PHYS 325: Lecture 14

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Harmonic & damped motion

Simple harmonic oscillator

Using f = ma

Let $x(t)$ be time dependent location. And $x_0 = L$ be the equilibrium point. Then

$$m\ddot{x} = -k(x - L) \tag{1}$$

Letting $x' = x - L$, then

$$m\ddot{x'} = -kx' \tag{2}$$

The solution is

$$x(t) = A\cos(\omega t + \phi_0) \tag{3}$$

Using energy conservation

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x - L)^2 = c \tag{4}$$

$$\partial_t E = m\ddot{x}\dot{x} + k(x - L)\dot{x} = 0 \tag{5}$$

Thus

$$m\ddot{x} = -k(x - L) \tag{6}$$

Energy of Oscillations

$$U = \frac{1}{2}ky^2 \iff \frac{1}{2}kC^2\cos^2(\omega t - \phi) \tag{7}$$

Non negative.

$$T = \frac{m}{2}C^2\omega^2\sin^2(\omega t - \phi) \tag{8}$$

Thus total energy is

$$E = \frac{kC^2}{2} \tag{9}$$