Fourier Convergence

Cliff Sun

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Suppose we define $S_m(x)$ to be following:

$$S_m(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}[A_n \cos(\frac{n\pi x}{l} + B_n \sin(\frac{n\pi x}{l}))] \tag{1}$$

Theorem 0.1. *If $\Psi(x)$ is differentiable at x_0, then $S_m(x_0) \to \Psi(x_0)$*

Theorem 0.2. *If $\Psi(x)$ is continously differentiable (that is $\Psi'(x)$ is continuous), then $S_m(x) \to \Psi(x)$ uniformly on $\mathbb{R}$.*

Theorem 0.3. *$S_m(x) \to \Psi(x)$ in L^2 on $(-l, l)$. Moreover:*

$$\int_{-l}^{l} |\Psi(x)|^2 dx = l[\frac{A_0^2}{2} + \sum_{n=1}^{\infty}(A_n^2 + B_n^2)] \tag{2}$$

For the heat equation, the Dirichlet boundary condition says that we must use the Sine Fourier Series. The Neumann Boundary Condition states that we must use the Cosine Fourier Series.