Keterkaitan

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IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

Capaian Pembelajaran

- Composite System
- Entanglement
- Non-locality
- Multiqubit system

Composite System

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Composite System

- Jika Alice mempunyai photon dengan state $|0\rangle$ dan $|1\rangle$, maka wavefunctionnya dapat berbentuk superposisi:

$$|\Psi_A\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$
$$\alpha_0^*\alpha_0 + \alpha_1^*\alpha_1 = 1$$

- Lalu bob mempunyai photon dengan state $|0\rangle$ dan $|1\rangle$, maka wavefunctionnya dapat berbentuk superposisi:

$$|\Psi_B\rangle = \beta_0|0\rangle + \beta_1|1\rangle$$
$$\beta_0^*\beta_0 + \beta_1^*\beta_1 = 1$$

- Dapatkah kita mengkonstruksi single composite system dari keduanya?

Tensor product

- Composite wavefunction dari 2 sistem dapat direpresentasikan secara Tensor :

$$|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$
$$|\Psi_{AB}\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes (\beta_0|0\rangle + \beta_1|1\rangle)$$
$$|\Psi_{AB}\rangle = a_0\beta_0|0\rangle|0\rangle + a_0\beta_1|0\rangle|1\rangle + a_1\beta_0|1\rangle|0\rangle + a_1\beta_1|1\rangle|1\rangle$$
$$|\Psi_{AB}\rangle = a_0\beta_0|00\rangle + a_0\beta_1|01\rangle + a_1\beta_0|10\rangle + a_1\beta_1|11\rangle$$

- Vektor ket sekarang memiliki 2 indeks untuk merepresentasikan suatu state:

$$|\Psi_{AB}\rangle = \sum_{a,b} c(a,b)|ab\rangle$$

- Vektor ket $|ab\rangle$ bersifat orthonormal, dimana:

$$\langle ab|a'b'\rangle = \delta_{aa'}\delta_{bb'}$$

$\langle 0|1\rangle = \delta_{01} = 0$

$\delta_{aa'} = 1 \rightarrow a = a'$

$\phantom{\delta_{aa'} = {}} 0 \rightarrow a \neq a'$

$\langle 00|00\rangle = \delta_{00}\delta_{00} = 1$

$\langle 01|00\rangle = \delta_{00}\delta_{10} = 0$

$\langle 11|10\rangle = \delta_{11}\delta_{10} = 0$

Uji Pemahaman

- Apakah hasil tensor product dari $|\Psi_{ab}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- Berapa peluang mengukur 01 pada wavefunction diatas?

$$|\Psi_{ab}\rangle = \frac{1}{2}\left(|0\rangle|0\rangle + |0\rangle|1\rangle + i|1\rangle|0\rangle + i|1\rangle|1\rangle\right)$$

$$= \frac{1}{2}\left(\underbrace{|00\rangle + |01\rangle + i|10\rangle + i|11\rangle}_{\text{biner}}\right) = \frac{1}{2}\left(\underbrace{|0\rangle + |1\rangle + i|2\rangle + i|3\rangle}_{\text{desimal}}\right)$$

$$P(01) = \langle\Psi_{ab}|01\rangle\langle 01|\Psi_{ab}\rangle = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(00) = \langle\Psi_{ab}|00\rangle\langle 00|\Psi_{ab}\rangle = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Tensor Product Matrices

- Untuk menggabungkan 2 matriks 2x2 menjadi matriks 4x4:

$$A \otimes B = \begin{pmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & A_{12}B_{11} & A_{12}B_{12} \\ A_{11}B_{21} & A_{11}B_{22} & A_{12}B_{21} & A_{12}B_{22} \\ A_{21}B_{11} & A_{21}B_{12} & A_{22}B_{11} & A_{22}B_{12} \\ A_{21}B_{21} & A_{21}B_{22} & A_{22}B_{21} & A_{22}B_{22} \end{pmatrix}$$

- Untuk menggabungkan 2 vektor kolom 2x1 menjadi vektor kolom 4x1:

$$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \otimes \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} \\ a_{11}b_{21} \\ a_{21}b_{11} \\ a_{21}b_{21} \end{pmatrix}$$

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Uji Pemahaman

- Apakah tensor product dari $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ dan $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$?
- Apakah tensor product dari $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ dan $X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

$$|+\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

$$H \otimes X = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \otimes \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0 & 1\\1 & 0 & 1 & 0\\0 & 1 & 0 & -1\\1 & 0 & -1 & 0\end{pmatrix}$$

Tensor Product Basis

- Bentuk basis ket:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$|0\rangle \otimes |0\rangle$ $\quad |0\rangle \otimes |1\rangle$ $\quad |1\rangle \otimes |0\rangle$ $\quad |1\rangle \otimes |1\rangle$

- Bentuk komposit:

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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Tensor Product Operator

- Bentuk basis operator:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Bentuk komposit:

$$Z \otimes X = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$X \otimes Z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$Z \otimes X \neq X \otimes Z$$

Operator pada Basis ket

- Sebuah operator $Z \otimes X$ bekerja pada basis ket $|01\rangle$

$$(Z \otimes X)|01\rangle = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

$$(Z \otimes X)(|0\rangle \otimes |1\rangle)$$

$$Z|0\rangle = |0\rangle$$

$$X|1\rangle = |0\rangle$$

Uji Pemahaman

- Apakah output dari operasi $(Z \otimes X)|00\rangle$?

$$|0\rangle \otimes |1\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |01\rangle$$

$$Z|0\rangle = |0\rangle$$

$$X|0\rangle = |1\rangle$$

Entanglement

Entangled States

- Quantum mechanics memungkinkan superposisi vektor lebih dari sekedar tensor product
- Bentuk komposit yang paling general adalah:

$$|\Psi_{AB}\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

- Dimana kondisi normalisasinya:

$$c_{00}^{*}c_{00} + c_{01}^{*}c_{01} + c_{10}^{*}c_{10} + c_{11}^{*}c_{11} = 1$$

Maximally Entangled States

- Terdapat sistem yang lebih entangled dari sistem lain (entanglement bukanlah binari antara ya dan tidak)
- Sebuah sistem disebut maximally entangled states jika:

$$|\Psi_{AB}\rangle = c_{01}|01\rangle + c_{10}|10\rangle$$

- Dimana tidak mungkin dibentuk dari produk 2 qubit:

$$|\Psi_{AB}\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$(a|0\rangle + b|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) = a\alpha|00\rangle + a\beta|01\rangle + b\alpha|10\rangle + b\beta|11\rangle$$

$$C_{00} = 0 = a\alpha$$

$$C_{11} = 0 = b\beta$$

$a, b = 0, 0$ ✗

$\alpha, \beta = 0, 0$ ✗

$a, \beta = 0, 0$ ✗

$\alpha, b = 0, 0$ ✗

Keterkaitan

entangled

- Sebuah system dikatakan terkait jika observasi yang satu akan sekaligus menentukan hasil observasi yang lain
- Secara wavefunction dapat ditulis:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|1\rangle|0\rangle + |0\rangle|1\rangle\right)$$

Uji Pemahaman

- Apakah qubit pertama dan kedua dari wavefunction berikut entangled?
  - $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$
  - $|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle + \frac{1}{\sqrt{4}}|11\rangle$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \overset{?}{=} \left(a|0\rangle + b|1\rangle\right) \otimes \left(c|0\rangle + d|1\rangle\right) = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$

$ac = \frac{1}{\sqrt{2}}$ $\quad ad = 0$ $\quad bc = 0$ $\quad bd = \frac{1}{\sqrt{2}} \rightarrow$ entangled

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle + \frac{1}{\sqrt{4}}|11\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$

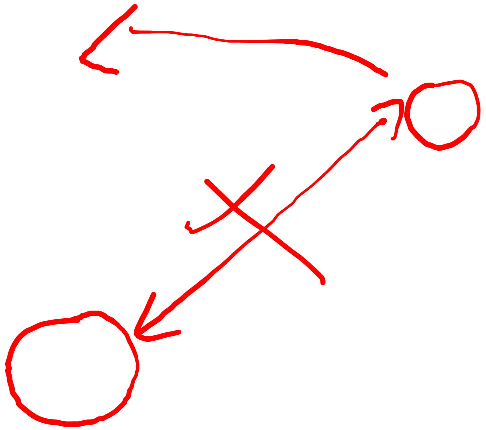
$ac = \frac{1}{\sqrt{2}}$ $\quad ad = \frac{1}{\sqrt{8}}$ $\quad bc = \frac{1}{\sqrt{8}}$ $\quad bd = \frac{1}{\sqrt{4}}$

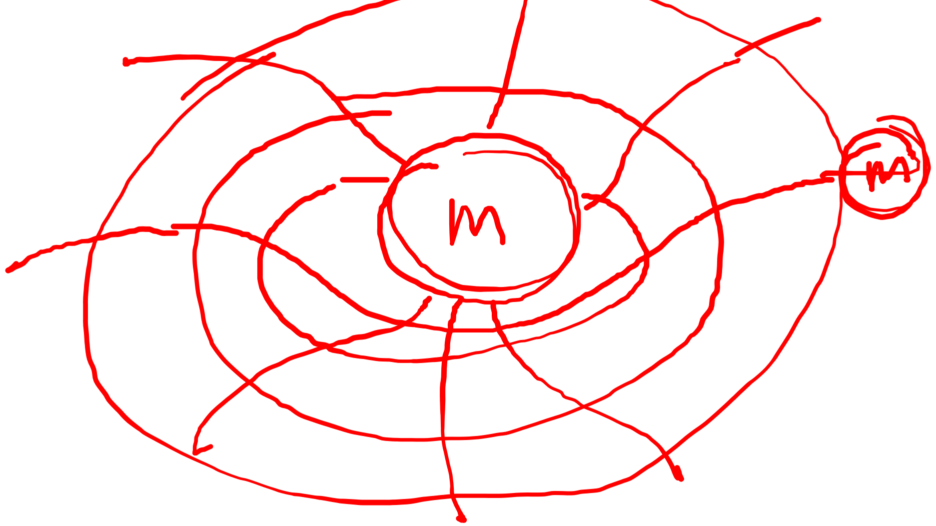
$\frac{\cancel{a}c}{\cancel{a}d} = \frac{1/\sqrt{2}}{1/\sqrt{8}} = 2$ $\qquad \frac{\cancel{b}c}{\cancel{b}d} = \frac{1/\sqrt{8}}{1/\sqrt{4}} = \frac{1}{\sqrt{2}} \rightarrow$ entangled

Non-Locality

Lokalitas

- Lokalitas adalah fondasi dari fisika klasik
- Lokal berarti suatu kejadian hanya dapat disebabkan oleh sesuatu yang berada di lokasi dan saat yang sama
- Gaya gravitasi Newton bersifat non-lokal dan medan gravitasi Einstein bersifat lokal

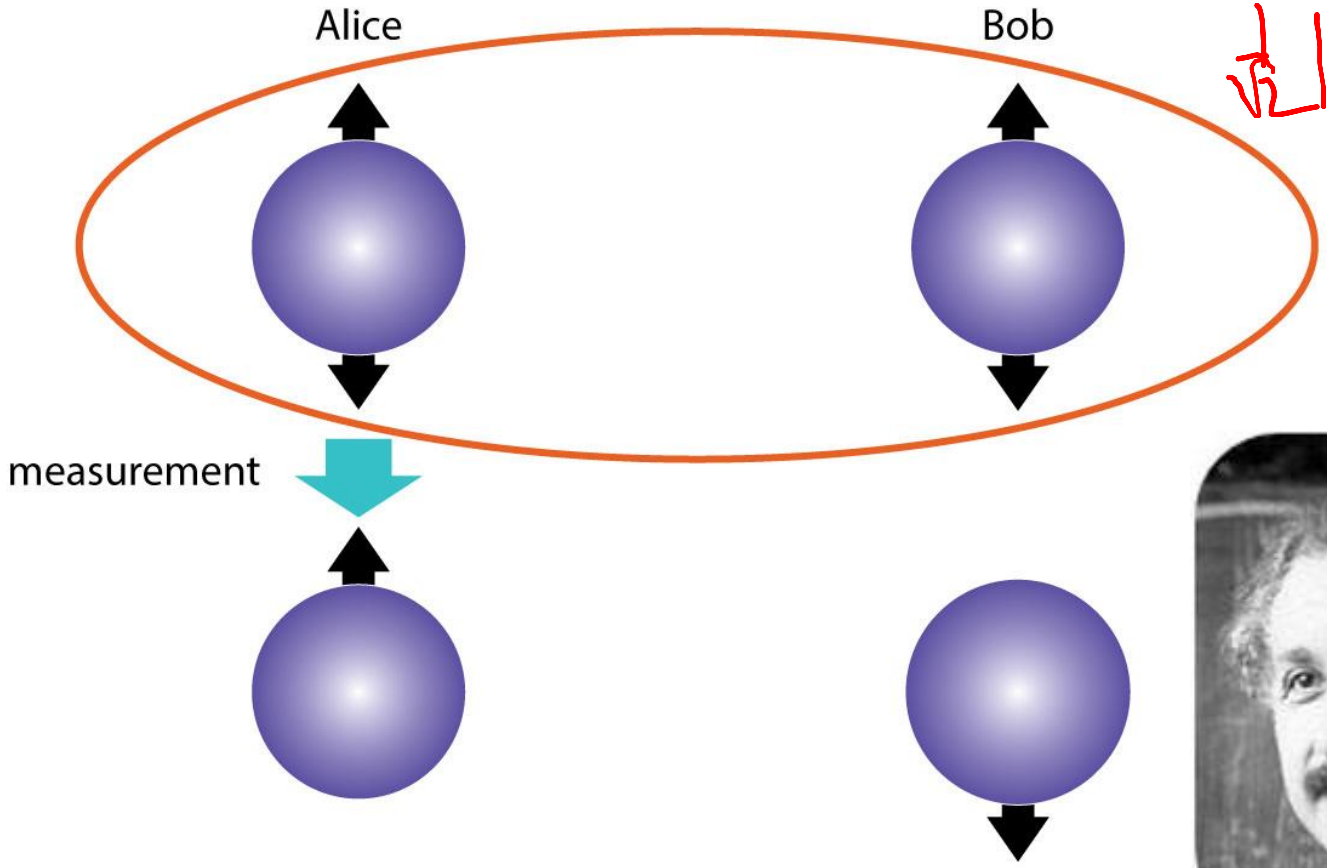




EPR Paradoks

- “Spooky Action at Distance”

Vodoo



$\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle$

qubit 1 qubit 2


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not ‘Complete’ Even Though ‘Correct.’

SEE FULLER ONE POSSIBLE

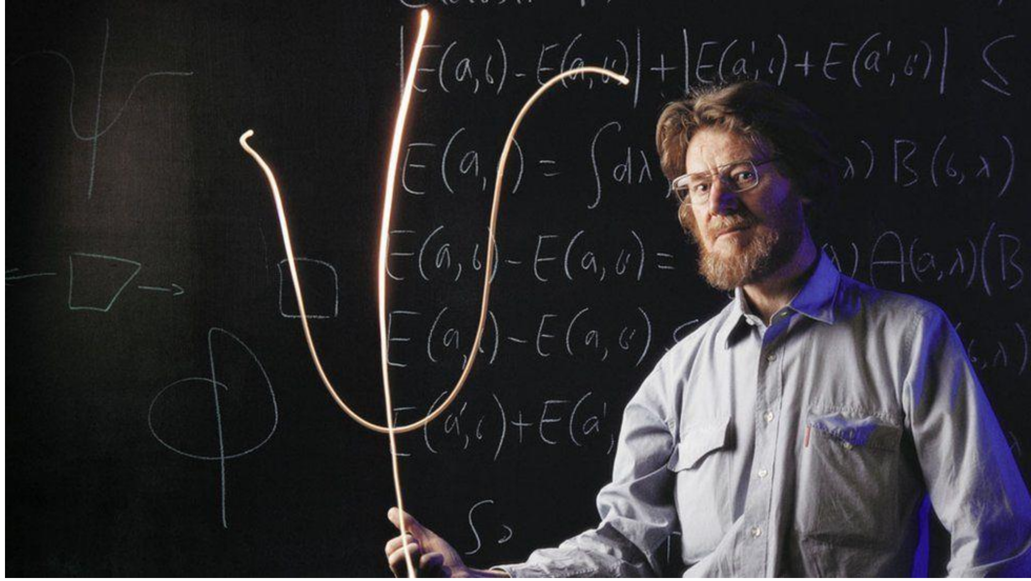
Believe a Whole Description of ‘the Physical Reality’ Can Be Provided Eventually.

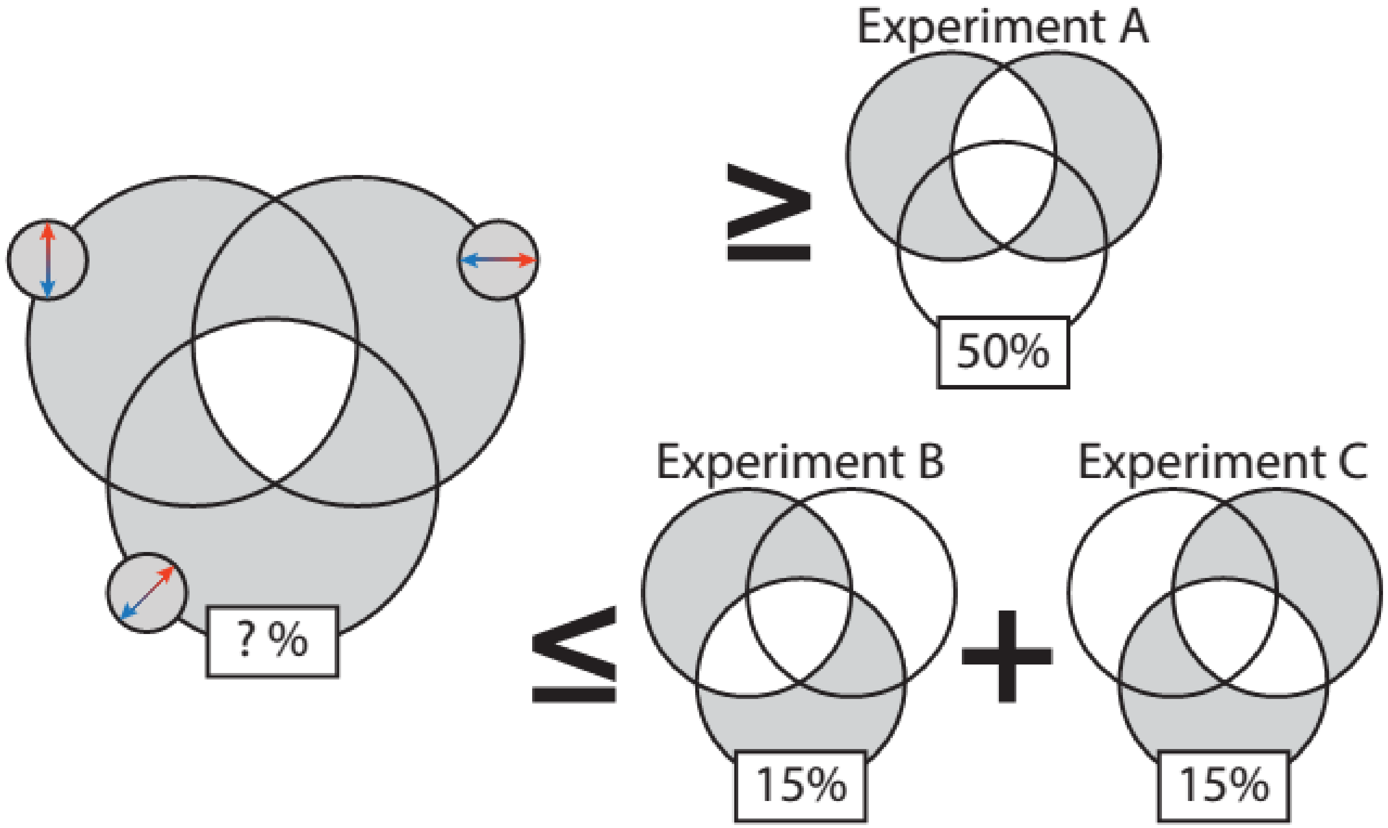
A. Einstein B. Podolsky N. Rosen

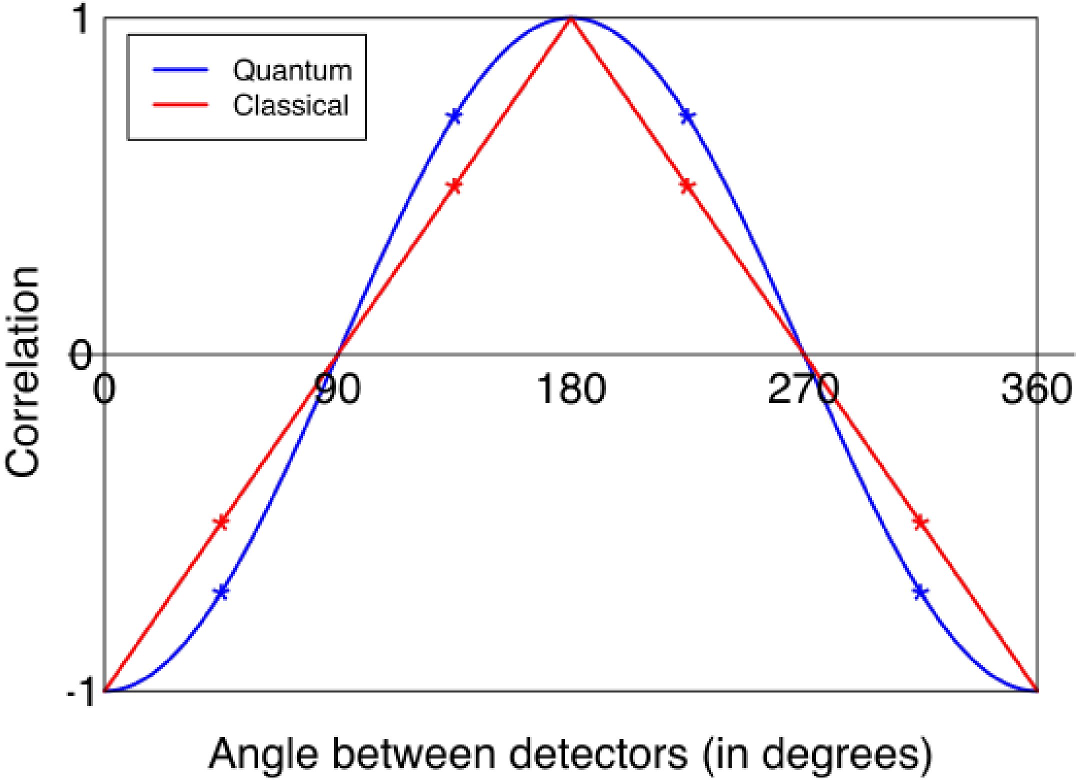
EPR Paradox

Bell Inequality

- https://www.youtube.com/watch?v=zcqZHYo7ONs

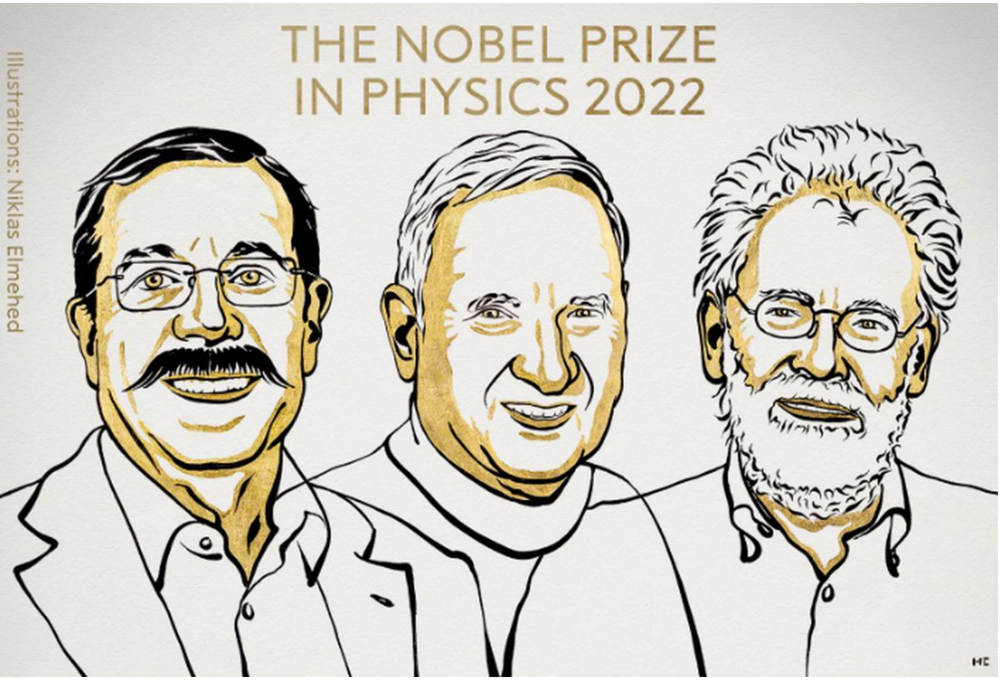


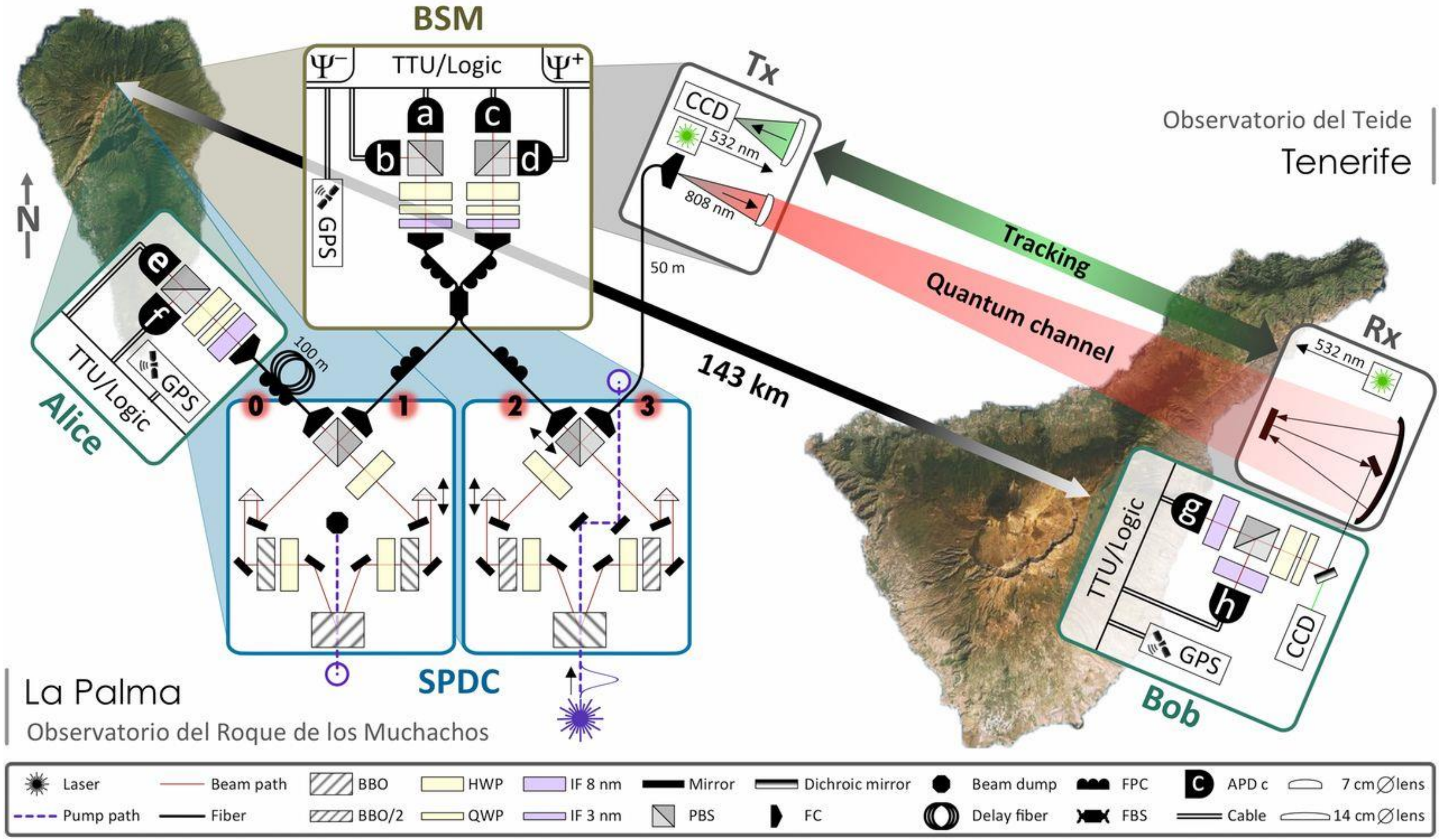




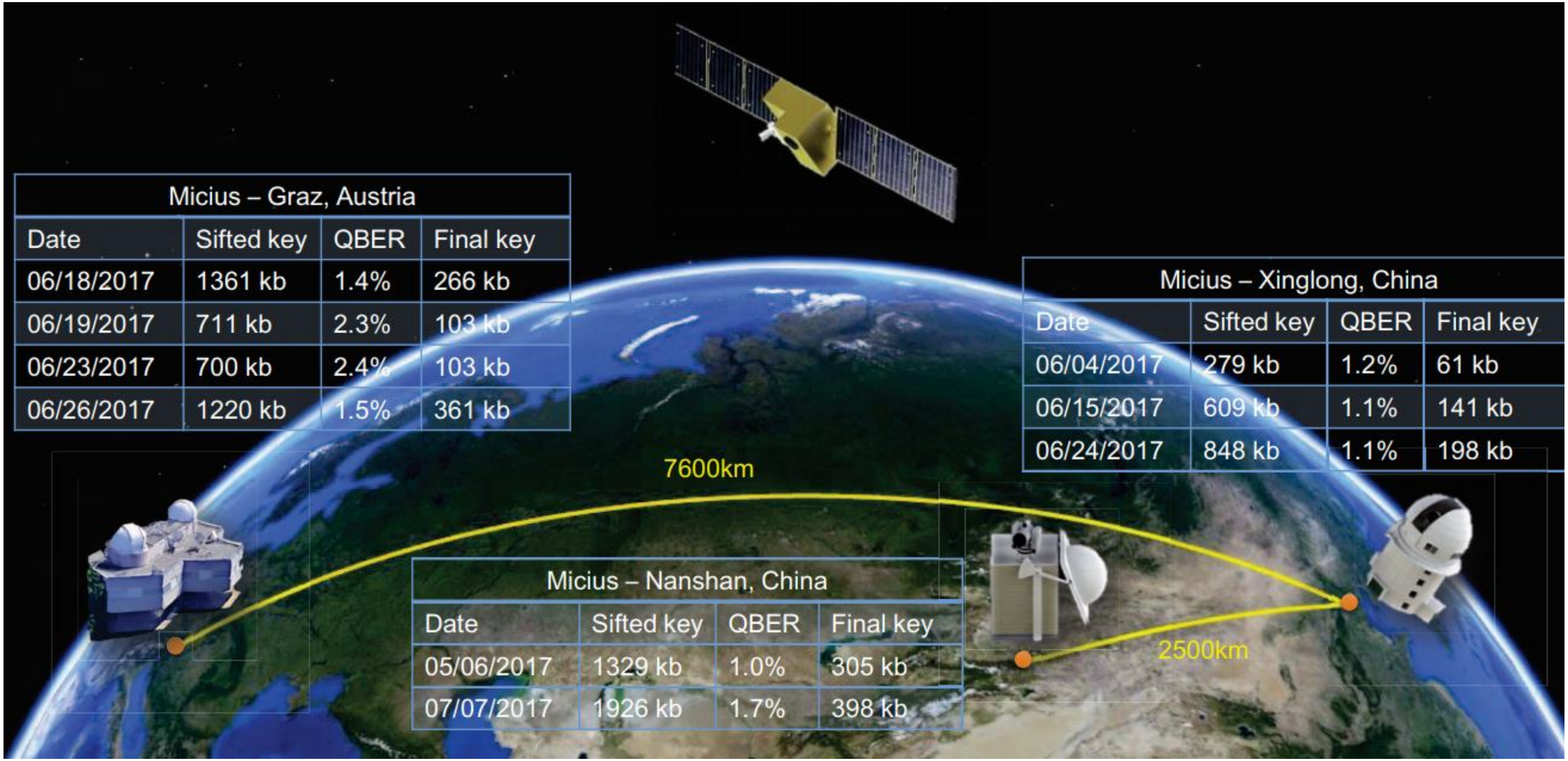
Demonstration of Entanglement

- https://www.pnas.org/doi/10.1073/pnas.1517007112





Longest Entanglement



Micius – Graz, Austria			
Date	Sifted key	QBER	Final key
06/18/2017	1361 kb	1.4%	266 kb
06/19/2017	711 kb	2.3%	103 kb
06/23/2017	700 kb	2.4%	103 kb
06/26/2017	1220 kb	1.5%	361 kb

Micius – Xinglong, China			
Date	Sifted key	QBER	Final key
06/04/2017	279 kb	1.2%	61 kb
06/15/2017	609 kb	1.1%	141 kb
06/24/2017	848 kb	1.1%	198 kb

Micius – Nanshan, China			
Date	Sifted key	QBER	Final key
05/06/2017	1329 kb	1.0%	305 kb
07/07/2017	1926 kb	1.7%	398 kb

Multiqubit System

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Single Qubit Gates on Multi-Qubit State

- Sekalipun kita menggunakan multi-qubit, tapi antar qubit bisa saja tidak berinteraksi (tiap operator hanya beroperasi pada qubit masing-masing secara independen)
- Contoh: Sebuah operator H beroperasi pada qubit q_0 dan operator X beroperasi pada qubit q_1

$$X|q_1\rangle \otimes H|q_0\rangle = (X \otimes H)|q_1 q_0\rangle$$

$X \otimes I$

- Operator kompositnya:

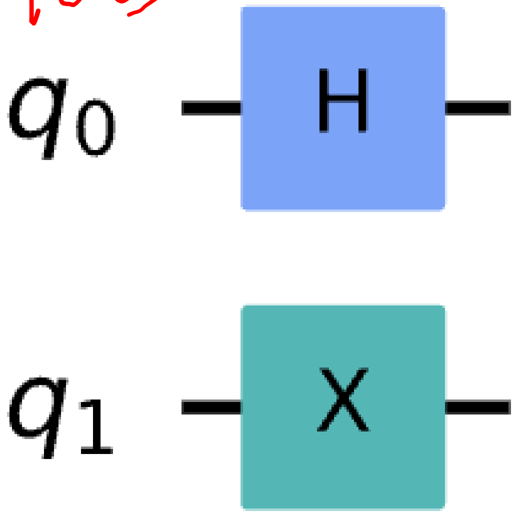
$$X \otimes H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

$|00\rangle$

- Misalkan semua qubit berada pada inisiasi awal, qubit kompositnya:

$$|q_1 q_0\rangle = |00\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$|00\rangle$



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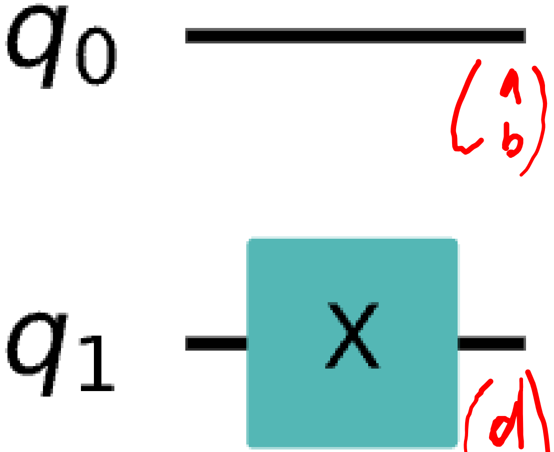
Uji Pemahaman

$X\,c|0\rangle + X\,d|1\rangle$

$c|1\rangle + d|0\rangle$

- Apakah bentuk komposit dari sirkuit kuantum berikut?
- Bandingkan state akhir bentuk komposit dan bentuk terpisah
  - Jika $|q_1 q_0\rangle = |00\rangle$ ✓
  - Jika $|q_0\rangle = a|0\rangle + b|1\rangle$ dan $|q_1\rangle = c|0\rangle + d|1\rangle$

$\begin{pmatrix} a \\ b \end{pmatrix}$  $\begin{pmatrix} a \\ b \end{pmatrix}$

$\begin{pmatrix} c \\ d \end{pmatrix}$ $\begin{pmatrix} d \\ c \end{pmatrix}$

$$X|q_1\rangle \otimes I|q_0\rangle = (X \otimes I)|q_1 q_0\rangle$$

$$|q_1\rangle \otimes |q_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$X \otimes I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$|q_1\rangle \otimes |q_0\rangle = \begin{pmatrix} c \\ d \end{pmatrix} \otimes \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ca \\ cb \\ da \\ db \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} ca \\ cb \\ da \\ db \end{pmatrix} = \begin{pmatrix} da \\ db \\ ca \\ cb \end{pmatrix} = \begin{pmatrix} d \\ c \end{pmatrix} \otimes \begin{pmatrix} a \\ b \end{pmatrix}$$

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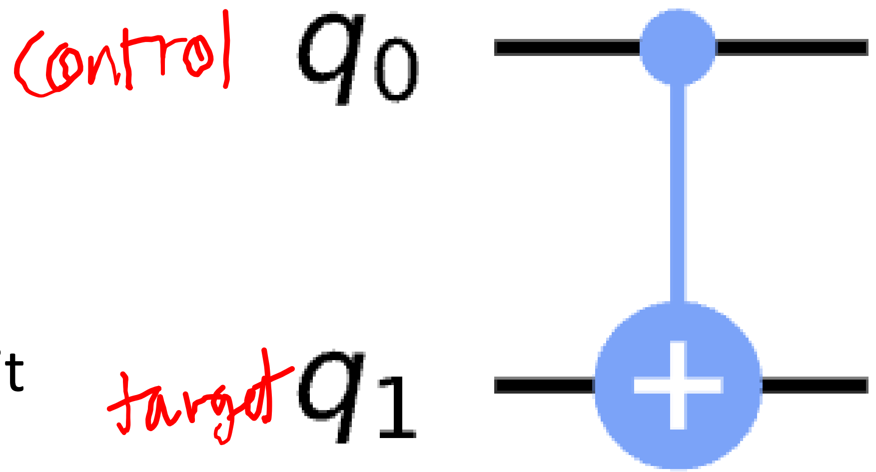
Multi-Qubit Gates

- Multiqubit gate adalah quantum gate yang beroperasi pada beberapa qubit sekaligus (terjadi interaksi antar qubit ketika operator ini digunakan)
- Contoh:
  - CNOT gate
  - CCNOT (Toffoli) gate
  - Swap gate
  - Cswap (Fredkin) gate

CNOT Gate



- Jika controlled qubit adalah 0, tidak merubah apa-apa
- Jika controlled qubit adalah 1, operasi X pada target qubit
- $CNOT = I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$
- $CNOT = (|0\rangle\langle 0| + |1\rangle\langle 1|) \otimes |0\rangle\langle 0| + (|0\rangle\langle 1| + |1\rangle\langle 0|) \otimes |1\rangle\langle 1|$
- $CNOT = |00\rangle\langle 00| + |10\rangle\langle 10| + |01\rangle\langle 11| + |11\rangle\langle 01|$
- $CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Input (t,c)	Output (t,c)
00	00
01	11
10	10
11	01

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$|00\rangle = |00\rangle$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$|01\rangle = |11\rangle$

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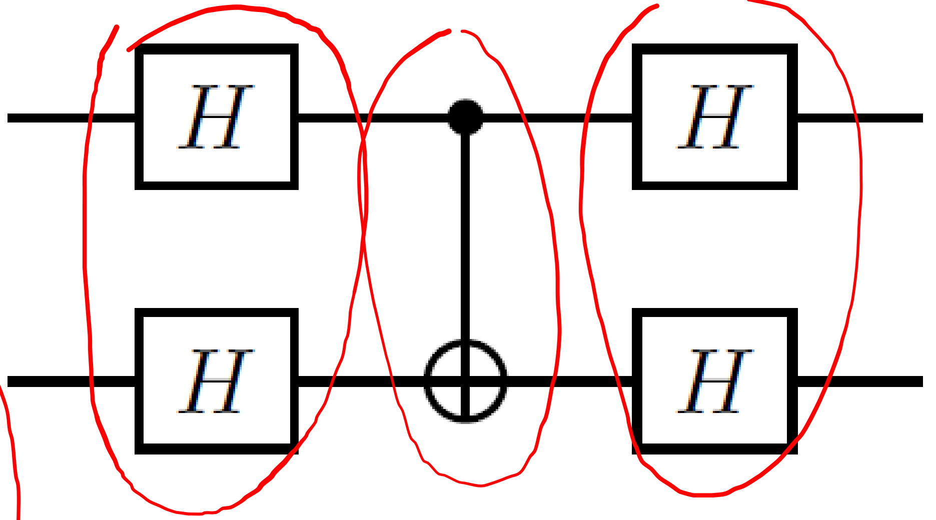
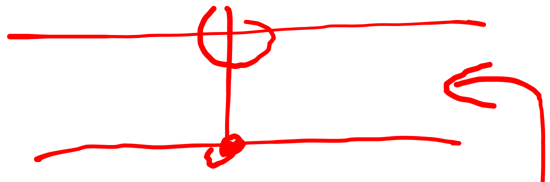
Uji Pemahaman

- Apakah hasil dari operasi berikut?



$$H\otimes H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}$$

$$CNOT = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix} = I\otimes|0\rangle\langle 0| + X\otimes|1\rangle\langle 1|$$



$$I\otimes|1\rangle\langle 1| + X\otimes|0\rangle\langle 0|$$

$$(H\otimes H)(CNOT)(H\otimes H) = \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\frac{1}{2} = \frac{1}{4}\begin{pmatrix}4 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 0 & 4\\ 0 & 0 & 4 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

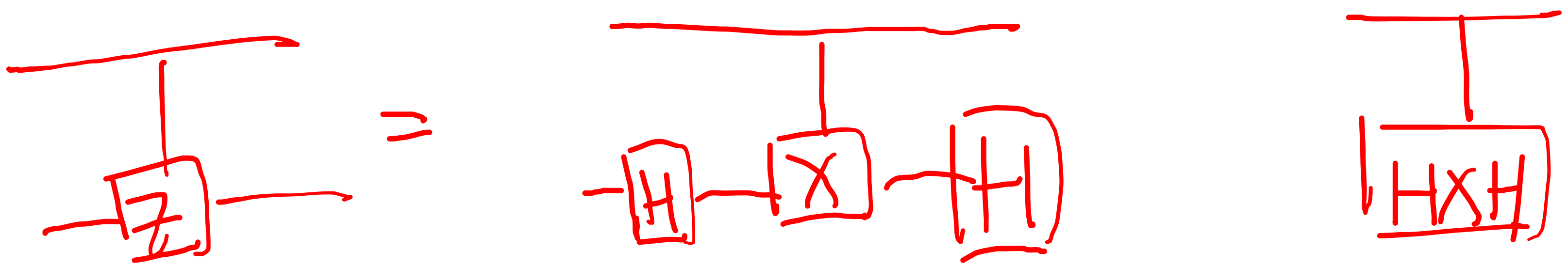
CU Gate

- Kita dapat mengontrol operasi gate apapun
- Jika controlled qubit adalah 0, tidak merubah apa-apa
- Jika controlled qubit adalah 1, operasi U pada target qubit
- $CU = I \otimes |0\rangle\langle 0| + U \otimes |1\rangle\langle 1|$
- Misalnya $CPhase = I \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$

Uji Pemahaman

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

- Buatlah Cphase gate dengan menggunakan CNOT gate



$$I\otimes|0\rangle\langle 0| + Z\otimes|1\rangle\langle 1| = (H\otimes I)\left(I\otimes|0\rangle\langle 0| + X\otimes|1\rangle\langle 1|\right)(H\otimes I)$$

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}\frac{1}{\sqrt{2}}$$

$$= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

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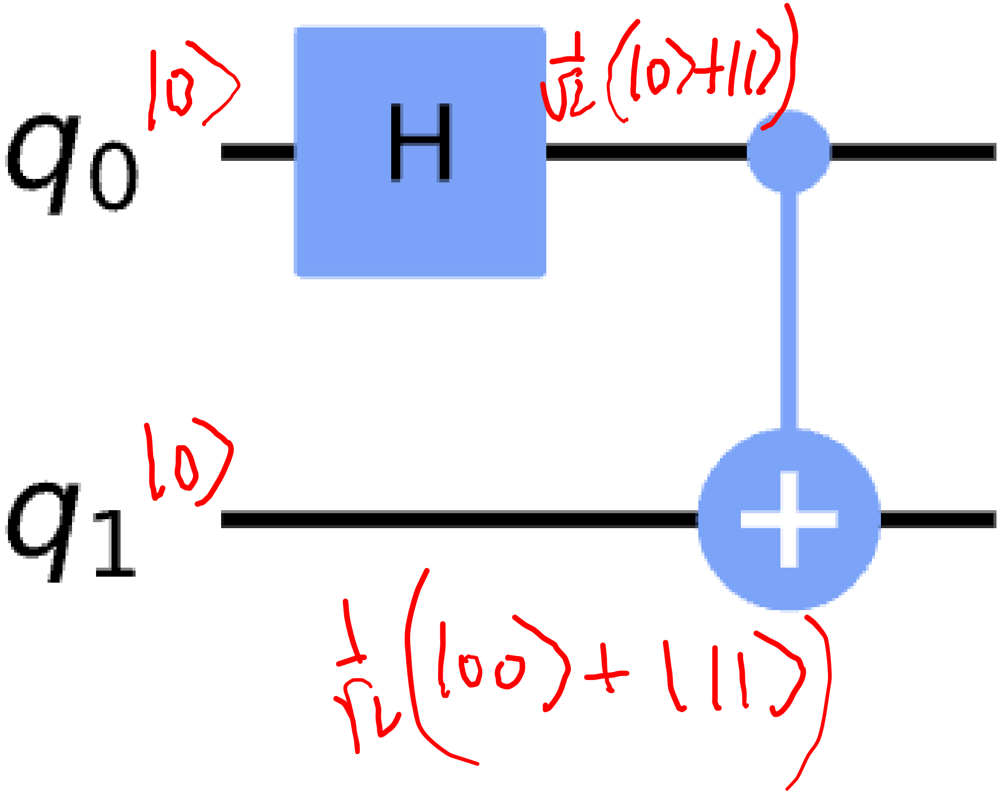
Entangled States

- $CNOT\,|0\,+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$|0\rangle \otimes |+\rangle$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$



Uji Pemahaman

- Berapa state akhir dari $CNOT(I \otimes H)|00\rangle)$?

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{|00\rangle} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{|0+\rangle}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \; \end{pmatrix} \cancel{\otimes} \begin{pmatrix} \; \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

entangled states

Bell States

$\{|0\rangle, |1\rangle\} \to \{|+\rangle, |-\rangle\}$

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\} \to \{|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle\}$

- Terdapat 4 bell states yang maximally entangled dan membentuk basis orthonormal:
- $|\phi^+\rangle = CNOT(I \otimes H)|00\rangle) = CNOT|0+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $|\phi^-\rangle = CNOT(I \otimes H)|01\rangle) = CNOT|0-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
- $|\psi^+\rangle = CNOT(I \otimes H)|10\rangle) = CNOT|1+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$
- $|\psi^-\rangle = CNOT(I \otimes H)|11\rangle) = CNOT|1-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$

$\langle\phi^-|\phi^+\rangle = \frac{1}{\sqrt{2}}(\langle 00| - \langle 11|)(|00\rangle + |11\rangle)\frac{1}{\sqrt{2}} = \frac{1}{2}(1+0+0-1) = 0$

$\langle\phi^+|\phi^+\rangle = \frac{1}{\sqrt{2}}(\langle 00| + \langle 11|)(|00\rangle + |11\rangle)\frac{1}{\sqrt{2}} = \frac{1}{2}(1+0+0+1) = 1$

$\langle\psi^+|\phi^-\rangle = \frac{1}{\sqrt{2}}(\langle 10| + \langle 01|)(|00\rangle - |11\rangle)\frac{1}{\sqrt{2}} = \frac{1}{2}(0+0+0+0) = 0$

Uji Pemahaman

- Apakah bentuk dari $\frac{1}{2}(|00\rangle + i|01\rangle + i|10\rangle + |11\rangle)$ dalam basis: $\{|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), |\psi^+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle), |\psi^-\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)\}$

$$\frac{1}{\sqrt{2}}\left(|\phi^+\rangle + i|\psi^+\rangle\right) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + i\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)\right)$$

$$= \frac{1}{2}\left(|00\rangle + |11\rangle + i|10\rangle + i|01\rangle\right)$$

Aktivitas

- Multiqubit system dan Entanglement di Qiskit
- https://learn.qiskit.org/course/ch-gates/multiple-qubits-and-entangled-states

Tuhan Memberkati

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