$\boxed{1}$

Leapfrog:

$$\frac{f(t+dt,x)-f(t-dt,x)}{2dt} = -v\,\frac{f(t,x+dx)-f(t,x-dx)}{2dx}$$

$$\frac{f(t+dt/2,x)-f(t-dt/2,x)}{dt} = -v\,\frac{f(t,x+\frac{dx}{2})-f(t,x-\frac{dx}{2})}{dx}$$

$$\to \frac{f(t+dt,x)-2f(t,x)+f(t-dt,x)}{dt^2} = v^2\,\frac{f(t,x+dx)-2f(t,x)+f(t,x-dx)}{dx^2}$$

$\to f(x,t) = \xi^t e^{ikx}$

$\to \alpha = v\dfrac{dt}{dx}$

$$LHS = \frac{\xi^{t+1}e^{ikx}-2\xi^t e^{ikx}+\xi^{t-1}e^{ikx}}{dt^2} = \left(\frac{\xi^{t+1}-2\xi^t+\xi^{t-1}}{dt^2}\right)e^{ikx}$$

$$RHS = \alpha^2\,\frac{\xi^t e^{ik(x+dx)}-2\xi^t e^{ikx}+\xi^t e^{ik(x-dx)}}{dx^2} = v^2\xi^t e^{ikx}\left(\frac{e^{ikdx}-2+e^{-ikdx}}{dx^2}\right)$$

$$e^{ikx}\left(\frac{\xi^{t+1}-2\xi^t+\xi^{t-1}}{dt^2}\right) = v^2\xi^t e^{ikx}\left(\frac{e^{ikdx}-2+e^{-ikdx}}{dx^2}\right)$$

$$\frac{\xi^{t+1}-2\xi^t+\xi^{t-1}}{\cancel{dt^2}} = \alpha^2\frac{\cancel{dx^2}}{\cancel{dt^2}}\,\xi^t\left(\frac{e^{ikdx}-2+e^{-ikdx}}{\cancel{dx^2}}\right)$$

$$\xi^2-2\xi+1 = \alpha^2\xi(2\cos(kdx)-2)$$

$$\xi^2-2\xi-2\alpha^2\xi\cos(kdx)+2\alpha^2\xi+1=0$$

$$\xi-2\xi\underbrace{(1-\alpha^2\cos(kdx)+\alpha^2)}_{C}+1=0$$

$$\xi-2C\xi+1=0$$

~~Since $|\xi| \leq 1$:~~

~~$|\xi|^2 = C^2 + (1-C^2) \leq 1$, where $C = 2i\sin(kdx)\alpha$~~

~~If $\alpha \leq 1$, then $(2i\sin(kdx)\alpha)^2 \leq 1$.~~

~~Therefore, when~~

if $\alpha \leq 1$:

$$|\alpha^2\cos(kdx)| \leq 1$$

$$\alpha^2 \leq 1$$

$$|1-\alpha^2\cos(kdx)+\alpha^2| \leq 1$$

$$|C| \leq 1$$

$$C^2 \leq 1$$

As $\xi - 2C\xi + 1 = 0$

$\Rightarrow \xi = C \pm \sqrt{C^2-1}$

Since $|C| \leq 1$ and $C^2 \leq 1$, $|\xi| \leq 1$

Therefore, if $\alpha = v\frac{dt}{dx} \leq 1$, $|\xi| \leq 1$, which means that the leapfrog scheme is stable.