PHYS 325: Lecture 16

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Forced oscillators

EOM:

$$m_{eff}\ddot{x} + c_{eff}\dot{x} + kx = F(t) \tag{1}$$

General Solution

Introduce linear operator:

$$\mathbb{L}[x] = F \tag{2}$$

Linear operator is a map of a function to another function. Note that linear operators obey this condition:

$$L(af + bg) = aL(f) + bL(g) \tag{3}$$

Force as power series

$$F_{ext} = a + bt + ct^2 + \cdots \tag{4}$$

$$L(x) = F_{ext} \tag{5}$$

Ansatz for $x_p(t)$

$$x_p(t) \sim \alpha + \beta y + \gamma t^2 + \cdots \tag{6}$$

Given some EOM with

$$F = F_0 \cos(\omega t) \tag{7}$$

Then the particular solution is

$$x = \frac{F_0}{k} e^{-i\theta} \left[1 + 2i\zeta \frac{\omega}{\omega_n} - \left(\frac{\omega}{\omega_n}\right)^2\right]^{-1} \tag{8}$$

$$\iff \tilde{G(\omega)} F_0 e^{-i\theta} \tag{9}$$

Note

$$\tilde{G(\omega)} = G(\omega) e^{-i\phi(\omega)} \tag{10}$$

$$G(\omega) = |\tilde{G}(\omega)| = \frac{1}{k} \left[(1 - \frac{\omega^2}{\omega_n^2} + (2\zeta \frac{\omega}{\omega_n})^2)\right]^{-\frac{1}{2}} \tag{11}$$

This is called the response function.