Notasi Bracket

Hendrik Santoso Sugiarto

IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

Capaian Pembelajaran

- Bracket
- Operator
- Bentuk Matriks
- Formalisme

Bracket

God's People for God's Glory
CALVIN
INSTITUTE OF TECHNOLOGY

Notasi Dirac

- Notasi dirac disebut juga dengan bracket notation, karena terdiri dari 2 macem vektor kompleks yaitu:
  - ket $|\beta\rangle$ yang menghuni state space
  - bra $\langle\alpha|$ yang menghuni dual space

$(\alpha_1 \alpha_2 \ldots \alpha_n)$ $\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$

- Inner product dari bra dan ket adalah bra(c)ket:

$$\langle\alpha|\beta\rangle = \langle\alpha| \cdot |\beta\rangle$$

- Proyeksi ket $|\beta\rangle$ pada basis $|\alpha\rangle$ menghasilkan sebuah inner product $\langle\alpha|\beta\rangle$ yang merupakan sebuah yang wavefunction yang terletak pada koordinat $\alpha \rightarrow \Psi(\alpha)$

Space

Karakteristik	Dirac Space	Euclidean Space
Penghuni	Objek kuantum	Objek klasik
Dimensi	Bisa sampai ∞	3
Aljabar	Kompleks	Real
Elemen	Ket $\lvert\psi\rangle$	Vektor
Penjumlahan	$\lvert\psi_1\rangle + \lvert\psi_2\rangle = \lvert\psi_3\rangle \in \mathbb{C}$	$\vec{r}_1 + \vec{r}_2 = \vec{r}_3 \in \mathbb{R}^3$
Perkalian skalar	$a\lvert\psi\rangle \in \mathbb{C}$	$a\vec{r} \in \mathbb{R}^3$
Perkalian matriks	$A\lvert\psi\rangle \in \mathbb{C}$	$A\vec{r} \in \mathbb{R}^3$
Inner product	$\langle\phi\lvert\psi\rangle \in \mathbb{C}$	$\vec{r}_1 \cdot \vec{r}_2 =\in \mathbb{R}^3$
Outer product	$\lvert\psi\rangle\langle\phi\rvert \in \mathbb{C}$	$\vec{r}_1 \otimes \vec{r}_2 =\in \mathbb{R}^3$

Ket Space

- Physical state direpresentasikan oleh sebuah complex vector yang disebut dengan ket
- State ket berisi informasi lengkap tentang seluruh physical state
- 2 ket dapat dijumlahnya dan hasilnya adalah ket yang lain

$$|\alpha\rangle + |\beta\rangle = |\gamma\rangle$$

- Sebuah ket dikalikan dengan sebuah bilangan kompleks akan menghasilkan ket lain:

$$|\alpha'\rangle = c|\alpha\rangle$$

- Jika $c = 0$ maka ket ini adalah null ket

CALVIN INSTITUTE OF TECHNOLOGY

Addition

- Komutatif

$$|\alpha\rangle + |\beta\rangle = |\beta\rangle + |\alpha\rangle$$

- Asosiatif

$$(|\alpha\rangle + |\beta\rangle) + |\gamma\rangle = |\alpha\rangle + (|\beta\rangle + |\gamma\rangle)$$

- Inverse

$$|\alpha\rangle + (-|\alpha\rangle) = 0$$

- Null ket

$$0 + |\alpha\rangle = |\alpha\rangle$$

Scalar Multiplication

- Asosiatif

$$a(b|\alpha\rangle) = (ab)|\alpha\rangle$$

- Distributif

$$(a+b)|\alpha\rangle = a|\alpha\rangle + b|\alpha\rangle$$
$$a(|\alpha\rangle + |\beta\rangle) = a|\alpha\rangle + a|\beta\rangle$$

- Identitas

$$1|\alpha\rangle = |\alpha\rangle$$

Uji Pemahaman

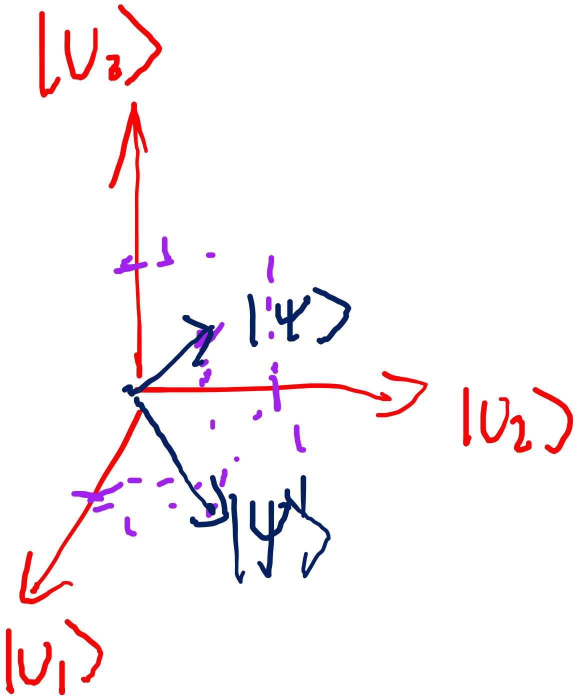
- Berapakah $(a + b)(|0\rangle + |1\rangle)$

$a|0\rangle + b|0\rangle + a|1\rangle + b|1\rangle$

Kombinasi Linear (Superposisi)

- Ket apapun dapat ditulis sebagai kombinasi linear N-dimensi ruang vektor ortonormal

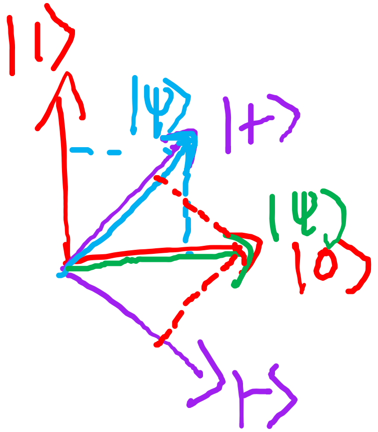
$$|\psi\rangle = \sum_{i}^{n} c_i|u_i\rangle = c_1|u_i\rangle + c_2|u_2\rangle + \cdots + c_n|u_n\rangle$$



Uji Pemahaman

$\frac{|+i\rangle + |-i\rangle}{2} = \frac{|0\rangle}{\sqrt{2}}$ $\qquad$ $\frac{|+i\rangle - |-i\rangle}{2i} = \frac{|1\rangle}{\sqrt{2}}$

- Apakah bentuk dari $\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$
  - pada basis $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ dan $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ $\quad |\psi\rangle = |+\rangle$
  - pada basis $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ dan $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$ $\quad |\psi\rangle = \left(\frac{1}{2} + \frac{1}{2i}\right)|+i\rangle + \left(\frac{1}{2} - \frac{1}{2i}\right)|-i\rangle$
- Apakah bentuk dari $\boxed{|\psi\rangle = |0\rangle}$
  - pada basis $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ dan $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ $\quad |\psi\rangle = \frac{|+\rangle + |-\rangle}{2}$
  - pada basis $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ dan $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$ $\quad |\psi\rangle = \frac{|+i\rangle + |-i\rangle}{2}$



Bra Space

- Untuk setiap ket, terdapat sebuah bra yang terletak pada dual space
- Bra adalah semacam bayangan cermin dari ket
- Sehingga terdapat one-to-one dual correspondence antara bra dan ket:

$$|\alpha\rangle \leftrightarrow \langle\alpha|$$
$$|\alpha'\rangle, |\alpha''\rangle, \ldots \leftrightarrow \langle\alpha'|, \langle\alpha''|, \ldots$$
$$|\alpha\rangle + |\beta\rangle \leftrightarrow \langle\alpha| + \langle\beta|$$
$$c_\alpha|\alpha\rangle + c_\beta|\beta\rangle \leftrightarrow c_\alpha^* \langle\alpha| + c_\beta^*\langle\beta|$$

CALVIN INSTITUTE OF TECHNOLOGY

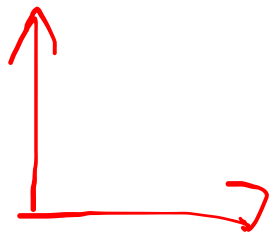
Inner Product

- Inner product dari bra dan ket adalah sebuah scalar kompleks:

$$\langle\alpha|\beta\rangle = \langle\alpha|\cdot|\beta\rangle$$
$$\langle\alpha|\beta+\gamma\rangle = \langle\alpha|\cdot(|\beta\rangle+|\gamma\rangle)$$

- Terdapat 2 sifat dari inner product:

$$\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^*$$
$$\langle\alpha|\alpha\rangle \geq 0$$



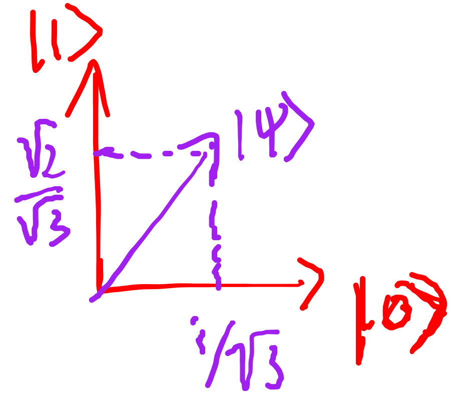
- 2 ket $|\alpha\rangle$ dan $|\beta\rangle$ disebut orthogonal jika $\langle\alpha|\beta\rangle$ =0
- Pada normalized ket dimana $|\tilde{\alpha}\rangle = \frac{|\alpha\rangle}{\sqrt{\langle\alpha|\alpha\rangle}}$, maka $\boxed{\langle\alpha|\alpha\rangle = 1}$

$$|\alpha\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\langle\alpha|\alpha\rangle = \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\,\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}\left(\cancel{\langle 0|0\rangle}^{1} + \cancel{\langle 0|1\rangle}^{0} + \cancel{\langle 1|0\rangle}^{0} + \cancel{\langle 1|1\rangle}^{1}\right) = 1$$

Uji Pemahaman



- Berapakah $\overset{=1}{\langle\psi|\psi\rangle}$, dimana $|\psi\rangle$ ternormalisasi
- Berapakah $\langle 0|\psi\rangle\langle\psi|0\rangle$, dimana $|\psi\rangle = \frac{1}{\sqrt{3}}\left(i|0\rangle + \sqrt{2}|1\rangle\right)$
- https://learn.qiskit.org/course/introduction/describing-quantum-computers

$$\langle 0| \frac{1}{\sqrt3}\left(i|0\rangle + \sqrt2|1\rangle\right) \frac{1}{\sqrt3}\left(-i\langle 0| + \sqrt2\langle 1|\right)|0\rangle$$

$$\left(\frac{i}{\sqrt3}\overset{1}{\cancel{\langle 0|0\rangle}} + \frac{\sqrt2}{\sqrt3}\overset{0}{\cancel{\langle 0|1\rangle}}\right)\left(\frac{-i}{\sqrt3}\overset{1}{\cancel{\langle 0|0\rangle}} + \frac{\sqrt2}{\sqrt3}\overset{0}{\cancel{\langle 1|0\rangle}}\right)$$

$$\frac{i}{\sqrt3}\cdot\frac{-i}{\sqrt3} = \frac{1}{3}$$

Outer Product

- Outer product dari ket dan bra adalah sebuah matriks:

$$|\beta\rangle\langle\alpha|$$

- Associative axiom:

matrix vect vect scalar

$$(|\beta\rangle\langle\alpha|)|\gamma\rangle = |\beta\rangle(\langle\alpha|\gamma\rangle)$$
$$(\langle\beta|)(X|\alpha\rangle) = (\langle\beta|X)(|\alpha\rangle)$$

- Jika $X = |\beta\rangle\langle\alpha|$ maka $X^\dagger = |\alpha\rangle\langle\beta|$

Uji Pemahaman

$\left(\frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}\cancel{|0\rangle\langle 1|} + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}\cancel{|1\rangle\langle 1|}\right)|0\rangle$

- Operator Hadamard adalah:

gate

$$H = \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$$

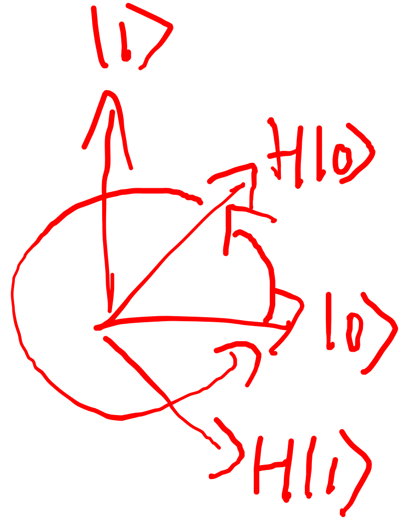
- Apakah hasil operator berikut terhadap:
  - $|0\rangle$
  - $|1\rangle$
  - $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
  - $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Manakah yang mengalami interferensi?

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$H|+\rangle = HH|0\rangle = |0\rangle$

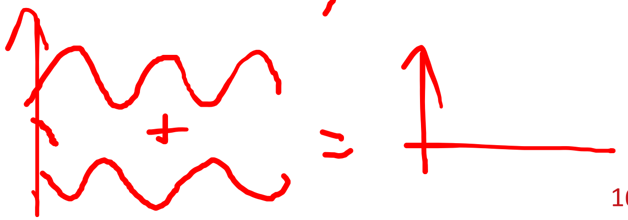
$HH \rightarrow I$



$H|+\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|\right)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$= \frac{1}{2}\left(|0\rangle\langle 0|0\rangle + |0\rangle\cancel{\langle 1|0\rangle} + |1\rangle\langle 0|0\rangle - |1\rangle\cancel{\langle 1|0\rangle} + |0\rangle\cancel{\langle 0|1\rangle} + |0\rangle\langle 1|1\rangle + |1\rangle\cancel{\langle 0|1\rangle} - |1\rangle\langle 1|1\rangle\right)$

$= \frac{1}{2}\left(|0\rangle + \cancel{|1\rangle} + |0\rangle - \cancel{|1\rangle}\right) = |0\rangle$



Operator

Operator

- Setiap observables (fenomena yang diukur), seperti momentum dan spin, dapat direpresentasikan oleh sebuah operator terhadap vector space ($A|\alpha\rangle$)
- Operator selalu beroperasi pada sebuah ket dari sisi kiri, dan menghasilkan ket baru:

$$|\psi\rangle = X|\alpha\rangle$$

- Operator selalu beroperasi pada sebuah bra dari sisi kanan, dan menghasilkan bra baru:

$$\langle\phi| = \langle\alpha|X$$

- Dimana dual correspondence antara dua operator ini adalah:

$$\langle\alpha|X \leftrightarrow X^{\dagger}|\alpha\rangle$$

$$\langle\alpha|X|\beta\rangle \leftrightarrow \langle\beta|X^{\dagger}|\alpha\rangle^{*}$$

CALVIN INSTITUTE OF TECHNOLOGY

Operator

- Sebuah operator disebut Hermitian jika:

$$X = X^{\dagger}$$

- Operators X dan Y adalah sama jika:

$$\mathrm{X}|\alpha\rangle = Y|\alpha\rangle$$

- Sebuah operator disebut null operator jika:

$$\mathrm{X}|\alpha\rangle = 0$$

- Operator pada kuantum selalu bersifat linear:

$$X(c_{\alpha}|\alpha\rangle + c_{\beta}|\beta\rangle) = c_{\alpha}X|\alpha\rangle + c_{\beta}X|\beta\rangle$$

Uji Pemahaman

complex $a+bi$

- Apakah nilai ekspektasi dari operator $M_0 = |0\rangle\langle 0|$ pada state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$?
- Apakah nilai ekspektasi dari operator $M_1 = |1\rangle\langle 1|$ pada state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$?

$$||\langle\psi|M_0|\psi\rangle||$$

$$\langle\psi|M_0|\psi\rangle = (\alpha^*\langle 0| + \beta^*\langle 1|)\,|0\rangle\langle 0|\,(\alpha|0\rangle + \beta|1\rangle)$$

$$= (\alpha^*\overset{1}{\langle 0|0\rangle}\langle 0| + \beta^*\overset{0}{\cancel{\langle 1|0\rangle}}\langle 0|)(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha^*\langle 0|0\rangle\alpha + \alpha^*\overset{0}{\cancel{\langle 0|1\rangle}}\beta = ||\alpha||^2$$

$$\alpha = a + bi$$

$$\alpha^* = a - bi$$

$$\alpha\alpha^* = a^2 - \cancel{abi} + \cancel{abi} + b^2$$

$$\langle M_1\rangle = \langle\psi|M_1|\psi\rangle = ||\beta||^2$$

Addition

- Beberapa operators dapat dijumlahkan
- Penjumlahan bersifat komutatif:
$$X + Y = Y + X$$
- Penjumlahan bersifat asosiatif:
$$X + (Y + Z) = (X + Y) + Z$$

Multiplication

- Beberapa operators dapat dikalikan
- Perkalian bersifat non-komutatif:

$$XY \neq YX$$

- Perkalian bersifat asosiatif:

$$X(YZ) = (XY)Z = XYZ$$

- Dual correspondence pada perkalian:

$$XY|\alpha\rangle = X(Y|\alpha\rangle) \leftrightarrow \left(\langle\alpha|Y^{\dagger}\right)X^{\dagger} = \langle\alpha|Y^{\dagger}X^{\dagger}$$
$$(XY)^{\dagger} = Y^{\dagger}X^{\dagger}$$

Eigen

- Terdapat bentuk ket khusus (eigenkets) dimana hasil dari sebuah operator adalah suatu nilai (eigenvalues):

$$\boxed{A|\alpha'\rangle = a'|a'\rangle}, A|\alpha''\rangle = a''|a''\rangle, \ldots$$

- Jika A adalah Hermitian, maka:

$$\langle a''|A = a''^{*}\langle a''|$$

- Hitung operator dari kiri dan kanan:

$$\langle a'|A|\alpha'\rangle = a'\langle a'||a'\rangle$$
$$\langle a'|A|\alpha'\rangle = a'^{*}\langle a'||a'\rangle$$
$$a = a^{*} \rightarrow real$$

$a = x + iy$

$a^{*} = x - iy$

- Hitung selisih inner product antar eigen:

$$0 = \langle a''|A|a'\rangle - \langle a''|A|a'\rangle = \left(a' - a''^{*}\right)\langle\alpha''|\alpha'\rangle$$
$$\boxed{\langle\alpha''|\alpha'\rangle = 0} \rightarrow \text{orthogonal}$$

Uji Pemahaman

$$\boxed{H|\psi\rangle = \lambda|\psi\rangle}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Berapakah eigenvalue dari operator Hadamard:
- $H = \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$

$$H|\psi\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|\right)(\alpha|0\rangle + \beta|1\rangle)$$

$$= \frac{1}{\sqrt{2}}\Big(\alpha|0\rangle\langle 0|0\rangle + \alpha|0\rangle\cancel{\langle 1|0\rangle}^{0} + \alpha|1\rangle\langle 0|0\rangle - \alpha|1\rangle\cancel{\langle 1|0\rangle}^{0} +$$

$$\beta|0\rangle\cancel{\langle 0|1\rangle}^{0} + \beta|0\rangle\langle 1|1\rangle + \beta|1\rangle\cancel{\langle 0|1\rangle}^{0} - \beta|1\rangle\langle 1|1\rangle\Big)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|0\rangle + \alpha|1\rangle + \beta|0\rangle - \beta|1\rangle\right) = \frac{1}{\sqrt{2}}\left((\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle\right) = \lambda\alpha|0\rangle + \lambda\beta|1\rangle$$

$$\frac{\alpha+\beta}{\alpha\sqrt{2}} = \lambda \qquad \frac{\alpha-\beta}{\beta\sqrt{2}} = \lambda \qquad \frac{\alpha+\beta}{\alpha\cancel{\sqrt{2}}} = \frac{\alpha-\beta}{\beta\cancel{\sqrt{2}}}$$

$$\lambda = 0$$

$$\lambda = -\frac{2}{\sqrt{2}}$$

$$\beta\alpha + \beta^2 = \alpha^2 - \beta\alpha$$

$$0 = \alpha^2 - 2\beta\alpha - \beta^2 = (\alpha-\beta)^2 \rightarrow \alpha = \pm\beta$$

Closure

- Setiap ket apapun $|\alpha\rangle$ dapat juga ditulis sebagai vektor pada basis N eigenkets dari observables A:

$$|\alpha\rangle = \sum_{a'} c_{a'}|a'\rangle = \sum_{a'} |a'\rangle\langle a'|\alpha\rangle$$

$c_{a'}$

- Dimana $c_{a'} = \langle a'|\alpha\rangle$
- Karena $|\alpha\rangle$ adalah ket apapun, maka kita memiliki completeness relation atau closure :

$$\sum_{a'} |a'\rangle\langle a'| = 1$$

identity operator

Projection

norm / probability

- Jika $|\alpha\rangle$ normalized, maka:

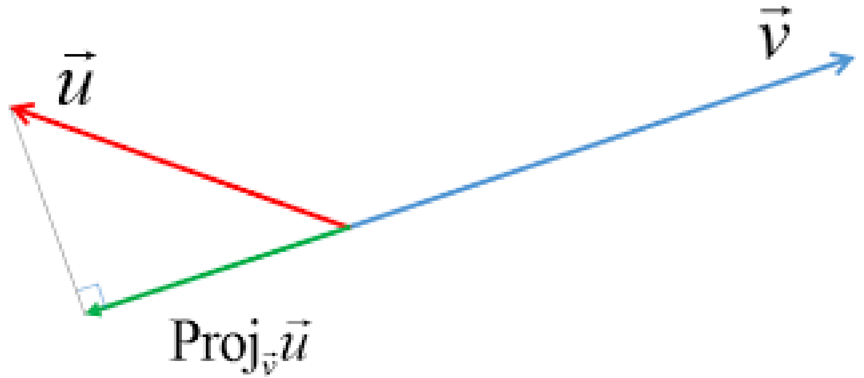
$$\langle\alpha|\alpha\rangle = \left\langle\alpha\left|\left(\sum_{a'}|a'\rangle\langle a'|\right)\right|\alpha\right\rangle = \sum_{a'}|\langle a'|\alpha\rangle|^2 = \boxed{\sum_{a'}|c_{a'}|^2 = 1}$$

born's rule

- Dimana $|a'\rangle\langle a'|$ adalah projection operator terhadap ket $|\alpha\rangle$ menuju base ket $|a'\rangle$:

$$(|a'\rangle\langle a'|)|\alpha\rangle = |a'\rangle\boxed{\langle a'|\alpha\rangle} = c_{a'}|a'\rangle$$

- Proyeksi ket $|\alpha\rangle$ terhadap basis observable $\underline{a'}$ adalah sebuah wavefunction pada koordinat $\underline{a'} \rightarrow \underline{\Psi(a')} = \langle a'|\alpha\rangle$



Uji Pemahaman

$|+\rangle$

- $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ terletak pada basis $|0\rangle$ dan $|1\rangle$
- Gunakan proyeksikan $|\psi\rangle$ terhadap basis $|+\rangle$ dan $|-\rangle$

$$\left(|+\rangle\langle +| + |-\rangle\langle -|\right)|+\rangle$$

$$|+\rangle\overset{1}{\langle +|+\rangle} + |-\rangle\overset{0}{\langle -|+\rangle}$$

$$|+\rangle$$

Bentuk Matriks

God's People for God's Glory
CALVIN
INSTITUTE OF TECHNOLOGY

Bentuk Matriks

$\sum_{all} \langle a'|X|a''\rangle\langle a''|\alpha\rangle$

- Sebuah operator X terhadap ket $|\alpha\rangle$ dapat direpresentasikan dalam bentuk matriks:

$$\langle a'|X|\alpha\rangle = \sum_{a''} \langle a'|X|a''\rangle\langle a''|\alpha\rangle$$

$$\begin{pmatrix} \langle a^{(1)}|X|a^{(1)}\rangle & \langle a^{(1)}|X|a^{(2)}\rangle & \dots \\ \langle a^{(2)}|X|a^{(1)}\rangle & \langle a^{(2)}|X|a^{(2)}\rangle & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \langle a^{(1)}|\alpha\rangle \\ \langle a^{(2)}|\alpha\rangle \\ \vdots \end{pmatrix} \begin{matrix} |0\rangle \\ |1\rangle \\ \end{matrix}$$

- Sebuah operator X terhadap bra $\langle\alpha|$ dapat direpresentasikan dalam bentuk matriks:

$$\langle\alpha| = \left(\langle\alpha|a^{(1)}\rangle \quad \langle\alpha|a^{(2)}\rangle \quad \dots\right) = \left(\langle a^{(1)}|\alpha\rangle^* \quad \langle a^{(2)}|\alpha\rangle^* \quad \dots\right)$$

$$\langle\alpha|X|a'\rangle = \sum_{a''} \langle\alpha|a''\rangle\langle a''|X|a'\rangle$$

$$\left(\langle a^{(1)}|\alpha\rangle^* \quad \langle a^{(2)}|\alpha\rangle^* \quad \dots\right) \begin{pmatrix} \langle a^{(1)}|X|a^{(1)}\rangle & \langle a^{(1)}|X|a^{(2)}\rangle & \dots \\ \langle a^{(2)}|X|a^{(1)}\rangle & \langle a^{(2)}|X|a^{(2)}\rangle & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Perkalian Matriks

- Jika matriks X Hermitian, maka:
$$\langle a''|X|a'\rangle = \left\langle a''\middle|X^\dagger\middle|a'\right\rangle^* = \langle a''|X|a'\rangle^*$$
- Jika matriks $Z = XY$, artinya:
$$\langle a''|Z|a'\rangle = \langle a''|XY|a'\rangle = \sum_{a'''}\langle a''|X|a'''\rangle\langle a'''|Y|a'\rangle$$
- Inner Product:
$$\langle\beta|\alpha\rangle = \sum_{a'}\langle\beta|a'\rangle\langle a'|\alpha\rangle = \left(\left\langle a^{(1)}\middle|\beta\right\rangle^* \quad \left\langle a^{(2)}\middle|\beta\right\rangle^* \quad \dots\right)\begin{pmatrix}\langle a^{(1)}|\alpha\rangle \\ \langle a^{(2)}|\alpha\rangle \\ \vdots\end{pmatrix}$$
- Outer Product:
$$|\beta\rangle\langle\alpha| = \begin{pmatrix}\langle a^{(1)}|\beta\rangle\left\langle a^{(1)}\middle|\alpha\right\rangle^* & \langle a^{(1)}|\beta\rangle\left\langle a^{(2)}\middle|\alpha\right\rangle^* & \dots \\ \langle a^{(2)}|\beta\rangle\left\langle a^{(1)}\middle|\alpha\right\rangle^* & \langle a^{(2)}|\beta\rangle\left\langle a^{(2)}\middle|\alpha\right\rangle^* & \dots \\ \vdots & \vdots & \ddots\end{pmatrix}$$

Uji Pemahaman

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\sum |v\rangle\langle v| = |0\rangle\langle 0| + |1\rangle\langle 1|$$

- Apakah closure relation dari computational basis $(|0\rangle, |1\rangle)$ dalam bentuk matriks?

$$\underset{|\psi'\rangle}{\begin{pmatrix} \\ \\ \end{pmatrix}} = \underset{H}{\begin{pmatrix} & \\ & \end{pmatrix}} \underset{|\psi\rangle}{\begin{pmatrix} \\ \\ \end{pmatrix}}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\begin{array}{c|cc} & \langle 0| & \langle 1| \\ \hline |0\rangle & 1 & 0 \\ |1\rangle & 0 & 1 \end{array} = I$$

$$|0\rangle = H\,|+\rangle$$

Uji Pemahaman

$$O|\psi\rangle = \lambda|\psi\rangle$$

$$(O - \lambda I)|\psi\rangle = 0$$

- Sebuah operator berbentuk matriks 3x3: $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$\lambda I = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

- Temukan eigenvector dan eigenvaluesnya

$$0 = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 0 + 0 - 0 - (-\lambda) - (-\lambda)$$

$$= -\lambda^3 + 2\lambda = 0$$

$$\lambda(2 - \lambda^2) = 0$$

$$\lambda = 0 \vee \lambda = 2 \vee \lambda = -2$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\vec{v} \qquad \vec{w} \qquad \vec{z}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

$$v_2 = 0, \quad v_1 + v_3 = 0, \quad v_2 = 0$$

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Formalisme

God's People for God's Glory
CALVIN
INSTITUTE OF TECHNOLOGY

Postulat Kuantum (Bransden & Joachain)

1. Seluruh informasi mengenai suatu sistem direpresentasikan oleh state vector $|\Psi\rangle$
2. Prinsip superposisi. $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$
3. Setiap dynamical variable dideskripsikan oleh linear operator.
4. Hasil dari pengukuran untuk variabel A adalah eigen value dari operator A
5. Ekspektasi dari hasil pengukuran berkali-kali terhadap variabel A terhadap sistem dengan wavefunction $|\Psi\rangle$ adalah $\langle A\rangle = \frac{\langle\Psi|A|\Psi\rangle}{\langle\Psi|\Psi\rangle}$
6. Wavefunction dapat diekspresikan secara kombinasi linear dari eigenfunction dari A
7. Perubahan wavefunction mengikuti persamaan schrodinger $i\hbar\frac{\partial|\Psi\rangle}{\partial t} = H|\Psi\rangle$, dimana H adalah operator energi total (Hamiltonian) dari sistem tersebut

CALVIN INSTITUTE OF TECHNOLOGY

Postulat 1

- Wavefunction $|\Psi\rangle$ dan $c|\Psi\rangle$ merepresentasikan state yang sama, biasanya nilai c dipilih agar wavefunction ternormalisasi:

$$\int |\Psi(\boldsymbol{r},t)|^2 d\boldsymbol{r} = 1$$

- Dimana $P(\boldsymbol{r},t) = |\Psi(\boldsymbol{r},t)|^2$ dapat diinterpretasi sebagai position probability density

CALVIN INSTITUTE OF TECHNOLOGY

Postulat 2

Kompleks

- Jika wavefunction $|\Psi_1\rangle$dan $|\Psi_2\rangle$ adalah state dari sistem maka:

$$|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$$

- Juga merupakan state dari sistem

CALVIN INSTITUTE OF TECHNOLOGY

Postulat 3

$$|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$$

- Operator A adalah linear jika memiliki sifat:

$$A(c_1|\Psi_1\rangle + c_2|\Psi_2\rangle) = c_1 A|\Psi_1\rangle + c_2 A|\Psi_2\rangle$$

$$A|\psi\rangle = Ac_1|\psi_1\rangle + Ac_2|\psi_2\rangle$$

$$f(x) = x^2 + \sin x + x$$

$$\frac{d}{dx}f(x) = \frac{d}{dx}x^2 + \frac{d}{dx}\sin x + \frac{d}{dx}x = 2x + \cos x + 1$$

Postulat 4

- Wavefunction $|\psi_n\rangle$ adalah eigenfunction dari operator A jika:

$$A|\psi_n\rangle = a_n|\psi_n\rangle$$

- Jika operator A adalah Hermitian maka eigenvalue a_n adalah bilangan real:

$A = A^\dagger$

$$\langle\psi_n|A|\psi_n\rangle = a_n\langle\psi_n|\psi_n\rangle$$
$$\langle(A\psi_n)|\psi_n\rangle = a_n^*\langle\psi_n|\psi_n\rangle$$
$$\boxed{a_n = a_n^*}$$

$\alpha + \beta i = \alpha - \beta i \rightarrow \beta = 0$

Mathematics: Hermitian Eigenvector Real Eigenvalue

$$\boxed{A|\psi_n\rangle = a_n|\psi_n\rangle}$$

eigen equation

Physics: Observable State of the System Measurement

Postulat 5

ekspektasi

- Jika wavefunction $|\Psi\rangle$ ternormalisasi maka $\boxed{\langle\Psi|\Psi\rangle = 1}$ dan $\boxed{\langle A\rangle = \langle\Psi|A|\Psi\rangle}$
- Jika operator A adalah Hermitian maka ekspektasi $\langle A\rangle$ adalah bilangan real

Postulat 6

- Untuk himpunan eigenfunctions $\{\psi_n\}$:

$$|\Psi\rangle = \sum_n c_n |\psi_n\rangle$$

Postulat 7

- Hamiltonian operator dari sebuah sistem adalah:

$$H = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m_i} + V(\boldsymbol{r}_1, \dots, \boldsymbol{r}_N, t)$$

- Dimana operator momentum $\boldsymbol{p}_i = i\hbar\nabla_i$

Uji Pemahaman

- Apakah hasil pengukuran yang mungkin dari operator pauli-z $\boxed{\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$ dan Hadamard $\boxed{H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} |\psi\rangle = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} |\psi\rangle \rightarrow \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0$$

$$0 = (1-\lambda)(-1-\lambda) = -1 - \cancel{\lambda} + \cancel{\lambda} + \lambda^2$$

$$\lambda^2 - 1 = 0 \rightarrow \lambda = 1 \vee \lambda = -1$$

$$\cancel{\frac{1}{\sqrt{2}}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} |\psi\rangle = \cancel{\frac{1}{\sqrt{2}}} \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} |\psi\rangle \rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$0 = (1-\lambda)(-1-\lambda) - 1 = \lambda^2 - 1 - 1 = \lambda^2 - 2$$

$$\lambda = 2 \vee \lambda = -2$$

Tuhan Memberkati