PHYS 435: Lecture 1

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Coulomb's Law:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_0}{|r-r'|^3}(r-r') \tag{1}$$

Because this is a linear phenomenon, that is doubling the charge yields a double in the electric field (because you are just adding another charge to the same position), this implies superposition. Then

$$\vec{E}(\vec{r}) = \sum \frac{1}{4\pi\epsilon_0}\frac{q_{0,i}}{|r-r'|^3}(r-r') \tag{2}$$

$$\implies \frac{1}{4\pi\epsilon_0}\int d^3r' \frac{(r-r')}{|r-r'|^3}\rho(r') \tag{3}$$

Gauss's law states:

$$\frac{1}{\epsilon_0}\int_V d^3r'\rho(r') = \int_{\partial V} d\vec{a}\cdot\vec{E}(\vec{r}) \tag{4}$$

Then,

$$\int_V d^3r'\vec{\nabla}\cdot\vec{E}(\vec{r}) = \int_{\partial V} d\vec{a}\cdot\vec{E}(\vec{r}) \tag{5}$$

Then

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0} \tag{6}$$

Faraday's Law: the <u>circulation</u> of any E field around any given point is equivalent to -1 times the time partial derivative of the B field.

$$\int_{\partial S} dl\cdot E = -\frac{d}{dt}\int_S da\cdot B \tag{7}$$

This is a non-local statement, that is calculating the value at specific point depends on the value at other points. We use stokes theorem:

$$\int_{\partial S} dl\cdot E = \int_S d\vec{a}\cdot\vec{\nabla}\times\vec{E} \tag{8}$$

Then

$$\int_S d\vec{a}\cdot\vec{\nabla}\times\vec{E} = -\int_S da\cdot\frac{dB}{dt} \tag{9}$$

Thus

$$vec\nabla\times\vec{E} = -\frac{\partial B}{\partial t} \tag{10}$$

Similarly,

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \tag{11}$$