Real numbers and least upper bounds

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Definition 0.1. *Let $A \in \mathbb{R}$, then*

1. *If there is some real number b such that $\forall a \in A$, $a < b$, then we say that b is an upper bound of A and A is bounded above.*

2. *If $\exists b \in \mathbb{R}$, such that $b \leq a$, $\forall a \in A$, then we say that b is a lower bound of A and A is bounded below.*

3. *A is bounded if it is bounded above and below.*

Axiom 0.2. *Every nonempty $A \subseteq \mathbb{R}$ which is bounded above has a least upper bound.*

Definition 0.3. *A Least Upper Bound is defined as b' such that for all $b \in B$ where B is the set of all upperbounds, $b' \leq b$.*

The reason why this axiom is called completeness is because it tells us that the real numbers has no "holes".

Notation

We call the least upper bound of set A its supremum, or $\sup A$. Similarly, the greatest lower bound of a set A its infimum or $\inf A$.