Probability Review

Klein
carlj.klein@gmail.com

1 Cards

Question1:
Inclusion-Exclusion ID
Hint: P(A $\bigcup$ B) = ?
Answer1:
P(A $\bigcup$ B) = P(A) + P(B) - P(AB)

Question2:
Define Mutually Exclusive
Answer2:
If AB = $\emptyset \rightarrowtail$ A and B are mutually exclusive

Question3:
Conditional Probability and Corollary
Answer3:
Definition: P(E|F) = $\frac{P(EF)}{P(F)}$
Corollary: P(EF) = P(E) * P(F|E)

Question4:
Multiplication Rule
Hint: extension of conditional probability: $\rightarrow P(E_1 *** E_n) =$?
Answer4:
$P(E_1 *** E_n) = P(E_1) * P(E_2 \mid E_1) * P(E_3 \mid E_2 * E_1) *** P(E_n \mid E_{n-1} *** E_1)$

Question5:
Law of Total Probability
Hint: Given a mutually exclusive and exhaustive set A, P(A_1) + ... + P(A_k) = 1, what can be deduced about the probability of an event B occurring?
Answer5:
P(B)

= P(BA_1) + ... + P(BA_k)
= P(A_1) * P(B | A_1) + ... + P(A_k) * P(B | A_k)
= $\sum_{i=1}^{k} P(A_i) * P(B \mid A_i)$

Question6:
Bayes' Theorem
Answer6:
Given a mutually exclusive and exhaustive set A, P(A_1) + ... + P(A_k) = 1, then
$P(A_j \mid B) =$
$\frac{P(A_j B)}{P(B)} =$
$\frac{P(A_j) * P(B|A_j)}{\sum_{i=1}^{k} P(A_i) * P(B|A_i)}$

Question7:
How does independence extend to intersections and conditional probability?
Answer7:
Given independent events, A and B:
P(AB) = P(A) * P(B)
P(A|B) = P(A) and P(B|A) = P(B)

Question8:
Cumulative Distribution Function (CDF)
Answer8:
F(x) = P(X $\leq$ x)

Question9:
Properties of the CDF
Answer9:

- 0 $\leq$ F(x) $\leq$ 1
- If x $\leq$ y $\rightarrow$ F(x) $\leq$ F(y)
- $lim_{x \rightarrow \infty} F(x) = 1$
- $lim_{xto-\infty} F(x) = 0$

Question10:
Probability Mass Function (pmf)
Answer10:
Note: for finate or coutnable infinite set (i.e. discrete)
p(x) = P(X=x)

Question11:
For discrete values, describe the following:

- E[X]
- E[X^2]
- E[g(X)]
- E[aX + b]
- E[aX + bY]

Answer11:

- E[X] = $\sum_x xP(X = x)$
- E[X^2] = $\sum_x x^2 P(X = x)$
- E[g(X)] = $\sum_x g(x)P(X = x)$
- E[aX + b] = aE[X] + b
- E[aX + bY] = aE[X] + bE[Y]

Question12:
For discrete values, describe the following:

- Var(X)
- Var(cX)
- Var(aX + b)
- sd(X)

Answer12:

- Var(X) = E[(X - μ)^2] = E[X^2] - (E[X])^2 = σ^2
- Var(cX) = c^2Var(X)
- Var(aX + b) = a^2Var(X)
- sd(X) = $\sqrt{Var(X)} = \sigma$

Question13:
Define moment and compare it to σ

Answer13:
E[X^k] = k^{th} raw moment
σ = mean = 1st raw moment

Question14:
Define covariance and provide the formula.
Answer14:
Cov(X, Y) =
E[(X - E[X])(Y - E[Y])] =
E[XY] - E[X]E[Y]

Question15:
Describe the following properties of covariance:

- Cov(X, X)
- Cov(aX, Y)
- Cov($\sum_{i=1}^{n} X_i$, $\sum_{j=1}^{m} Y_j$)

Answer15:

- Cov(X, X) = E[X^2] - $(E[X])^2$ = σ^2 = Var(X)
- Cov(aX, Y) = aCov(X, Y)
- Cov($\sum_{i=1}^{n} X_i$, $\sum_{j=1}^{m} Y_j$) = $\sum_{i=1}^{n} \sum_{j=1}^{m}$ Cov(X_i, Y_j)

Question16:
Uniform Distribution (discrete)
Provide the following:

- pdf
- E[X]
- Var(X)

Answer16:

- P(X=x) = $\frac{1}{n}$
- E[X] = $\frac{n+1}{2}$ = $\frac{b+a}{2}$

- Var(X) = $\frac{n^2+1}{12} = \frac{(b-a+1)^2+1}{12}$

Question17:
Bernoulli Distribution (discrete)
Provide the following:

- pdf
- E[X]
- Var(X)

Answer17:
Example: flipping a coin for a single experiment, 0 represents heads (probability p) and 1 represents tails ()probability 1 - p).

- $$P(X = x) = \begin{cases} p; x = 0 \\ 1 - p; x = 1 \end{cases}$$
- E[X] = p
- Var(X) = p(1 - p)

Question18:
Binomial Distribution (discrete)
Provide the following:

- pdf
- E[X]
- Var(X)

Answer18:
Example: Multiple experiments where each experiment only has 2 outcomes (flipping a coin multiple times).
The distribution is the sum of Bernoulli's, number of successes with probability p in n independent trials.

- P(X=x) = $\binom{n}{x} p^x p(1-p)^{n-x}$
- E[X] = np
- Var(X) = np(1 - p)

Question19:
Geometric Distribution - "Starting at 0" (discrete)
Provide the following:

- pdf

- E[X]
- Var(X)

Answer19:
Example: x is the number of failures before a success (i.e. can be 0).

- P(X=x) = p(1-p)^x
- E[X] = $\frac{1-p}{p}$
- Var(X) = $\frac{1-p}{p^2}$

Question20:
Geometric Distribution - "Starting at 1" (discrete)
Provide the following:

- pdf
- E[X]
- Var(X)

Answer20:
Example: x is the number of trials necessary for a success, including the success (i.e. cannot be 0).

- P(X=x) = p(1-p)$^{x-1}$
- E[X] = $\frac{1}{p}$
- Var(X) = $\frac{1-p}{p^2}$