## Question 3

Oilco has oil fields in Los Angeles and San Diego. The Los Angeles field can produce 400,000 barrels per day and the San Diego field can produce 500,000 barrels per day.

Oil is sent from the fields to a refinery, in either Dallas or Houston (assume each refinery has unlimited capacity). To refine 100,000 barrels costs 700atDallasand900 at Houston.

Refined oil is shipped to customers in New York and Chicago. New York customers require 300,000 barrels per day and Chicago customers require 400,000 barrels per day.

The costs of shipping 100,000 barrels of oil (refined or unrefined) between cities are shown below.

From/To(\$)	Dallas	Houston	New York	Chicago
Los Angels	300	110	-	-
San Diego	420	100	-	-
Dallas	-	-	450	550
Houston	-	-	470	530

```
In [ ]: using JuMP, HiGHS

# defining model
oilco = Model(HiGHS.Optimizer)
```

Out[ ]: A JuMP Model

Feasibility problem with:

Variables: 0

Model mode: AUTOMATIC

CachingOptimizer state: EMPTY\_OPTIMIZER

Solver name: HiGHS

```
numedges = 12
In [ ]:
         #Define the incidence matrix A for our fully connected network
         A = [1 \ 1]
                    1 0 0
                              0
                                 0
                                    0
                                       0
                                          0
                                             0
                                                 0
                       1
               0 0
                     0
                           1
                              1
                                 0
                                    0
                                       0
                                          0
                                                 0
             -1
                  0
                     0 -1
                           0
                              0
                                1
                                          0
                                    1
                                        1
              0 -1
                     0 0 -1
                              0 0
                                    0
                                       0
                                          1
                                              1
                                                 1
                    0
                       0
                           0
                              0 -1
                                    0
                                       0 -1
              0 0 0 0
                              0
                                0 -1
                                       0 0 -1
                           0
              0 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0 \quad -1
             1
         #Supply (first 2), relay (middle 2), demand (last 3)
         b = [400000, 500000, 0, 0, -300000, -400000, -200000]
         # costs of shipping per 100000 barrels
         costs = [300 110 0 420 100 0 1150 1250 0 1370 1430 0];
         Α
        7×12 Matrix{Int64}:
Out[ ]:
                   1
                               0
                                   0
                                        0
                                            0
                                                0
                                                    0
                                                         0
           1
               1
                       0
           0
               0
                   0
                      1
                          1
                              1
                                                         0
          -1
               0
                   0
                     -1
                           0
                              0
                                  1
                                       1
                                            1
                                                0
                                                    0
                                                         0
             -1
           0
                   0
                      0
                          -1
                               0
                                   0
                                       0
                                            0
                                                1
                                                    1
                                                        1
                              0
                   0
                     0
                          0
                                  -1
                                      0
                                          0
                                              -1
           0
              0
                                                   0
                                                        0
                   0
                           0
                               0
                                   0 -1
                                            0
                                               0 - 1
                                                        0
           0
               0
                       0
                           0
                                        0
                                                    0 -1
               0 - 1
                       0
                              -1
                                    0
                                          -1
                                                0
         @variable(oilco, x[1:numedges] >= 0)
In [ ]:
         @constraint(oilco, supplyanddemand, A*x .== b)
         @objective(oilco, Min, sum(costs[i]*x[i] for i in 1:numedges))
```

print(oilco)

```
\min \ \ 300x_1 + 110x_2 + 420x_4 + 100x_5 + 1150x_7 + 1250x_8 + 1370x_{10} + 1430x_{10}
Subject to x_1 + x_2 + x_3 = 400000.0
              x_4 + x_5 + x_6 = 500000.0
               -x_1-x_4+x_7+x_8+x_9=0.0
               -x_2 - x_5 + x_{10} + x_{11} + x_{12} = 0.0
               -x_7 - x_{10} = -300000.0
               -x_8-x_{11}=-400000.0
               -x_3 - x_6 - x_9 - x_{12} = -200000.0
              x_1 \ge 0.0
              x_2 \ge 0.0
              x_3 \geq 0.0
              x_4 > 0.0
              x_5 \geq 0.0
              x_6 \ge 0.0
              x_7 \geq 0.0
              x_8 \ge 0.0
              x_9 \ge 0.0
              x_{10} \ge 0.0
              x_{11} \geq 0.0
              x_{12} \geq 0.0
```

```
optimize! (oilco)
In [ ]:
        Presolving model
        5 rows, 10 cols, 20 nonzeros
        4 rows, 10 cols, 17 nonzeros
        Presolve: Reductions: rows 4(-3); columns 10(-2); elements 17(-7)
        Solving the presolved LP
        Using EKK dual simplex solver - serial
          Iteration
                           Objective
                                         Infeasibilities num(sum)
                        8.4500000000e+08 Pr: 3(1.4e+06) 0s
                  0
                  5
                        1.0470000000e+09 Pr: 0(0) 0s
        Solving the original LP from the solution after postsolve
        Model
                status
                           : Optimal
        Simplex iterations: 5
        Objective value : 1.0470000000e+09
        HiGHS run time
                                       0.00
        value.(x)
In [ ]:
```