Data Structure: Binary Search Tree

Yongjun Park
Hanyang University

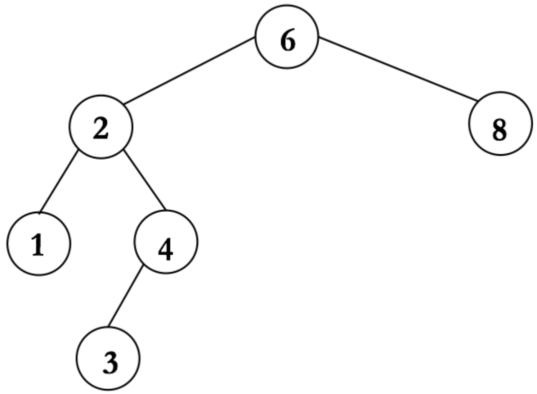
Binary Search Tree

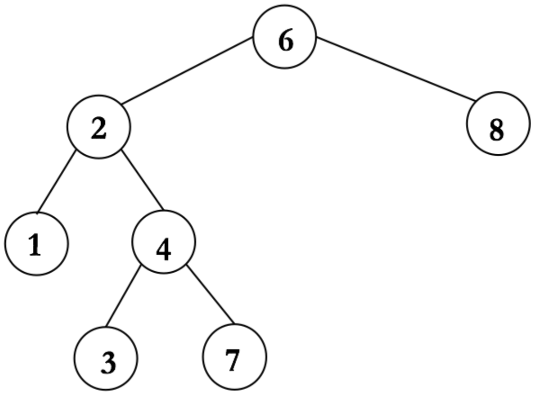
- one of the most fundamental problems in data structure design
  - there is a set of records $R_1, R_2, \ldots, R_n$, which are associated with distinct key values $X_1, X_2, \ldots, X_n$, respectively.
  - given a search key x, find the record if it occurs in the set.

- for every node X in the tree,
  - the values of all the keys in its left subtree are smaller than the key value in X
  - the values of all the keys in its right subtree are larger than the key value in X

Binary Search Tree

- which one can be the binary search tree?





Search in a linear array

- sequential search
  - simply store the keys in a linear array and search sequentially
  - insertion: $O(1)$, searching $O(n)$

- binary search
  - searching: $O(\log n)$, insertion/deletion: $O(n)$

Binary Search Tree

A BST ADT can process the following requests

- insert(x,T):
  - insert x into T
  - if x already exists, do appropriate action (e.g., do nothing, return error message, increment reference count.)

- delete(x,T):
  - delete x from T
  - if x does not exist, issue an error message

- find(x,T): search x in T

return either True/False or the pointer to the record

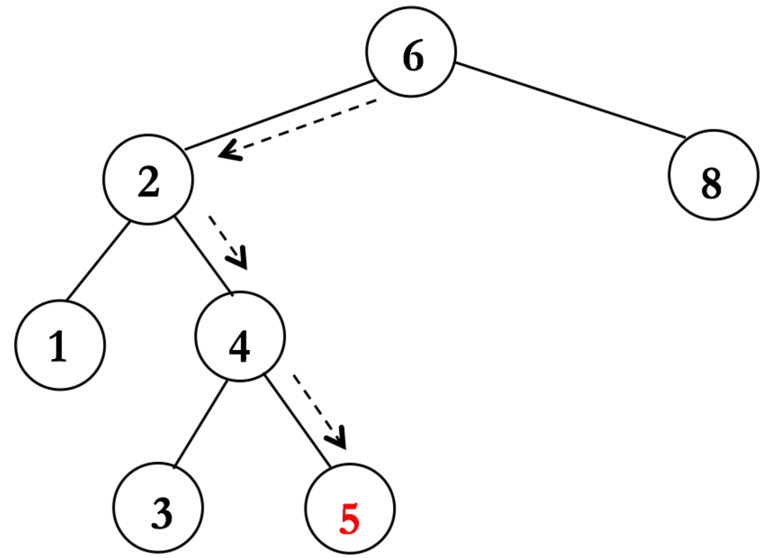
Binary Search Tree: data structure

```
struct TreeNode;
typedef struct TreeNode*   SearchTree;
typedef struct TreeNode*   Node;

struct TreeNode
{
   ElementType Element;
   SearchTree  Left;
   SearchTree  Right;
}
```

Binary Search Tree: Find

```
Node Find( ElementType X, SearchTree T )
{

   if  ( T == NULL )
              return NULL;

   if  ( X < T->Element )
              return Find( X, T->Left );

   else if ( X > T->Element )
              return Find( X, T->Right );

   else    /* X == T->Element */
              return T;
}
```

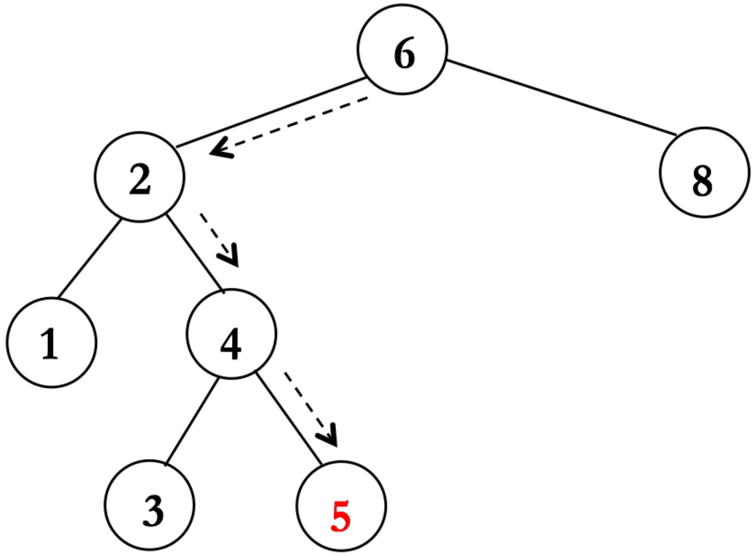
Find 5



Binary Search Tree: Find

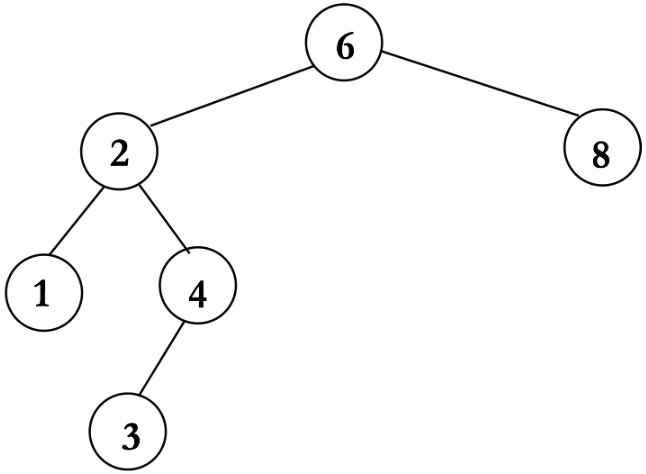
```
Node Find( ElementType X, SearchTree T )
{

   if  ( T == NULL )
             return NULL;

   if  ( X < T->Element )
             return Find( X, T->Left );

   else if ( X > T->Element )
             return Find( X, T->Right );

   else    /* X == T->Element */
             return T;
}
```

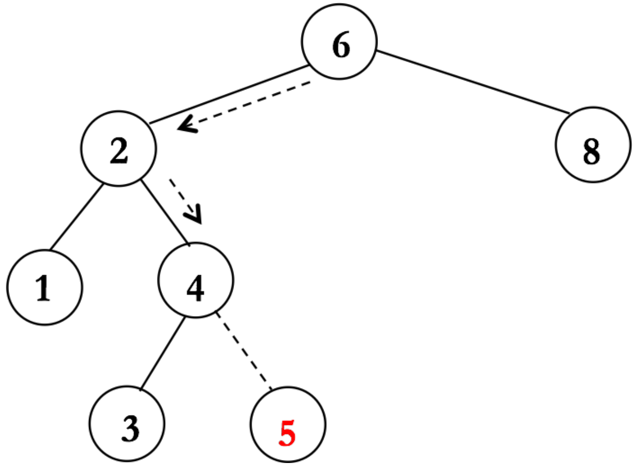
Find 2.5?



Binary Search Tree: Insert

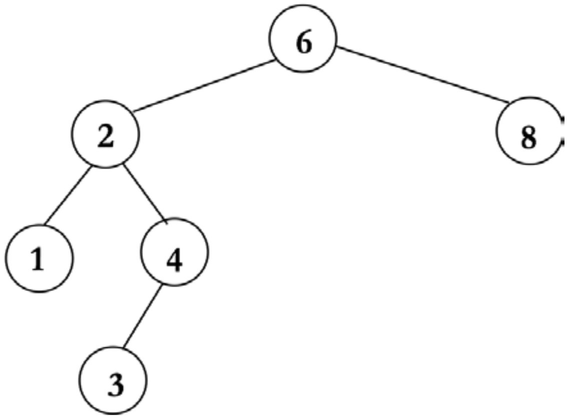
insertion of 5





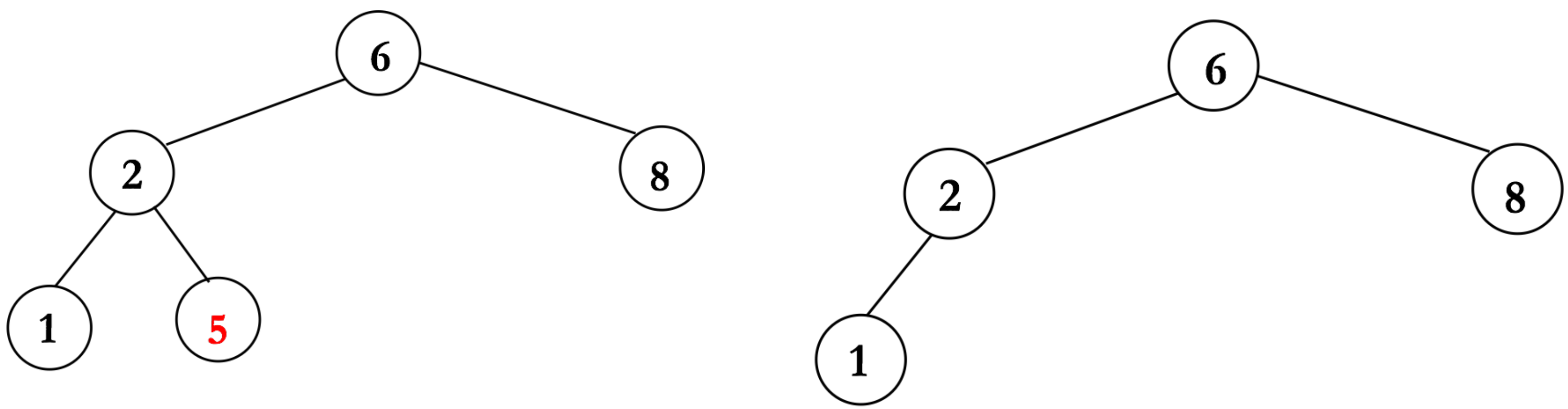
Binary Search Tree: Insert

```
SearchTree  Insert ( ElementType X, SearchTree T )
{
        if( T == NULL ) {

                T = malloc( sizeof( struct TreeNode ) );
                if( T == NULL )
                        FatalError( "Out of space!!!" );
                else
                {
                        T->Element = X;
                        T->Left = T->Right = NULL;
                }
        } else if( X < T->Element ) {
                T->Left = Insert( X, T->Left );

        } else if( X > T->Element )
                T->Right = Insert( X, T->Right );

        /* Else X is in the tree already; we'll do nothing */

        return T;  /* Do not forget this line! */
}
```



Binary Search Tree: Delete

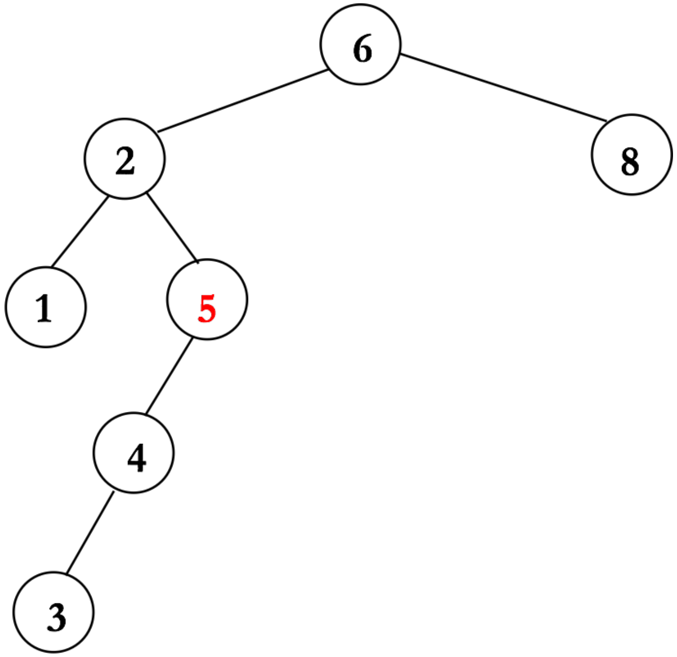
If the node to be deleted is a leaf , just delete it!

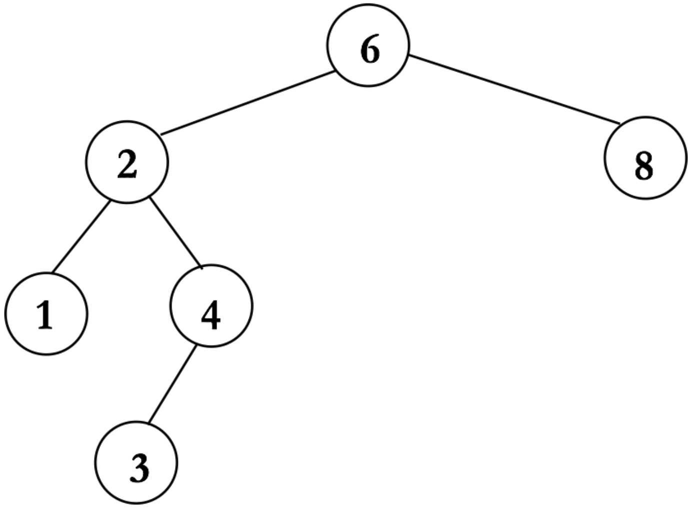


Binary Search Tree: Delete

If the node to be deleted has one child,

the child of the node is connected to the parent of the node

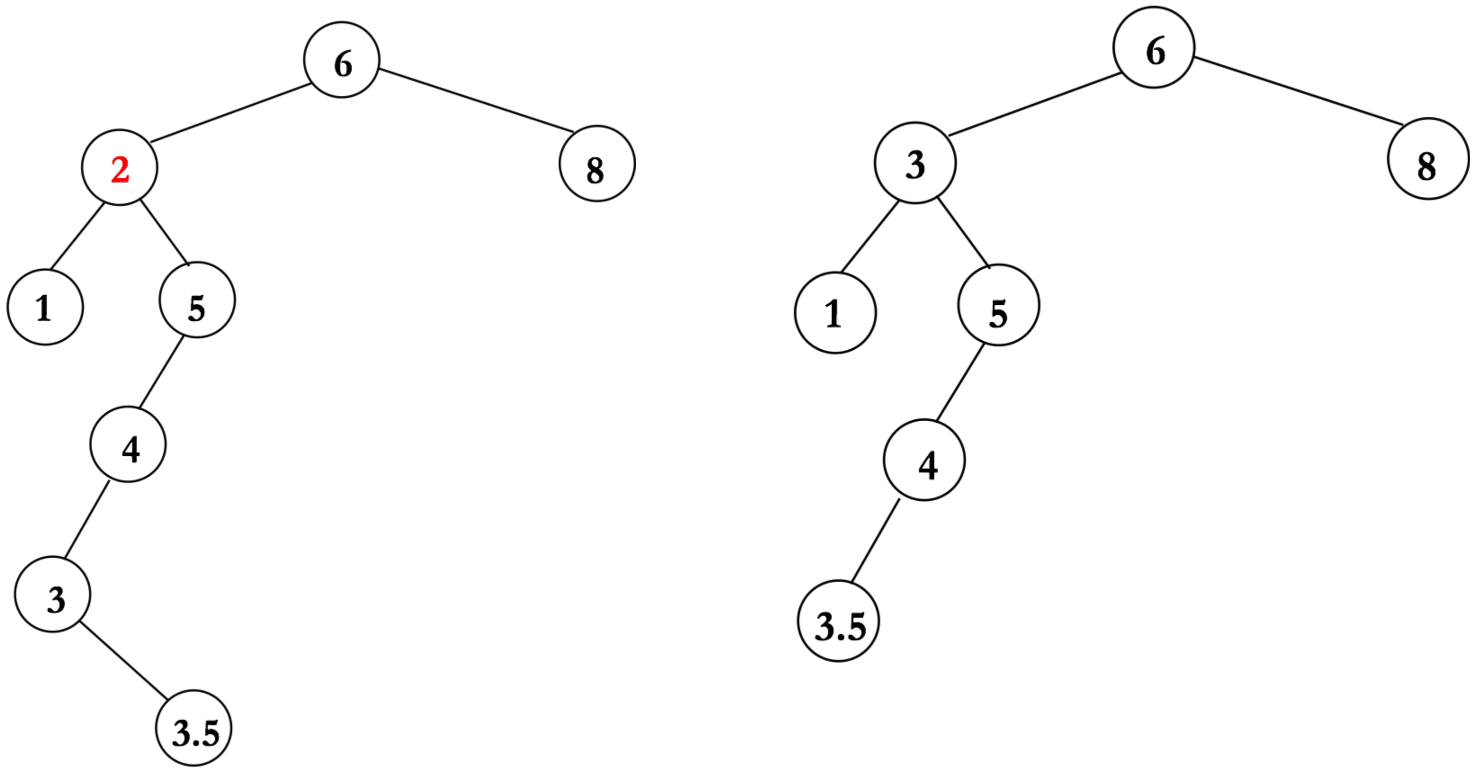




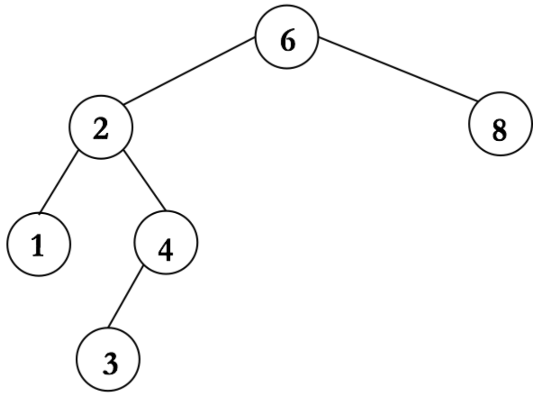
Binary Search Tree: Delete

If the node to be deleted has both children,

it is replaced with the smallest node in the right subtree.



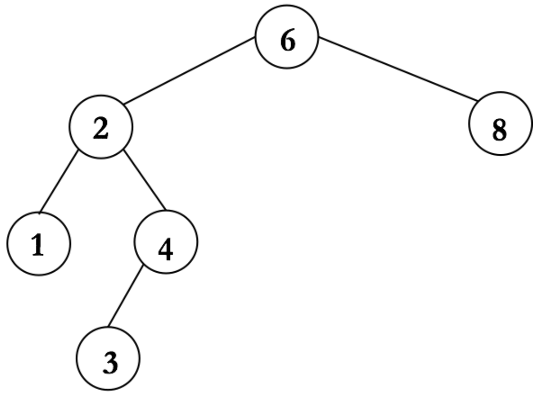
Binary Search Tree: FindMin



recursive implementation

```
Node  FindMin( SearchTree T )
{
    if( T == NULL )
        return NULL;

    else if  ( T->Left == NULL )
        return T;

    else  /* T->Left != NULL */
        return FindMin( T->Left );
}
```

Binary Search Tree: FindMax



nonrecursive implementation

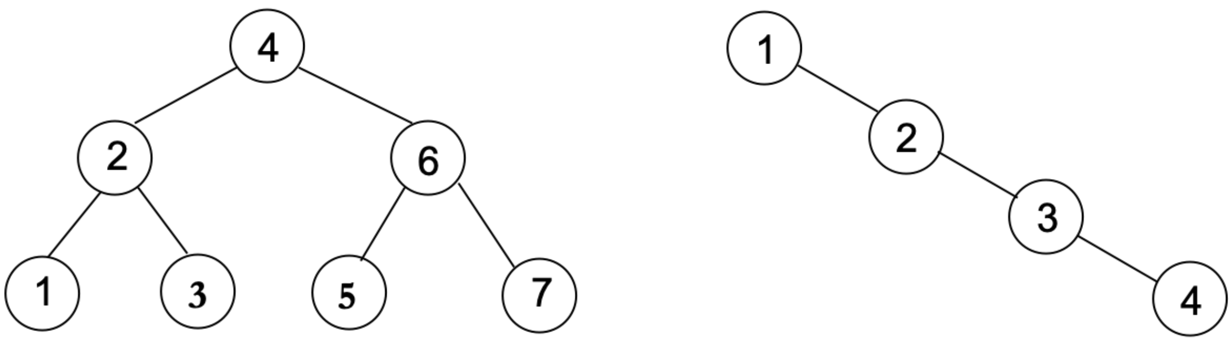
```
Node  FindMax( SearchTree T )
{
        if (T == NULL)
                return NULL

        else
         while( T->Right != NULL )
                 T = T->Right;

    return T;
}
```

Binary Search Tree: delete

```
SearchTree  Delete( ElementType X, SearchTree T )
{
        Node TmpCell;

        if   ( T == NULL )
                Error( “Element not found” );

        else if   ( X < T->Element )          /* Go Left */
                T->Left = Delete( X, T->Left );

        else if   ( X > T->Element )          /* Go Right */
                T->Right = Delete( X, T->Right );

        else if   ( T->Left && T->Right )      {    /* found the node to be deleted */

         TmpCell = FindMin( T->Right );
                 T->Element = TmpCell->Element;
                 T->Right = Delete( T->Element, T->Right );

        }  else {                                  /* 1 or 0 child */
                TmpCell = T;
                     if( T->Left == NULL )
                        T = T->Right;
                else if( T->Right == NULL )
                        T = T->Left;
                     free(TmpCell);
        }
    return T;
}
```

Analysis of binary search tree

- The runtime of find(), insert(), and delete() is proportional to the height of the tree.
- What is the height of the tree with N nodes?
  - Worst case: Linear tree $O(n)$
  - Best case: Complete binary tree $O(\log n)$



Best Case: Insert 4, 2, 6, 1, 3, 5, 7 Worst Case: insert 1, 2, 3, 4 …

  - Average case depends on the distribution of insertion/deletion
    - Assumption: insertions only
    - The order in which the keys are inserted is completely random.
      => will average all possible n! insertion orders.

Binary Search Tree: balanced binary tree

- non-random insertion to BST can produce unbalanced trees
- can we rebalance the tree so that the tree always has $O(\log n)$ height?
- We need *balance* information for each node.