Excercise 13

① $\frac{\partial u_E}{\partial t} - f v_E = A_V \frac{\partial^2 u_E}{\partial z^2}$

$\frac{\partial v_E}{\partial t} + f u_E = A_V \frac{\partial^2 v_E}{\partial z^2}$

$t_n = t_0 + n\Delta t$
$z_k = z_0 + k\Delta z$

$\therefore u_{Ek}^n = (z_k, t_n)$
$\therefore v_{Ek}^n = (z_k, t_n)$

Breaking it up:

$\frac{\partial u_E}{\partial t} \approx \frac{u_k^{n+1} - u_k^n}{\Delta t}$

$\frac{\partial v_E}{\partial t} \approx \frac{v_k^{n+1} - v_k^n}{\Delta t}$

$\frac{\partial^2 u_E}{\partial z^2} \approx \frac{u_{k-1}^n - 2u_k^n + u_{k+1}^n}{(\Delta z)^2}$

$\frac{\partial^2 v_E}{\partial z^2} \approx \frac{v_{k-1}^n - 2v_k^n + v_{k+1}^n}{(\Delta z)^2}$

$\frac{u_k^{n+1} - u_k^n}{\Delta t} = A_V\left(\frac{u_{k-1}^n - 2u_k^n + u_{k+1}^n}{(\Delta z)^2}\right) + f v_E$

$\therefore u_k^{n+1} = u_k^n + \Delta t\left[A_V\left(\frac{u_{k-1}^n - 2u_k^n + u_{k+1}^n}{(\Delta z)^2}\right) + f v_E\right]$

$\therefore v_k^{n+1} = v_k^n + \Delta t\left[A_V\left(\frac{v_{k-1}^n - 2v_k^n + v_{k+1}^n}{(\Delta z)^2}\right) - f u_E\right]$

② Isolating the diffusion process:

$\Delta t\left[A_V\left(\frac{u_{k-1}^n - 2u_k^n + u_{k+1}^n}{(\Delta z)^2}\right) + \cancel{f v_E}\right]$

$= \frac{\Delta t \cdot A_V (u_{k-1}^n - 2u_k^n + u_{k+1}^n)}{(\Delta z)^2}$

Non dimensional $\therefore (u_{k-1}^n - 2u_k^n + u_{k+1}^n)$ not included

$\therefore C = \frac{\Delta t\, A_V}{(\Delta z)^2}$

(3) $C = \frac{A_v \Delta t}{(\Delta z)^2}$

$A_v = 10^{-4} \; m^2/s$

Stability : $C \leq 0.5$

$\Delta t = 200s$

$\Delta z = 2m$

$\therefore C = \frac{(10^{-4} \; m^2/s)(200 s)}{(2 m)^2}$

$\therefore C = 5 \times 10^{-3}$

∴ stable because C is close to 0. 0 indicates stability (not efficiency) so ∴ @ these values system stable.