MTH 447: Lecture # 22

Cliff Sun

March 31, 2025

Power Series

Definition 0.1. *Let a_n be a sequence. Then*

$$\sum_{n=0}^{\infty} a_n x^n \tag{1}$$

is called a Power Series with coefficients a_n.

Theorem 0.2. *Let $\sum_{n=0}^{\infty} a_n x^n$. Define*

$$\beta = \limsup |a_n|^{\frac{1}{n}} \tag{2}$$

Let $R = 1/\beta$, then for $|x| < R$, then

$$\sum a_n x^n \text{ converges absolutely} \tag{3}$$

For all $|x| > R$, then

$$\sum a_n x^n \text{ diverges} \tag{4}$$

For $x = R, -R$, we don't know.

Proof. $\limsup |a_n x^n|^{1/n} = \limsup |a_n|^{1/n} |x|$.

$$\to |x|\beta$$

If β is finite, then if $x > 1/\beta$, then it diverges. If $x < 1/\beta$, then it converges. $\square$