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INSTITUTE OF TECHNOLOGY
God's People for God's Glory

Fisika Kuantum

Hendrik Santoso Sugiarto

IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

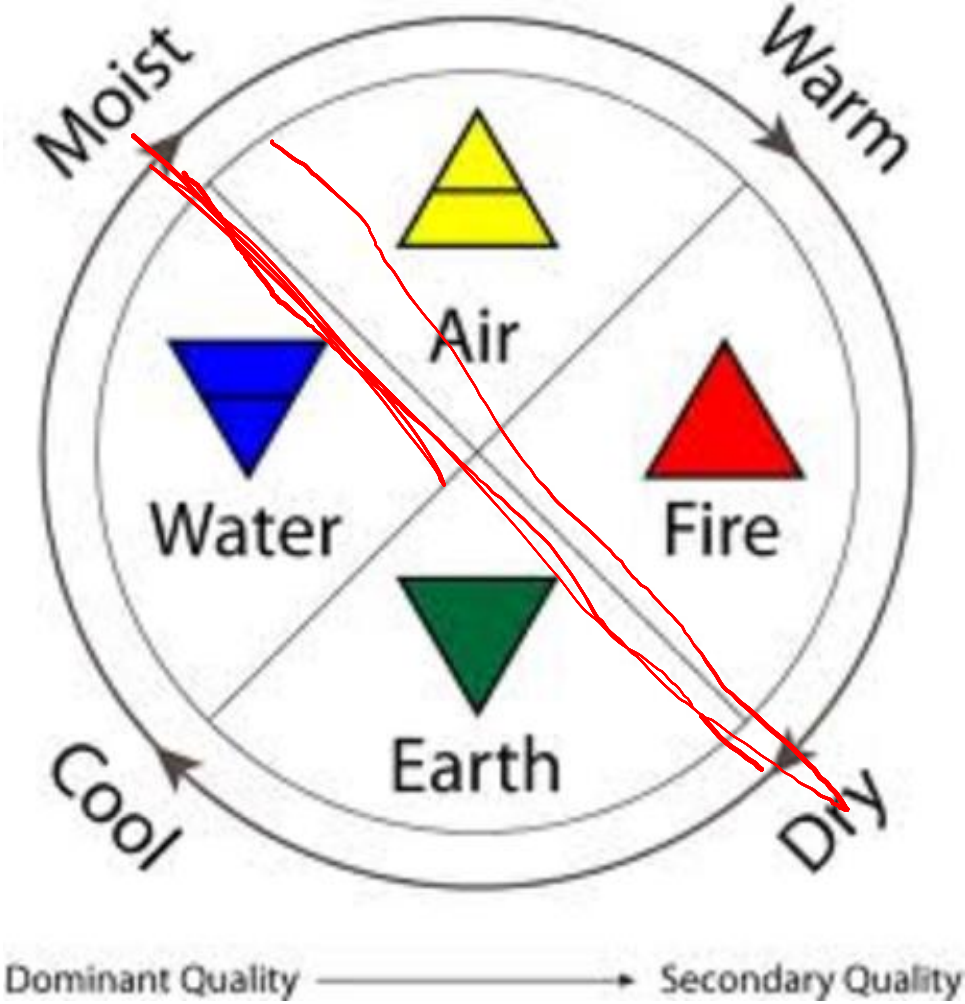
Capaian Pembelajaran

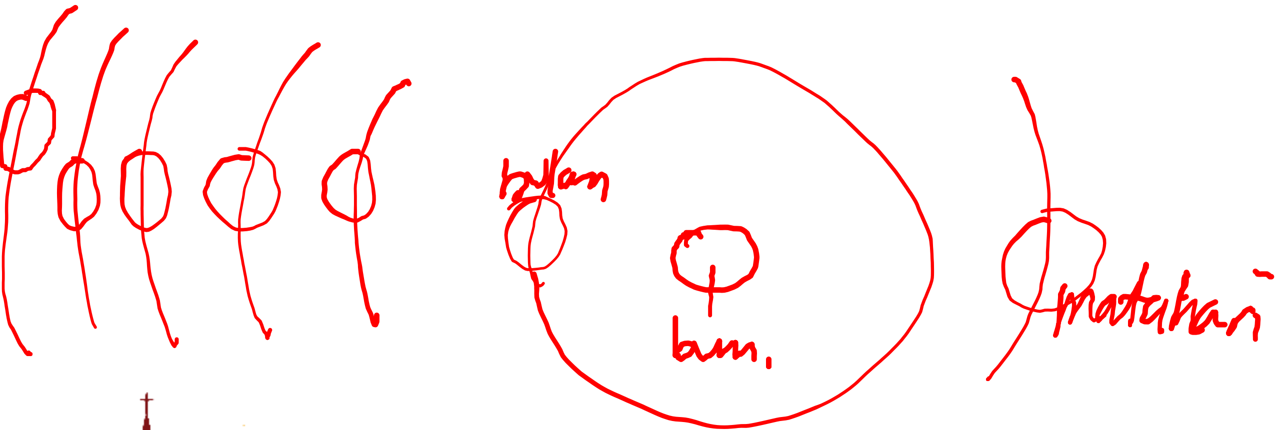
- Breakdown of Classical Physics
- Old Quantum Theory:
  - Planck: Quantization
  - Einstein: Photoelectric Effect
  - Bohr: Atomic Orbit
  - De Broglie: Wave-Particle Duality
- Modern Quantum Theory:
  - Schrodinger: Wave Mechanics
  - Born: Probabilistic Interpretation
  - Pauli: Exclusion Principle
  - Heisenberg: Uncertainty Principle & Matrix Mechanics
  - Dirac: Bracket Notation & Relativistic Quantum Mechanics
  - Standard Model
- Wavefunction
- Komutator

Breakdown of Classical Physics

Ancient Physics

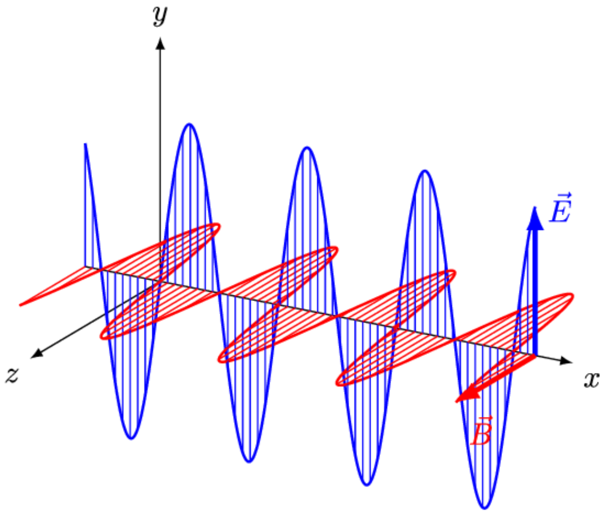
- Konteks: Greek Philosophy
- Kontributor utama: Aristotle, Ptolemy
- Asumsi dasar: surga bumi
  - Form-Matter (celestial-terrestial)
  - Teleologis
  - Filosofis
- Kontribusi:
  - Mengajukan penjelasan filosofis lepas dari mitologi





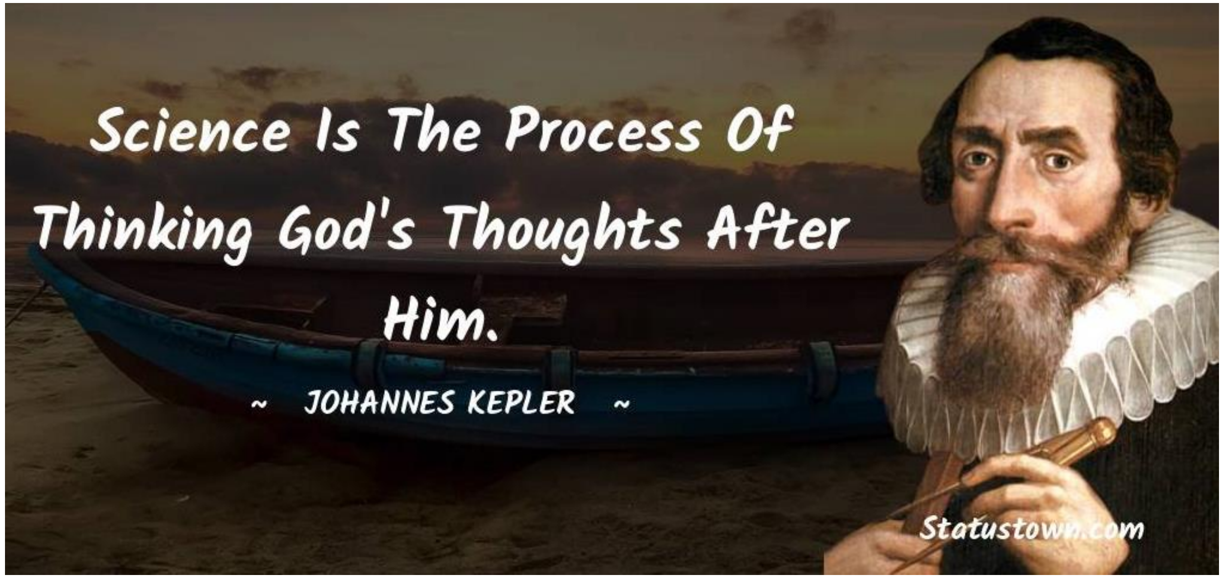
Classical Physics

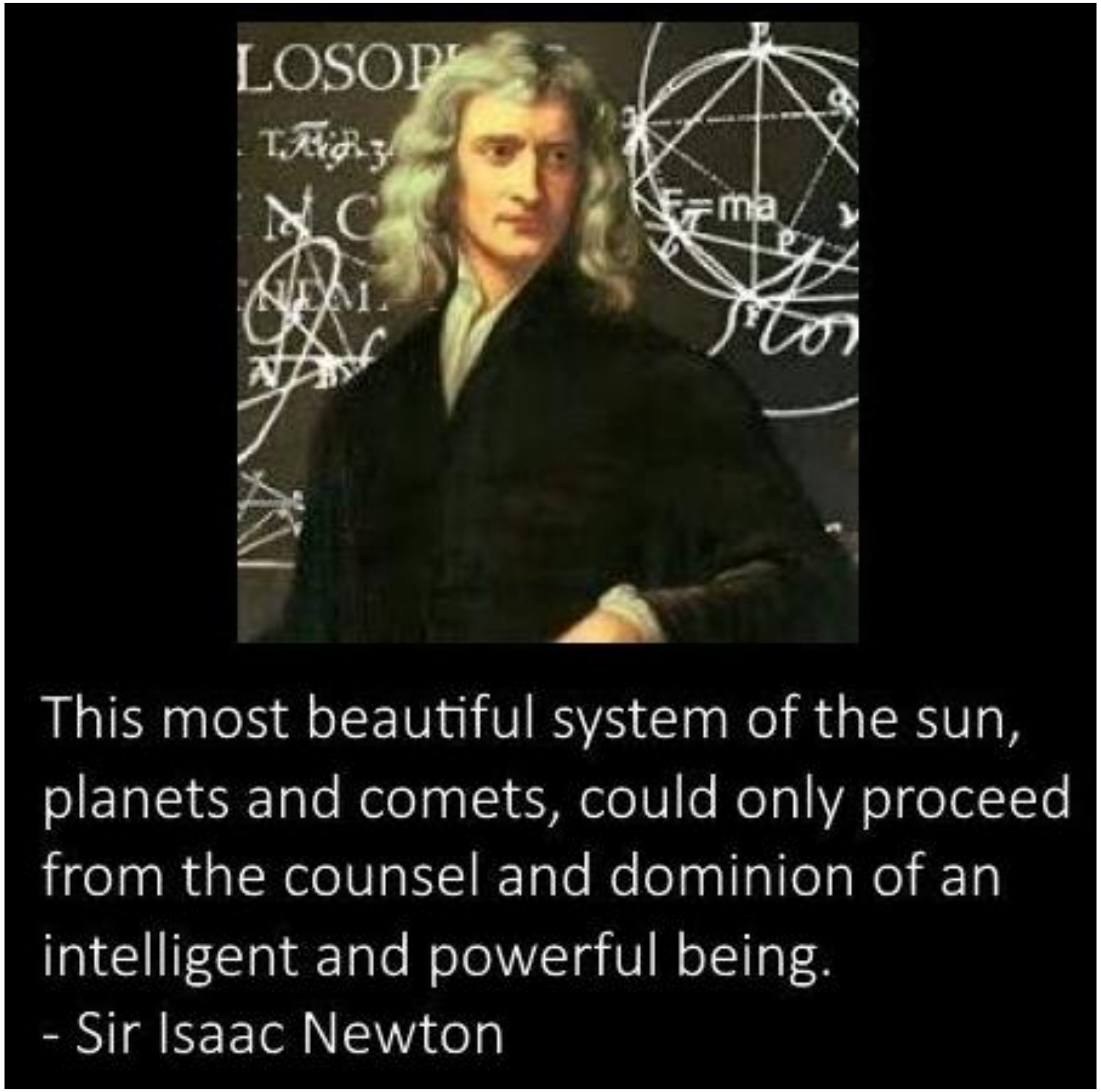
- Konteks: Reformation & Enlightenment
- Kontributor utama: Newton, Lagrange, Hamilton, Maxwell
- Asumsi dasar:
  - Universal & Deterministik
  - Interaksi lokal
  - Empiris
- Kontribusi:
  - Newton's Law: menyatukan hukum langit dan bumi
  - Maxwell's Equation: Menyatukan listrik, magnet, dan cahaya



Asumsi

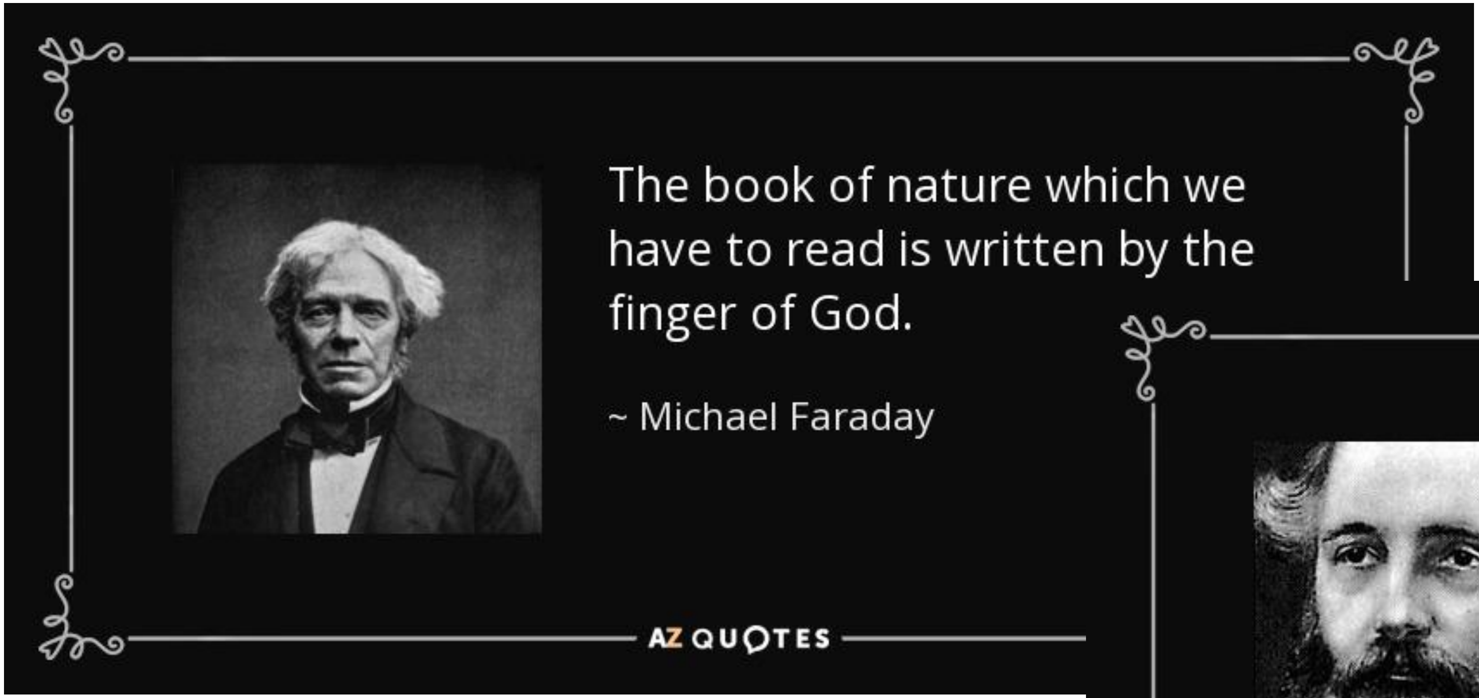
- Fisika klasik digerakan oleh asumsi adanya pencipta → universal

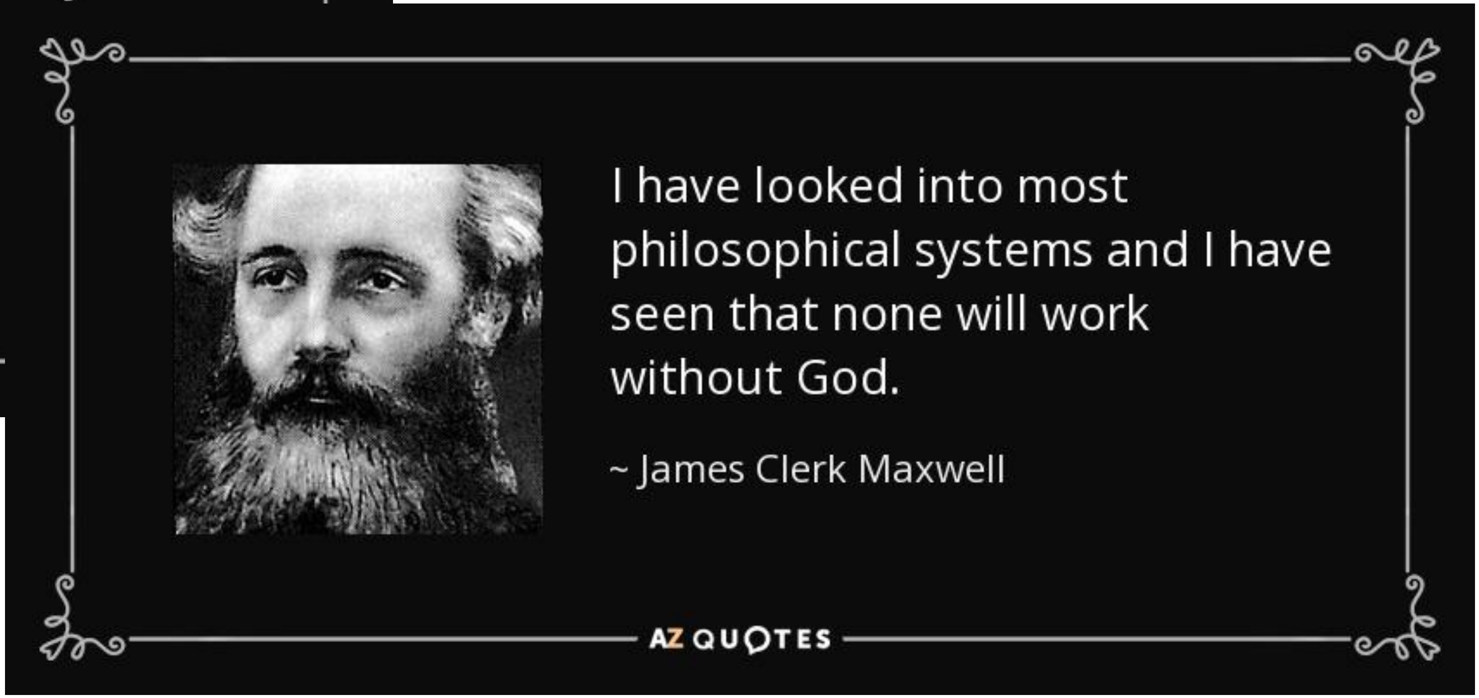




Asumsi

- Penemuan electromagnetism juga digerakan asumsi yang sama

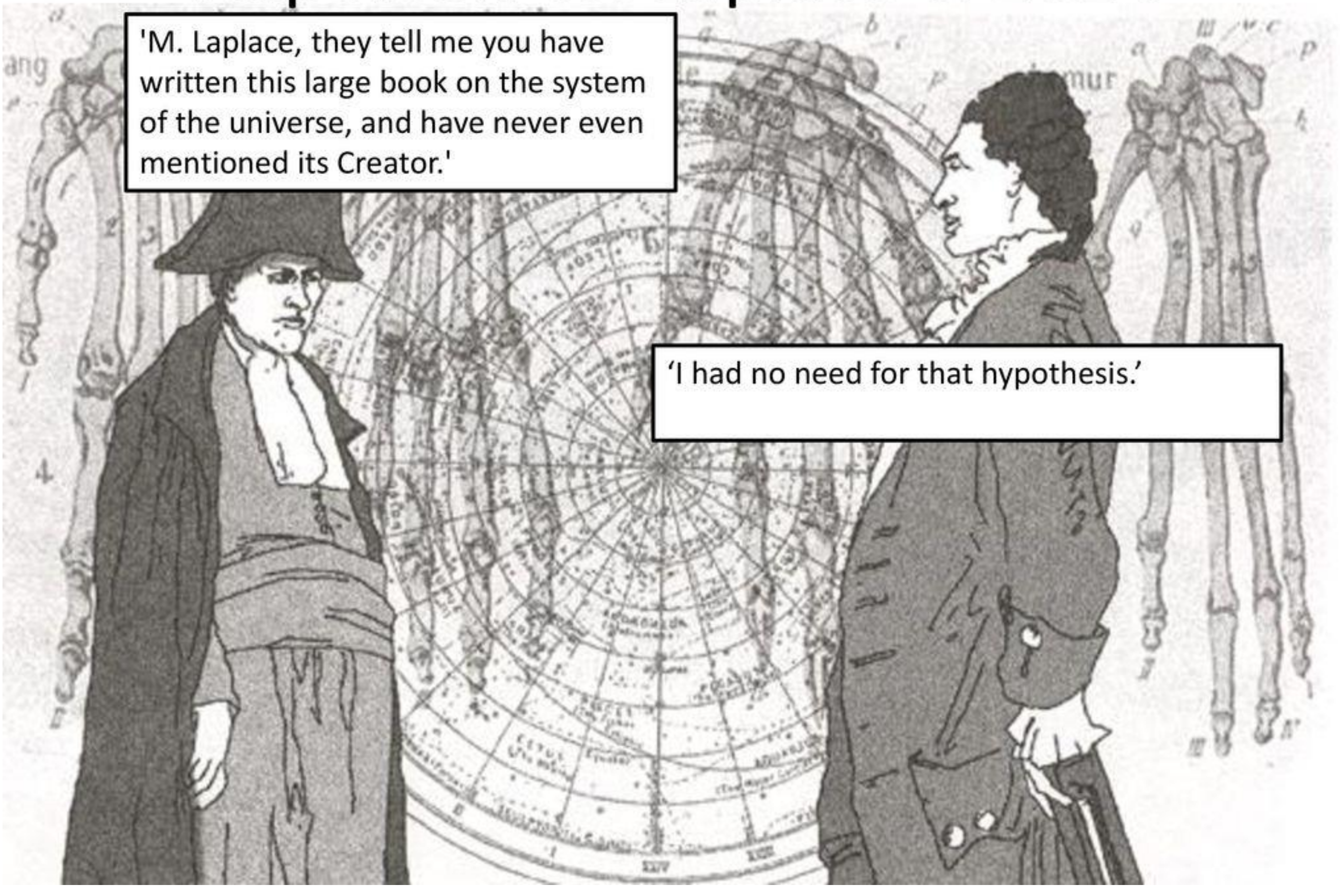




Enlightenment and Autonomy

- Enlightenment menekankan otonomi ciptaan akan Pencipta
- Tuhan digeser jauh keatas (Deism)

Napoleon Vs. Laplace c. 1814



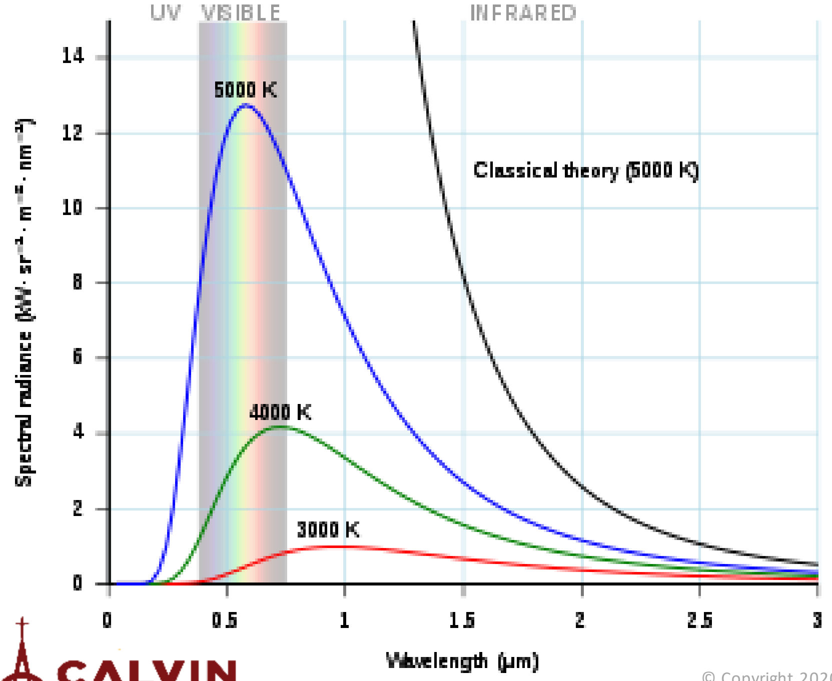
Runtuhnya Fisika Klasik

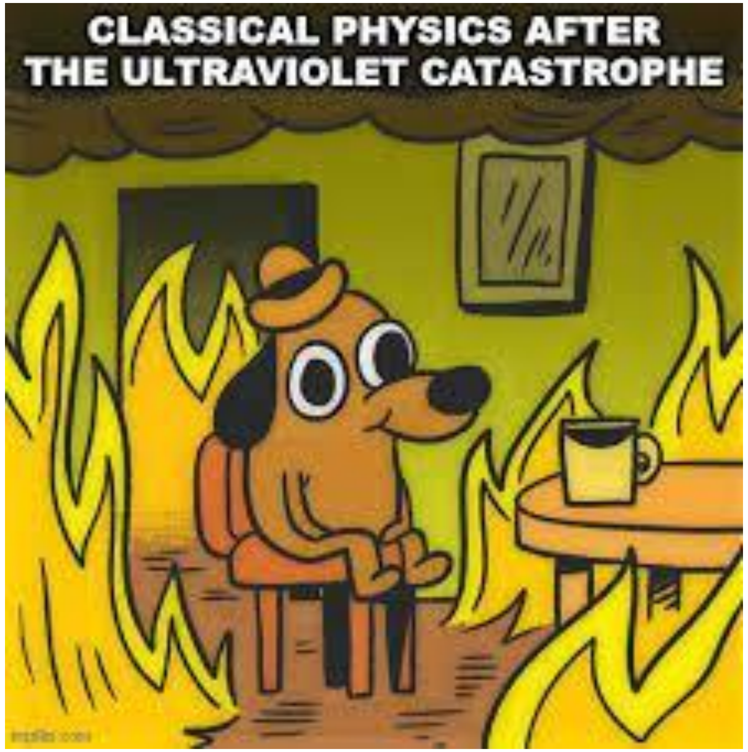
(1900an)

- Bencana ultraviolet
- Paradoks pada stabilitas orbit elektron
- Eksperimen celah ganda
- Efek fotoelektrik
- Polarisasi cahaya
- Eksperimen Stern-Gerlach
- Dll

Ultraviolet Catasthrope

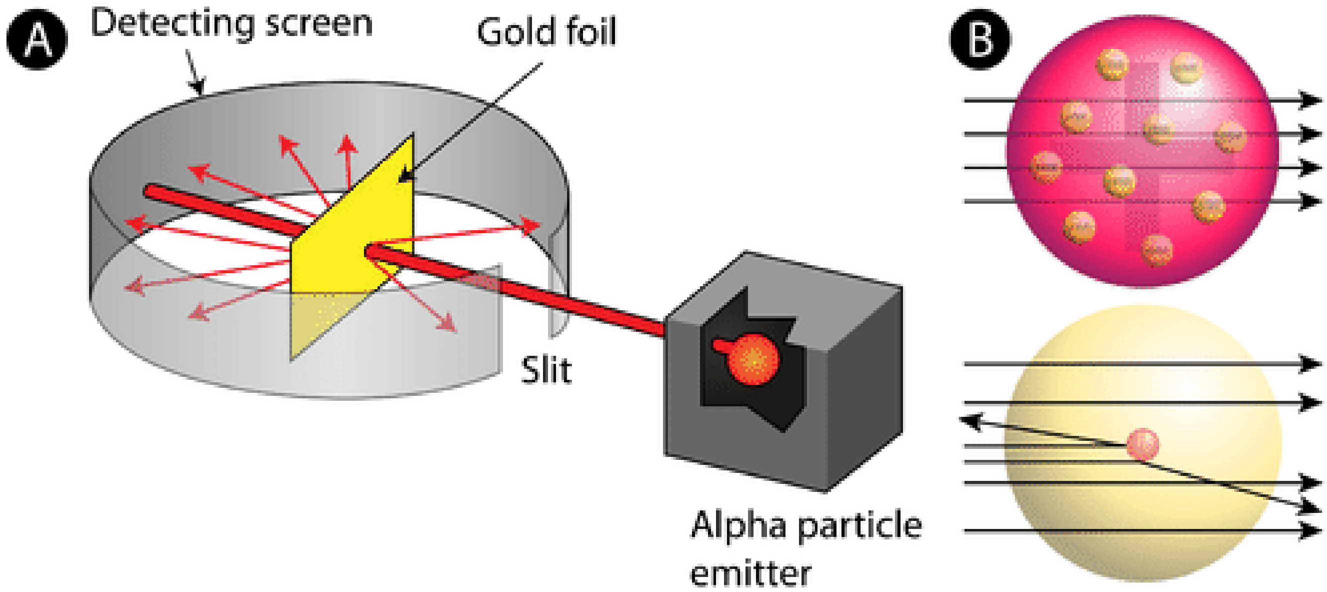
- Benda panas akan bersinar (menghasilkan radiasi)
- Benda yang sinarnya hanya berasal dari radiasi diri sendiri (bukan refleksi) disebut dengan “black body” → contoh matahari, bintang
- Pada 1900 Rayleigh dan Jeans menghitung frekuensi radiasi dari benda panas
  - hasilnya frekuensi ultraviolet yang sangat besar

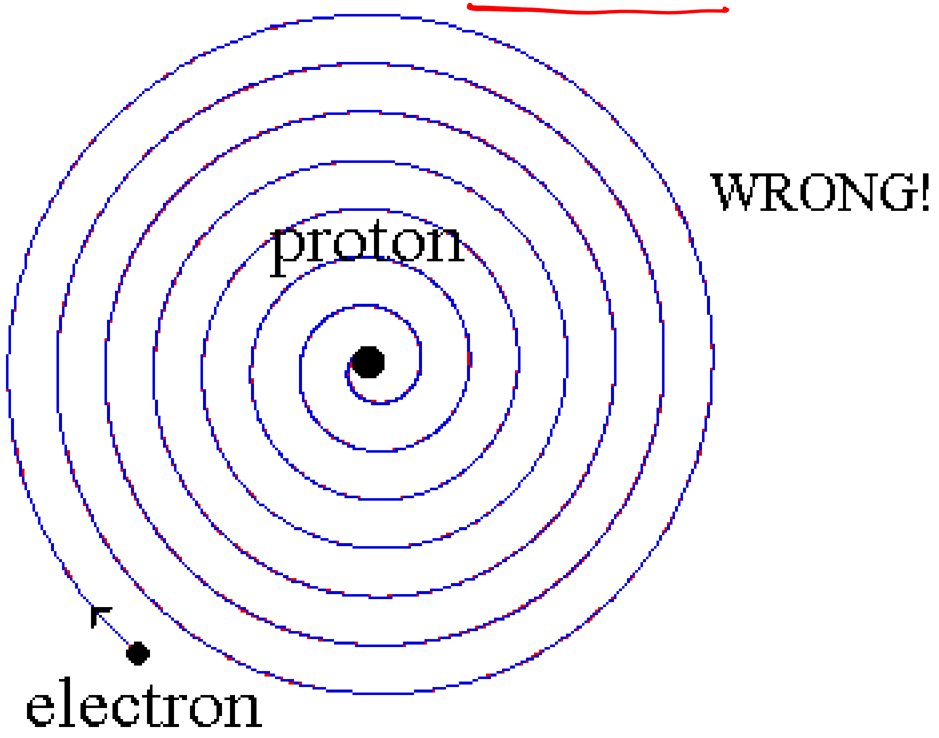




Orbitals Stability

- Rutherford: elektron mengelilingi inti atom seperti planet mengelilingi bintang
- Tapi menurut hukum maxwell, electron yang berakselerasi akan menghasilkan radiasi dan kehilangan energi
- Sehingga menurut prediksi fisika klasik, elektron akan jatuh ke dalam inti atom





Double Slit Experiment

- Jika beberapa partikel ditembakan melalui dua lubang akan membentuk pola interferensi, sekalipun hanya 1 partikel per tembakan

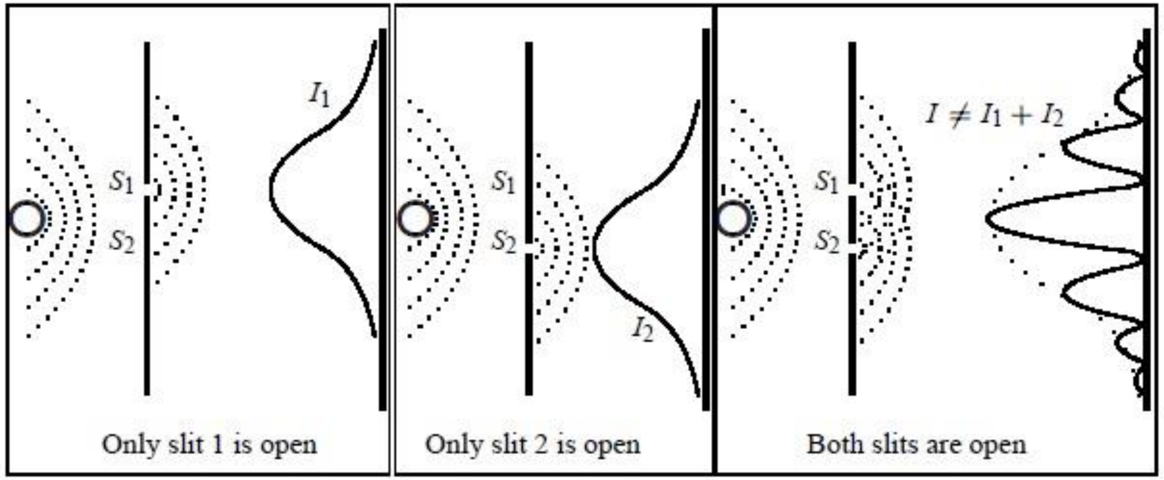
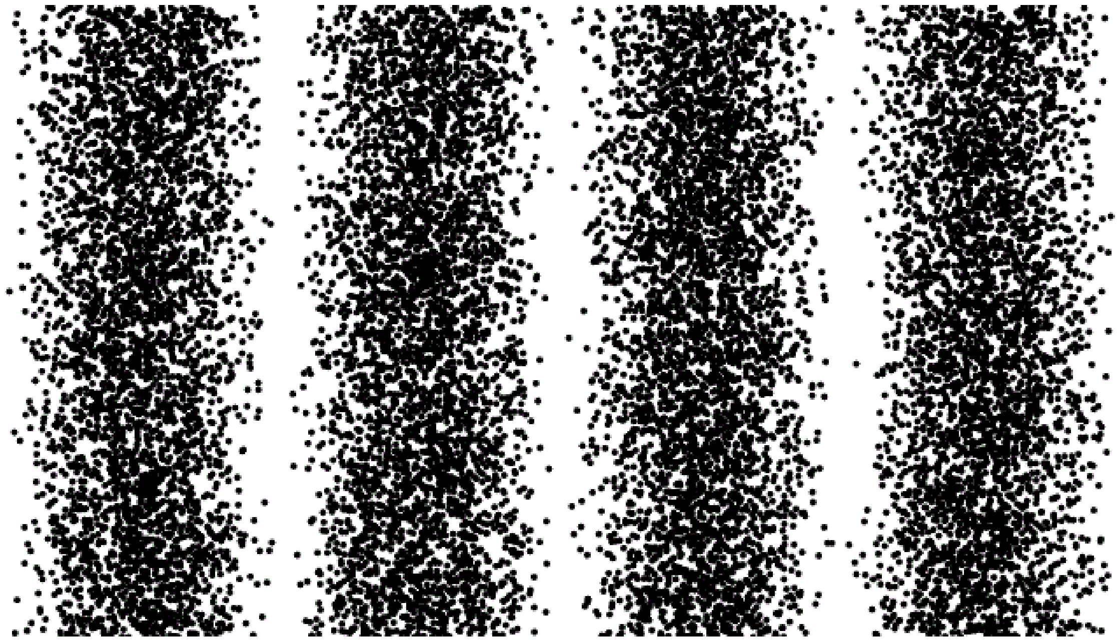
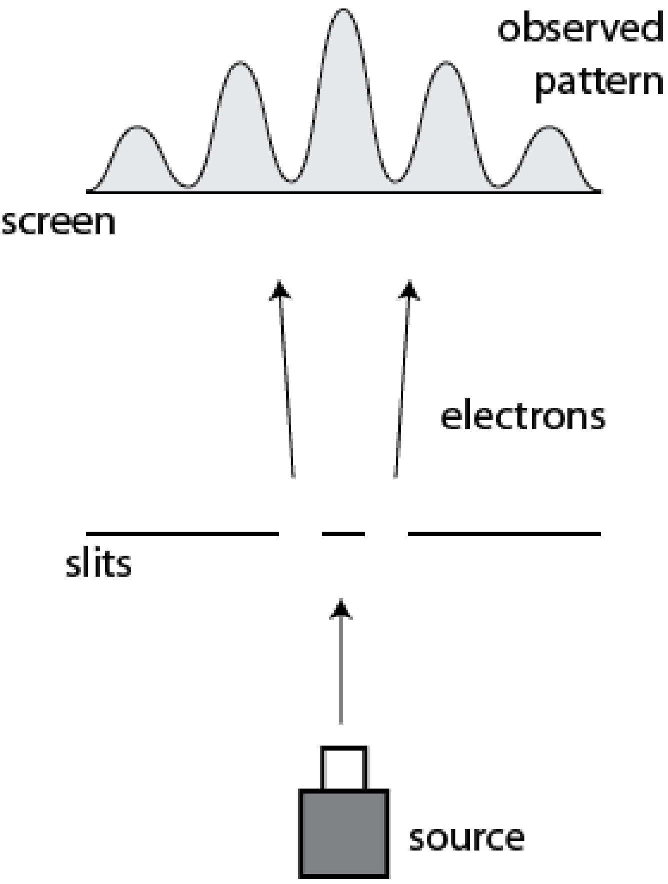


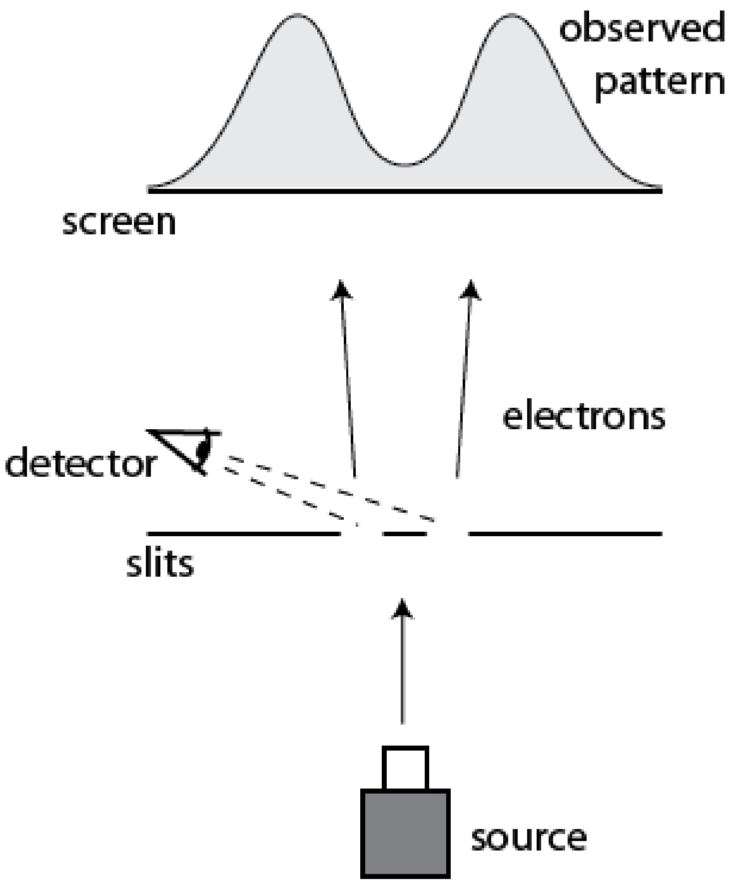
Figure 1.9 The double-slit experiment: S is a source of *waves*, I_1 and I_2 are the intensities recorded on the screen when only S_1 is open, and then when only S_2 is open, respectively. When both slits are open, the total intensity is no longer equal to the sum of I_1 and I_2; an *oscillating* term has to be added.

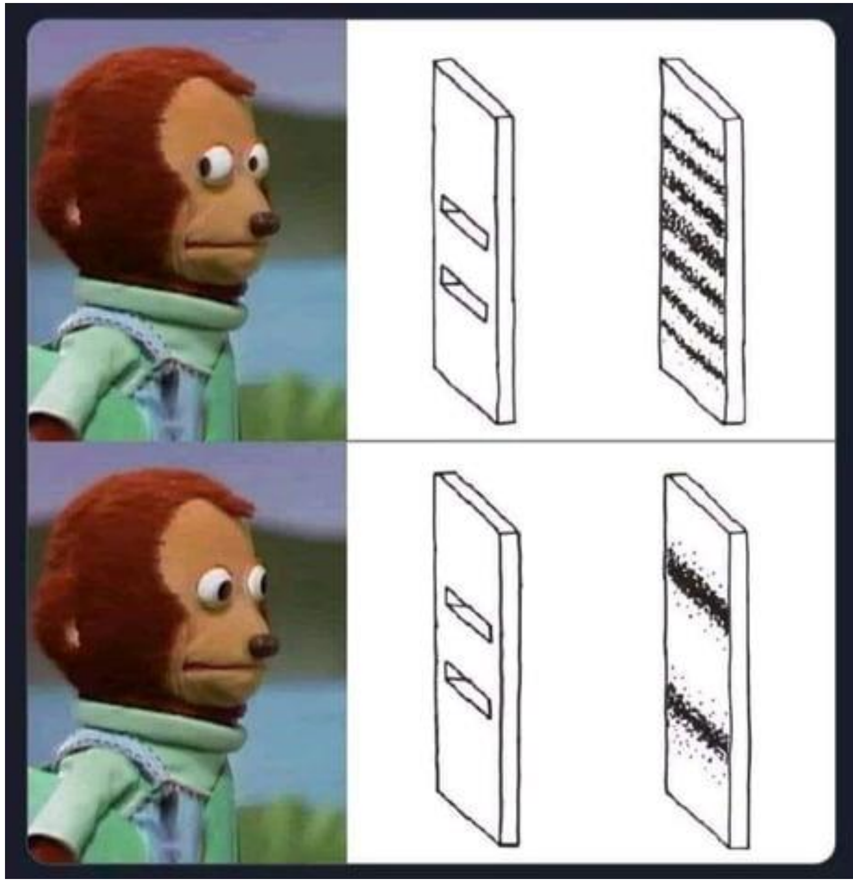


Observer

- Hasil interferensi berubah tergantung posisi observasi

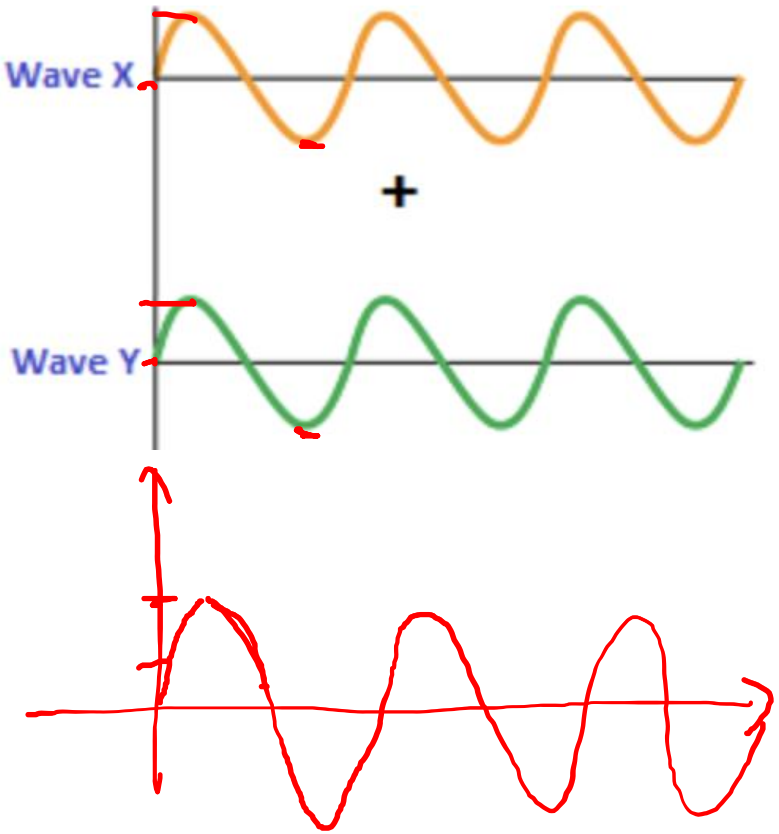


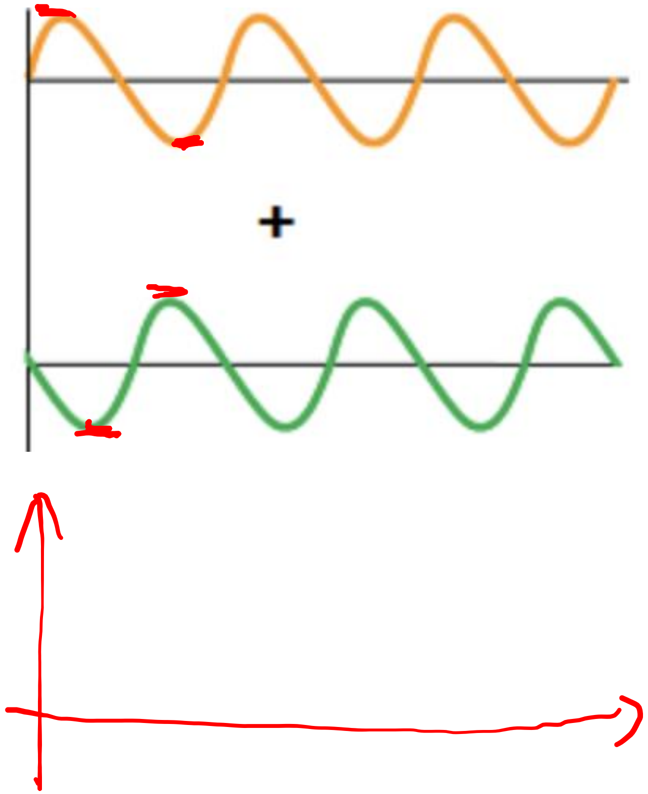




Uji Pemahaman

- Salah satu sifat gelombang adalah interferensi. Apakah hasil akhir dari resultan 2 gelombang berikut?





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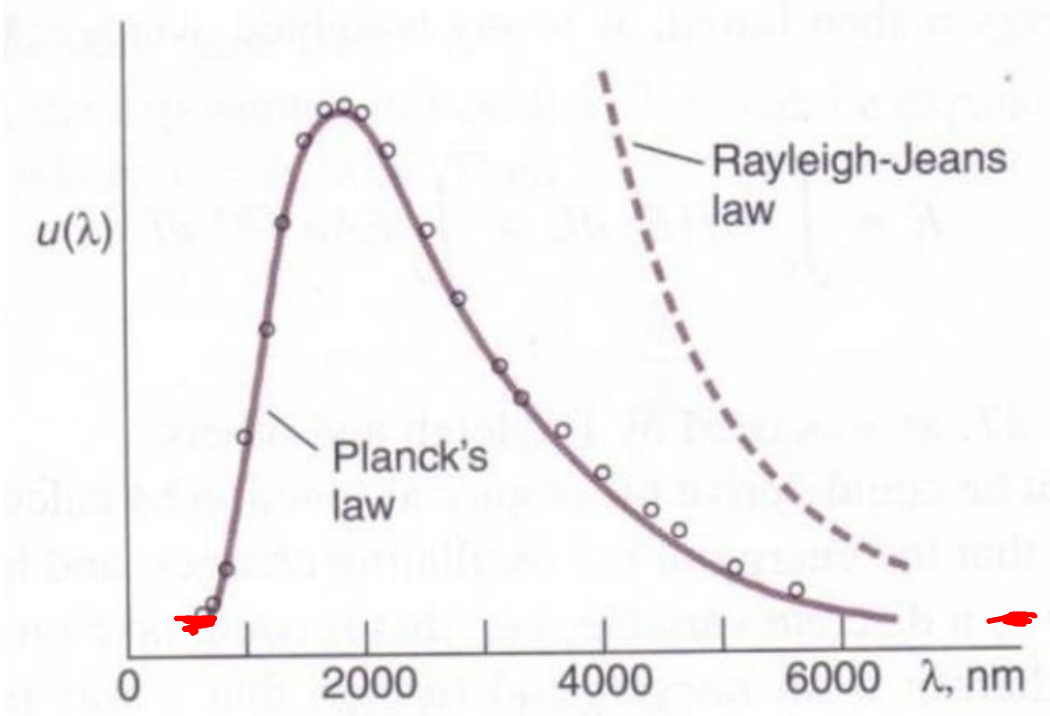
Old Quantum Theory

Planck: Quantization

→ cahaya = partikel

- Selama ini cahaya dimengerti sebagai gelombang electromagnet
- Planck: cahaya dipancarkan secara paket-paket kecil (quantized)
- Energy berbanding lurus dengan frekuensi:
  - 1 paket energi bernilai $E = hf$,
  - n paket energi bernilai $E_n = nhf$
- Energy rata-rata adalah $\overline{E_n} = \sum_{n=0}^{\infty} E_n p(n)$
- $\overline{E_n} = \dfrac{\sum_{n=0}^{\infty} E_n e^{-\frac{E_n}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{E_n}{kT}}} = \dfrac{\sum_{n=0}^{\infty} nhf\, e^{-\frac{nhf}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{nhf}{kT}}}$
- Misal $x = e^{-\frac{hf}{kT}}$
- $\overline{E_n} = \dfrac{hf\left(x+2x^2+3x^3+\cdots\right)}{(1+x+x^2+\cdots)} = \dfrac{hfx}{1-x} = \boxed{\dfrac{hf}{e^{hf/kT}-1}}$

$\dfrac{\times(1+2x+3x^2+\cdots)}{(1+x+x^2+\cdots)}$



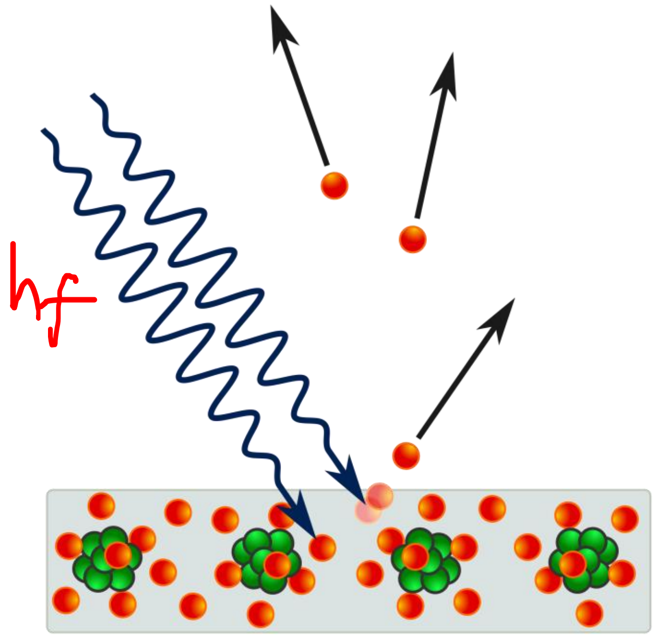
Quantization Lain

- Informasi: satuan terkecil adalah 1 byte
- Jaringan: satuan terkecil adalah 1 sel
- Populasi: satuan terkecil adalah 1 manusia
- Uang: satuan terkecil adalah koin 100 rupiah / permen

Einstein: Photoelectric Effect

- Pada 1887: Hertz menyinari keping metal dengan cahaya dan terjadi spark
- Metal yang berbeda membutuhkan frekuensi minimum yang berbeda
  - Menambah intensitas→menghasilkan lebih banyak electron, tanpa menambah energinya
  - Meningkatkan frekuensi→menghasilkan energi lebih tinggi, tanpa menambah jumlahnya
- Einstein menggunakan konsep quanta dari planck untuk menjelaskan fenomena ini
- Cahaya adalah kumpulan partikel yang energinya mengikuti $E = hf$
- $K_{max} = \underline{hf} - \underline{hf_0}$ > 0

$= 0 \rightarrow$ tdk ada elektron



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Bohr: Atomic Orbit

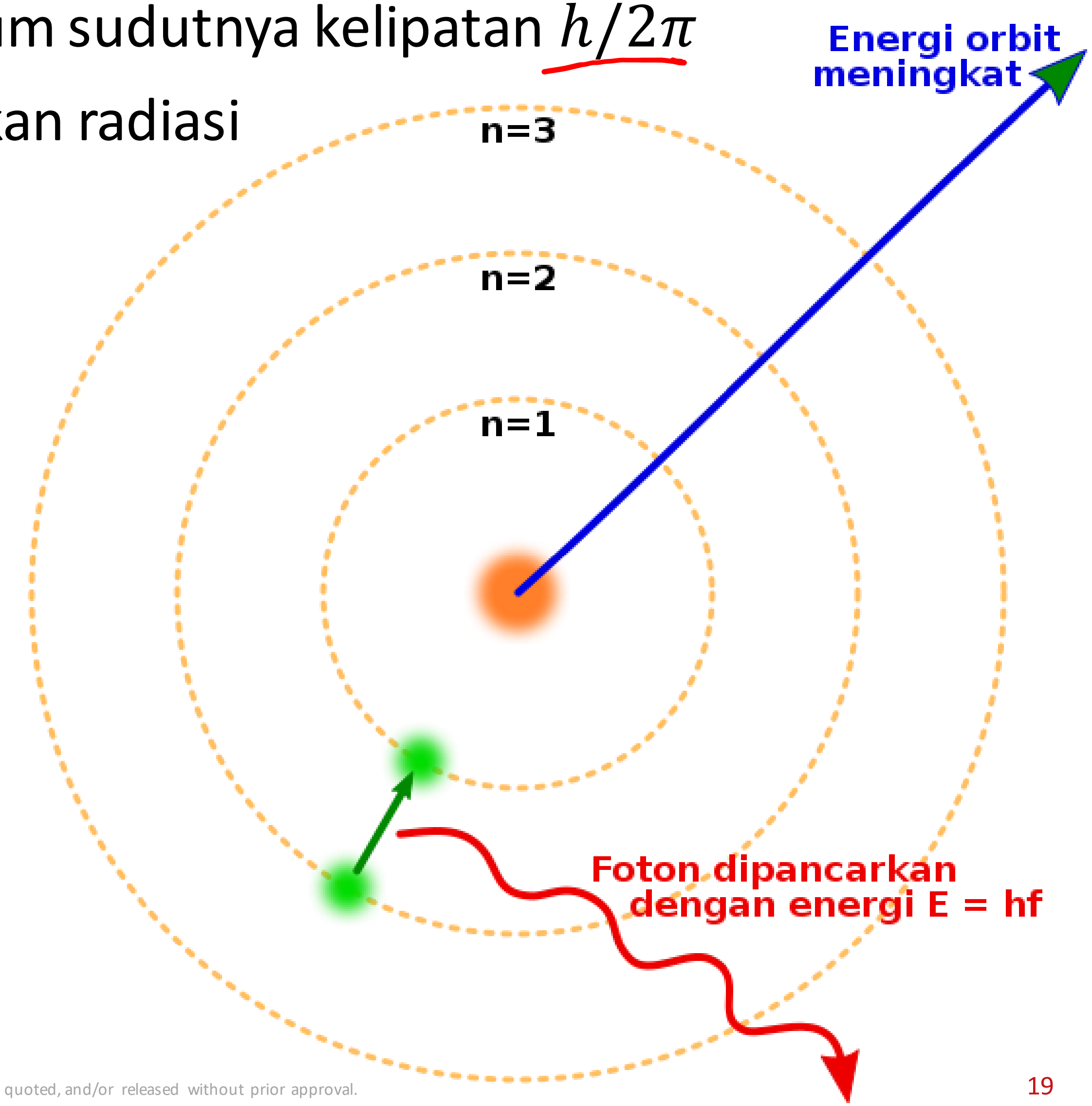
$$\frac{h}{2\pi} = mvr$$

$$v = \frac{h}{2\pi}\frac{1}{mr}$$

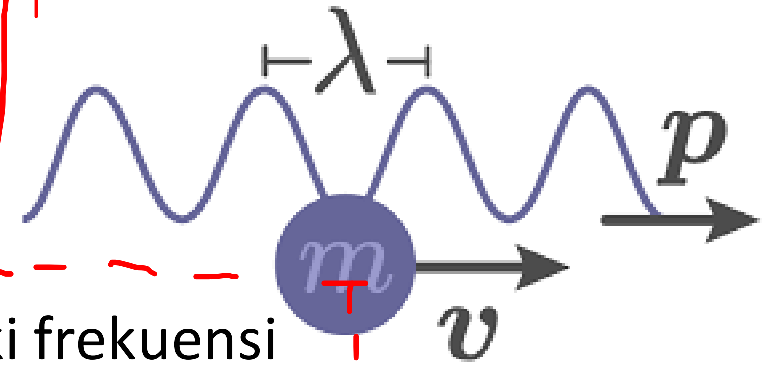
L

- Electron bergerak dalam orbit yang momentum sudutnya kelipatan $h/2\pi$
- Selama di orbit ini, electron tidak memancarkan radiasi
- $\frac{ke^2}{r^2} = \frac{mv^2}{r}$, $L = mvr = \frac{nh}{2\pi} \rightarrow \boxed{v = \frac{n\hbar}{mr}}$
- $f = \frac{E_i - E_f}{h} \propto \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\frac{1}{2^2} - \frac{1}{3^2} = \frac{1}{4} - \frac{1}{9} = \frac{5}{36}$$



De Broglie: Wave Particle Duality



- Segala sesuatu memiliki sifat gelombang sekaligus partikel
- De Broglie: gelombang memiliki momentum, partikel memiliki frekuensi
- $E = hf,\ c = \lambda f,\ E = \frac{hc}{\lambda} = pc$
- $\boxed{\lambda = \frac{h}{p}}$ → makin kecil λ, makin mirip partikel (misalnya: proton)

Paradoks	Gelombang	Partikel
Lokasi	Menyebar	Terletak di suatu posisi
Interferensi	Berpola interferensi	Tidak ada pola interferensi
Superposisi	Akumulasi bisa lebih besar atau lebih kecil dari masing-masing gelombang individu	Akumulasi sama dengan jumlah masing-masing partikel individu

Uji pemahaman

- Berapakah energi terkecil yang dimiliki cahaya? hf

$$E = nhf$$
$$= 1hf = hf$$

Modern Quantum Theory

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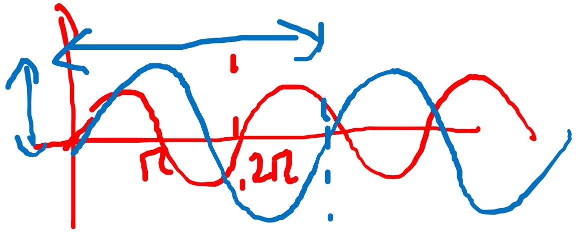
Piccard
Henriot
Ehrenfest
Herzen
Donder
Schrödinger
Verschaffelt
Pauli
Heisenberg
Fowler
Brillouin
Debye
Knudsen
Bragg
Kramers
Dirac
Compton
Broglie
Born
Bohr
Planck
Curie
Lorentz
Einstein
Langevin
Guye
Wilson
Richardso
Langmuir

Schrodinger: Wave Mechanics



Ψ psi

F = ma

- Jika di fisika klasik gerakan partikel diatur oleh hukum newton, apakah yang mengatur Gerakan gelombang-partikel dalam fisika kuantum?

$p^2 = \frac{h^2}{\lambda^2}$ $\hbar = \frac{h}{2\pi}$

old quantum

- 3 Asumsi:
  - Konsisten dengan postulat de Broglie→ $\lambda = \frac{h}{p}$ dan Planck→ $f = \frac{E}{h}$

$\omega = 2\pi f$

classical

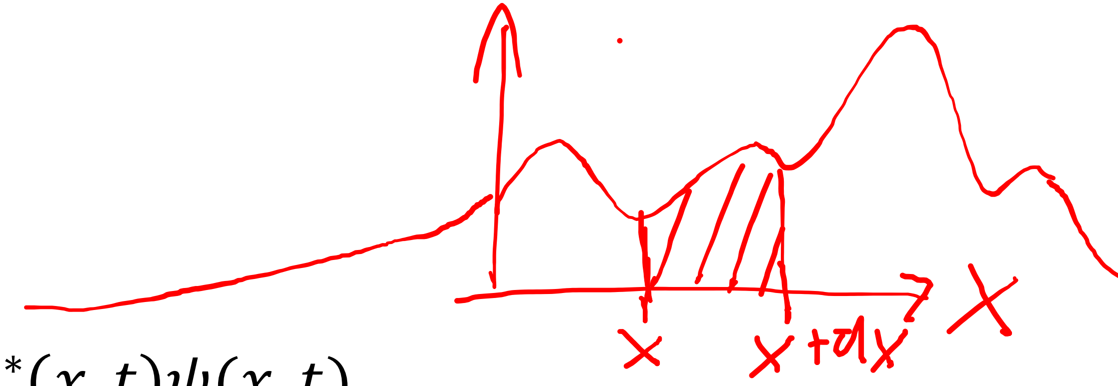
  - Konsisten dengan kekekalan energi→ $E = \frac{p^2}{2m} + V$
  - Konsisten dengan sifat gelombang: solusi linear→$\psi(x,t) = c_1\psi_1(x,t) + c_2\psi_2(x,t)$
- Kekekalan energi: $hf = \frac{h^2}{2m\lambda^2} + V \rightarrow \hbar\omega = \frac{\hbar^2 k^2}{2m} + V(x,t)$, dimana $\boxed{k = \frac{2\pi}{\lambda}}$, $\boxed{\omega = 2\pi f}$
- Gunakan fungsi gelombang $\psi(x,t) = e^{i(kx-\omega t)} = \cos(kx - \omega t) + i\sin(kx - \omega t)$
- $\frac{\partial\psi(x,t)}{\partial t} = -i\omega e^{i(kx-\omega t)}, \frac{\partial\psi(x,t)}{\partial x} = ike^{i(kx-\omega t)}, \frac{\partial^2\psi(x,t)}{\partial x^2} = -k^2 e^{i(kx-\omega t)}$

eigen vector

- Schrodinger Equation: $\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}} \rightarrow H\psi(x,t) = E\psi(x,t)$
- Dimana Hamiltonian (energi kinetik+potensial), $H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$

eigen value

Born: Probabilistic Interpretation



- Makhluk apakah wave function ini?
- Born: peluang menemukan partikel: $P(x,t) = \psi^*(x,t)\psi(x,t)$
- Peluang menemukan partikel diantar x dan $x + dx$: $P(x,t)dx = \psi^*(x,t)\psi(x,t)dx$
- Posisi rata-rata: $\bar{x} = \int_{-\infty}^{\infty} xP(x,t)dx = \int_{-\infty}^{\infty} \psi^*(x,t)x\psi(x,t)dx$
- Dengan postulat: $p \leftrightarrow \boxed{-i\hbar\frac{\partial}{\partial x}}$ dan $\mathrm{E} \leftrightarrow \boxed{i\hbar\frac{\partial}{\partial t}}$, dimana p dan E adalah operator
- Momentum rata-rata: $\bar{p} = \int_{-\infty}^{\infty} \psi^*(x,t)p\psi(x,t)dx = \int_{-\infty}^{\infty} -i\hbar\psi^*(x,t)\frac{\partial}{\partial x}\psi(x,t)dx$
- Energi rata-rata: $\bar{E} = \int_{-\infty}^{\infty} \psi^*(x,t)E\psi(x,t)dx = \int_{-\infty}^{\infty} i\hbar\psi^*(x,t)\frac{\partial}{\partial t}\psi(x,t)dx$
- $\frac{\partial\psi(x,t)}{\partial t} = -i\omega\psi(x,t) = -\frac{iE}{\hbar}\psi(x,t)$, $\frac{\partial\psi(x,t)}{\partial x} = ik\psi(x,t) = \frac{ip}{\hbar}\psi(x,t)$

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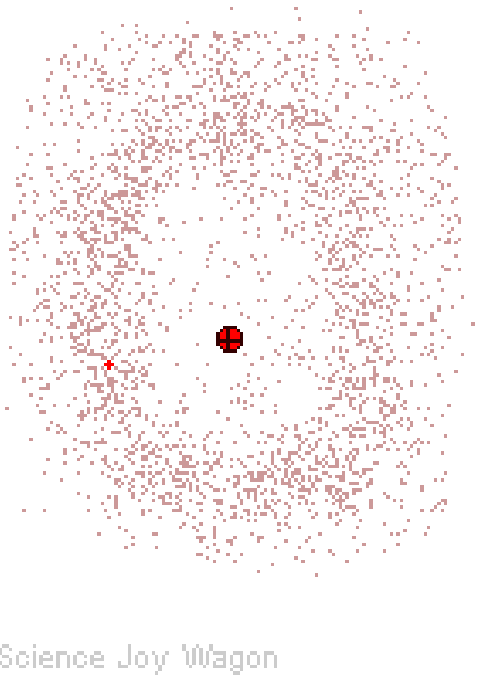
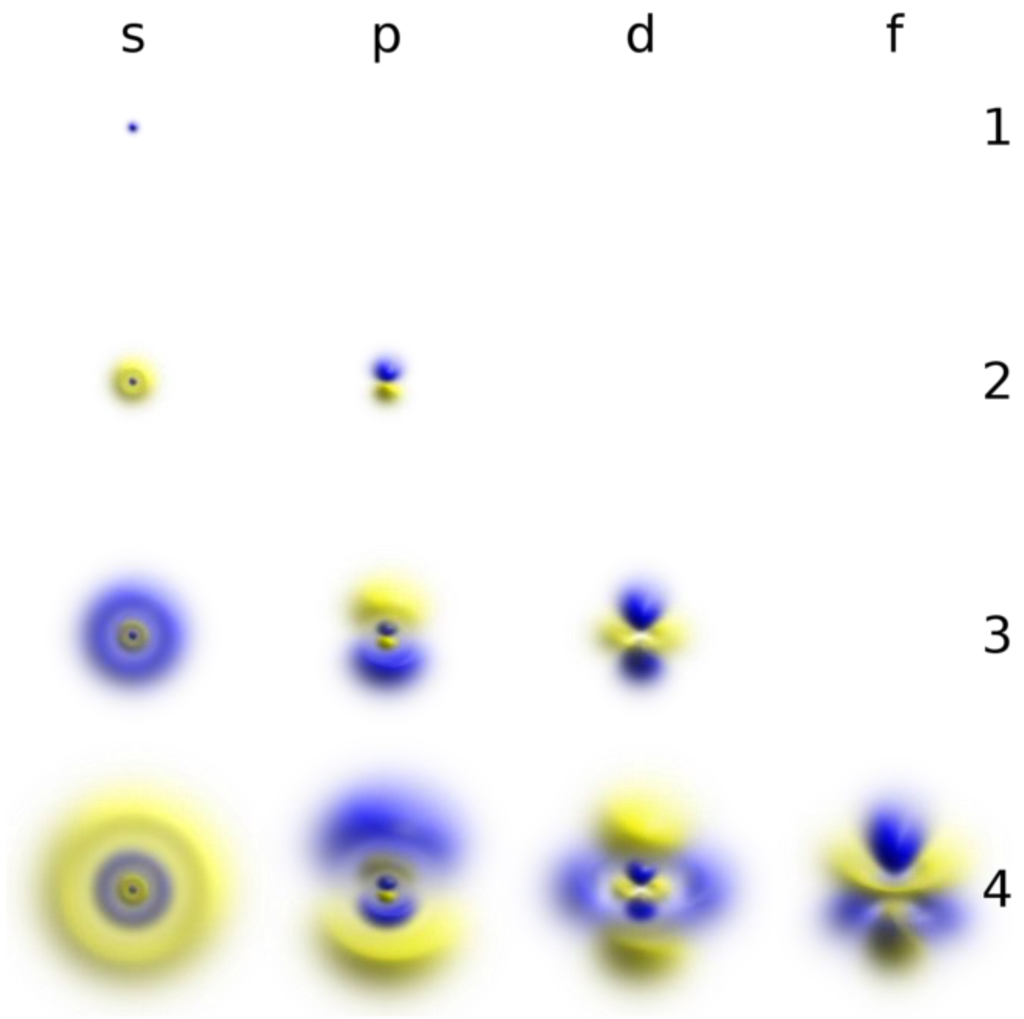
Pauli: Exclusion Principle

- Pauli: Setiap partikel identic dengan spin=1/2 (fermion), tidak dapat menempati quantum state yang sama
- Orbitals atom mengikuti schrodinger equation





Uji Pemahaman

real $\qquad$ img

- Sebuah qubit memiliki wave function $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$ Berapakah peluang mengukur 1?

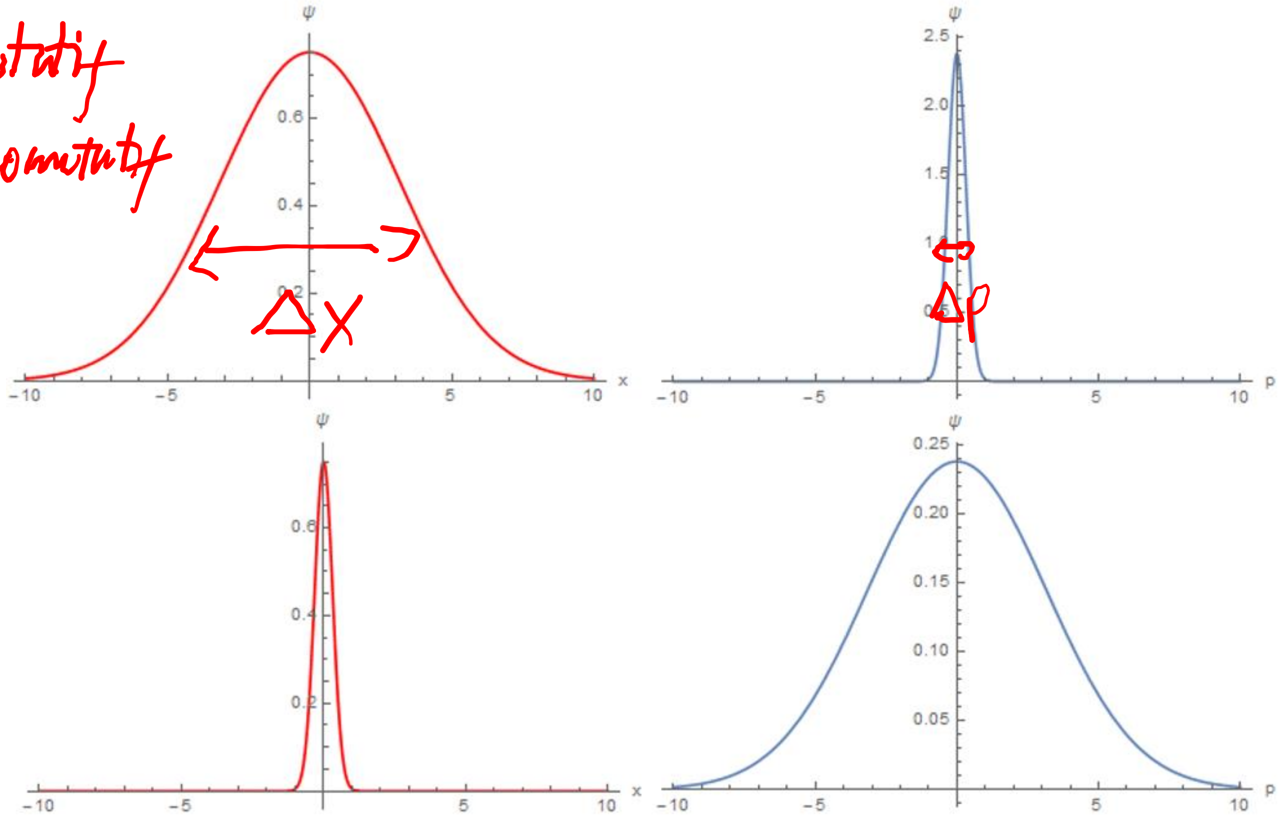
$$\psi^* = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$P = \psi^* \psi$$

$$\begin{pmatrix} P(0) \\ P(1) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Heisenberg: Uncertainty Principle

- Mengatur fundamental limit akurasi 2 variable fisika yang tidak komutatif (misalnya posisi dan momentum)→ $\Delta\hat{p}\Delta\hat{x} \geq \hbar/2$
- Bentuk umum: $\Delta\hat{A}\Delta\hat{B} \geq \frac{1}{2}|\langle[A,B]\rangle|$
- Dimana $[A,B] = AB - BA$ $= 0$ → komutatif
  $\neq 0$ → tdk komutatif
- $\Delta\hat{p}\Delta\hat{x} \geq \frac{1}{2}|\langle[p,x]\rangle| = \frac{\hbar}{2}$



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Heisenberg: Matrix Mechanics

- Hubungan yang bersifat tidak komutatif membuat banyak orang kebingungan, sampai mereka menyadari suatu teori matematika yang jarang digunakan yaitu matriks
- Mekanisme quantum mechanics dengan menggunakan wave function (Schrodinger picture) dapat direpresentasikan secara matrix (Heisenberg picture)
- Heisenberg Equation:

HA-AH

$$\frac{dA}{dt} = -\frac{i}{\hbar}[H, A] + \frac{\partial A}{\partial t}$$

- Dimana A adalah observables

	Schrodinger Picture	Heisenberg Picture
Wave function ψ	Berubah	Konstan
Observables / Operators x, p, E	Konstan	Berubah

Uji Pemahaman

- Apakah pauli-y gate, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ dan pauli-x gate, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ komutatif?

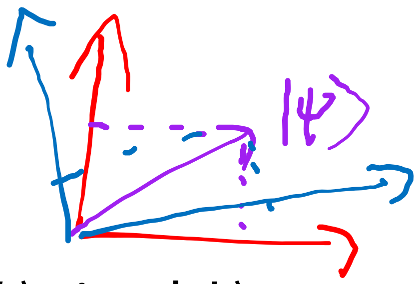
$$\sigma_y \sigma_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

$$\neq$$

$$\sigma_x \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Dirac: Bracket Notation

- Notasi bracket $\langle\psi|\psi\rangle$ terdiri dari bra $\langle\psi|$ dan ket $|\psi\rangle$ yang merupakan wave function (complex vector) yang hidup di Hilbert Space
- Ket $|\psi\rangle$ dapat direpresentasikan dalam vektor basis lain: $|\psi\rangle = \sum_i^N c_i|u_i\rangle$



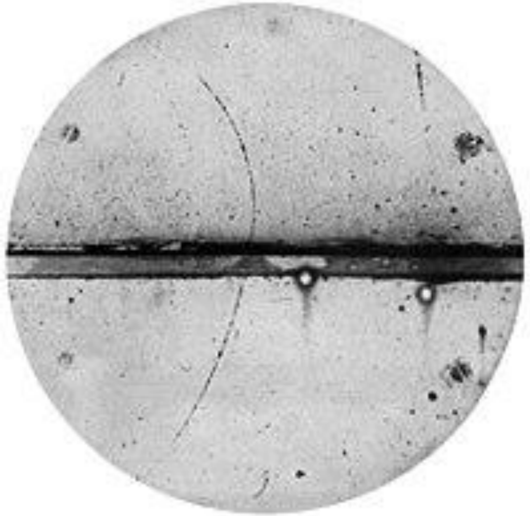
P(x,t)

- $\langle\phi|\psi\rangle = \begin{pmatrix} b_1^* \\ \vdots \\ b_N^* \end{pmatrix}^T \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix} = \int \phi^*(x,t)\psi(x,t)dx$ adalah dot product antara $|\phi\rangle$ dan $|\psi\rangle$
- Maka peluang i adalah $\langle u_i|\psi\rangle = c_i^* c_i$, dimana $|\langle\psi|\psi\rangle|^2 = 1$
- Jika terdapat operator A: $\langle\psi|A|\psi\rangle$ dimana $A|\psi\rangle$ merupakan operasi A terhadap $|\psi\rangle$
- Formalisme notasi ini akan dibahas lebih detail di pertemuan berikutnya

Dirac: Relativistic Quantum Mechanics

- Persamaan Schrodinger bersifat non-relativistic
- Persamaan Dirac: $\left(i\gamma^{\mu}\partial_{\mu} - (mc/\hbar)^2\right)\psi = 0$
- Memprediksi keberadaan positron
- Bentuk medan relativistik membuka paradigma baru: quantum field theory

Cloud chamber photograph by C. D. Anderson of the first positron ever identified. A 6 mm lead plate separates the chamber. The deflection and direction of the particle's ion trail indicate that the particle is a positron.



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Uji Pemahaman

- Basis qubit R dan L didefinisikan sebagai:
  - $|R\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$
  - $|L\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$
- Apakah representasi superposisi qubit $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ dalam basis R dan L?

$$\frac{|R\rangle + |L\rangle}{2} = \frac{|0\rangle}{\sqrt{2}}$$

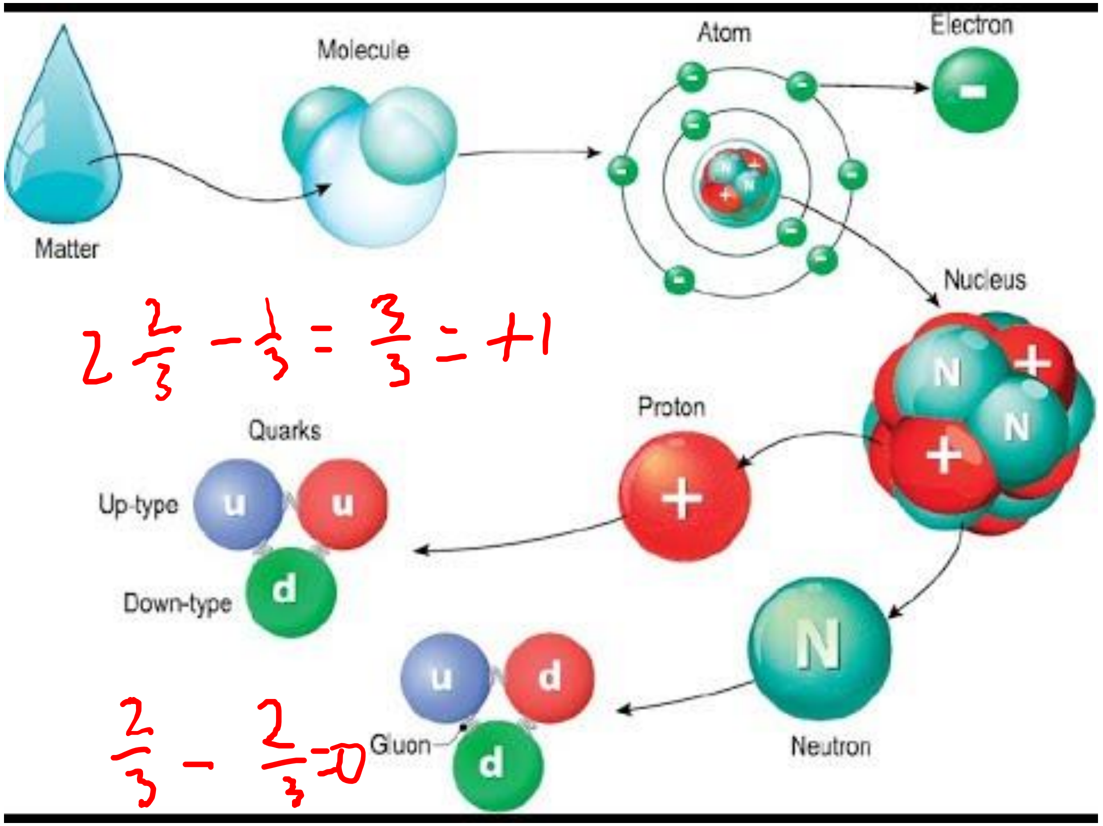
$$\frac{|R\rangle - |L\rangle}{2i} = \frac{|1\rangle}{\sqrt{2}}$$

$$= \frac{|R\rangle + |L\rangle}{2} + \frac{|R\rangle - |L\rangle}{2i}$$

$$= \frac{1}{2}\left[(1-i)|R\rangle + (1+i)|L\rangle\right]$$

Standard Model

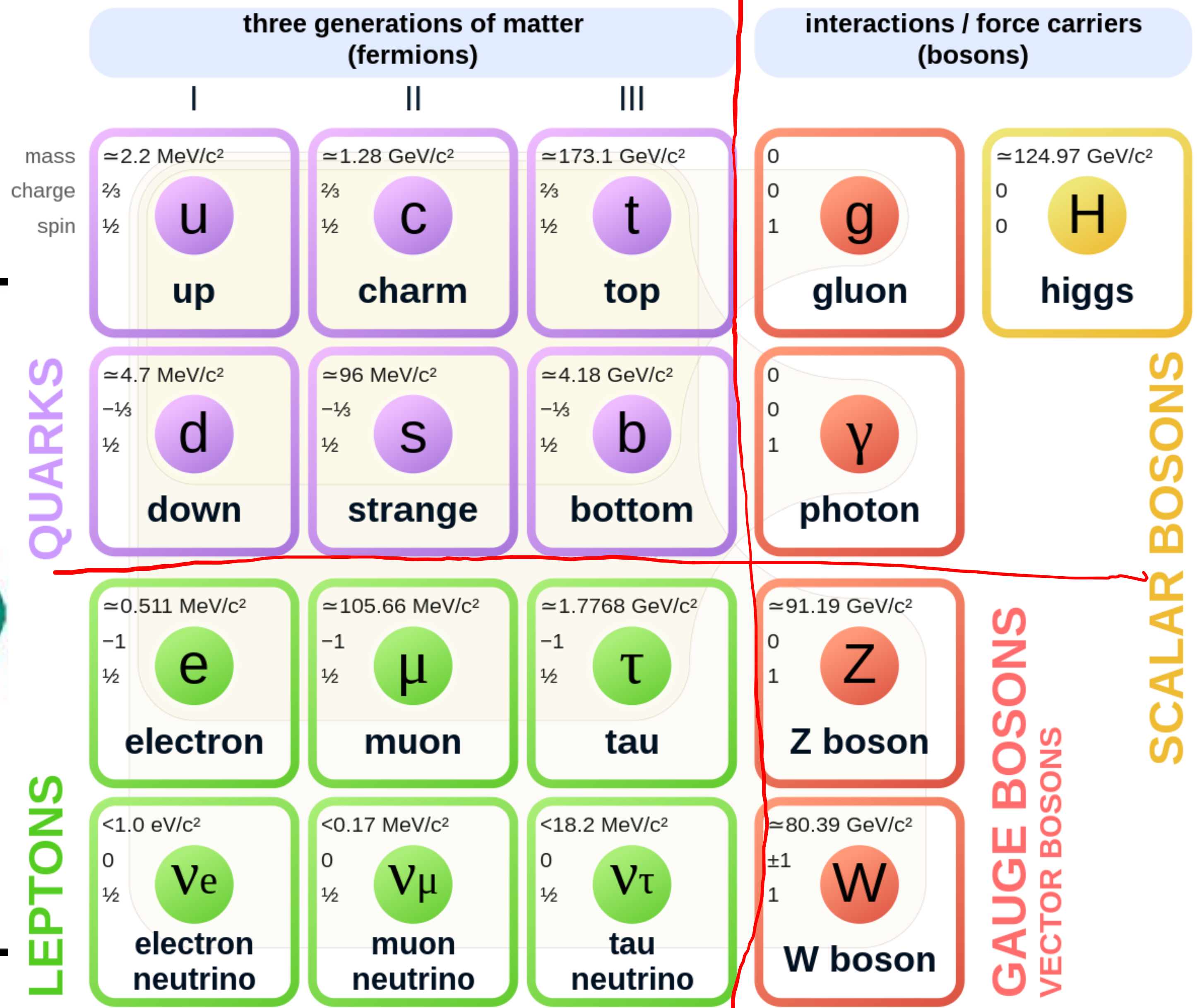
- Materi dibentuk oleh fermion
- Interaksi dibentuk oleh boson



$2\frac{2}{3} - \frac{1}{3} = \frac{3}{3} = +1$

$\frac{2}{3} - \frac{2}{3} = 0$

Standard Model of Elementary Particles



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Aktivitas

- Menghitung peluang pengukuran di qiskit
- https://learn.qiskit.org/course/introduction/what-is-quantum

WaveFunction

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WaveFunction

- Teori kuantum dimulai dengan penemuan:
  - Sifat kuanta (paket) dari radiasi: radiasi planck, efek fotoelektrik
  - Sifat superposisi dari partikel: pola interferensi
- Maka terdapat 2 pendekatan teori kuantum:
  - Konsep wave-particle duality: de Broglie
    - Menghasilkan pendekatan wavefunction sebagai fungsi gelombang: Schrodinger
  - Konsep diskrit: Bohr
    - Menghasilkan pendekatan wavefunction sebagai vektor kompleks: Heisenberg

WaveFunction sebagai Fungsi Gelombang

konjugat wavefunction wavefunction

- Interpretasi Probabilistic (Born):
  - Peluang menemukan partikel dengan volume $d\boldsymbol{r} \equiv dxdydz$ pada lokasi $\boldsymbol{r} \equiv (\mathrm{x}, \mathrm{y}, \mathrm{z})$ pada waktu t adalah:

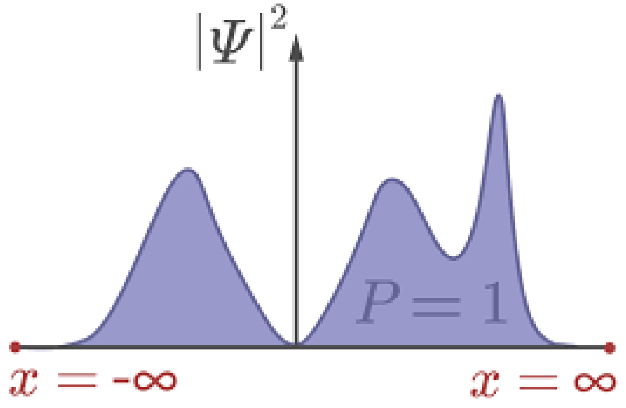
$$P(\boldsymbol{r}, t) = |\Psi(\boldsymbol{r}, t)|^2 d\boldsymbol{r} = \Psi^*(\boldsymbol{r}, t)\Psi(\boldsymbol{r}, t) d\boldsymbol{r}$$

  - Dimana total peluang adalah 1:

$$\int |\Psi(\boldsymbol{r}, t)|^2 d\boldsymbol{r} = 1$$

  - Total peluang tidak berubah (probability conservation):

$$\frac{\partial}{\partial t} \int P(\boldsymbol{r}, t) d\boldsymbol{r} = 0$$



$$\Psi = e^{i(kx-\omega t)} = \cos(kx-\omega t) + i\sin(kx-\omega t)$$

$$\Psi^* = e^{-i(kx-\omega t)}$$

WaveFunction sebagai Fungsi Gelombang

- Superposisi & Interferensi (sifat gelombang):
  - Berbeda dengan gelombang klasik (suara, cahaya), Ψ adalah fungsi kompleks

    Real

    Imaginary
  - Linear kombinasi dari beberapa wavefunction juga merupakan wavefunction dari system

$$\Psi = c_1\Psi_1 + c_2\Psi_2$$

- Momentum & Energi (sifat partikel):
  - Plane wave untuk gerakan free particle pada 1 dimensi:

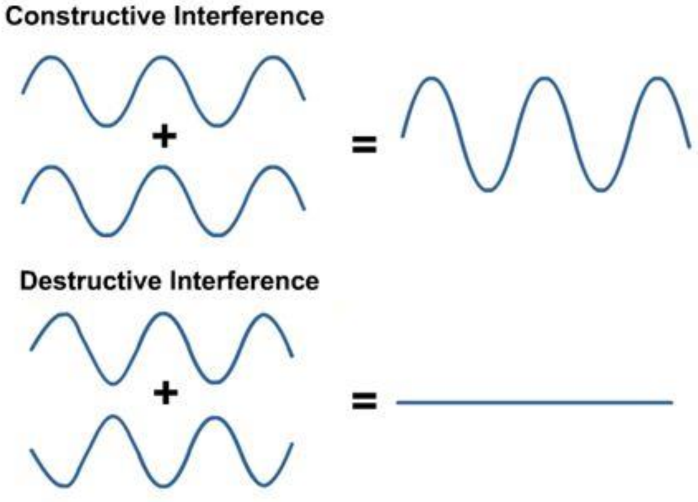
$$\Psi(x) = Ae^{\{i[kx-\omega t]\}}$$

  - Maka operator momentum & energi terhadap $\Psi(x)$:

eigen value

$$\boxed{-i\hbar\frac{\partial}{\partial x}}\Psi(x,t) = \hbar k\Psi(\mathrm{x},\mathrm{t}) = p_x\Psi(\mathrm{x},\mathrm{t})$$

$$\boxed{i\hbar\frac{\partial}{\partial t}}\Psi(x,t) = \hbar\omega\Psi(\mathrm{x},\mathrm{t}) = E\Psi(\mathrm{x},\mathrm{t})$$



Fourier Transform

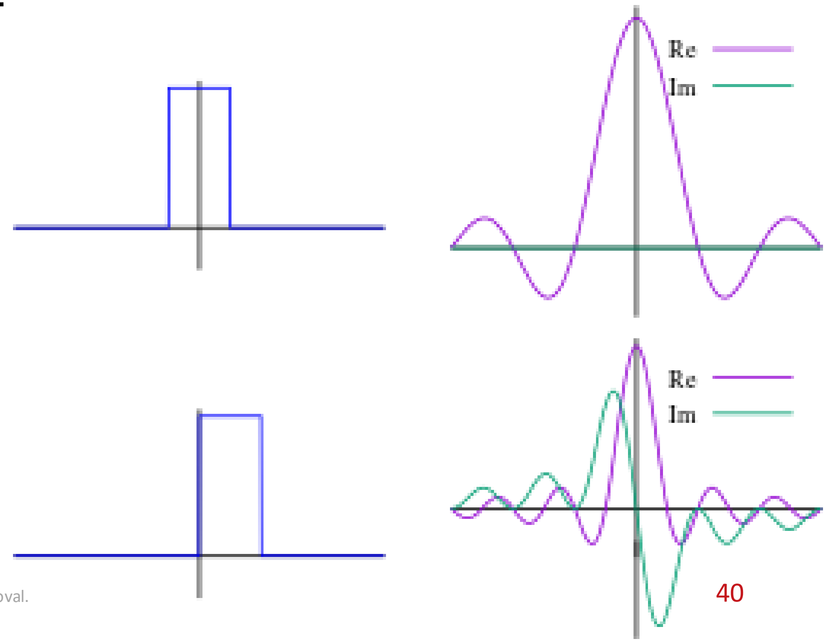
- Fourier transform dari wavefunction $\Psi(x,t)$ adalah:

$$\Phi(p_x,t) = \frac{\int_{-\infty}^{\infty} e^{-\frac{ip_x x}{\hbar}} \Psi(x,t)dx}{2\pi\hbar}$$

- Dimana $\Phi(p_x,t)$ merupakan wavefunction pada momentum space dan mutual fourier transform dari $\Psi(x,t)$

$$\Psi(x,t) = \frac{\int_{-\infty}^{\infty} e^{\frac{ip_x x}{\hbar}} \Phi(p_x,t)dp_x}{2\pi\hbar}$$

- Maka Posisi-Momentum adalah complementary variables
- Waktu-Energi juga merupakan complementary variables



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Persamaan Schrodinger

- Persamaan umum:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = H\Psi(\boldsymbol{r},t)$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,y,z,t) = \left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(x,y,z)\right]\Psi(x,y,z,t)$$

- 1D:

$$\underbrace{i\hbar\frac{\partial}{\partial t}}_{E}\Psi(x,t) = \underbrace{\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right)+V(x)\right]}_{H}\Psi(x,t)$$

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Classical Limit (Ehrenfest Theorem)

- Dengan menggunakan persamaan Schrodinger:
  - Perubahan ekspektasi posisi:

$$\frac{d}{dt}\langle x\rangle = \frac{d}{dt}\int \Psi^*(\boldsymbol{r},t)x\Psi(\boldsymbol{r},t)d\boldsymbol{r} = \int \Psi^*(\boldsymbol{r},t)x\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t} + \int \frac{\partial\Psi^*(\boldsymbol{r},t)}{\partial t}x\Psi(\boldsymbol{r},t)d\boldsymbol{r}$$

$$\frac{d}{dt}\langle x\rangle = \frac{i\hbar}{2m}\int [\Psi^* x(\nabla^2\Psi) - (\nabla^2\Psi^*)x\Psi]d\boldsymbol{r} = -\frac{i\hbar}{m}\int \Psi^*\frac{\partial\Psi}{\partial x}d\boldsymbol{r}$$

$$\frac{d}{dt}\langle x\rangle = \frac{\langle p_x\rangle}{m}$$

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} \longrightarrow mv = p$$

  - Perubahan ekspektasi momentum:

$$\frac{d}{dt}\langle p_x\rangle = \frac{d}{dt}\int \Psi^*(\boldsymbol{r},t)\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi(\boldsymbol{r},t)d\boldsymbol{r} = -i\hbar\left[\int \Psi^*\frac{\partial}{\partial x}\frac{\partial\Psi}{\partial t}d\boldsymbol{r} + \int \frac{\partial\Psi^*}{\partial t}\frac{\partial\Psi}{\partial x}d\boldsymbol{r}\right]$$

$$\frac{d}{dt}\langle p_x\rangle = \frac{\hbar^2}{2m}\int \left[\Psi^*\left(\nabla^2\frac{\partial\Psi}{\partial x}\right) - (\nabla^2\Psi^*)\frac{\partial\Psi}{\partial x}\right]d\boldsymbol{r} - \int \Psi^*\left[\frac{\partial}{\partial x}(V\Psi) - V\frac{\partial\Psi}{\partial x}\right]d\boldsymbol{r} = -\int \Psi^*\frac{\partial V}{\partial x}\Psi d\mathbf{r}$$

$$\frac{d}{dt}\langle p_x\rangle = -\left\langle\frac{\partial V}{\partial x}\right\rangle$$

- Menjadi hukum Newton:

$$\frac{d\boldsymbol{x}}{dt} = \frac{\boldsymbol{p}}{m}$$

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F} = -\boldsymbol{\nabla}\mathrm{V}$$

$$F = \frac{d(mv)}{dt} = ma$$

Energy Eigenvalue

$$\frac{\partial}{\partial t}\left[\psi(r)f(t)\right] = \psi(r)\,\frac{\partial f(t)}{\partial t}$$

- Jika E merupakan eigenvalue dari persamaan schrodinger:

$$\boxed{i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t)} = H\Psi(\boldsymbol{r},t) = E\Psi(\boldsymbol{r},t)$$

- Jika potensial V tidak bergantung pada waktu, maka komponen posisi dan waktu dari $\Psi(\mathrm{x},\mathrm{t})$ separable:

$$\Psi(\boldsymbol{r},t) = \psi(\boldsymbol{r})f(t)$$

- Maka terdapat 2 persamaan eigen:

$$\cancel{\psi(\boldsymbol{r})}i\hbar\frac{\partial}{\partial t}f(t) = \cancel{\psi(\boldsymbol{r})}Ef(t)$$

$$\cancel{f(t)}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})\right]\psi(\boldsymbol{r}) = \cancel{f(t)}E\psi(\boldsymbol{r})$$

sebanarnya eigen value

eigenfunction

- Dimana:
  - $f(t)$ adalah eigenfunction dari operator energi $i\hbar\frac{\partial}{\partial t}$
  - $\psi(\boldsymbol{r})$ adalah eigenfunction dari operator Hamiltonian $\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})\right]$
  - Dan keduanya memiliki eigenvalue E
- Probability density:

$$P(\boldsymbol{r},t) = \Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = \psi^*(\boldsymbol{r})\psi(\boldsymbol{r})e^{-\frac{i}{\hbar}(E-E^*)t}$$

- Probability conservation:

$$\frac{\partial}{\partial t}\int P(\boldsymbol{r},t)d\boldsymbol{r} = 0$$

$$(E - E^*)\int \Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t)d\boldsymbol{r} = 0$$

- Maka $(E - E^*)$ harus 0 → eigenvalue E adalah bilangan real

$E = a + bi$

$E^* = a - bi$

$E - E^* = 0$

$a + bi - a + bi = 0$

$2bi = 0$

$b = 0$

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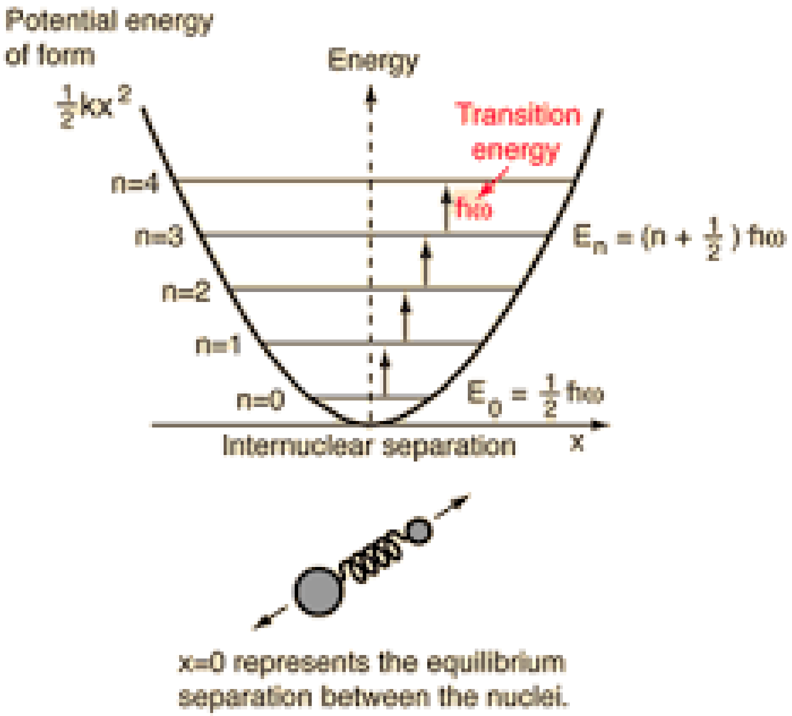
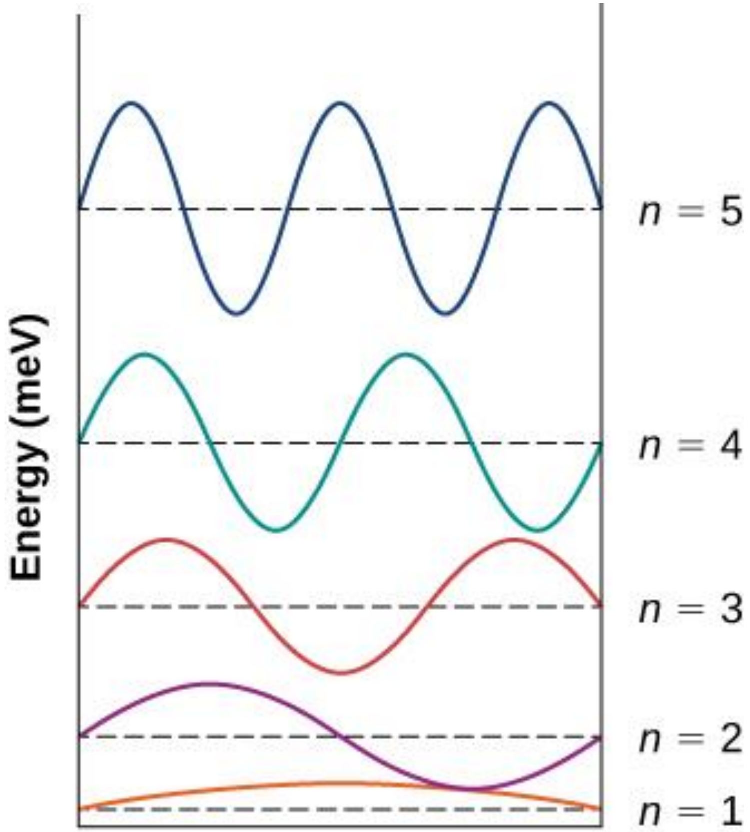
Kuantisasi Energi

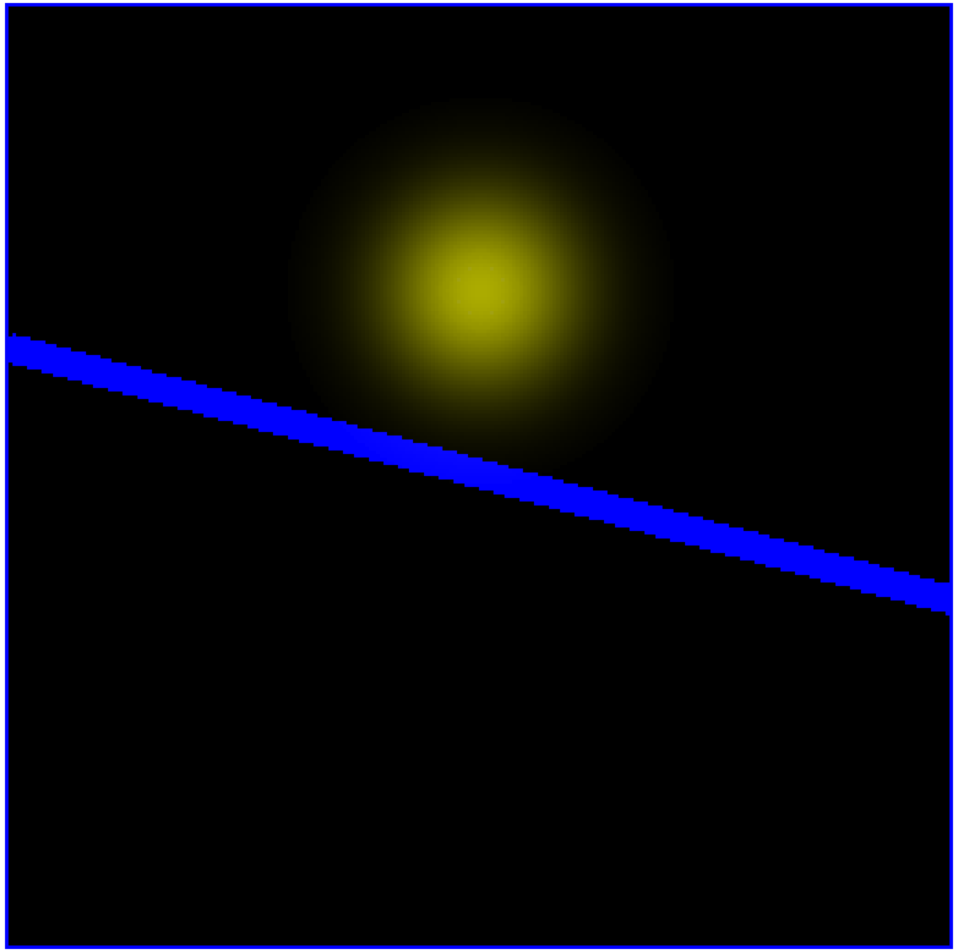
- Beberapa jenis potensial menghasilkan kuantisasi energi:
  - Infinite Potential ($V = \begin{cases} 0, 0 \leq x \leq L \\ \infty, x < 0, x > L \end{cases}$)
  - Harmonic Oscillator ($V = \frac{1}{2} k x^2$)





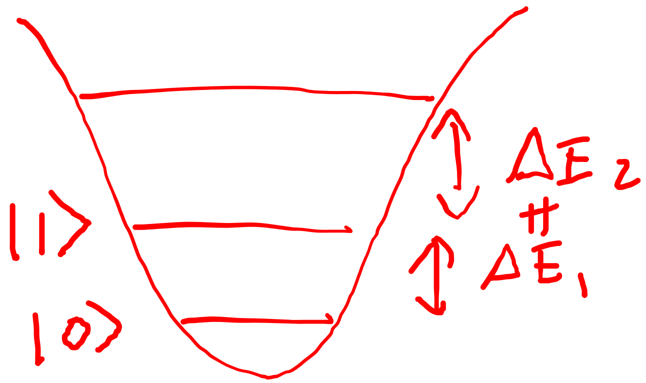
Quantum Tunneling

- Ketika wave packet menabrak dinding, sebagian terpantulkan dan sebagian menembus dinding (tunneling)



Uji Pemahaman

- Superconducting Qubit (IBM) menggunakan mekanisme anharmonic oscillator untuk membuat qubit. Mengapa bukan harmonic oscillator yang dipilih?



Komutator

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Komutator

- Persamaan Schrodinger:

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi \rightarrow \frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}H\Psi$$

- Ekspektasi operator A:

$$\langle A\rangle = \int \Psi^*(\boldsymbol{r},t)A\Psi(\boldsymbol{r},t)d\boldsymbol{r}$$

- Perubahan terhadap A:

$$\frac{d}{dt}\langle A\rangle = \frac{d}{dt}(\Psi^* A\Psi) = \frac{d\Psi^*}{dt}A\Psi + \Psi^* A\frac{d\Psi}{dt} = \frac{i}{\hbar}\Psi^*(HA - AH)\Psi = \frac{i}{\hbar}\Psi^*[H,A]\Psi$$

- Dimana komutator didefinisikan sebagai:

$$[A,B] = AB - BA$$

- 2 operator disebut komutatif jika $\langle AB - BA\rangle = 0$
  - Misal: energi-momentum

$$\Psi^*\left(i\hbar\frac{\partial}{\partial t}\left(-\frac{\hbar^2}{2m}\nabla^2\right)\Psi\right) - \Psi^*\left(-\frac{\hbar^2}{2m}\nabla^2\left(i\hbar\frac{\partial}{\partial t}\right)\Psi\right) = (Ep - pE)\Psi^*\Psi = 0$$

- 2 operator disebut anti-komutatif jika $\langle AB - BA\rangle \neq 0$
  - Misal: posisi-momentum

$$\Psi^*\left(\left(-\frac{\hbar^2}{2m}\nabla^2\right)(x\Psi)\right) - \Psi^* x\left(-\frac{\hbar^2}{2m}\nabla^2\Psi\right) \neq 0$$

Sifat Komutator

$AA - AA = 0$

$AB - BA = -(BA - AB)$

$= -[B, A]$

$$[A, A] = 0$$
$$[A, B] = -[B, A]$$
$$[A, c] = 0 \ (c \ is \ number)$$
$$[A + B, C] = [A, C] + [B, C]$$
$$[A, BC] = [A, B]C + B[A, C]$$
$$\big[A, [B, C]\big] + \big[B, [C, A]\big] + [C, [A, B]] = 0$$

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Uji Pemahaman

$$[A, A] = 0$$
$$[A, B] = -[B, A]$$
$$[A, c] = 0\ (c\ is\ number)$$
$$[A + B, C] = [A, C] + [B, C]$$
$$[A, BC] = [A, B]C + B[A, C]$$
$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$$

- Buktikan sifat komutator sebelumnya

$$[A, A] = AA - AA = 0$$

$$[A, B] = AB - BA = -(BA - AB) = -[B, A]$$

$$[A, c] = Ac - cA = c(A - A) = 0$$

$$[A+B, C] = (A+B)C - C(A+B) = AC + BC - CA - CB = AC - CA + BC - CB = [A, C] + [B, C]$$

$$[A, BC] = ABC - BCA = ABC - BAC + BAC - BCA = [A, B]C + B[A, C]$$

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = A(BC - CB) - (BC - CB)A + B(CA - AC) - (CA - AC)B + C(AB - BA) - (AB - BA)C$$

$$= \cancel{ABC} - \cancel{ACB} - \cancel{BCA} + \cancel{CBA} + \cancel{BCA} - \cancel{BAC} - \cancel{CAB} + \cancel{ACB} + \cancel{CAB} - \cancel{CBA} - \cancel{ABC} + \cancel{BAC} = 0$$

Poisson Bracket

$\{A, B\}$

energi kinetik

energi potensial

- Classical Hamiltonian:

$$H = \frac{p^2}{2m} + V(x)$$

- Poisson Bracket pada basis posisi dan momentum adalah:

$$\{A, B\} = \frac{\partial A}{\partial x}\frac{\partial B}{\partial p} - \frac{\partial B}{\partial x}\frac{\partial A}{\partial p}$$

- Poisson Bracket terhadap Hamiltonian adalah:

$$\frac{dQ}{dt} = \{Q, H\} = \frac{\partial Q}{\partial x}\frac{\partial H}{\partial p} - \frac{\partial H}{\partial x}\frac{\partial Q}{\partial p}$$

$$\{p, H\} = \frac{\partial p}{\partial x}\frac{\partial H}{\partial p} - \frac{\partial H}{\partial x}\frac{\partial p}{\partial p} = -\frac{\partial V(x)}{\partial x} = \frac{dp}{dt}$$

gaya

$$\{x, H\} = \frac{\partial x}{\partial x}\frac{\partial H}{\partial p} - \frac{\partial H}{\partial x}\frac{\partial x}{\partial p} = \frac{p}{m} = \frac{dx}{dt}$$

kecepatan

- Untuk conserved quantity Q:

$$\frac{dQ}{dt} = \{Q, H\} = 0$$

$$\{H, H\} = 0 \rightarrow energy\ conservation$$

$$\{p, H\} = 0 \rightarrow momentum\ conservation$$

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Dari klasik ke kuantum

$\{\}$ → $[]$

- Komutator [] adalah versi kuantum dari Poisson Bracket {}:

$$\{A, B\} \to \frac{1}{i\hbar}[A, B]$$

$$\{x, p\} = 1 \to [x, p] = i\hbar$$

- Persamaan Heisenberg:

$$\frac{dQ}{dt} = \{Q, H\} \to \frac{dQ}{dt} = \frac{1}{i\hbar}[Q, H] = \frac{i}{\hbar}[H, Q]$$

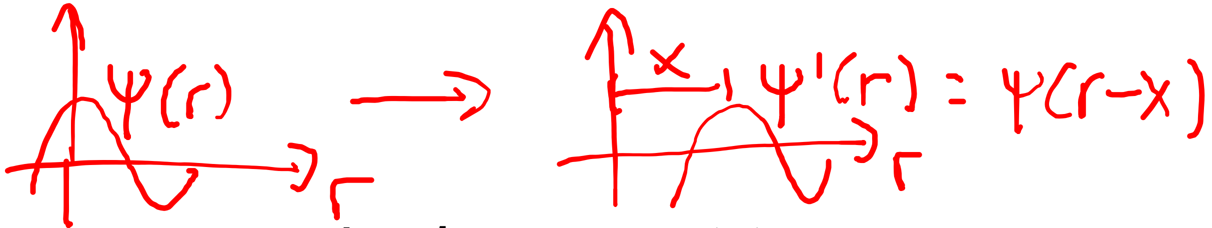
- Energy Conservation:

$$\{H, H\} = 0 \to [\mathrm{H}, \mathrm{H}] = 0$$ → $HH - HH = 0$

- Momentum Conservation (jika $V(x) = 0$):

$$\{p, H\} = 0 \to [p, H] = 0$$ → $pH - Hp = 0$

Spatial Translations



- Operator U_T mengubah wavefunction ψ menjadi ψ': $\psi'(\boldsymbol{r}) = U_T(\boldsymbol{x})\psi(\boldsymbol{r})$
- Dimana ψ' adalah wavefunction sistem yang digeser sejauh $\boldsymbol{x} \rightarrow \psi'(\boldsymbol{r}) = \psi(\boldsymbol{r} - \boldsymbol{x})$
- Misalkan kita geser sebesar infinitesimal $\delta\boldsymbol{x}$

$$\psi'(\boldsymbol{r}) = \psi(\boldsymbol{r} - \boldsymbol{x}) = \psi(\boldsymbol{r}) - \delta\boldsymbol{x} \cdot \nabla\psi(\boldsymbol{r}) = (I - \delta\boldsymbol{x} \cdot \nabla)\psi(\boldsymbol{r}) = U_T(\delta\boldsymbol{x})\psi(\boldsymbol{r})$$

- $U_T(\delta\boldsymbol{x}) = I - \delta\boldsymbol{x} \cdot \nabla = I - \frac{i}{\hbar}\delta\boldsymbol{x} \cdot \boldsymbol{p}$, dimana $\boldsymbol{p} = -i\hbar\nabla$ adalah momentum operator
- Misalkan kita menggeser n kali dimana $n \rightarrow \infty$

$$U_T(\boldsymbol{x}) = \lim_{n\rightarrow\infty}\left(I - \frac{i}{\hbar}\frac{(\boldsymbol{x} \cdot \boldsymbol{p})}{n}\right)^n = \boxed{e^{-\frac{i}{\hbar}\boldsymbol{x}\cdot\boldsymbol{p}}}$$

- Jika Hamiltonian **invariant terhadap translasi** apapun, maka:

$$H' = U_T(\boldsymbol{x})HU_T^\dagger(\boldsymbol{x}) = \left(I - \frac{i}{\hbar}\delta\boldsymbol{x} \cdot \boldsymbol{p}\right)H\left(I + \frac{i}{\hbar}\delta\boldsymbol{x} \cdot \boldsymbol{p}\right) = H - \frac{i}{\hbar}\delta\boldsymbol{x} \cdot \overset{0}{[\boldsymbol{p}, H]} = H = H'$$

- Maka $[\boldsymbol{p}, H] = 0 \rightarrow$ **momentum conservation**

Simetri

$\Psi' = U_s \Psi$

- Untuk operator U_S apa saja yang dapat mengubah wavefunction Ψ menjadi wavefunction Ψ' secara simetri: $\Psi = U_S \Psi$
- Dimana $U_S(\delta\boldsymbol{\theta}) = I + i\epsilon F_S$ adalah transformasi simetri terhadap kordinat θ
- Selama H **invariant terhadap operasi simetri**, maka:

$$H' = U_S(\boldsymbol{\theta}) H U_T^{\dagger}(\boldsymbol{\theta}) = (I + i\epsilon F_S) H (I - i\epsilon F_S) = H + i\epsilon[F_S, H] = H$$

- Maka $[F_S, H] = 0 \rightarrow$ **Operator F_S akan conserved**

F_S Conserved

Symmetry → Conservation

Transformations	Unobservables	Conservation laws and selection rules
Translation in space $\boldsymbol{r} \rightarrow \boldsymbol{r} + \Delta$	Absolute position in space	Momentum
Translation in time $t \rightarrow t + \tau$	Absolute time	Energy
Rotation $\boldsymbol{r} \rightarrow \boldsymbol{r}'$	Absolute direction in space	Angular momentum
Space inversion $\boldsymbol{r} \rightarrow -\boldsymbol{r}$	Absolute left or right	Parity
Time reversal $t \rightarrow -t$	Absolute sign of time	Kramers degeneracy
Sign reversion of charge $e \rightarrow -e$	Absolute sign of electric charge	Charge conjugation
Particle substitution	Distinguishability of identical particles	Bose or Fermi statistics
Gauge transformation $\psi \rightarrow e^{iN\theta}\psi$	Relative phase between different normal states	Particle number

Noether

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Dirac Notation

$\rightarrow \psi(r)$

- Notasi dirac dapat digunakan untuk memudahkan hidup kita:

$$\langle \Psi_1 | \Psi_2 \rangle = \int \Psi_1^* \Psi_2 d\boldsymbol{r}$$

- Untuk kasus diskrit, integral bisa menjadi summation:

$$\int \Psi^* \Psi d\boldsymbol{r} = 1 \rightarrow \sum \Psi^* \Psi = 1$$

- $\sum \quad \Psi^* \Psi$ adalah definisi dari inner product dari 2 vektor kompleks
  - Jika Ψ adalah proyeksi sebuah vektor kompleks ($|\Psi\rangle$) terhadap posisi ($\boldsymbol{r} = x, y, z$), $\Psi = \langle \boldsymbol{r} | \psi \rangle$
  - Maka $\sum \quad \Psi^* \Psi = \sum \; \langle \Psi | \boldsymbol{r} \rangle \langle \boldsymbol{r} | \psi \rangle \; = \langle \Psi | \Psi \rangle$
- Detail dari notasi ini akan kita pelajari di pertemuan berikutnya

Aktivitas

- Mengamati perilaku operator gate di qiskit
- https://quantum-computing.ibm.com/composer/docs/iqx/guide/creating-superpositions

Tuhan Memberkati