

# MTH 447: Lecture 17

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**Definition 0.1.** A repeating decimal is a sequence such that  $\exists n, p$  with

$$d_{n+kp} = d_n \quad \forall k \in \mathbb{N} \quad (1)$$

Consider two finite sequences  $d = (d_1, d_2, \dots, d_n)$  and  $e = (e_1, e_2, \dots, e_p)$ , then

$$0.d_1d_2\dots d_ne_1\dots e_pe_1\dots e_p\dots \quad (2)$$

**Theorem 0.2.** Consider the fraction  $a/b = k$ , and  $a, b \in \mathbb{Z}$ . Assume we have that two non negative integers  $n_1, n_2$  such that

$$a10^{n_1} \pmod{b} = a10^{n_2} \pmod{b} \quad (3)$$

Then the decimal sequence starting at  $n_1$  equals the decimal sequence starting at  $n_2$ .