PHYS 486: Lecture # 17

Cliff Sun

September 30, 2025

Basic Rules

1. physical state $|\psi\rangle$ in Hilbert Space. Must be normalizable
2. Observables with real eigenvalues $\hat{A}|\alpha_n\rangle = a_n|\alpha_n\rangle$
3. Born's rule: $Pr(a_n) = |\langle\alpha_n|\psi\rangle|^2$
4. Expectation values: $\hat{A} = \left\langle\psi\middle|\hat{A}\psi\right\rangle$
5. Stationary states: $\hat{H}\psi = E\psi$
6. Time evolution of the stationary states: $\psi_n \to \psi_n \exp(-iE_n/\hbar t)$

Typical Problems (cont. and dis. basis)

1. Validity of states, is it normalized, real eigenvalues of observables
2. Bases: inner product, write states in eigenbasis of some operator
3. Basis transformation operator:
$$\sum_k |k\rangle\langle k| \tag{1}$$
4. Probabilities for measurement outcomes. Statistics for measurement outcome.
$$\langle A\rangle,\ \sigma_A^2 \tag{2}$$

Wave mechanics

Continuous basis $|x\rangle$, $|p\rangle$. (1-dimensional)

1. SWE: $H = \frac{p^2}{2m} + V(x) \iff -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \hat{V}(x)$
2. Inner product: $\langle\psi|\varphi\rangle = \int dx \psi^*(x)\varphi(x)$
3. Actual problem: choose $V(x)$. Need ψ, ψ' continuous. If $V \to \infty$, then only ψ needs to be continuous.

Typical Tasks

1. Given some potential, find s.s. and energies.
2. Find $\langle x\rangle$, $\langle p\rangle$, σ_x, σ_p. Given some $\psi(x)$.
3. Probabilities for "range" of outcomes.

Specific problems

1. Free Particle, $V = 0$. The wave function must be continuous. SS. Plane waves are not physical. BUT, good basis.
2. Consequence: No such thing as a completely still standing of a free particle.
3. Infinite square well: Only has bound states (states that are trapped in this well) $|k\rangle$ and $E_k \propto k^2$
4. Finite square well: Both ψ and ψ' need to be continuous.