PHYS 325: Lecture 4

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September 5, 2024

Lecture Span

- Recap (potential curve)
- Simple Harmonic Oscillator
- Drag Forces

Potential Curves

To analyze extrema points of potential, all we do is

$$\left(\frac{dU}{dx}\right)_{x_0} = 0 \tag{1}$$

So what happens if we perturb the particle near the extremas?

Minimum

Check if the 2nd derivative of the potential at the extrema is greater than 0. Than this means that the particle is trapped in a local minima, and any perturbations to the particle will result in it settling in the minimum for time sufficiently high. This is called stable.

Maximum

If the 2nd derivative of the potential is less than 0, then any pertubation will result in the particle leaving the extrema. This is called unstable.

Saddle point

If the 2nd derivative of the potential at the extrema is 0, then we call the system marginally stable, since it may be stable in one direction but not in another.

Simple Harmonic Oscillator

Suppose we're given some graph with a local minima near x_0,

We first taylor expand the potential around x_0, that is

$$U(x) \approx U(x_0) + U'_{x_0}(x - x_0) + \frac{1}{2}U''_{x_0}(x - x_0)^2 + \cdots \tag{2}$$

Since we are in an extrema, we know that $U'_{x_0} = 0$, and letting $U''_{x_0} = k$, we obtain

$$U(x) \approx U(x_0) + \frac{1}{2}k(x - x_0)^2 \tag{3}$$

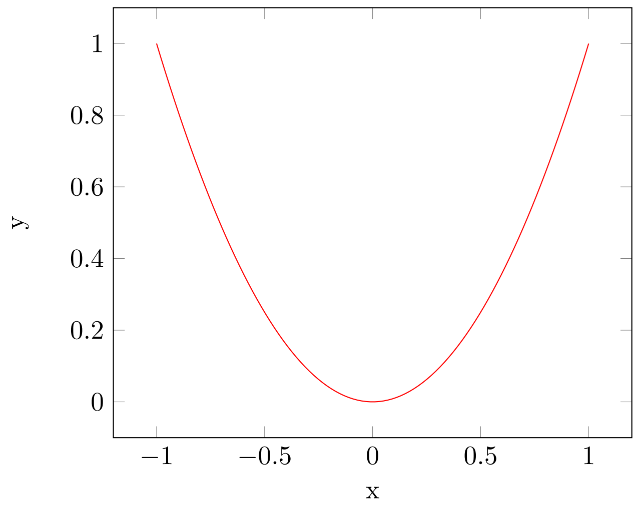
We choose $U(x_0) = 0$ and shift our coordinate axis $x' = x + x_0$, we transform the problem to

$$U(x)' \approx \frac{1}{2}kx'^2 \tag{4}$$

Note that this looks like the spring potential. We now solce the following differential equation:

$$F = ma = m\ddot{x} \tag{5}$$

Local minima zoomed in around $x_0 = 0$



When restricting ourselves to this local minima, we obtain

$$F(x) \approx F_{x_0} + F'_{x_0}(x - x_0) + F''_{x_0}(x - x_0)^2 + \cdots \tag{6}$$

We write $F = -\frac{dU}{dx}$ we write $F_{x_0} = -\frac{dU}{dx}_{x_0} = 0$. Similarly, $F' = k$ we compute this

$$F(x) \approx -kx' \tag{7}$$

by shifting the coordinate axis $x' = x + x_0$. So we see that

$$m\ddot{x} = -kx \tag{8}$$

or

$$m\ddot{x} + kx = 0 \tag{9}$$

This is the equation for the simple harmonic oscillator. The solution is

$$x(t) = A\sin(\sqrt{\frac{k}{m}} + \phi_0) \tag{10}$$

or

$$x(t) = Ae^{i\sqrt{\frac{k}{m}}\ t} \tag{11}$$

Where the phase is encoded within the amplitude of the complex solution. Note that the frequency $\omega = \sqrt{\frac{k}{m}}$.

Note that the period is $\frac{2\pi}{\omega}$

Drag Forces

We first note that $\vec{F} = \vec{F}(\vec{v})$, or that forces are purely a function of velocity. This could be friction, air resistance, etc.

Types of Drag Forces

1. Stoke's Drag (linear drag), we describe the drag as

$$F = -cv \tag{12}$$

   Note the minus sign, as it means that the drag acts in the opposite direction of velocity. This includes

   - Laminar flow
   - Very viscous fluids (Honey, etc.)
   - Small velocities

2. Newtonian drag (nonlinear), we describe the drag as

$$F = -kv^2 \tag{13}$$

   This differential equation can describe turbulent flow. This is valid for

   - Large velocities
   - Less viscous fluids (e.g. air)

   What type is this applicable?

   - Reynold's number: $R_e = \frac{\rho v L}{\mu}$ Where $\mu =$ viscosity and $L =$ size. If this number is small (e.g < 2300) then we get laminar flow. Else, we get turbulent flow.

Example 1: Linear Drag

Setup

Particle m and initial velocity $v_0 = v_0 \neq 0$, we derive $\vec{v}(t)$ at late times.

Strategy

1. Choose coordinates: 1D along the x axis, and let $v = \dot{x}$
2. Force $F(v) = -cv$, we also introduce a new constant $\kappa = \frac{c}{m}$. We rewrite $F = -mkv$
3. in EOM, we start with N2L, we get

$$m\dot{v} = -mkv \iff \dot{v} = -kv \tag{14}$$

   The solution is

$$v(t) = v_0 e^{-kt} \tag{15}$$