PHYS 325: Lecture 7

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Lecture Span

- Bead on a whirling rod

Bead on a whirling rod

Given

1. Rod is spinning with angular velocity ω
2. Thus $\phi(t) = \omega t$
3. Solve for $\vec{r}(t)$

Strategy

1. Draw a sketch
2. Choose coordinates
3. Write out positions
4. Velocity and acceleration vectors
5. Plug into $F = ma$
6. Solve Diff Eq

Solving

$$a(t) = \left[\ddot{r} - r\dot{\phi}^2\right] e_r + \left[r\ddot{\phi} + 2\dot{r}\dot{\phi}\right] \tag{1}$$

Plug into $F = ma$

$$F_n \vec{e_\phi(t)} = m\left[\ddot{r} - r\dot{\phi}^2\right] e_r + m\left[r\ddot{\phi} + 2\dot{r}\dot{\phi}\right] e_\phi \tag{2}$$

Thus

$$e_r : 0 = m\left[\ddot{r} - r\dot{\phi}^2\right] \tag{3}$$

$$e_\phi : F_n = m\left[r\ddot{\phi} + 2\dot{r}\dot{\phi}\right] \tag{4}$$

For e_r,

$$0 = \ddot{r} - r\ddot{\phi} \tag{5}$$

$$\ddot{r} = r\omega^2 \tag{6}$$

Let $r = e^{\lambda t}$

$$\lambda^2 = \omega^2 \iff \lambda = \pm\omega \tag{7}$$

$$r(t) = Ae^{\lambda t} + Be^{-\lambda t} \tag{8}$$

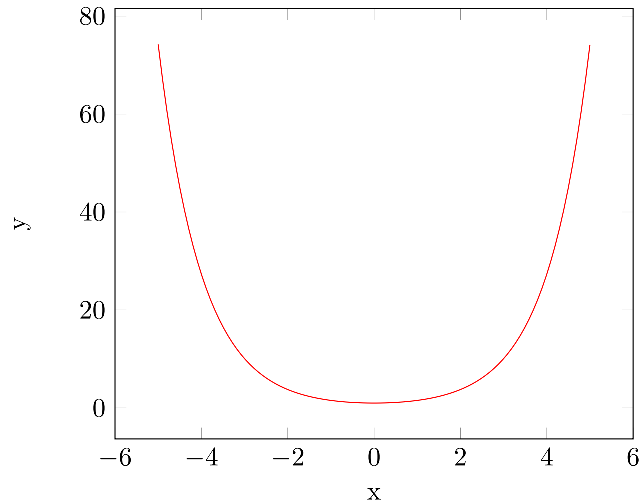
Let

$$r(t=0) = r_0 \wedge v(t=0) = v_0 \tag{9}$$

$$A = \frac{1}{2}(r_0 + \frac{v_0}{\omega}) \tag{10}$$

$$B = \frac{1}{2}(r_0 - \frac{v_0}{\omega}) \tag{11}$$

$r(t)$

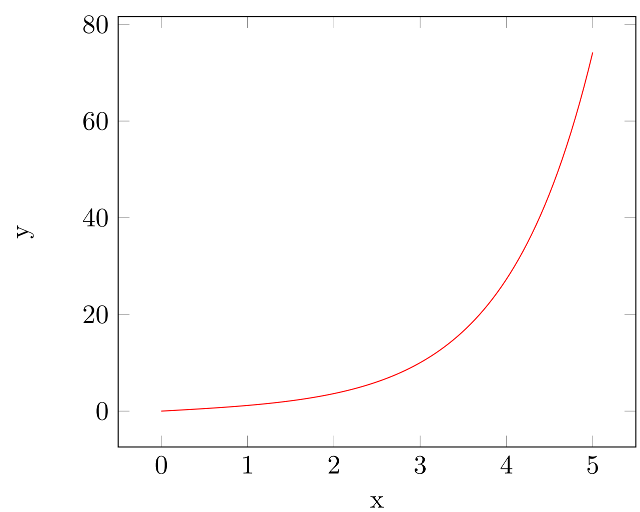


$$F_n = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \tag{12}$$

$$\frac{F_n}{2m\omega} = \dot{r} \tag{13}$$

$$\frac{F_n}{2m\omega^3} = \sinh(\omega t) \tag{14}$$

Force in a spinning rod



Fixed Bead on a spinning loop

Loop of fixed radius R, spinning about its vertical axis at rate Ω. Bead of mass m, free to move in large loop, in gravitational field. Remember, our basis vectors are time-dependent, using spherical coordinates. We have that

$$e_r = \sin\theta\cos\phi e_x + \sin\theta\sin\phi e_y + \cos\theta e_z \tag{15}$$

$$e_\theta = \cos\theta\cos\phi e_x + \cos\theta\sin\phi e_y - \sin\theta e_z \tag{16}$$

$$e_\phi = \sin\phi e_x + \cos\phi e_y \tag{17}$$

Given

$$r(t) = R = \text{const} \tag{18}$$

$$\phi(t) = \Omega t \tag{19}$$

$$\vec{r}(t) = r(t) e_r(t) \tag{20}$$

$$v(t) = \dot{r} e_r + r \dot{e_r} \iff r\dot{\theta} e_\theta + r\Omega \sin\theta e_\phi \tag{21}$$

$$a(t) = \text{ hell nah} \tag{22}$$