PHYS 486 Final Exam Cheat Sheet

Cliff Sun

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Important stuff to know!

Spin

Bosons have a spin of ± 1 and Fermions have a spin of $\pm 1/2$. As well

$$S^2 |s, m\rangle = \hbar^2 s(s+1) |s, m\rangle$$
$$S_z |s, m\rangle = \hbar m |s, m\rangle$$
$$S_\pm |s, m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)} |s, m \pm 1\rangle$$

Note that $m \in [-s, s]$. As well,

$$[S_x, S_y] = i\hbar S_z$$
$$[S_y, S_z] = i\hbar S_x$$
$$[S_z, S_x] = i\hbar S_y$$

$\mathbf{s} = 1/2$

We will work in the eigenbasis of S_z. We define the spin up ($\lambda = 1$) as

$$\chi_+ \iff \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left| \frac{1}{2}, \frac{1}{2} \right\rangle$$

and

$$\chi_- \iff \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

The relations:

$$S^2 \chi_\pm = \frac{3}{4} \hbar^2 \chi_\pm = \frac{3}{4} \hbar^2 \mathbb{I} \tag{1}$$

$$S_z \chi_\pm = \pm \frac{\hbar}{2} \chi_\pm \tag{2}$$

$$S_x = \frac{\hbar}{2} \sigma_x \quad S_y = \frac{\hbar}{2} \sigma_y \quad S_z = \frac{\hbar}{2} \sigma_z \tag{3}$$

For two particles, the notation is

$$|s_1, s_2, m_1, m_2\rangle \tag{4}$$

$$S_x = \frac{\hbar}{2} (|0\rangle \langle 1| + |1\rangle \langle 0|) \tag{5}$$

$$S_y = \frac{\hbar}{2} (i |0\rangle \langle 1| - i |1\rangle \langle 0|) \tag{6}$$

$$S_z = \frac{\hbar}{2}\left(\left|0\right\rangle\left\langle 0\right| - \left|1\right\rangle\left\langle 1\right|\right) \tag{7}$$

$$a_- \left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle \tag{8}$$

$$a_+ \left|\alpha\right\rangle = \alpha^*\left|\alpha\right\rangle \tag{9}$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a_- + a_+\right) \tag{10}$$

$$\hat{p} = i\sqrt{\frac{\hbar\omega m}{2}}\left(a_+ - a_-\right) \tag{11}$$