6.2/6.3 - Power Sets and Indexed Collections of Sets

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Theorem 0.1. *If $A \subseteq B$, then $P(A) \subseteq P(B)$*

Proof. Suppose that $A \subseteq B$, then to prove that $P(A) \subseteq P(B)$, choose c to be an element of $P(A)$. That is $c \in A$. Thus, $c \in A \in B$, thus it follows that $c \in P(B)$. This proves the theorem that $P(A) \in P(B)$. □

How do power sets relate to Cartesian Products? That is Is $P(A \times B) = P(A) \times P(B)$?

Suppose that that $|A| = a$ and $|B| = b$, then $P(A \times B) = 2^{ab}$ and $P(A) \times P(B) = 2^{a+b}$.

However, suppose that A and B are disjoint, that is they don't have any elements in common. Then the cardinality of $P(A) \times P(B)$ and $P(A \cup B)$ have the same cardinality. Therefore, there must be a bijection between them.

Indexed Collections of Sets

Let I be any set, call it an "Indexed Set". Then, for $n \in I$, suppose that we have some set called A_n. Generally, I could be the natural numbers, the real numbers, etc. Given this set-up, we can write down some definitions:

Theorem 0.2. *The collection of all of these sets is called an "Index Collection Of Sets", written in mathematical notation is the following:*

$$\{A_n : n \in I\} \tag{1}$$

Theorem 0.3. $\cup_{n \in I} A_n = \{x : x \in A_n \textit{for some} n \in I\}$
$\cap_{n \in I} A_n = \{x : x \in A_n \forall n \in I\}$
The collection $\{A_n : n \in I\}$ is pairwise disjoint if we have that $A_n \cap A_m$ for $m \neq n$ in I.

Simply speaking, this $\cup$ represents the union of all sets, where $\cap$ is the intersection of all sets.

General fact:

1. For any $m \in I$, A_m is always a subset of the union of the Sets
2. For any $n \in I$, the intersection of all of the sets will be a subset of A_n