PHYS 325: Lecture 2

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August 29, 2024

Lecture Span

- Vector Recap
- Newton's 3 Laws
- 1 Dimensional Particle Dynamics

Vector Recap

Vector Differentiation

Given a vector space in $\mathbb{R}^3$ with a set of basis: $(\vec{e_x}, \vec{e_y}, \vec{e_z})$

$$\vec{r} = x\vec{e_x} + y\vec{e_y} + z\vec{e_z} \tag{1}$$

$$\dot{\vec{r}} = \dot{x}\vec{e_x} + \dot{y}\vec{e_y} + \dot{z}\vec{e_z} \tag{2}$$

Which is equivalent to

$$\vec{v} = v_x\vec{e_x} + v_y\vec{e_y} + v_z\vec{e_z} \tag{3}$$

Similarly

$$\vec{a} = \frac{d\vec{v}}{dt} \iff \frac{d^2\vec{r}}{dt^2} \tag{4}$$

Leibniz/Product Rule

$$\frac{d}{dt}(f(t)g(t)) = \dot{f}g + f\dot{g} \tag{5}$$

Dot product

$$\frac{d}{dt}(\vec{v} \cdot \vec{w}) = \vec{v} \cdot \dot{\vec{w}} + \dot{\vec{v}} \cdot \vec{w} \tag{6}$$

Cross Produc

$$\frac{d}{dt}(\vec{r} \times \vec{w}) = \vec{r} \times \dot{\vec{w}} + \dot{\vec{r}} \times \vec{w} \tag{7}$$

Newton's 3 Laws

1. 1st law: object in motion stays in motion
2. 2nd law: $\vec{F} = m\vec{a} \iff m\ddot{\vec{r}}$ (an ordinary differential equation!)
3. 3rd law: Equal and opposite reactions

Consider a box of mass m being pushed on by a constant force F_0 at x_0 with an initial velocity v_0. Find the box's displacement. Answer:

$$x(t) = \frac{F_0}{2m}t^2 + v_0t + x_0 \tag{8}$$

Linear Momentum

$$\vec{p} = m\vec{v} \tag{9}$$

$$\dot{\vec{p}} = m\dot{\vec{v}} \iff m\vec{a} \iff \vec{F} = \dot{\vec{p}} \tag{10}$$

Inertial Frames

One system in 2 reference frames

1. Cartesian coordinate with origin at rest. Particle at rest $\vec{a} = 0$
2. Rotating reference frame, particle in rest has a non-zero acceleration. Newton's Laws don't hold in non-zero acceleration frames.

Definition 0.1. *Inertial Frames is a reference frame in which Newton's Laws can predict its motion. In other words, particles move with a constant velocity if there is no force acting on them. That is*

$$\ddot{\vec{r}} = 0 \implies \vec{F} = 0 \tag{11}$$

Inertial Frames are not unique, i.e there exists more than one inertia frames.

Take inertial frame S with $\vec{r}$, then we can construct another inertial frame S' with another $\vec{r}'$. They include

- Boosts
- Translations
- Rotations

Translations

$$\vec{r'} = \vec{r} + \vec{k} \tag{12}$$

Boost

$$\vec{v'} = \vec{v} + \vec{k} \tag{13}$$

Rotation

$$\vec{r'} = A\vec{r} \tag{14}$$

Proof. Suppose that $\ddot{\vec{r}} = 0$, then $\ddot{\vec{r'}} = A\ddot{\vec{r}} \iff A0 \iff 0$. Because A is not time dependent. □