Fourier Transform and Complex Numbers

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April 23, 2024

One Minute Paper 0.1. *Before: MTH 442, After:* $i = e^{i\frac{\pi}{2}}$

Complex Numbers

Magnitude of Complex Numbers

$$|z| = \sqrt{\bar{z}z} \tag{1}$$

Euler's Identity

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \tag{2}$$

$$z = re^{i\theta} \text{ polar form in Complex Coordinates} \tag{3}$$

Suppose the wave equation

$$u_{tt} - v^2 u_{xx} = 0 \tag{4}$$

Then we guess

$$f = Ae^{iwt}e^{ikx} \tag{5}$$

We plug our guess into the wave equation

$$A(iw)^2 e^{iwt}e^{ikx} = v^2 A(ik^2)e^{iwt}e^{ikx} \tag{6}$$

$$w^2 = v^2k^2 \tag{7}$$

This is called the dispersion relation.
In general, the law of superposition holds for linear PDE's. Thus, an integral also holds

$$f(x,t) = \int_{-\infty}^{\infty} dk \tilde{f}(k)e^{ikx}e^{ivkt} \tag{8}$$

This is called the fourier transform of $\widehat{f}$.