

# PHYS 325: Lecture 18

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October 29, 2024

## Lecture Span

- Greens functions
- Delta functions
- Fourier Stuff

## Fourier Series

Applied for periodic functions.

$$F(t) = \frac{a_0}{2} + \sum_{p=1}^{\infty} a_p \cos(p\Omega t) + \sum_{p=1}^{\infty} b_p \sin(p\Omega t) \quad (1)$$

Note that

$$a_0 = \frac{2}{T} \int_T F(t) dt \quad (2)$$

$$a_p = \frac{2}{T} \int_T F(t) \cos(p\Omega t) dt \quad (3)$$

and

$$b_p = \frac{2}{T} \int_T F(t) \sin(p\Omega t) dt \quad (4)$$

Note:  $\cos(mx)$  and  $\cos(nx)$  are "orthogonal" for  $m \neq n$ , that is

$$\langle \cos(mx), \cos(nx) \rangle_T = 0 \quad (5)$$

Same argument for  $\sin([m, n]x)$ .

## Gibbs Phenomenon

Overshooting followed by undershooting. This is prevalent in Fourier series fitting.

## Summary

EOM:

$$m\ddot{x} + c\dot{x} + kx = F(t) \quad (6)$$

Then for  $\Omega = \frac{2\pi}{T}$ , then

$$F(t) = \frac{a_0}{2} + \sum a_p \cos(p\Omega t) + \sum b_p \sin(p\Omega t) \quad (7)$$

The steady state solution  $x(t)$  is the solution for  $t \gg 0$ .