7.2-7.3; Equivalence Relations and Partitions

Cliff Sun

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To recap, if R is an Equivalence relation, then

$$xRy \iff \text{x and y have something in common} \tag{1}$$

Then abstractly, each piece in A has something in commmon with each other.

Definition 0.1. *Let $\sim$ be an Equivalence relation on a set A.*

1. *For an $x \in A$, the equivalence class of x is $[x] = \{z \in A : z \sim x\}$*
2. *The quotient of A by $\sim$ is the set of all equivalence classes for each element $x \in A$*

Partitions

Definition 0.2. *Let X be a set, and let $\{A_n : n \in I\}$ be a collection of non-empty subsets of X. We call this a paritition if 2 things are true*

1. *The Union of all A_n's is X*
2. *They are all pairwise disjoint, in other words. $\forall m, n \in I$, either $A_m = A_n$ or $A_m \cap A_n = \emptyset$*

Given this defintion, we can relate it to Equivalence relations by stating

$$m \sim n \iff \text{m and n are in the same piece of the partition} \tag{2}$$

Lemma 0.3. *Let $\sim$ be an Equivalence relation on a set A, Then the following are Equivalent:*

$$\forall x, y \in A, x \sim y \iff [x] = [y] \tag{3}$$