PHYS 435: Lecture 1

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Infinite Line of charge

Suppose a infinite line of charge. How do we calculate the E field at a specific point? We take an Gaussian cylinder around some length L around the Line. We obtain that at a distance s, we get

$$2\pi E(s)sL = \frac{\lambda \cdot L}{\epsilon_0} \tag{1}$$

We solve for

$$E(s) = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s} \tag{2}$$

Solving for potential energy gets

$$\frac{\lambda}{2\pi\epsilon_0}\ln\frac{a}{s} \tag{3}$$

This is the weakest analytical divergence. We can derive a PDE for the potential energy function:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{4}$$

Let $E = -\vec{\nabla}V$, then

$$\nabla^2 V(r) = -\frac{\rho}{\epsilon} \tag{5}$$

The total work done by moving point charges into each other is

$$U = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{|\vec{r_2} - \vec{r_1}|} \tag{6}$$

The continuous analogue of this is

$$U = \frac{1}{2}\int_S d^3r d^3r' \frac{1}{4\pi\epsilon_0}\frac{\rho(r)\rho(r')}{|r - r'|} \tag{7}$$

Let the potential energy function be

$$V(r') = \frac{1}{4\pi\epsilon_0}\int d^r \frac{\rho(r)}{|r - r'|} \tag{8}$$

Then

$$U = \frac{1}{2}\int d^3r' \rho(r')V(r') \tag{9}$$

Since

$$\rho(r') = -\epsilon_0 \nabla^2 V(r') \tag{10}$$

Then our integral turns into

$$U = -\frac{\epsilon_0}{2}\int d^3r' V(r')\nabla^2 V(r') \tag{11}$$

First evaluating the x partial derivative, then we can generalize:

$$U = -\frac{\epsilon_0}{2}\int d^r V(r)\partial_x\left[\partial_x V(r)\right] \tag{12}$$

We first evaluate:

$$\partial_x\left[V(r)\partial_x V(r)\right] = \partial_x V(r)\cdot\partial_x V(r) + V\partial_x^2 V \tag{13}$$

$$\partial_x\left[V(r)\partial_x V(r)\right] - \partial_x V(r)\cdot\partial_x V(r) = V\partial_x^2 V \tag{14}$$

$$\implies -\frac{\epsilon_0}{2}\int d^3r\left[\partial_x(V(r)\partial_x(V(r))) - E_x^2\right] \tag{15}$$

We generalize this:

$$\implies \frac{\epsilon_0}{2}\int d^3r\left[\vec{\nabla}\cdot(V(r)E(r)) + \vec{E}\cdot\vec{E}\right] \tag{16}$$

We note that

$$\vec{\nabla}\cdot(V(r)E(r)) = 0 \tag{17}$$

Then

$$u = \int d^r\frac{\epsilon_0}{2}E^2 \tag{18}$$