

PHYS 326: Lecture 11

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Non-linear dynamics - Continued

Driven Damped Pendulum

$$mL^2\ddot{\theta} = -bc^2\dot{\theta} - mgL \sin \theta + LF(t) \quad (1)$$

Choose harmonic forcing $F(t) = F_0 \cos(\omega t)$. The phase space is $\{\theta, \dot{\theta}, \phi = \omega t\}$.

Definition 0.1. Poincare sections are defined as the following:

1. Wait for transients to decay
2. Plot Phase-Plane positions $(\theta, \dot{\theta})$ at 1 cycle intervals.

You plot **points** every $\frac{n2\pi}{\omega}$.

Definition 0.2. Chaos is

1. Divergence from initial conditions
2. The attractor is a fractal, ("strange attractor")

Definition 0.3. A fractal is an object that is

1. Self-similar (same features at all scales)
2. Its dimension is a non integer.

Whats the dimension of a fractal?

Definition 0.4. The dimension is defined by how mass scales with linear size.

For example, for a square. If you split the square in fourths, etc. How does the mass increase? For r sections, it follows that the mass would increase by r^2 . Thus the dimension

$$\dim = \frac{\ln(r^2)}{\ln(r)} = 2 \quad (2)$$

For the Koch curve, for $r = 3$, the mass is $m = 4$. Then

$$\dim = \frac{\ln(m)}{\ln(r)} = 1.26 \quad (3)$$