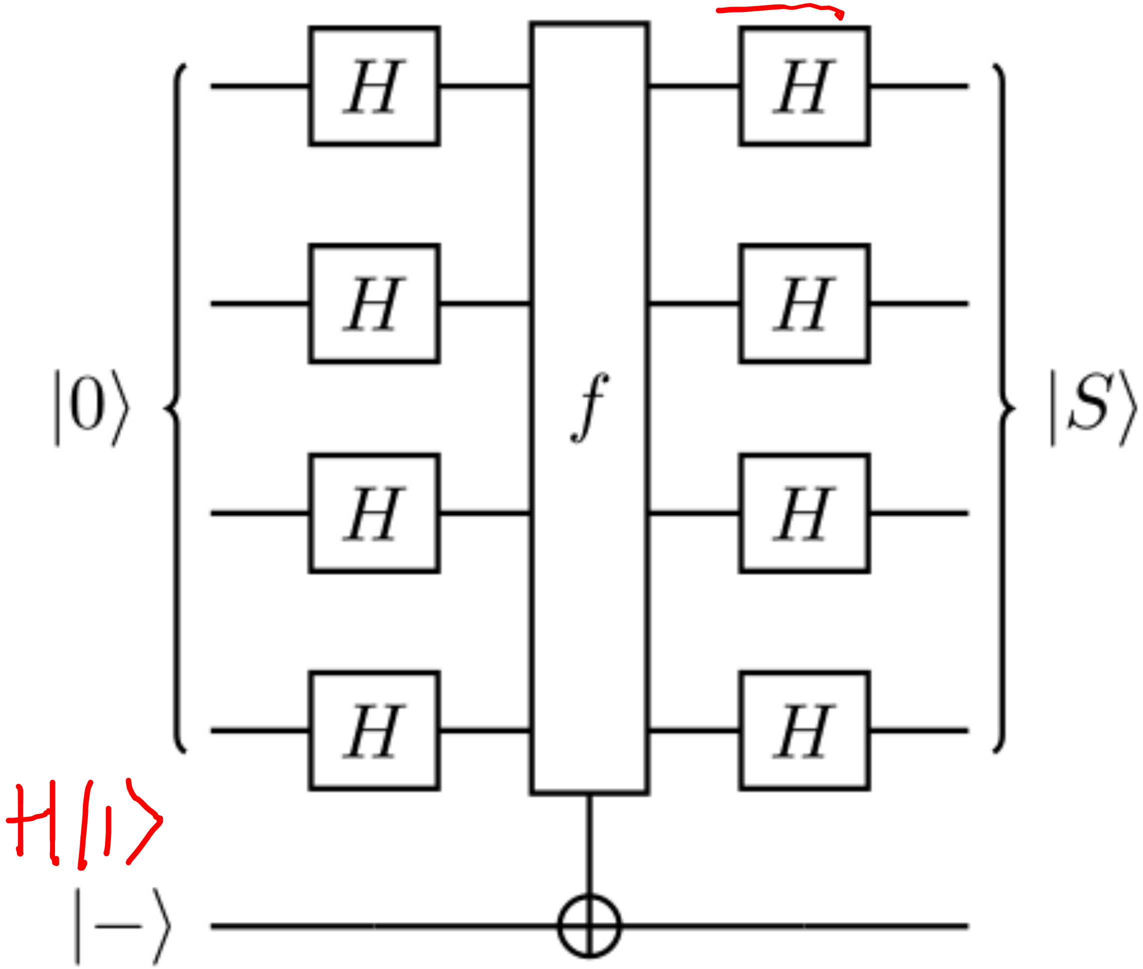
$f(000001) = \quad = 1$

s

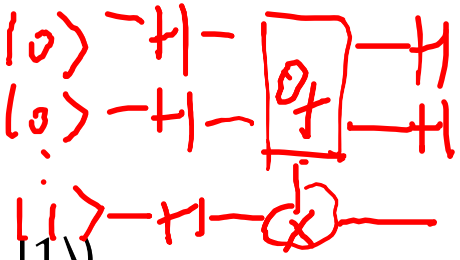
$n = 6$ kali coba

Solusi Kuantum

- Dengan komputer kuantum, kita dapat memecahkan masalah ini hanya sekali memanggil fungsi f



Bernstein-Vazirani Algorithm



- Siapkan 2 register (pertama: n-qubit diinisiasi $|0\rangle$, kedua: 1-qubit diinisiasi $|1\rangle$)

$$|\psi_0\rangle = |1\rangle|0\rangle^{\otimes n}$$

- Gunakan Hadamard gate pada setiap qubit

$|-\rangle$

$$|\psi_1\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(|0\rangle - |1\rangle)|x\rangle$$

$(-1)^0 = 1$

- Gunakan oracle $|y\rangle|x\rangle \to |y \oplus f(x)\rangle|x\rangle = |y \oplus s \cdot x \bmod 2\rangle|x\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(|0 \oplus s \cdot x \bmod 2\rangle - |1 \oplus s \cdot x \bmod 2\rangle)|x\rangle = \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(-1)^{s\cdot x}(|0\rangle - |1\rangle)|x\rangle$$

$(-1)^1 = -1$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$$|\psi_3\rangle = H^{\otimes n}\frac{1}{2^n}\sum_{x=0}^{2^n}(-1)^{s\cdot x}|x\rangle = |s\rangle$$

$HH|\psi\rangle = I|\psi\rangle$

$H^{\otimes n}H^{\otimes n}|s\rangle = H^{\otimes n}\frac{1}{\sqrt{2^{n+1}}}\sum_{k}^{2^{n-1}}(-1)^{s\cdot x}|x\rangle = |s\rangle$

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Contoh 2-qubit

$s = 11$

- Siapkan 2 register (pertama: 2-qubit diinisiasi $|00\rangle$, kedua: 1-qubit diinisiasi $|1\rangle$)

$$|\psi_0\rangle = |1\rangle|00\rangle$$

- Gunakan Hadamard gate pada setiap qubit

$H|1\rangle$ $\quad H|0\rangle \otimes H|0\rangle$

$|-\rangle$

$$|\psi_1\rangle = \frac{1}{\sqrt{2^3}}(|0\rangle - |1\rangle)(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

- Gunakan oracle $|y\rangle|x\rangle \rightarrow |y \oplus f(x)\rangle|x\rangle = (|0 \oplus 11 \cdot x \bmod 2\rangle - |1 \oplus 11 \cdot x \bmod 2\rangle)|x\rangle$

$$|\psi_2\rangle = \frac{1}{2}\sum_{x=0}^{2^2-1} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)(-1)^{11\cdot x}|x\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\frac{1}{2}\left((-1)^{11\cdot 00}|00\rangle + (-1)^{11\cdot 01}|01\rangle + (-1)^{11\cdot 10}|10\rangle + (-1)^{11\cdot 11}|11\rangle\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$s = 11$

$$|\psi_3\rangle = H^{\otimes 2}\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) = |11\rangle$$

$H|1\rangle \otimes H|1\rangle = |-\rangle|-\rangle = \frac{1}{2}(|0\rangle - |1\rangle)\otimes(|0\rangle - |1\rangle) = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$

Contoh 3-qubit s=101

$$|\psi_1\rangle = H|1\rangle \otimes H|0\rangle \otimes H|0\rangle \otimes H|0\rangle$$

$$= |-\rangle \otimes |+\rangle \otimes |+\rangle \otimes |+\rangle$$

$$|\psi_2\rangle = O_f|\psi_1\rangle = \sum_{x}^{8} \left(|0 \oplus s\cdot x \bmod 2\rangle - |1 \oplus s\cdot x \bmod 2\rangle\right)|x\rangle \frac{1}{4}$$

$$= \left(|0\rangle - |1\rangle\right) \sum_{x}^{8} \frac{1}{4}(-1)^{s\cdot x}|x\rangle$$

$$= |-\rangle \frac{1}{4}\Big[\underset{+}{(-1)^{101\cdot 000}}|000\rangle + \underset{-}{(-1)^{101\cdot 001}}|001\rangle + \underset{+}{(-1)^{101\cdot 010}}|010\rangle + \underset{-}{(-1)^{101\cdot 011}}|011\rangle +$$
$$\underset{-}{(-1)^{101\cdot 100}}|100\rangle + \underset{+}{(-1)^{101\cdot 101}}|101\rangle + \underset{-}{(-1)^{101\cdot 110}}|110\rangle + \underset{+}{(-1)^{101\cdot 111}}|111\rangle\Big]$$

$$|\psi_3\rangle = H^{\otimes 3}|\psi_2\rangle = |101\rangle \longrightarrow s = 101 \longrightarrow O(1)$$

Aktivitas

- 3-qubit BV algorithm

Algoritma Simon

God's People for God's Glory
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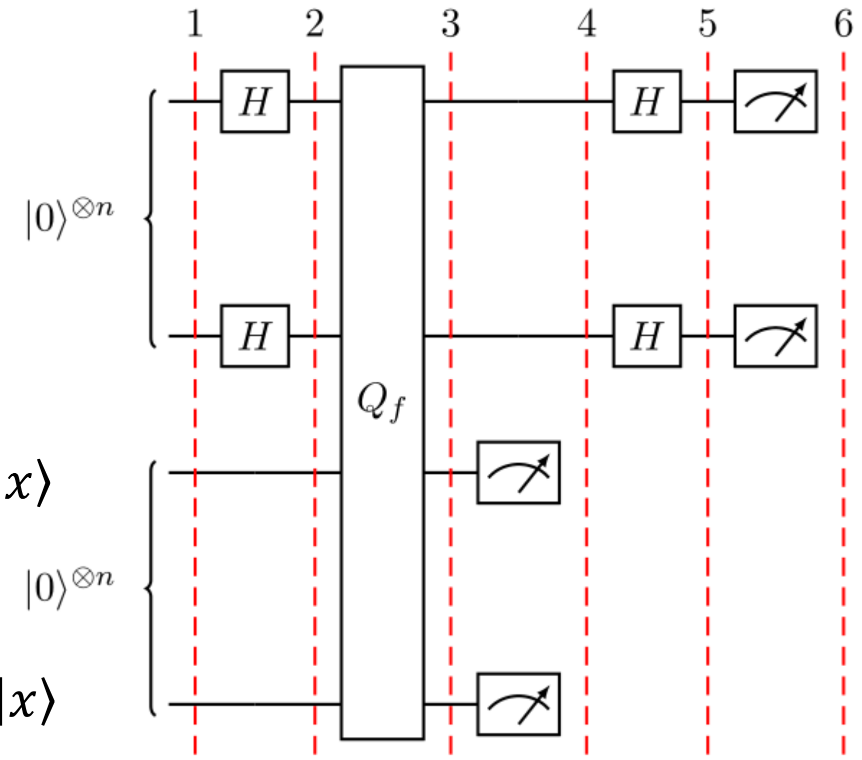
Simon's Problem

- Algoritma Simon adalah algoritma kuantum pertama yang menunjukan peningkatan eksponensial dibanding algoritma klasik
- Jika terdapat blackbox f yang memiliki 2 kemungkinan: one-to-one atau two-to-one:
  - One-to-one: contoh fungsi dengan 4 input
    $$f(1) \to 1, f(2) \to 2, f(3) \to 3, f(4) \to 4$$
  - Two-to-one: contoh fungsi dengan 4 input
    $$f(1) \to 1, f(2) \to 2, f(3) \to 1, f(4) \to 2$$
- Dapatkah kita memprediksi apakah f itu one-to-one atau two-to-one

Solusi Klasik

- Untuk akurasi 100% kita perlu cek $2^{n-1} + 1$ kali:
  - $f(00 \ldots 0) \rightarrow x_1$
  - $f(00 \ldots 1) \rightarrow x_2$
  - …
  - $f(11 \ldots 1) \rightarrow x_{2^n}$

Simon Algorithm



- Siapkan 2 register

$$|\psi_1\rangle = |0\rangle^{\otimes n}|0\rangle^{\otimes n}$$

- Gunakan Hadamard gate pada register pertama

$$|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |0\rangle^{\otimes n}|x\rangle$$

- Gunakan oracle$|y\rangle|x\rangle \rightarrow |y \oplus f(x)\rangle|x\rangle$

$$|\psi_3\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |f(x)\rangle|x\rangle$$

- Ukur register kedua, maka register pertama akan menjadi

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|x\rangle + |y\rangle) = \frac{1}{\sqrt{2}}(|x\rangle + |x \oplus b\rangle)$$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$$|\psi_5\rangle = \frac{1}{\sqrt{2^{n+1}}} \sum_{x\in\{0,1\}^n} [(-1)^{x\cdot z} + (-1)^{y\cdot z}]|z\rangle$$

- Ukur register pertama, dan menghasilkan output jika:

$$(-1)^{x\cdot z} = (-1)^{y\cdot z} \rightarrow x \cdot z = (x \oplus b) \cdot z \rightarrow b \cdot z = 0 \; mod \; 2$$

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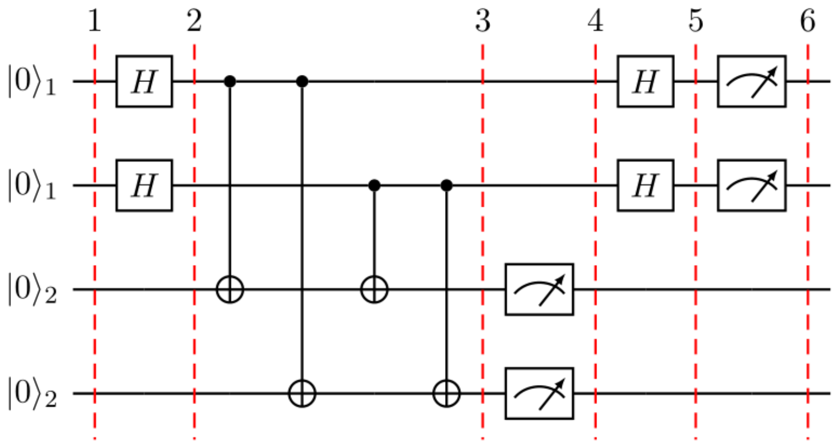
Contoh 2-qubit (b=11)



- Siapkan 2 register

$$|\psi_1\rangle = |00\rangle|00\rangle$$

- Gunakan Hadamard gate pada register pertama

$$|\psi_2\rangle = |00\rangle\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

- Gunakan oracle$|y\rangle|x\rangle \rightarrow |y \oplus f(x)\rangle|x\rangle$

$$|\psi_3\rangle = \frac{1}{2}(|00\rangle|00\rangle + |11\rangle|01\rangle + |11\rangle|10\rangle + |00\rangle|11\rangle)$$

- Ukur register kedua, maka register pertama akan menjadi

$$|11\rangle \rightarrow |\psi_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|00\rangle \rightarrow |\psi_4\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- Gunakan Hadamard gate pada setiap qubit di register pertama

$$|11\rangle \rightarrow |\psi_5\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|00\rangle \rightarrow |\psi_5\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

- Ukur register pertama, dan menghasilkan output jika:

$$|11\rangle \rightarrow b \cdot 11 = 0 \,\&\, b \cdot 00 = 0 \rightarrow b = 11$$

Aktivitas

- 3-qubit (b=110) S-Algorithm

Tuhan Memberkati