PHYS 326: Lecture # 14

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Rigid Body Motion (cont'd)

Principal Axis of Inertia

Recall,

$$\vec{L} = I\vec{\omega} \neq L \not\parallel \omega \tag{1}$$

But, it's always possible to define 3 orthogonal directions in which I is diagonal. These 3 orthogonal axis are called the "3 principal axis of inertia". That is

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{2}$$

Parallel Axis Theorem

For each m_a, we have that

$$x_a = x_{cm} + x'_a \tag{3}$$
$$y_a = y_{cm} + y'_a \tag{4}$$
$$z_a = z_{cm} + z'_a \tag{5}$$

Then

$$I_{zz} = \sum_a m_a(x_a^2 + y_a^2) \tag{6}$$
$$= \sum_a m_a((x_{cm} + x'_a)^2 + (y_{cm} + y'_a)^2) \tag{7}$$
$$= \sum_a m_a(x_{cm}^2 + y_{cm}^2) + \underbrace{2\sum_a m_a(x'_a x_c m + y'_a y_c m)}_{0} + \underbrace{\sum_a m_a((x'_a)^2 + (y'_a)^2)}_{I_{zz}} \tag{8}$$

Thus

$$I_{zz}^O = M(x_{CM}^2 + y_{CM}^2) + I_{zz}^{CM} \tag{9}$$
$$I_{xx}^O = M(z_{CM}^2 + y_{CM}^2) + I_{xx}^{CM} \tag{10}$$
$$I_{yy}^O = \dots \tag{11}$$

Rigid Body Motion: Torque-Free

$$\Gamma_o(t) = \frac{dL_O}{dt} = \frac{d}{dt}\left(I_O\omega\right) \tag{12}$$

Note, "O" is a fixed point. Let's work in a coordinate system where I_O is diagonal. However, we are now working in a rotating coordinate system. Then

$$\vec{v} = \sum v_i \hat{e}_i(t) \tag{13}$$

Then

$$\frac{dv}{dt} = \sum \frac{dv_i}{dt}\hat{e}(t) + \omega \times v \tag{14}$$

Then, choose that I_{CM} is diagonal, but

$$\Gamma(t) = \frac{d}{dt}\left(I\omega\right) = I\frac{d\omega}{dt} + \omega \times \left(I\omega\right) = 0 \tag{15}$$