MTH 417: Lecture # 17

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Can we give the set of left cosets of H in G

$$G/H = \{aH | a \in H\} \tag{1}$$

a group structure with respect to the operation in G. We proceed with the equivalence class. For $a \in X$, the equivalence class $\sim$

$$[a] = \{b \in X | b \sim a\} \tag{2}$$

Proposition 0.1. *Suppose $\sim$ be an equivalence relation on X. Then, for $x, y \in X$, then*

$$x \sim y \iff [x] = [y] \tag{3}$$

This is proving the equivalence class from the ground up.

Corollary 0.2. *Let $\sim$ be an equivalence relation on X. Then for $x, y \in X$, either $[x] = [y]$ or $[x] \cap [y] = \emptyset$.*

Proof. If $[x] \cap [y] \neq \emptyset$, then $z \in [x] \cap [y]$. If $z \in [x]$, then $z \sim x \sim y$. Therefore, $[x] = [y]$. $\square$

Theorem 0.3. *Let N be a normal subgroup relative to G. Then there exists a unique group structure on the set of cosets G/N which means the quotient function:*

$$\pi : X \to X/\sim = \{[x]\} \tag{4}$$

Into a group homomorphism.