Line Integration, Maxwell's Equations

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One Minute Paper 0.1. *Before: Surface Integral - Gauss's Law; Line Integral - Ampere's Law*

After: Line Integral

Recall: Line integrals are moving across a higher dimensional area and calculating the summations of all the dot products.

$$\int_C F \cdot dr \iff \int_C F \cdot \frac{dr}{dt} dt \tag{1}$$

Fundamental Theorem of Vector Calculus:

$$\int_C \nabla F \cdot dr = f(b) - f(a) \tag{2}$$

This curve goes from a to b, both are vectors. Next is the surface integral:

$$\int_S F \cdot dS \iff \int \int F(r(u,v)) \cdot |\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial u}| dudv \tag{3}$$

Such that

$$dS = \Delta u \Delta v \tag{4}$$

In the generalized coordinates of u and v. In general, we have that

$$\int_R d\omega = \int_{\partial R} \omega \tag{5}$$

For divergence, we have that

$$\int_V (\nabla \cdot F) dV = \int_{\partial V} F \cdot dS \tag{6}$$

Similarly, for curl, we have that

$$\int_S (\nabla \times F) \cdot dS = \int_{\partial S} F \cdot dr \tag{7}$$

$$\epsilon^{ijk} = 1, -1, 0 \tag{8}$$

$$\text{1 if } ijk \text{ is an even permutation of (1,2,3), -1 if odd, etc.} \tag{9}$$

Even permutation means an even amount of rotations relative to $(1, 2, 3)$, etc. So curl is This

$$\nabla \times F \iff \epsilon^{ijk} \partial_j F_k \tag{10}$$

Delta functions

$$\int_V \delta^(3)(x) dV = \begin{cases} 1, & \text{if V contains the origin} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Maxwell's Equations

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{12}$$

$$\nabla \cdot B = 0 \tag{13}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{14}$$