A Dynamic Theory of Deterrence and Compliance*

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Abstract

Classical thinkers like Bentham saw deterrence as a dynamic process in which the punishment of violators today helped deter potential violators in the future by increasing their subjective probability of apprehension. Modern deterrence theory took a different path, analyzing deterrence as a static equilibrium framework in which the subjective and objective probabilities of apprehension are equal. We follow Bentham's lead, assuming that the subjective probability is determined by the recent history of violations and apprehensions. Further, we assume that the objective probability is increasing in the quantity of enforcement resources and decreasing in the number of violations. Together these assumptions imply that deterrence is a dynamic stochastic process with the potential for disruptive positive feedback. We use simulation techniques to explore deterrence policy in a parameterized version of the more general process. We find that managing the dynamics of positive feedback is the core problem in developing and implementing effective deterrence policy. Our dynamic theoretical framework helps clarify and unify aspects of the research on optimal deterrence capacity, crime waves, and the efficacy of crackdowns and hot spot policing strategies.

Keywords: Crime, Compliance, Perceptual Deterrence, Cost of Crime, Crackdowns
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1 Introduction

All societies have myriad schemes to deter antisocial behavior. A typical scheme involves a law or regulation forbidding some problematic behavior, prescribes a punishment for the behavior, and mandates resources devoted to detecting and deterring the behavior. We forbid cheating on examinations in our university classes, carefully monitor student work, and threaten to fail or place students in front of an honor board if cheating is found to have occurred. Regulators mandate food handling standards for restaurants, license their operation subject to those standards, and then threaten to revoke that license if food inspectors find that the standards have been violated. States set speed limits, establish fines for exceeding the limit, and deploy police with cameras to catch speeders. Modern nation states pay for public goods through taxation, punishing cheaters based on the periodic auditing of their citizen's tax returns.

For classical thinkers the goal of moral legislation was to produce compliance absent punishment. Beccaria argued it was "better to prevent crimes than to punish them," and that the "end of punishment, therefore, is no other than to prevent others from committing the like offence" (Beccaria, 1764).[1] An explicitly utilitarian formulation of deterrence starts with Bentham (1780), who argued that the judicious deterrence of rational criminals required that a violation's associated sanction produce an expected loss in utility that just exceeds its expected gain. For Bentham and Beccaria, as well as other classical thinkers, deterrence was a quintessentially dynamic process. The dynamics of deterrence concerned Bentham explicitly, "How can a past robbery weaken the force with which the political sanction tends to prevent a future robbery?" His answer? Through the beliefs of future violators, "sanction tends to prevent a robbery by proclaiming some particular kind of punishment against anyone who commits it; the real value of such punishment will of course be lessened by real uncertainty as to whether it will be inflicted." For Bentham, uncertainty of sanction—"proportionally

[1] See Kleiman (2010b) for a normative and positive argument for reducing incarceration through deterrence.

increased by every instance in which a man is known to commit the offense, without undergoing the punishment"—was a principal threat to the efficacy of moral legislation (Bentham, 1780, p. 124).

While the classical tradition was to view the deterrence of antisocial behavior (ASB) as a dynamic process, the modern deterrence literature remains dominated by static models.[2] Elements of Bentham's utilitarian framework were formalized in the static equilibrium model of Becker (1968), which remains the core analytical framework for the modern literature on the economics of crime.[3] In their review of the "workhorse model" of deterrence, Draca and Machin (2015, p. 392) note that "the model is less amenable to more complex extensions in other directions. One limitation is that it is static. This very clearly misses an important aspect of criminal behavior...". We take the view that managing deterrence is inherently a dynamic undertaking.

In Section 2, expanding on Bentham's insight, we assume that the subjective probabilities of apprehension that govern peoples' choices with respect to antisocial behavior are determined by the recent history of violations and apprehensions. We show that this assumption implies that ASB is a dynamic stochastic process. The objective probability of apprehension is conceptually distinct from the subjective probability. Because the objective probability of apprehension is decreasing in the number of violations there is a potential for positive feedback in the dynamic process: an unusually large (small) number of violations in any period decreases (increases) the objective probability of apprehension in that period, and this tends to decrease (increase) the subjective probability of apprehension in the next period further increasing (decreasing) the number of violations. We then add additional assumptions to produce a model in which the deterrence process is a regular Markov chain in the space of

[2] Reviews cite, as an exception to this rule, a small literature focusing on the interaction of discounting and sentence length. See, for example, Polinsky and Shavell (1999) and Lee and McCrary (2017). However, this literature is more accurately described as focusing on the intertemporal effects of incarceration. While an important issue, it does not concern the *dynamics* of crime explicitly.

[3] Although Schelling (1960) had a substantial impact on deterrence mechanisms for national security and the analysis of entry deterrence strategies in the industrial organization literature, his early work in *The Strategy of Conflict* had little impact on the crime deterrence literature.

recent histories. We explore the Markov chain using simulation. For a number of reasons we assume that the penalty for violating is the same in all periods, so our policy analysis focuses on varying the quantity of enforcement resources, R.

In Section 3, we analyze *passive* policies in which the same R is deployed in all periods. When the deterrence policy is passive, the frequency distribution over the number of violations in a period is either unimodal or bimodal. When R is relatively small it is unimodal and the expected number of violations is large. When R is relatively large it is again unimodal but the expected number of violations is small. For a small intermediate range of values of R, the frequency distribution is bimodal. Looking over the entire range of deterrent spending, we discover a massive nonconvexity in the expected number of violations as a function of R. Naturally, the optimal R depends on the assumed cost parameters. However, across a broad range of parameter values, the optimal R is just large enough so that the distribution is unimodal with all of its mass in the neighborhood of the lower mode and the number of violations small.

In Section 4 we explore *active* policies in which the R deployed in a period is dependent on the number of violations in the previous period. The simplest active policy is a two bin policy which deploys a relatively small R in periods where the number of violations in the preceding period was less than or equal to some bin boundary BE, and a relatively large R when the number exceeds BE. We show that the cost of antisocial behavior when BE and the two Rs are chosen optimally is significantly lower than the cost associated with the optimal passive policy. This is intuitive—a dynamic policy is more effective at managing a dynamic process. Perhaps not so intuitive, more complex dynamic policies, with three bins, reduce the cost of antisocial behavior only modestly relative to the simplest active policy.

In Section 5 we discuss how our work fits into the literature on crime waves, reserve deterrence capacity, and *crackdowns* or *hot spot policing*. We assume that one unit of the enforcement resource, no more and no less, is required to investigate one violation. Hence enforcement capacity in any period is equal to the R deployed in that period. Our index of *reserve capacity* is the expected fraction of enforcement resources available in a period that are surplus to investigative needs in that period. Perhaps surprisingly, in the neighborhood of the optimal policy, whether it is passive or active, the value of this index exceeds 0.60—so in a typical period, less than one half of available enforcement resources are used to investigate violations. It is tempting to think of this as *excess capacity.* That interpretation would be accurate if the purpose of enforcement resources was to apprehend violators. But it is not. The purpose is to deter violations, and in a dynamic setting where positive feedback is a problem, a high rate of reserve capacity is necessary for effective deterrence. In Section 6 we conclude.

2 Deterrence as a Dynamic Stochastic Process

Models of deterrence specify an act that is proscribed, a punishment for apprehended violators, and the resources devoted to apprehension. Two probabilities are at the heart of any perceptual deterrence mechanism: the *subjective* and *objective* probabilities of apprehension, q and p respectively. A person's subjective probability governs their choice to violate or comply with a prohibition against some ASB. The objective probability is the probability violators are actually apprehended. The punishment, F, and enforcement resources, R, are taken as policy parameters.[4] Some structure allows us to more fully articulate the important distinction between p and q, as well as the choice it presents to anyone modelling deterrence.[5]

[4] Some models cast F and p, not F and R, as the relevant policy variables. It is clear from both the empirical and theoretical literatures, however, that policy makers are not able to "choose" p. Chalfin and McCrary (2017, p. 10) note that "Perceptual deterrence is important because the vast majority of the empirical-deterrence literature operationalizes Becker's model of crime by studying the responsiveness of crime to particular policy variables, such as the number or productivity of police or the punitiveness of sanctions." See also Apel (2013) and Montmarquette et al. (1985).

[5] Two approaches dominate the perceptions literature: (i) survey methods, do people report accurate knowledge of the size and probability of sanction; and, (ii) revealed behavior, do populations respond as if they have accurate knowledge of the size and probability of sanction? This research finds that: (i) q correlates with p but with errors (i.e, $q \neq p$); (ii) q is not directly responsive to deterrence spending; (iii) measured errors in q shrink for agents with experience in the criminal justice system; and, (iv) the most reliable predictors of q are local and/or personal histories. Lochner (2007, p. 457-58) finds that "Individuals who engage in crime while avoiding arrest tend to reduce their probability of arrest; those who are arrested raise their perceived probability." Matsueda et al. (2006) finds evidence both that $p \neq q$ and that people

2.1 Modeling deterrence

Consider an infinitely lived community with N individuals willing to engage in antisocial behavior. The behavior is proscribed by regulation or statute. Every potential violator, at the start of every period t, first formulates their subjective probability of apprehension; they all then simultaneously choose whether to *violate* the law or *comply* with the law. Finally, violators are either apprehended or escape punishment.

The choice of person i in period t depends on their utility from violating, g_i^t, their subjective probability of apprehension, q_i^t, possible sanctions, F^t, and their utility from complying. Subsequent to the decision of all N potential violators, the objective probability of apprehension, p^t, is jointly determined by the quantity of enforcement resources, R^t, and the number of violations, v^t,

$$p^t = P(R^t, v^t).$$

The function P embodies the technology of apprehension: it is increasing in R^t and decreasing in v^t.[6]

What is the relationship between q_i^t and p^t? It is often assumed—sometimes explicitly, most often implicitly—that in equilibrium all of the subjective probabilities q_i^t are equal to the unique objective probability p^t.[7] This assumption is often defended by an appeal to fulfilled expectations: it should be the case that the expectations on which violators and compliers

update q as Bayesians. Surveying the literature, Apel (2013, p. 85) concludes that "A number of studies provide support for the Bayesian updating model with respect to crime and punishment experiences...".

[6] There is clear evidence that increasing police resources in a jurisdiction reduces crime. For reviews of the literature see Chalfin and McCrary (2017), and Nagin (2013). Chalfin and McCrary (2017) note that "These strategies consistently demonstrate that police do reduce crime. However, the estimated elasticities display a wide range, roughly —0.1 to —2, depending on the study and the type of crime." With the elasticities "favoring a larger effect of police on violent crimes than on property crimes, with especially large effects of police on murder, robbery, and motor vehicle theft" (p. 14).

[7] For example, see Becker (1968) and Ehrlich (1973), or, more recently, Eaton and Wen (2008) and Bond and Hagerty (2010). Kleck (2016, p. 767) argues that "a host of...studies defending the deterrence doctrine" assume that "there is a close connection between objective risks of legal punishment of crime and individual perceptions of those risks" and that this assumption is often left unstated and is almost certainly wrong. He goes on to note that "Economists...seem to be especially oblivious to (a) the fact that the assumption is essential to their conclusions and (b) the existence of evidence casting grave doubt on it" (Kleck, 2016, p. 767). See aslo Kleck and Barnes (2013) and Kleck and Barnes (2014).

base their choices are confirmed. Or, that when the dust settles, no one regrets their choice. Or, most simply, that the choices people make constitute a Nash equilibrium. This approach commits one to an *equilibrium analysis* of deterrence.[8] In such an equilibrium—described by the number of people who chose to violate, v^*—the objective probability of apprehension is $P(R, v^*)$. So assuming $p = q$ is equivalent to assuming that $q = P(R, v^*)$. This is problematic.[9]

First, at the point when people are forming q_i^t, v^t is unobservable; as p^t depends on v^t, it too must be unobservable.[10] Clearly, this is a problem for the fulfilled expectations approach. It could be argued that potential violators actually calculate the equilibrium before formulating q_i^t. This, however, requires that each potential violator knows the level of enforcement resources, the technology of apprehension, as well as the preferences and anti-social opportunities of all other potential violators. This seems unreasonable. A second problem arises when there is more than one equilibrium level of violations.[11] In such a case, the modeler must require that all potential violators coordinate on the same equilibrium.[12]

If the equilibrium approach is rejected, how should one model the relationship between the subjective and objective probabilities of apprehension? There is, of course, no a priori correct approach. Importantly, rejecting the equilibrium approach does not require accepting

[8] Piliavin et al. (1986, p. 102, emphasis added) explicitly reject this equilibrium approach, arguing that models "must include the expected rewards from alternative courses of legal or illegal action; it must consider the expected costs of these actions; and it must consider those expectations as *subjectively perceived by the actor*, not as *objectively inhering in the actions.*"

[9] In Section A.1 of the appendix we explore the fulfilled expectations, or equilibrium, approach to modeling deterrence. That analysis reveals some interesting and productive complementarities between the equilibrium and dynamic models.

[10] Sah (1991) assumes that p^t is unobservable at the point people decide whether to violate. In part, this simply reflects the problem of information acquisition. But it also reflects the fact that p^t is endogenous to people's choices, "The reason is that for any given public expenditure on the 'criminal apprehension system,' a higher crime participation rate leads to fewer resources being spent on apprehending each criminal..." (p, 1275). He goes on to note that, for this reason, high levels of crime today may reduce q in the future.

[11] The idea that positive feedback may produce multiple equilibria in models of deterrence informs a large literature. Kleiman (1993) introduced the term *enforcement swamping* to describe this sort of dynamic, "Once the capacity to punish is saturated, the rate of punishment per crime must fall as the number of crimes rises...Diminished deterrence will tend to generate more criminal activity, further reducing punishment per crime, and so on."

[12] In Bond and Hagerty (2010), for example, people coordinate on the Pareto superior equilibrium.

that people make systematic and/or persistent errors.[13] One intuitive possibility, the one we pursue, is that people simply rely on the public history of violations and apprehensions. This approach commits us to modelling deterrence as a *dynamic stochastic process*.[14]

More specifically, in any period t, we assume that q_i^t for all potential violators is determined by the history of violations and apprehensions in the z most recent periods. We call this the *z-history*, or zh^t,

$$zh^t = ((v^{t-z}, ..., v^{t-1}), (a^{t-z}, ..., a^{t-1})),$$

where a^t is the number of violators apprehended in period t. Observing zh^t, individuals formulate q_i^t, which, in combination with the utility of their antisocial opportunity, g_i^t, and the sanctions faced, determines their decision to violate. As the z-history links current ASB to past ASB, the process is necessarily *dynamic*. As every apprehension in the z-history, $a^k \in zh^t \mid k \in \{t-z, t-z+1, ..., t-1\}$, is drawn from a binomial distribution in which each of the v^k violators is apprehended with probability p^k, the process is necessarily *stochastic*.

Proposition 1 *When subjective probabilities of apprehension are determined by the history of violations and apprehensions, the time series of violations and apprehensions is a* ***dynamic stochastic process****.*

2.2 Antisocial behavior as a finite Markov chain

We model this process as a Markov chain in the space of possible z-histories, $\boldsymbol{Z}$. $\boldsymbol{Z}$ is the set of all z-histories $((v^{t-z}, ..., v^{t-1}), (a^{t-z}, ..., a^{t-1}))$ such that for all $k \in \{t-z, ..., t-1\}$, $0 \leq v^k \leq N$ and $0 \leq a^k \leq v^t$. The z-history involves z pairs (v^k, a^k), and each pair

[13] While there are models of this sort, in our dynamic model the time path of q is always approximately equal to that of p. See, Figures 3.2 and 4.1, footnote 23 elaborates.

[14] While there are models that incorporate dynamic elements and aspects of our approach—see, for example, Sah (1991), Glaeser et al. (1996), Schrag and Scotchmer (1997), and Lui (1986)—we are unaware of any fully dynamic models of deterrence. Our approach probably most closely resembles that of Sah (1991), who models high levels of violations as driven by low values of q and the positive feedback that exists between both factors. People exposed to high violation jurisdictions tend to have lower values of q and so "Past crime breeds future crime" (Sah, 1991, p. 1282)

is an element of a set with $(1 + 2 + ... + (N + 1))$ elements,[15] so the cardinality of $\boldsymbol{Z}$ is $L = (1 + 2 + ... + (N + 1))^z$.

We adopt the following six assumptions. Assumptions $1-5$ produce a model of deterrence that is a finite Markov chain; 6 ensures that choosing to violate is an antisocial act.

Assumption 1: In every period t, each person $i \in \{1, ..., N\}$ is presented with an antisocial opportunity which produces utility g_i^t if exercised; g_i^t is generated by a random draw from the probability density function $\phi_i(g_i)$. The utility of comply is 0.

Assumption 2: In every period t, each person $i \in \{1, ..., N\}$ formulates their subjective probability of apprehension according to, $q_i^t = Q_i(zh^t)$. Q_i is decreasing in each element of $(v^{t-z}, ..., v^{t-1})$, increasing in each element of $(a^{t-z}, ..., a^{t-1})$, and strictly bounded by 0 and 1.

Assumption 3: Policies for deterrence spending and sanctions are mappings from states in $\boldsymbol{Z}$ to R and F respectively: (i) $R^t = PR(zh^t)$, and (ii) $F^t = PF(zh^t)$.

Assumption 4: People are risk neutral. Hence, in any period t, person $i \in \{1, ..., N\}$ will choose violate if $g_i^t - q_i^t \cdot PF(zh^t) \geq 0$; they comply otherwise.

Assumption 5: In every period t the sequence of events is: (i) people form q_i^t; (ii) people realize g_i^t; (iii) people simultaneously choose violate or comply; (iv) each violator is apprehended with probability $P(PR(zh^t), v^t)$.

Assumption 6: The gross utility cost to the community from person i choosing to violate is $\lambda \cdot g_i^t$ with $\lambda > 1$. The fact that λ exceeds 1 ensures that choosing to violate is an antisocial act. The net social cost is therefore $(\lambda - 1) \cdot g_i^t$.

Proposition 2 *Assumptions 1 through 5 ensure the time series of violations and apprehensions is generated by a **finite Markov chain** with state space $\boldsymbol{Z}$.*

[15] The 1 in the sum corresponds to the case in which $v^k = 0$ and $a^k = 0$, the 2 to the case in which $v^k = 1$ and $a^k = 0$ or 1, and so on.

We use $T(zh^t, zh^{t+1})$ to denote the probability of transitioning from state zh^t to state zh^{t+1} in one period, and $\boldsymbol{T} = [T(zh^t, zh^{t+1})]$ to denote the entire transition matrix. Since a Markov chain is completely described by its transition matrix, we prove Proposition 2 by showing how to calculate $T(zh^t, zh^{t+1})$ for any arbitrarily chosen pair of states. Suppose the state at some time t is

$$\widehat{zh}^t = ((\hat{v}^{t-z}, ..., \hat{v}^{t-1}), (\hat{a}^{t-z}, ..., \hat{a}^{t-1})).$$

At time $t+1$ the state will be one of the following

$$zh^{t+1} = ((\hat{v}^{t-z+1}, ..., \hat{v}^{t-1}, v^t), (\hat{a}^{t-z+1}, ..., \hat{a}^{t-1}, a^t)),$$

where $v^t \in \{0, ..., N\}$ and $a^t \in \{0, ..., v^t\}$. Elements $((\hat{v}^{t-z+1}, ..., \hat{v}^{t-1}), (\hat{a}^{t-z+1}, ..., \hat{a}^{t-1}))$ in zh^{t+1} are predetermined by history; v^t is determined by the choices of potential violators, a^t by the operation of the enforcement mechanism in period t. So the probability of transitioning from state $\widehat{zh}^t$ to zh^{t+1}, $T(\widehat{zh}^t, zh^{t+1})$, is the product of two conditional probabilities. The probability that, conditional on being in state $\widehat{zh}^t$, exactly v^t of the N potential violators choose violate. To calculate this probability, $\pi_i(\widehat{zh}^t)$, we need the probability that person $i \in \{1, ..., N\}$ will choose violate in state $\widehat{zh}^t$. In state $\widehat{zh}^t$, given g_i^t, person i's expected utility of violate is $g_i^t - Q_i(\widehat{zh}^t) \cdot PF(\widehat{zh}^t)$, and the expected utility of comply is 0. So i will choose violate if and only if

$$g_i^t \geq Q_i(\widehat{zh}^t) \cdot PF(\widehat{zh}^t).$$

Since g_i^t is a random draw from $\phi_i(g)$, $\pi_i(\widehat{zh}^t)$ is

$$\pi_i(\widehat{zh}^t) = \int_{Q_i(\widehat{zh}^t) \cdot PF(\widehat{zh}^t)}^{\infty} \phi_i(g) dg.$$

Given $(\pi_1(\widehat{zh}^t), ..., \pi_N(\widehat{zh}^t))$, the calculation of $\pi_i(\widehat{zh}^t)$ is straightforward. All that remains is to find the probability that, conditional on being in state $\widehat{zh}^t$ with v^t violations, a^t violators are apprehended. This is simply the binomial probability that a^t of the v^t violators are apprehended when each is apprehended with probability $P(PR(\widehat{zh}^t), v^t)$. The product of the two conditional probabilities is $T(\widehat{zh}^t, zh^{t+1})$.

2.3 A computable version of the dynamic model

To create a computable version of the dynamic model we specify: (i) the function P that determines the objective probability of apprehension, (ii) the N functions Q that determine the subjective probabilities of apprehension, and (iii) the N functions ϕ that determine the utilities of choosing the antisocial act.

Given R^t and v^t, the objective probability of apprehension is,

$$P(R^t, v^t) = \gamma \cdot min(1, \frac{R^t}{v^t}).$$

This function is based on two assumptions: first, it takes one unit of enforcement resources, no more and no less, to investigate a violation; second, when a violation is investigated the violator is apprehended with probability γ. If more than one unit of R is used in an investigation the probability of apprehension is still γ, it is 0 if less than one unit is used. Violations may only be investigated once. Since the maximum number of violations that can be productively investigated is just R, the apprehension technology has a well defined capacity: (i) when $v \leq R$, every violation is investigated and each violator apprehended with probability γ; (ii) when $v > R$, R randomly chosen violators are investigated and each violator is apprehended with probability $\gamma \cdot (R/v)$. Given this technology of apprehension, if $R^t > N$ the probability of apprehension is constant and equal to γ, so we impose the restriction $0 \leq R^t \leq N$.[16]

[16] This natural investigative capacity is an attractive feature of the enforcement technology. It results, however, from the fact that the technology is rigid in much the same way that a Leontief production function is rigid. An alternative specification assumes that the probability that a violator is apprehended is continuous in r, $\gamma(1 - 1/\epsilon^r)$, where $\epsilon > 1$ and $0 < \gamma < 1$. This probability is increasing and concave in r and bounded above by γ. Maximizing the expected number of apprehensions given R^t requires allocating R^t/v^t to each violation. With this allocation, the probability that any violator is apprehended is $\gamma(1-1/\epsilon^{R^t/v^t})$. In Section 3.4 we show that our results are robust to this enforcement technology.

All people have a common subjective probability of apprehension given by $Q(zh^t)$, where

$$Q(zh^t) = \frac{\alpha + a^{t-z} + a^{t-z+1} + ... + a^{t-1}}{\alpha + \beta + v^{t-z} + v^{t-z+1} + ... + v^{t-1}}.$$

Here we are assuming that people update their subjective probability of apprehension as Bayesians with priors α and β.[17]

Lastly, in every period all N values of g_i^t are drawn from the same normal probability density function $\phi(g)$, with mean μ and standard deviation σ.

Given that $\phi(g)$ is a normal distribution, the probability density $\phi(g)$ is strictly positive for all g. This implies that the probability, $\pi(zh^t)$, that any person will choose violate in any state is strictly positive, which implies that the Markov chain for the computable model is regular.

Proposition 3 *The finite Markov chain for the computable model is **regular**.*

The defining feature of a regular Markov chain is that, regardless of the state of the process at time t, after a sufficient lapse of time it is possible to be in any state. To prove Proposition 3 we show that z periods is sufficient. In Section 2.2 we showed that in the transition from any $zh^t \in \boldsymbol{Z}$ to any $zh^{t+1} \in \boldsymbol{Z}$, zh^t determines all of the component parts of zh^{t+1} except for v^t and a^t. For any $v^t \in \{0, 1, ..., N\}$, the probability that there will v^t violations in period t is the binomial probability that v^t of the N potential violators will choose violate given that each of them will choose violate with probability π^t. In the computable model, since $\pi^t > 0$, this probability is strictly positive. In addition, given v^t,

[17] Peoples' priors with respect to the subjective probability of apprehension are described by a beta distribution with shape parameters α and β,

$$h(p; \alpha, \beta) = \frac{p^{\widehat{\alpha}-1}(1-p)^{\widehat{\beta}-1}}{B(\alpha, \beta)}$$

where $B(\alpha, \beta)$ is the beta function. Priors are chosen so as to eliminate the possibility of systematic divergence of q from p when the number of violations is small. Specifically, we choose α and β so that $q = \gamma$ for the null z-history. For $\gamma = 0.8$, $\alpha = 1$ and $\beta = 0.25$, so that $\alpha/(\alpha + \beta) = 0.8$. Given zh^t, the posterior distribution is also a beta distribution, $h(p; \widehat{\alpha}, \widehat{\beta})$, with shape parameters $\widehat{\alpha} = \alpha + a^{t-z} + ... + a^{t-1}$ and $\widehat{\beta} = \beta + v^{t-z} + ... + v^{t-1}$. The subjective probability of apprehension, $q^t = Q(zh^t)$, is the mean of the posterior distribution.

for all $a^t \in \{0, 1, ..., v^t\}$, the binomial probability that a^t violators will be apprehended is also strictly positive. In other words, in one period, it is possible to go from state zh^t to any state that is not ruled out by the fact that zh^t predetermines some components of zh^{t+1}. But zh^t predetermines none of the components of zh^{t+z}, so the probability of going from any state to any other in z periods is strictly positive.

Because $p^t = P(R^t, v^t)$ is a decreasing function of v^t there is a potential in the dynamic stochastic process for positive feedback. A random positive perturbation of v^t drives down p^t, the resulting decline in $E(a^t)$ drives $E(q^{t+1})$ down and, in turn, $E(v^{t+1})$ up. The same sort of positive feedback occurs in response to a negative perturbation in v^t. A reduction in v^t drives up p^t, the resulting increase in $E(a^t)$ drives $E(q^{t+1})$ up and, in turn, $E(v^{t+1})$ down. In both cases the positive feedback is attributable to the fact that p^t is decreasing in v^t. There are two sources of perturbations in v^t. One is the fact that the individual g_i^ts are random variables. The other is the fact that, given v^{t-1} and p^{t-1}, a^{t-1} is a random variable with a binomial distribution over the set $\{0, 1, ..., v^{t-1}\}$. Realizations at the lower end of this set drive q^t down and v^t up, triggering the positive feedback; realizations at the upper end have the opposite effect.

Observation: *The potential for* ***disruptive positive feedback*** *exists in the computable model.*

Given the transition matrix $\boldsymbol{T}$ for the computable model and the state at time $\widetilde{t}$, we can calculate the probability of any state j at time $t > \widetilde{t}$. Represent the initial state by a probability vector $\boldsymbol{H}^{\widetilde{t}} = (h_1^{\widetilde{t}}, h_2^{\widetilde{t}}, ..., h_L^{\widetilde{t}})$, where $h_j^{\widetilde{t}}$ is 1 if at time $\widetilde{t}$ the state is j and 0 otherwise. For any time $t > \widetilde{t}$, element h_j^t of $\boldsymbol{H}^t = \boldsymbol{H}^{\widetilde{t}} \cdot \boldsymbol{T}^{t-\widetilde{t}}$ is the probability of state j at time t. Given the regularity of the Markov chain, if $t - \widetilde{t} \geq z$, every element of $\boldsymbol{H}^t$ is positive. As t increases without bound, each row of $\boldsymbol{T}^{t-\widetilde{t}}$ approaches the same probability vector $\boldsymbol{D} = (d_1, d_2, ..., d_L)$, where $\boldsymbol{D}$ is the unique probability vector such that $\boldsymbol{D} \cdot \boldsymbol{T} = \boldsymbol{D}$. Consequently, $\boldsymbol{H}^t$ also approaches $\boldsymbol{D}$. Every element of $\boldsymbol{D}$ is, of course, strictly positive. $\boldsymbol{D}$ is the *primary stationary distribution* of the Markov chain.

Using $\boldsymbol{D}$ we can derive the stationary distribution of the number of violations in a period, $\widehat{\boldsymbol{D}} = (\widehat{d}_0, \widehat{d}_1, ..., \widehat{d}_N)$. Element $\widehat{d}_m$ of $\widehat{\boldsymbol{D}}$ is defined as follows:

$$\widehat{d}_m = \sum_{j|v^{t-1}=m} d_j$$

The summation is over all states $j \in \{1, ..., L\}$ such that the number of violations in period $t-1$ is m. Two interpretations of $\widehat{d}_m$ are worth mentioning: it is the limiting probability that the Markov chain will be in a state with m violations in the most recent period; alternatively, it is the *fraction of time* that the process can be expected to be in a state in which there were m violations in the most recent period.

The transition matrix $\boldsymbol{T}$ contains everything one might wish to know regarding the dynamics of the computable model. Unfortunately, for interesting parameterizations, the number of states is so large that calculating and manipulating $\boldsymbol{T}$ directly is simply not feasible. For example, for $N = 100$ and $z = 2$ the number of states is $5150^2 = 25,503,500$. So we instead use simulation techniques to explore the computable model. Our primary window on the deterrence process is a convergence simulation that yields an approximation of the stationary distribution of violations $\widehat{D}$ (see Section 3.3, and Appendixes A.2 and A.3).[18]

2.4 Baseline parameter values

The kernel of the computational model has seven parameters. For six of them we choose *baseline* values, exploring deviations from this baseline one parameter at a time: the number of potential violators is $N = 100$; the value of the sanction is $F = 1$; the values of the parameters of the normal distribution $\phi(g)$ are $\mu = 0.6$ and $\sigma = 0.2$; the length of the z-history is $z = 2$;[19] the probability of a successful investigation is $\gamma = 0.8$. As detailed in

[18]Simulation results were produced and then replicated in Matlab and Python. Interested readers will find codebases for all computations and figures at `https://github.com/cliffbekar/dynamic_deterrence`.

[19] Computational costs are increasing dramatically in N and z. This, in part, informs our choice to analyze a relatively short z-history.

footnote 17, people's Bayesian priors are determined by choice of γ. The seventh parameter in the kernel is the quantity of enforcement resources, R; it is the primary policy variable of interest and has no fixed value.[20]

We focus on policies PR in which R^t is a function of the number of violations in the preceding period, v^{t-1},

$$R^t = PR(v^{t-1}).$$

Given that both R^t and v^{t-1} are elements of the set $\{0, 1, ..., N\}$, the number of enforcement policies of this sort is $(N+1)^2$. For the baseline parameterization there are more than 10,000 policies. We have no hope of exploring all of them, so our strategy is to start with simple policies and build from there. We start with the $N+1$ policies that assign the same R to every v^{t-1}; we call these *passive* policies. Our analysis of passive policies suggests that more complex *active* policies may be more effective. The active policies we explore involve partitioning the set of possible values of v^{t-1} into different bins, and assigning a different R to each bin. Our exploration of sanction policy functions PF is more limited. We consider only passive policies, those that assign the same F to all states.

3 Passive Enforcement Policies

In this section we explore the implications of passive policies in the baseline parameterization of the model. We first focus on the implications for violations, and then for costs. Except in Section 3.4, we use a single passive sanction policy: $PF(v^{t-1}) = 1$. In contrast, we analyze all of the relevant passive enforcement policies, $PR(v^{t-1}) = R$, $R \in \{0, 1, ..., N\}$.

[20] In Section 3.5 we add two parameters to price ASB, in Sections 3 and 4, we ultimately add five more related to policy analysis.

3.1 Some simulated histories of antisocial behavior

Figure 3.1 presents simulated data on violations for five carefully chosen passive enforcement policies in which R is 37, 40, 41, 42, and 46. The left-hand panels present the raw number of violations over 5000 periods for each value of R, the right-hand panels present the corresponding frequency distributions of the number of violations in a period. Since there are very few periods in which the number of violations is in the interval $[40, 60]$, we partition violations into a good bin $GB = \{0, 49\}$, and a bad bin $BB = \{50, 100\}$.

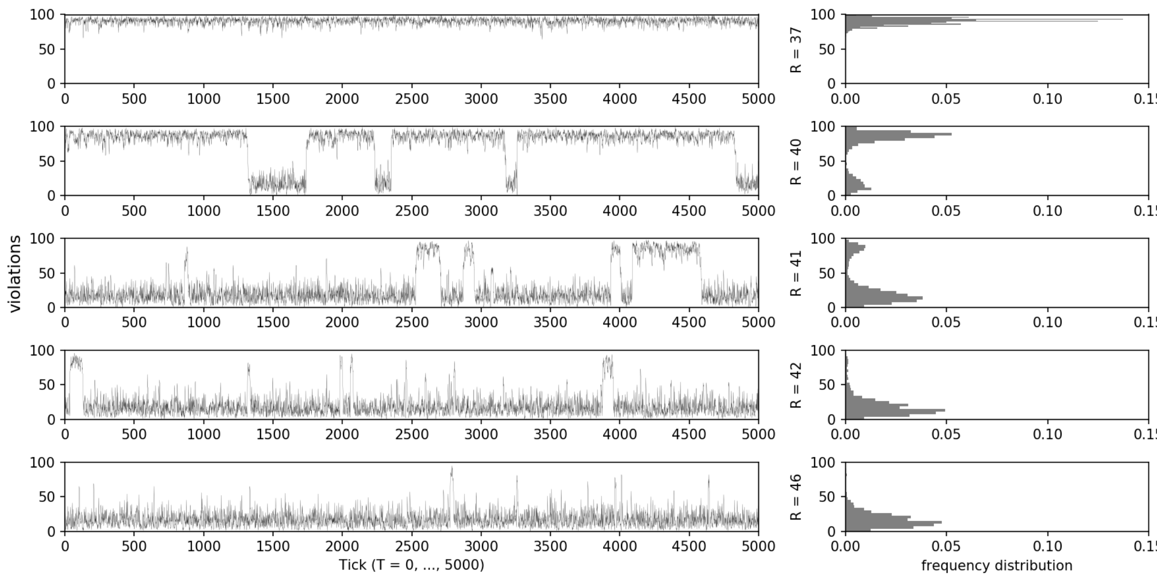

Figure 3.1: Raw time series of violations for chosen values of R.

First, consider the frequency distributions. Typically, more than 80% of the population choose violate when $R = 37$ (modal violations are approximately 95 violations per period), only in a handful of periods is the number of violations less than 80. The system spends almost all of the time in BB, producing a unimodal frequency distribution. For smaller values of R, not shown in the figure, the same pattern prevails, but the concentration of violations at the upper end of the interval is even more pronounced. When $R = 40$ the distribution is bimodal, with one mode at approximately 85 violations and another at 15. When $R = 42$, we see that the migration of mass from very high numbers of violations to very low numbers is nearly complete—in roughly 19 out of 20 periods there are fewer than

35 violations. When we get to $R = 46$ the transition is complete—the frequency distribution is again unimodal—virtually all periods are in GB with a mode of 15 violations per period. For larger values of R, not shown in the figure, the same pattern prevails. Remarkably, almost all of the migration of mass from BB to GB occurs as R is increased from 40 to 42. When $R = 40$, almost all periods are in BB; when $R = 42$ almost all periods are in GB.

Now consider the time series plots in the left-hand panels. Violations clearly persist from one period to the next. If there are many violations in a period there is a high probability that there will be many in the next; if there are few violations in a period there is a high probability that there will be few in the next. For the intermediate values of R, there are long runs of time when there are many violations and long runs when there are few. These intervals are separated by short periods in which the system transitions from one to the other. The transition from a run where the number of violations is relatively small to a run where it is relatively large is driven by *positive feedback*.

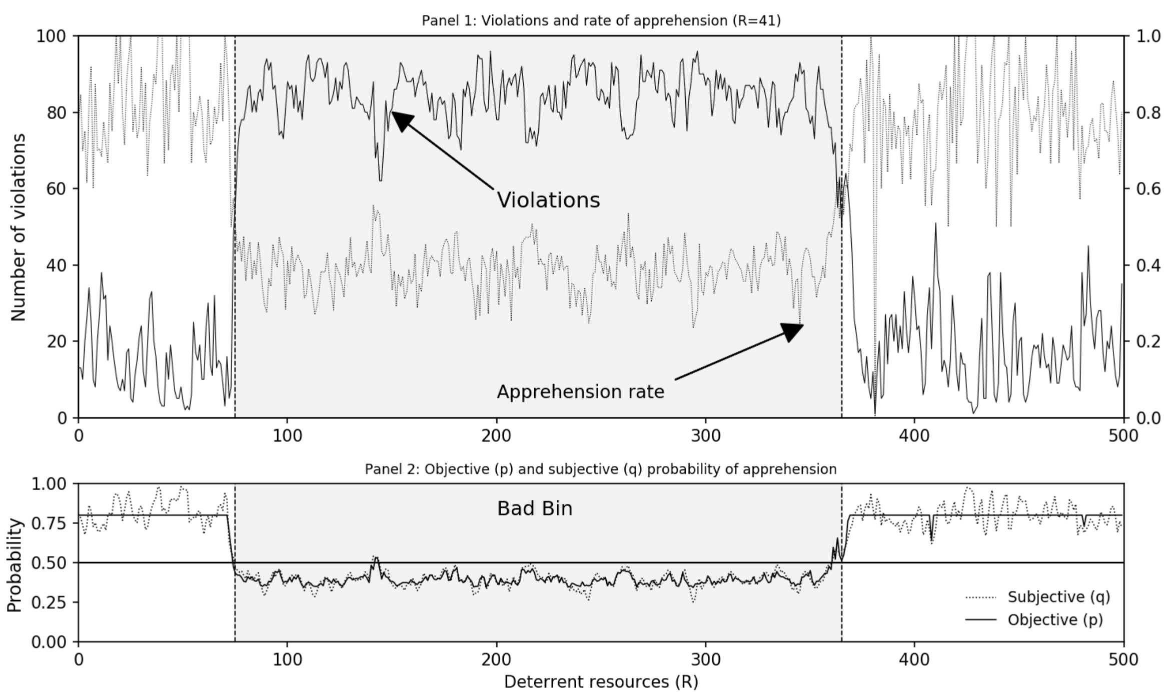

Figure 3.2: The durability of crime waves and perceptions of apprehension.

Figure 3.2 presents a more revealing plot of 500 periods for $R = 41$ and illustrates

Proposition 4. Around period 75, random fluctuations in the apprehension rate produce an unusually low q. This increases v, triggering a process of positive feedback—the increase in v drives down p, which drives down apprehensions, which drives down q, and this increases v yet again—transitioning the system to BB in less than 5 periods.[21] Starting around period 275, random fluctuations in the apprehension rate reverse the process, driving a transition of the system back to GB. A central finding in Section 4 is that deterrence policy can be much improved by focusing on the management of this positive feedback. Specifically, an active policy of crackdowns[22] allows policy makers to quickly intervene after an increase in violations to nudge q back up and so minimize the time spent in BB while leveraging the stickiness of GB.[23]

3.2 Positive feedback and quasi-attractors

A Markov chain has an *attractor* if there exists a set of states that once entered is never left. In this sense the *persistence* of an attractor is *perfect.* Figure 3.1 displays collections of states that exhibit significant, though not perfect, persistence. We call these sets of states *quasi-attractors*, or *q-attractors*. Both GB and BB are q-attractors.

To estimate the persistence of GB, P_{GB}, we calculate: (i) the total number of times that v^t is in GB, $\#GB$; and (ii) conditional on v^t being in GB, the total number of times v^{t+1} is in GB, $\#(GB \rightarrow GB)$. Our estimate of P_{GB} is then $\#(GB \rightarrow GB)/\#GB$. Naturally, P_{BB} is estimated in the same way. The *expected durations of stay* are inversely related to persistence, respectively $ED_{GB} = 1/(1 - P_{GB})$ and $ED_{BB} = 1/(1 - P_{BB})$.

[21] Kleiman (1993) introduced the term *enforcement swamping* to describe this sort of dynamic, "Once the capacity to punish is saturated, the rate of punishment per crime must fall as the number of crimes rises...Diminished deterrence will tend to generate more criminal activity, further reducing punishment per crime, and so on."

[22] We use the term crackdown to refer to policies that dynamically allocate deterrent resources between different states. These active policies capture elements of policing strategies analyzed by Sherman (1990) and Koper (1995). We explore crackdowns more fully in Section 4.

[23] The subjective probability of apprehension (q^t) closely tracks the objective probability of apprehension (p^t) over the entire period. This illustrates an important though not terribly subtle point concerning the relationship between the objective and subjective probabilities of apprehension—it is not necessary to impose the condition that the two are equal to have a model in which they are approximately so.

In Panel 1 of Figure 3.3 we report our estimates of $log_{10}(ED_{GB})$ and $log_{10}(ED_{BB})$ for $39 \leq R \leq 43$ (we report logs due to the large variation in both estimates). ED_{BB} decreases and ED_{GB} increases as R goes from 39 to 43. In Panel 2 we report the time spent by the system in BB and GB. When $R < 39$ the proportion of time spent in BB is very close to 1, but as R increases from 39 to 43 it falls steadily and is very close to 0 for $R > 43$. The evolution of the proportion of time spent in GB is just the opposite.[24]

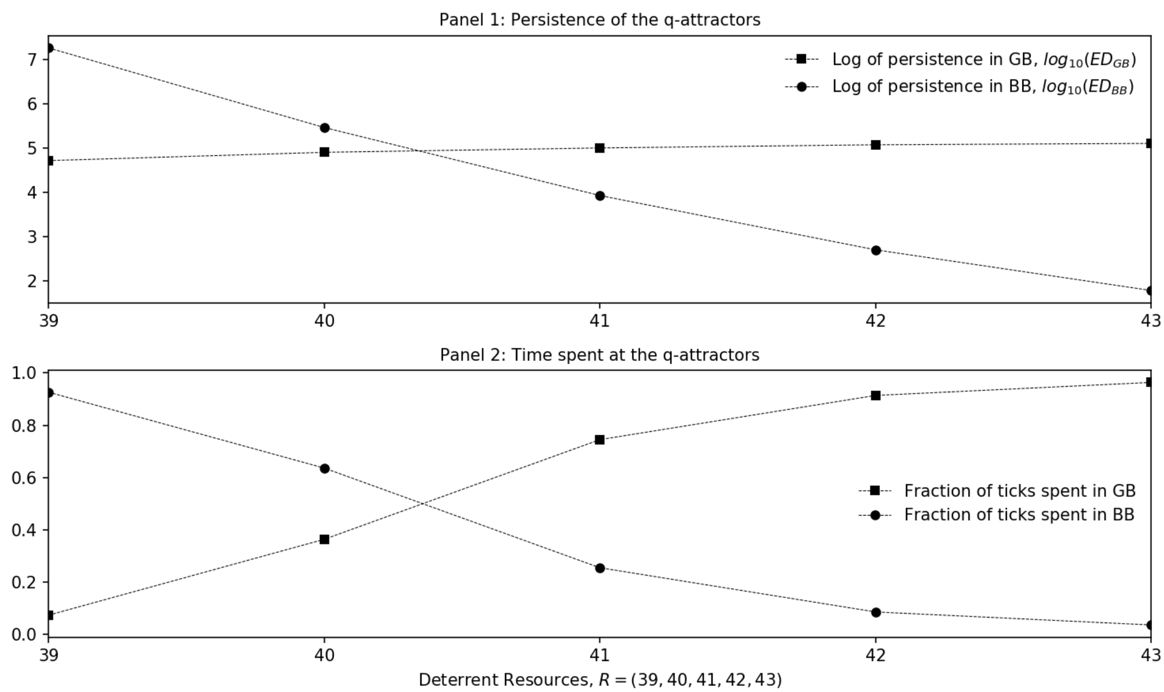

Figure 3.3: Persistence of violations in each regime.

In Panel 1 of Figure 3.4 we use a long simulation (10 million periods) to identify the focal points of the q-attractors for $R = 39$. For every $v \in \{0, ..., N\}$ we calculate and plot the mean value of v^{t+1} conditional on v^t, $E(v^{t+1}|v^t)$. The number of violations will increase from one period to the next if $E(v^{t+1}|v^t) > v^t$, and decrease if $E(v^{t+1}|v^t) < v^t$. The arrows along the horizontal axis indicate these dynamics. The focal point for GB is $v \approx 17$; for BB it is $v \approx 88$. The positive slope of the dotted line in Figure 3.4 reveals an interesting fact about the dynamic model—the time series of violations generated by the model exhibits

[24] When $R \leq 39$, it is very difficult to accurately estimate P_{GB}. P_{BB} is so close to 1 that once a simulation enters BB it tends to stay there for many millions of periods.

positive serial correlation. That there is autocorrelation in the time series of v_t is necessarily a general property of any model in which the qs in the system are determined by its past history. This is illustrated for our model by the 10 period lag structure of autocorrelation reported in the inset in Panel 1.

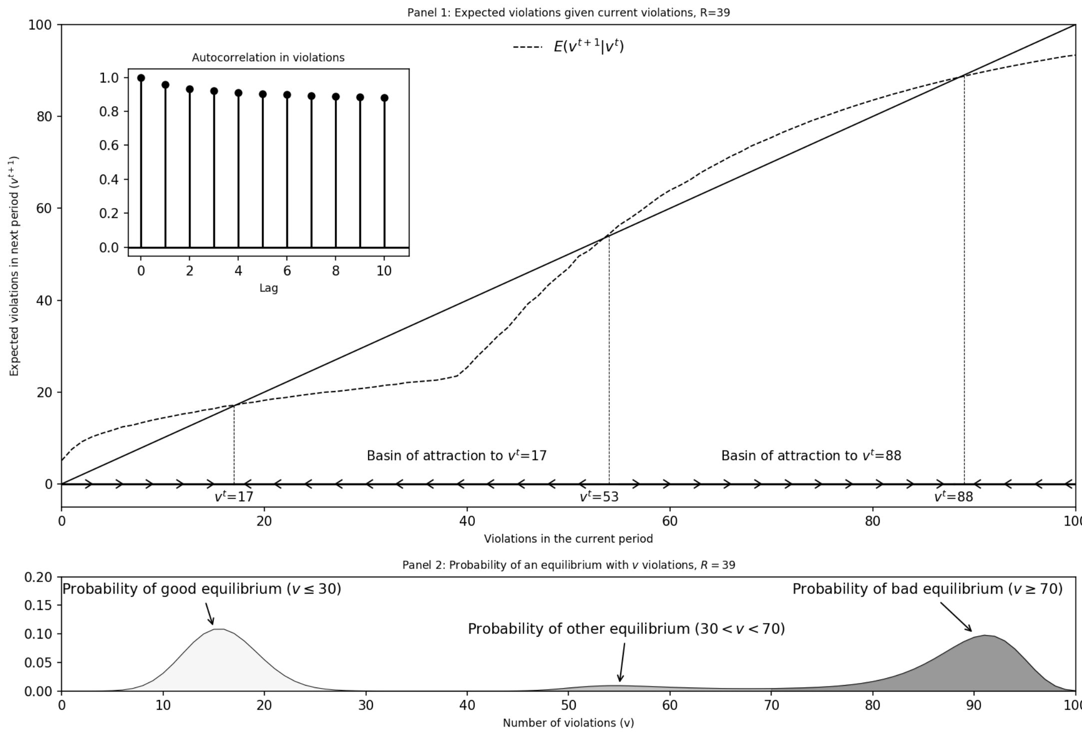

Figure 3.4: The dynamics of antisocial behavior for $R = 39$.

We explore the Nash equilibria of a static version of the dynamic model in Section A.1 of the Appendix. In that model q must always equal p. In Panel 2 of Figure 3.4 we reproduce Panel 1 of Figure A.1. Panel 2 plots the probability that that a full information static equilibrium exists for each $v \in \{0, ...N\}$ for $R = 39$. It is clear from the two panels that there is an interesting link between the static model and the dynamic model. Notice that the two modal values in Panel 2, consistent with a bimodal distribution of violations, are remarkably similar to the focal points in Panel 1. It is almost as if the dynamic model is randomly picking equilibria of the static model. However, given the dynamics of the Markov process—notably the positive feedback effects and the resulting serial correlation in violations—this is clearly not what is happening in the dynamic model.[25]

3.3 Violations in the stationary distribution

Panel 1 of Figure 3.5 plots the expected number of violations in the stationary distribution, $E(v|R)$ for $R \in \{0, ..., N\}$. Three distinct regimes are apparent: (i) an *unruly regime* ($0 \leq R \leq 38$) where $E(v|R) > 90$; (ii) a *compliant regime* ($43 \leq R \leq 100$) where $E(v|R) < 20$; and, (iii) a *transition regime* ($39 \leq R \leq 42$) where the expected number of violations decreases rapidly as R increases. These distinct regimes, and the resultant cliff of positive feedback separating the unruly and compliant regimes, has emerged in all of the many simulations we have run.

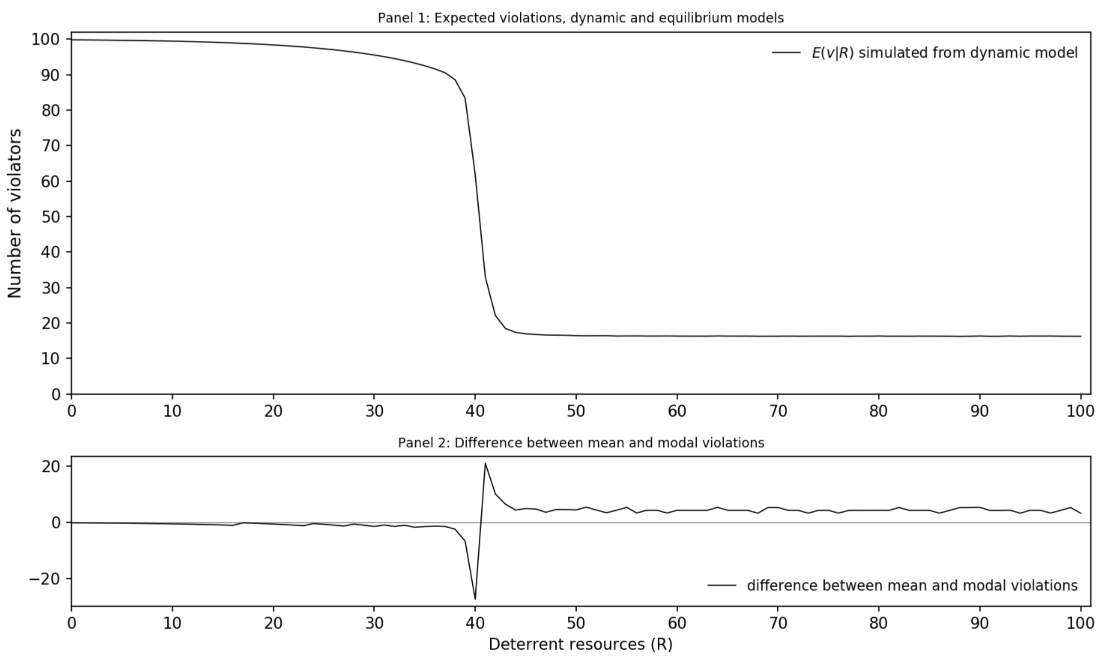

Figure 3.5: $E(v|R)$ for baseline parameterization, the "Cliff of Positive Feedback."

Panel 2 plots the mean number of violations minus the modal number of violations in the

[25] The similarity is noteworthy however—a number of models producing multiple equilibria ascribe the selection of those equilibria to positive feedback effects. Kleiman (1993) argues that enforcement swamping may produce tipping points through a positive feedback effect. In modeling crime waves, Tabarrok (1997) argues that there is a feedback effect from the number of violations to the probability of apprehension. However, these models, as well as other static models that produce multiple equilibria, are not actually modelling the process of positive feedback.

stationary distribution for every value of R. In the compliant regime the mean number of violations exceeds the modal number of violations, indicating that the frequency distributions are left skewed. In the unruly regime the opposite is true. In the transition regime the modal value is initially greater than the mean, this reverses dramatically as the frequency distributions transition from right to left skewed. These patterns are consistent with the frequency distributions in Figure 3.1 and the first panel of Figure 3.3.

3.4 Robustness exercises

It might be thought that the cliff-like geometry in Figure 3.5 is driven by the particular specifications we assumed in Section 2.3 in order to implement a computable version of the dynamic model. We provide evidence that, instead, this geometry of deterrence is a deep characteristic of the model; robust to a range of specifications and assumptions. Figure 3.6 presents results for six comparative exercises. The solid line in each plot is $E(v|R)$ for our baseline parameterization from Figure 3.5, which serves as our reference plot.

Panel 1 plots $E(v|R)$ when N is decreased from 100 to 50 (in this panel alone we plot per-capita values, $E(v|R)/N$ and R/N). Panel 2 plots $E(v|R)$ when the z-history is 4 periods long instead of 2. Panel 3 plots $E(v|R)$ for a tighter distribution of criminal opportunities, $\sigma = 0.15$ instead of 0.20. Panel 4 plots $E(v|R)$ for a different technology of apprehension (see footnote 16). Panel 5 plots $E(v|R)$ when the the q_i^ts are person specific, and equal to $Q(zh^t) + \delta_i^t$, where δ_i^t is a random draw from a uniform distribution with support $[-.2, +.2]$. Panel 6 plots $E(v|R)$ when the g_i^ts are drawn, not from a normal distribution, but from a uniform distribution with support $[0.0, 1.2]$. Clearly, the positive feedback producing the cliff in Figure 3.5 is a generic feature of a wide class of dynamic deterrence models. It is robust to all six specifications. An unruly regime and a compliant regime—q-attractors in clearly defined bins of good and bad behavior—separated by a transition regime manifest in a wide range of plausible specifications and parameterizations.[26]

[26] The cliff is also unaltered for variations in how we generate antisocial opportunities. Alternative models

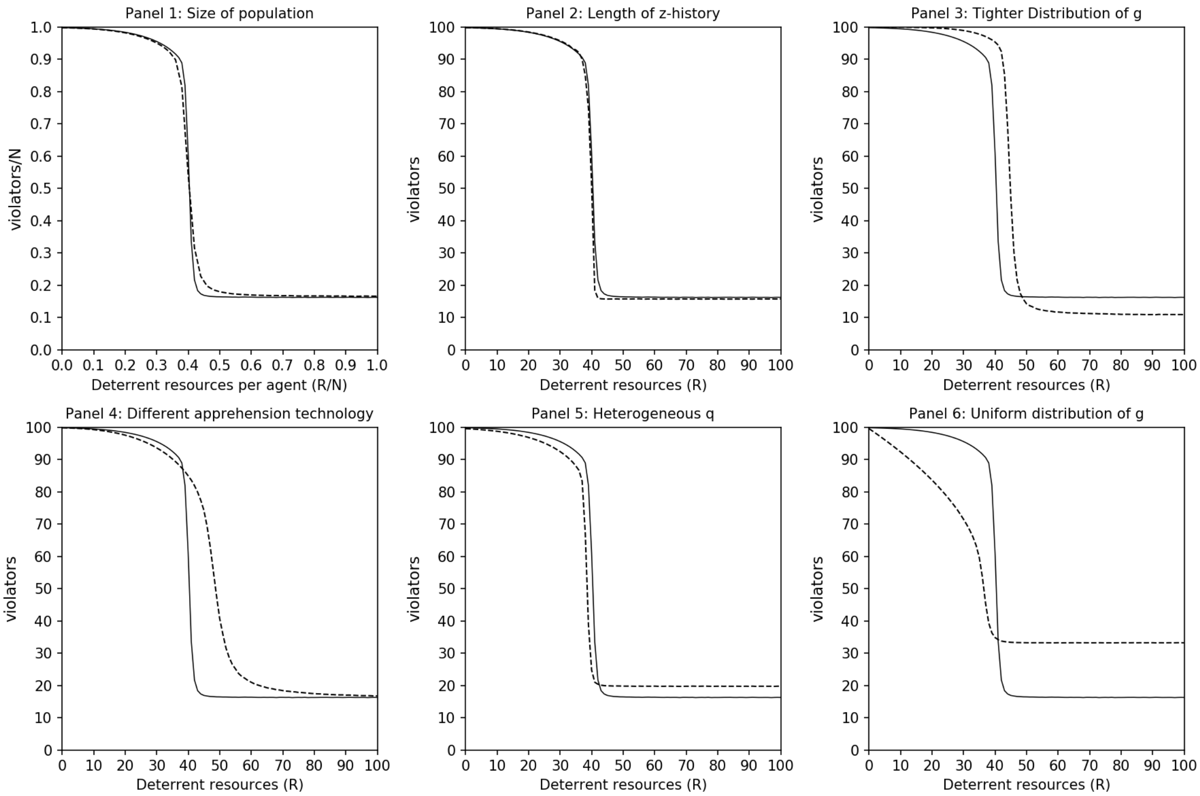

Figure 3.6: Robustness tests, cliff variants on z, N, σ, q, and $P(v, R)$.

Another issue concerns the robustness of our results to alternative levels of sanction. In the passive policies considered to this point the sanction F was fixed, and R varied. We now examine passive policies in which both R and F vary. We restrict our attention to passive policies for which $R \in \{0, ..., N\}$ and $F \in \{0, 0.05, ..., 1.45, 1.5\}$. This results in a set of passive polices with $101 \cdot 31 = 3131$ elements. Figure 3.7 is a surface plot of $E(v|R, F)$ for this set of polices. Two cross-sections are highlighted. The first is produced by holding the baseline sanction constant and varying R. This reproduces the "cliff of positive feedback" from Figure 3.5. The second is produced by holding deterrence spending constant at its cost minimizing level (see Section 3.5 below) and varying F. This reveals a similar cliff of positive feedback.

Both cross-sections, as well as the surface itself, reveal the same regimes we saw in Figure 3.5. In fact, every cross-section we can generate for all but small fixed values of R, or,

for generating antisocial opportunities are discussed in Appendix A.4.

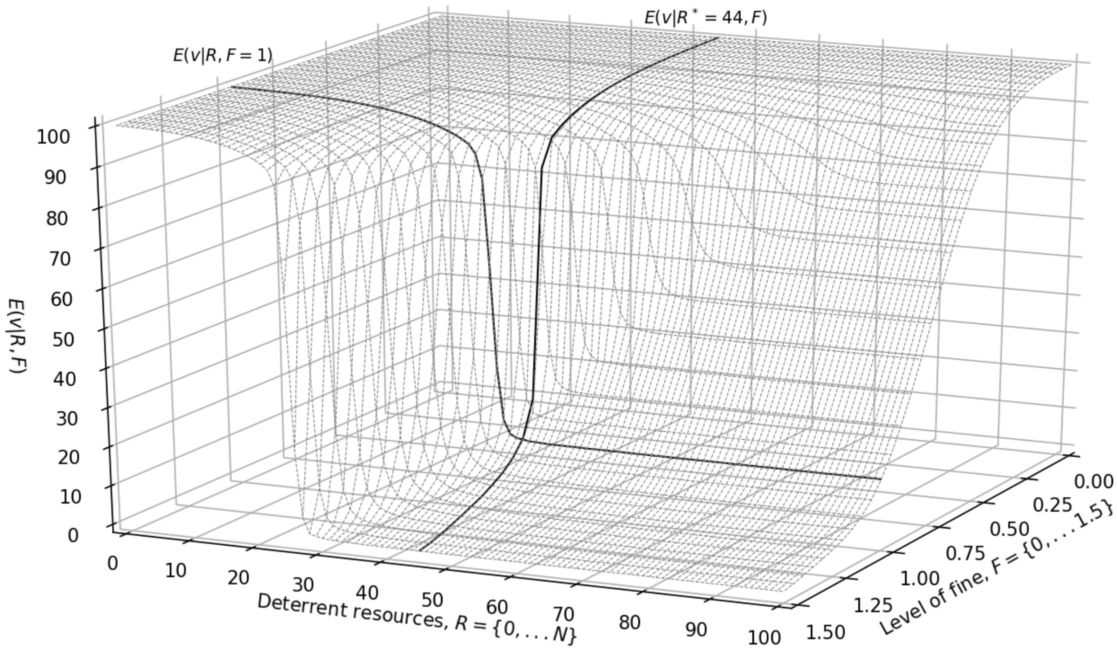

Figure 3.7: Expected violations for the entire passive policy space, $E(v|R, F)$.

alternatively, every cross-section for all but small fixed values of F, generates the same cliff-like features: (i) the unruly regime consists of a plateau around the top edge of the figure; (ii) the compliant regime is the rectangular plateau at the bottom; and, (iii) the transitional regime is a cliff separating the two. The cliff of positive feedback is a pervasive feature of our dynamic process.

3.5 The costs of antisocial behavior and optimal policies

We now consider the impact of passive policies on the costs of ASB.[27] We focus on policies in which $F = 1$ and R varies from 0 to N. We assume it is costless to sanction apprehended violators, and that F is a transfer. So, in any period t, there are two sorts of costs: those associated with enforcement and those associated with violations. Enforcement costs are

[27] Typically one would like to choose a policy for every state so as to minimize the discounted present value of the cost of ASB over some time horizon. Given the number of states in our baseline model, and our reliance on simulation, it is not feasible to solve that optimization problem. Further, one cannot simplify this cost minimization problem by grouping states, since the objective function for that problem is not well defined (the discounted present value is well defined only for individual states, not groups of states). So we instead focus on the expected cost of ASB in the stationary distribution. In this section we choose the passive enforcement policy that minimizes that expected cost.

simply $\rho \cdot R$, where ρ is the cost of one unit of the composite enforcement resource. If person i chooses violate, the net social cost is $(\lambda - 1) \cdot g_i^t$. Defining V^t as the set of all violators in period t comprised of every person i for whom $g_i^t \geq Q(zh^t) \cdot F$, the cost of ASB in period t is,

$$C^t = \rho \cdot R + (\lambda - 1) \cdot \sum_{i \,\in\, V^t} g_i^t.$$

Let the simulation converge in period $t = \Gamma$, the expected per period cost of ASB in the stationary distribution is then

$$E(C|R) = \frac{1}{\Gamma} \sum_{t=1}^{t=\Gamma} C^t.$$

The *optimal* passive policy is the R that minimizes $E(C|R)$. We set the cost of investigating a single violation to $\rho = 2$ and the fine to $F = 1$. In Figure 3.8 below we plot $E(C|R)$ over $R = \{0, ..., N\}$ for three carefully chosen values of λ. Each illustrates a class of deterrence problems.

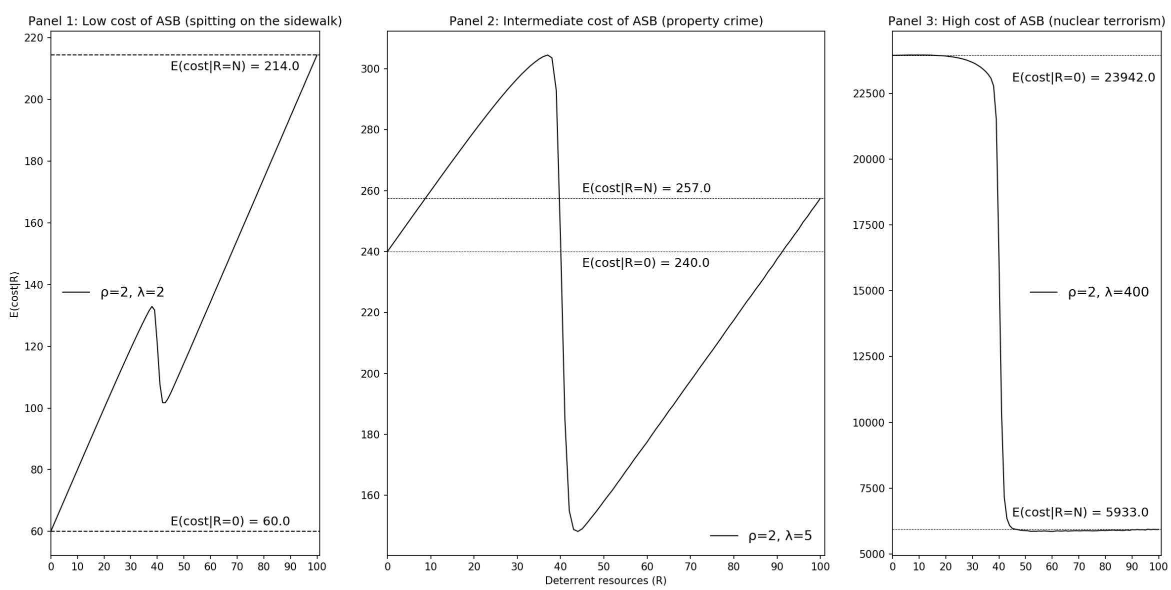

Figure 3.8: Cost of antisocial behavior for $\rho = 2$ and $\lambda = 2, 5, 400$.

The two horizontal lines in each panel indicate the costs of no enforcement (originating at $E(C^t|R = 0)$) and maximal enforcement (originating at $E(C^t|R = N)$). Panel 1 captures the *'spitting on the sidewalk'* case, where the cost of ASB is so low that zero enforcement is optimal ($R^* = 0$ minimizes costs). Panel 3 captures the *'nuclear terrorist'* case, where the cost of ASB is so high that maximal enforcement is optimal ($R^* = N$ minimizes costs).

Panel 2 depicts the case where it is cost effective to deter some ASB but not to eliminate it completely, which we take to be the more typical case confronting policy makers. When $\rho = 2$ and $\lambda = 5$ we get an interior solution for deterrence efforts, $R^* = 44$, which sits at the left edge of the compliant regime. The gross features of the cost function seen in Panel 2 of Figure 3.8 are the same for a wide range of parameter values (i.e., for $2 \lessapprox \frac{\lambda}{\rho} \lessapprox 50$). Notice that, except when λ is very large, in most of the unruly regime, the cost of ASB increases with R. If we look back to Figure 3.5 we see what accounts for this curious fact—in the unruly regime expected violations fall slowly in R, so slowly that the cost reductions from increasing compliance are swamped by the increase in enforcement costs. Importantly, this means that when a passive policy is used there is the potential for the policy maker to get stuck at the local minimum of $R = 0$. This is the case since, in the neighborhood of $R = 0$, small increases in R actually increases the cost of ASB. For the remainder of the paper we add $\rho = 2$ and $\lambda = 5$ to our list of baseline parameters.[28]

The cost estimate obtained from any single convergence simulation is a random variable. This raises a question: how reliable are the results of any single simulation? We use Monte Carlo methods to estimate the variance in our simulated results and to get a sense of how costs change in the neighborhood of the optimal deterrent policy. We ran 150 independently seeded convergence simulations for each of the three lowest cost passive policies, $R = (43, 44, 45)$. These results indicate that the noise in our costs estimates is within acceptable bounds. The second column reports the mean cost over the 150 estimates, the third column the standard deviation of the 150 estimates, and fourth column the standard error of mean cost estimates.

[28] We explicitly follow Becker (1968) in assuming F is a transfer. Relaxing this assumption, and including a term in the cost function to reflect the potential costs of imposing sanctions—funding courts and prosecutions, building and maintaining prisons, the spillover costs on the families of violators, the lost production of incarcerated violators—increases the the non-concavity of costs in Figure 3.8. Essentially, the only substantive effect is to heighten the cost advantage associated with being in the the compliant regime.

Table 3.1: Cost of passive policies in the neighborhood of the optimal deterrent policy.

R	Mean of $E(C^t \vert R)$ over 150 runs	**SD**	**SE**
43	148.74	0.4113	0.034
44	148.23	0.1921	0.016
45	149.23	0.1344	0.011

3.6 Useful indices relating to antisocial behavior

Given the paucity of data on the cost of antisocial behavior, it is sensible to examine the ways in which more readily observable indices correlate with it. We focus on the *investigation rate* and the *apprehension rate* as functions of R. The capacity to investigate violations is R, so the investigation rate is $\frac{E(v|R)}{R}$; it is the probability that a violation randomly selected from the stationary distribution of violations is investigated. The apprehension rate is $\frac{E(a|R)}{E(v|R)}$; it is the probability that the person who committed a violation randomly selected from the stationary distribution is apprehended. Given our technology of apprehension, the probability that the violator is apprehended when the violation is investigated is γ (0.8 in the baseline parameterization), so the apprehension rate is just $\gamma \cdot \frac{E(v|R)}{R}$.

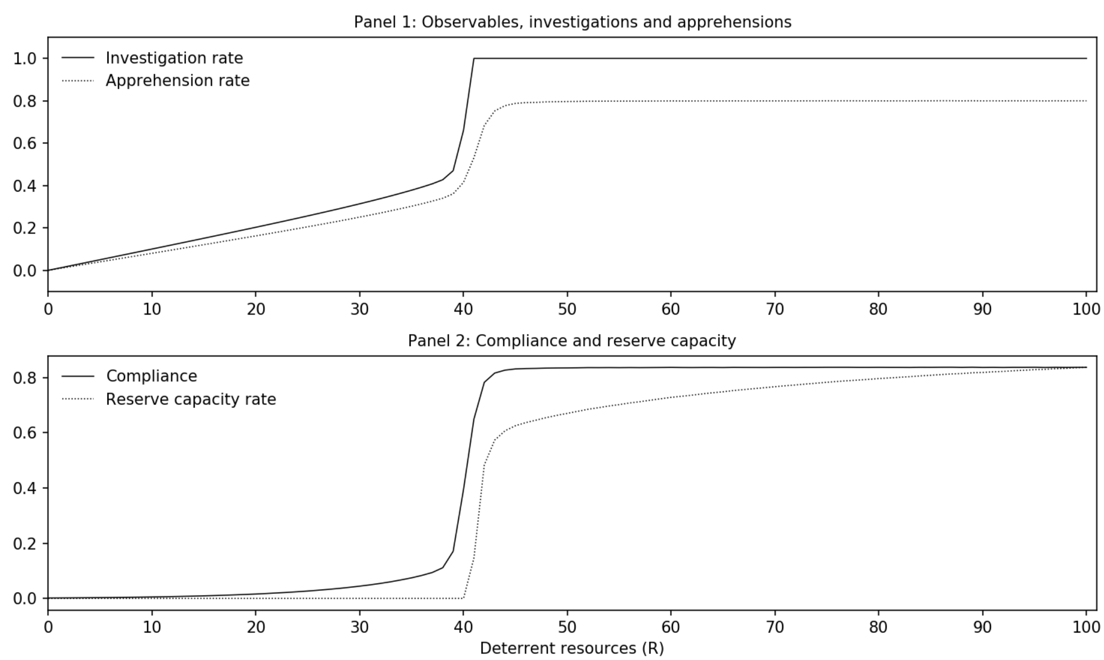

Figure 3.9: Useful indices related to the cost of antisocial behavior.

We have plotted both rates in the top panel of Figure 3.9. They rise steadily throughout the unruly regime and rapidly in the transitional regime. Both are also rising throughout the compliant regime, but the rate of increase is so small that it is imperceptible. This is a reflection of the fact that throughout the compliant regime both rates are very close to their maximum acheivable values, 1 for the investigation rate and $\gamma = .8$ for the apprehension rate.

In Panel 2 we have also plotted the expected *reserve capacity* rate, defined as the fraction of available enforcement resources that are not used to investigate a violation. It is equal to $\max(0, \frac{R-E(v|R)}{R})$. Throughout the unruly regime the rate approaches 0—because $E(v|R)$ is so much larger than R, in virtually every period all available enforcement resources are deployed to investigate a violation. It rises rapidly to 0.59 in the transitional regime, and 0.83 in the compliant regime.

In Section 3.5, we saw that except in the spitting on the sidewalk case, where the best thing to do is to ignore the behavior, the optimal enforcement policy is in the compliant regime. Panel 1 of Figure 3.9 is telling us that when it makes sense to deploy resources to deter ASB, the optimal policy entails an investigation rate that is virtually 1, an apprehension rate that very close to the maximum achievable rate, and a reserve capacity rate that is 0.60 or higher. This last finding is remarkable: when the enforcement policy is optimal, 60% or more of available enforcement resources are not typically deployed to investigate a crime.

That looks a lot like excess or wasted enforcement capacity. This interpretation of reserve capacity would be accurate if the purpose of the deterrence effort was to apprehend violators. But that is not the case. The purpose is to deter potential violators, or the same thing looked at from a different perspective, to promote compliance. In Panel 2 of Figure 3.9 we have plotted the *rate of compliance*, the probability that a randomly chosen person in a randomly chosen period will choose comply over violate. Comparing the reserve capacity and compliance rates it clear that to induce a high rate of compliance, a high rate of reserve capacity is required. That reserve capacity is needed to maintain the high subjective probability of apprehension that induces the high rate of compliance.

4 Active Enforcement Policies

There are two q-attractors when the enforcement policy is passive, one good and one bad. The optimal passive policy requires setting R so the fraction of time spent in the bad q-attractor approaches 0. In other words, the optimal passive policy sets R just large enough to effectively extinguish the bad q-attractor.

There is an important insight in this: deterrence is about managing positive feedback. This suggests that active policies—those in which the R deployed in any period t is dependent on violations in the previous period—will outperform the best passive policy. Clearly, the set of active policies is so large that we cannot explore them all. We begin with a simple policy of *crackdowns* and build from there.[29]

4.1 Crackdown policies

A crackdown policy deploys one R when v^{t-1} is small and another when it is large. It is described by three parameters, $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}})$[30] : (i) $\mathbb{BE}$ is a bin edge that defines the *good bin*, GB, and the *bad bin*, BB; (ii) $\mathbb{R}_{\mathbb{GB}}$ is the quantity of enforcement resources deployed when v^{t-1} is in $GB = \{v^{t-1}|0 \leq v^{t-1} \leq \mathbb{BE}\}$; and, (iii) $\mathbb{R}_{\mathbb{BB}}$ is the quantity deployed when v^{t-1} is in $BB = \{v^{t-1}|\mathbb{BE}+1 \leq v^{t-1} \leq N\}$. In the spirit of cracking down on ASB when it gets out of hand, we assume that $\mathbb{R}_{\mathbb{BB}} \geq \mathbb{R}_{\mathbb{GB}}$.

To illustrate how a crackdown works, Figure 4.1 presents various time series for 100 periods of simulated ASB for crackdown policy $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}}) = (50, 37, 59)$. The solid line in panel 1 is v^t, the dashed line the apprehension rate, $\frac{a^t}{v^t}$, and the horizontal line $\mathbb{BE}$.

[29] Our crackdown terminology is inspired by Sherman (1990, p.2) who defines a crackdown as "a sharp increase in enforcement resources" which communicates "a far more powerful threat of apprehension and punishment than does 'normal' policing."

[30] In introducing crackdown policies we have used a different font to distinguish the crackdown policy parameters from those already defined and those soon to be defined with the introduction of refined crackdown policies in Section 4.3. For the parameters of refined crackdown policies we use another font.

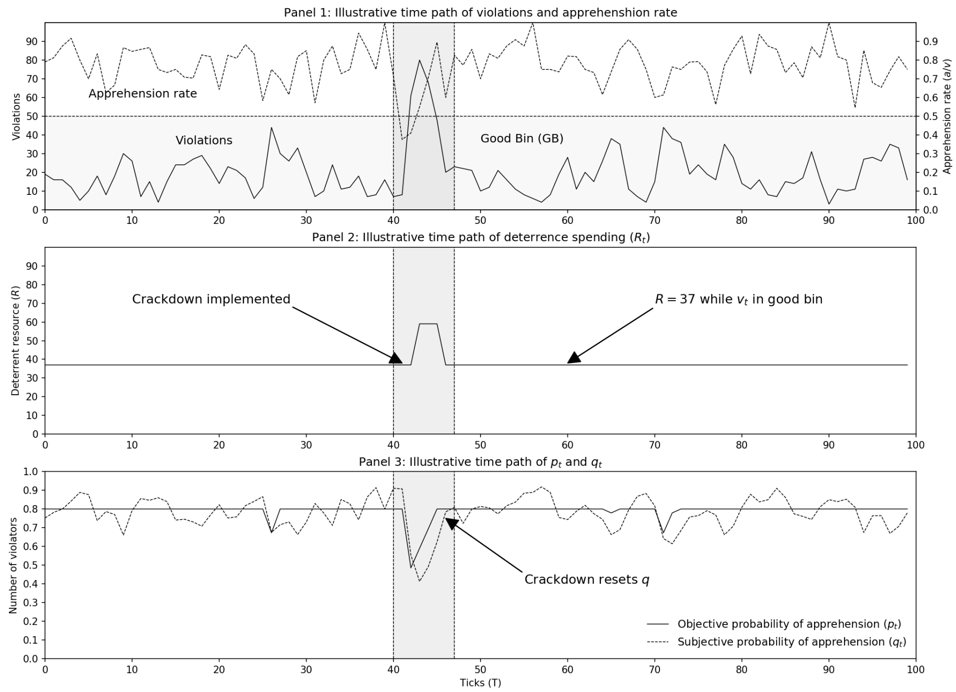

Figure 4.1: Illustrating the mechanics of a crackdown policy.

Panel 2 plots R^t, and panel 3 p^t and q^t. Around period 42, v^t enters BB and stays there for approximately 4 periods. Entry into BB is precipitated by an anomalously low apprehension rate which drives q^t down and v^t up. Entry into BB triggers an increase in R^t from 37 to 59, this increases the apprehension rate and, with a lag, drives q^t up and hence v^t down. By resetting the subjective probability of apprehension, the crackdown precipitates a quick exit from BB.

Now let us see what a well chosen crackdown policy can do. In Figure 3.4 we examined the dynamics of deterrence when the policy was passive and R was 39. We saw that there were two q-attractors with basins of attraction centered on 53 violations. For purposes of comparison we have reproduced that figure in panel 2 of Figure 4.2 and we have added the bimodal frequency distribution of violations in the stationary distribution for that passive policy. In panel 1 we have constructed the analogous figure for crackdown policy

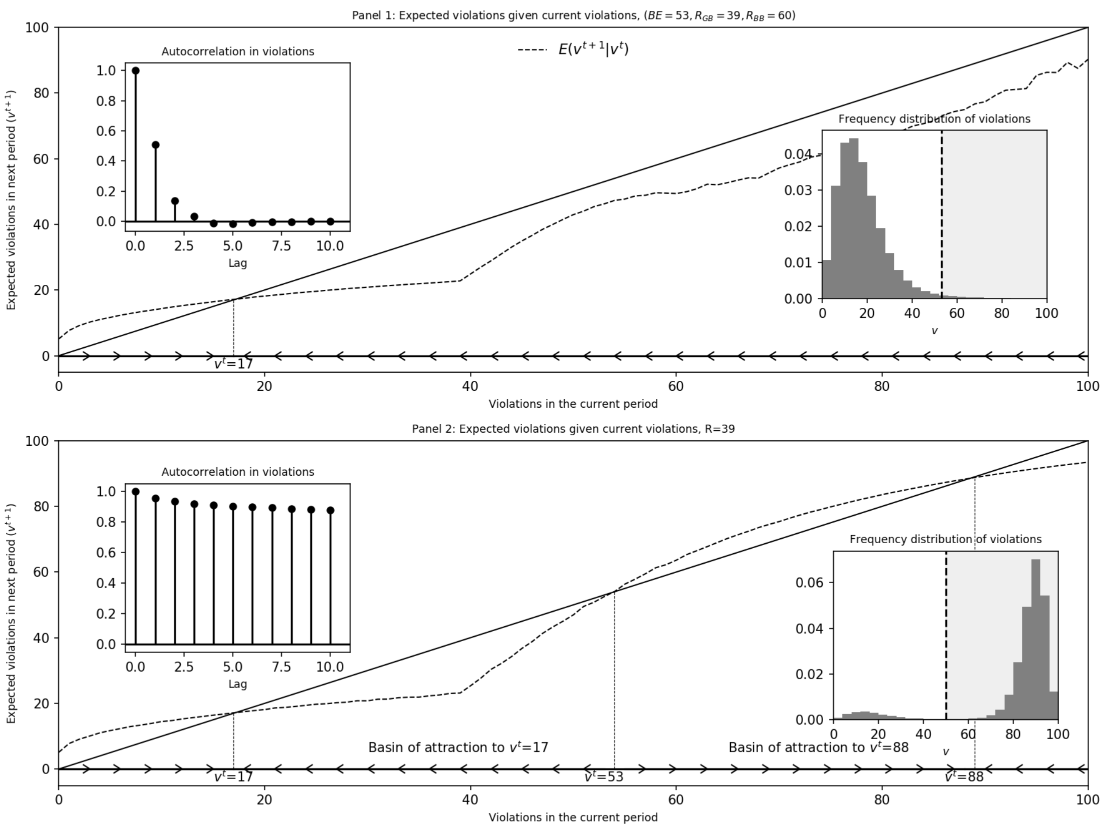

Figure 4.2: Extinguishing BB with crackdown.

$(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}}) = (53, 39, 60)$. Notice that in this crackdown policy the bin edge is $\mathbb{BE} = 53$; the only difference between the policies used in the two panels is that when $v^t > 53$, R is 60 with the crackdown policy and just 39 with the passive policy. Comparing the two panels we see a number of implications of cracking down on antisocial behavior when the number of violations exceeds 53. The bad q-attractor is extinguished. The frequency distribution of violations is now unimodal with virtually all of its mass concentrated in the good bin $GB = \{v^t | 0 \leq v^t \leq 53\}$. And for all of the lags illustrated, the autocorrelation coefficients are dramatically reduced.

4.2 Finding the optimal crackdown policy

Finding the passive policy that minimized the cost of ASB in the stationary distribution was straight forward. We simply estimated the cost of every one of the 101 possible policies. The number of crackdown policies is so vast that this method is simply not feasible. Further, the cost functions associated with active policies often reveal local non-convexities and significant ranges where costs are "flat." For example, for a given values of $\mathbb{R}_{\mathbb{GB}}$ and $\mathbb{R}_{\mathbb{BB}}$, it is often the case that a wide range of bin edges produce essentially the same expected costs.

We use a two stage procedure to overcome these issues and identify the approximately optimal $(\mathbb{BE}^*, \mathbb{R}_{\mathbb{GB}}{}^*, \mathbb{R}_{\mathbb{BB}}{}^*)$. In the first stage we identify the neighborhood of the optimal policy with a directed search routine for 1000 randomly chosen initial policies. To speed the search, the routine employs a loose convergence criterion. In the second stage, having identified the neighborhood of the optimal policy, and thus significantly reduced our search grid, we use Monte Carlo methods and our standard convergence simulation to generate improved estimates of the cost of every policy in the neighborhood. Section A.5 of the appendix provides a detailed description of our procedure.

In stage 1 we concluded that the optimal crackdown policy was in the set of policies $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}})$ such that $30 \leq \mathbb{BE} \leq 39$, $29 \leq \mathbb{R}_{\mathbb{GB}} \leq 32$, and $53 \leq \mathbb{R}_{\mathbb{BB}} \leq 65$. In stage 2, using our standard convergence simulation procedure, we generated 150 independent estimates of the cost of ASB for each policy in this neighborhood. Our improved cost estimates are the mean values of these 150 estimates for each policy. These estimates indicate that the optimal crackdown policy, $(\mathbb{BE}^*, \mathbb{R}_{\mathbb{GB}}{}^*, \mathbb{R}_{\mathbb{BB}}{}^*)$, is either (35,30,57) or (34,30,56)—the cost estimate for both is \$130.33 per period. In Section 3.5, our cost estimate for the optimal passive was \$148.23. So the best crackdown policy is significantly better than the best passive policy.

Table 4.1 reports the *minimum cost* of all crackdown policies $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}})$ such that $30 \leq \mathbb{BE} \leq 39$, and $29 \leq \mathbb{R}_{\mathbb{GB}} \leq 32$. The minimization is done with respect to $\mathbb{R}_{\mathbb{BB}}$; we report the value of $\mathbb{R}_{\mathbb{BB}}$ that produces the minimum cost in the associated cell. For example, the cell in the upper-left corner of the table indicates that the minimum costs of crackdown policies in which $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}) = (30, 29)$ is \$131.28, and the cost minimizing $\mathbb{R}_{\mathbb{BB}}$ is 54. As the standard errors of these estimates are all very small (i.e., $\leq$ \$0.017), it is clear that the optimal policy

Table 4.1: Estimated Costs of Crackdown Policies in the Neighborhood of the Optimum.

		$\mathbb{BE}$									
		30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
$\mathbb{R}_{\mathbb{GB}}$	**29**	131.28 **54**	130.91 **56**	130.66 **56**	130.57 **58**	130.62 **57**	130.81 **58**	131.17 **60**	131.62 **60**	132.16 **62**	132.77 **62**
	30	131.52 **55**	131.06 **55**	130.68 **56**	130.45 **57**	130.33 **56**	130.33 **57**	130.45 **59**	130.66 **60**	131.03 **61**	131.43 **62**
	31	131.97 **53**	131.11 **54**	131.11 **55**	130.76 **56**	130.52 **57**	130.37 **57**	130.35 **58**	130.41 **58**	130.58 **59**	130.82 **60**
	32	132.61 **53**	132.16 **53**	131.71 **55**	131.40 **55**	131.10 **56**	130.84 **57**	130.72 **58**	130.65 **59**	130.67 **59**	130.80 **60**

Note: The **bold** number in each cell is the associated $\mathbb{R}_{\mathbb{BB}}$ of each policy. The standard errors for each cost estimate are in the range 0.013 to 0.017. So the deviation around each point estimate of costs in each cell is in the second decimal point (e.g. costs in the first cell range at most between 131.297 and 131.263.)

is either (35,30,57) or (34,30,56). Every other cost estimate is more than 2 standard errors higher than \$130.33. Notice how flat the cost function is in this neighborhood. The lowest cost reported in the table is \$130.80 and the highest is \$132.77, a difference of just 1.5%. It is even flatter in the $\mathbb{R}_{\mathbb{BB}}$ dimension (not presented in the table). For example, the cost estimate for policy $(39, 31, 60)$ is \$130.82, and the cost estimates for four related policies, (39,31,57), (39,31,58), (30,31,59) and (30,31,61), are all within one standard error of \$130.82.

Accounting for the fact that the cost of the optimal passive policy, $E(C|44) = \$148.23$, is 12.1% higher than the cost of the optimal crackdown policy, $E(C|35, 30, 57) = \$130.33$, is relatively straightforward. Consider first the expected cost of enforcement resources per period: with the optimal passive policy it is $\rho * R^* = 2 * \$44 = \$88$; with the optimal crackdown policy it is \$60 when the process is in GB and \$114 when it is in BB, but 90% of all periods are in GB and just 10% in BB, so the weighted average is roughly \$65.40. The optimal crackdown policy therefore lowers enforcement costs by \$22.60 per period. Next consider the expected cost per period arising from violations: with the optimal passive policy it is \$60; with the optimal crackdown policy it is \$125 when the process in BB and \$58 when it is in GB, and the weighted average is roughly \$64.70. Relative to the optimal passive policy, the optimal crackdown policy spends \$22.60 per period less on enforcement but it involves \$4.70 more per period in costs associated with violations, so the net cost advantage of the crackdown policy is \$17.90 per period, or 12.1%.

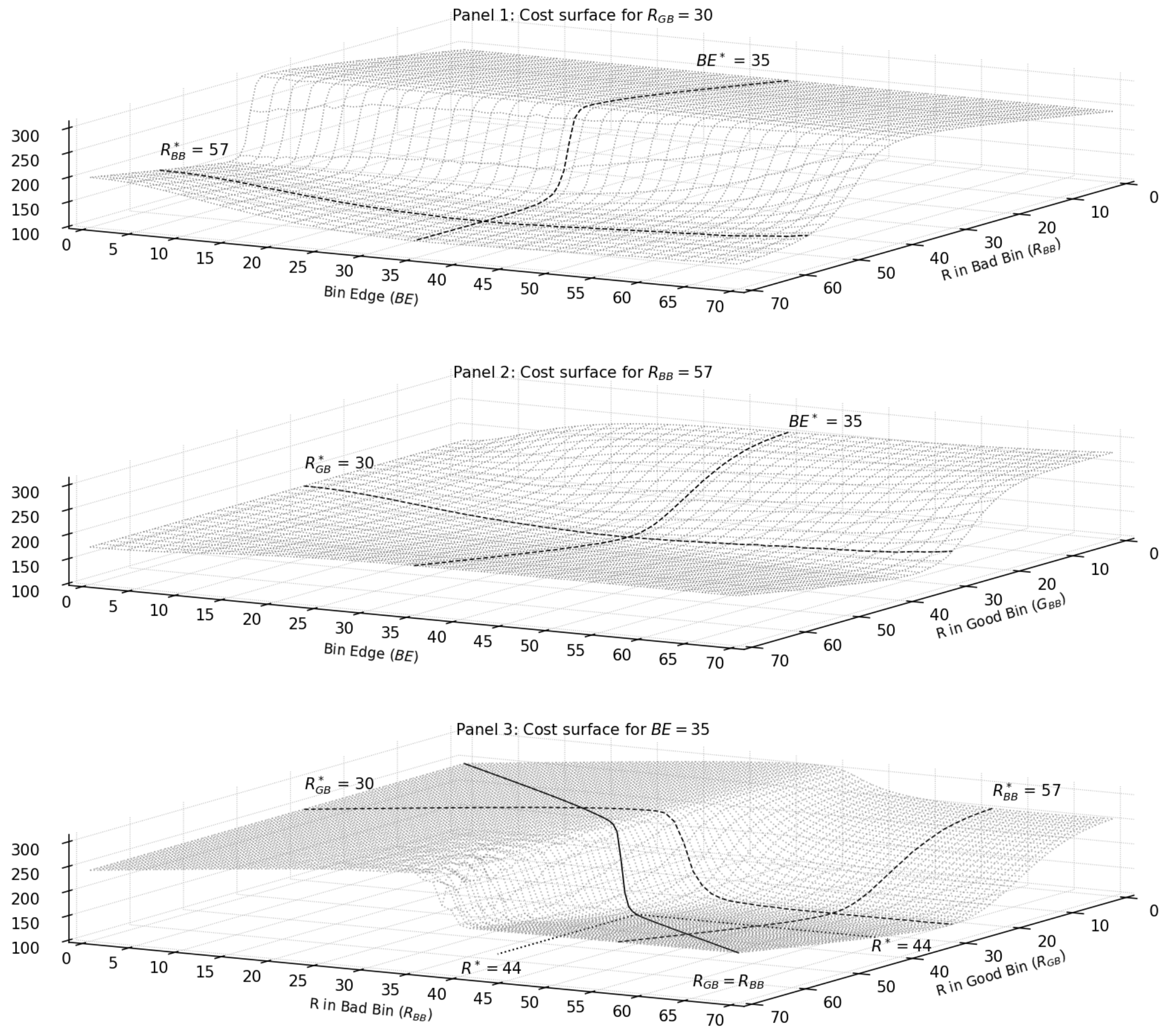

Figure 4.3: Surface plots of costs in $\mathbb{BE}$, $\mathbb{R}_{\mathbb{BB}}$, and $\mathbb{R}_{\mathbb{GB}}$ space

Figure 4.3 presents three surface plots of the per period expected cost of ASB for the crackdown policy. In each panel, one of the three parameters of the crackdown policy is held constant at its optimal value and the other parameters vary, and the optimal policy is identified as the intersection of two cross-sections traced out by the dotted lines. In the bottom panel, where $\mathbb{BE}$ is fixed at its optimal value, 35, we have also identified the optimal passive policy, $R^* = 44$, and the cross section through that policy were $\mathbb{R}_{\mathbb{GB}} = \mathbb{R}_{\mathbb{BB}}$. In all three panels we see a familiar pattern: large high cost unruly regimes separated from low cost compliant regimes by a cliff like structure produced by positive feedback in the transition regimes. This indicates that the policy maker faces the familiar challenge of managing positive feedback in selecting each element of the crackdown policy.

The top panel of Figure 4.4 presents three plots of the costs of ASB for the three cross sections highlighted in the third panel of Figure 4.3. The bottom panel of Figure 4.4 presents the corresponding indices of reserve enforcement capacity and the apprehension rate. The plots in the first column of the figure should be familiar. They pertain to the cross section in which $\mathbb{R}_{\mathbb{GB}} = \mathbb{R}_{\mathbb{BB}}$. The point we made in Section 3.6 was that when the cost of ASB is relatively small reserve capacity is high ($\frac{R-v}{v} \geq 0.59$) and the apprehension rate is very close to the maximum possible rate ($\gamma = .8$). The second and third columns illustrate the fact that the same can be said of any crackdown policy that produces a relatively low cost of ASB.

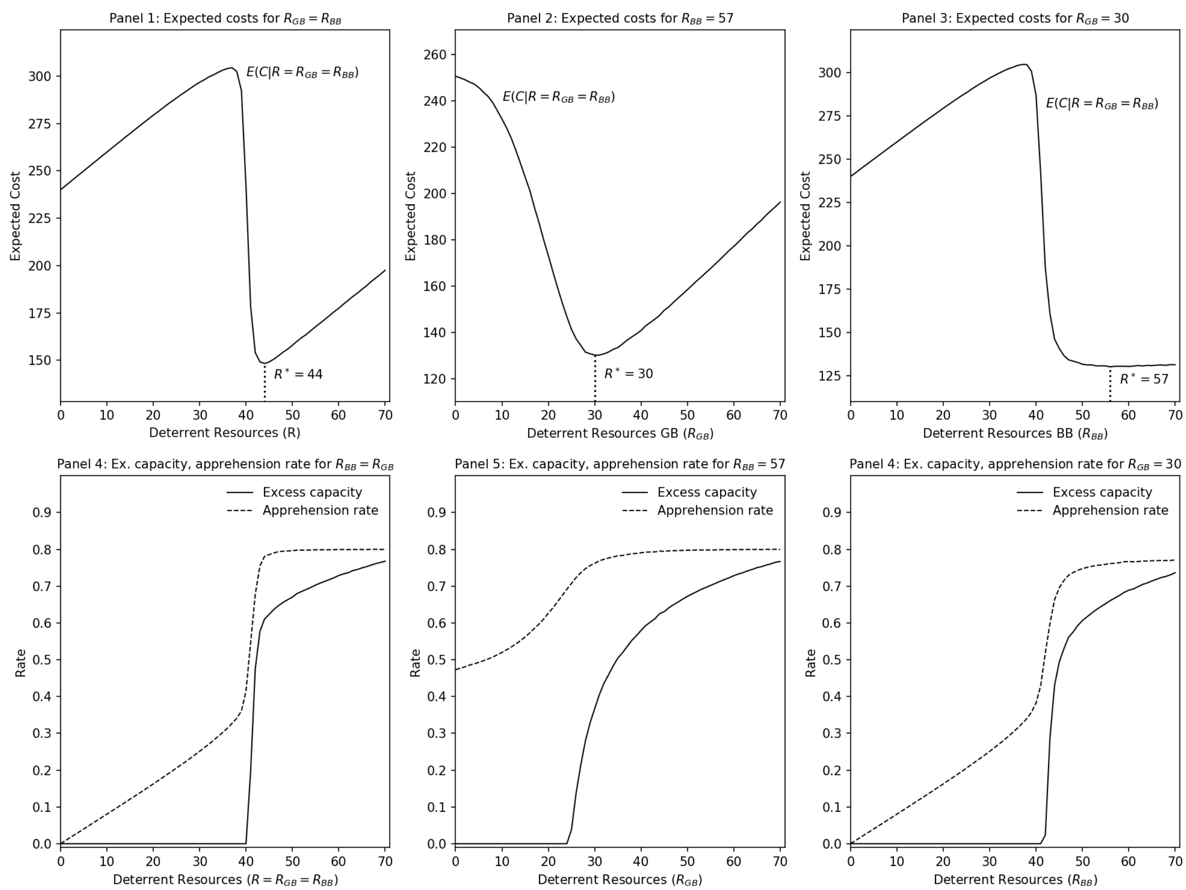

Figure 4.4: Cross sections of costs and efficiency indexes for $\mathbb{BE} = 35$

It is not surprising that the optimal crackdown policy is superior to the optimal passive policy. As a matter of logic, it could not be inferior since the set of passive policies is a proper subset of the set of crackdown policies. As an intuitive matter it seems clear that it ought to be superior because deterrence involves managing two q-attractors, and to do so two policy instruments are required. The question then arises: is there a refined crackdown policy of some description that is demonstrably superior to the optimal crackdown policy? Perhaps surprisingly, it is far from clear there is.

4.3 Refined crackdown policies

A refined crackdown policy is described by five parameters, $(\mathsf{BE}, \mathsf{R}_{\mathsf{GB}}, \mathsf{SBE}, \mathsf{R}_{\mathsf{BB}_1}, \mathsf{R}_{\mathsf{BB}_2})$. The bin edge BE defines the good bin, $GB = \{v^{t-1} | 0 \leq v^{t-1} \leq \mathsf{BE}\}$; R_{GB} is the quantity of enforcement resources deployed when v^{t-1} is in GB. The bin edge SBE defines two bins, $BB_1 = \{v^{t-1} | \mathsf{BE} < v^{t-1} \leq \mathsf{SBE}\}$ and $BB_2 = \{v^{t-1} | \mathsf{SBE} < v^{t-1} \leq N\}$. $\mathsf{R}_{\mathsf{BB}_1}$ is the quantity of enforcement resources deployed when v^{t-1} is in BB_1; $\mathsf{R}_{\mathsf{BB}_2}$ is the quantity deployed when v^{t-1} is in BB_2. We assume that $\mathsf{BE} \leq \mathsf{SBE}$ and $\mathsf{R}_{\mathsf{GB}} \leq \mathsf{R}_{\mathsf{BB}_1} \leq \mathsf{R}_{\mathsf{BB}_2}$.

Table 4.2: The Value of Refinement.

		BE									
		30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
R_{GB}	**29**	1.07 9,18	0.73 11	0.62 6	XXX	XXX	XXX	XXX	XXX	XXX	XXX
	30	XXX	XXX	0.68 2,7,15	0.49 3,4,10	0.42 1,5,8,13,14,21,22	0.17 20	XXX	XXX	XXX	XXX
	31	XXX	XXX	XXX	0.55 17	0.42 12,24	0.20 19,25	0.24 16,23	XXX	XXX	XXX
	32	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX

To find the approximate cost of the optimal refined crackdown policy, we use a two stage procedure similar to the one we used to find the optimal crackdown policy. Details of the procedure can be found in Section A.5 of the Appendix. In the first stage we identify the neighborhood of the optimal refinement by using a straight forward extension of the directed search routine used in Section 4.2. For each of the 1000 randomly chosen policies, the directed search routine identifies a candidate policy. In the second stage, we rank the 1000 candidate policies in ascending order of their cost estimates. We then generate tight costs estimates for the first 25 policies in this ranking, running 150 independent convergence simulations for each policy and using the mean cost as our cost estimate for the policy.

You may have noticed that the first two elements in our notation for a refined crackdown policy, BE and $\mathsf{R_{GB}}$, are the same as those used for the first two elements of a crackdown policy. This reflects a conscious choice to frame a refined crackdown policy as a refinement of a crackdown policy with the same $\mathbb{BE}$ and $\mathbb{R}_{\mathbb{GB}}$ in which the bad bin, BB, is partitioned into two bins, BB_1 and BB_2, with different quantities of the enforcement resource. We chose to frame them in this way to highlight the fact the 25 least costly refined crackdown policies we identify in stage 1 of our search procedure are in fact refinements of one of the least costly crackdown policies we identified in the previous section, and presented in Table 4.1. Key results for the 25 refined crackdown policies with the lowest tight cost estimates are reported in Table 4.2. Notice that the skeletons of Tables 4.1 and 4.2 are identical.

All 25 of the best refined crackdown policies appear in just 11 of the 40 cells in Table 4.2, and 13 of them are in cells (32, 30), (33, 30) and (34, 30). In the top line of each cell we have listed the percentage difference in cost between the best refined crackdown policy and crackdown policy for that cell (from Table 4.1). The second line of each cell identifies the refined crackdown policy by its rank—with rank 1 being the lowest cost estimate. Policies 1, 5, 8, 13, 14, 21 and 22 are listed in cell (34, 30). All are refinements of the crackdown policy (34, 30, 56) from Table 4.1, and all produce a lower cost. This is not surprising – with more instruments to control ASB we expect to see a lower cost of ASB. What may be surprising is that the reduction in cost is relatively small — that is what the entry in the first line of each cell addresses. The cost estimate for the refined crackdown policy with rank 1 is only 0.41% lower than the costs estimate for optimal crackdown policy (34, 30, 56).

In Table 4.3 we report the policies that are ranked 1, 2 and 3, our costs estimates for them, and the standard errors of the cost estimates. When we look at the policies themselves we see that the refinements make intuitive sense. The lowest cost refined crackdown policy is $(\mathsf{BE}, \mathsf{R_{GB}}, \mathsf{SBE}, \mathsf{R_{BB1}}, \mathsf{R_{BB2}}) = (34, 30, 44, 47, 64)$ and it is a refinement of crackdown policy $(\mathbb{BE}, \mathbb{R}_{\mathbb{GB}}, \mathbb{R}_{\mathbb{BB}}) = (34, 30, 56)$. When crackdown policy $(34, 30, 56)$ is used, 56 units of enforcement resource are deployed in the bad bin, $\{v^{t-1}|34 < v^{t-1} \leq 100\}$. When refined crackdown policy $(34, 30 : 44, 47, 64)$ is used the bad bin is partitioned into a two two bins, the first is $\{v^{t-1}|34 < v^{t-1} \leq 44\}$ and the second is $\{v^{t-1}|44 < v^{t-1} \leq 100\}$, and quantity of enforcement resources deployed in the first is 47, somewhat less 56, and quantity deployed in the second is 64, somewhat greater than 56. In the bad bin, the proximate objective of policy is to drive violations down and back into GB, and it makes intuitive sense that the quantity of resources needed to do that efficiently is smaller when violations are close to the lower bound of BB than when violations are close to the upper bound.

Table 4.3: Top ranked refined crackdown policies.

Rank	BE	$\mathsf{R_{GB}}$	SBE	$\mathsf{R_{BB_1}}$	$\mathsf{R_{BB_2}}$	**Cost**	**Std Error**
#1	34	30	44	47	64	129.79	0.013
#2	32	30	37	42	60	129.79	0.013
#3	33	30	43	49	64	129.82	0.015

In Section 4.2, to identify the optimal crackdown policy, we generated a tight cost estimate for every policy in the neighborhood that we identified using the stage one directed search routine. That approach is not feasible for refined crackdown policies. We can use the directed search routine to identify the neighborhood of the optimum, but there are so many policies in that neighborhood that it is not feasible to generate a tight cost estimate for each of them. In addition, in three of the five relevant dimensions, SBE, $\mathsf{R_{BB_1}}$ and $\mathsf{R_{BB_2}}$, the cost function is so flat that it would take thousands of convergence simulations to get cost estimates that would allow us to identify the optimal policy.

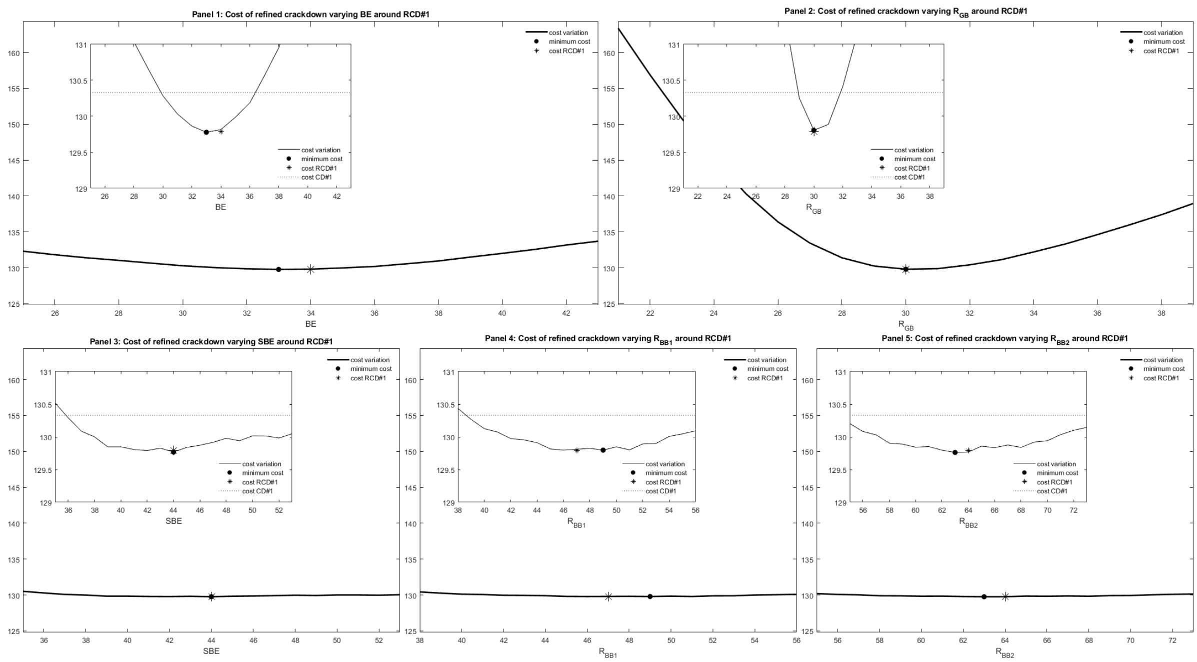

Figure 4.5: Cross sections of costs in the neighborhood of $RCD\#1$

Nevertheless, the results reported in Figure 4.5 suggest that the cost of ASB for the optimal policy is very close to the cost estimates in Table 4.3. The cost estimates reported in the figure are based on 50 independent convergence simulations for each policy examined (the standard errors of these estimates are roughly 0.03). In each panel, cost estimates are reported for 19 policies centered on policy #1: four of the five parameters of the policy are fixed at their values in policy #1, and the fifth varies up and down from its value in policy #1 by 9 units. The asterisk indicates policy #1, and the dot the policy with the lowest cost estimate. There is considerable curvature in the cost function when $\mathsf{R_{GB}}$ and BE vary, which means that the optimal values of these parameters are much easier to pin down, and that is exactly what we saw in Table 4.3. But over the range of values examined for parameters SBE, $\mathsf{R_{BB_1}}$ and $\mathsf{R_{BB_2}}$, the cost function is very flat and it is much more difficult to pin down the optimal values of theses parameters. This flatness reflects the fact that there are many ways to extinguish BB, and, it does matter much in terms of costs precisely how that is achieved because so little time is spent there. The inserts in each panel present the same information but the scale on the vertical axis is different. The more fine grained scale allows us to see how the cost estimates for the refined crackdown policies compare to our best estimate of the minimum cost achievable with the simpler crackdown policy—the dotted reference line in each insert represents that cost.

From Table 4.1, it is clear that the cost of ASB for the optimal crackdown policy is no larger than \$130.50 and results in Table 4.3 suggest that the cost of the optimal refined crackdown policy is no smaller than \$129.50. Apparently, the optimal refinement reduces the cost of ASB by less than 1%.

4.4 The big picture

In our baseline model, the best crackdown policy is substantially better that the best passive policy. When a passive policy is used, there is just one tool, one R, to manage the two q-attractors. Optimality requires that the one R be large enough so that the bad q-attractor is extinguished. That is costly because it means that the R used when the system is in the good q-attractor is larger than would be needed if the policy maker did not need to worry about the possibility of getting caught in the bad q-attractor. When a crackdown policy is used there two tools, or two Rs, and the larger of the two can be used to extinguish the bad q-attractor which allows the policy maker to economize on enforcement resources by using a much smaller R when the system is in the good q-attractor. In contrast, although there is an additional tool, the optimal refined crackdown policy is only marginally better than the optimal crackdown policy. When there are just two q-attractors, it appears that two tools allow the policy maker to realize most of cost reduction that active policies could achieve. This is remarkable, and, at the same time, intuitive.

5 A Dynamic Perspective on Deterrence

To this point we have focused on the geometry of ASB produced by our Markov chain. Propositions 1 and 2 suggests that the analysis may apply to a much larger set of dynamic stochastic systems. This motivates our attempt here to situate our results into the broader deterrence literature.[31] We focus on three literatures where we think our dynamic theoretical framework offers clarity and/or unifies elements of existing research: optimal deterrent capacity, the persistence of crime, and the efficacy of crackdowns.

5.1 Perceptions and "excess" deterrent capacity

Summarizing his policy recommendations for modernizing U.S. deterrent policy, Kleiman (2010a, p. 175-76) focuses on two issues regarding policing. First, "high" levels of policing produce compliance and, therefore, should not be seen as excessive, "Add police to areas that are under-policed, as measured by a high ratio of crimes to officers." Second, managing deterrence means managing the perceptions of potential violators, "Emphasize swiftness and certainty of punishment rather than severity...directly communicate deterrent threats." Our dynamic framework provides a clear motivation for both. As we have seen, the effective management of ASB, with either passive or active policies, requires substantial reserve capacity in order to manage the subjective probability of apprehension. This is most easily seen in Figures 3.9 and 4.4 which make clear that an optimal policy of either type implies setting R^t significantly above $E(v^t|R^t)$.

In Sections 3 and 4 we found that the optimal deterrence policy requires an investigation rate approaching 100%, which implies maximizing the apprehension rate. The U.S. is unique amongst industrialized jurisdictions in its reliance on punishment rather than apprehension to produce deterrence.[32] While it incarcerates far more of its population than other industrialized countries—roughly 0.7% of its population currently resides in jail—a potential violator

[31] Important work that anticipated our approach here includes the work of Tabarrok (1997)'s on crime waves, Sah (1991) on how history helps determine the perceptions of apprehension, and Schrag and Scotchmer (1997) on the self-reinforcing nature of crime.

[32] President Obama's Council of Economic Advisors started their 2016 report on the economics of criminal justice reform by noting that the U.S. incarcerates more individuals per-capita than any other industrialized country in the world, and that the incarcerated were arrested by one of the smallest police forces in the world. Relative to the world per-capita averages, the U.S. employs one and a half times more prison guards to oversee three times the prison population while employing only two-thirds as many police. As the ratio of police to citizens was flat over the last three decades, the Presidential task force concluded that it was unlikely that the rise in incarceration were due to "over policing."

still faces a lower expected sanction per crime than those in other industrialized countries, or even the U.S. of the 1950s.[33] One reason for this, Kleiman (2010a) argues, is that policy makers responded to the rise in crime of the 1970s—despite extensive evidence that violations respond only weakly to severity of sanction—by increasing the severity of punishment. This focus on sanction led to a decline, often deliberately so, in the certainty of apprehension. Kleiman (2010a, p. 12) estimates that during the crime wave of the 1970s, despite harsher sanctions, the probability of apprehension had fallen so far that the typical burglar could expect to spend only 4 days in prison for their crime. Scholars analyzing the empirics of crime mostly agree that perceptions drove the increase in crime. Mathur (1978, p. 464) finds, controlling for policing in order to isolate the effects of apprehension, that the "certainty of punishment for all crimes except larceny in 1970 varies inversely with crime rates for both 1960 and 1970." That the U.S. might be under policed but over sanctioned is consistent with a large literature that finds a negative relationship between increased policing resources and crime.[34] For example, Chalfin and McCrary (2013) find that increasing expenditures on police by $1.00 produces a $1.60 reduction in the cost of crime; Klick and Tabarrok (2010) argue that the estimated elasticities for deterrent spending support a doubling of policing resources in the U.S. from a cost benefit point of view.

A significant portion of the literature establishing the negative relationship between policing and crime takes the form of event studies, "In my view the most convincing evidence comes from the abrupt-change type of study focusing on cases in which the regime change is

[33] From a report in the New York Times, ""The United States today is the only country I know of that spends more on prisons than police," said Lawrence W. Sherman, an American criminologist on the faculties of the University of Maryland and Cambridge University in Britain. "In England and Wales, the spending on police is twice as high as on corrections. In Australia it's more than three times higher. In Japan it's seven times higher. Only in the United States is it lower, and only in our recent history....". Dr. Ludwig and Philip J. Cook, a Duke University economist, calculate that nationwide, money diverted from prison to policing would buy at least four times as much reduction in crime. They suggest shrinking the prison population by a quarter and using the savings to hire another 100,000 police officers."

[34] The empirical literature on the issue is large, but see, for example, Levitt (1997), Lim and Galster (2009), McCrary (2002), Evans and Owens (2007), Chalfin and McCrary (2013), Mello (2019), and Weisburst (2019). The estimated elasticities for crime reduction from increased policing range widely and differ by methodology (Nagin, 2013). Klick and Tabarrok (2010, p. 5) find that a 50% increase in police presence reduced crime by 15% (producing an estimated elasticity of 0.3). They argue this finding is broadly consistent with the range of elasticities (0.2 and 0.9) from the extant literature. See also Lin (2009).

clearly attributable to an event unrelated to the crime rate" (Nagin, 2013, p. 88). A classic in the literature is Andenæs (1974), who documents that "in September 1944, German soldiers occupying Denmark arrested the entire Danish police force....crime rates rose immediately but not uniformly. The frequency of street crimes such as robbery, whose control depends heavily on visible police presence, rose sharply. By contrast, crimes such as fraud were less affected" (Nagin, 2013, p. 88). Other event studies take the form of overwhelming police response to terrorist events or other disasters; all involve a narrowly focused surge of "excess" deterrent capacity into a region/location. The deployment of overwhelming deterrent resources is an important element of military doctrine regarding stabilization efforts—"a good way to think about the force requirement is roughly similar to the way that one would think about policing a civil society" (Dobbins and Quinlivan, 2003, p. 28). It is accepted that an adequate stabilization force has a minimum of 20 troops per thousand residents. Civil policing requires around 4 per thousand, the U.S. employs 2.3 (Dobbins and Quinlivan, 2003).

Kleiman (1993) identifies "tipping points" in which people simultaneously decide to violate knowing the number of possible sanctions that might be imposed (e.g., available prison cells). The model produces a well known result from the empirical literature: over a range of violations the costs of crime are unresponsive to increasing the severity of sanctions, which only serves to increase the sanction applied to those who happen to get caught. As one approaches the tipping point, however, increasing the number of sanctions even a little can tip the system into a low crime equilibria, "the violation rate goes from unity (a violation every time) to zero, and the total amount of fines collected also falls to zero...This illustrates economist Thomas C. Schelling's principle that the perfect threat never needs to be carried out. Having more punishment available can lead to imposing less actual punishment" (Kleiman, 2010a, p. 50).[35] Dynamic tipping behavior is a ubiquitous feature of our model.

[35] For $F = 1$, the punitive ratio, or sanction per apprehended violator ${}^{a*F}/_{v}$, is just our apprehension rate ${}^{a}/_{v}$. So consider again Figure 3.9, which plots the apprehension rate and clearly confirms Kleinman's findings for the dynamic case. Over most of the unruly regime the punitiveness ratio increases with no associated fall in expected violations or the cost of crime. As we approach the transition regime we see a dramatic,

The geometry of dynamic tipping points is driven by positive feedback, and, we argue, is a natural byproduct of modeling ASB as a dynamic stochastic process. This is seen throughout our analysis but is perhaps most clear in the ”cliffs” of Figures 3.5 and 3.7.

Increased policing, in Kleinman's view, and in our dynamic analysis, reduces crime through its effects on the subjective probability of apprehension. While the literature testing this link directly is modest relative to that testing the link between police and crime, it consistently supports a long standing regularity in the empirical criminology literature that certainty of sanction is more important to deterrence than severity of sanction. Gómez et al. (2021) find that the public placement of visible cameras, even when they provided no evidence relevant to prosecution, lowered ASB through their effect on perceptions. This is consistent with a literature, starting with Doleac and Sanders (2015), that finds that improved lighting conditions reduces crime for similar reasons. Mathur (1978, p. 465) found that "certainty and severity of punishment are effective deterrents to most crimes. But the more severe the punishment is for a particular crime, the lesser is the probability that a criminal, on the average, will be punished. Our findings also suggest that the criminal justice system would be well advised to pay more attention to the certainty of punishment as certainty is more important than severity." Using a dynamic specification that allows them to control for the feedback from crime to measures of deterrence, Bun et al. (2020, p. 2305) find that "criminal activity is highly responsive to the prospect of arrest and conviction, but much less responsive to the prospect or severity of imprisonment, if at all." Perhaps the most well known scholar on the empirics of perception and deterrence is Lochner. Modelling beliefs as formed through personal and local arrest histories, he finds that "there is robust evidence in favor of deterrence theory based on an individual's perceived probability of arrest (and not actual local arrest rates)" (Lochner, 2007, p. 459).

We believe that a fully dynamic analysis of crime, one which adequately captures the effects of positive feedback, serves as a productive complement to much of the empirical

and mostly simultaneous, fall in punitiveness, violations, and the cost of crime. Another way to frame these findings is that the U.S. has "flirted with the cliff" and found itself in a persistent high crime equilibrium.

and case study work done by economists and criminologists on policing levels. In fact, the consensus in the deterrence literature points directly to one of our central findings: an optimal deterrent policy requires maximizing the apprehension rate by driving the rate of investigations towards 100%. This requires a healthy reserve capacity of deterrent resources.

5.2 The persistence of crime and crime waves

Lochner (2007, p. 444) notes that "when the probability of arrest is learned from others or through one's own experiences, the effects of policy tend to be lagged and long-lasting, even if policy changes are temporary." In proposition 1 we argue that when subjective probabilities are determined by the history of the system, the system is a dynamic stochastic process. Any such process must produce some level of autocorrelation in ASB. That most empirical measures of ASB display some degree of autocorrelation is well known and widely accepted. Jacob et al. (2007, p.3) note that "The persistence of of criminal activity is well documented. Higher crime today in any particular area is associated with higher crime tomorrow." Jacob et al. (2007, p. 491) note that an analysis of this persistence—inconsistent with any static equilibrium theory but a necessary implication of crime as a dynamic stochastic process—has been hampered by the static nature of the workhorse model of deterrence, "economic models of criminal behavior have generally been constructed and tested using a static framework. While these models have been very useful in understanding some features of criminal behavior, they are not well suited to explaining how criminal behavior changes over time." The centrality of persistence as an empirical regularity, with no adequate dynamic framework for its analysis, has produced a large and active literature trying to locate its source. Potential explanations of persistence include business cycle effects,[36] habit formation, social osmosis, and neighborhood effects. No single explanation has emerged as a consensus cause of persistence. Our analysis suggests that, in addition to any other possible sources driving persistence, there is some natural underlying rate of persistence in ASB. That autocorrelation

[36] For example, Raphael and Winter-Ebmer (2001) and Greenberg (2017) both find a significant effect of unemployment on crime and its persistence.

in violations emerges naturally in any system where subjective probabilities are determined by the history of the system is most clearly seen in Figure 3.4.

One manifestation of persistence in crime is the existence of "crime waves," the spontaneous emergence of periods of high and low crime. In his analysis of the 1970s, Kleiman (2010a, p. 13) notes a source of positive feedback that is complementary to our own, "the falling punishment-per-crime ratio was partly the result of that crime wave, since for any fixed amount of prison capacity more crimes meant fewer cells per crime...That is the vicious circle of 'enforcement swamping,' in which rising levels of crime so overtax the capacity to punish as to make the uptrend self-reinforcing." Philipson and Posner (1996, p. 405) note that "The language of epidemiology has long been a feature of discussions of crime. Talk of crime 'waves,' of crime 'spreading' from one area or population to another, and of 'epidemics' of crime is not new." They develop a dynamic model in which private individuals invest in crime prevention which predicts cycles of crime analogous to the cycles produced in predator prey relationships. Bond and Hagerty (2010) model a situation in which enforcement policies may induce people to switch from low level violations to more serious violations in a coordinated manner. Perhaps most consistent with our analysis, Tabarrok (1997) models a situation in which the probability of apprehension is declining in the number of violators. He finds that ASB emerges in self-sustaining waves, "The model applies not only to crime strictly but also to phenomena like riots, strikes, and revolutions. In each of these cases, the probability of being punished is a decreasing function of the total amount of activity." However, none of these existing models actually produce a time series of violations in which waves of crime spontaneously emerge, persist, and, just as spontaneously, end. Our model naturally produces such waves of ASB. This is most clearly illustrated in the time series of Figures 3.1 and 3.2.

There is significant evidence that persistence is driven, at least in part, by perceptions. Curry et al. (2016) use Canadian data to estimate the effect of clearance rates, controlling for the size of the police force, on future levels of crime. They find that "All else being equal, an increase in the clearance rate is correlated with a reduction in crime..." (Curry et al., 2016, p. 508). Using high frequency monthly data from New York, Corman and Mocan (2000) finds that a documented increase in crime during the 1980s, thought by some to be related to a rise in drug use, is instead better understood as the result of declining arrest rates. Controlling for the size of the police force and outside economic opportunities, Corman and Mocan (2000) finds that lagged apprehension rates were the most important link between crime in successive periods.

There is abundant evidence on autocorrelation in crime and for the existence of self-sustaining crime waves. Despite decades of empirical work there is no agreed upon explanation for either. We have seen multiple instances (see Figures 3.1 and 3.2) where random fluctuations in the system may kick it out of a low violation equilibrium and into a naturally persistent high violation equilibrium. Our model allows us to potentially explain and estimate the level of autocorrelation in crime as well as the length and persistence of crime waves. Demonstrating that our model is qualitatively consistent with the empirical data is a first step; the next step is to empirically calibrate our simulations and test for quantitative consistency.

5.3 Crackdowns and other active deterrent policies

An important insight provided by our model is that a system with two q-attractors is best managed using two levels of deterrent resources. Active policies outperform passive policies. Policy makers can benefit from a strategy of dynamically concentrating deterrence, cracking down on ASB when violations spike.[37] Crackdowns have been implemented and studied in neighborhoods, specific city blocks, even intersections.[38] The literature finds that: (i) the short-term rapid deployment of overwhelming deterrent resources to problematic ju-

[37] Sherman (1990, p. 2) defines crackdowns as "a *geographically focused* increase in the dosage of police presence, which can approximate a temporary state of full enforcement of every law on the books...crackdowns attempt to communicate a far more powerful threat of apprehension and punishment than does 'normal' policing."

[38] See, for example, Banerjee et al. (2019), Hansen (2015), Weisburd (2021), Perc et al. (2013), Fu and Wolpin (2018), Ratcliffe et al. (2015), and Poutvaara and Priks (2006).

risdictions may result in durable reductions in crime; and, (ii) a primary channel of effect rendering such interventions durable is the slow rate of change in agent perceptions of sanction. In our model the effectiveness of crackdowns results from their durable effect on the perceptions of potential violators. This durability emerges through the z-history, producing a natural persistence in people's subjective probability of apprehension.[39]

This result is central to the crackdown literature. Sherman (1990, p. 10) argues that the effect of crackdowns persists because the "perceived risk of apprehension could influence decisions not to commit offenses after the risk (or the communicated threat) is actually reduced, at least until such time as other evidence shows the quasi-rational actors that the risk has returned to its prior level." An empirical analysis of crackdowns finds that "risk perceptions may decay only slowly, even when the actual police effort has been returned to normal" (Sherman, 1990, p. 3). This renders crackdowns durable, so that "By constantly changing crackdown targets, police may reduce crime more through residual deterrence than through initial deterrence."[40] Koper (1995) builds on Sherman's work explicitly, finding that longer crackdowns increase residual deterrence. The argument is that increased police visibility, as measured by time spent on location, alters people's estimates of the probability of apprehension (at a diminishing rate). Koper (1995, p. 668) finds that longer hot spot deployments "lengthens survival times, probably through a combination of driving away some troublesome persons and making others more cautious for some time afterward." He summarizes his findings by noting that "The policy implication of this study is that police can maximize crime and disorder reduction at hot spots by making proactive, medium-length

[39] Banerjee et al. (2019) make a similar argument, although they have a slightly different focus than we do here. They model how learning about the pattern of crackdown deployments themselves might help endogenously determine violation strategies. However, their empirical findings overlap with ours in potentially interesting ways. Specifically, they find that crackdowns produce residual deterrence by altering people's subjective apprehension risk and that "when we alter our assumptions to require citizens' beliefs about police activity to be correct in expectation, we find that the optimal police strategy becomes one of a short, intense crackdown mostly done in one location" (p. 44).

[40] Analyzing a set of 15 high visibility crackdowns, Sherman (1990, p. 35-6) finds mixed results, claiming durable reductions in crime were achieved in 13 of the cases. The evidence for crackdowns largely rests on the qualitative analysis of case studies (primarily a meta-analysis of deliberate policing interventions). For a discussion see Nagin (2013), especially section 4.2), and MacDonald et al. (2016).

stops at these locations on a random, intermittent basis in a manner similar to Sherman's (1990) crackdown-backoff rotation strategy" (Koper, 1995, p. 668). Analyzing the hot spot deployment of police to the London Underground, Ariel et al. (2020, p. 125) find that "The findings indicate that platform patrols cause large reductions in both crime- and citizen-generated calls for service in these areas that had never been proactively patrolled by uniformed police. These effects were so long lasting and so spatially diffusive that the actual patrol time had geometric effects on crime."

We find that active policies can significantly reduce the costs of managing ASB. This thinking is central to Kleinman's review of deterrence policy, arguing that crackdowns are not just a theoretical curiosity but a core principle behind a number of successful policy initiatives and pilot projects.[41] In fact, the dynamic concentration of deterrent resources is a key element of his policy prescription to modernize U.S. deterrence efforts.[42] Comparing static with passive enforcement strategies, Kleiman (2010b, p. 64) finds that "the same initial conditions that led to the high violation equilibrium under random sanctioning [passive enforcement] lead to the low-violation equilibrium under dynamic concentration...Dynamic concentration makes a little punishment go a long way." We have shown, and the literature suggests, that crackdowns are an effective general framework for an active deterrence policy. It is important, however, to realize that their implementation depends on a range of parameters. Reviewing the details of actual crackdowns, Kleinman notes some important details in translating the theory of crackdowns into real world policy: the costs of concentrating resources (low when police move dynamically from one corner to another, high when concentration requires dynamic hiring efforts), the persistence of a crackdown's effect (short for roadblocks targeting DUI offenders, long for crackdowns on drug sales), the technology of

[41] See Kleiman (2010a) and Kleiman (2010b), but also Bratton (2011)

[42] Kleiman and Kilmer (2009) model how enforcement swamping may be overcome, incarceration sharply reduced, and durable reductions in violations achieved by dynamically concentrating deterrent resources. See also Lim and Galster (2009), and Bratton (2011). Kleiman (2010a) reviews a number of crackdowns, noting their effectiveness at reducing crime by producing a persistent increase in the subjective probability of apprehension (see chapters 4, 6, and 7). Lui (1986, p. 216) argues that understanding corruption as a dynamic process helps explain why we see periodic crackdowns on corrupt behavior.

monitoring (effective when using GPS tracking, less effective when targeting DUI), and the ability of violators to displace their activity to nearby jurisdictions (low for burglary, relatively high for drug dealing).[43] After an extensive review Kleiman (2010a, p. 64) concludes that "dynamic concentration turns out to be the deterrence version of the Miracle of the Loaves and Fishes."[44]

5.4 Overview

Much is known about ASB and its management. Much more remains unexplained, motivating a large and active literature. While the workhorse model of deterrence is useful for understanding some aspects of the overall picture, it does not provide a unifying theoretical framework. Reviews of the literature repeatedly suggest that a major limitation of the existing theoretical framework is its static nature. We agree, and argue that our model, along with other implementations of our general dynamic framework, has the potential to unify significant elements of the literature, many of which are seen as inherently dynamic by researchers working in the field.

6 Conclusions

Bentham was right. Deterrence is, inherently, a dynamic process. This is clearly indicated by the stubbornly persistent autocorrelation in violations data, and the failure to explain that autocorrelation with determinates that are themselves autocorrelated. Our dynamic stochastic deterrence process, which is based squarely on Bentham's view of how subjective probabilities of apprehension are formed, produces the sort of autocorrelation seen in the violations data, and is broadly consistent with many of the themes in the empirical literature on crime.

[43] See Lazzati and Menichini (2016) for how one might deal with issues related to crackdowns and displacement.

[44] See Jabri (2021) for an interesting application of crackdowns which use predictive algorithms to allocate deterrent resources to potentially high crime areas preemptively.

We do not want to claim too much. Ours is just one of many possible dynamic deterrence processes that model subjective probabilities as being determined by the recent history of violations and apprehensions, and we have not thoroughly explored the parameter space of our model. On the other hand, we do not want to claim too little. The interesting features of our dynamic process are all associated with the three regime phenomenon we saw in Figure 3.5, where we reported the expected number of violations in the stationary distribution for the set of passive enforcement policies: there is a compliant regime where the number of violations is relatively small, an unruly regime where the number is relatively large, and a transitional regime that shows up as a cliff like structure connecting the compliant and unruly regimes. As we saw in Figures 3.6 and 3.7, over a broad range, the three regime phenomenon is robust to different parameterizations of the process, and it is robust to variations in how key features of the process are modeled.

What's next? The real test of the value of dynamic processes like ours will come when they are used in empirical work to interpret real data. We used simulation techniques to explore the Markov chain that our assumptions produced. But a complete analysis of our model is clearly beyond the scope of any single paper. Our purpose was to initiate the theoretical and computational analysis of the dynamics of deterrence. We have only scratched the surface.

A Appendix

We have implemented all of our numerical simulations in two wholly separate codebases, Matlab and Python. All reported results have been independently replicated in each codebase. One member of our team wrote and maintained the Matlab codebase, another wrote and maintained the Python codebase. Both codebases are available at `https://github.com/cliffbekar/dynamic_deterrence`.

A.1 An equilibrium model of deterrence

There are N potential violators in a community. Each has the opportunity to commit an antisocial act proscribed by law. If person i violates the law their utility is g_i, a random drawing from the density function $\phi(g)$, and if they comply their utility is 0. Deterrence policy involves an R and an F, and the probability of apprehension is $P(R, v)$. When we want to emphasize the dependence of p on v, we write this probability as $P(v)$. We are interested in the Nash equilibria of the static game in which the N potential violators simultaneously choose violate or comply. We discover a number of results that are helpful in understanding our dynamic model. In this sense, the equilibrium and dynamic models are complementary.

We start by showing that for any realization of $\boldsymbol{G} = (g_1, ..., g_N)$, there is at least one equilibrium. For this purpose, we sort the elements of $\boldsymbol{G}$ from largest to smallest, indexing them so that $g_1 \geq g_2 \geq ... \geq g_N$. Note the following: since $P(v)$ is weakly decreasing in v, it is the case that $P(1) \cdot F \geq P(2) \cdot F \geq ... \geq P(N) \cdot F$. These inequalities allow us to express the necessary and sufficient conditions for three sorts of equilibrium.

1. A Nash equilibrium with $v = 0$ exists $\Leftrightarrow g_1 < P(1) \cdot F$. If person 1 prefers comply given that no one else chooses violate, persons 2 through N also prefer comply.

2. A Nash equilibrium with $v = N$ exists $\Leftrightarrow g_N \geq P(N) \cdot F$. If person N prefers violate given that everyone else chooses violate, persons 1 through $N - 1$ also prefer violate.

3. A Nash equilibrium with $0 < v < N$ exists $\Leftrightarrow g_v \geq P(v) \cdot F$ and $g_{v+1} < P(v+1) \cdot F$. If person v prefers violate given that $v - 1$ others choose violate, persons 1 through $v - 1$ also prefer violate; if person $v + 1$ prefers comply given that v others choose violate, persons $v + 2$ through N also prefer comply.

It is intuitively clear that at least of one of the three sets of conditions holds, so there is at least one equilibrium.[45] But there may be more than one. Consider, for example, the

[45] Formally, if $g_i < P(i) \cdot F$ for all $i \in (1, ..., N\}$, then $g_1 < P(1) \cdot F$ and there is an equilibrium in

possibility that there is one equilibrium with no violations and a second with N. The first requires that $g_1 < P(1) \cdot F$ and the second that $g_N \geq P(N) \cdot F$. These conditions can be rewritten as $g_1/P(1) < F \leq g_N/P(N)$. Since $g_N > g_1$ and $P(1) > P(N)$, it is possible that both conditions are satisfied.

Proposition 4 *Given a set of realizations* $(g_1, g_2, ..., g_N)$*, there is at least one Nash equilibrium, and there may be more than one.*

What is the probability an equilibrium with v violations exists? To answer this question, we no longer sort the elements of $\boldsymbol{G}$. Each element of $\boldsymbol{G}$ is simply a random draw from $\phi(g)$ and it is no longer the case that $i < j$ dictates that $g_i \geq g_j$. For there to be an equilibrium with $0 < v < N$ violations, two conditions must be met: (i) exactly v of the N realizations must satisfy $g_i \geq P(v) \cdot F$ and (ii) the remaining $(N - v)$ realizations must satisfy $g_i < P(v + 1) \cdot F$. The probability of an equilibrium with v violations is the product of the probability that (i) is satisfied and, conditional on (i) being satisfied, the probability that (ii) is satisfied.

Proposition 5 *The probability that there will be an equilibrium with exactly* v *violations is*

$$\frac{N!}{v!(N-v)!}(1 - \Phi(P(v) \cdot F))^v \cdot (\Phi(P(v+1) \cdot F))^{N-v}$$

In Figure A.1 we use Proposition 5 to illustrate certain features of the equilibrium model for our *baseline parameterization:* $N = 100, F = 1, \mu = .6, \sigma = .2, \gamma = .80$. In the top panel of the figure, for all $0 \leq v \leq 100$, we have plotted the probability of an equilibrium with v violations for the case in which $R = 39$. Notice the two prominent modes in this figure, one at $v = 15$ and the other at $v = 94$. More generally, bimodality is readily apparent when $10 \leq R \leq 50$. Outside this range, only one mode is apparent: when $R < 10$, there is a one mode at $v = 100$; when $R > 50$ there is a one at $v = 16$.

Given R, we can calculate the expected number of equilibria by summing the probabilities of equilibrium over the entire range of possible values of v. The top line in second panel of Figure A.1 plots the expected number of equilibria for all $0 \leq R \leq 100$. The other lines plot the expected number of equilibria in three bins. *Good equilibria* are those in bin $0 \leq v \leq 30$; *bad equilibria* are those in bin $70 \leq v \leq 100$; *other equilibria* are those in bin $30 < v < 70$.

which everyone chooses comply. So suppose it is not the case that $g_i < P(i) \cdot F$ for all $i \in (1, ..., N\}$. Then there exists a smallest $i \in \{1, ..., N\}$ such that $g_i \geq P(i) \cdot F$, which we denote by $\widetilde{i}$. Either there is no $j \in (\widetilde{i} + 1, ..., N\}$ such that $g_j < P(j) \cdot F$, or there is a smallest $j \in (\widetilde{i} + 1, ..., N\}$ such that $g_j < P(j) \cdot F$, which we will denote by $\widetilde{j}$. In the first case, there is an equilibrium in which everyone chooses violate. In the second, there is an equilibrium in which the first $\widetilde{j} - 1$ people choose violate, and the others choose comply. Hence, there is at least one equilibrium.

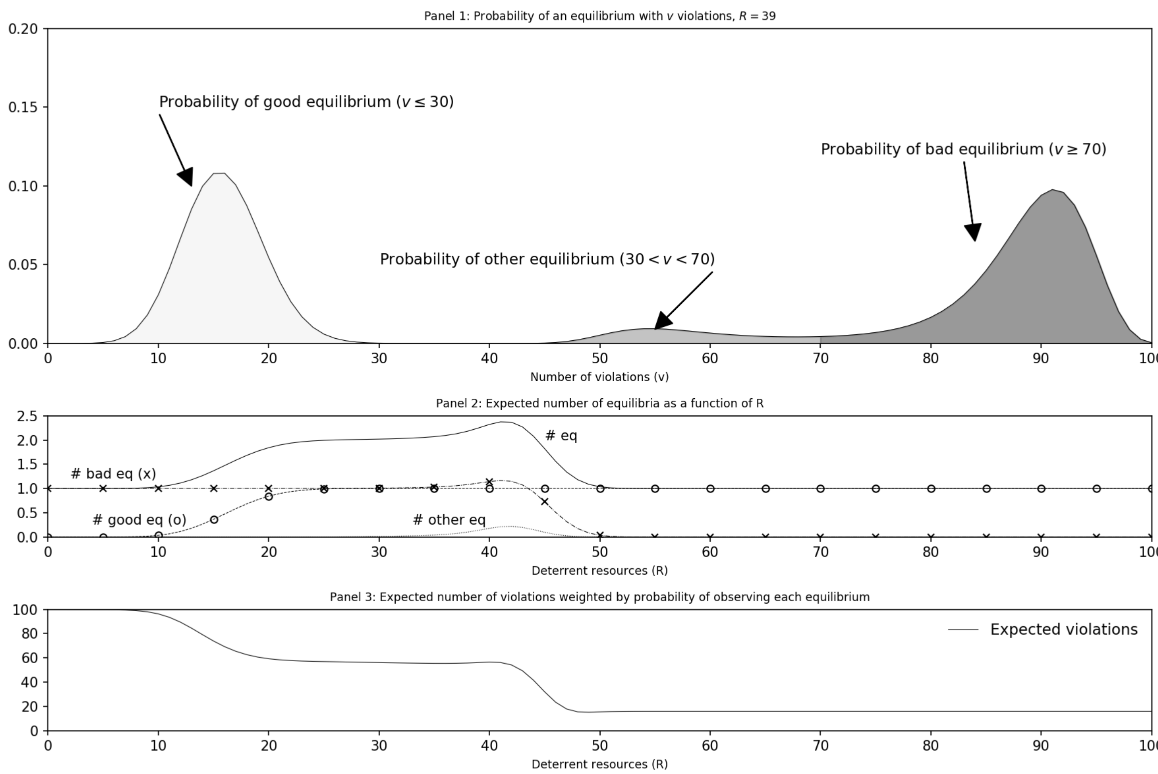

Figure A.1: Number of equilibria and their probability

There is always at least 1 equilibrium, with at least one bad equilibrium for $R \leq 44$ and one good equilibrium for $R \geq 24$. For $24 < R < 44$ there are least two equilibria (we could expect as many as 2.4). Notice that the probability of an equilibrium in the middle bin, $30 < v < 70$, is very close to 0 for $R \leq 36$ and $R \geq 46$ and quite small for $36 < R < 46$. Our dynamic model produces a similar result: in both models it is rare to see a period in which the number of violations falls outside the good and bad bins defined above.

For any R we can use Proposition 5 to construct a probability distribution over $0 \leq v \leq 100$ for the experiment in which one equilibrium is selected from the set of possible equilibria. In the bottom panel of Figure A.1, we have used that probability distribution to calculate, for every $0 \leq R \leq 100$, the expected number of violations for a randomly selected equilibrium. This is one useful point of reference for results generated by our dynamic model. The pattern seen in the bottom panel of the figure is quite distinctive: three ranges of R where the relationship is very nearly flat, separated by two where violations fall rapidly with R. The pattern produced by our dynamic model is quite different (see Figure 3.5).

We are interested in the static equilibrium model because it provides a useful point of reference for our dynamic model. The informational assumptions that the analysis entails are so extreme that the model has limited appeal as vehicle for understand real ASB. Rational choice requires that people know the probability of apprehension, $P(v)$. That probability is, of course, an endogenous variable. The only way people can know $P(v)$ is to find the Nash equilibrium for themselves, and that requires that they all know $\boldsymbol{G} = (g_1, ..., g_N)$, R, F and the function $P(R, v)$. That is just not reasonable.

A.2 The simulator

The number of states in $\boldsymbol{Z}$ for $z = 1$ and $N \leq 100$ is manageable, meaning it is feasible to produce the associated transition matrix $\boldsymbol{T}$ and use it to directly compute $\boldsymbol{D}$. But for $z > 1$ and larger N we must use numerical simulation methods. We now describe the simulation and our benchmarking efforts.

In each period of the simulation:[46]

1. Every potential violator is assigned a g drawn randomly from $\phi(g)$.
2. Using the current z-history, a common subjective probability of apprehension q is calculated.
3. People choose to violate if and only if their $g_i^t \geq q^t \cdot F$.
4. Violators are apprehended with probability $P(R^t, v^t)$, where v^t is the number of violators in the current period.
5. The number of violations, apprehensions, and the sum of the gains from violating, the sum of the realized draws on $\phi(g)$ for those who violated, are recorded.
6. The frequency distribution of violations and apprehensions is calculated for the current period and recorded.
7. The z-history is updated and the simulation moves to the next period.

We calculate the frequency distribution of violations and apprehensions and a running total of the gains from violating. The frequency distribution of violations is $\boldsymbol{FV}(nv_0^t, nv_1^t, ..., nv_N^t)$, where nv_j^t is the number of instances in the first t periods of the simulation in which there were exactly j violations. The frequency distribution of apprehensions is $\boldsymbol{FA}(na_0^t, na_1^t, ..., na_N^t)$,

[46] Initiating a simulation requires values for: the quantity of enforcement resources (R); people's look back horizon (z); the number of potential violators (N); the mean (μ) and standard deviation (σ) of the private gains from committing a violation; the probability a violator is apprehended if investigated (γ); the utility penalty paid by apprehended violators (F); and, the population's Bayesian priors regarding apprehension (α, β). Unless noted, we set these parameters according to our baseline model (see Section 2.4).

where na_j^t is the number of instances in the first t periods of the simulation in which there were exactly j apprehensions. The running total of the gains violators is,

$$G^t = \sum_{j=1}^{t} (\sum_{k|g_k^j \geq q^j \cdot F} g_k^j)$$

The inner sum is for all individuals k who choose violate in period j, that is, for all k such that g_k^j is greater than or equal to $q^j \cdot F$.

The simulation proceeds in blocks of $50,000$ ticks. At the end of each block n we use $\boldsymbol{FV}$ to calculate the relative frequency distribution of violations $\boldsymbol{RFV}^n = (rv_0^n, ..., rv_N^n)$, where rv_j^n is the proportion of periods in which there were exactly j violations in the first $50000 \cdot n$ periods of the simulation. As n approaches infinity $\boldsymbol{RFV}^n$ approaches $\widehat{\boldsymbol{D}}$. The question is: How large must n be to ensure that $\boldsymbol{RFV}^n$ is a *good* approximation of $\widehat{\boldsymbol{D}}$? To answer this we need a convergence criterion.

At the end of each simulation block, for all $n \geq 2$, we calculate the following test statistic:

$$TEST^n = \sum_{j=0}^{j=N} |rv_j^n - rv_j^{n-1}|$$

$TEST^n$ is a measure, decreasing in n, of the *distance* between $\boldsymbol{RFV}^n$ and $\boldsymbol{RFV}^{n-1}$. $TEST^n$ does not decline monotonically, which complicates any test of convergence. After extensive experimentation we settled on the following criterion: we require that $TEST^n \leq 0.01$ for 5 consecutive blocks of n. Denoting the minimum n for which the convergence criterion is satisfied by n^* yields our approximation of the stationary distribution, $\boldsymbol{RFV}^{n^*} \sim \widehat{\boldsymbol{D}}$.[47]

A.3 Benchmarking the simulator

We employ Monte Carlo simulations and the relationship between $\widehat{\boldsymbol{D}}$ and $\boldsymbol{RFV}^{n^*}$ to: (i) cross validate the output from our Python and Matlab codebases; and, (ii) benchmark the accuracy of the simulator against a known transition matrix.[48] The computational intensity of producing both $\widehat{\boldsymbol{D}}$ and $\boldsymbol{RFV}^{n^*}$ increases geometrically with the size of the state space. It is only practical, therefore, to implement both approaches for a relatively small number of agents ($N = 50$) and a single look back period ($z = 1$). For these parameters, and for each element of $R = \{5, 21, 45\}$, we generate 1000 instances of $\boldsymbol{RFV}^{n^*}$. We then construct

[47] An important confounding issue for finding n^* is that the *shape* of $\widehat{\boldsymbol{D}}$ changes dramatically as R changes. For low values of R, $\widehat{\boldsymbol{D}}$ is unimodal and right skewed, for high values unimodal and left skewed, for intermediate values it becomes bimodal. This renders n^* non-monotonic in R (see the last row of Table 1).

[48] We are, of course, implicitly testing the validity of our convergence criterion.

the following distance metric:

$$ATEST = \widehat{\boldsymbol{D}} - \boldsymbol{RFV}^{n^*} = \sum_{j=0}^{j=N} |\widehat{d}_j - rv_j^{n^*}|$$

In Table 4.1 we report the mean, standard deviation, and maximal value of $ATEST$ along with the mean n^*. It is apparent that the two platforms deliver results that are essentially the same, and that $\boldsymbol{RFV}^{n^*}$ very closely approximates $\widehat{\boldsymbol{D}}$.

Table A.1: Benchmarking the simulator

	Matlab Codebase			Python Codebase		
	Value of R					
	5	21	45	5	21	45
Mean of ATEST	0.0017	0.0098	0.0078	0.0018	0.0115	0.0087
Std Dev of ATEST	0.0009	0.0017	0.0011	0.0009	0.0019	0.0011
Max value ATEST	0.0061	0.0193	0.0116	0.0057	0.0198	0.0128
Mean n^*	6.0010	7.8430	7.0870	6.0010	7.8120	6.0010

A.4 Alternative methods for generating antisocial opportunities

Here we show that the gross features of the stationary distributions of violations are robust to an alternative specification of the antisocial opportunities of each person, the case where the opportunities are unchanging over time. To establish this we run our convergence simulation for our baseline parameters and $g_i^t = g_i \ \forall \ t$. While the resulting Markov chain is more cumbersome to work with, it preserves the characteristics presented in Section 2.2 and Figure 3.5.

In panels 1 and 2 of Figure A.2 the opportunity set for agents is not randomly drawn from $\phi(g)$ each period, but is instead drawn once for all i agents and held constant for all t and all values of $R \in (0, ..., N)$. The original cliff from Section 3.5 is included as the dashed line in panel 1. Having eliminated any inter-temporal randomness produced by the draw of antisocial opportunities, we again reproduce the geometry of Figure 3.5.

A.5 Search algorithm for active policies described

We use a two stage procedure to identify the optimal crackdown and refined crackdown policies. In the first stage, we use a directed search routine with a loose convergence criterion

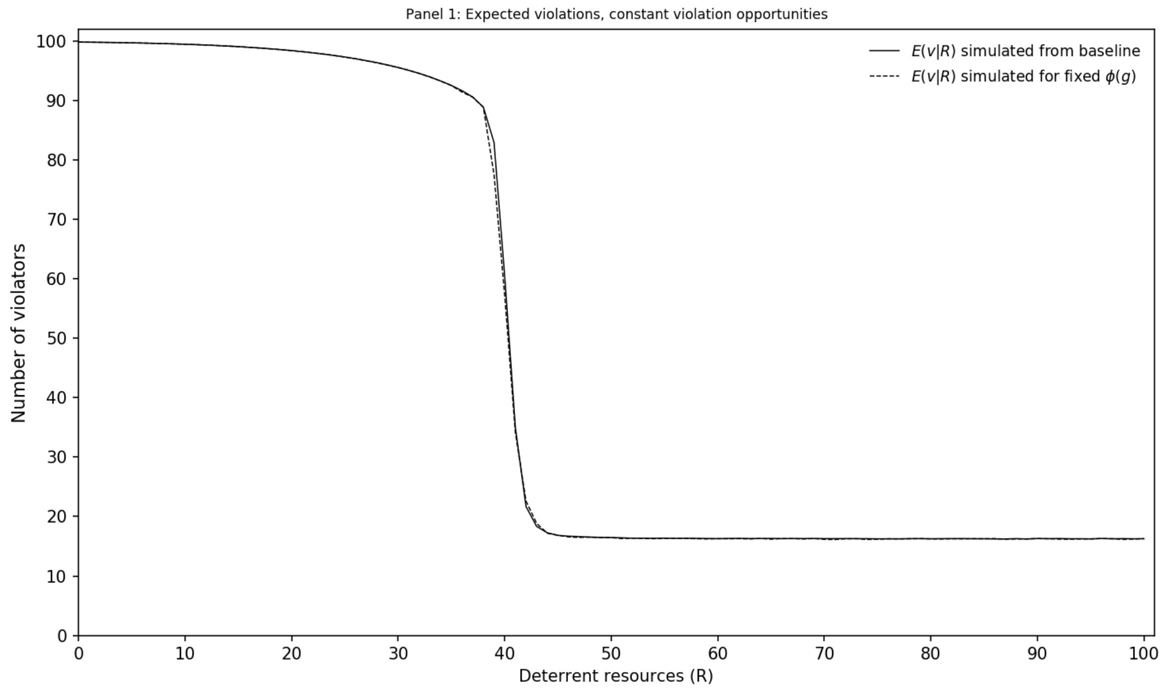

Figure A.2: The "Cliff" with constant opportunities $g_i^t = g_i \ \forall \ t$

to identify the neighborhood in which the optimal policy lies, and in the second stage we use Monte Carlo methods to generate tight estimates of the cost of policies in the neighborhood.

The algorithm at the core of the directed search routine is this. For a given crackdown policy at stage s, $CD^s = (\mathbb{BE}^s, \mathbb{R_{GB}}^s, \mathbb{R_{BB}}^s)$, a modified policy $CD^{s+1} = (\mathbb{BE}^{s+1}, \mathbb{R_{GB}}^{s+1}, \mathbb{R_{BB}}^{s+1})$, is generated using a three step procedure: in the first step, $\mathbb{BE}^{s+1}$ is chosen from the set $\{\mathbb{BE}^s - 5, \mathbb{BE}^s - 4, ..., \mathbb{BE}^s + 5\}$ to minimize the cost of ASB for policy $(\mathbb{BE}^{s+1}, \mathbb{R_{GB}}^s, \mathbb{R_{BB}}^s)$; in the second step, $\mathbb{R_{GB}}^{s+1}$ is chosen from the set $\{\mathbb{R_{GB}}^s - 5, \mathbb{R_{GB}}^s - 4, ..., \mathbb{R_{GB}}^s + 5\}$ to minimize the cost of ASB for policy $(\mathbb{BE}^{s+1}, \mathbb{R_{GB}}^{s+1}, \mathbb{R_{BB}}^s)$; in the third step, $\mathbb{R_{BB}}^{s+1}$ is chosen from the set $\{\mathbb{R_{BB}}^s - 5, \mathbb{R_{BB}}^s - 4, ..., \mathbb{R_{BB}}^s + 5\}$ to minimize the cost of ASB for policy $(\mathbb{BE}^{s+1}, \mathbb{R_{GB}}^{s+1}, \mathbb{R_{BB}}^{s+1})$. For each of the 33 policies considered, we ran a convergence simulation and used it to estimate cost. The search is terminated when the criteria $(E(C|CD^s) - E(C|CD^{s+1}))/E(C|CD^s) < .02$ is met. Each of the 1000 simulations was seeded with an initial policy randomly chosen from the set $\{(\mathbb{BE}, \mathbb{R_{GB}}, \mathbb{R_{BB}}) | 0 \leq \mathbb{BE} \leq 100, 0 \leq \mathbb{R_{GB}} \leq 100, \mathbb{R_{GB}} \leq \mathbb{R_{BB}} \leq 100\}$ and for each we used the directed search algorithm to a generate a terminal policy. From initial policy to terminal policy, the average number of policies evaluated was 165. To identify the neighborhood of the optimal crackdown policy, we ranked the 1000 terminal policies from lowest to highest cost. The first 78 policies in this ranking, and 98 of the first 100, were in the set $\{(\mathbb{BE}, \mathbb{R_{GB}}, \mathbb{R_{BB}}) | 30 \leq \mathbb{BE} \leq 39, 29 \leq \mathbb{R_{GB}}, 3 \leq \mathbb{R_{BB}} \leq 65\}$. This is neighborhood of the optimal policy.

The search for the optimal refined crackdown policies was analogous to that for the crackdown policy, with the differences being the need for two additional steps in the procedure to search over the two additional parameters, $\mathsf{BE2}$ and $\mathsf{R_{BB1}}$, of the refined crackdown policy. The algorithm for the refined crackdown policy search is, given a refined crackdown policy at stage s, $CD^s = (\mathsf{BE}^s, \mathsf{BE2}^s, \mathsf{R_{GB}}^s, \mathsf{R_{BB1}}^s, \mathsf{R_{BB2}}^s)$, a modified policy $CD^{s+1} = (\mathsf{BE}^{s+1}, \mathsf{BE2}^{s+1}, \mathsf{R_{GB}}^{s+1}, \mathsf{R_{BB1}}^{s+1}, \mathsf{R_{BB1}}^{s+1})$, is generated using a five step procedure analogous to that of the three step procedure of the crackdown policy search. For each of the 55 policies considered, we again ran a convergence simulation and used it to estimate the cost. The termination criteria for the refined crackdown policy search was the same as that used for the crackdown policy search. The 1000 simulations were randomly seeded with initial refined crackdown policies chosen from the set $\{(\mathsf{BE}, \mathsf{BE2}, \mathsf{R_{GB}}, \mathsf{R_{BB1}}, \mathsf{R_{BB2}}) | 0 \leq \mathsf{BE} \leq 100, \mathsf{BE} \leq \mathsf{BE2} \leq 100, 0 \leq \mathsf{R_{GB}} \leq 100, \mathsf{R_{GB}} \leq \mathsf{R_{BB1}} \leq \mathsf{R_{BB2}}, \mathsf{R_{BB1}} \leq \mathsf{R_{BB2}} \leq 100\}$. From initial policy to terminal policy, the average number of policies evaluated was 440 for each of the 1000 simulations. The neighborhood of the optimal refined crackdown policies was determined in the same way as for the crackdown policy.

The estimate of cost that comes out of one convergence simulation is, obviously, a random variable. In the neighborhood of the optimal policy, the cost function is so flat that in order to reliably identify the optimum policy, many independent cost estimates are needed. Accordingly, for every policy in the neighborhood of the optimum we ran 150 convergence simulations and calculated the cost of ASB in the steady state for each of them. Our cost estimate is the mean value of these 150 cost estimates. Typically the standard deviation of cost for the 150 simulations is close to 0.20, and hence the standard error of the estimate is close to $0.016 = .2/\sqrt{150}$.

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