PHYS 325: Lecture 22

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Non-rotating reference frames

Consider

1. 2 Frames accelerating relative to each other
2. O: rest frame with fixed unit vectors
3. O': accelerated frame with unfixed basis vectors

Define

1. $\vec{r}_{OA}$ from O to point A.
2. $\vec{r}_{OO'}$ from O to accelerated frame O'.
3. $\vec{r}_{O'A}$ from O' to point A.

Thus

$$\vec{r}_{OA} = \vec{r}_{OO'} + \vec{r}_{O'A} \tag{1}$$

Velocity is

$$\vec{v}_{OA} = \vec{v}_{OO'} + \vec{v}_{O'A} \tag{2}$$

Acceleration is also defined similarly. So

$$F = ma_{OA} \tag{3}$$

Thus

$$F = m(a_{OO'} + a_{O'A}) \tag{4}$$

Thus

$$F - ma_{OO'} = ma_{O'A} \tag{5}$$

We define

$$F_{\text{fictious}} = ma_{OO'} \tag{6}$$

And

$$F_{\text{true}} = ma_{O'A} \tag{7}$$

Thus

$$F_{\text{tot}} = F_{\text{fictious}} + F_{\text{true}} = ma_{O'A} \tag{8}$$

This is newton's 2nd law in an accelerated frame.

Rotating Frames

Rest frame O is fixed. Consider a rotating reference frame O' with $\vec{\omega}$. Then let

1. $\vec{L}$ is fixed on O'
2. Then $\dot{\vec{L}} = \vec{\omega} \times \vec{L}$

Then we define the vectors as the following:

1. $\vec{r}_{OA} = \vec{r}_{OO'} + \vec{r}_{O'A}$
2. $\vec{v}_{OA} = \vec{v}_{OO'} + \vec{v}_{O'A}$ We define this further:

$$\vec{v}_{OO'} = \frac{d}{dt} r\vec{_{OO'}} = \frac{d}{dt}(\vec{r}_{OO'}\vec{e}_x + \dots) \tag{9}$$

Note that

$$\dot{\vec{e}_x} = 0, \dots \tag{10}$$

Thus

$$\vec{v}_{OO'}\vec{e}_x + \dots \tag{11}$$

Note

$$\vec{v}_{O'A} = \frac{d}{dt}(x'\vec{e}_{x'} + \dots) \tag{12}$$

These basis vectors do change in time, so

$$= \dot{x}\vec{e}_{x'} + \dots + x\dot{\vec{e}}_{x'} + \dots \tag{13}$$

Note that

$$\dot{\vec{e}}_{x'} = \omega times \vec{e}_{x'} \tag{14}$$

$$= v_{\text{apparant}} + \omega \times \vec{r}_{O'A} \tag{15}$$

Thus

$$\vec{v}_{\text{rest frame}} = \vec{v}_{OO'} + \vec{v}_{\text{apparant}} + \omega \times \vec{r}_{O'A} \tag{16}$$

Acceleration

$$\vec{a}_{OA} = \frac{d}{dt}\vec{v}_{OO'} + \frac{d}{dt}v_{\text{app}} + \frac{d}{dt}(\omega \times r_{O'A}) \tag{17}$$

We note that

$$\frac{d}{dt}v_{\text{app}} = \frac{d}{dt}(v_{\text{app},x'}\vec{e}_{x'} + \dots) \tag{18}$$

$$\vec{a}_{\text{app}} + \omega \times \vec{v}_{\text{app}} \tag{19}$$

And

$$\frac{d}{dt}(\omega \times r_{O'A}) = \omega \times r_{O'A} + \omega \times (v_{app} + \omega \times r_{O'A}) \tag{20}$$

$$a_{OA} = a_{OO'} + a_{app} + \omega \times (\omega \times r_{O'A}) + 2\omega \times v_{app} + \omega \times r_{O'A} \tag{21}$$

We define the following:

1. $a_{OO'}$: relative acceleration between frames O and O'.
2. a_{app}: apparant acceleration of A in O'.
3. $\omega \times (\omega \times r_{O'A})$: generalized centripetal acceleration
4. $2\omega \times v_{app}$ is the coriolis "force".
5. $\dot{\omega} \times r_{O'A}$ is the Euler term

Dynamics in non-inertial reference frames

Write N2L in non-inertial reference frame,

1. account for fictious forces