Excercise 13

① $\frac{\partial u_E}{\partial t} - f V_E = A_V \frac{\partial^2 u_E}{\partial z^2}$

$\frac{\partial V_E}{\partial t} + f u_E = A_V \frac{\partial^2 V_E}{\partial z^2}$

$t_n = t_0 + n\Delta t$

$u_i = u_0 + i\Delta u$
$V_i = V_0 + i\Delta V$

$\frac{\partial u_E}{\partial t} \approx \frac{u_0^{n+1} - u_0^n}{\Delta t}$

$\frac{\partial V_E}{\partial t} \approx \frac{V_0^{n+1} - V_0^n}{\Delta t}$

$\frac{\partial^2 u_E}{\partial z^2} \approx \frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}$

$\frac{\partial^2 V_E}{\partial z^2} \approx \frac{V_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}$

$\frac{u_0^{n+1} - u_0^n}{\Delta t} = A_V\left(\frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}\right) + fV_E$

$\therefore u_0^{n+1} = u_0^n + \Delta t\left[A_V\left(\frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}\right) + fV_E\right]$

$\therefore V_0^{n+1} = V_0^n + \Delta t\left[A_V\left(\frac{V_{0-1}^n - 2V_0^n + V_{0+1}^n}{(\Delta z)^2}\right) - fu_E\right]$

② $A_V \cdot \Delta t\left(\frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{\Delta z^2}\right)$ } Diffusion process

$u_0^{n+1} = u_i^n + \Delta t\left[\overbrace{A_V\left(\frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}\right)}^{\text{Diffusion}} + fV_E\right]$

$\therefore C = \frac{\Delta t \cdot A_V}{(\Delta z)^2}\left(\frac{u_{0-1}^n - 2u_0^n + u_{0+1}^n}{(\Delta z)^2}\right)$

$\cdot$ constant Courant based on constants $= \frac{A_V \Delta t}{(\Delta z)^2}$

③ $C = \frac{A_v \Delta t}{(\Delta z)^2}$

$A_v = 10^{-4}\ m^2/s$

stability : $C \leq 0.5$

$\Delta t = 200s$

$\Delta z = 2m$

$$\therefore C = \frac{(10^{-4} \frac{m^2}{s})(200s)}{(2m)^2}$$

$\therefore C = 5 \times 10^{-3}$

$\therefore$ stable because closer C is to 0, higher stability (not efficiency). $\therefore$ at these values, system is stable.