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God's People for God's Glory

Superposisi

Hendrik Santoso Sugiarto

IBDA4221 – Selected Topic in Computer Technology

Quantum Computing

Capaian Pembelajaran

- Superposisi
- Pengukuran
- Interpretasi
- Single Qubit Gate

Superposisi

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Superposisi

- Salah satu sifat wavefunction dalam persamaan schordinger adalah superposisi

$$H|\Psi\rangle = i\hbar \frac{d}{dt}|\Psi\rangle$$

- Jika $|\Psi_1\rangle$ dan $|\Psi_2\rangle$ adalah solusi dari persamaan schrodinger, maka $a|\Psi_1\rangle + b|\Psi_2\rangle$ dengan nilai a dan b berapapun juga adalah solusi
- $a|\Psi_1\rangle + b|\Psi_2\rangle$ adalah superposisi dari $|\Psi_1\rangle$ dan $|\Psi_2\rangle$

Uji Pemahaman

- Jika $|\Psi_1\rangle$ dan $|\Psi_2\rangle$ adalah solusi dari persamaan schrodinger $H|\Psi\rangle = i\hbar\frac{d}{dt}|\Psi\rangle$
- Buktikan bahwa $\boxed{a|\Psi_1\rangle + b|\Psi_2\rangle}$ juga adalah solusi dari persamaan schrodinger

$$|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle$$

$$H\left(a|\psi_1\rangle + b|\psi_2\rangle\right) = i\hbar\frac{d}{dt}\left(a|\psi_1\rangle + b|\psi_2\rangle\right)$$

$$aH|\psi_1\rangle + bH|\psi_2\rangle = a\,i\hbar\frac{d}{dt}|\psi_1\rangle + b\,i\hbar\frac{d}{dt}|\psi_2\rangle$$

$$a\underbrace{\left(H - i\hbar\frac{d}{dt}\right)}_{0}|\psi_1\rangle + b\underbrace{\left(H - i\hbar\frac{d}{dt}\right)}_{0}|\psi_2\rangle = 0$$

Superposisi bersifat relatif

- Misalkan $|\Psi_3\rangle$ dan $|\Psi_4\rangle$ merupakan superposisi dari $|\Psi_1\rangle$ dan $|\Psi_2\rangle$:

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle + |\Psi_2\rangle)$$

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle - |\Psi_2\rangle)$$

- Maka $|\Psi_1\rangle$ dan $|\Psi_2\rangle$ juga merupakan superposisi dari $|\Psi_3\rangle$ dan $|\Psi_4\rangle$:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|\Psi_3\rangle + |\Psi_4\rangle)$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|\Psi_3\rangle - |\Psi_4\rangle)$$

Uji Pemahaman

- Tunjukan bahwa $|0\rangle$ dan $|1\rangle$ juga merupakan superposisi dari $|+\rangle$ dan $|-\rangle$. Dimana$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ dan $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$|+\rangle + |-\rangle = \frac{1}{\sqrt{2}}(2|0\rangle + 0|1\rangle)$$

$$\frac{\sqrt{2}}{2}(|+\rangle + |-\rangle) = |0\rangle$$

$$\sqrt{2}\sqrt{2}$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|+\rangle - |-\rangle = \frac{1}{\sqrt{2}}(0|0\rangle + 2|1\rangle)$$

$$\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) = |1\rangle$$

Physical Meaning

- Suatu fenomena kuantum dapat direpresentasikan oleh persamaan berikut:

Mathematics: Hermitian Eigenvector Real Eigenvalue

$$H|a\rangle = \lambda|a\rangle$$

Physics: Observable State of the System Measurement

Eigenket

- Setiap ket apapun dapat direpresentasikan sebagai superposisi dari eigenket

$$H|\psi\rangle = H\sum_i |\psi_i\rangle = \sum_i \lambda_i |\psi_i\rangle$$

$$H|\psi\rangle = \lambda|\psi\rangle$$

$$= \lambda_1|\psi_1\rangle + \lambda_2|\psi_2\rangle + \lambda_3|\psi_3\rangle + \cdots$$

Uji Pemahaman

- Apakah hasil operasi dari X-gate ($X = |0\rangle\langle 1| + |1\rangle\langle 0|$) terhadap state:
  - $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\left(|1\rangle + |0\rangle\right) = |+\rangle$
  - $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ $\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(|1\rangle - |0\rangle\right) = -|-\rangle$
  - $|\psi\rangle = a|+\rangle + b|-\rangle$ $X|\psi\rangle = aX|+\rangle + bX|-\rangle = a|+\rangle - b|-\rangle$

$X|0\rangle = \left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)|0\rangle = |1\rangle$

$X|1\rangle = \left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)|1\rangle = |0\rangle$

$X|+\rangle = |+\rangle$

$X|-\rangle = -|-\rangle$

Pengukuran

Pengukuran

- Di dalam sistem kuantum:
  - Eigenvalue adalah nilai-nilai yang **mungkin** diperoleh ketika mengukur sebuah sistem (postulat 4)
  - Mustahil memprediksi hasil pengukuran suatu sistem → **Sebelum mengukur**, hanya bisa memprediksi peluangnya saja (postulat 5)
- Physical quantity A → Observable A:

yg diukur

$$A|u_n\rangle = \lambda_n|u_n\rangle$$

$$P(\lambda_n) = \frac{|\langle u_n|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}$$

Peluang

- Untuk normalized wavefunction ($\langle\Psi|\Psi\rangle = 1$) → $P(\lambda_n) = |\langle u_n|\Psi\rangle|^2$
- Jika kita mengukur variable A, maka kita akan memperoleh nilai $\lambda_1, \lambda_2, \ldots$ dengan peluang $P(\lambda_1), P(\lambda_2), \ldots$

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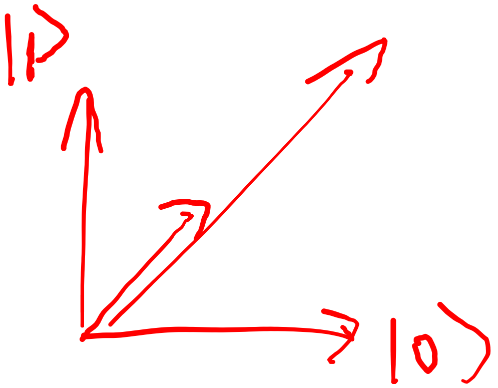
Perubahan Fase

scala fase

$e^{-i\theta} e^{i\theta}$

$e^{(i\theta - i\theta)} = e^0 = 1$

- Peluang untuk mengukur λ_m pada ket $|\Psi\rangle$ adalah $P_\psi(\lambda_m) = \frac{|\langle u_m|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}$
- Peluang untuk mengukur λ_m pada ket $|\Psi'\rangle = re^{i\theta}|\Psi\rangle$ adalah:
- $P_{\Psi'}(\lambda_m) = \frac{|\langle u_m|\Psi'\rangle|^2}{\langle\Psi'|\Psi'\rangle} = \frac{|\langle u_m|re^{i\theta}|\Psi\rangle|^2}{\langle\Psi|re^{-i\theta}re^{i\theta}|\Psi\rangle} = \frac{r^2|\langle u_m|\Psi\rangle|^2}{r^2\langle\Psi|\Psi\rangle}$
- $|\Psi\rangle$ dan $|\Psi'\rangle$ menghasilkan pengukuran yang sama
- Pengukuran bersifat independent terhadap scaling and phase change pada ket



State Collapse

- Sebuah operator dan sistem memenuhi relasi eigen:

$$A|u_n\rangle = \lambda_n|u_n\rangle$$

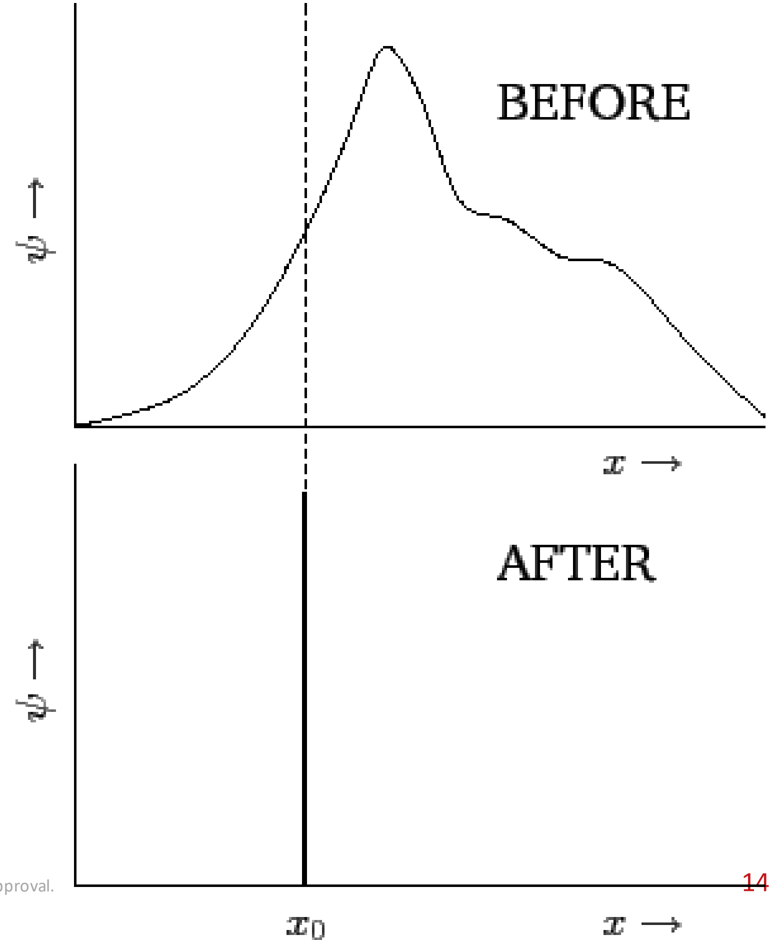
- Jika pengukuran A pada state $|\Psi\rangle$ menghasilkan eigenvalue λ_n maka state dari sistem tersebut setelah diukur akan berubah ke $|u_n\rangle$
- Kegiatan mengukur merubah state dari sebuah system:

$$|\Psi\rangle \xrightarrow{A:\lambda_n} |u_n\rangle$$

$$|\psi\rangle = \lambda_1|u_1\rangle + \lambda_2|u_2\rangle + \lambda_3|u_3\rangle + \cdots$$

$$\downarrow$$

$$|\psi\rangle = 1\,|u_3\rangle$$



Uji Pemahaman

- Berapa peluang mengukur 1 pada state $|\psi\rangle = |0\rangle$?
- Berapa peluang mengukur 1 jika kita mengoperasikan Hadamard terlebih dahulu pada state $|\psi\rangle = |0\rangle$?

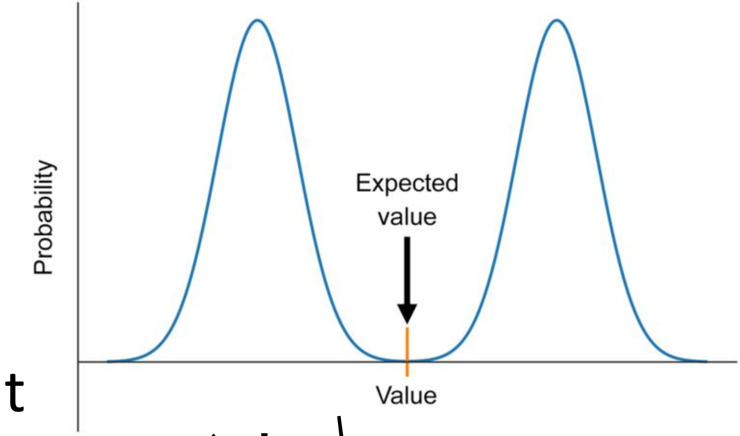
$$P(1) = |\langle 1|\psi\rangle|^2 = \langle 1|\psi\rangle\langle\psi|1\rangle = \langle 1|0\rangle\langle 0|1\rangle = 0$$

$$|\psi'\rangle = H|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$P(1) = |\langle 1|\psi'\rangle|^2 = \langle 1|\psi'\rangle\langle\psi'|1\rangle = \langle 1|\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)|1\rangle$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

Expectation Values



- Expectation value ≠ measurement outcome
- Expectation values → Rata-rata dari N pengukuran independent

$$\langle\Psi|A|\Psi\rangle = \langle\Psi|IAI|\Psi\rangle = \left\langle\Psi\left|\left(\sum_n |u_n\rangle\langle u_n|\right)A\left(\sum_m |u_m\rangle\langle u_m|\right)\right|\Psi\right\rangle$$

$$= \sum_{n,m}\langle\Psi|u_n\rangle\langle u_n|\mathrm{A}|u_m\rangle\langle u_m|\Psi\rangle = \sum_{n,m}\langle\Psi|u_n\rangle\langle u_n|\underline{\lambda_m}|u_m\rangle\langle u_m|\Psi\rangle$$

λ_m $\quad \delta_{nm}$

$$= \sum_{n,m}\lambda_m\langle\Psi|u_n\rangle\delta_{nm}\langle u_n|\Psi\rangle = \sum_n \lambda_n\,|\langle u_n|\Psi\rangle|^2 = \sum_n \lambda_n\,P(\lambda_n)$$

- Expectation values → weighted mean

$$\delta_{nm} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

$\langle 0|0\rangle = 1$

$\langle 0|1\rangle = 0$

Mean Square Deviation

$$\langle A^2 \rangle$$

$$\langle 2A\langle A\rangle\rangle = 2\langle A\rangle^2$$

$$2\langle A\rangle\langle A\rangle = 2\langle A\rangle^2$$

- How far from the average:

$$\sigma_A = A - \langle A \rangle$$

- Mean square deviation:

$$\langle \sigma_A^2 \rangle = \langle (A - \langle A \rangle)^2 \rangle$$

$$\langle \sigma_A^2 \rangle = \langle A^2 + \langle A \rangle^2 - 2A\langle A \rangle \rangle = \langle A^2 \rangle - \langle A \rangle^2$$

- Root mean square deviation:

$$\Delta A = \sqrt{\langle \sigma_A^2 \rangle} = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$

Uji Pemahaman

$$|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- Operator $M_0 = |0\rangle\langle 0|$ mengukur output dari $H|0\rangle$, dimana operator Hadamard $H = \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$. Apakah nilai dari $\langle M_o\rangle$ dan $\langle M_0^2\rangle$?

$$\langle M_0\rangle = \langle\psi|M_0|\psi\rangle = \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\,|0\rangle\langle 0|\,\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}(1+0)(1+0) = \frac{1}{2}$$

$$M_0^2 = |0\rangle\langle 0|\,|0\rangle\langle 0| = |0\rangle\langle 0| = M_0$$

$$\langle M_0^2\rangle = \frac{1}{2}$$

$$\Delta M_0 = \sqrt{\langle M_0^2\rangle - \langle M_0\rangle^2} = \sqrt{\frac{1}{2} - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2} - \frac{1}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Relasi 2 Operator

$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|\psi\rangle = 1|+\rangle + 0|-\rangle$

- Pengukuran 2 operator berbeda terhadap $|\Psi\rangle$:

$$A|u_n\rangle = \lambda_n|u_n\rangle, \qquad |\Psi\rangle = \sum_n c_n|u_n\rangle, \qquad c_n = \langle u_n|\Psi\rangle$$

$$B|v_m\rangle = \mu_m|v_m\rangle, \qquad |\Psi\rangle = \sum_n d_m|v_m\rangle, \qquad d_m = \langle v_m|\Psi\rangle$$

- Deviation Operator:

$$|\Psi_A\rangle = \sigma_A|\Psi\rangle \to \langle\Psi_A|\Psi_A\rangle = \langle\Psi|\sigma_A^\dagger\sigma_A|\Psi\rangle = \langle\sigma_A^2\rangle$$
$$|\Psi_B\rangle = \sigma_B|\Psi\rangle \to \langle\Psi_B|\Psi_B\rangle = \langle\Psi|\sigma_B^\dagger\sigma_B|\Psi\rangle = \langle\sigma_B^2\rangle$$
$$\langle\Psi_A|\Psi_B\rangle = \langle\sigma_A\sigma_B\rangle$$

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Heisenberg Uncertainty Principle Revisited

- Relasi komutator operator deviasi:

$$[\sigma_A, \sigma_B] = \sigma_A\sigma_B - \sigma_B\sigma_A = (A - \langle A \rangle)(B - \langle B \rangle) - (B - \langle B \rangle)(A - \langle A \rangle) = AB - BA$$
$$= [A, B]$$

- Komutator+antikomutator:

$$[\sigma_A, \sigma_B] + \{\sigma_A, \sigma_B\} = \sigma_A\sigma_B - \sigma_B\sigma_A + \sigma_A\sigma_B + \sigma_B\sigma_A = 2\sigma_A\sigma_B$$

$$\sigma_A\sigma_B = \frac{1}{2}[\sigma_A, \sigma_B] + \frac{1}{2}\{\sigma_A, \sigma_B\} = \frac{1}{2}[A, B] + \frac{1}{2}\{\sigma_A, \sigma_B\}$$

$$|\langle \sigma_A\sigma_B \rangle|^2 = \left| \underbrace{\frac{1}{2}\langle [A, B] \rangle}_{\text{ekspektasi komutator}} + \underbrace{\frac{1}{2}\langle \{\sigma_A, \sigma_B\} \rangle}_{\text{ekspektasi anti-komutator}} \right|^2$$

ekspektasi komutator

ekspektasi anti-komutator

Heisenberg Uncertainty Principle

- Schwars inequality:

$$\langle \Psi_A | \Psi_A \rangle \langle \Psi_B | \Psi_B \rangle \geq |\langle \Psi_A | \Psi_B \rangle|^2$$
$$\langle \sigma_A^2 \rangle \langle \sigma_B^2 \rangle \geq |\langle \sigma_A \sigma_B \rangle|^2$$
$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle + \langle \{\sigma_A, \sigma_B\} \rangle|$$

- Imaginary & real component:

$$[A, B]^\dagger = -[A, B] \rightarrow antiHermitian \rightarrow imaginary$$
$$\{A, B\}^\dagger = \{A, B\} \rightarrow Hermitian \rightarrow real\ (+ve\ or\ 0)$$

- Heisenberg uncertainty principle:

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

Position-Momentum Uncertainty

- Heisenberg Uncertainty Principle:

$$\Delta A \Delta B \geq \frac{1}{2}|\langle[A,B]\rangle|$$

- Position-momentum uncertainty:

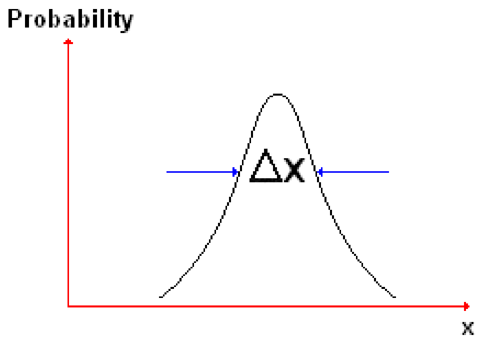
$$\Delta \mathrm{x} \Delta p \geq \frac{1}{2}|\langle[x,p]\rangle| = \frac{1}{2}|\langle i\hbar\rangle| = \frac{\hbar}{2}$$

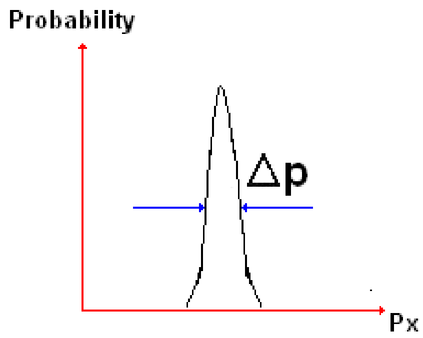
- Position projection on $|\Psi\rangle$:

$$\psi(x) = \langle x|\Psi\rangle$$

- Momentum projection on $|\Psi\rangle$:

$$\phi(p) = \langle p|\Psi\rangle$$





- Fourier transform:

$$\phi(p) = f\big(\psi(x)\big) \leftrightarrow \psi(x) = f(\phi(p))$$

Uji Pemahaman

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

- Apakah kita dapat mengukur qubit dengan $M_0 = |0\rangle\langle 0|$ dan $M_+ = |+\rangle\langle +|$ sekaligus?

komutator

$$M_0 M_+ - M_+ M_0 = |0\rangle\langle 0|+\rangle\langle +| - |+\rangle\langle +|0\rangle\langle 0|$$
$$= \frac{1}{\sqrt{2}}|0\rangle\langle +| - |+\rangle\frac{1}{\sqrt{2}}\langle 0|$$
$$= \frac{1}{\sqrt{2}}\left(|0\rangle\langle +| - |+\rangle\langle 0|\right)$$
$$= \frac{1}{2}\left(\cancel{|0\rangle\langle 0|} + |0\rangle\langle 1| - \cancel{|0\rangle\langle 0|} - |1\rangle\langle 0|\right)$$
$$= \frac{1}{2}\left(|0\rangle\langle 1| - |1\rangle\langle 0|\right)$$

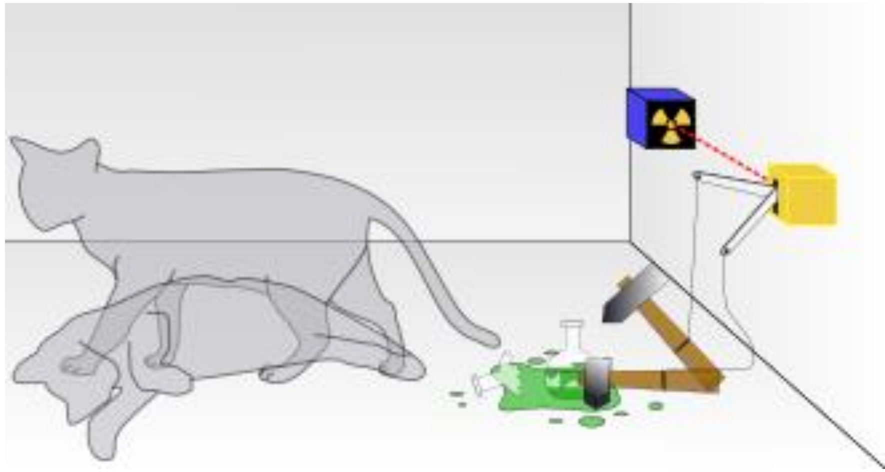
tidak komute!

Interpretasi

Schrodinger Cat

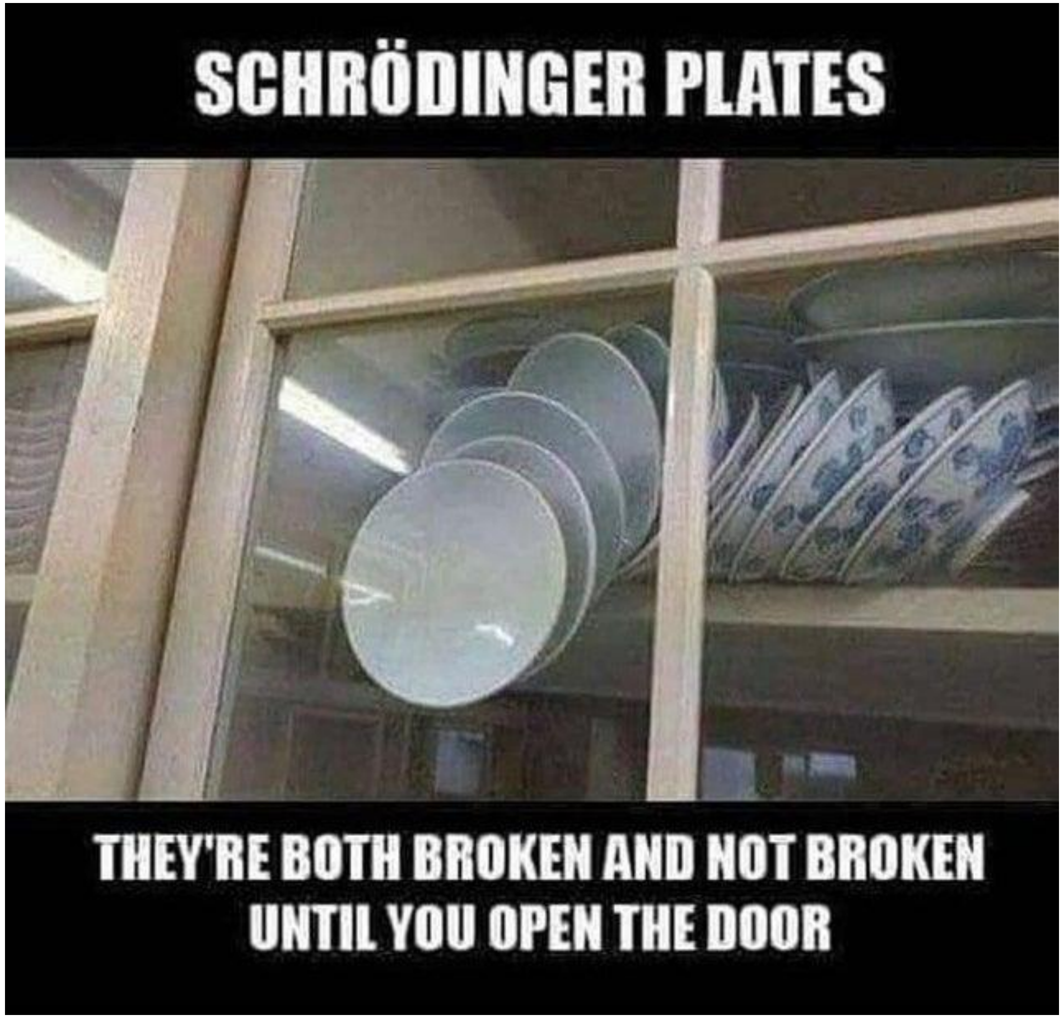
- Tahun 1035 Schrodinger membuat sebuah thought experiment yang menggambarkan masalah dari interpretasi kopenhagen:
    - Seekor kucing di ruang baja yang terkunci yang berisi atom radioaktif yang dapat mengaktifkan suatu mekanisme pelepasan asam beracun
    - Proses peluruhan radioaktif mengikuti probabilitas kuantum
    - Penafsiran Kopenhagen: *kucing itu dalam keadaan hidup dan mati* sampai peristiwa itu telah diamati.
- Wavefunction dari kucing tersebut adalah:

$$|\Psi\rangle = a|dead\rangle + b|alive\rangle$$



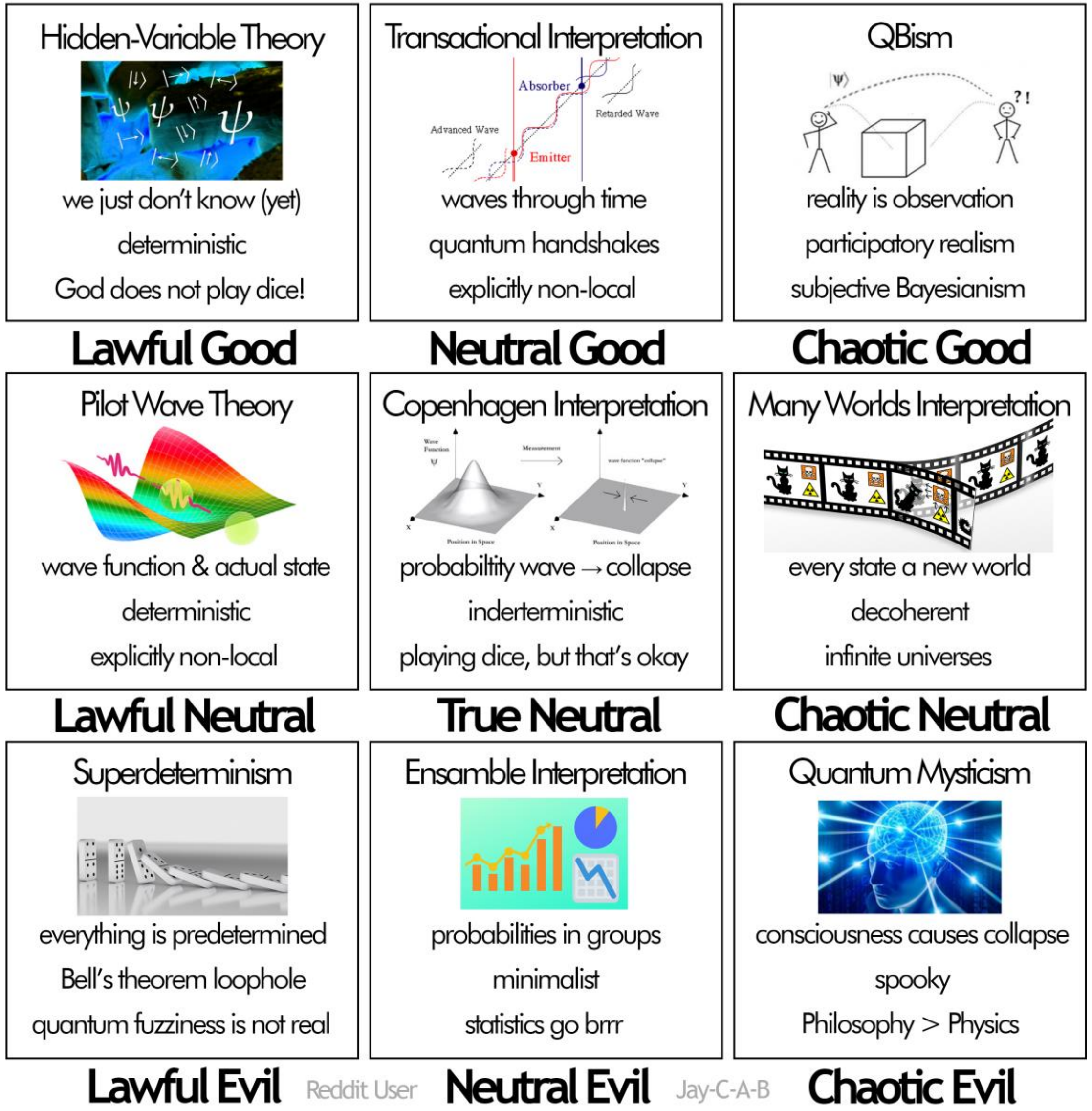
Apakah teori kuantum meniadakan realita objektif?

- Apakah kucing tersebut benar-benar mati sekaligus hidup?



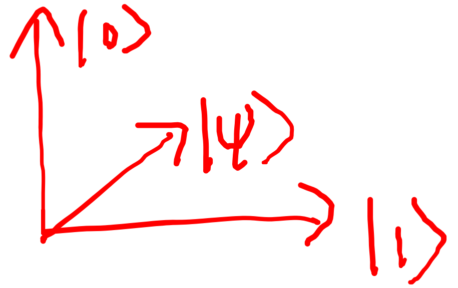
Philosophical Interpretation

- Copenhagen Interpretation Mayoritas
- Consciousness interpretation
- Pilot-wave interpretation objective
- Many world interpretation
- Superdeterminism
- Etc



Single Qubit Gate

Qubit



- Unit dalam pemrosesan informasi kuantum:
  - Sebelum mengukur, kita memiliki qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  - Setelah mengukur, kita memiliki statistik pengukuran yang diukur dalam bit (0 atau 1)
- Untuk dapat melakukan komputasi dalam bit, kita memerlukan sistem kuantum dalam ruang Hilbert 2 dimensi
- State qubit dapat ditulis sebagai: $|\psi\rangle \in \mathcal{H}_2 = span\{|0\rangle, |1\rangle\}$
- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,\ dimana\ \alpha, \beta \in \overset{\text{kompleks}}{\mathbb{C}},\ \ dengan\ basis\ |0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$
- Operator untuk mengukur 0: $M_0 = |0\rangle\langle 0|$, mengukur 1: $M_1 = |1\rangle\langle 1|$
- Peluang mengukur 0: $\langle\psi|M_0|\psi\rangle = |\alpha|^2$, Peluang mengukur 1: $\langle\psi|M_1|\psi\rangle = |\beta|^2$

$\langle\psi|0\rangle\langle 0|\psi\rangle = |\langle\psi|0\rangle|^2 = \alpha\alpha^*$ $\qquad$ $\langle\psi|1\rangle\langle 1|\psi\rangle = |\langle\psi|1\rangle|^2 = \beta\beta^*$

Sifat Qubit

$\alpha = a + bi = Re^{i\phi}$

- Qubit adalah benda kuantum yang memiliki superposisi state 0 dan 1:
- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle)$, dimana $\alpha, \beta \in \mathbb{C}$ (kompleks)
- Kita dapat menggunakan parameter real dengan relative phase:
- $|\psi\rangle = e^{i\phi_1}\alpha|0\rangle + e^{i\phi_2}\beta|1\rangle)$, dimana $\alpha, \beta, \phi_1, \phi_2 \in \mathbb{R}$

$$\frac{\alpha \cancel{e^{i\phi_1}}|0\rangle + \beta e^{i\phi_2}|1\rangle}{\cancel{e^{i\phi_1}}}$$

- $|\psi\rangle = \alpha|0\rangle + e^{i\phi}\beta|1\rangle)$, dimana $\phi = \phi_1 - \phi_2 \in \mathbb{R}$
- Qubit ternormalisasi: $\boxed{1 = \langle\psi|\psi\rangle} = |\alpha|^2\langle 0|0\rangle + |\beta|^2 e^{-i\phi}e^{i\phi}\langle 1|1\rangle = \underbrace{|\alpha|^2 + |\beta|^2}_{\cos^2\theta + \sin^2\theta = 1} = 1$
- $\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle,}$ dimana $\phi, \theta \in \mathbb{R}$
- Ketika diukur, qubit akan collapse ke salah satu pilihan state

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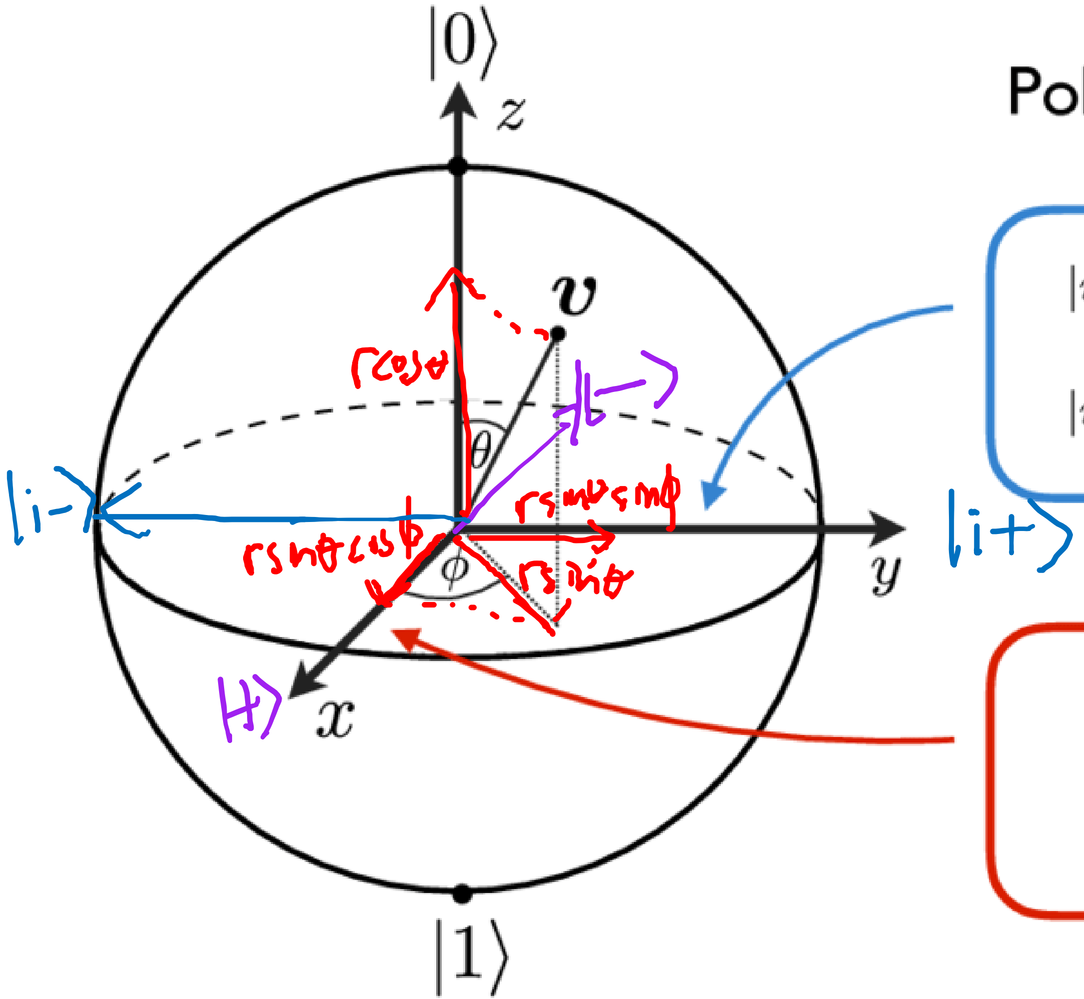
$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$

Bloch Sphere

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

- Qubit dapat direpresentasikan oleh bloch sphere: $r = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$
- $|0\rangle: \theta = 0 \rightarrow r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
- $|1\rangle: \theta = \pi \rightarrow r = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$
- $|+\rangle: \theta = \frac{\pi}{2}, \phi = 0 \rightarrow r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
- $|-\rangle: \theta = \frac{\pi}{2}, \phi = \pi \rightarrow r = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
- $|i+\rangle: \theta = \frac{\pi}{2}, \phi = \frac{\pi}{2} \rightarrow r = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
- $|i-\rangle: \theta = \frac{\pi}{2}, \phi = \frac{3\pi}{2} \rightarrow r = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$



Pole states:

$$|i+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|i-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

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Hadamard Gate

- $H = \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$
- $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
- $|\psi'\rangle = e^{-\frac{i}{\hbar}\delta H}|\psi\rangle$

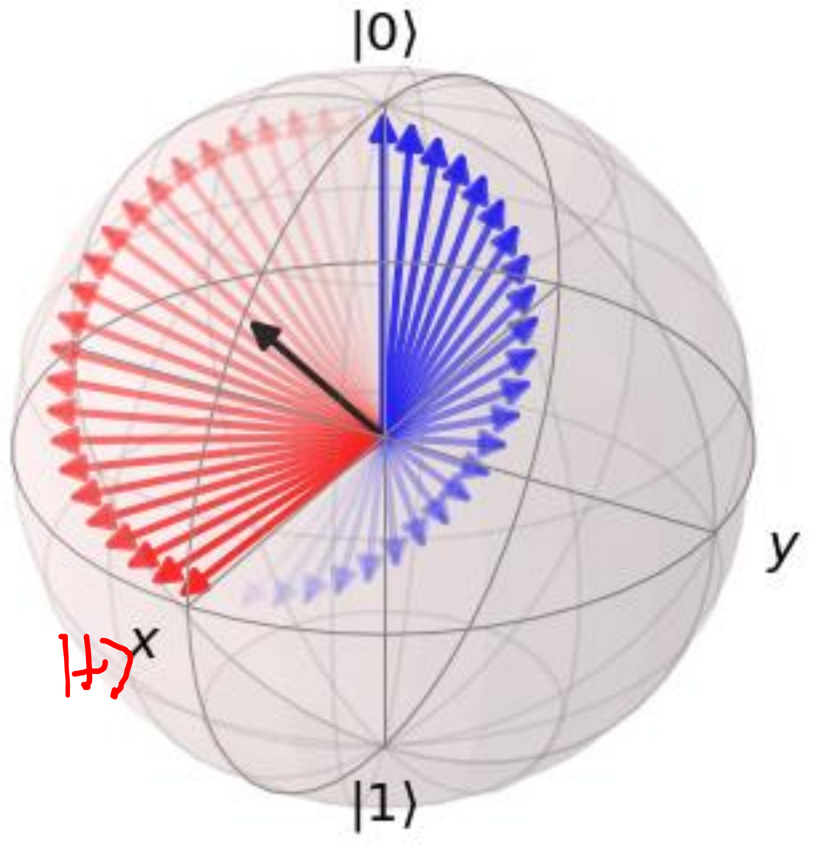
$HH|\psi\rangle = |\psi\rangle$

$H|0\rangle = |+\rangle$

$H|+\rangle = |0\rangle$

$HH|0\rangle = |0\rangle \longrightarrow HH = I$

$HH|+\rangle = |+\rangle$

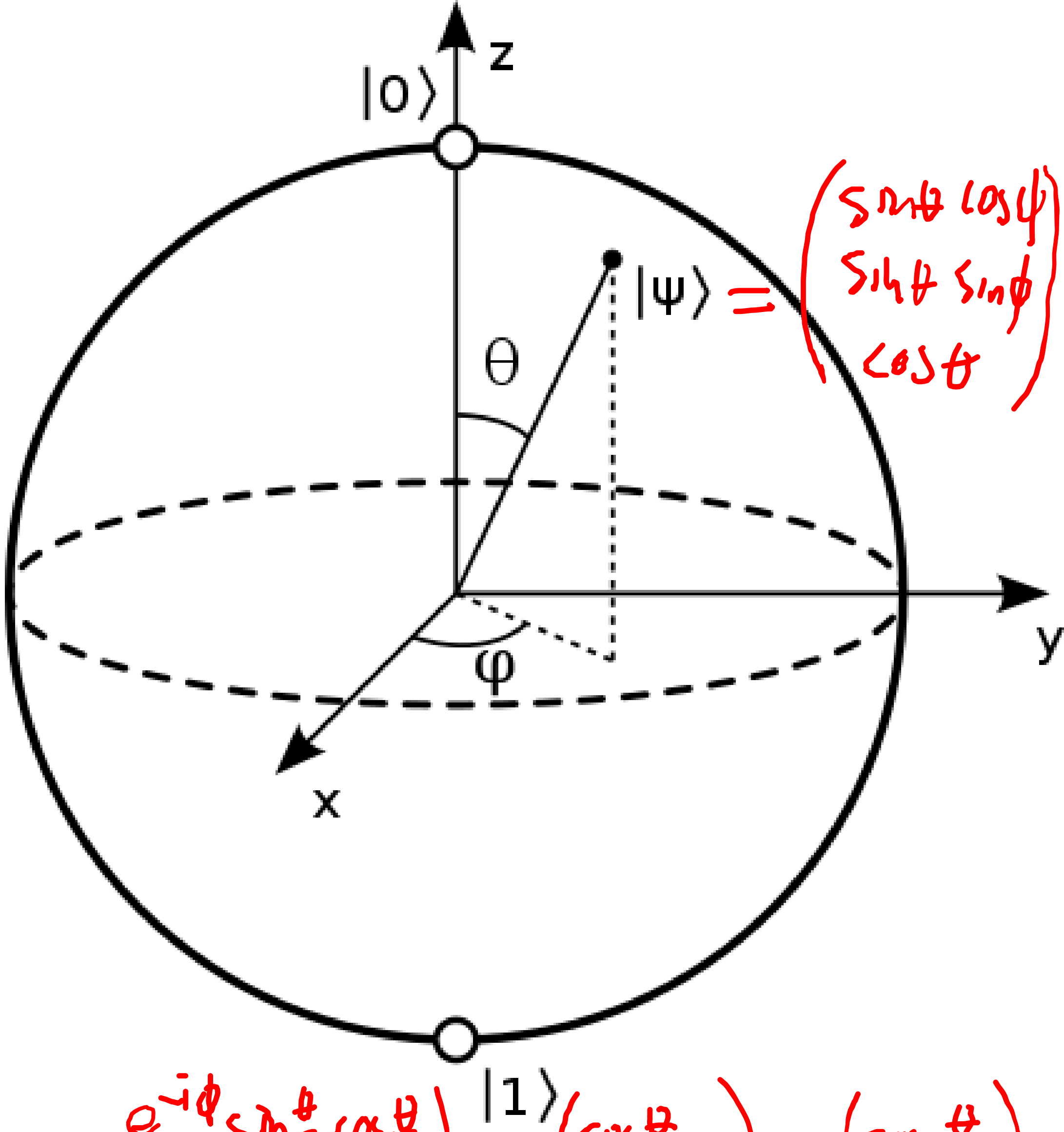


Aktivitas

- Eksplorasi state qubit di qiskit
- https://learn.qiskit.org/course/ch-states/representing-qubit-states

Bloch Sphere Coordinate

- $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
- $\rho = |\psi\rangle\langle\psi| = \begin{bmatrix} \cos^2\frac{\theta}{2} & e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{bmatrix}$
- Bloch vector: $r = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$
- X basis: $|+\rangle, |-\rangle$
- Y basis: $|i+\rangle, |i-\rangle$
- Z basis (computational basis): $|0\rangle, |1\rangle$



$|\psi\rangle = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

$\rho|\psi\rangle = |\psi\rangle\underbrace{\cancel{\langle\psi|\psi\rangle}}_{1} = |\psi\rangle$

$\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$

Measurement Operator

- Measure-0: $M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = |0\rangle\langle 0|$
- Measure-1: $M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = |1\rangle\langle 1|$

Pauli Operator

Not Gate

$$XX^\dagger = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

- Identity: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |0\rangle\langle 0| + |1\rangle\langle 1|$
- Pauli-x: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |+\rangle\langle +| - |-\rangle\langle -|$
- Pauli-y: $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = |i+\rangle\langle i+| - |i-\rangle\langle i-|$
- Pauli-z: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$
- $X = X^\dagger \to XX^\dagger = X^\dagger X = XX = I$
- $\rho = \frac{1}{2}(I + r \cdot \sigma) = \frac{1}{2}(I + r_x X + r_y Y + r_z Z)$ $= |\psi\rangle\langle\psi|$
- $\rho = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{r_x}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \frac{r_y}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + \frac{r_z}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 + r_z & r_x - ir_y \\ r_x + ir_y & 1 - r_z \end{bmatrix}$
- $\rho = \frac{1}{2}\begin{bmatrix} 1 + \cos\theta & \sin\theta\cos\phi - i\sin\theta\sin\phi \\ \sin\theta\cos\phi + i\sin\theta\sin\phi & 1 - \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\frac{\theta}{2} & e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{bmatrix}$

$$r = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

Uji Pemahaman

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

- Apakah hasil dari $X|0\rangle$ dan $X|1\rangle$? $X|0\rangle = |1\rangle, \; X|1\rangle = |0\rangle \rightarrow$ Not
- Apakah hasil dari $Y|0\rangle$ dan $Y|1\rangle$?
- Apakah hasil dari $Z|0\rangle$ dan $\mathrm{Z}|1\rangle$?

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\begin{pmatrix} 0 \\ 1 \end{pmatrix} = i|1\rangle, \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -i|0\rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

$$\left(|+\rangle\langle +| - |-\rangle\langle -|\right)|0\rangle = |+\rangle\underbrace{\langle +|0\rangle}_{\frac{1}{\sqrt{2}}} - |-\rangle\underbrace{\langle -|0\rangle}_{1/\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$= |1\rangle$$

Hadamard Operator

- Hadamard: $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$
- $H = H^\dagger \rightarrow HH^\dagger = H^\dagger H = HH = I$

$$\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)$$

$$\frac{1}{2}\left(|0\rangle\langle 0| + \cancel{|0\rangle\langle 1|} + |0\rangle\langle 0| - \cancel{|0\rangle\langle 1|} + \cancel{|1\rangle\langle 0|} + |1\rangle\langle 1| - \cancel{|1\rangle\langle 0|} + |1\rangle\langle 1|\right)$$

$$\frac{1}{2}\left(2|0\rangle\langle 0| + 2|1\rangle\langle 1|\right) = |0\rangle\langle 0| + |1\rangle\langle 1| = I$$

Uji Pemahaman

- Apakah hasil dari $H|0\rangle$ dan $H|1\rangle$?
- Apakah hasil dari $H|+\rangle$ dan $H|-\rangle$?

$$\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |-\rangle$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |0\rangle$$

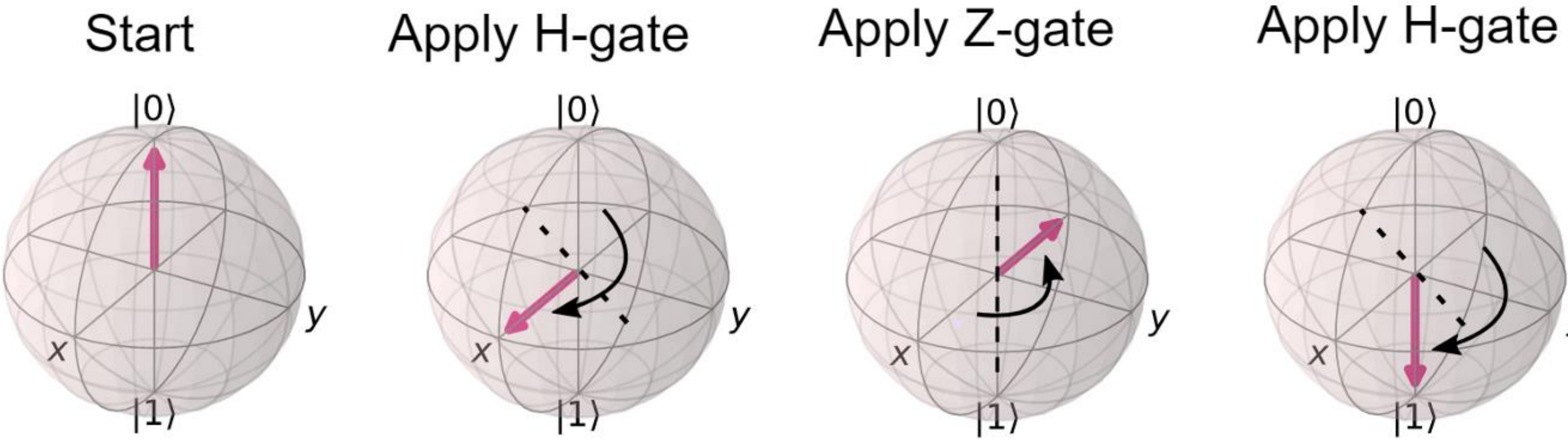
$$\frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |1\rangle$$

Multiple Gates

$X = HZH$

$Z = HXH$

- $X = |+\rangle\langle+| - |-\rangle\langle-| = \overbrace{H|0\rangle\langle0|H - H|1\rangle\langle1|H}$ $= H\overbrace{(|0\rangle\langle0| - |1\rangle\langle1|)}^{Z}H^\dagger = HZH$
- $HXH = H(HZH)H = IZI = Z$



Uji Pemahaman

$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$

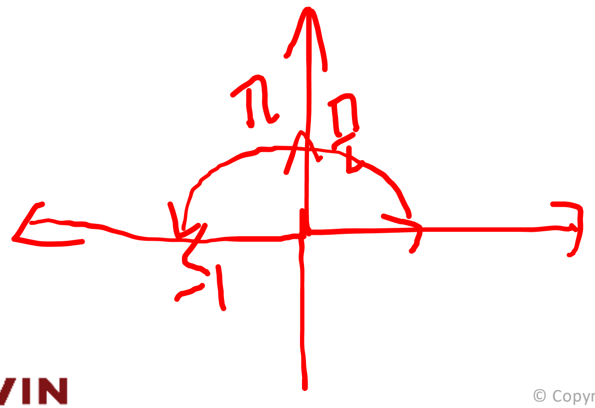
- Apakah hasil dari $HZH|0\rangle$ dan $HZH|1\rangle$?
- Operator apakah yang ekuivalen dengan ZYZ dan ZXZ?

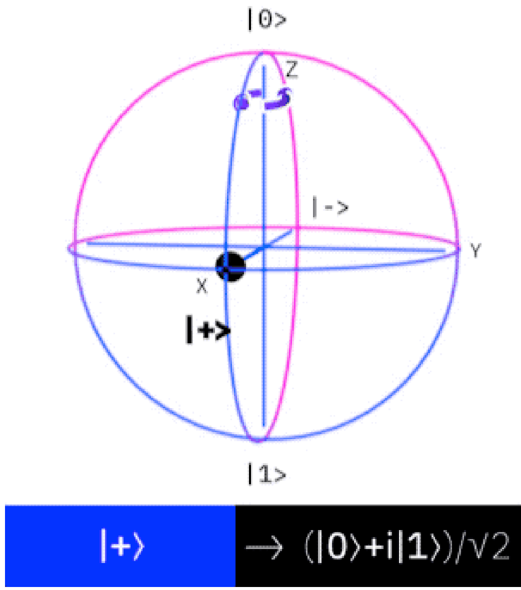
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = Y^\dagger$$

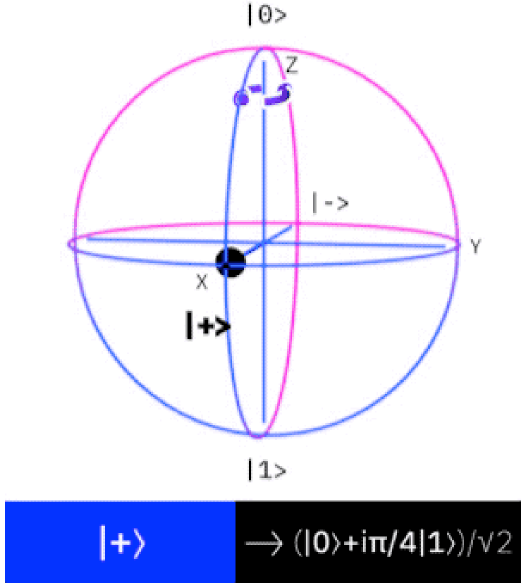
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -X$$

Phase Operator

- Phase-α : $P(\alpha) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix} = |0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1|$
- $P(\pi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$
- $P\left(\frac{\pi}{2}\right) = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = S = \sqrt{Z}$
- $P\left(\frac{\pi}{4}\right) = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{i} \end{bmatrix} = T = \sqrt{S} = \sqrt[4]{Z}$







Uji Pemahaman

$$P(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix} \qquad HP(\alpha)H = \frac{1}{2}\begin{pmatrix} 1+e^{i\alpha} & 1-e^{i\alpha} \\ 1-e^{i\alpha} & 1+e^{i\alpha} \end{pmatrix}$$

- Apakah peluang pengukuran qubit 0 dan 1 pada $H|0\rangle$?
- Apakah peluang pengukuran qubit 0 dan 1 pada $HP(\alpha)H|0\rangle$?
- Apakah hasil dari $S|+\rangle$ dan $S|-\rangle$?

$$|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$P(0) = \langle\psi|0\rangle\langle 0|\psi\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$P(1) = \langle\psi|1\rangle\langle 1|\psi\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\hookrightarrow \left(\frac{1+e^{i\alpha}}{2}\right)|0\rangle + \left(\frac{1-e^{i\alpha}}{2}\right)|1\rangle$$

$$P(0) = \left(\frac{1+e^{i\alpha}}{2}\right)\left(\frac{1+e^{-i\alpha}}{2}\right) = \frac{2+e^{i\alpha}+e^{-i\alpha}}{4}$$

$$= (1+\cos\alpha)/2$$

$$P(1) = (1-\cos\alpha)/2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = |i+\rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = |i-\rangle$$

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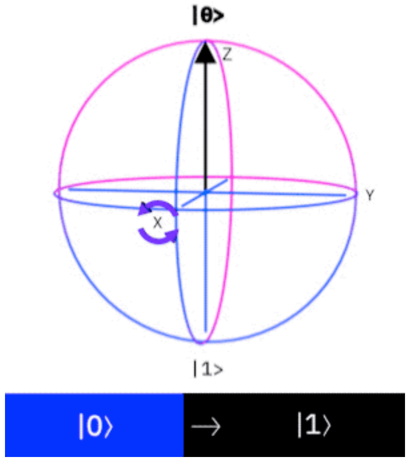
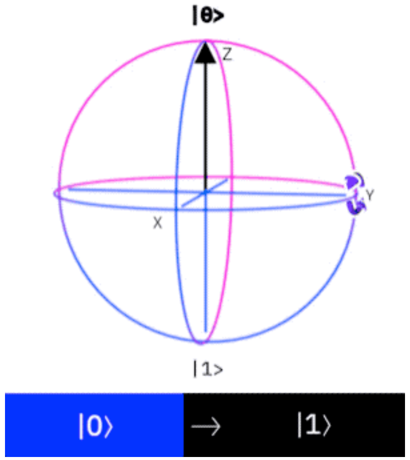
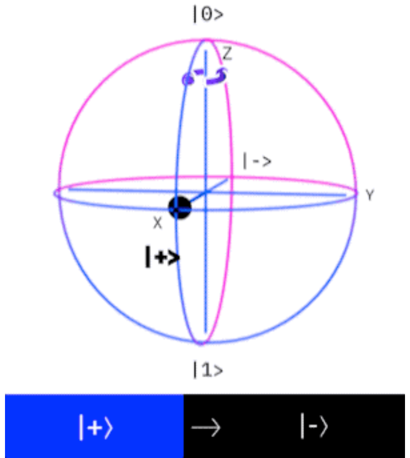
Rotation Operator

- Rotation-X: $R_x(\alpha) = \begin{bmatrix} \cos\frac{\alpha}{2} & -i\sin\frac{\alpha}{2} \\ -i\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{bmatrix}$
- Rotation-Y: $R_y(\alpha) = \begin{bmatrix} \cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2} \\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{bmatrix}$
- Rotation-Z: $R_z(\alpha) = \begin{bmatrix} e^{-\frac{i\alpha}{2}} & 0 \\ 0 & e^{\frac{i\alpha}{2}} \end{bmatrix}$







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Uji Pemahaman

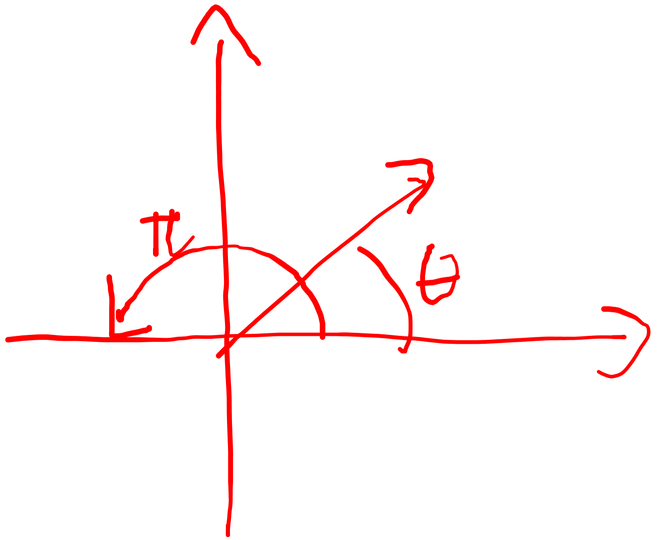
- Bagaimana merepresentasikan Z-gate dengan rotation operator?

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_z(\alpha) = \begin{pmatrix} e^{-i\frac{\alpha}{2}} & 0 \\ 0 & e^{+i\frac{\alpha}{2}} \end{pmatrix}$$

$$R_z(\ ) \neq Z$$

$$P(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix} \Rightarrow P(\pi) = Z$$



U-gate Operator

- U-gate: $U(\theta, \phi, \alpha) = \begin{bmatrix} \cos\frac{\theta}{2} & -e^{i\alpha}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\alpha)}\cos\frac{\theta}{2} \end{bmatrix}$
- $U\left(\frac{\pi}{2}, 0, \pi\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H$
- $U(0,0,\alpha) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix} = P(\alpha)$
- U-gate apapun dapat dibentuk dengan hanya rotasi 2-axis apapun

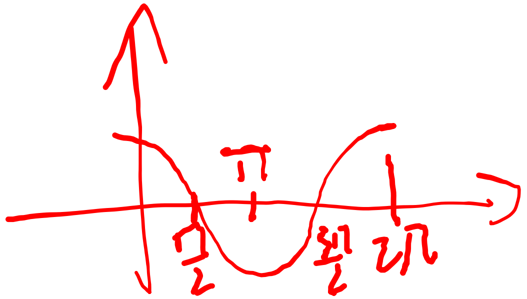
Uji Pemahaman

- Apakah parameter U-gate yang dapat merepresentasikan X-gate dan Y-gate?

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = U(\pi, 0, \pi)$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = U\left(\pi, \frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$U = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\alpha}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\alpha)}\cos\frac{\theta}{2} \end{pmatrix}$$



Aktivitas

- Operasi single qubit gates di qiskit
- https://learn.qiskit.org/course/ch-states/single-qubit-gates
- https://quantum-computing.ibm.com/composer/docs/iqx/guide/introducing-qubit-phase
- https://quantum-computing.ibm.com/composer/docs/iqx/guide/advanced-single-qubit-gates

Tuhan Memberkati