PHYS 325: Lecture 26

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December 5, 2024

Definition 0.1. *A <u>Phase space</u> is a space of all possible behaviors for the system.*

Recap: a functional is

$$S[\vec{q}_i(t)] = \int_{t_1}^{t_2} dt L(q, \dot{q}; t) \tag{1}$$

We note some things:

1. If we didn't specify 2 fixed end points, maybe $q(t_1)$ but not $q(t_2)$, then

$$\frac{\partial L}{\partial \dot{q}} \delta q \Big|_{t_1}^{t_2} \neq 0 \tag{2}$$

So then, to "fix" an endpoint, we impose the restriction that

$$\frac{\partial L}{\partial \dot{q}} \Big|_{t_2} = 0 \tag{3}$$

Brachistochrone Problem

We note that the total time T is

$$T = \int_\gamma dt \tag{4}$$

Where γ is the curve that the particle takes. We have that

$$dt = \frac{dl}{v} \tag{5}$$

Where dl is a small step in the curve and v is the velocity of the particle. We can derive the velocity of the particle using conservation of energy:

$$\frac{1}{2}mv^2 + mgy(t) = mgy(t_0) \tag{6}$$

Then

$$v^2 = 2g(y(t_0) - y(t)) \tag{7}$$

Thus

$$v = \sqrt{2g(y(t_0) - y(t))} \tag{8}$$

We define $y(t_0) = 0$, thus

$$v = \sqrt{2g(y(t))} \tag{9}$$

Where $y(t)$ is defined as positive from below the x,y axis.

$$T = \int_{t_1}^{t_2} \frac{dl}{v} \iff \int_{t_1}^{t_2} \frac{\sqrt{dx^2 + dy^2}}{\sqrt{2g(y)}} \tag{10}$$

We choose $t = y$, $q = x(y)$, and $\dot{q} = x'(y)$, thus

$$\int_{t_1}^{t_2} \frac{\sqrt{x'^2 + 1}}{\sqrt{2gy}} \tag{11}$$

We use the euler lagrangian equations to get

$$\frac{1}{\sqrt{2gy}}\frac{x'}{\sqrt{x'^2+1}} = c \tag{12}$$

This implies that

$$x' = \pm\frac{\sqrt{2gy}c}{\sqrt{1-2gyc^2}} \tag{13}$$

Thus

$$x = \frac{1}{2c}\sqrt{\frac{2}{gy} - 4c^2} + \frac{1}{2gc^2}\arctan\left[\sqrt{\frac{1}{2gyc^2} - 1}\right] \tag{14}$$