PHYS 325: Lecture 17

Cliff Sun

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Harmonic Force

EOM:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\omega t - \theta) \tag{1}$$

Solution:

$$x(t) = x_h(t) + x_p(t) \tag{2}$$

$$x_p(t) = F_0 G(\omega) \cos(\omega t - \theta - \phi(\omega)) \tag{3}$$

Note that

$$G(\omega) = \frac{1}{k}\left[(1 - (\frac{\omega}{\omega_n}^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2)\right]^{-\frac{1}{2}} \tag{4}$$

Phase is

$$\phi(\omega) = \arctan(\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}) \tag{5}$$

Analyze $x_p(t)$

$x_p(t)$ has a resonance frequency, such that the amplitude of the particular solution is maximized. Thus,

$$D_w(G(\omega_0)) = 0 \tag{6}$$

Thus,

$$w_0 = \pm\omega_n\sqrt{1 - 2\zeta^2} \tag{7}$$

Periodic Force & Fourier Series

Periodic Force:

$$F(t + T) = F(t) \tag{8}$$

with period $T = \frac{2\pi}{\Omega}$. We can describe the solution as a sum of harmonic functions.

Fourier Series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\Omega t) = \sum_{p=1}^{\infty} b_p \sin(p\Omega t) \tag{9}$$

Note

- $n, p \in \mathbb{N}$

with

$$a_0 = \frac{2}{T}\int_0^T F(t)dt \tag{10}$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos(n\Omega t)dt \tag{11}$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\Omega t)dt \tag{12}$$