PHYS 436 Lecture Notes

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Lecture 11: 9/17

What does a monochromatic E&M plane wave carry?

1. Energy Density u
2. E&M Wave carries energy, i.e. the enegry current density $S = 1/\mu_0(E \times B) = \hat{k}cu$
3. Momentum

Note, light intensity is the magnitude of the averaged energy current density.

Radiation pressure for perfect absorption

1. Consequence of EM momentum
2. For cross section A and length of $c\Delta t$. The average momentum is

$$\Delta p = \langle g \rangle (A \cdot c\Delta t) \iff P = |F|/A = \langle g \rangle c \tag{1}$$

   Note, P is pressure.

E&M Wave in Isotropic Linear Medium

Maxwells equations are

$$\nabla \cdot E = 0 \tag{2}$$
$$\nabla \cdot B = 0 \tag{3}$$
$$\nabla \times E = -\partial_t B \tag{4}$$
$$\nabla \times B = \mu\epsilon\partial_t E \tag{5}$$

Note, $\mu = \mu_0(1 + \chi_m)$ and etc for ϵ.

Lecture 12: 9/19

E&M wave in a linear media, has that

$$\epsilon_0 \to \epsilon \quad \mu_0 \to \mu \tag{6}$$

Lecture 13: 9/22

Deriving Fresnel equation for normal incidence:

1. E, B are parallel to the surface
2. The boundary conditions are in the lecture notes.

We now study generalized incidence and try to derive the Fresnel Equation for this. The Electric field is

$$E_R = E_{RO} \exp(i(k_R \cdot r - \omega t)) \tag{7}$$
$$B_R = \hat{k}_R \times \frac{n_1}{c} E_R \tag{8}$$

$$E_I = E_{IO} \exp(i(k_I \cdot r - \omega t)) \tag{9}$$
$$B_I = \hat{k}_I \times \frac{n_1}{c} E_I \tag{10}$$

The boundary conditions are

$$(D_1 - D_2) \cdot n = 0 \tag{11}$$
$$(B_1 - B_2) \cdot n = 0 \tag{12}$$
$$(E_1 - E_2) \times n = 0 \tag{13}$$
$$(H_1 - H_2) \times n = 0 \tag{14}$$

Lecture 14: 9/24

Midterm, 1st question chapter 8, 2 questions on chapter 9.

Lecture 15: 9/26

Electric field in conductors. Maxwell's equations become

$$
\begin{aligned}
\nabla \cdot E &= \rho_f/\epsilon_0 &(15)\\
\nabla \cdot B &= 0 &(16)\\
\nabla \times E &= -\partial_t B &(17)\\
\nabla \times B &= \mu_0 J + \mu_0\epsilon_0\partial_t E &(18)
\end{aligned}
$$

Assuming thqt $J_f = \sigma E$, we can then assume that the free charge is constant. We prove this:

$$
\begin{aligned}
\partial_t \rho &= \nabla \cdot J \quad \text{Ohm's Law}\\
&= \sigma \nabla \cdot E \quad \text{Gauss's Law}\\
&= \sigma \frac{\rho}{\epsilon}\\
\rho(t) &= \rho(t=0)\exp\left(-\frac{\sigma}{\epsilon_0}t\right)
\end{aligned}
$$

Note that if $\sigma = 1$, then $1/\epsilon_0 \sim 10^{12}$, which means that the time dependence of the charges can be ignored. Then the modified wave equation becomes:

$$
\begin{aligned}
\nabla \times (\nabla \times E) &= -\partial_t(\nabla \times B)\\
-\nabla^2 E &= -\partial_t(\mu_0\epsilon_0\partial_t E + \mu_0\sigma E)\\
&= \mu_0\epsilon_0\partial_t^2 E + \mu_0\sigma\partial_t E
\end{aligned}
$$

Plugging in $E_0\exp(i(kz-\omega t))$, we obtain

$$k^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega \tag{19}$$

Then our solution becomes

$$\exp(-\kappa z)\exp(i(kz-\omega t)) \tag{20}$$