7.2:7.3

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Theorem 0.1. *A relation from A to B is a subset*

$$R \subseteq A \times B \tag{1}$$

In other words

$$aRb \iff (a, b) \in R \tag{2}$$

Theorem 0.2. *A function is a relation that satisfies the vertical line test.*

$$f : A \to B \tag{3}$$

is the statement that for every a in the domain of f, there exists some value b such that (a, b) *passes the vertical line test.*

The image is a function from P(A) to P(B). Similarly, the preimage/inverse image is from P(B) to P(A).

In notion it would be

$$f(U) = \{f(u) : u \in U\} \tag{4}$$

$$f^{-1}(V) = \{a \in A : f(a) \in V\} \tag{5}$$

respectively. Such that U is a set of A and V is a set of B. Notice how these functions spit out a set.

What does it mean for a function to be equal to each other?

1. Same domain, same codomain, and graph
2. Same domain and same graph, different codomains

7.3: Equivalence Relations

Assume

$$R = \{(a, b) : a \equiv b \mod n\} \tag{6}$$

Then $a \equiv a \mod n$, $a \equiv b \implies b \equiv a$, $a \equiv b \equiv c \implies a \equiv c$.

1. R is reflexive if a R a for a in A
2. R is symmetric if a R b $\implies$ b R a
3. R is transitive if a R b $\wedge$ b R c $\implies$ a R c

An equivalence relation satisfies all three requirements. They tell us that a & b have something in common.