PHYS 325: Lecture 6

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September 12, 2024

Lecture Span

- Previous lecture
- Curvilinear Coordinates (non-Cartesian Coordinates)

Notes

1. Midterm 1 is in **1 month**, on October 10th, 12:00-13:30
2. Midterm 1 covers Lecture 1 to Lecture 13

Previous lecture

Building from previous lecture, we considered a charged particle in a homogenous magnetic field. We found that

$$\dot{v_x} = \omega v_Y \tag{1}$$

$$\dot{v_y} = -\omega v_x \tag{2}$$

Such that

$$\omega = \frac{qB}{m} \tag{3}$$

We would also "complexify" the velocity, where we introduce a new complex variable

$$\eta = v_x + iv_y \tag{4}$$

Thus

$$\dot{\eta} = \dot{v_x} + i\dot{v_y} \tag{5}$$

Thus we insert equations 1 & 2

$$\dot{\eta} = (\omega v_y - i\omega v_x) \iff -i\omega(v_x + iv_y) \iff i\omega\eta \tag{6}$$

Then we solve this using separation of variables:

$$\frac{1}{\eta} d\eta = -i\omega dt \tag{7}$$

$$ln(\eta) = -i\omega t + C \tag{8}$$

$$\eta = Ae^{-i\omega t} \tag{9}$$

Such that

$$A = Ce^{-i\delta} \tag{10}$$

Where our final solution condenses into

$$\eta = Ce^{-i(\omega t + \delta)} \tag{11}$$

Where δ is phase. We can show that the amplitude of the velocity stays constant:

$$A = \sqrt{v_x^2 + v_y^2} \tag{12}$$

By taking the time derivative, that is

$$\frac{d}{dt} T = \frac{1}{2} m \frac{d}{dt}(v^2) = 0 \implies v^2 = const \tag{13}$$

We take the time derivative:

$$\frac{1}{2} m 2v \cdot \dot{v} \implies v \cdot (F) \implies \vec{v} \cdot (q(v \times B)) = 0 \tag{14}$$

Thus, the amplitude of the velocity is constant.

Trajectory

We can now get the trajectory from the velocity through

$$x(t) = \int v_x dt = \int A\cos(\omega t - \delta) = \frac{A}{\omega}\sin(\omega t - \delta) + C_x \tag{15}$$

$$y(t) = \int v_y dt = \int A - \sin(\omega t - \delta) = \frac{A}{\omega}\cos(\omega t - \delta) + C_y \tag{16}$$

$$z(t) = \int v_z dt = v_{z,0}t + z_0 \tag{17}$$

Such that

$$A = \sqrt{v_x^2 + v_y^2} \tag{18}$$

Curvilinear Coordinates

2-D Polar Coordinates (r, ϕ)

$$x = r\cos(\phi(t)) \tag{19}$$

$$y = r\sin(\phi(t)) \tag{20}$$

$$r = x^2 + y^2 \tag{21}$$

$$\phi(t) = \arctan(\frac{x}{y}) \tag{22}$$

3D Cylindrical Coordinates (r, ϕ, z)

$$x = r\cos(\phi(t)) \tag{23}$$

$$y = r\sin(\phi(t)) \tag{24}$$

$$z = z \tag{25}$$

3D Spherical Coordinates (r, θ, ϕ)

$$\theta \in (\frac{\pi}{2}, -\frac{\pi}{2}) \tag{26}$$

$$\phi \in (0, 2\pi] \tag{27}$$

$$x = r\cos\phi\sin\theta \tag{28}$$

$$y = r\sin\phi\sin\theta \tag{29}$$

$$z = r\cos\theta \tag{30}$$

Bead on whirling stick

Set-up

- A rigid stick whirling with frequency ω
- a Bead sliding along stick with no friction

We choose 2D Polar Coordinates to do this problem: