PHYS 486: Lecture # 31

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Exchange Force

For distinguishable particles, the wave function is

$$\Psi = \psi_a(r_1)\psi_b(r_2) \tag{1}$$

But for indistinguishable particles, the wave function is

$$\Psi_\pm = A(\psi_a(r_1)\psi_b(r_2) \pm \psi_a(r_2)\psi_b(r_1)) \tag{2}$$

Where $+$ is a boson and $-$ is a fermion. We compute

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1 \rangle^2 + \langle x_2 \rangle^2 + \langle x_1 x_2 \rangle \tag{3}$$

For a boson, this is

$$\langle (\Delta x)^2 \rangle - 2|\langle x \rangle_{ab}|^2 \tag{4}$$

For a fermion, this is

$$\langle (\Delta x)^2 \rangle + 2|\langle x \rangle_{ab}|^2 \tag{5}$$

Where

$$|\langle x \rangle_{ab}| = \int x\psi_a\psi_b^* dx \tag{6}$$

Note that this term only appears if there is an overlap between ψ_a and ψ_b. Therefore, for a boson, the particles are a bit closer than distinguishable particles and vice versa for fermions.