MTH 447: Lecture 9

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February 10, 2025

Non-convergent Sequences

Definition 0.1. *The sequence* (x_n) *diverges to* ∞ *if* $\forall M \in \mathbb{N}$, $\exists N$ *such that*

$$n > N \implies x_n > M \tag{1}$$

Equivalently, let (x_n) be a sequence. For any M, define

$$S_M = \{n \in \mathbb{N} \big| x_n \leq M\} \tag{2}$$

But $x_n \to \infty \iff \forall M \in \mathbb{R}$, S_m is finite.

Theorem 0.2. *If* $x_n \to \infty$*, and* $y_n \geq x_n$*, for all* n*, then* $y_n \to \infty$*.*

Definition 0.3. *We say that* $x_n \to -\infty$*, if* $\forall M \in \mathbb{R}$, $\exists N$ *such that*

$$n > N \implies x_n < M \tag{3}$$

Theorem 0.4. *Limit theorems for infinity.*

1. *If* $x_n \to \infty$*, and* $y_n \to L$*, then* $x_n + y_n \to \infty$*.*
2. *If* $x_n \to \infty$*, and* $y_n \to \infty$*, then* $x_n + y_n \to \infty$
3. *If* $x_n \to \infty$ *and* $y_n \to L$*, then* $x_n y_n \to \infty$
4. *If* $x_n \to \infty$ *and* $y_n \to \infty$*, then* $x_n y_n \to \infty$
5. $x_n > 0$, $x_n \to \infty \iff \frac{1}{x_n} \to 0$.