Practical Machine Learning

Module 1: Modelling and Linear Regression

intelligent layer

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Content and objectives

Content

- Review of modelling
- Linear Regression
- Gradient Descent

Objectives

- Understand where the model fits into the machine learning workflow
- Understand why we need a good model
- Understand types of modelling

Content and objectives

Content

- Review of modelling
- Linear Regression
- Gradient Descent

Objectives

- Understand when we use regression
- Introduce a simple 1D linear model
- Understand how to evaluate the cost of predictions against observations

Content and objectives

Content

- Review of modelling
- Linear Regression
- Gradient Descent

Objectives

- Understand why we use gradient descent
- Understand the operation of gradient descent
- Understand how gradient descent can go wrong, and why

Modelling

We want to find a RELATIONSHIP between INPUTS and TARGET

$$y = f(\mathbf{x}) + \epsilon$$

target function inputs noise

Modelling

Why do we need a GOOD MODEL?

A PREDICTION

$$y^* = f_{est}(\mathbf{x}^*)$$

B INFERENCE

- Which input features are most strongly related to y?
- Are relationships +ve or –ve?
- What is the nature of f? (Linear / non-linear etc.)

Modelling

PARAMETRIC and NON-PARAMETRIC modelling

A PARAMETRIC

$$f(\mathbf{x}) = \theta_0 + x_1\theta_1 + x_2\theta_2 \cdots$$

B NON-PARAMETRIC

- f is not fixed
- complexity of f adapts to the complexity of the data

Modelling

SUPERVISED vs. UNSUPERVISED learning

A SUPERVISED

- Access to target variable
- e.g. linear regression or classification

B UNSUPERVISED

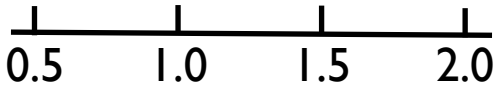
- No access to target variable
- e.g. clustering or dimensionality reduction

Linear Regression with one variable

Regression

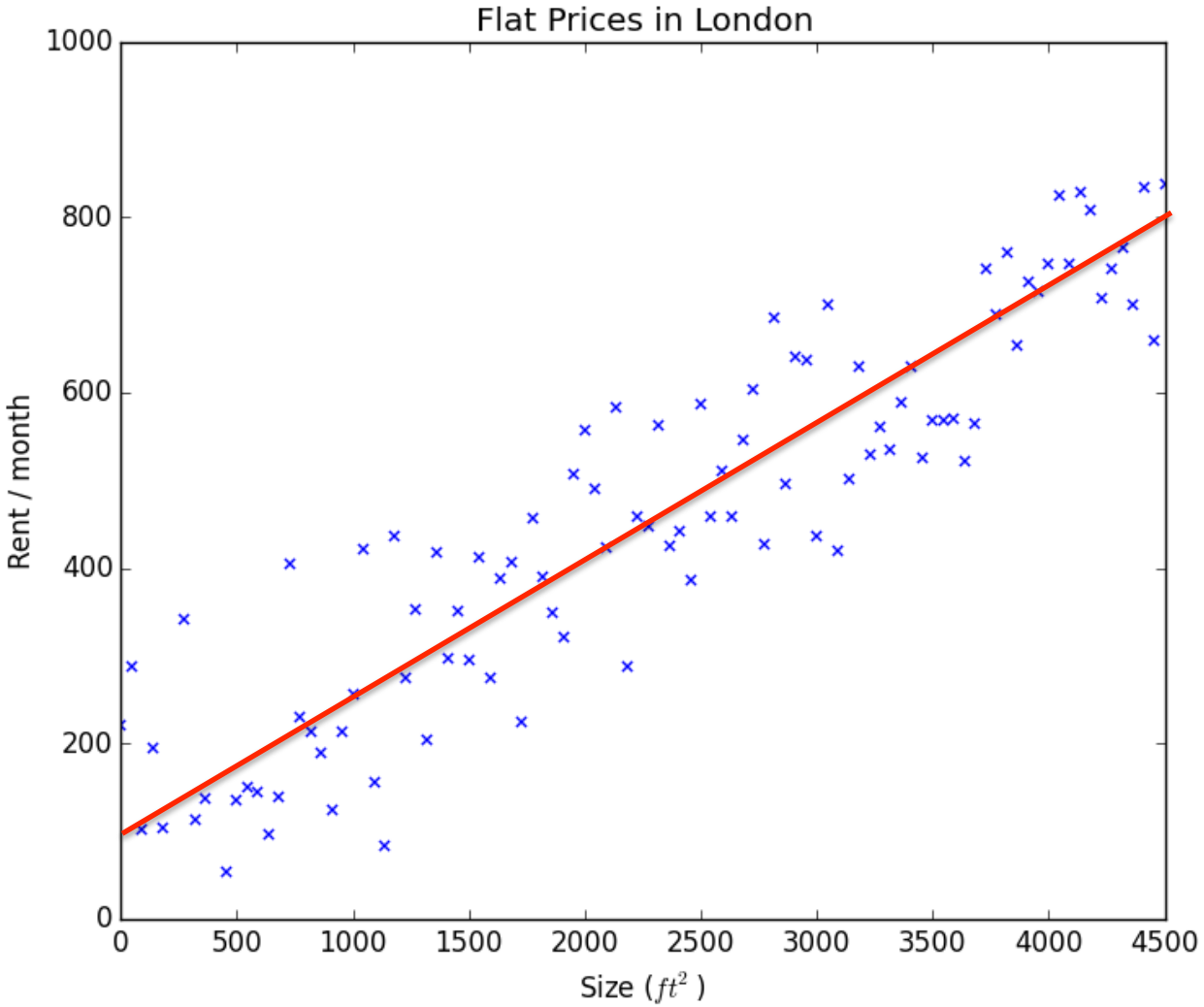
TARGET variable takes on NUMERICAL VALUES

$$f(\mathbf{x}) \longrightarrow$$

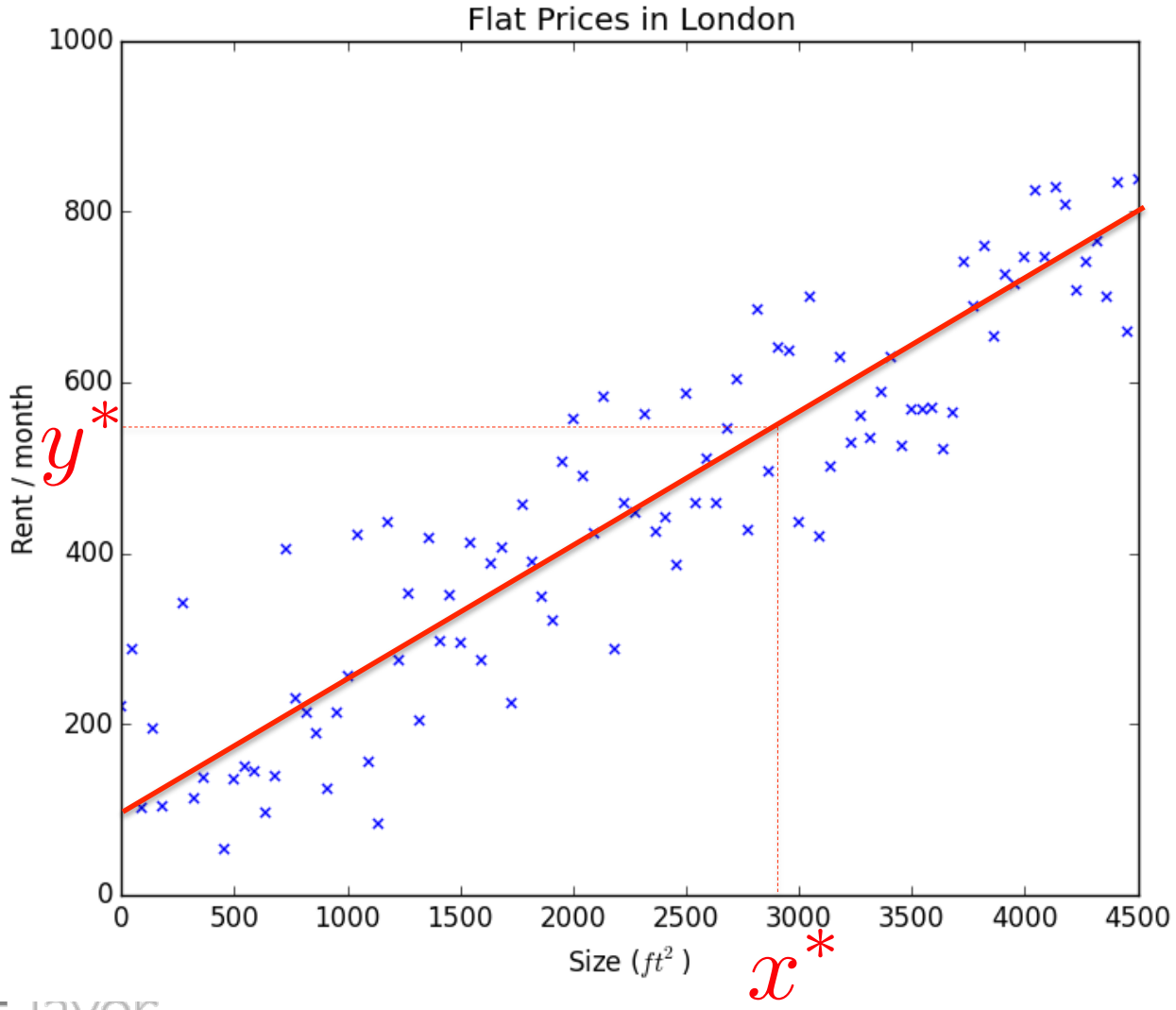


	mpg	cylinders	displacement	horsepower	weight	acceleration	modelyear	origin
0	18	8	307	130	3504	12.0	70	1
1	15	8	350	165	3693	11.5	70	1
2	18	8	318	150	3436	11.0	70	1
3	16	8	304	150	3433	12.0	70	1
4	17	8	302	140	3449	10.5	70	1

Linear Regression with 1 variable



Linear Regression with 1 variable



Linear Regression with one variable

Size in ft^2 **(x)**	Rent / month (£) **(y)**
3121	450
1746	415
2443	398
…	…

Notation

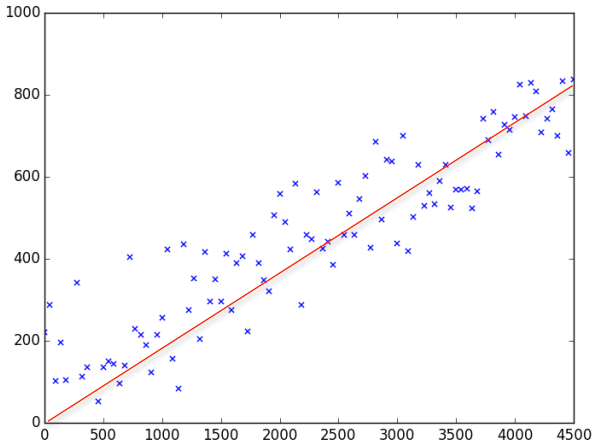
m = No. of training examples
x = input variable (feature)
y = output variable (target)

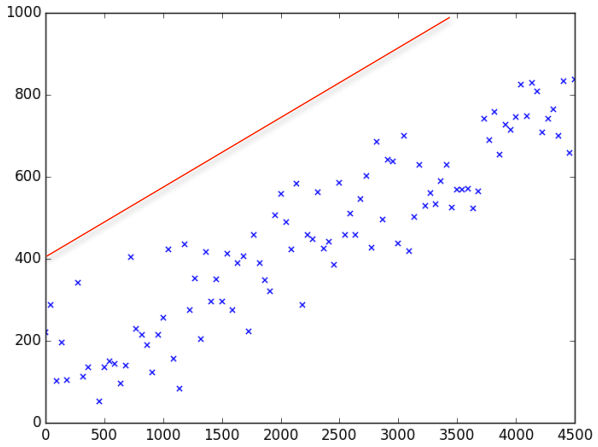
(x,y) = one training example
$(x^{(i)},y^{(i)})$ = i^{th} example

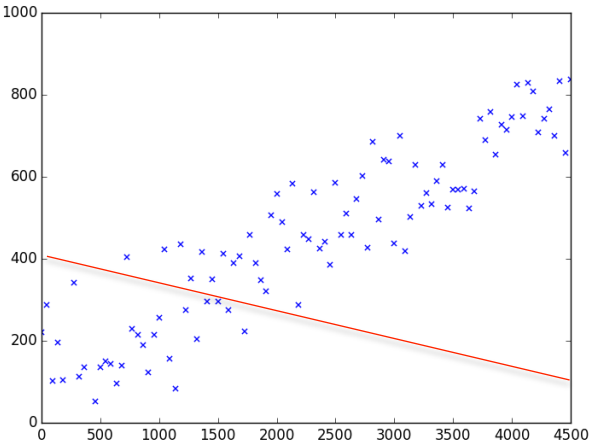
Linear Regression with 1 variable

The Model

$$f_\theta(x) = \theta_0 + \theta_1 x$$



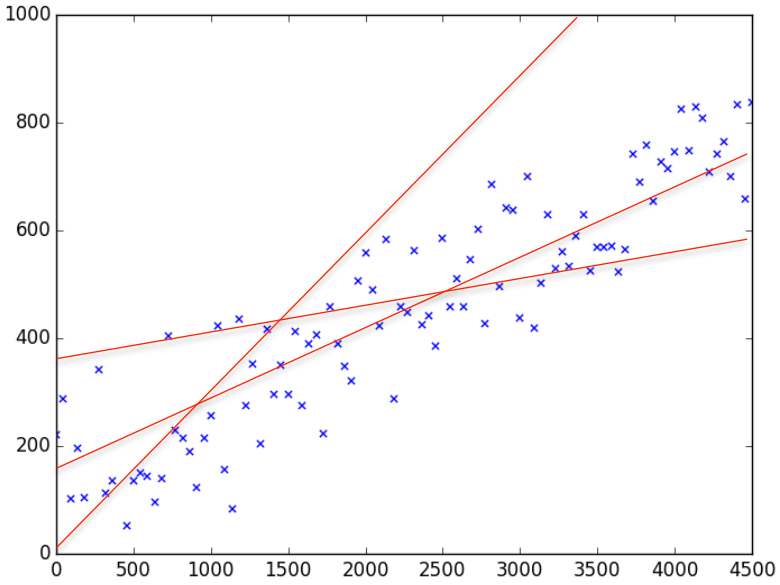




How to choose θ ?

Choose θ_1, θ_0 so that f(x) is close to y for all our (x,y)

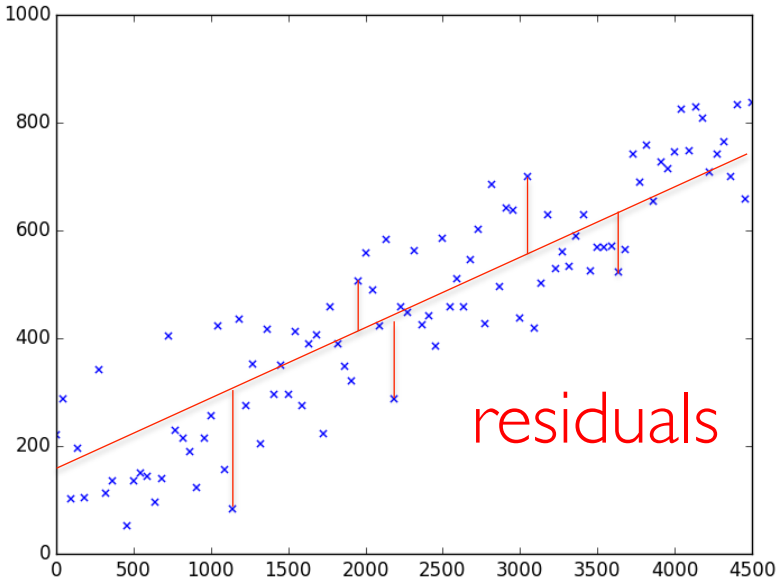
Size in ft^2 **(x)**	Rent / month (£) **(y)**
3121	450
1746	415
2443	398
…	…



How to choose θ ?

Introduce a COST FUNCTION

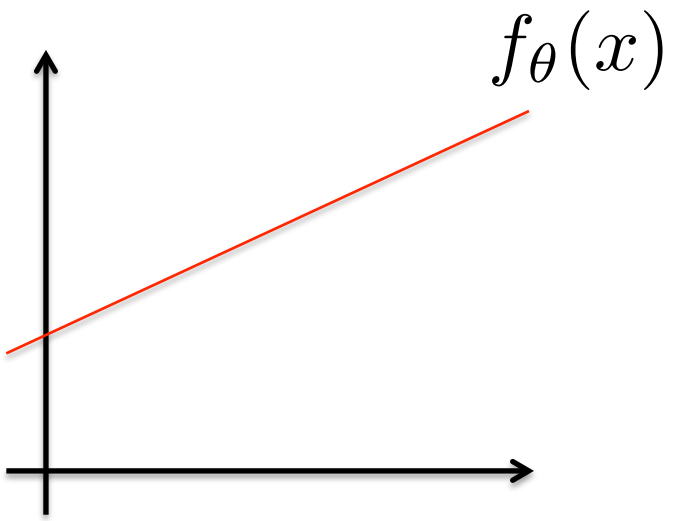
$$C = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$



Cost Function

Function:

$$f_\theta(x) = \theta_0 + \theta_1 x$$



Parameters:

$$\theta_0, \theta_1$$

Cost Function:

$$C(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

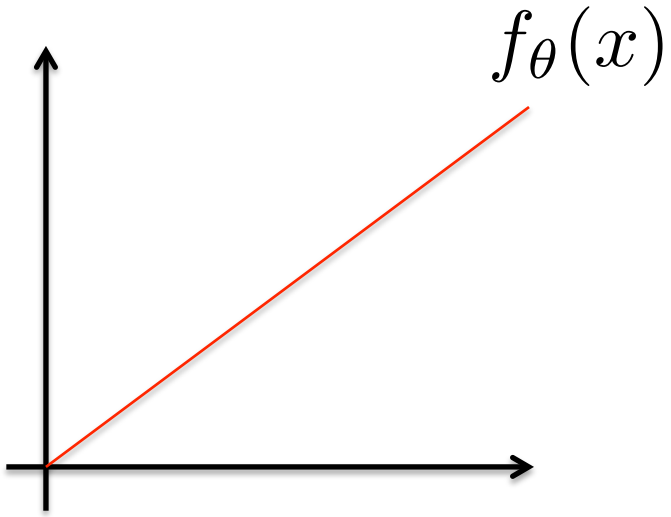
Goal:

$$\underset{\theta_0, \theta_1}{\text{minimize}}\ C(\theta_0, \theta_1)$$

Simplified Cost Function

Function:

$$f_\theta(x) = \theta_1 x$$



Parameters:

$$\theta_1$$

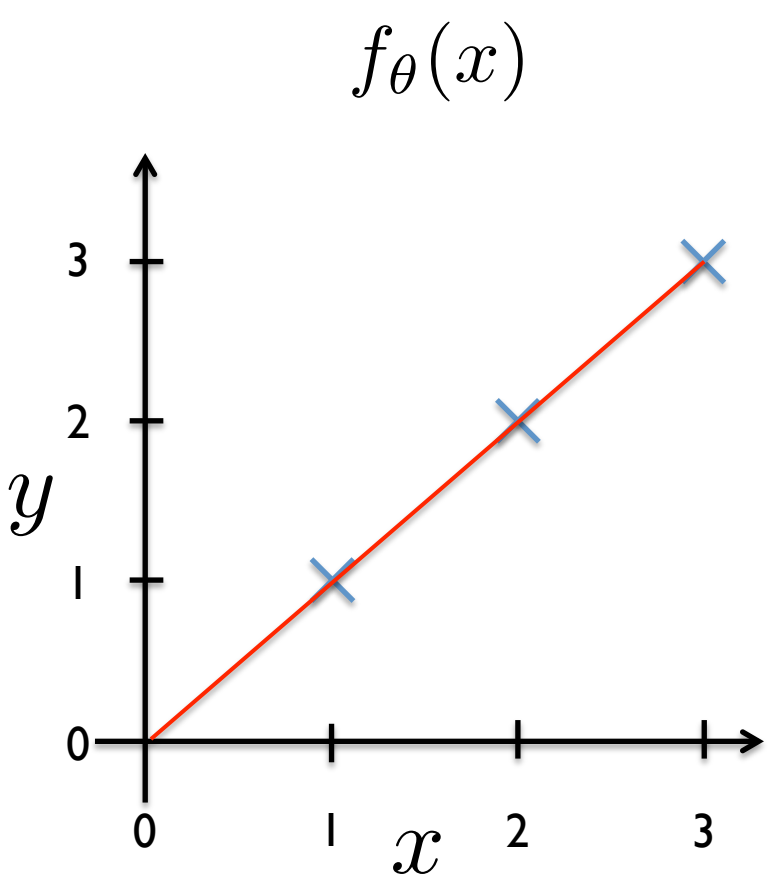
Cost Function:

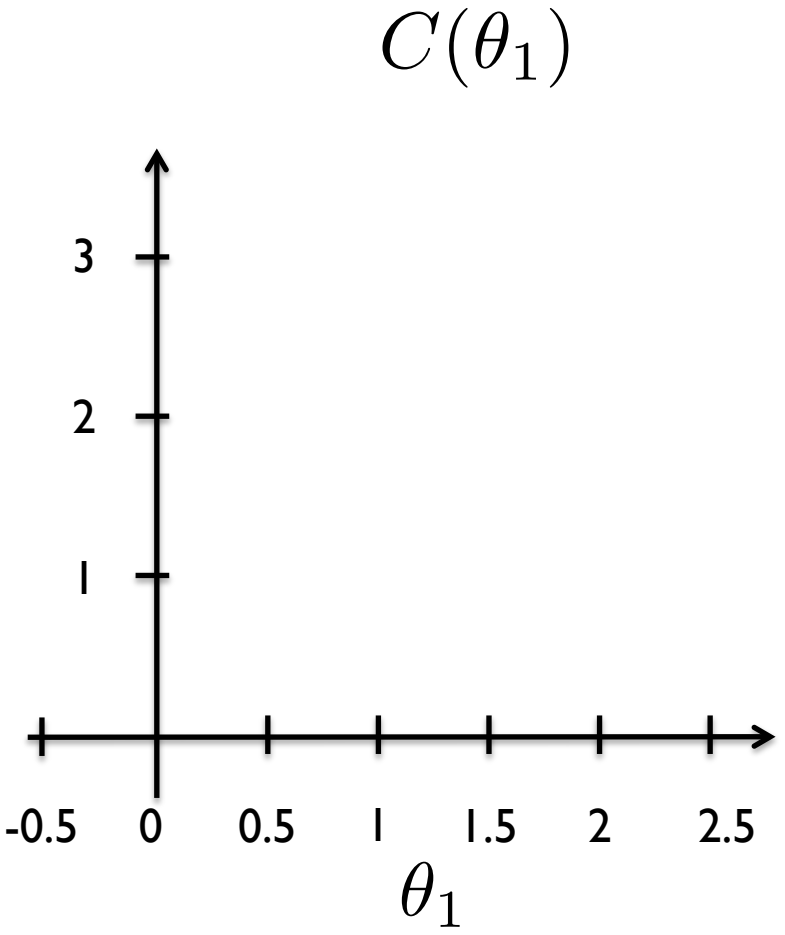
$$C(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

Goal:

$$\underset{\theta_1}{\text{minimize}}\ C(\theta_1)$$

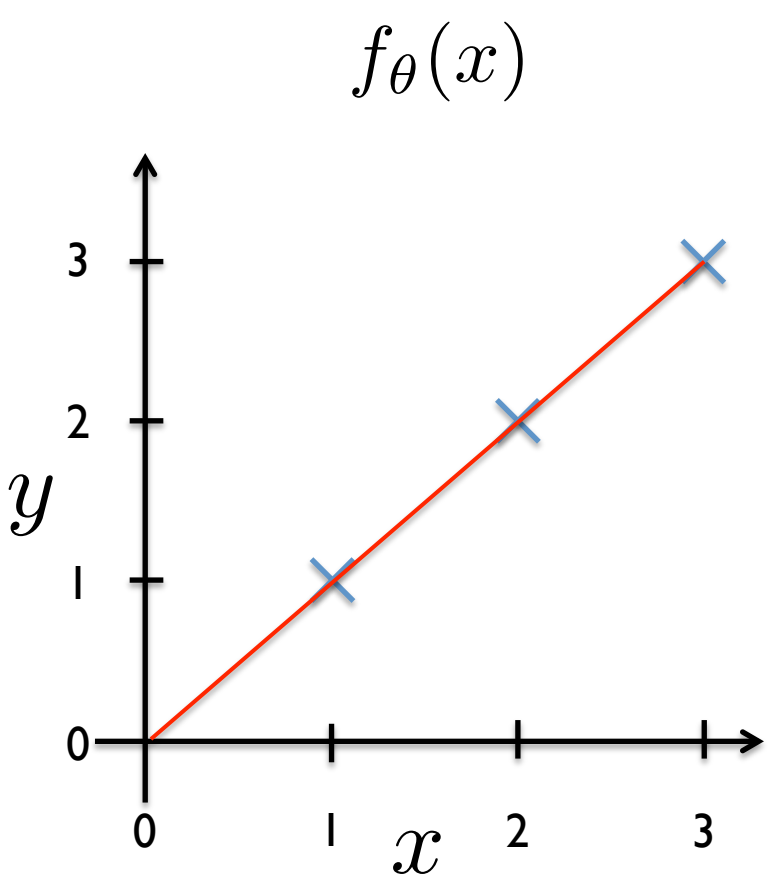
Simplified Cost Function

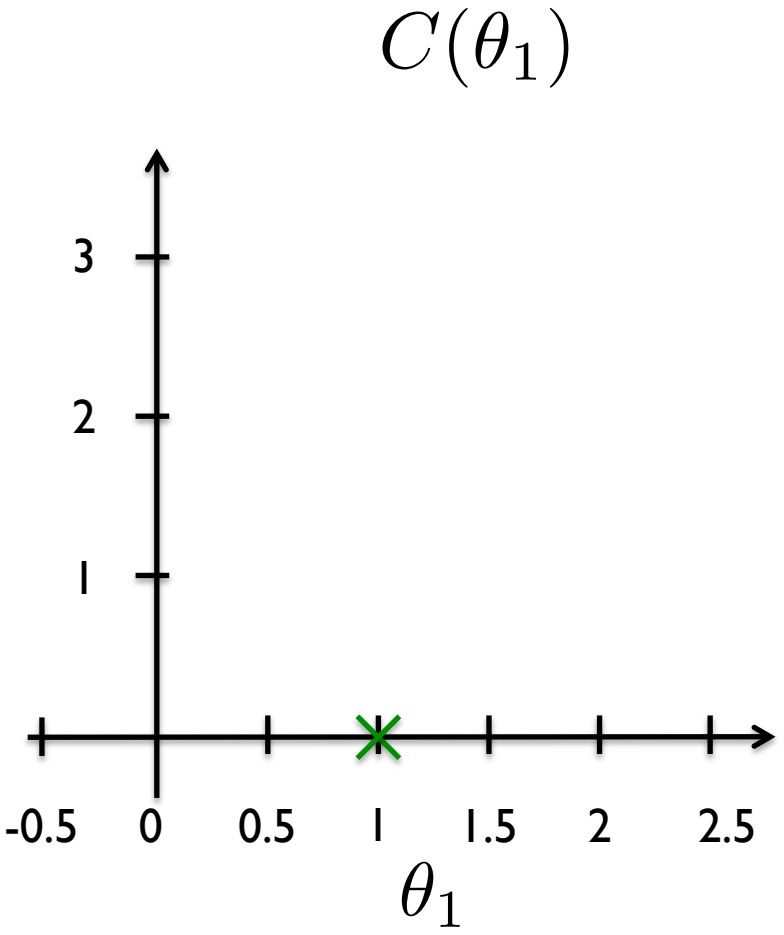




$$C(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

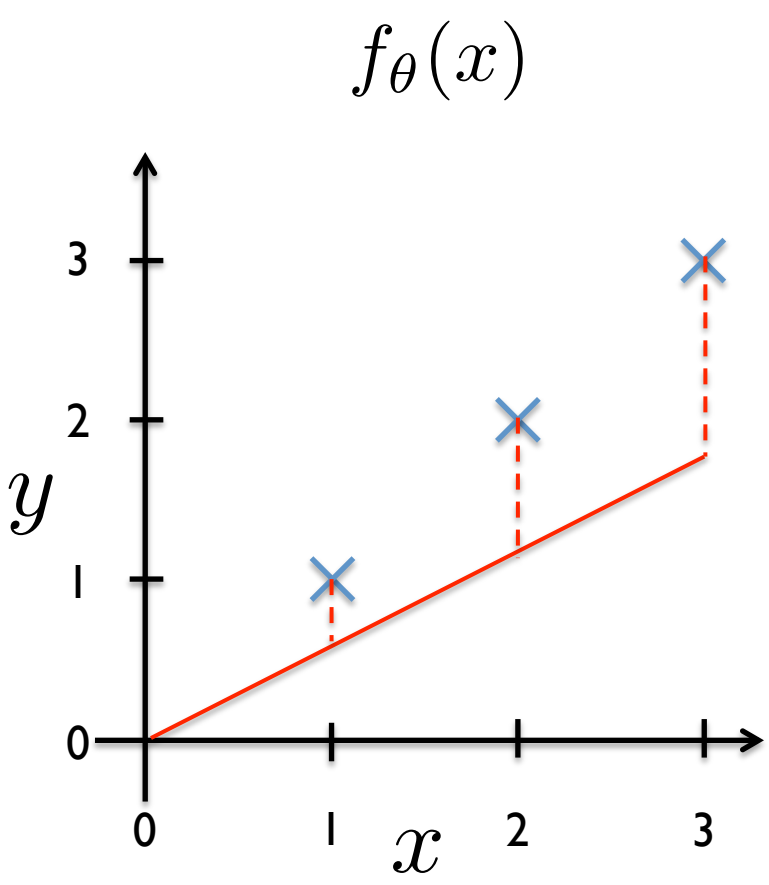
Simplified Cost Function

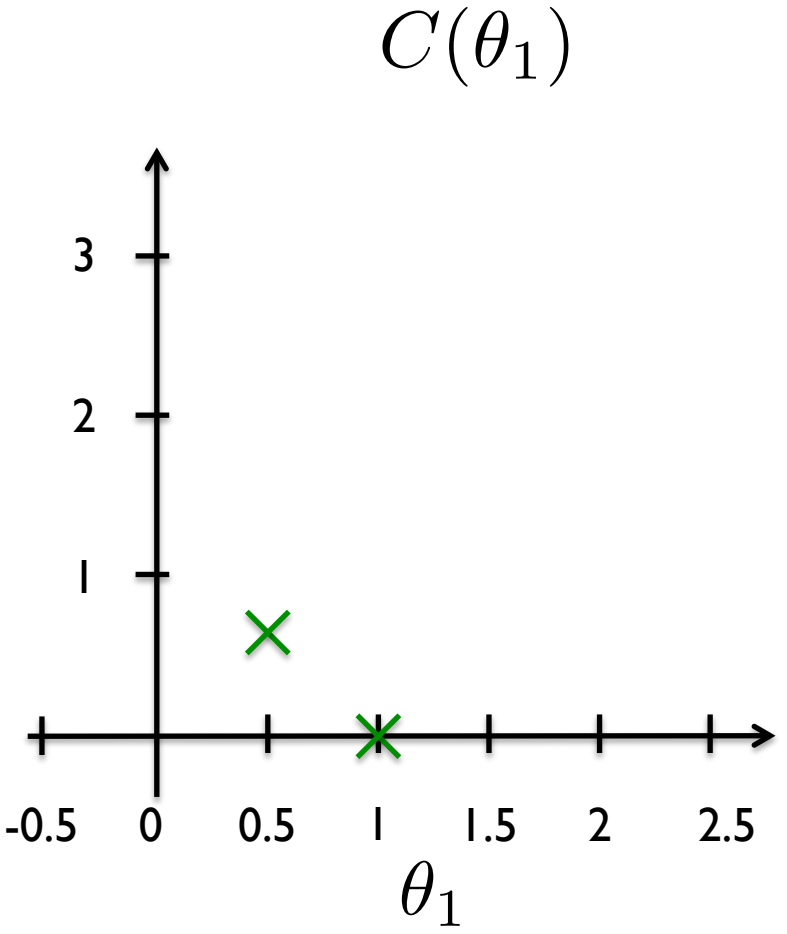




$$C(\theta_1 = 1) = \frac{1}{2m}(0^2 + 0^2 + 0^2)$$

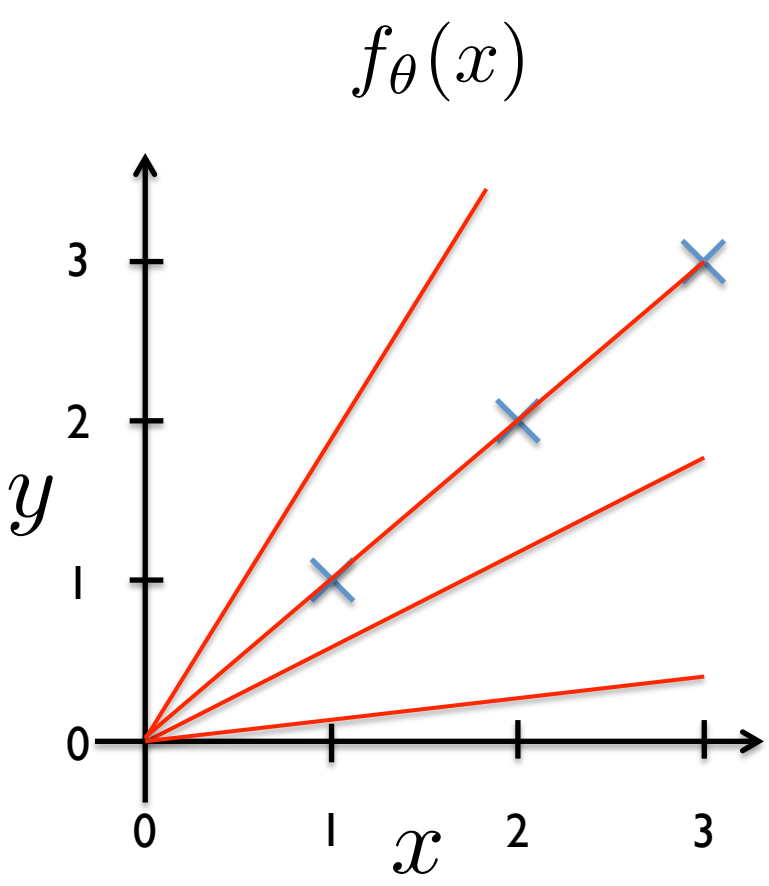
Simplified Cost Function

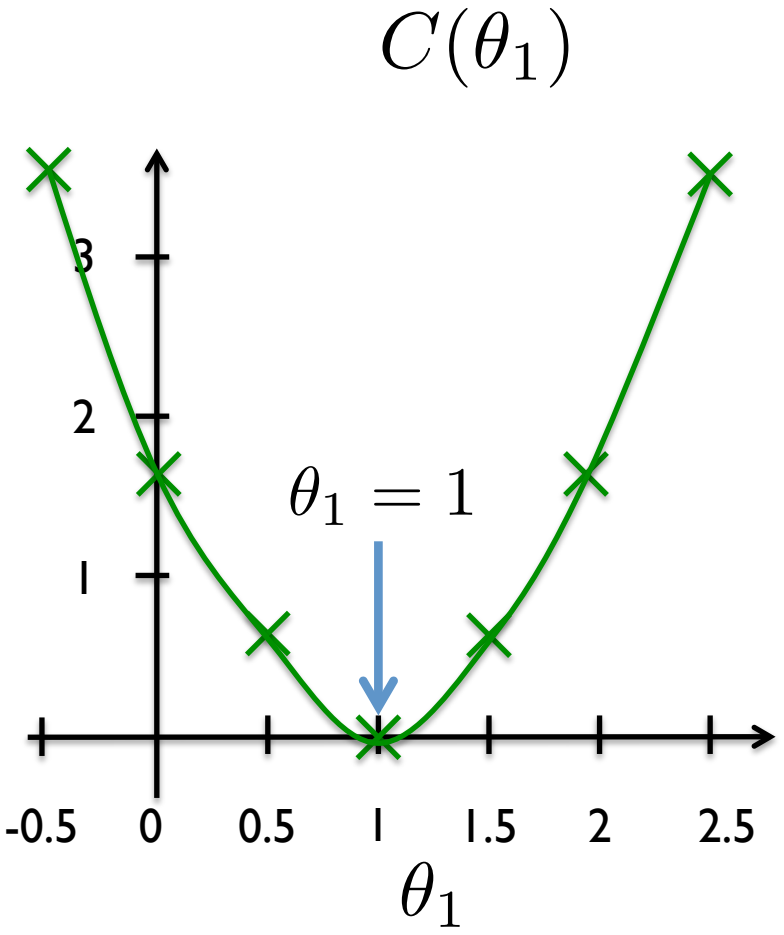




$$C(\theta_1 = 0.5) = \frac{1}{2m}(0.5^2 + 1^2 + 1.5^2) \approx 0.58$$

Simplified Cost Function

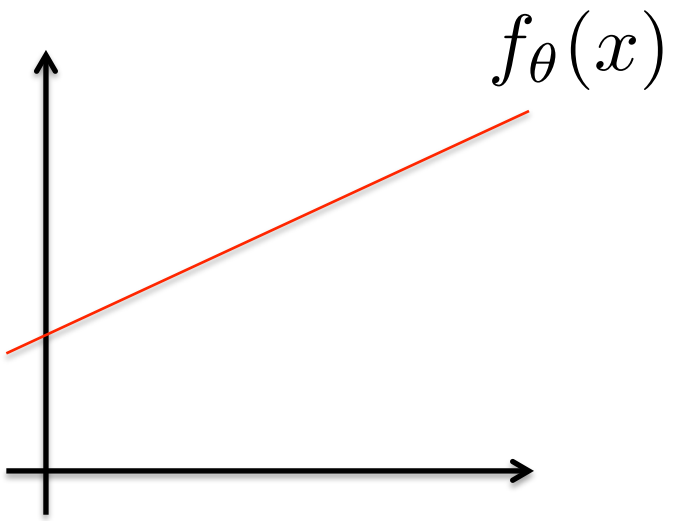




Cost Function

Function:

$$f_\theta(x) = \theta_0 + \theta_1 x$$



Parameters:

$$\theta_0, \theta_1$$

Cost Function:

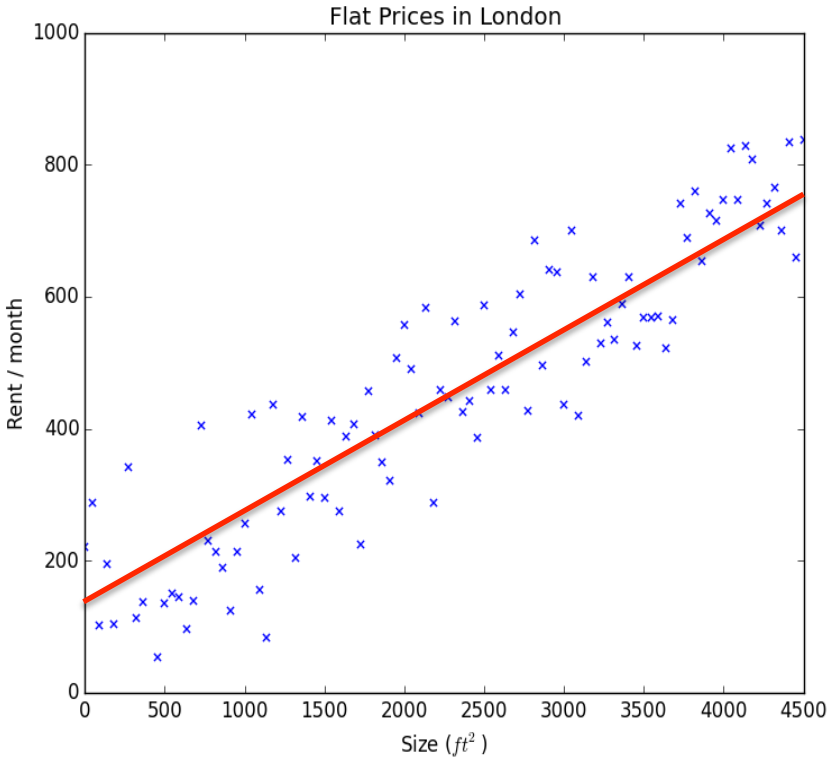
$$C(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

Goal:

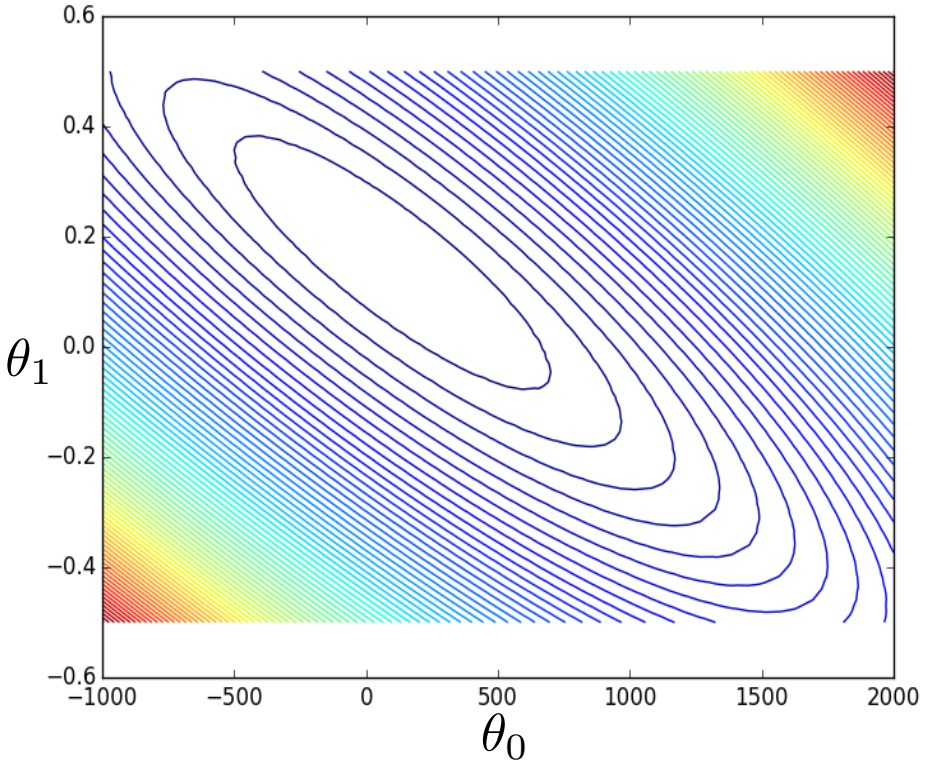
$$\underset{\theta_0, \theta_1}{\text{minimize}}\ C(\theta_0, \theta_1)$$

Cost Function

$f_\theta(x)$

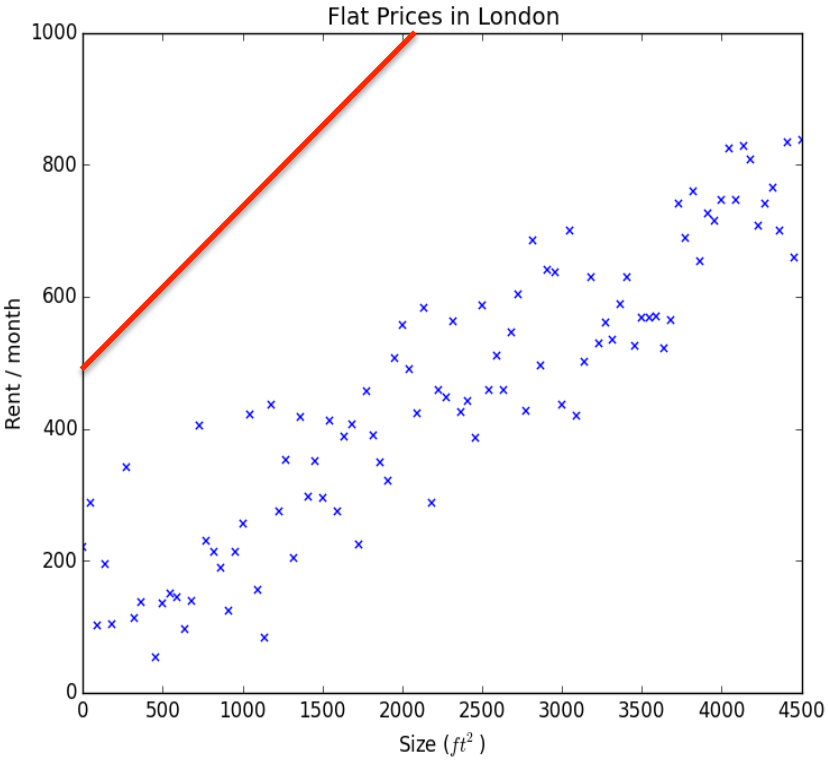


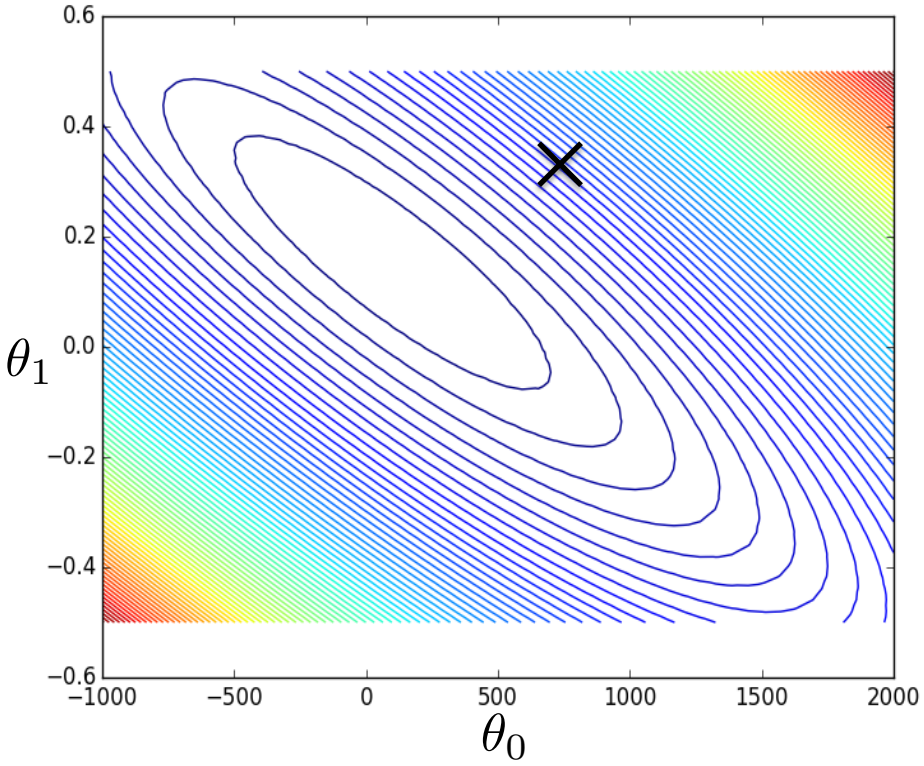
$C(\theta_0, \theta_1)$



Cost Function

Points on cost surface correspond to different functions





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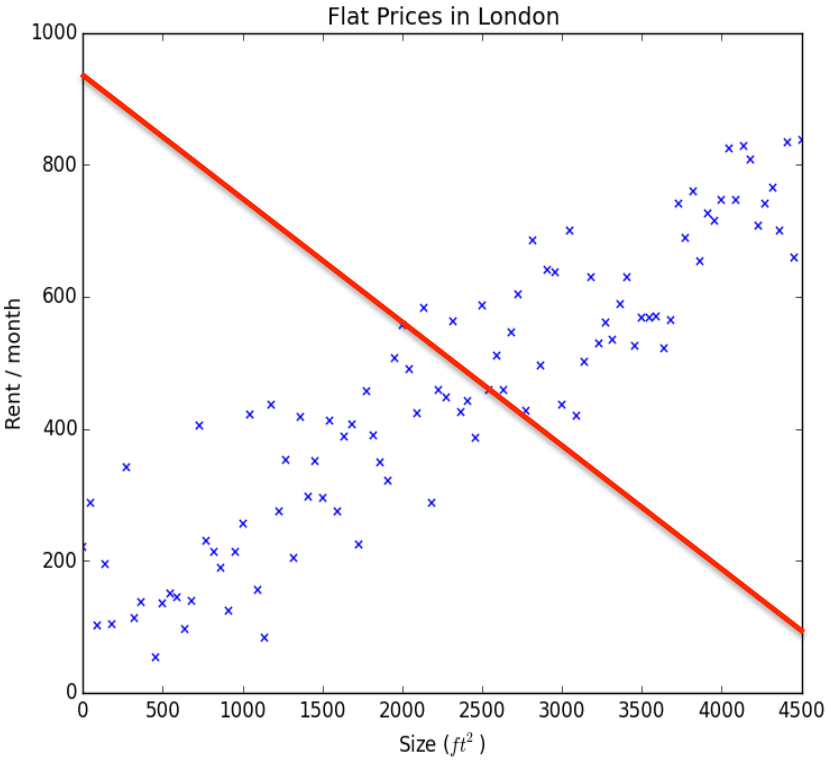
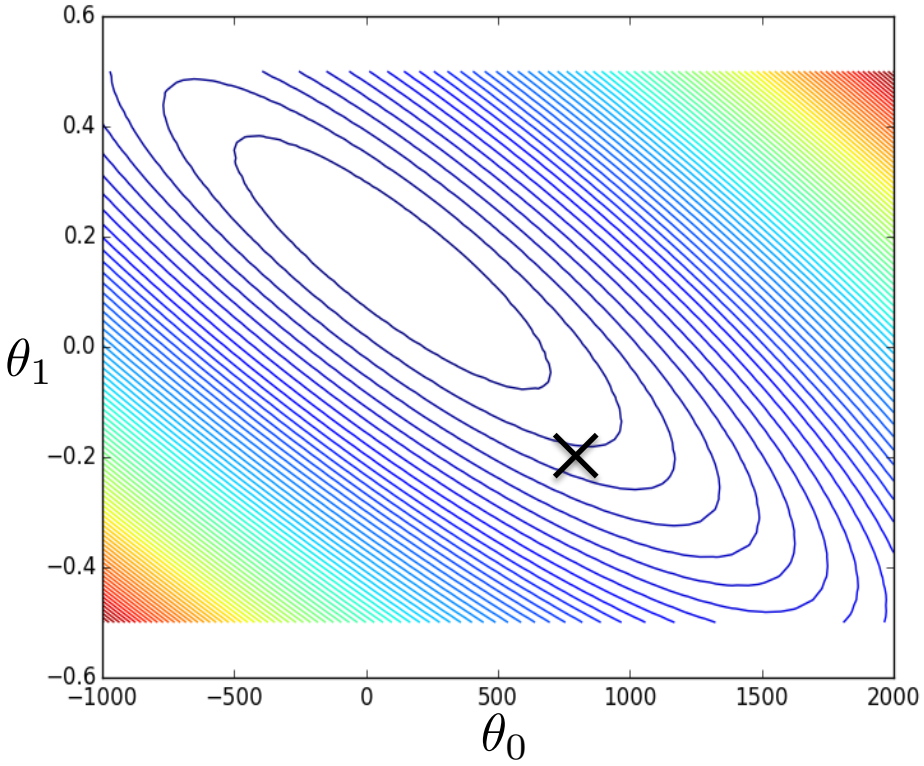
Cost Function

Points on cost surface correspond to different functions





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Cost Function

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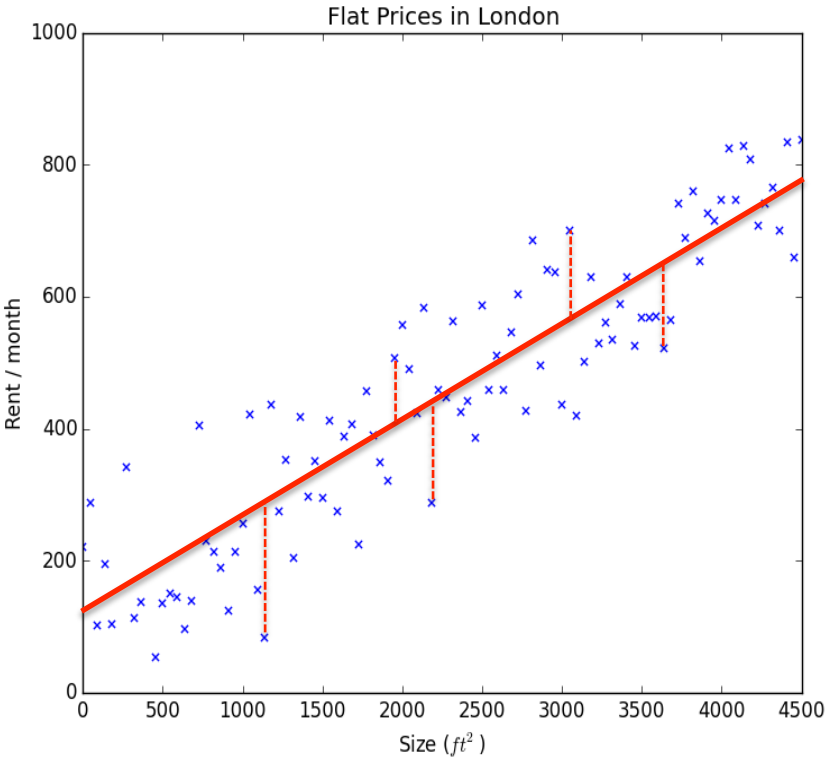

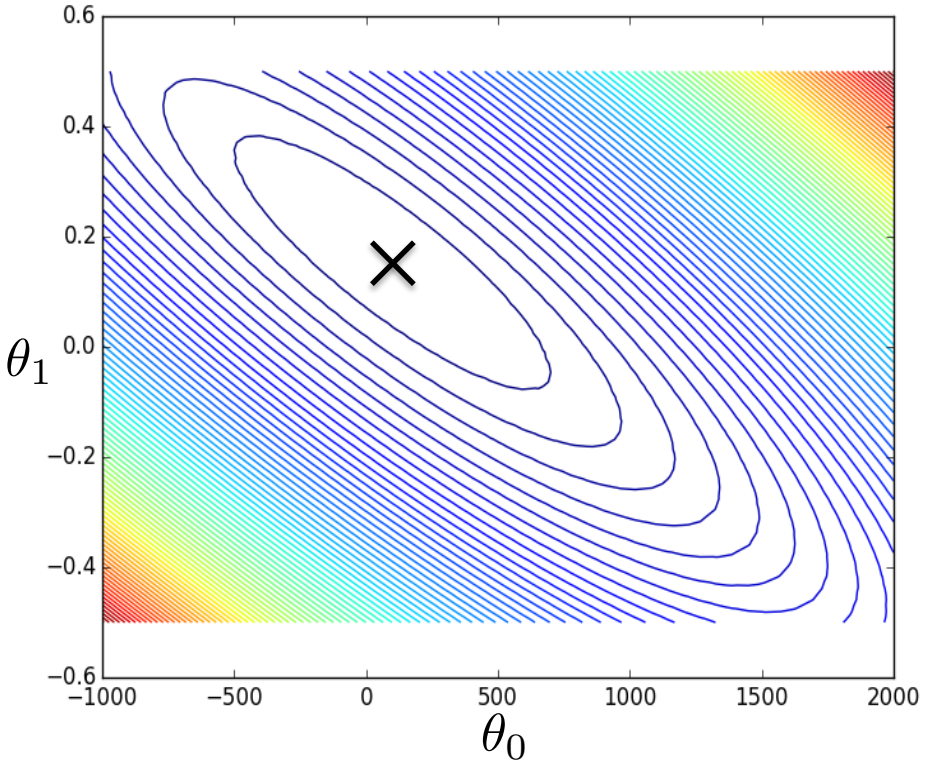

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EXERCISE

Time: ? mins

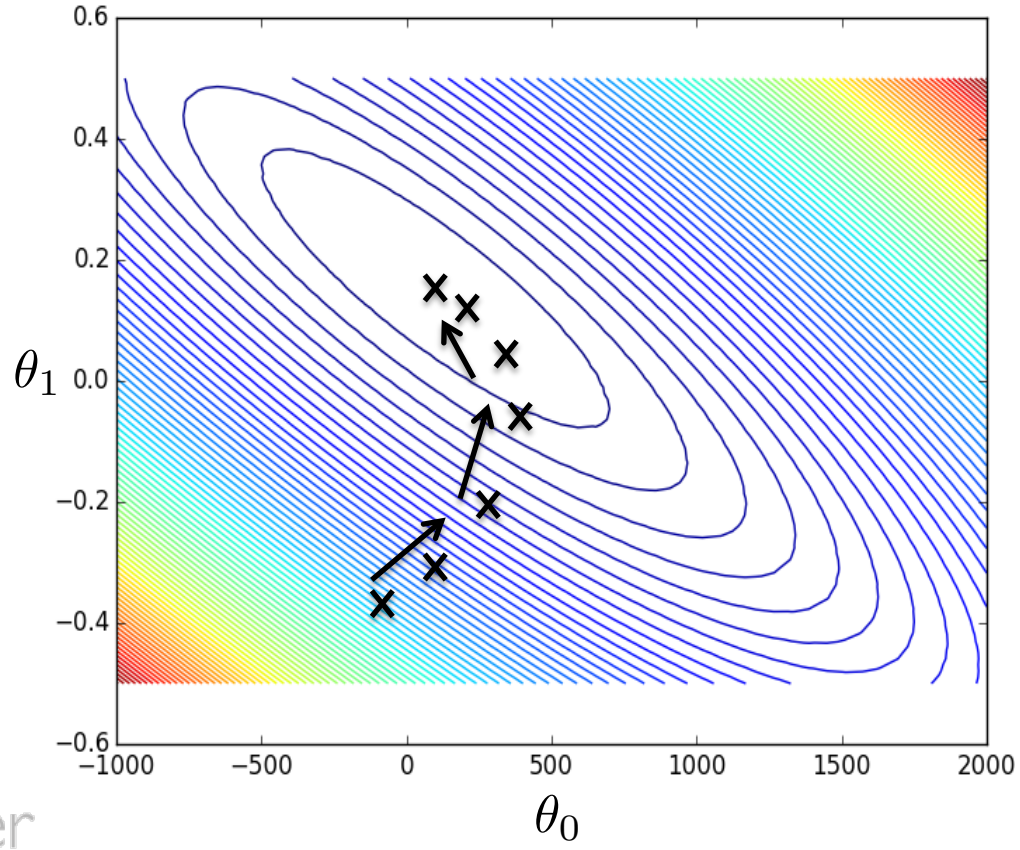
Linear Regression Cost Function

- Open *Linear Regression iPython Notebook* and complete exercises up to Gradient Descent

Gradient Descent

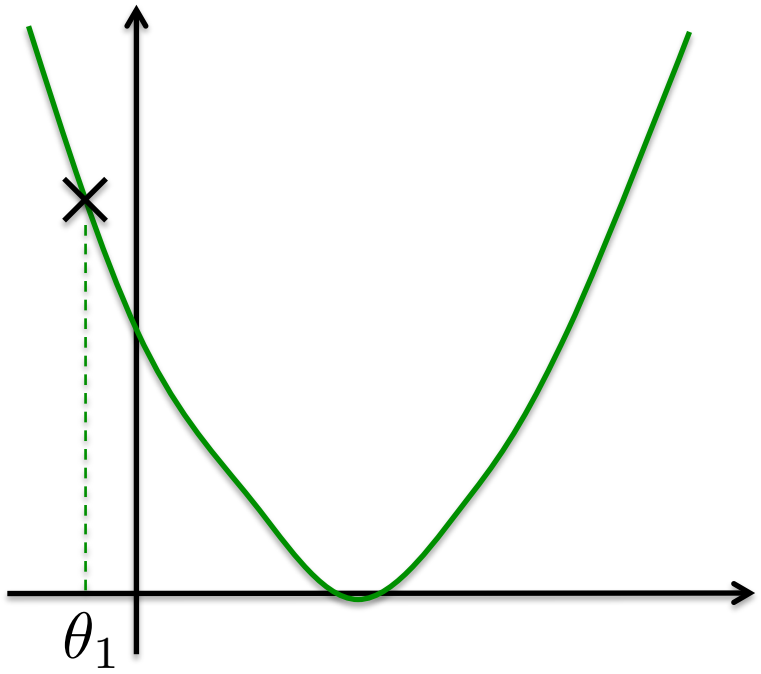
Minimising the Cost Function

Keep changing θ_1 and θ_0 so that C reduces



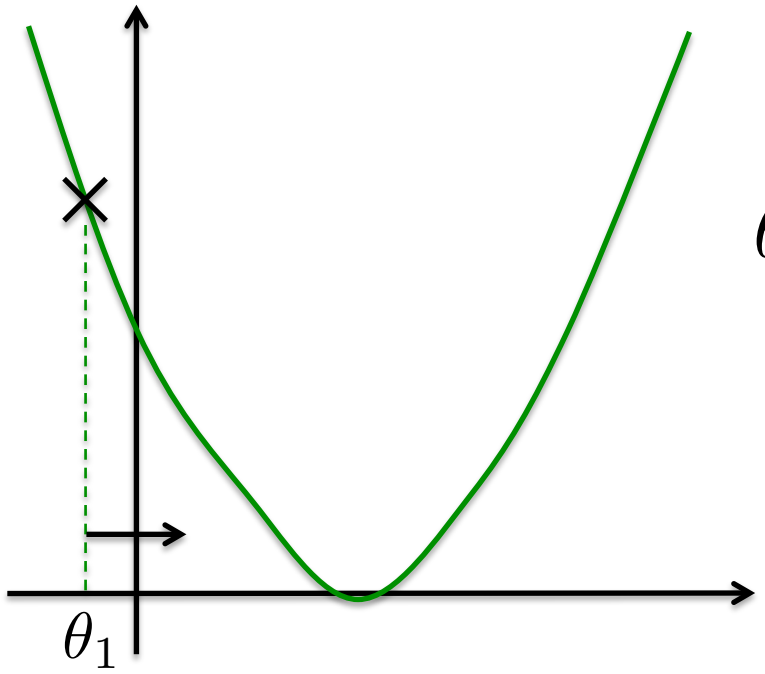
Minimising the Cost Function

Keep changing θ_1 and θ_0 so that C reduces



Minimising the Cost Function

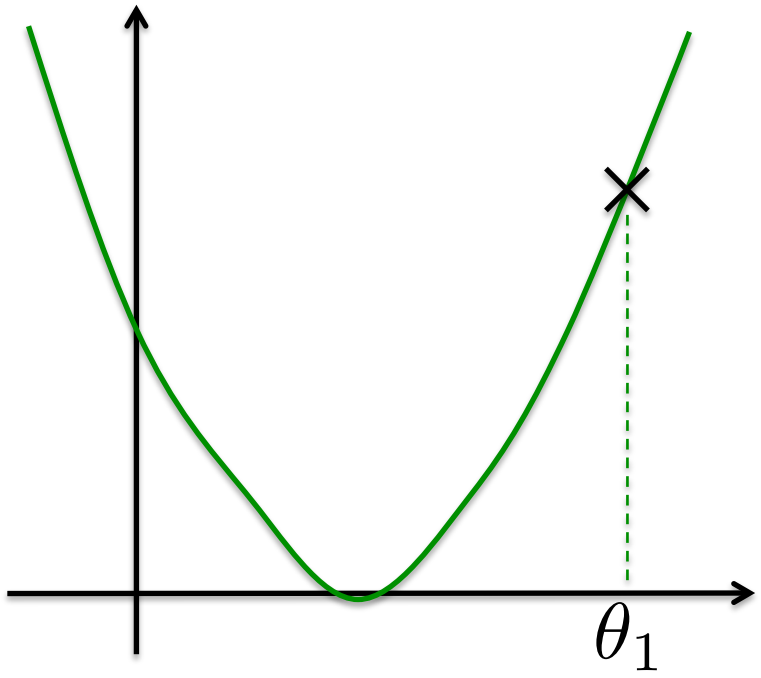
Keep changing θ_1 and θ_0 so that C reduces



$$\theta_1 := \theta_1 + \text{positive number}$$

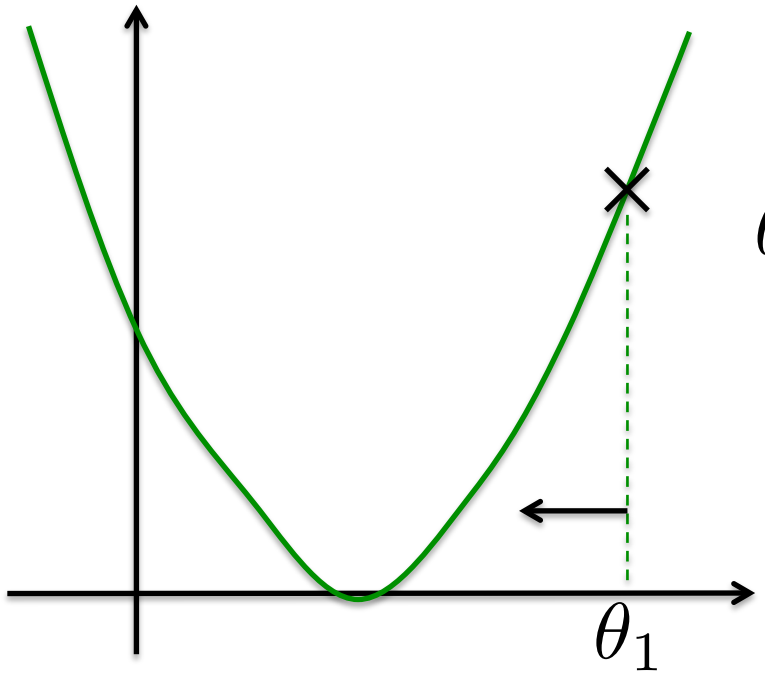
Minimising the Cost Function

Keep changing θ_1 and θ_0 so that C reduces



Minimising the Cost Function

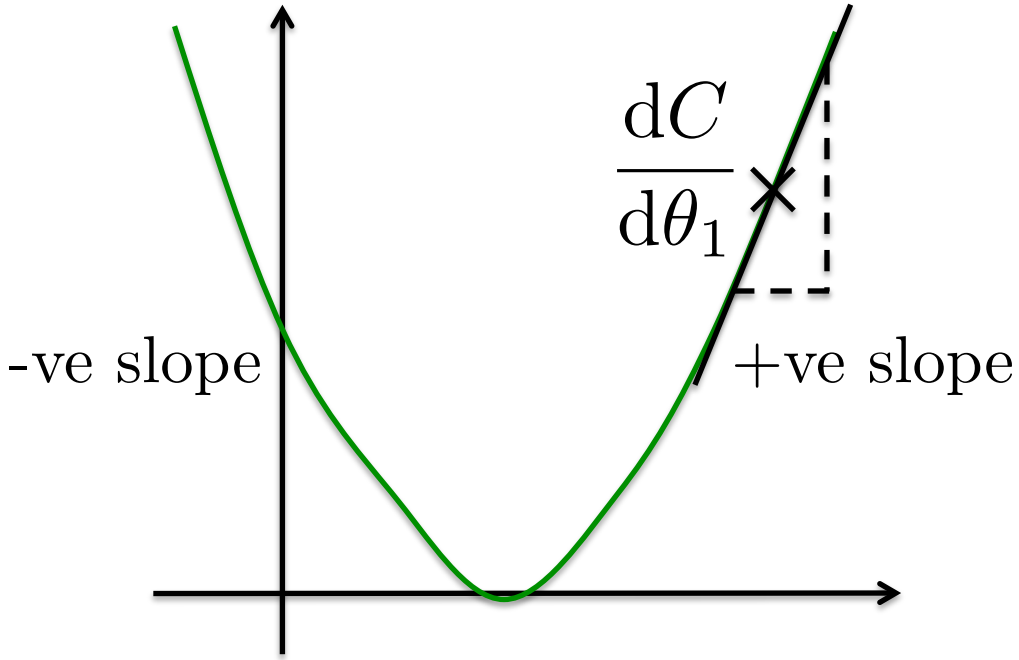
Keep changing θ_1 and θ_0 so that C reduces



$$\theta_1 := \theta_1 + \text{negative number}$$

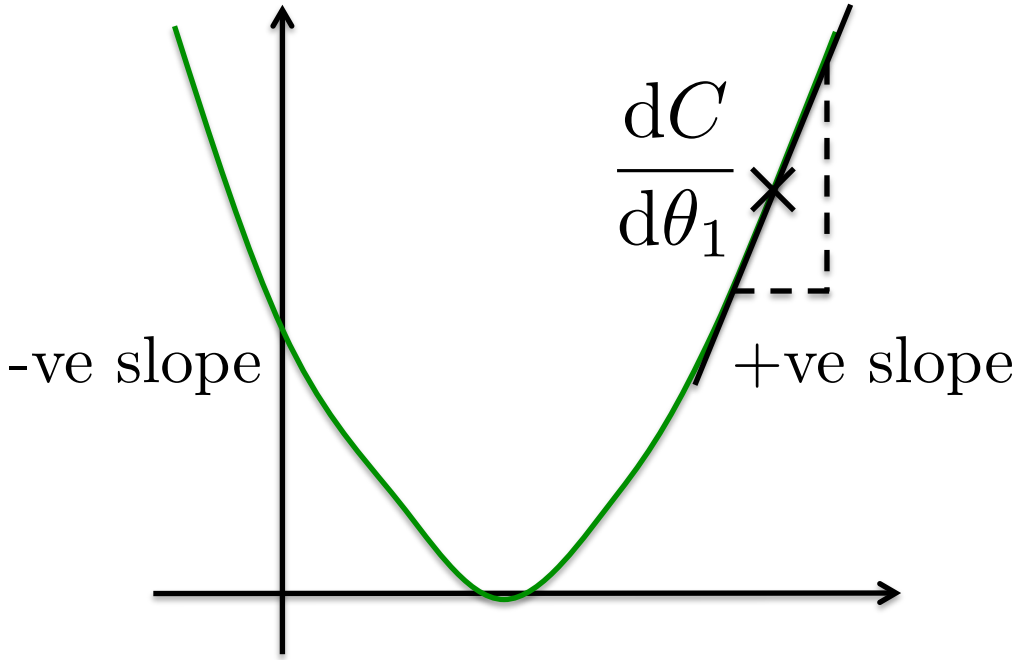
Minimising the Cost Function

Gradient of line tells us which direction to move



Minimising the Cost Function

Gradient of line tells us which direction to move

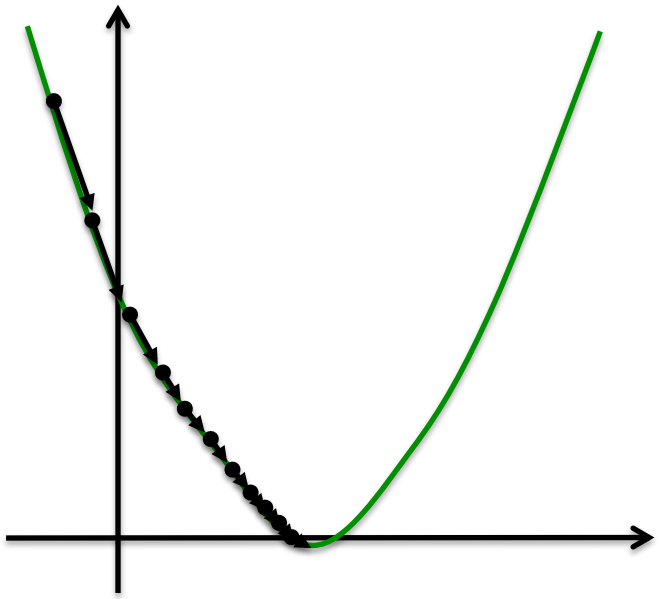


$$\theta_1 := \theta_1 - \alpha \frac{\mathrm{d}C}{\mathrm{d}\theta_1}$$

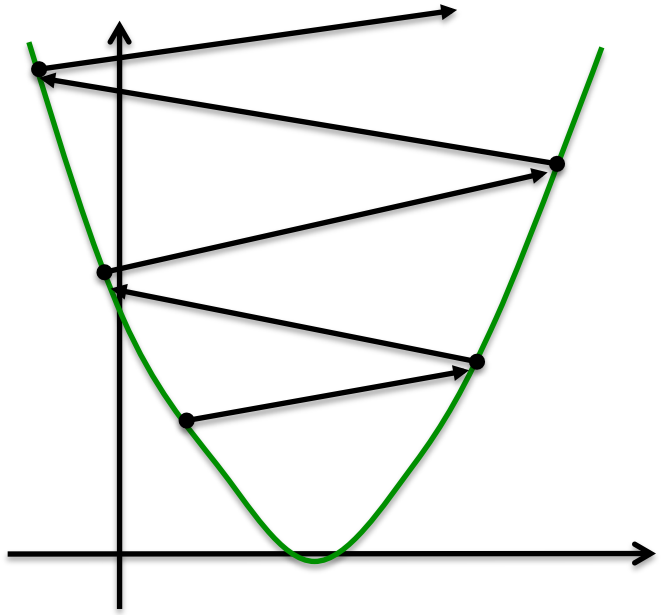
-ve α!

Choosing rate parameter

Convergence of algorithm depends on α



Too small = slow convergence

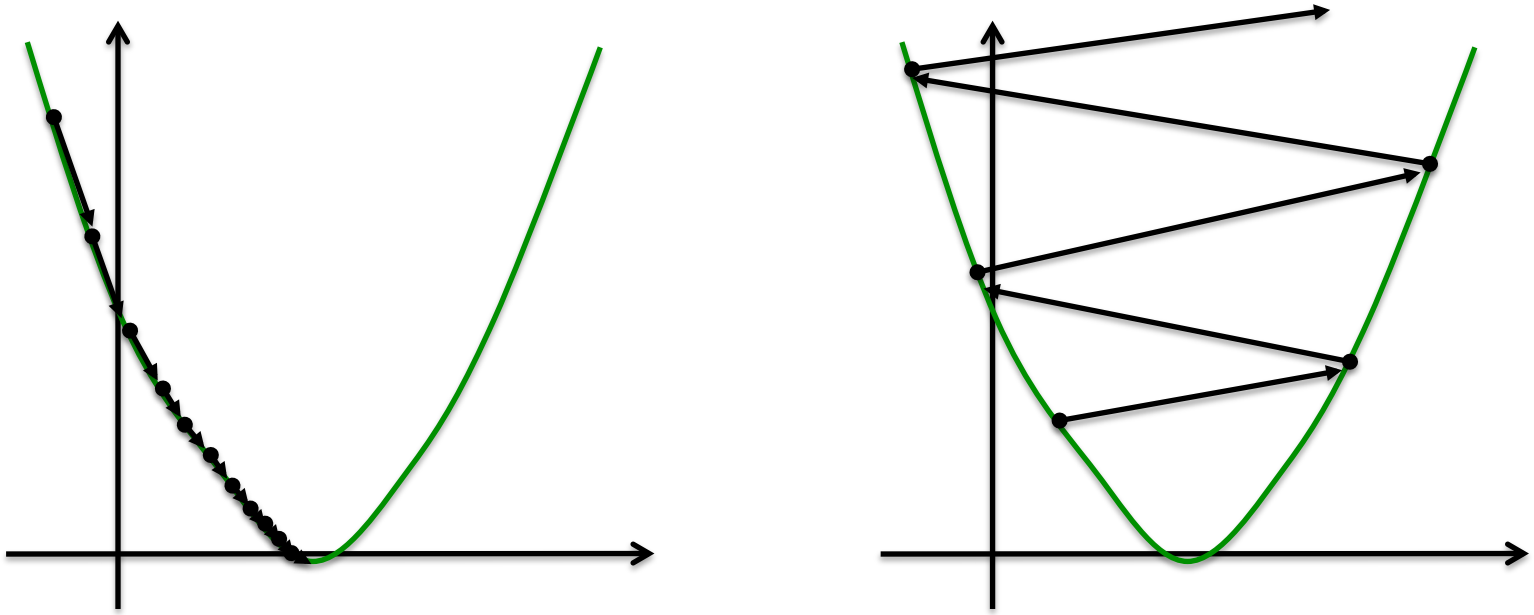


Too big = divergent

$$\theta_1 := \theta_1 - \alpha \frac{\mathrm{d}C}{\mathrm{d}\theta_1}$$

Choosing rate parameter

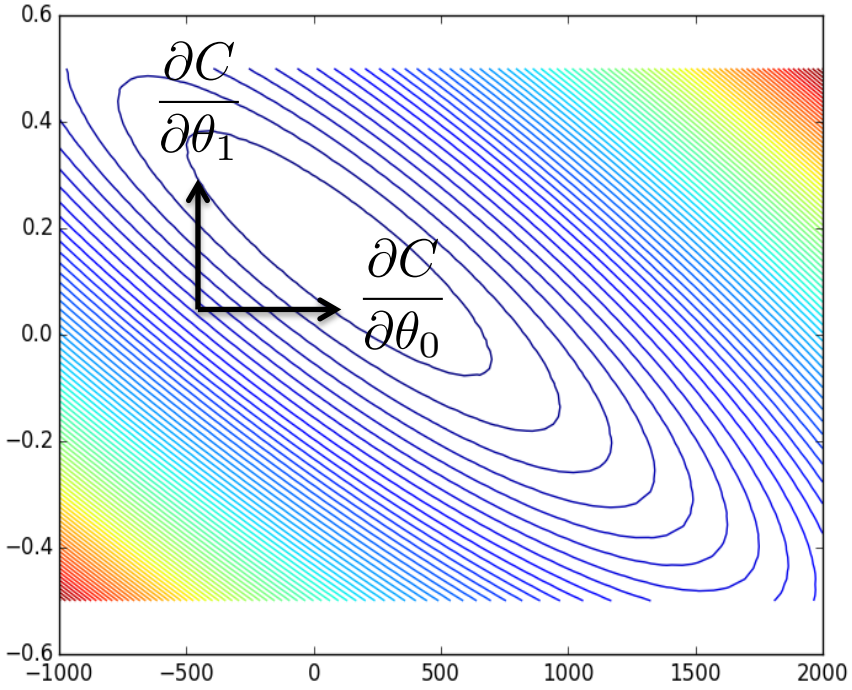
Convergence of algorithm depends on α



NOTE - Gradient Descent will naturally take smaller steps as we approach a local minimum

Full Problem

We must take PARTIAL DERIVATIVES



$$\theta_0 := \theta_0 - \alpha \frac{\partial C}{\partial \theta_0}$$

$$\theta_1 := \theta_1 - \alpha \frac{\partial C}{\partial \theta_1}$$

Gradient Descent Algorithm

Linear Model:

$$f_\theta(x) = \theta_0 + \theta_1 x \qquad C(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

Gradient Descent Algorithm:

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial C}{\partial \theta_j} \qquad (\text{for } j = 0 \text{ and } j = 1)$$

}

Formula for the update equations

$$C(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)^2$$

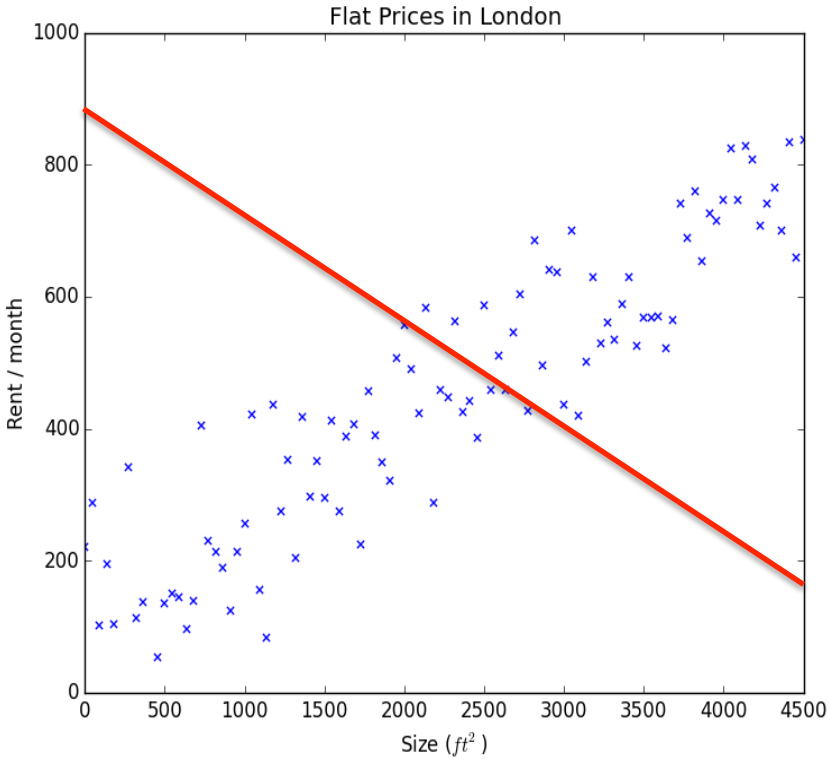
$$= \frac{1}{2m} \sum_{i=1}^{m} (\theta_0 + \theta_1 x^i - y^i)^2$$

$$\theta_0 : \quad \frac{\partial C}{\partial \theta_0} = \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i)$$

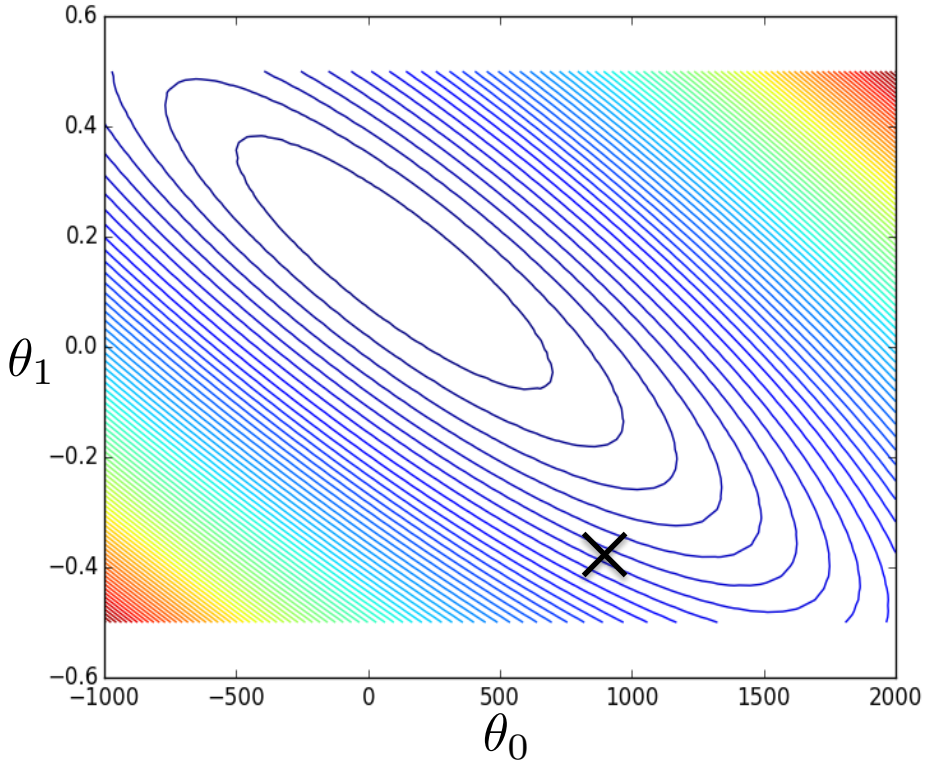
$$\theta_1 : \quad \frac{\partial C}{\partial \theta_1} = \frac{1}{m} \sum_{i=1}^{m} (f_\theta(x^i) - y^i) \cdot x^i$$

Using Gradient Descent

$f_\theta(x)$

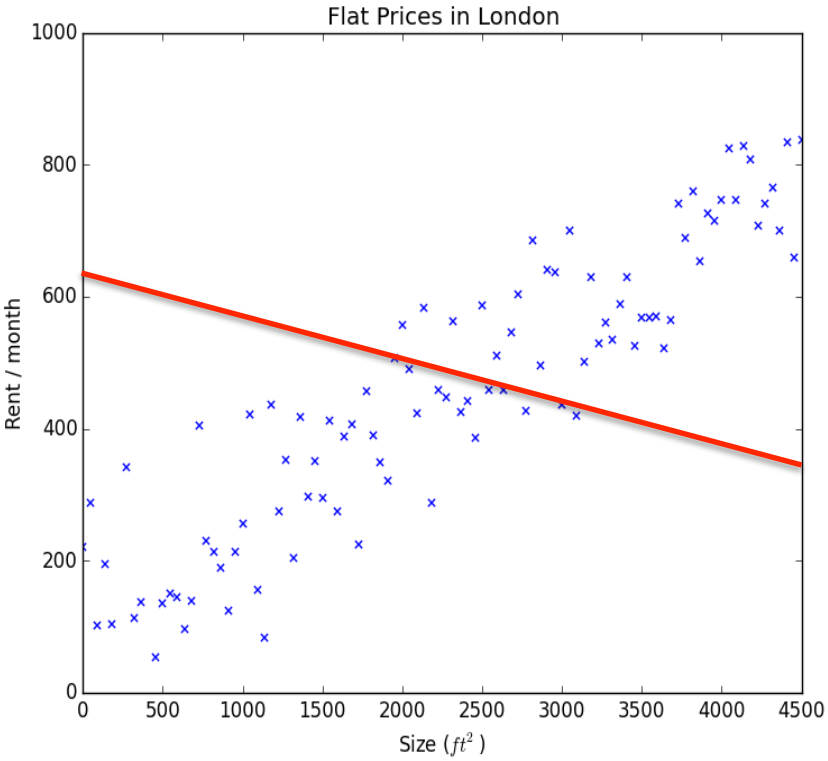


$C(\theta_0, \theta_1)$

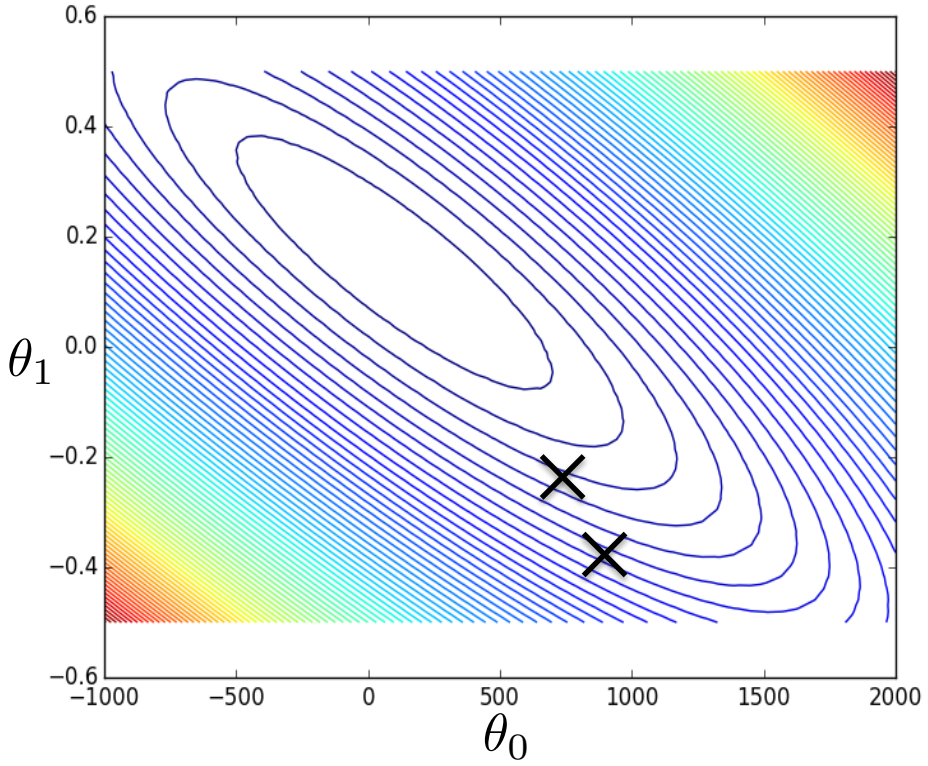


Using Gradient Descent

$f_\theta(x)$

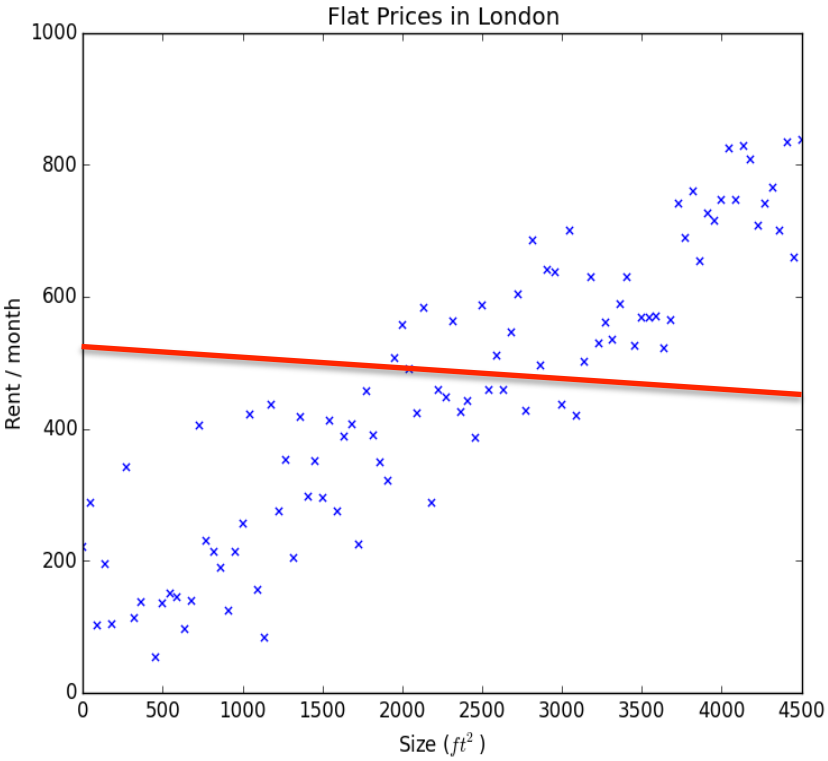


$C(\theta_0, \theta_1)$

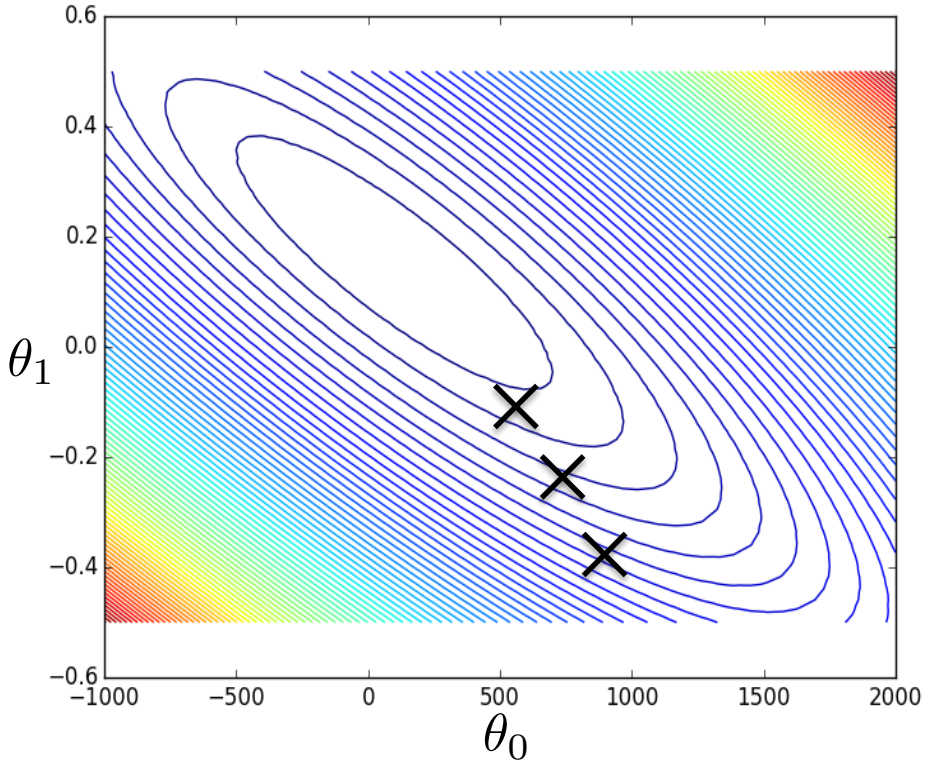


Using Gradient Descent

$f_\theta(x)$



$C(\theta_0, \theta_1)$



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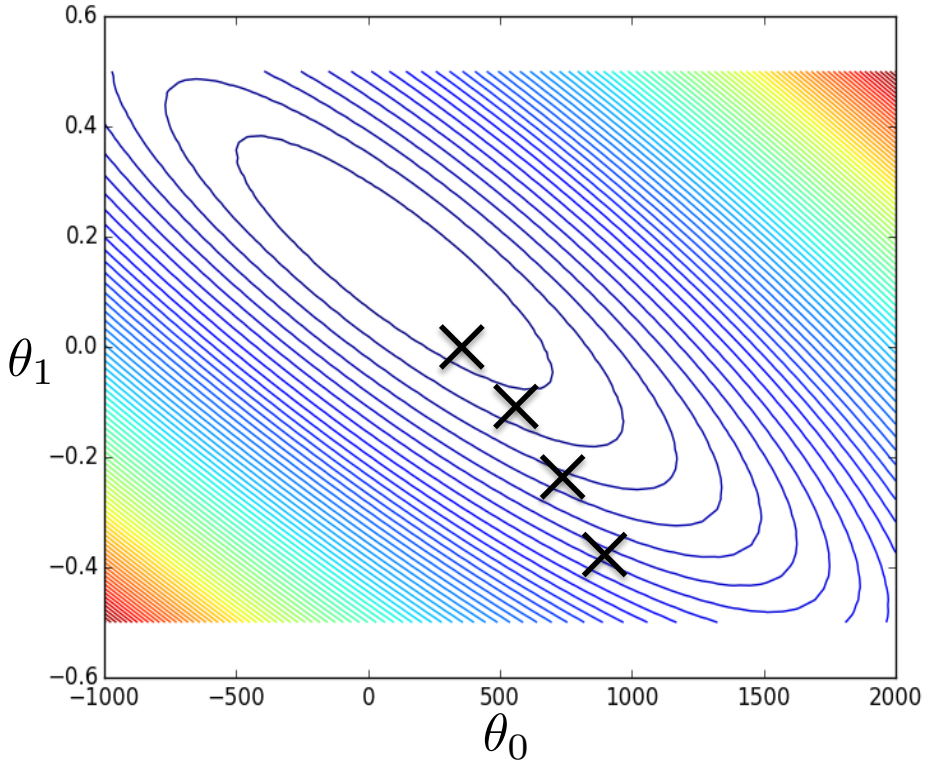
Using Gradient Descent

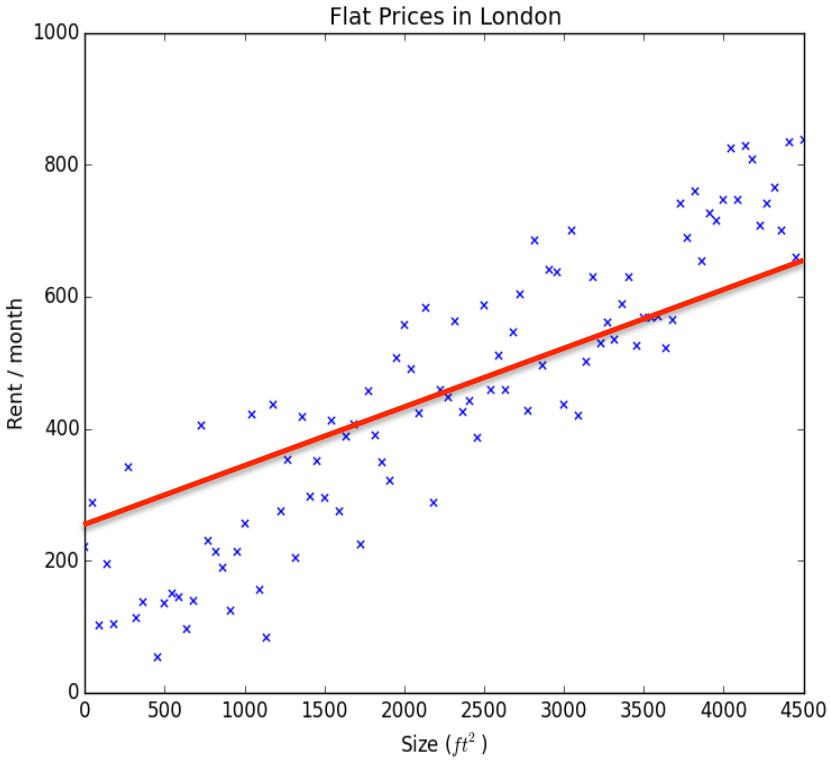
$f_\theta(x)$

$C(\theta_0, \theta_1)$

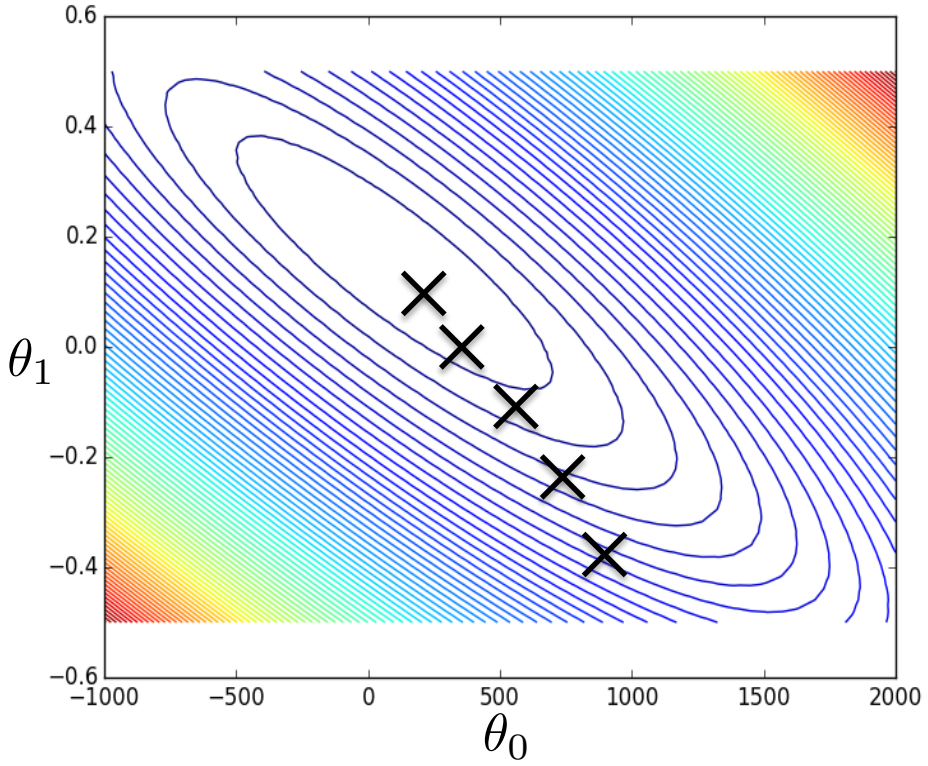


Using Gradient Descent

$f_\theta(x)$



$C(\theta_0, \theta_1)$



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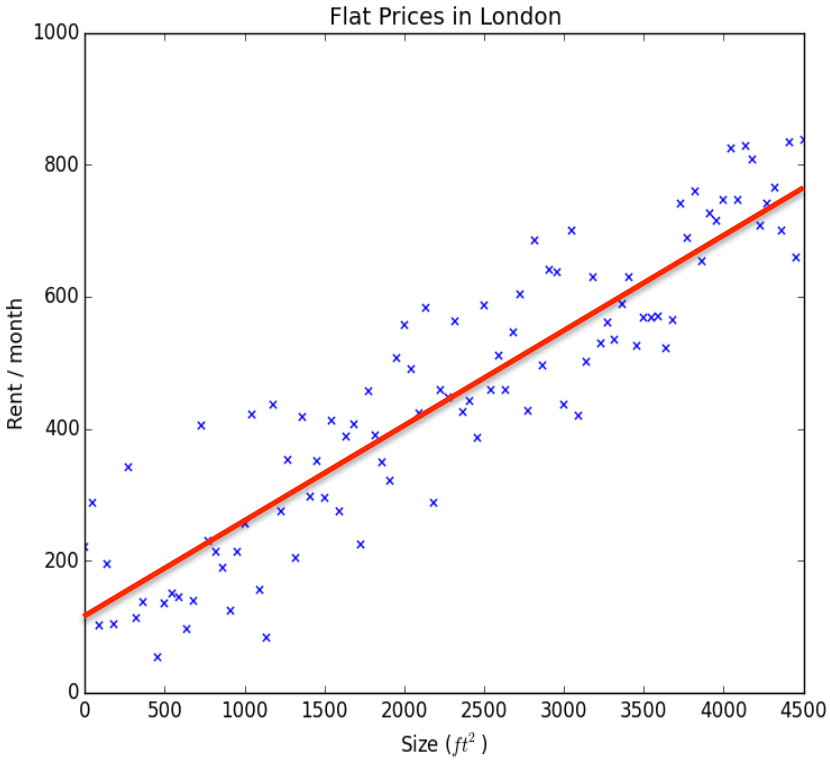
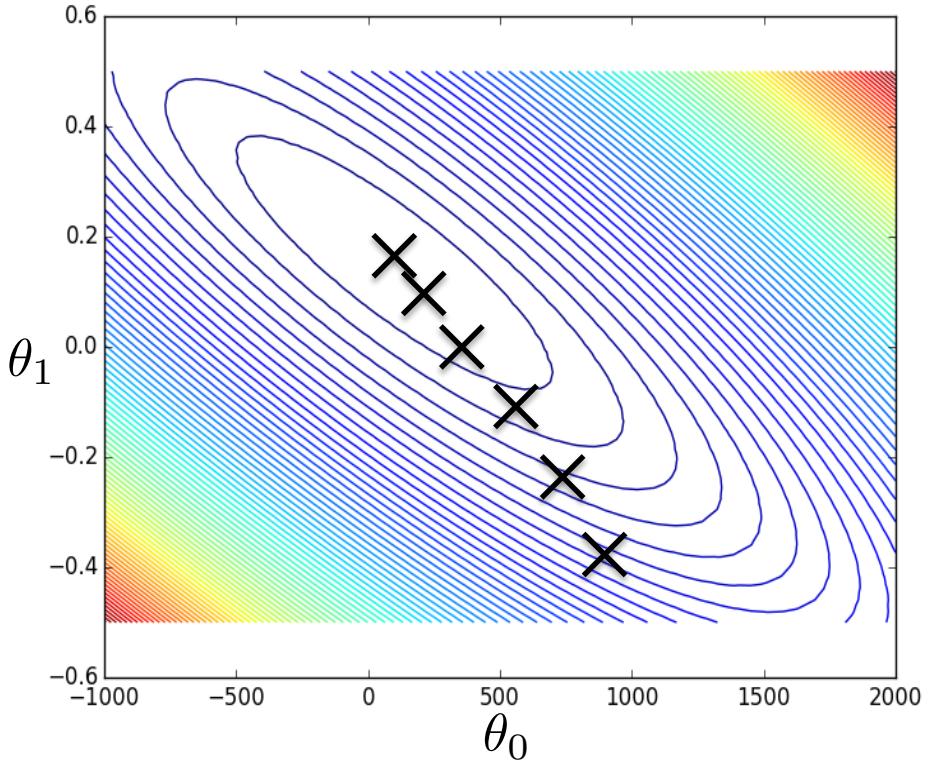
Using Gradient Descent

$f_\theta(x)$



$C(\theta_0, \theta_1)$



EXERCISE

Time: ? mins

Linear Regression Gradient Descent

- Finish exercises in *Linear Regression iPython Notebook*