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Fondasi

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IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

Capaian Pembelajaran

- Information Theory
- Complex Number

Information Theory

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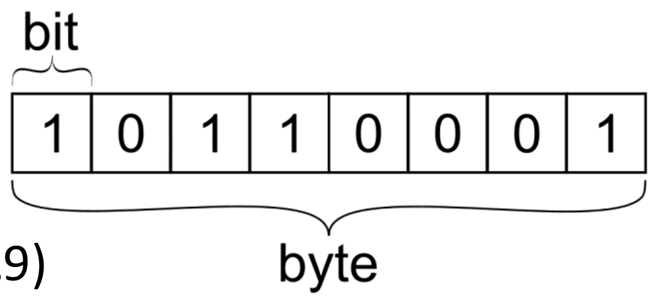
Bits pada Digital Computer

- Semua jenis informasi dapat direpresentasikan dalam bentuk paling sederhana dengan 2 simbols saja: 0 dan 1 (bit = binary digit)



- Contoh:
- Dalam sistem desimal:
  - Angka adalah kumpulan string dari kombinasi 10 digits (0,1,2,3,4,5,6,7,8,9)
  - 213 berarti $200 + 10 + 3 = (2 \times 10^2) + (1 \times 10^1) + (3 \times 10^0)$

    ratusan puluhan satuan
- Dalam sistem binary:
  - Angka diekspresikan melalui kelipatan $2,4,8,16,32, \ldots$
  - $213 = (1 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 11010101$

    128 64 32 16 8 4 2 1
- Huruf, angka, symbol, teks dapat direpresentasikan menggunakan system binary: https://www.ibm.com/docs/en/aix/7.2?topic=adapters-ascii-decimal-hexadecimal-octal-binary-conversion-table

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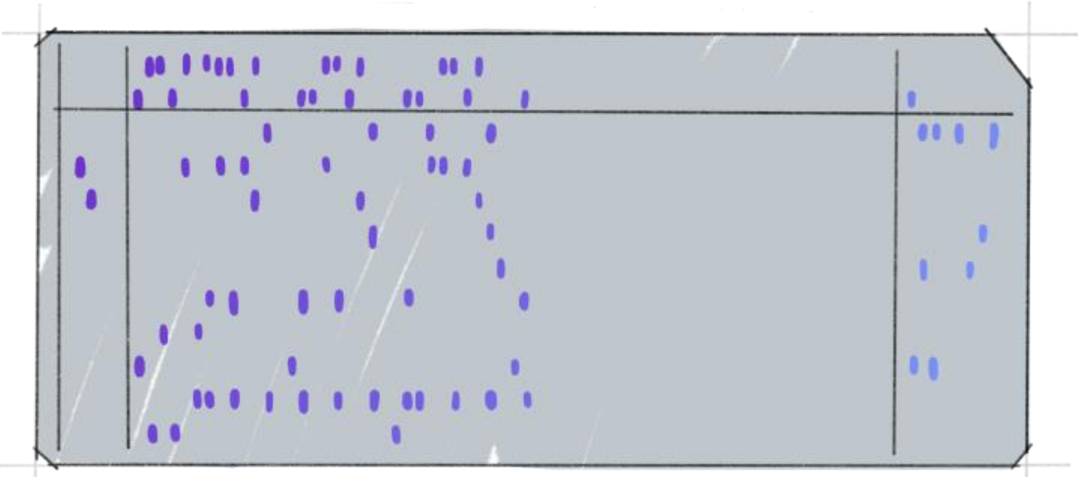
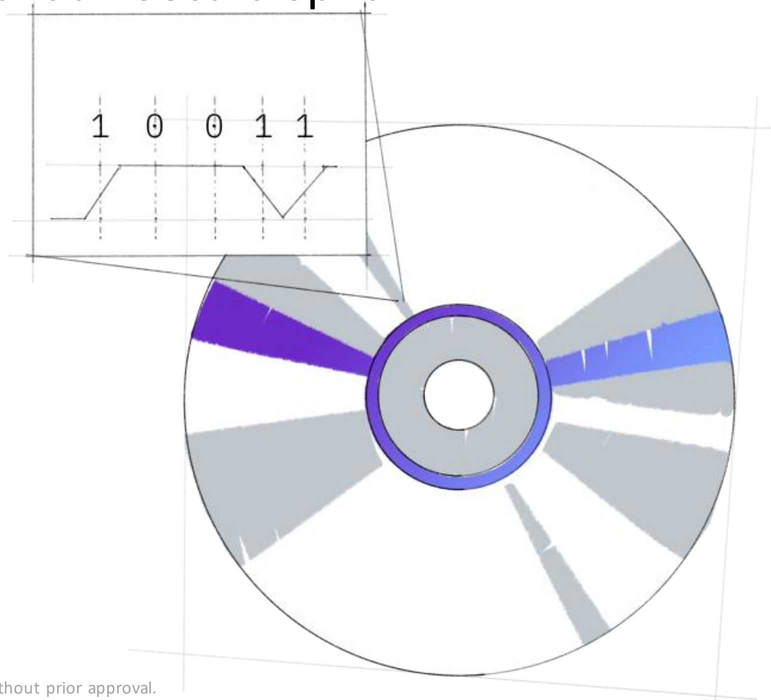
Aktivitas: Bermain Bits

- from qiskit_textbook.widgets import binary_widget
- binary_widget(nbits=5)

Menyimpan Bits

- Punched cards
  - Komputer awal menyimpan bits dengan melubangi kertas
  - Kertas dibagi menjadi banyak grid dan setiap grid merepresentasikan bit
  - 1 jika berlubang, 0 jika tidak ada lubang
- Compact disks
  - CD popular di tahun 80an dimana laser akan menyisir permukaan secara spiral
  - 1 jika permukaan miring, 0 jika permukaan datar

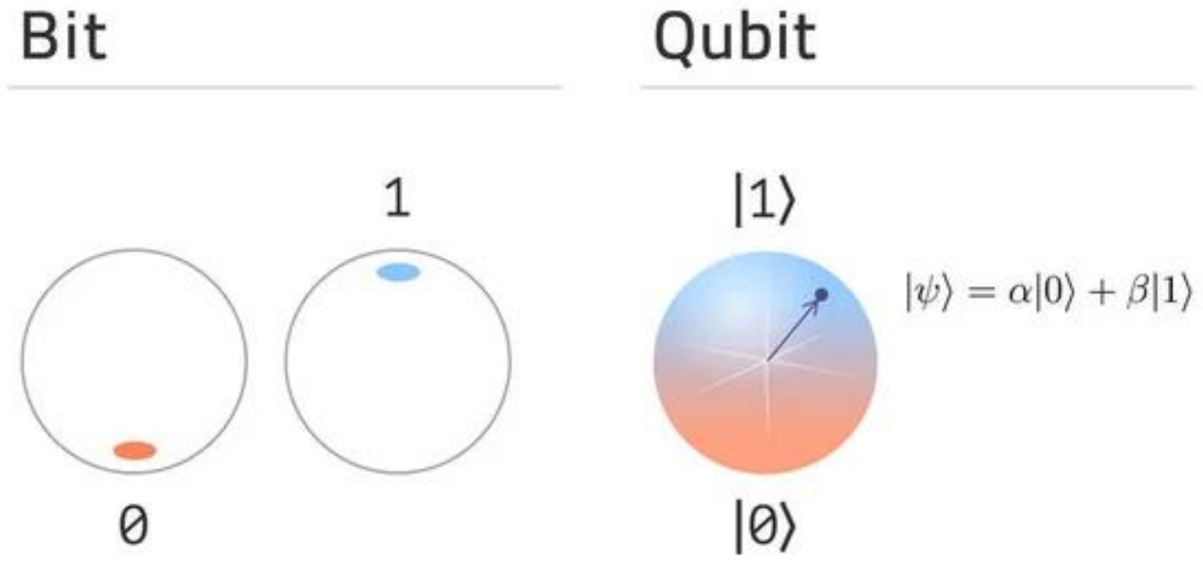




Qubits pada Quantum Computer

- Qubit menyimpan informasi secara binary (seperti bit), hanya saja qubit memiliki sifat quantum
- 1-bit bernilai antara 1 atau 0
- 1-qubit bernilai 0 dan 1 sekaligus (dalam bentuk superposisi)
- 1-qubit dapat menyimpan informasi 2^1 bit
- $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

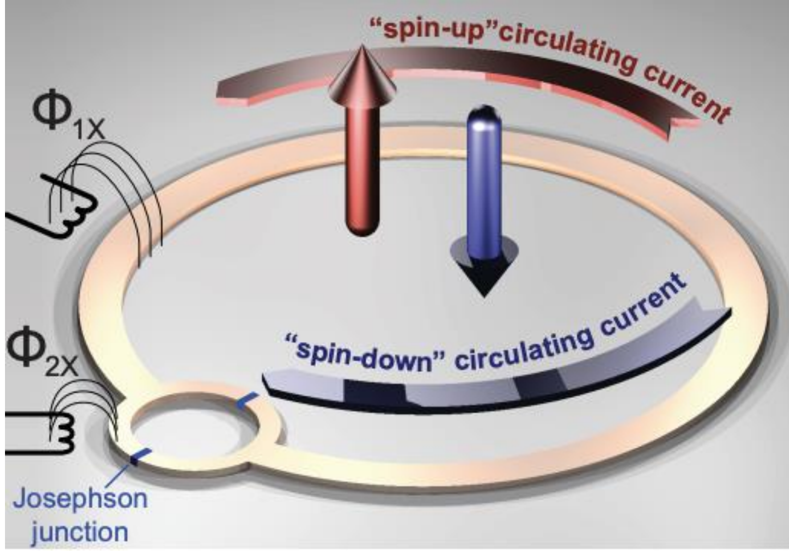
10
11
01
00

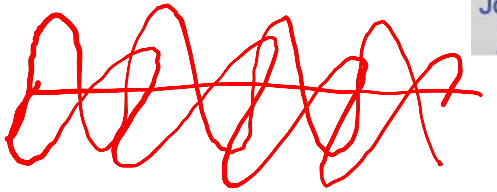


Menyimpan Qubits

- Electron orbitals
  - Apakah shell terisi electron atau tidak
- Spin
  - Apakah spin up atau down
- Polarized photon
  - Apakah photon terpolarisasi horizontal atau vertikal
- Superconducting Junction
  - Apakah arus berputar searah atau berlawanan jarum jam
- Topological Anyon — matter / antimatter
  - Apakah bersifat boson atau fermion

→ fermi exclussion principle





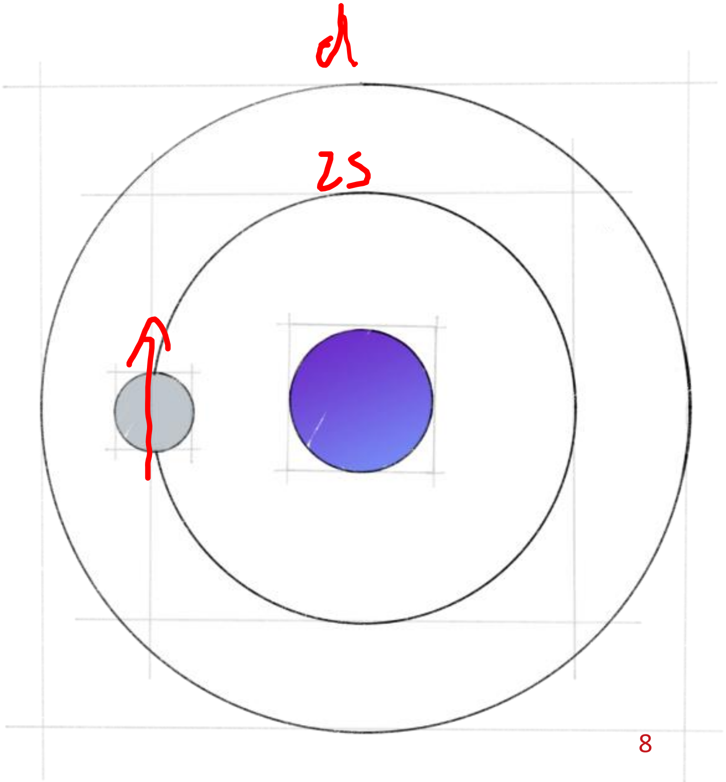


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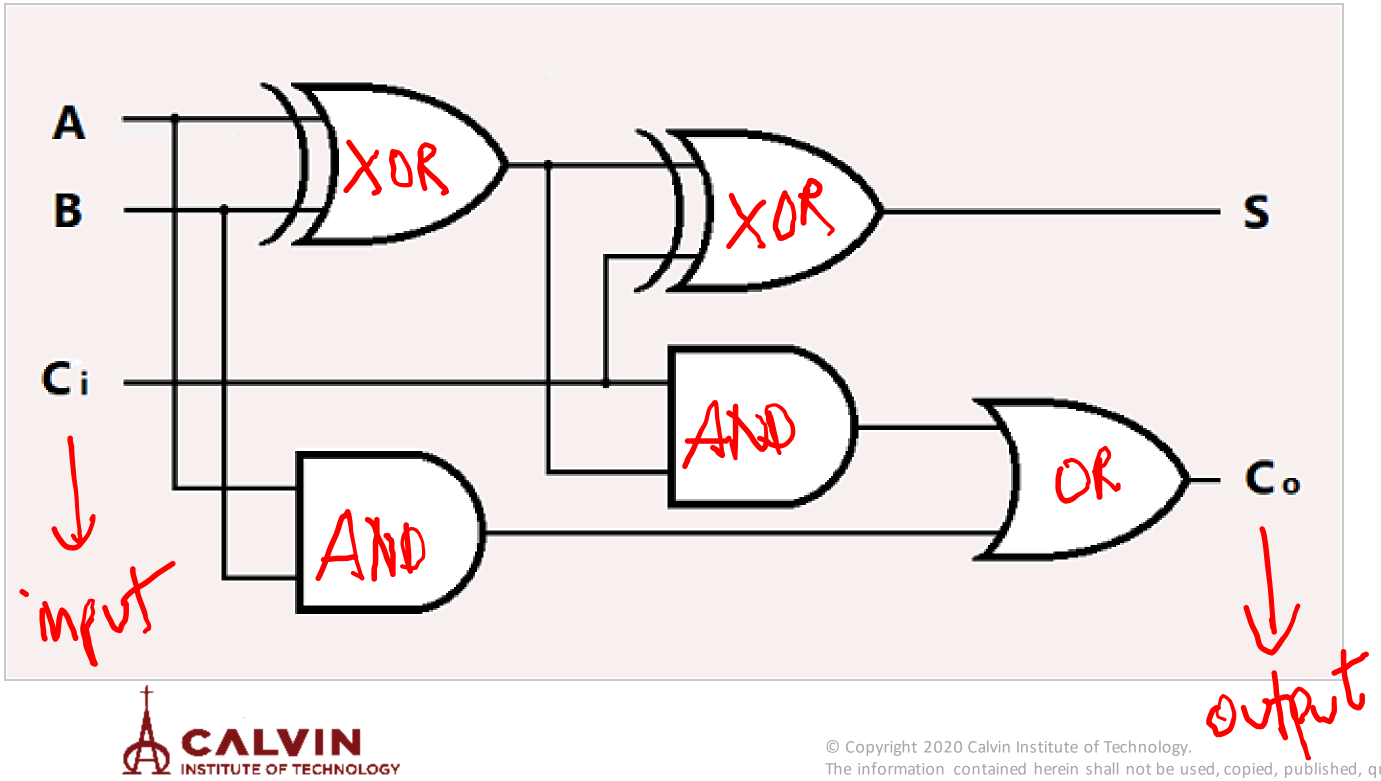
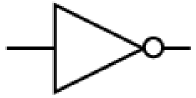
Diagram sirkuit

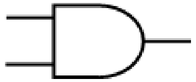
- Komputasi: input → operation →output
- Proses ini dapat direpresentasikan dalam bentuk circuit diagram (input di kiri, output di kanan, dan operasinya diantaranya)
- Operasi-operasi komputasi ini di sebut juga gates





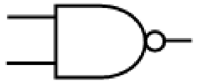
NOT

Input	Output
I	F
0	1
1	0



AND

Inputs		Output
A	B	F
0	0	0
1	0	0
0	1	0
1	1	1



NAND

Inputs		Output
A	B	F
0	0	1
1	0	1
0	1	1
1	1	0

OR

Inputs		Output
A	B	F
0	0	0
1	0	1
0	1	1
1	1	1

NOR

Inputs		Output
A	B	F
0	0	1
1	0	0
0	1	0
1	1	0

EXCLUSIVE OR

Inputs		Output
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

EXCLUSIVE NOR

Inputs		Output
A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

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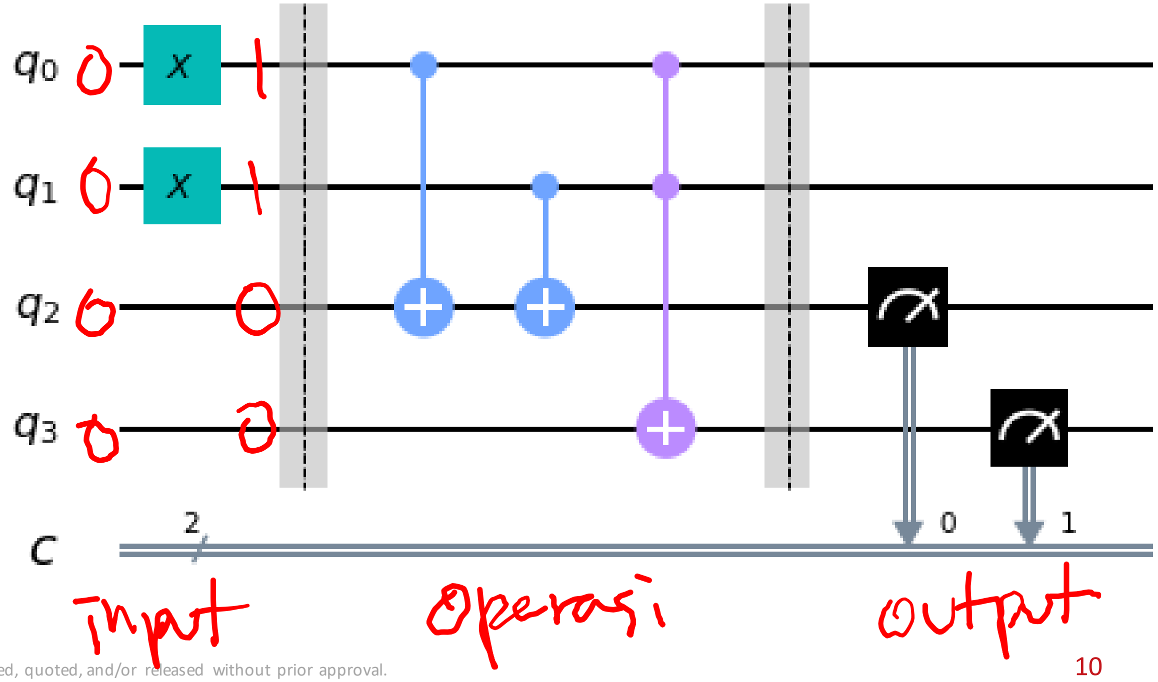
Quantum Circuit

- Mirip seperti classical circuit, quantum circuit menerima input qubit dan melakukan operasi quantum untuk mengolahnya menjadi output qubit
- Pada quantum circuit, output tidak dapat diketahui secara langsung, tetapi harus diukur (measure) terlebih dahulu
- Qubit diproses dari paling atas (kanan): $|0011\rangle \rightarrow q_0 = 1, q_1 = 1, q_2 = 0, q_3 = 0$



Contoh: Sirkuit Penjumlahan

12
9 +
21

- Binary addition

```
    1000111111101
+   0001110011110
+               1
=   ??????????011
```

A=1 Cin=1
B=1

```
    1000111111101
+   0001110011110
+               1
=   ?????????1011
```

11111111
1010110011

...

- Half adder

0+0 = 00

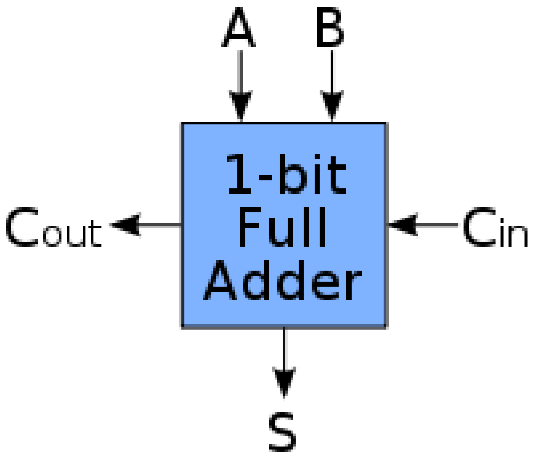
0+1 = 01

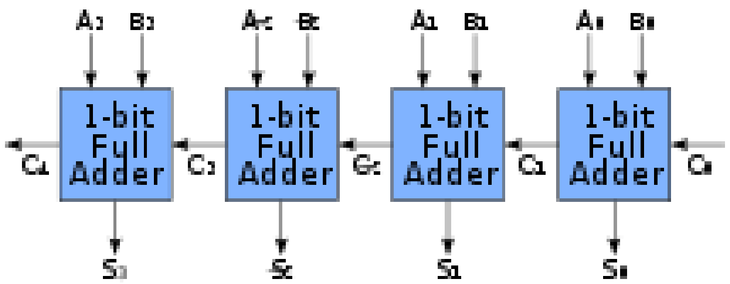
1+0 = 01

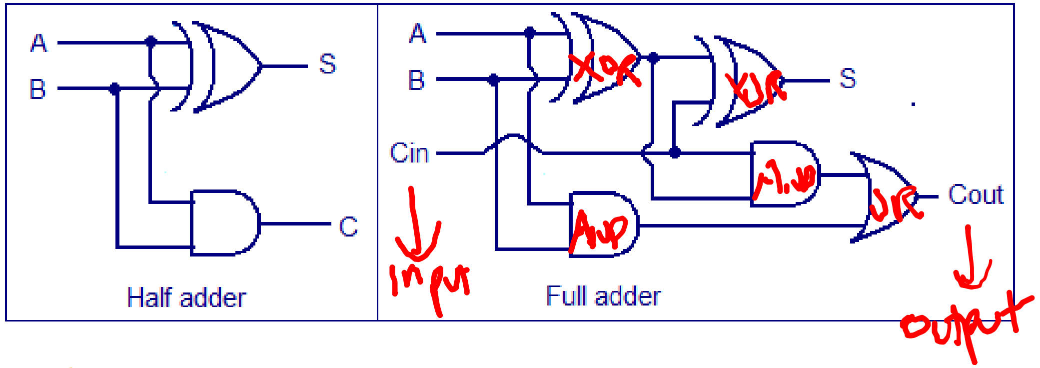
1+1 = 10

Full adder

Inputs			Outputs	
A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1





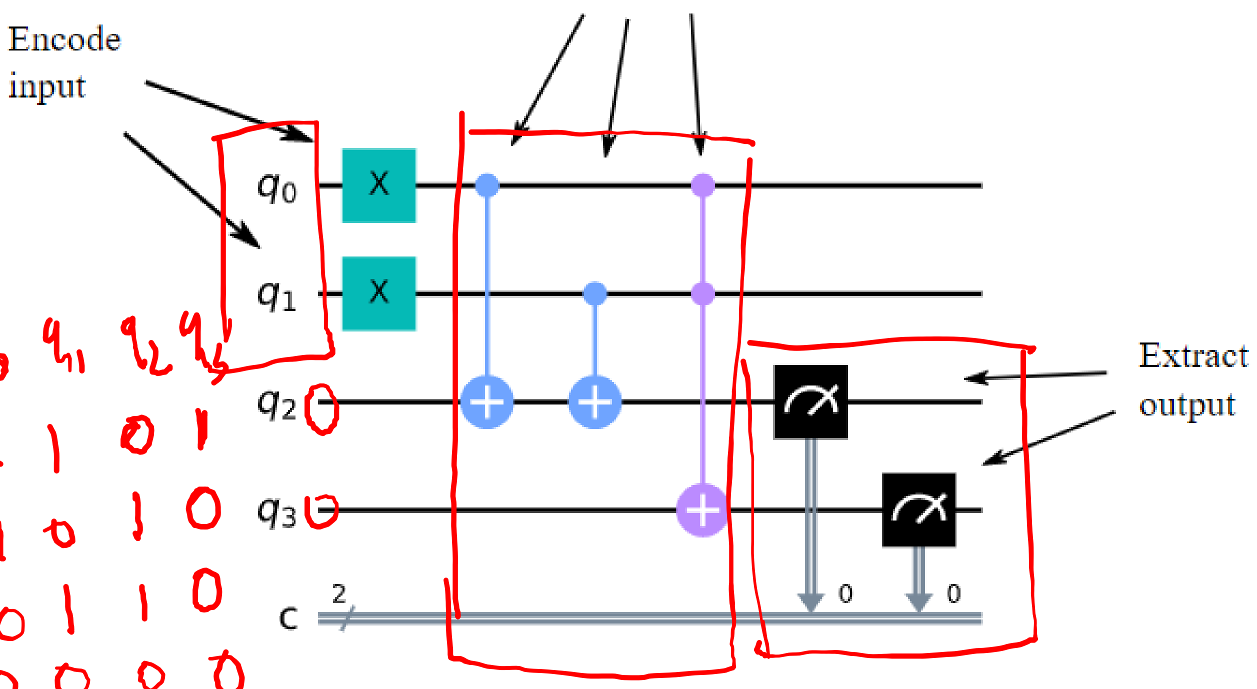


Half adder
Full adder

Penjumlahan di Sirkuit Kuantum

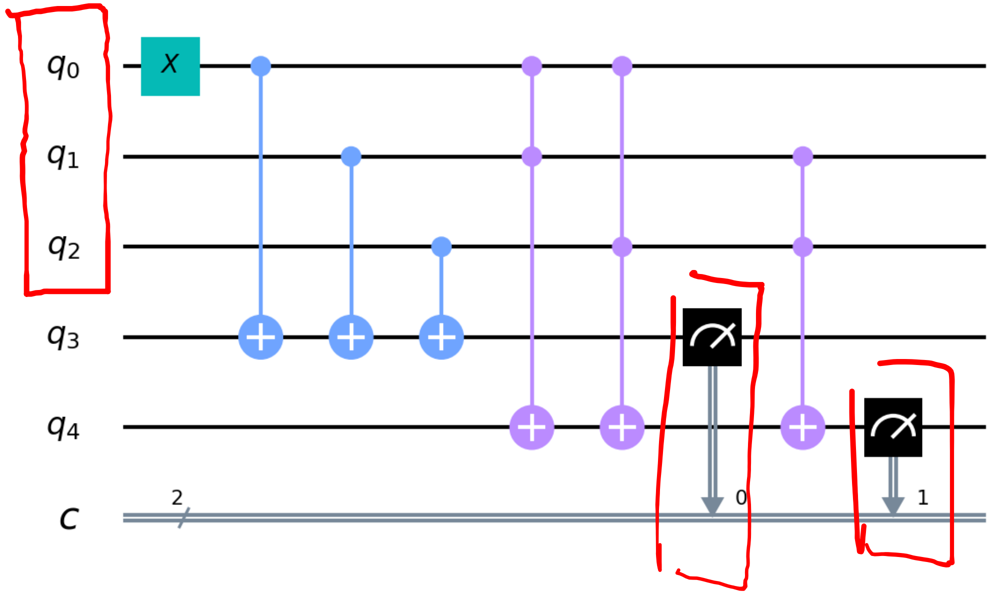
- Bit kanan half-adder sama dengan XOR gate
- Bit kiri half-adder hanya teraktivasi jika keduanya 1 (Toffoli gate)

Half-adder



q_0	q_1	q_2	q_3
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	0

Full-adder



Aktivitas: Membuat Adder

Complex Number

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Imaginary Number

- Descartes menyebutnya "imaginary" sebagai ejekan
- $x^2 = -1 \rightarrow$ *no real solution*
- $x = \pm\sqrt{-1}$
- $i = \sqrt{-1}$
- $i^2 = -1$

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

$\mathbb{Q} = \{1/3, 1/2, \dots\}$

$\mathbb{I} = \{\sqrt{3}, \pi\}$

$10a = 3.333\dots$

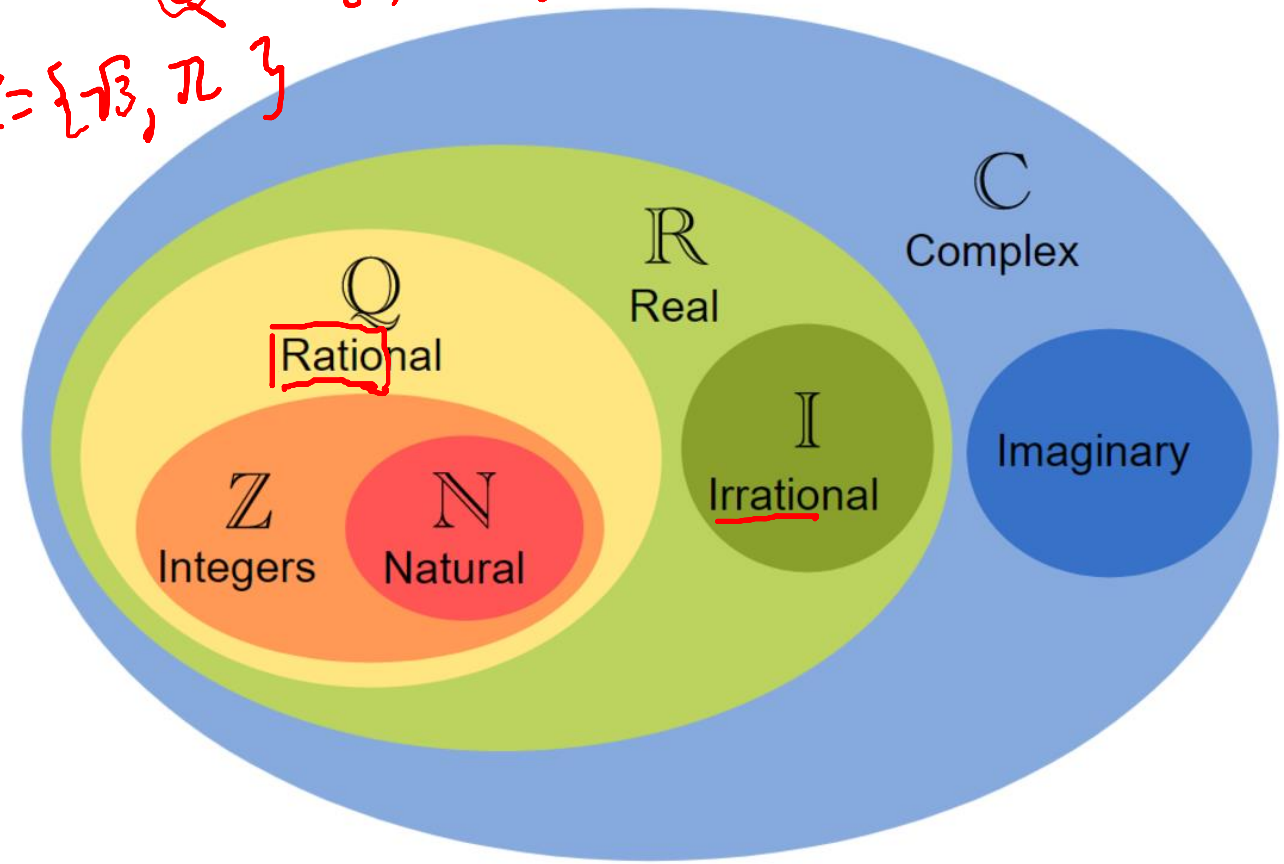
$a = 0.333\dots$

$9a = 3$

$a = 3/9 = 1/3$

$x \times x = \boxed{-1}$

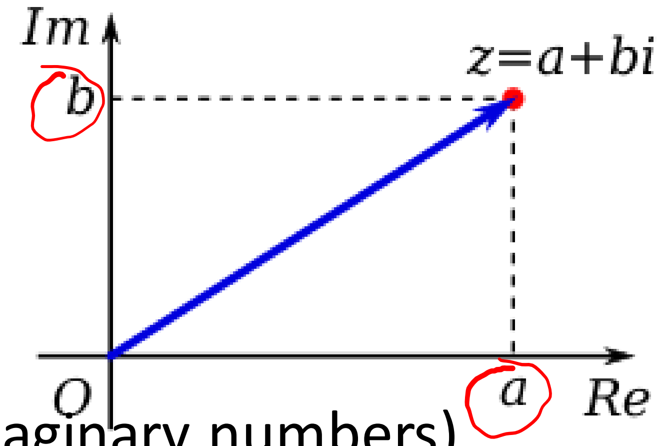
$x = \pm\sqrt{-1}$



Complex Number

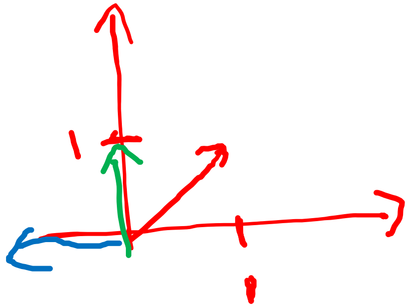
real imaginary

$$z = a + bi$$



- If $b = 0$, then the complex number $a + bi$ reduces to a
- If a $= 0$, then the complex number $a + bi$ reduces to bi (pure imaginary numbers)

$1 + i - 1 = i$



Addition

- $(a + bi) + (c + di) = (a + c) + (b + d)i$
- $(a + bi) - (c + di) = (a - c) + (b - d)i$

Multiplication

- $k(a + bi) = (ka) + (kb)i$
- $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Rotasi

- Bilangan kompleks $a + bi$ bisa menyatakan fungsi rotasi sudut berapapun
  - $f(z) = (a + bi)z$
- Bilangan imajiner i menyatakan fungsi rotasi 90^0
  - $f(z) = iz$

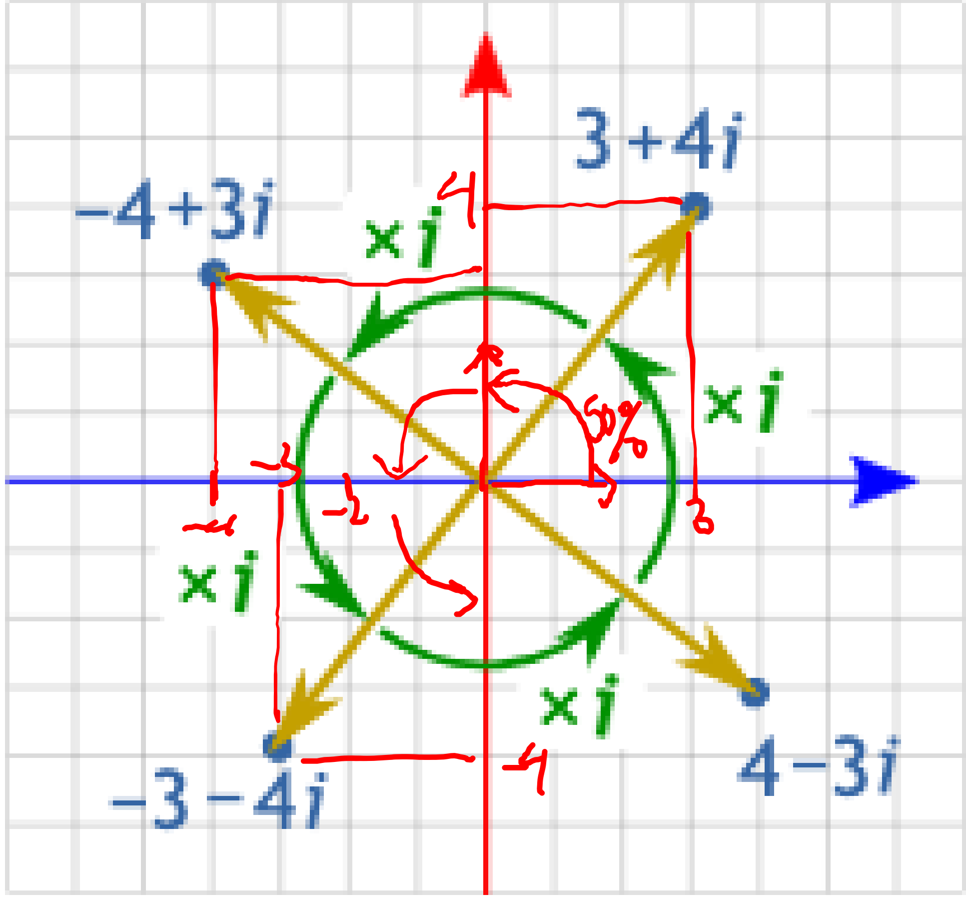
$z = 2$ $\quad f(z) = 2i$

$f(f(z)) = -2$

$f(f(f(z))) = -2i$

$z = 3+4i$ $\quad f(z) = 3i - 4$

$f(f(z)) = -3 - 4i$



Uji Pemahaman

- Jika $A = \begin{bmatrix} 1 & -i \\ 1+i & 4-i \end{bmatrix}$ dan $B = \begin{bmatrix} i & 1-i \\ 2-3i & 4 \end{bmatrix}$
- $A + B =$
- $iA =$
- $AB =$

$$A+B = \begin{bmatrix} 1+i & 1-2i \\ 3-2i & 8-i \end{bmatrix}$$

$$iA = \begin{bmatrix} i & 1 \\ i-1 & 4i+1 \end{bmatrix}$$

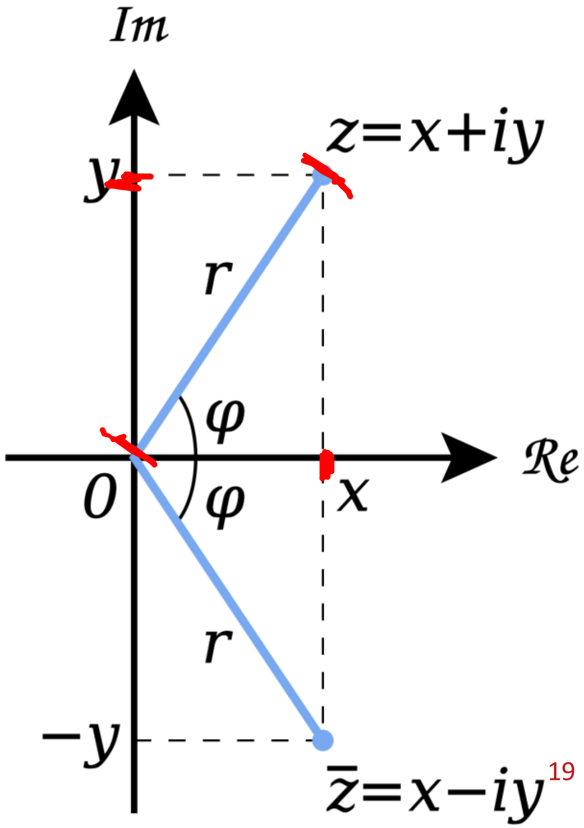
$$AB = \begin{bmatrix} 1 & -i \\ 1+i & 4-i \end{bmatrix} \begin{bmatrix} i & 1-i \\ 2-3i & 4 \end{bmatrix}$$

$$= \begin{bmatrix} i-2i-3 & 1-i-4i \\ i-1+8-12i-2i-3 & 1-i+i+1+8-4i \end{bmatrix}$$

Complex Conjugates

z^*

- Jika $z = a + bi$ maka complex conjugate dari z didefinisikan dengan $\bar{z} = a - bi$
- $\overline{z_1 + z_2} = \overline{(a_1 + a_2) + (b_1 + b_2)i} = (a_1 + a_2) - (b_1 + b_2)i = (a_1 - b_1 i) + (a_2 - b_2 i) = \bar{z_1} + \bar{z_2}$
- $\overline{z_1 z_2} = \overline{(a_1 a_2 + a_2 b_1 i + b_2 a_1 i - b_1 b_2)} = (a_1 a_2 - b_1 b_2) - (a_2 b_1 + b_2 a_1)i = (a_1 - b_1 i)(a_2 - b_2 i) = \bar{z_1}\bar{z_2}$
- $z\bar{z} = (a + bi)(a - bi) = a^2 + b^2 = |z|^2$
- $\bar{\bar{z}} = \overline{\overline{a + bi}} = \overline{a - bi} = a + bi = z$

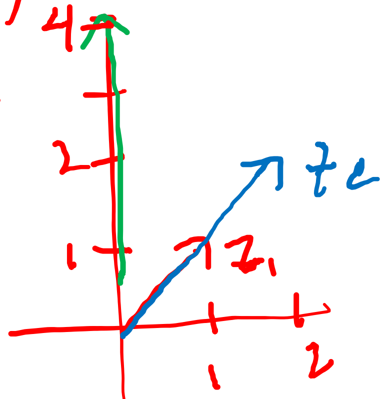


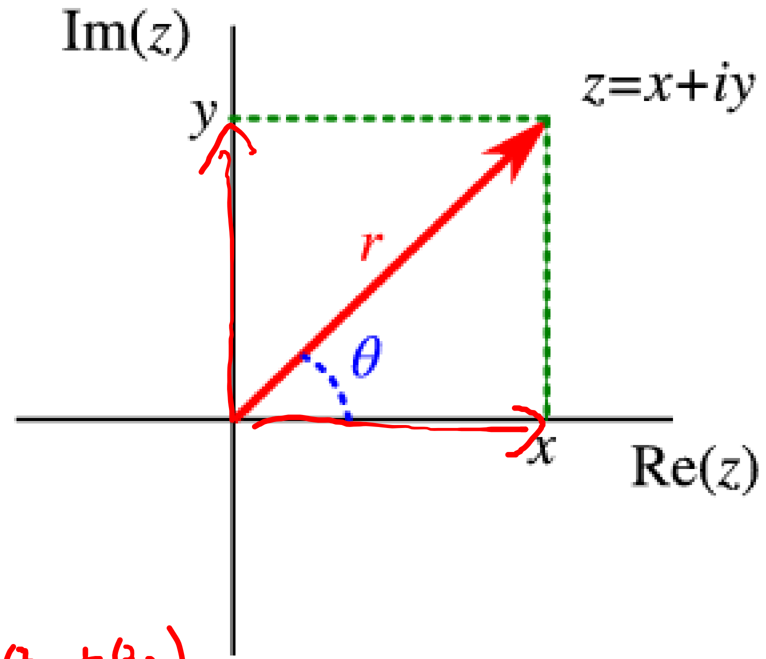
Polar Form

$(1+i)(2+2i)$

$\cancel{2}+2i+2i-\cancel{2}$

$4i$





$\sqrt{1^2+1^2}$ $\sqrt{2^2+2^2}$

$\sqrt{2\cdot 8}=4$ $\cos(\theta_1+\theta_2)$

- $z = x + iy$
- $r = |z|$
- $x = r cos\theta, y = r sin\theta$
- $z = r(cos\theta + i sin\theta)$
- $\theta = \arg z$
- $z_1 z_2 = r_1(cos\theta_1 + i sin\theta_1) r_2(cos\theta_2 + i sin\theta_2) = \mathrm{r}_1\mathrm{r}_2[(cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2) + i(sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2)] = r_1 r_2[\cos(\theta_1 + \theta_2) + i sin(\theta_1 + \theta_2)]$
- $|z_1 z_2| = |z_1||z_2|$
- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

Uji Pemahaman

$$\overline{\left(\frac{a_1+b_1 i}{a_2+b_2 i}\right)}$$

- Buktikan bahwa $\overline{\left(\frac{z_1}{z_2}\right)} = \bar{z}_1/\bar{z}_2$
- Buktikan bahwa $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

$$\frac{z_1 = r_1(\cos\theta_1 + i\sin\theta_1)}{z_2 = r_2(\cos\theta_2 + i\sin\theta_2)}$$

DeMoivre's Formula

- $z^n = r^n[\cos(\theta + \theta + \cdots + \theta) + isin(\theta + \theta + \cdots + \theta)] = r^n(\cos n\theta + i \sin n\theta)$
- Jika $r = 1 \rightarrow (\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$
- $z = \underline{r(\cos\theta + i\sin\theta)} = r\underline{e^{i\theta}}$ $\quad re^{i\theta n} = r(\cos n\theta + i \sin n\theta)$
- $\bar{z} = re^{-i\theta} = r(\cos\theta - i\sin\theta)$
- $z_1 z_2 = r_1 e^{i\theta_1} r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1+\theta_2)} = r_1 r_2[\cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)]$
- $\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

$e^{i\pi} = \cos\pi - i\sin\pi = -1 + 0i = -1$

$e^{i\pi} = -1$

Euler Identity

- Deret Maclaurin $g(x) = \sum_{n=0}^{\infty} \frac{d^n f(0)}{dx^n} \frac{x^n}{n!}$ → polynomial → factorial

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!}$$

- Euler Identity:

$$\boxed{e^{i\pi} = -1}$$

$e^{i\begin{bmatrix} 2 & 3 \\ i & -1 \end{bmatrix}}$

- Eksponensial matriks:

$$e^{i\gamma H} = \sum_{n=0}^{\infty} \frac{(i\gamma H)^n}{n!}$$

- Dimana H adalah sebuah matriks

Complex Vector Spaces

vector

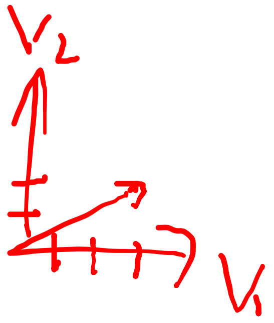
$\frac{-i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

- Complex vector didefinisikan:

$$\boldsymbol{w} = k_1\boldsymbol{v}_1 + k_2\boldsymbol{v}_2 + \cdots + k_r\boldsymbol{v}_r$$

- Dimana $k_1, k_2, \ldots, k_r$ adalah bilangan kompleks
- Sebuah vector $\boldsymbol{u} \in C^n$ dapat ditulis secara notasi vector:

$$\boldsymbol{u} = (u_1, u_2, \ldots, u_n)$$



- Atau notasi matriks:

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

3V1 + 2V2

(3, 2)

- Dimana:

$$u_1 = a_1 + b_1 i, \qquad u_2 = a_2 + b_2 i, \qquad \ldots, \qquad u_n = a_n + b_n i$$

Complex inner product

$|0\rangle$

$\begin{pmatrix} 1 \\ i \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix} = 1 + 1 = 2$

- Complex euclidean inner product didefinisikan:

$$\boldsymbol{u} \cdot \boldsymbol{v} = u_1\overline{v_1} + u_2\overline{v_2} + \cdots + u_n\overline{v_n}$$

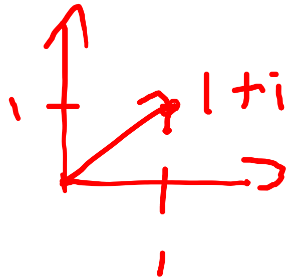
$\begin{pmatrix} 1 \\ i \end{pmatrix} \cdot \begin{pmatrix} 1 \\ i \end{pmatrix} = 1 - 1 = 0$

- Bandingkan dengan definisi:

$$\boldsymbol{u} \cdot \boldsymbol{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

- Identifikasi kelemahannya untuk kasus khusus $\boldsymbol{u} = (i, 1)$:

$$\boldsymbol{u} \cdot \boldsymbol{u} = 0$$



- Inner product dalam kuantum:

$$\langle v|u\rangle = \begin{pmatrix} v_1^* & \ldots & v_n^* \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} = v_1^*u_1 + \cdots + v_n^*u_n$$

- Dimana z^* adalah notasi kuantum untuk complex conjugate dari z

Uji Pemahaman

- Misalkan:
- $\boldsymbol{u} = (i, 1+i, -2)$ dan $\boldsymbol{v} = (2+i, 1-i, 3+2i)$
- $\boldsymbol{u} + \boldsymbol{v} =$ $(2+2i, 2, 1+2i)$
- $i\boldsymbol{u} =$ $(-1, i-1, -2i)$
- $\boldsymbol{u} \cdot \boldsymbol{v} =$ $(-i, 1-i, -2)\begin{pmatrix} 2+i \\ 1-i \\ 3+2i \end{pmatrix} = (-2i+1) + (-2i) + (-6-4i)$

$= -8i - 5$

$(1-i)(1-i) = 1 - i - i - 1$

Complex Outer Product

- Complex outer product didefinisikan:

$$\boldsymbol{u} \otimes \boldsymbol{v} = \begin{pmatrix} u_1 \overline{v_1} & \cdots & u_1 \overline{v_n} \\ \vdots & \ddots & \vdots \\ u_n \overline{v_1} & \cdots & u_n \overline{v_n} \end{pmatrix}$$

- Outer product dalam kuantum:

$$|u\rangle\langle v| = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \begin{pmatrix} v_1^* & \cdots & v_n^* \end{pmatrix} = \begin{pmatrix} u_1 v_1^* & \cdots & u_1 v_n^* \\ \vdots & \ddots & \vdots \\ u_n v_1^* & \cdots & u_n v_n^* \end{pmatrix}$$

Conjugate Transpose

dagger $\dagger$

$$\begin{pmatrix} -i & 1 \\ 2 & 3 \end{pmatrix}^{\dagger} = \begin{pmatrix} i & 2 \\ 1 & 3 \end{pmatrix}$$

- Conjugate transpose (Hermitian transpose):

$$A^{\dagger} = \bar{A}^{T}$$

- Sifat:
  - $\left(A^{\dagger}\right)^{\dagger} = A$
  - $(kA)^{\dagger} = \bar{k}A^{\dagger}$
  - $(A+B)^{\dagger} = A^{\dagger} + B^{\dagger}$
  - $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$

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Unitary Matrices

$AA^{-1} = I$

- Sebuah matriks riil disebut orthogonal jika:
$$A^{-1} = A^T$$
- Sebuah matriks kompleks disebut unitary jika:
$$A^{-1} = A^\dagger$$
- Sebuah matriks kompleks disebut Hermitian jika:
$$A = A^\dagger$$
- Sebuah matriks kompleks disebut normal jika:
$$AA^\dagger = A^\dagger A$$
- Berdasarkan definisi diatas, maka Hermitian matriks adalah normal, dan unitary matriks adalah normal

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^\dagger = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}^\dagger = \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

Uji Pemahaman

- Buktikan matriks Pauli-y $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ dan matriks Hadamard $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ adalah sebuah Hermitian

$$A = A^{\dagger}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

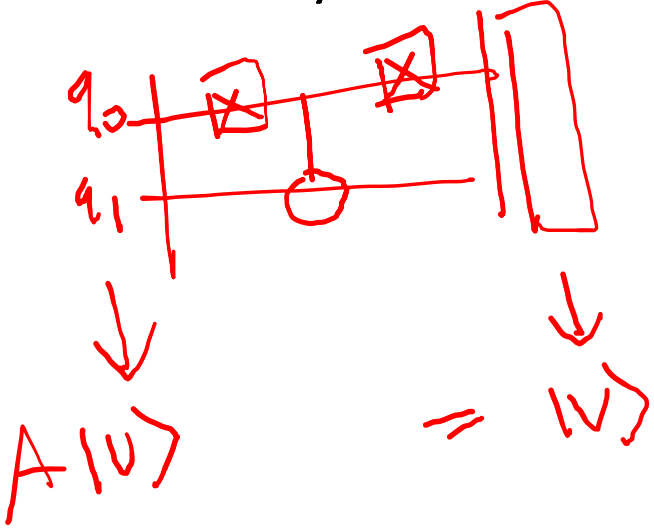
Linear Transformation

- Operasi matriks memproses suatu vector kompleks menjadi vektor kompleks lainnya

$$\boldsymbol{v} = A\boldsymbol{u}$$

- Unitary matriks memproses suatu wavefunction menjadi wavefunction lainnya

$$|v\rangle = A|u\rangle$$



$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$$

Eigenvalue dan Eigenvector

matriks

skalar

- Relasi dalam bentuk seperti ini:

$$\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}$$

Vektor

- Atau dalam notasi dirac:

$$A|v\rangle = \lambda|v\rangle$$
$$(A - I\lambda)|v\rangle = 0$$

- Dimana A adalah sebuah matriks dan λ adalah sebuah angka
- Solusi dari persamaan ini adalah:

$$\det(A - I\lambda) = 0$$

- Jika kita mempunyai relasi ini $A|v\rangle = \lambda|v\rangle$, maka relasi eksponensial eigen adalah:

$$\mathrm{e}^{A}|v\rangle = \sum_{n=0}^{\infty} \frac{(A)^n|v\rangle}{n!} = \sum_{n=0}^{\infty} \frac{\lambda^n|v\rangle}{n!} = e^{\lambda}|v\rangle$$

Uji Pemahaman

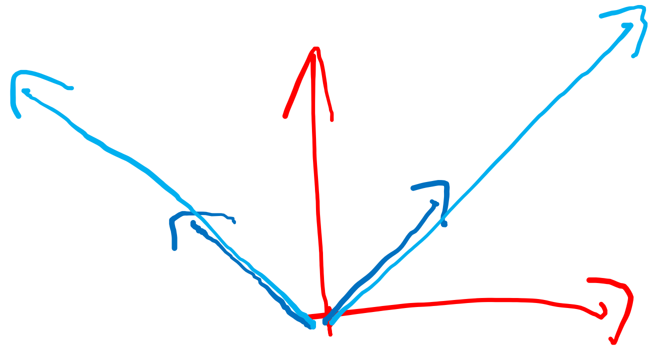
$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

- Temukan eigenvalue dan eigenvector dari matriks Pauli-z $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$AV = \lambda V$$

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right]\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0$$



$$-1-\lambda+\lambda+\lambda^2 = 0$$

$$\lambda^2 = 1 \rightarrow \lambda = 1 \vee -1$$

Summary

- Bits: 0101100
- Gates: operasi terhadap bits
- Algoritma: kombinasi beberapa gates
- $z = a + bi$
- $\bar{z} = a - bi$
- $z = r(cos\theta + isin\theta)$
- Vektor kompleks: vektor dengan bilangan kompleks
- Eigen value → observables (λ) dan eigen vector → ket ($|\psi\rangle$)

Tuhan Memberkati