7.5-7.6

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7.5

Definition 0.1. *Let $\sim_n$ be the relation*

$$\equiv \mod n \tag{1}$$

on $\mathbb{Z}$. Then

$$Z/\sim_n = \{[0], [1], \ldots, [n-1]\} \tag{2}$$

Our goal today is to view this as a number system, including the operations $+_n$ and $\cdot_n$.
We define these operations to be

Definition 0.2. *For integers a and b, we define the following:*

1. $[a] +_n [b] = [a + b]$
2. $[a] \cdot_n [b] = [a \cdot b]$
3. *The ring of $\mathbb{Z}_n$ or $\mathbb{Z}/n$ is the set $\mathbb{Z}/\sim_n$ equipped with the operations stated above. The term Ring is from abstract algebra and roughly means a set of elements which can be added, subtracted, and multiplied.*

To define the operation of $X \cdot_n Y$, we first choose two representatives from X and Y, and multiply them together. Then the equivalence class of this new term xy is the result of $X \cdot_n Y$. WLOG.

Theorem 0.3. *$+_n$ and $\cdot_n$ are well-defined. That is if $x \equiv x' \mod n$ and $y \equiv y' \mod n$, then $x + y \equiv x' + y' \mod n$ and $x \cdot y \equiv x' \cdot y' \mod n$*

Next, suppose X and Y are sets and $\sim$ is an equivalence relation. Then, let's try to define a function f such that

$$f : X/\sim \to Y \tag{3}$$

1. For each $x \in X$, we define $f([x]) \in Y$ by referring to x.
2. Prove that if $[x] = [x']$, then $f([x]) = f([x'])$, that is f is well-defined.