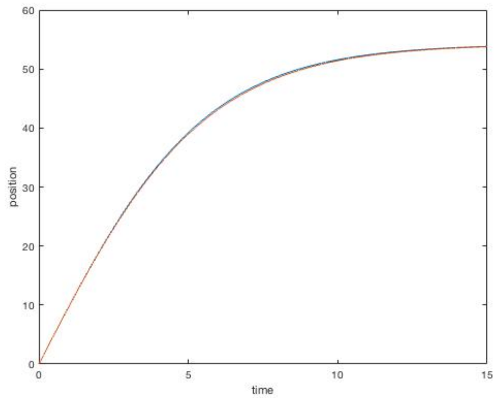
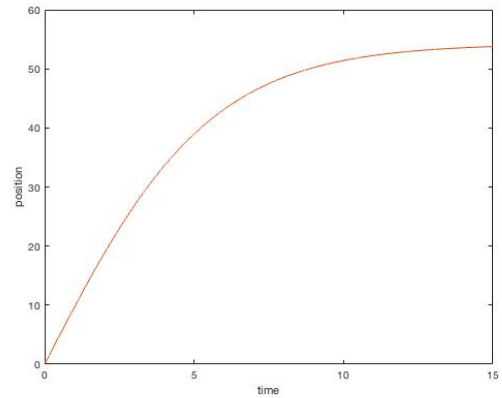
Homework 1

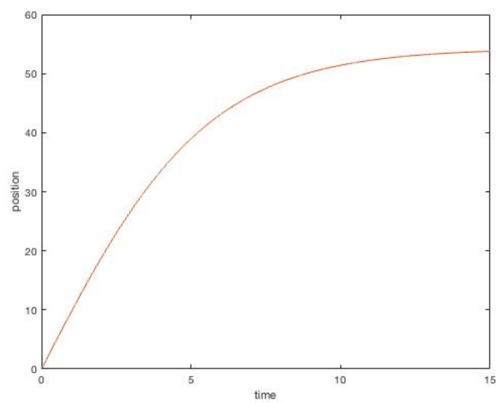
1 Graphs representing Euler, RK2 and RK4



(a) *Euler*



(b) *RK*2



(c) *RK*4

dt	Euler	RK2	RK4
0.1	0.18152173	0.00217328	8.639x10^{-8}
0.05	0.0904899	5.3806x10^{-4}	5.35093x10^{-9}
0.025	0.04517788	1.339x10^{-4}	3.329887x10^{-10}
0.0125	0.0225722	3.3396x10^{-5}	2.09823x10^{-11}

2 Order of Accuracy

$$y^{n+1} = y^n + dt\frac{dy}{dt} + \frac{dt^2}{2}\frac{d^2y}{dt^2} + \frac{dt^3}{6}\frac{d^3y}{dt^3} + \frac{dt^4}{24}\frac{d^4y}{dt^4} + O(dt^5) \tag{1}$$

$$y^{n-1} = y^n - dt\frac{dy}{dt} + \frac{dt^2}{2}\frac{d^2y}{dt^2} - \frac{dt^3}{6}\frac{d^3y}{dt^3} + \frac{dt^4}{24}\frac{d^4y}{dt^4} + O(dt^5) \tag{2}$$

$$y^{n+1} - y^{n-1} = 2dt\frac{dy}{dt} + 2\frac{dt^3}{6}\frac{d^3y}{dt^3} + O(dt^5) \tag{3}$$

$$\frac{y^{n+1} - y^{n-1}}{2dt} = \frac{dy}{dt} + \frac{dt^2}{6}\frac{d^3y}{dt^3} + O(dt^4) \tag{4}$$

We can see that: $\frac{dy}{dt}$ matches $y' = f(t, y)$

This term: $\frac{dt^2}{6}\frac{d^3y}{dt^3}$: is bigger than $O(dt^4)$.

$\frac{dt^2}{6}$ is a constant; $\frac{d^3y}{dt^3}$ is a third order derivative, which means it is a bounded by a second order derivative. Making the order of accuracy, **Second Order**.