Maxwell's Equations and Special Relativity

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One Minute Paper 0.1. *Before: Lorentz Transformations, identities, same physics in different reference frames, divergence of B = 0, fourier transform*

After: Fourier Transform

E&M wave equation:

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{1}$$

We use natural units:

$$\nabla^2 E = \frac{\partial^2 E}{\partial t^2} \tag{2}$$

But in what frame? So we define a 4-gradient operator:

$$\partial_\mu = (\partial_t, \partial_x, \cdots) \tag{3}$$

This transforms like a 4-vector. Then we define a 4-laplacian called the d'Alembertian

$$\partial_\mu \partial^\mu \equiv g_{\mu\mu}\partial_\mu\partial_\mu \iff \partial_t^2 - \nabla^2 \tag{4}$$

In natural units, the wave equation is the following:

$$\partial_\mu \partial^\mu E = 0 \tag{5}$$

and

$$\partial_\mu \partial^\mu B = 0 \tag{6}$$

Thus this wave equation is invariant. Thus, the speed of the wave is invariant. Now we fourier transform the wave:

$$E(t, x) = E_0 e^{-ik_\mu x^\mu} \tag{7}$$

Taking the derivative yields

$$\partial_\nu (e^{-ik_\mu x^\mu}) = -k_\nu e^{-ik\cdot x} \tag{8}$$

Similarly:

$$\partial^\nu \partial_\nu = -k^2 e^{-ik\cdot x} \tag{9}$$

But since we have that

$$\partial^\nu \partial_\nu E = 0 \tag{10}$$

Thus

$$k^2 = 0 \tag{11}$$

In other words, light is massless. Thus E&M waves are described by massless photons. Thus

k is proportional to the momentum 4-vector of the wave