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Keterkaitan

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IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

Capaian Pembelajaran

- Composite System
- Entanglement
- Non-locality
- Multiqubit system

Composite System

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Composite System

- Jika Alice mempunyai photon dengan state $|0\rangle$ dan $|1\rangle$, maka wavefunctionnya dapat berbentuk superposisi:

$$|\Psi_A\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$
$$\alpha_0^*\alpha_0 + \alpha_1^*\alpha_1 = 1$$

- Lalu bob mempunyai photon dengan state $|0\rangle$ dan $|1\rangle$, maka wavefunctionnya dapat berbentuk superposisi:

$$|\Psi_B\rangle = \beta_0|0\rangle + \beta_1|1\rangle$$
$$\beta_0^*\beta_0 + \beta_1^*\beta_1 = 1$$

- Dapatkah kita mengkonstruksi single composite system dari keduanya?

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Tensor product

- Composite wavefunction dari 2 sistem dapat direpresentasikan secara Tensor :

$$|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$
$$|\Psi_{AB}\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes (\beta_0|0\rangle + \beta_1|1\rangle)$$
$$|\Psi_{AB}\rangle = a_0\beta_0|0\rangle|0\rangle + a_0\beta_1|0\rangle|1\rangle + a_1\beta_0|1\rangle|0\rangle + a_1\beta_1|1\rangle|1\rangle$$
$$|\Psi_{AB}\rangle = a_0\beta_0|00\rangle + a_0\beta_1|01\rangle + a_1\beta_0|10\rangle + a_1\beta_1|11\rangle$$

- Vektor ket sekarang memiliki 2 indeks untuk merepresentasikan suatu state:

$$|\Psi_{AB}\rangle = \sum_{a,b} c(a,b)|ab\rangle$$

- Vektor ket $|ab\rangle$ bersifat orthonormal, dimana:

$$\langle ab|a'b'\rangle = \delta_{aa'}\delta_{bb'}$$

Uji Pemahaman

- Apakah hasil tensor product dari $|\Psi_{ab}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- Berapa peluang mengukur 01 pada wavefunction diatas?

Tensor Product Matrices

- Untuk menggabungkan 2 matriks 2x2 menjadi matriks 4x4:

$$A \otimes B = \begin{pmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}$$

- Untuk menggabungkan 2 vektor kolom 2x1 menjadi vektor kolom 4x1:

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \otimes \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} \\ a_{11}b_{21} \\ a_{21}b_{11} \\ a_{21}b_{21} \end{pmatrix}$$

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Uji Pemahaman

- Apakah tensor product dari $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ dan $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$?
- Apakah tensor product dari $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ dan $X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

Tensor Product Basis

- Bentuk basis ket:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Bentuk komposit:

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Tensor Product Operator

- Bentuk basis operator:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Bentuk komposit:

$$Z \otimes X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$X \otimes Z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$Z \otimes X \neq X \otimes Z$$

Operator pada Basis ket

- Sebuah operator $Z \otimes X$ bekerja pada basis ket $|01\rangle$

$$(Z \otimes X)|01\rangle = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

Uji Pemahaman

- Apakah output dari operasi $(Z \otimes X)|00\rangle$?

Entanglement

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Entangled States

- Quantum mechanics memungkinkan superposisi vektor lebih dari sekedar tensor product
- Bentuk komposit yang paling general adalah:

$$|\Psi_{AB}\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

- Dimana kondisi normalisasinya:

$$c_{00}^{*}c_{00} + c_{01}^{*}c_{01} + c_{10}^{*}c_{10} + c_{11}^{*}c_{11} = 1$$

Maximally Entangled States

- Terdapat sistem yang lebih entangled dari sistem lain (entanglement bukanlah binari antara ya dan tidak)
- Sebuah sistem disebut maximally entangled states jika:

$$|\Psi_{AB}\rangle = c_{01}|01\rangle + c_{10}|10\rangle$$

- Dimana tidak mungkin dibentuk dari produk 2 qubit:

$$|\Psi_{AB}\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

Keterkaitan

- Sebuah system dikatakan terkait jika observasi yang satu akan sekaligus menentukan hasil observasi yang lain
- Secara wavefunction dapat ditulis:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle|0\rangle + |0\rangle|1\rangle)$$

Uji Pemahaman

- Apakah qubit pertama dan kedua dari wavefunction berikut entangled?
  - $|\Psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$
  - $|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle + \frac{1}{\sqrt{4}}|11\rangle$

Non-Locality

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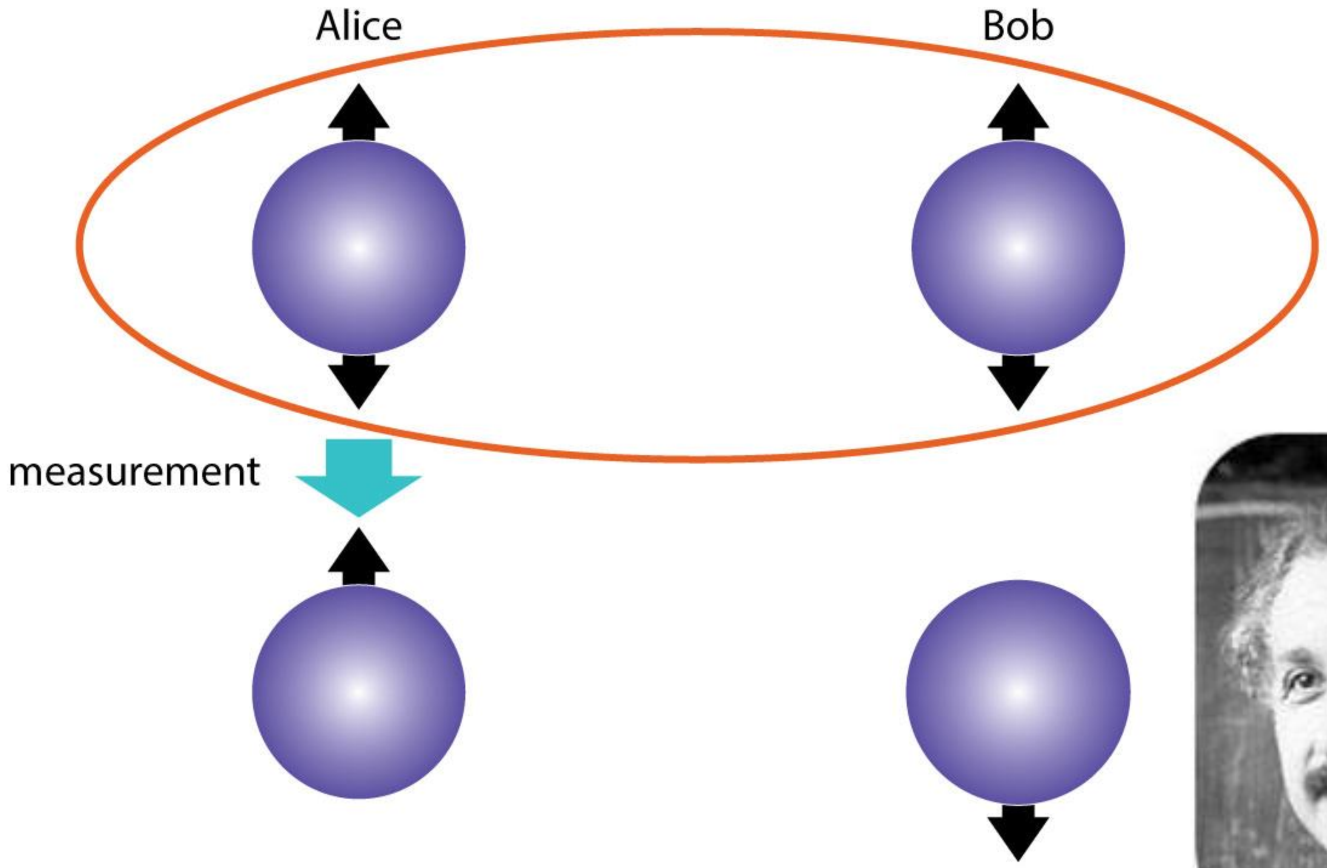
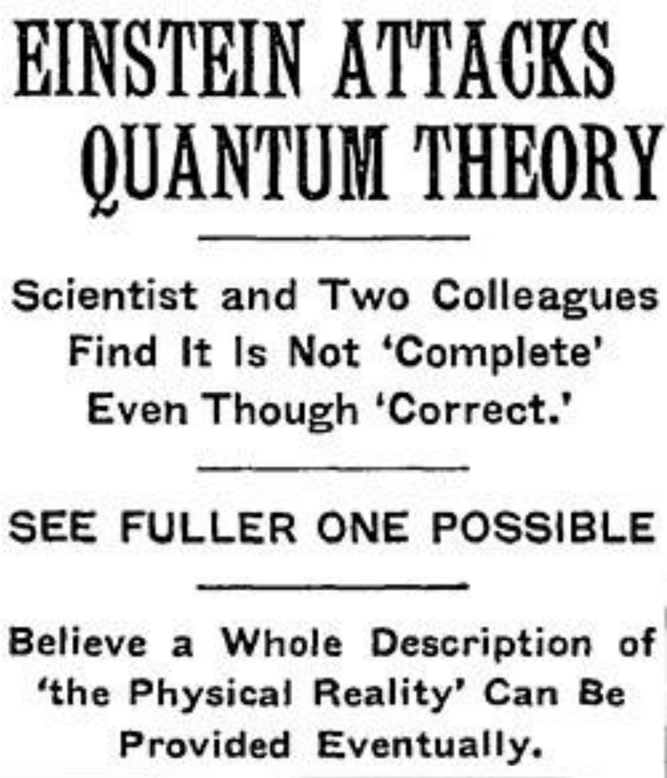
Lokalitas

- Lokalitas adalah fondasi dari fisika klasik
- Lokal berarti suatu kejadian hanya dapat disebabkan oleh sesuatu yang berada di lokasi dan saat yang sama
- Gaya gravitasi Newton bersifat non-lokal dan medan gravitasi Einstein bersifat lokal

EPR Paradoks

- “Spooky Action at Distance”



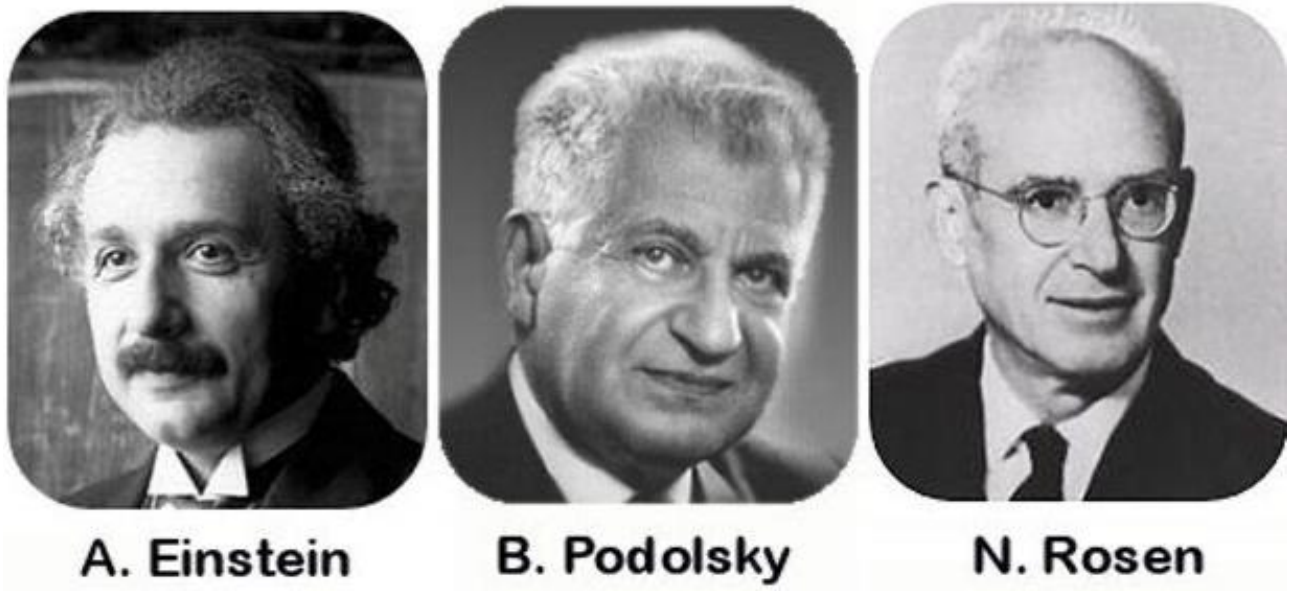


EINSTEIN ATTACKS QUANTUM THEORY

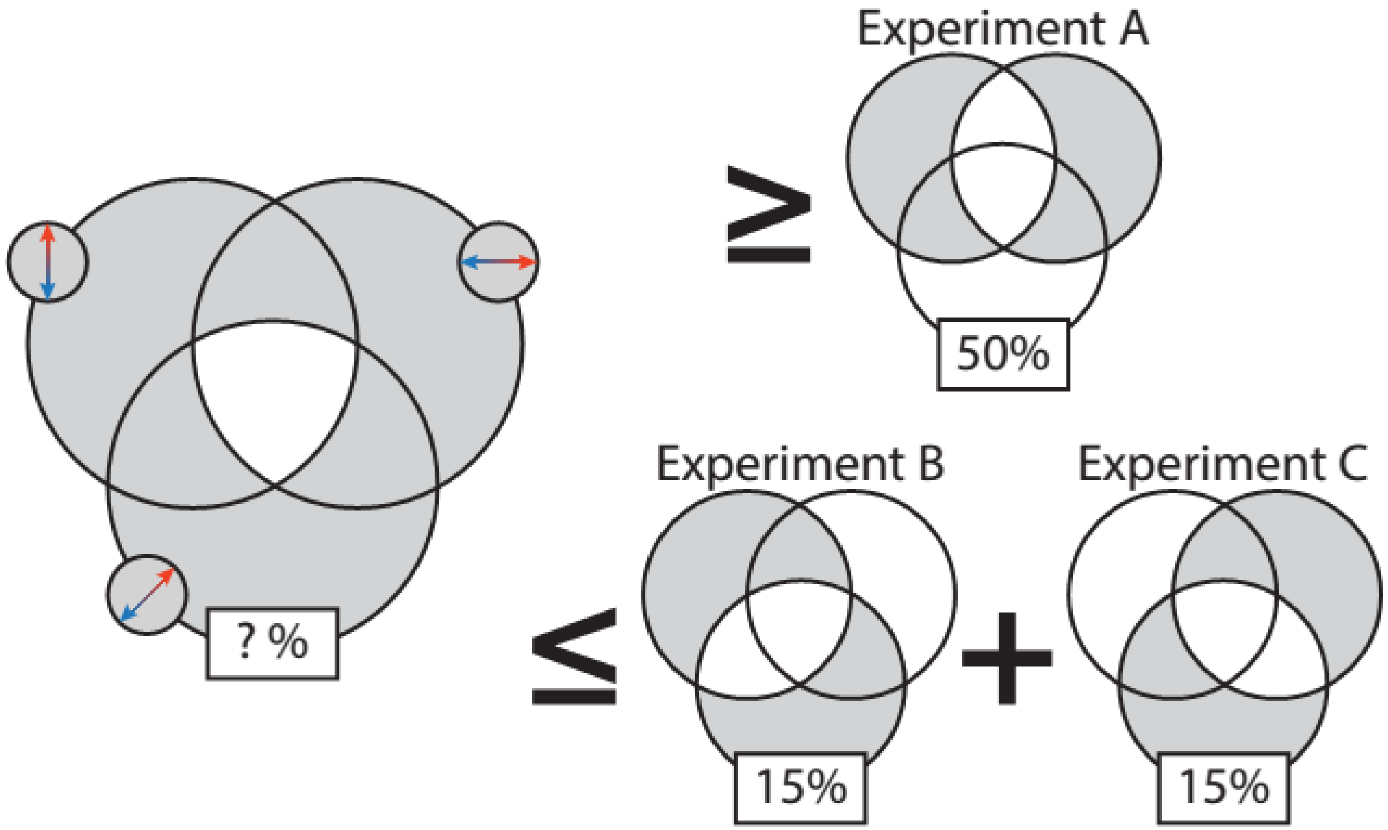
Scientist and Two Colleagues Find It Is Not ‘Complete’ Even Though ‘Correct.’

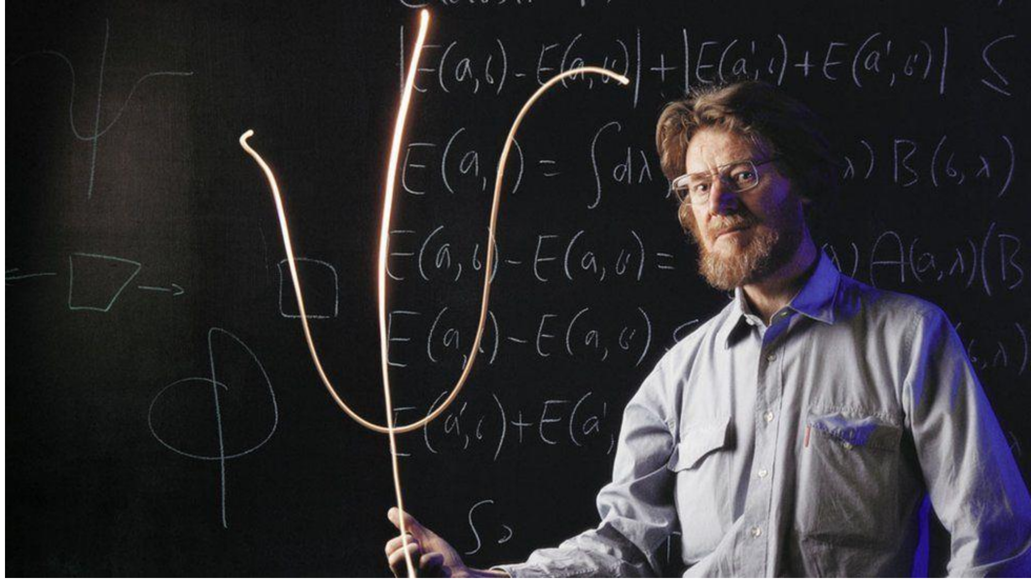
SEE FULLER ONE POSSIBLE

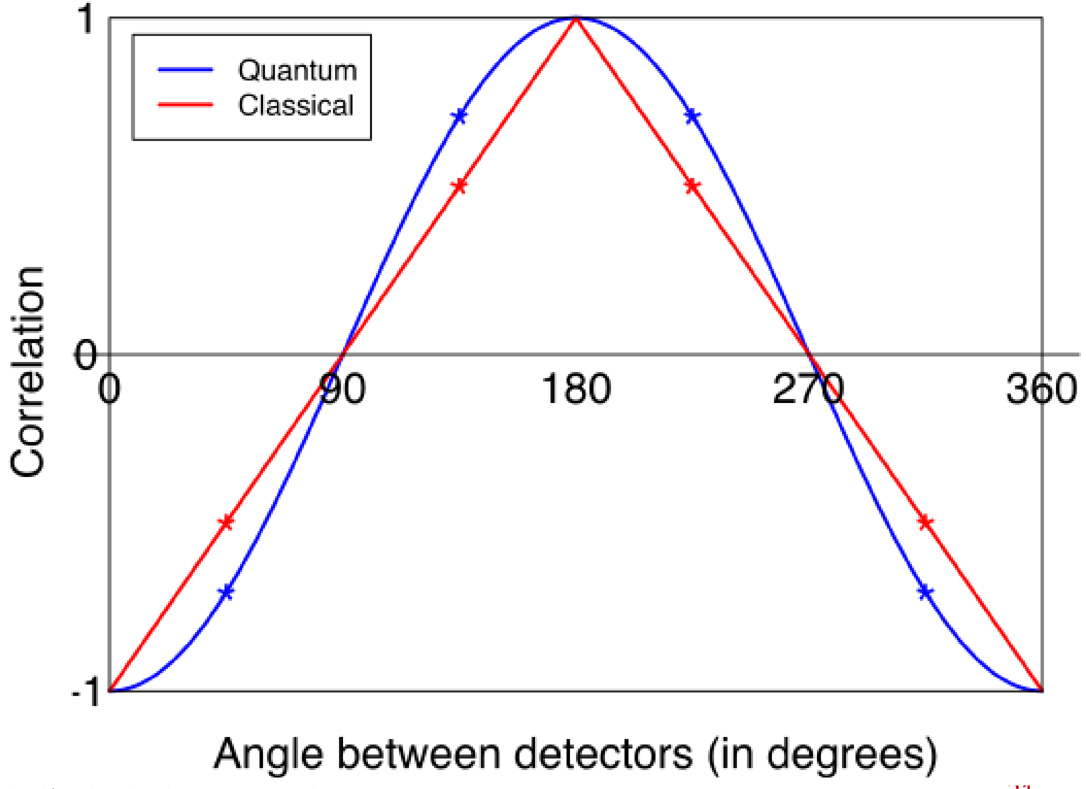
Believe a Whole Description of ‘the Physical Reality’ Can Be Provided Eventually.



Bell Inequality







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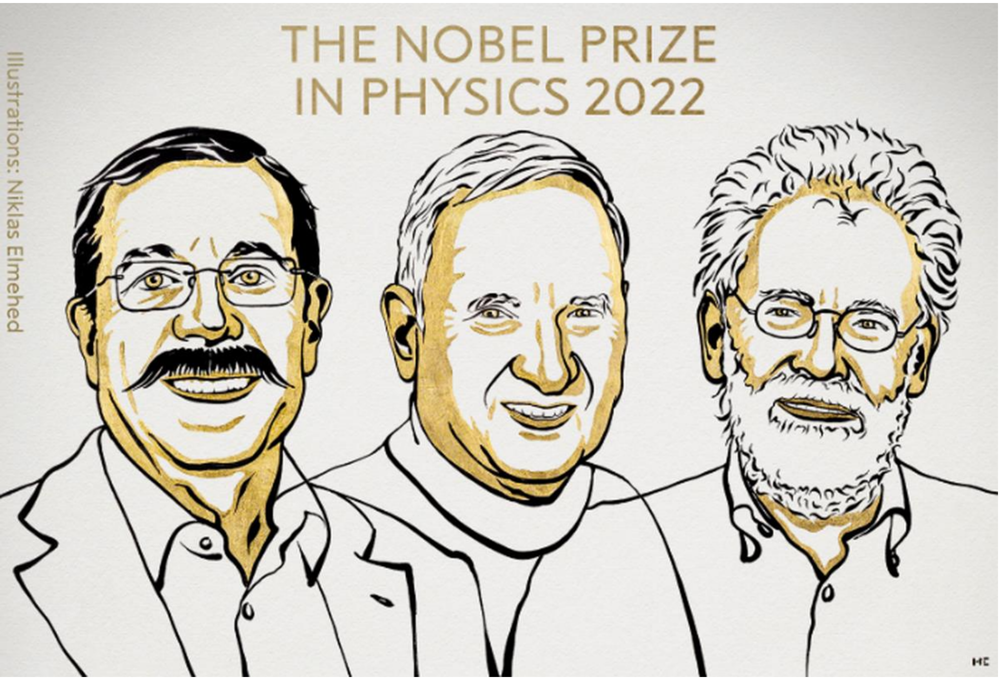
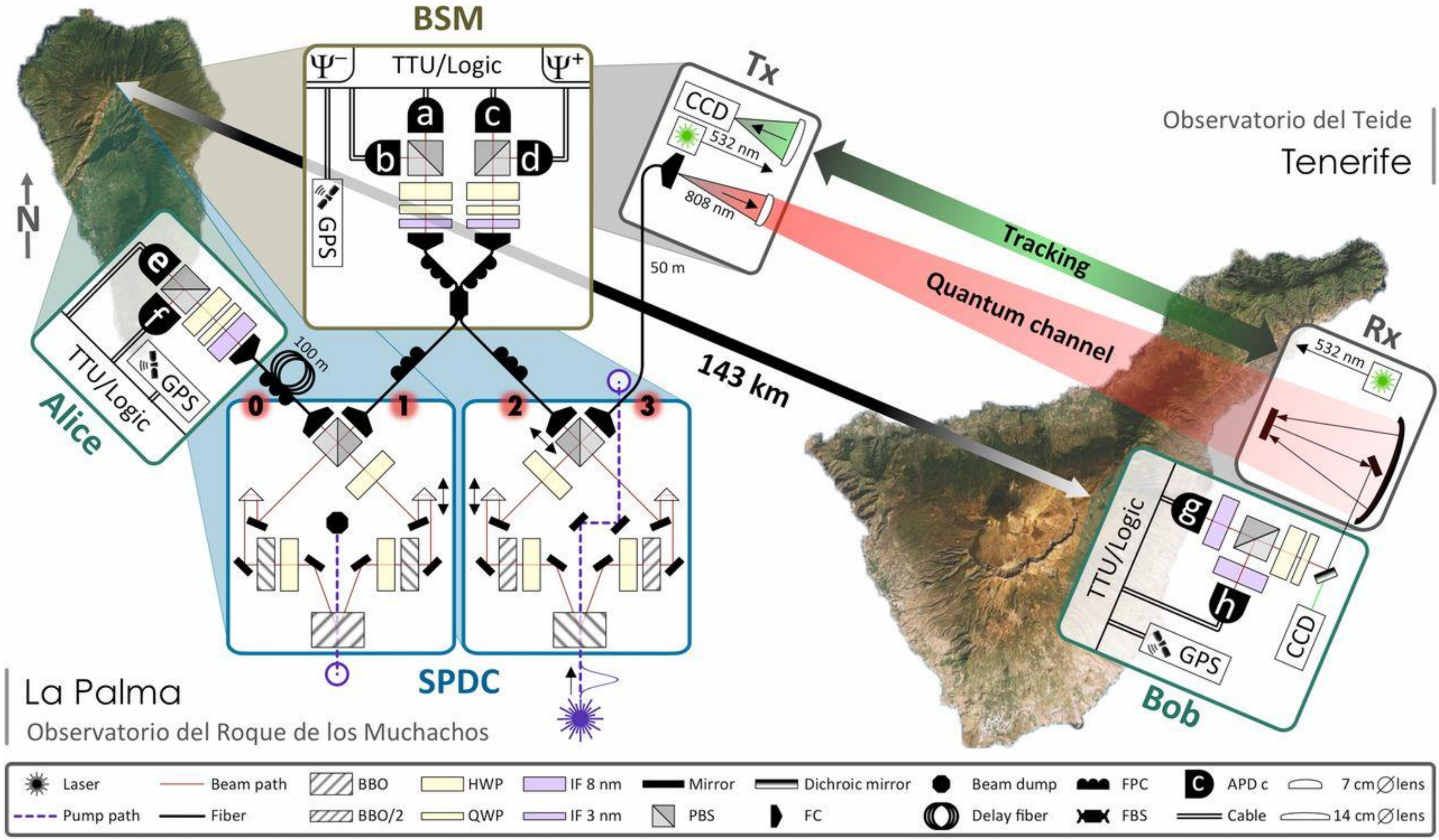
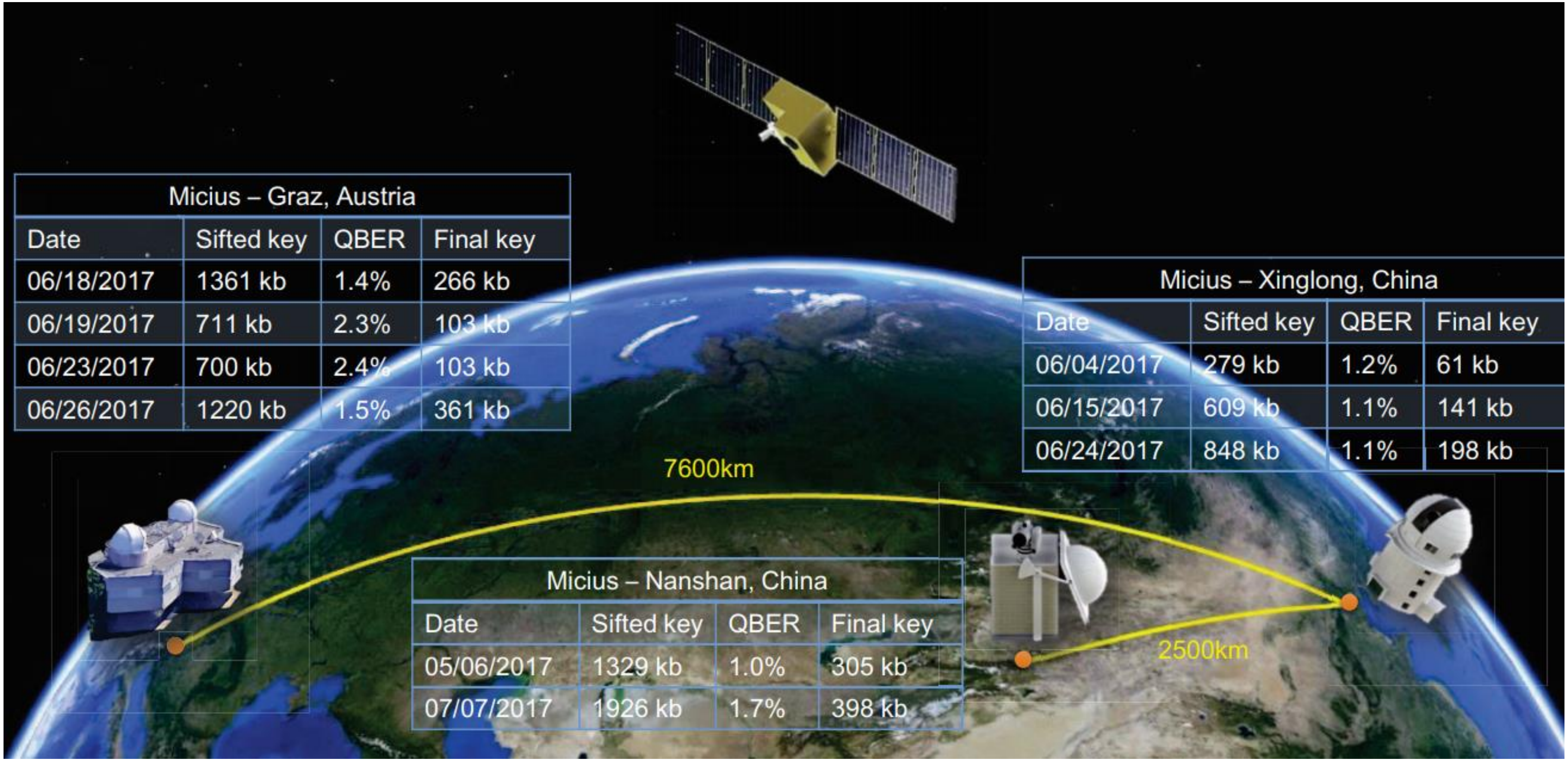
Demonstration of Entanglement

- https://www.pnas.org/doi/10.1073/pnas.1517007112





Longest Entanglement



Micius – Graz, Austria			
Date	Sifted key	QBER	Final key
06/18/2017	1361 kb	1.4%	266 kb
06/19/2017	711 kb	2.3%	103 kb
06/23/2017	700 kb	2.4%	103 kb
06/26/2017	1220 kb	1.5%	361 kb

Micius – Xinglong, China			
Date	Sifted key	QBER	Final key
06/04/2017	279 kb	1.2%	61 kb
06/15/2017	609 kb	1.1%	141 kb
06/24/2017	848 kb	1.1%	198 kb

Micius – Nanshan, China			
Date	Sifted key	QBER	Final key
05/06/2017	1329 kb	1.0%	305 kb
07/07/2017	1926 kb	1.7%	398 kb

Multiqubit System

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Single Qubit Gates on Multi-Qubit State

- Sekalipun kita menggunakan multi-qubit, tapi antar qubit bisa saja tidak berinteraksi (tiap operator hanya beroperasi pada qubit masing-masing secara independen)
- Contoh: Sebuah operator H beroperasi pada qubit q_0 dan operator X beroperasi pada qubit q_1

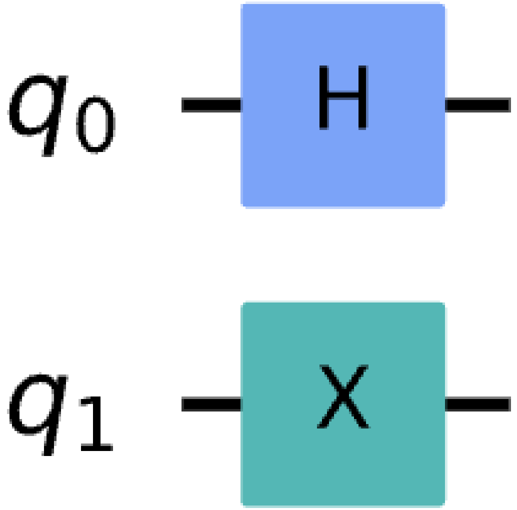
$$X|q_1\rangle \otimes H|q_0\rangle = (X \otimes H)|q_1 q_0\rangle$$

- Operator kompositnya:

$$X \otimes H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

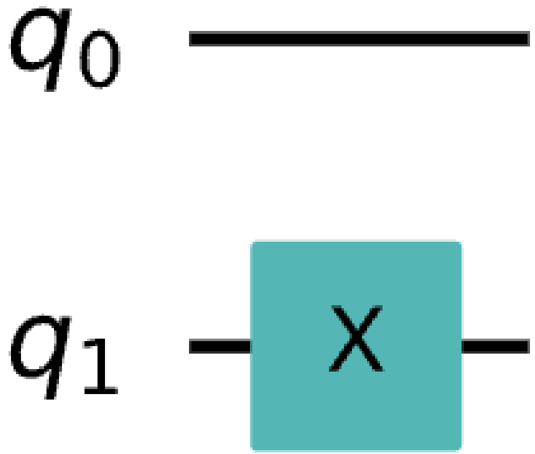
- Misalkan semua qubit berada pada inisiasi awal, qubit kompositnya:

$$|q_1 q_0\rangle = |00\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Uji Pemahaman

- Apakah bentuk komposit dari sirkuit kuantum berikut?
- Bandingkan state akhir bentuk komposit dan bentuk terpisah
  - Jika $|q_1 q_0\rangle = |00\rangle$
  - Jika $|q_0\rangle = a|0\rangle + b|1\rangle$ dan $|q_1\rangle = c|0\rangle + d|1\rangle$

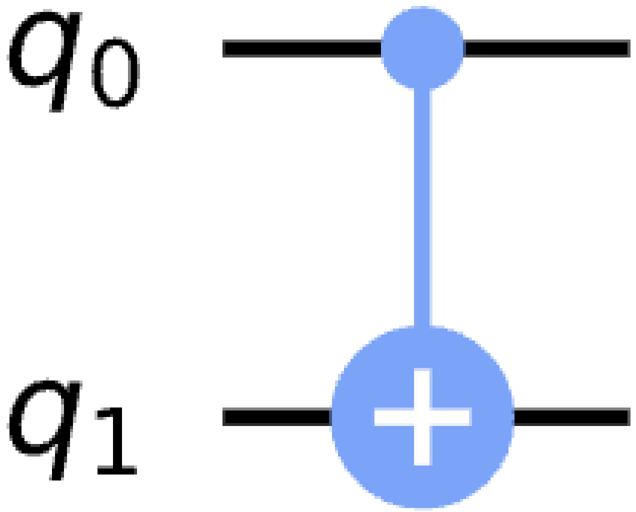


Multi-Qubit Gates

- Multiqubit gate adalah quantum gate yang beroperasi pada beberapa qubit sekaligus (terjadi interaksi antar qubit ketika operator ini digunakan)
- Contoh:
  - CNOT gate
  - CCNOT (Toffoli) gate
  - Swap gate
  - Cswap (Fredkin) gate

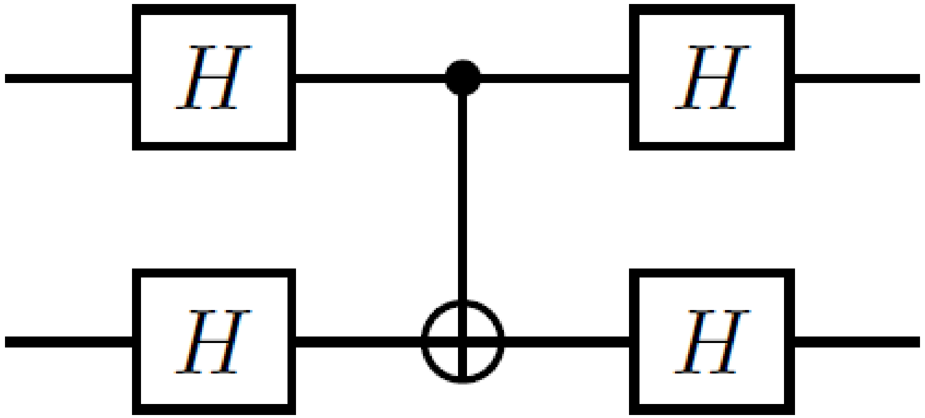
CNOT Gate

- Jika controlled qubit adalah 0, tidak merubah apa-apa
- Jika controlled qubit adalah 1, operasi X pada target qubit
- $CNOT = I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$
- $CNOT = (|0\rangle\langle 0| + |1\rangle\langle 1|) \otimes |0\rangle\langle 0| + (|0\rangle\langle 1| + |1\rangle\langle 0|) \otimes |1\rangle\langle 1$
- $CNOT = |00\rangle\langle 00| + |10\rangle\langle 10| + |01\rangle\langle 11| + |11\rangle\langle 01|$
- $CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$



Input (t,c)	Output (t,c)
00	00
01	11
10	10
11	01

Uji Pemahaman



- Apakah hasil dari operasi berikut?

CU Gate

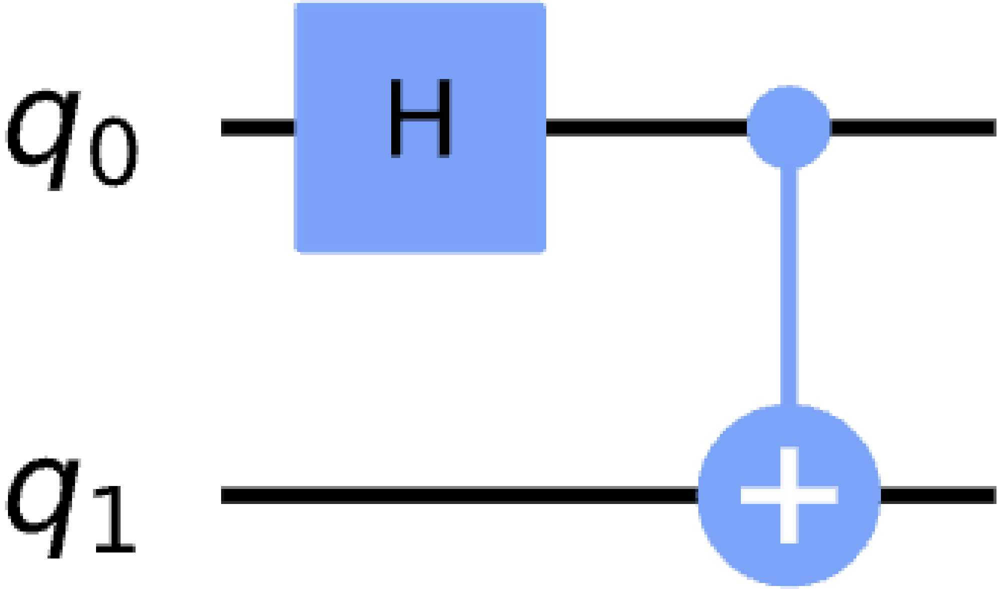
- Kita dapat mengontrol operasi gate apapun
- Jika controlled qubit adalah 0, tidak merubah apa-apa
- Jika controlled qubit adalah 1, operasi U pada target qubit
- $CU = I \otimes |0\rangle\langle 0| + U \otimes |1\rangle\langle 1|$
- Misalnya $CPhase = I \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$

Uji Pemahaman

- Buatlah Cphase gate dengan menggunakan CNOT gate

Entangled States

- $CNOT\ |0\ +\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$



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Uji Pemahaman

- Berapa state akhir dari $CNOT(I \otimes H)|00\rangle)$?

Bell States

- Terdapat 4 bell states yang maximally entangled dan membentuk basis orthonormal:
- $|\phi^+\rangle = CNOT(I \otimes H)|00\rangle\rangle = CNOT|0+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $|\phi^-\rangle = CNOT(I \otimes H)|01\rangle\rangle = CNOT|0-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
- $|\psi^+\rangle = CNOT(I \otimes H)|10\rangle\rangle = CNOT|1+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$
- $|\psi^-\rangle = CNOT(I \otimes H)|11\rangle\rangle = CNOT|1-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$

Uji Pemahaman

- Apakah bentuk dari $\frac{1}{2}(|00\rangle + i|01\rangle + i|10\rangle + |11\rangle)$ dalam basis: $\{|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), |\psi^+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle), |\psi^-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)\}$

Aktivitas

- Multiqubit system dan Entanglement di Qiskit
- https://learn.qiskit.org/course/ch-gates/multiple-qubits-and-entangled-states

Tuhan Memberkati