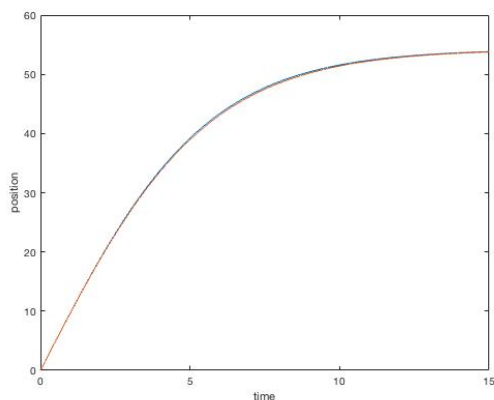
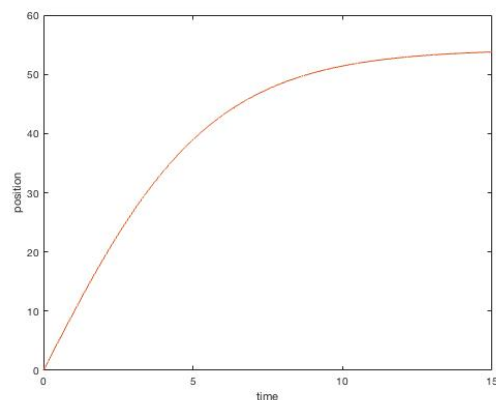
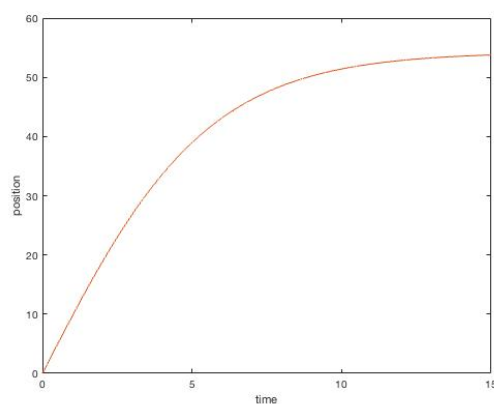


Homework 1

1 Graphs representing Euler, RK2 and RK4

(a) *Euler*(b) *RK2*(c) *RK4*

dt	Euler	RK2	RK4
0.1	0.18152173	0.00217328	8.639×10^{-8}
0.05	0.0904899	5.3806×10^{-4}	5.35093×10^{-9}
0.025	0.04517788	1.339×10^{-4}	3.329887×10^{-10}
0.0125	0.0225722	3.3396×10^{-5}	2.09823×10^{-11}

2 Order of Accuracy

$$y^{n+1} = y^n + dt \frac{dy}{dt} + \frac{dt^2}{2} \frac{d^2y}{dt^2} + \frac{dt^3}{6} \frac{d^3y}{dt^3} + \frac{dt^4}{24} \frac{d^4y}{dt^4} + O(dt^5) \quad (1)$$

$$y^{n-1} = y^n - dt \frac{dy}{dt} + \frac{dt^2}{2} \frac{d^2y}{dt^2} - \frac{dt^3}{6} \frac{d^3y}{dt^3} + \frac{dt^4}{24} \frac{d^4y}{dt^4} + O(dt^5) \quad (2)$$

$$y^{n+1} - y^{n-1} = 2dt \frac{dy}{dt} + 2 \frac{dt^3}{6} \frac{d^3y}{dt^3} + O(dt^5) \quad (3)$$

$$\frac{y^{n+1} - y^{n-1}}{2dt} = \frac{dy}{dt} + \frac{dt^2}{6} \frac{d^3y}{dt^3} + O(dt^4) \quad (4)$$

We can see that: $\frac{dy}{dt}$ matches $y' = f(t, y)$, this term is bigger than $O(dt^4)$, so this term determines the order of accuracy.

This term: $\frac{dt^2}{6} \frac{d^3y}{dt^3}$: is bigger than $O(dt^4)$.

$\frac{dt^2}{6}$ is a constant; $\frac{d^3y}{dt^3}$ is a third order derivative, which means it is bounded by a second order derivative. Making the order of accuracy, **Second Order**.