PHYS 487 Lecture # 1

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PHYS 487 Plan

We will use techniques and examples to analyze realistic simple systems.

Outline

1. Symmetries + conservation laws
   (a) Central symmetric problems (rotational symmetries)
   (b) Periodic potentials (crystal)
2. Time-independent Perturbation Theory
   (a) Fine-structure of hydrogen
3. Dynamics
   (a) Emission and absorption of light
4. Multi-particle quantum systems: approximations to place boundaries
   (a) Ground state energies estimation
5. Perhaps some bonus topics on some cool stuff

QM Speedrun Recap

1. Physical system $|\psi\rangle \in \mathcal{H}$
2. Linearity/Superposition principle $|\psi\rangle = \sum_k c_k |\psi_k\rangle$ where $|\psi_k\rangle$ are the basis vectors of the same operator.
3. Inner product: $\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$ such that $\langle\psi|\psi\rangle \leq 0$
4. Observables: $\hat{O}$ operators; Linearity $\hat{O}|\psi\rangle = \hat{O}\left(\sum_k c_k |\psi_k\rangle\right) = \sum_k c_k \left(\hat{O}|\psi_k\rangle\right)$; Hermitian: $\hat{O}^\dagger = \hat{O}$, defined as $\left\langle\psi\middle|\hat{O}^\dagger\psi\right\rangle = \left\langle\hat{O}\psi\middle|\psi\right\rangle$
5. Eigenvalues/vectors via diagonalization: $\hat{O}|\alpha_n\rangle = a_n|\alpha_n\rangle$. Where a_n are the possible measurement outcomes and $|\alpha_n\rangle$ is a complete basis set for $|\psi\rangle$.
6. Born's Rule: $|c_k|^2$ is the probability for outcome c_k.
7. Wavefunction Collapse: $|\psi\rangle \to |\psi_k\rangle$ after measuring.
8. Incompatibility: Observables A, B. If $[A, B] \neq 0$, then there doesn't exist an eigenbasis where both A and B are simulatenously diagonal. This means that the measurement of A will disturb that of B and vice versa.
9. Uncertainty principle: recall $\langle A\rangle = \langle\psi|A\psi\rangle$ and $\sigma_A^2 = \langle A^2\rangle - \langle A\rangle^2$, then

$$\sigma_A^2\sigma_B^2 \leq \left(\frac{1}{2i}\langle[A, B]\rangle\right)^2 \tag{1}$$

Example Systems

1. Continuous systems: $\mathcal{H} = L^2$. $\langle\psi|\varphi\rangle = \int dx \varphi(x)^* \psi(x)$. In the 1-d position basis, $\psi(x) = \langle x|\psi\rangle$

When expressing an operator acting on a function in the position basis, consider (note that $\langle x|\hat{x}\psi\rangle$ can be thought of as (first act $\hat{x}$ on ψ, then find this vector in the position basis))

$$\begin{aligned}\langle x|\hat{x}\psi\rangle &= \int dx' \langle x|\hat{x}|x'\rangle \langle x'|\psi\rangle \\ &= \int dx' x' \delta(x - x')\psi(x) \\ &= x'\psi(x')\end{aligned}$$

The expectation value is

$$\int dx \psi^*(x) x \psi(x) \tag{2}$$

Momentum is

$$\langle x|\hat{p}|\psi\rangle = -i\hbar \frac{\partial}{\partial x}\psi(x) \tag{3}$$

This is interpretated as acting the momentum operator on ψ (abstract), then making it more concrete by turning this abstract concept into the position basis. Then, the Hamiltonian is

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) \tag{4}$$

Discrete Systems

$\mathcal{H} = \mathbb{C}^n$,

$$\vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{pmatrix}$$

And $\langle c|d\rangle = \sum_k c_k^* d_k$

Simplest Physical System - Spin 1/2

$$|\chi\rangle = c_1 \underbrace{|\uparrow\rangle}_{\begin{pmatrix}1\\0\end{pmatrix}} + c_2 \underbrace{|\downarrow\rangle}_{\begin{pmatrix}0\\1\end{pmatrix}} \tag{5}$$

Recall that the electron is $|\psi\rangle |\chi\rangle$. Operators for spin-1/2:

$$S_z = \frac{\hbar}{2}\sigma_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

S_x, S_y are defined similarly.

Angular momentum

$$L = r \times p \tag{7}$$

Where $r = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}$ and $p = -i\hbar \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$. For angular momentum, $[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$. Eigenstates $|j, m\rangle$ where j corresponds to the total angular momentum (J^2) and m is the z-projection of the angular momentum J_z. Then

$$J^2 |j,m\rangle = \hbar^2 j(j+1) |j,m\rangle \tag{8}$$

$$J_z |j,m\rangle = \hbar m |j,m\rangle \tag{9}$$

Note, $j = 0, 1/2, 1, 3/2, \dots$ and $m = -j, -j+1, \dots, j$. Ladder operators $J_\pm = J_x \pm iJ_y$ raise and lower m by ± 1.

Dynamics

$$i\hbar\partial_t |\psi\rangle = \hat{H} |\psi\rangle \tag{10}$$

And the stationary states are the eigenstates of the Hamiltonian: $\hat{H} |\psi_n\rangle = E_n |\psi_n\rangle$ where $|\psi_n\rangle$ is the stationary state. Given $|\psi(0)\rangle$, then

1. Find $|\psi_n\rangle$
2. $|\psi(0)\rangle = \sum_n c_n |\psi_n\rangle$
3. $|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |\psi_n\rangle$

Examples to recall

1. 1D ∞ square well
2. QHO
3. 3D coulomb potential
4. Spin 1/2 (qubit)