PHYS 325: Lecture 15

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Lecture Span

- Damped harmonic oscillator

The EOM for damped harmonic oscillators is

$$m\ddot{y} + c\dot{y} + ky = 0 \tag{1}$$

The "normal" form:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0 \tag{2}$$

Then the eigen frequency, or the normal frequency is

$$\omega_n = \sqrt{\frac{k_{eff}}{m_{eff}}} \tag{3}$$

and

$$\zeta = \frac{c}{2m\omega_n} \tag{4}$$

Guess

$$y \sim e^{\lambda t} \tag{5}$$

insert in EOM

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \tag{6}$$

Then

$$\lambda_\pm = -\omega_n(\zeta \pm \sqrt{\zeta^2 - 1}) \tag{7}$$

Undamped

Let $\zeta = 0$, then

$$\lambda_\pm = \pm i\omega_n \tag{8}$$

Overdamped

Let $\zeta > 1$

$$\lambda_\pm = -\omega_n(\zeta \pm \sqrt{\zeta^2 - 1}) \tag{9}$$

Note that

$$\lambda_\pm < 0 \tag{10}$$

Critical damping

Let $\zeta = 1$, then

$$\lambda_{\pm} = -omega_n \tag{11}$$

Then

$$y(t) = A_1 e^{\omega_n t} + A_2 t e^{-\omega_n t} \tag{12}$$

Underdamping

Let $0 < \zeta < 1$, then

$$\lambda_p m = -\omega_n(\zeta \pm \sqrt{\zeta^2 - 1}) \tag{13}$$

Then

$$\lambda_p m = \omega_n \zeta \pm i\sqrt{1 - \zeta^2} \tag{14}$$

Versions we can work with:

$$y(t) = e^{-\zeta \omega t}(A\cos(\omega t) + B\sin(\omega t)) \tag{15}$$

What happens if $\zeta < 0$? Then double yo work dumbass.

Forced oscillator

Oscillator with external driving force. Note, energy is <u>not</u> conserved. Then

$$\frac{dE}{dt} + P_{\text{diss}} = \frac{dW}{dt}_{\text{ext}} \iff F_{\text{ext}} \cdot v \tag{16}$$

$$m_{\text{eff}} \frac{d^2\vec{x}}{dt^2} + c_{\text{eff}} \frac{d\vec{x}}{dt} + k_{\text{eff}} \vec{x} = \vec{F_{\text{ext}}}(t) \tag{17}$$

No damping means that

$$P_{\text{diss}} = 0 \tag{18}$$

We also have

$$T = \frac{1}{2}mv^2 \iff \frac{1}{2}mL^2\dot{\theta}^2 \tag{19}$$

$$U = mgh \iff mgL(1 - \cos\theta) \tag{20}$$

Note we have that

$$F \cdot v \iff L\dot{\theta}F \cdot e_\theta \iff L\dot{\theta}F\cos\theta \tag{21}$$

Thus we have that

$$\frac{1}{2}mL^2 2\dot{\theta}\ddot{\theta} + mgL\dot{\theta}\sin\theta = L\dot{\theta}F\cos\theta \tag{22}$$

$$mL\ddot{\theta} + mg\sin\theta = F\cos\theta \tag{23}$$

For small angles:

$$mL\ddot{\theta} + mg\theta = F(t) \tag{24}$$

Cart on a cart

- Big cart as acceleration $\ddot{y}$
- Displacement of big truck $= y$, displacement of everything is $y + x$
- also another force $-kx$

Then

$$m(\ddot{x} + \ddot{y}) = -kx \tag{25}$$

then

$$m\ddot{x} + kx = -m\ddot{y} \tag{26}$$