PHYS 325: Lecture 12

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Lecture Span

- Two body problem (in Newtonian Gravity)

Two body problem

Note that there are a total of 6 differential equations.

Simplifications

1. Work in the center of mass frame. Introduce the total mass $M = m_1 + m_2$ and $\mu = \frac{m_1 m_2}{M}$ is reduced mass. And
$$\vec{r} = \vec{r}_2 - \vec{r}_1 \tag{1}$$
Center of mass =
$$\vec{C} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M} \tag{2}$$
Thus
$$m_1\ddot{\vec{r_1}} + m_2\ddot{\vec{r_2}} = -\frac{Gm_1m_2}{|r_2 - r_1|^2}\left(\frac{(\vec{r_1} - \vec{r_2} + \vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}}\right) \iff 0 \tag{3}$$
$$\iff (m_1 + m_2)\ddot{\vec{C}} = 0 \tag{4}$$
Thus
$$\ddot{\vec{C}} = 0 \tag{5}$$
So the EOM of $\vec{r}$ is
$$\ddot{r} = -\frac{G\mu}{|\vec{r}|^2}\frac{\vec{r}}{|\vec{r}|} \tag{6}$$
Thus the relative position of $\vec{r}$ is governed by the same EOM as a test mass μ in the gravitational field of M.

Energy

$$E = T + U = \frac{1}{2}m_1\dot{r_1}^2 + \frac{1}{2}m_2\dot{r_2}^2 - \frac{Gm_1m_2}{(r_2 - r_1)^2} \tag{7}$$
$$E = \frac{1}{2}\mu\dot{r}^2 - \frac{GM_\mu}{r} + \frac{1}{2}M\dot{C}^2 \tag{8}$$