MTH 447: Lecture 10

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Definition 0.1. *Let (x_n) be an increasing sequence, then we say it is* <u>*increasing*</u> *if*

$$x_{n+1} \geq x_n \ \forall n \tag{1}$$

It is strictly increasing if

$$x_{n+1} > x_n \ \forall n \tag{2}$$

Similar definitions for decreasing. Monotone is increasing or decreasing.

Theorem 0.2. *Bounded + monotone $\implies$ convergent.*

Proof. Suppose (x_n) is increasing and is bounded above.

$$S = \{x_n | n \in \mathbb{N}\} \tag{3}$$

This set is bounded above. Then $L = \sup S$ exist and is a real number. Claim, $x_n \to L$. Note, $x_n \leq L$ for all n. Then $\forall \epsilon > 0$, there exists k such that

$$x_k > L - \epsilon \tag{4}$$

Proof. Suppose not, but that implies that $L - \epsilon$ is the new supremum. Thus, this is a contradiction. □

Since x_n is increasing, then

$$x_n \geq x_k \geq L - \epsilon \tag{5}$$

Then

$$L - \epsilon \leq x_n \leq L \leq L + \epsilon \tag{6}$$

□