

# PHYS 325: Lecture 14

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## Harmonic & damped motion

### Simple harmonic oscillator

Using  $\mathbf{f} = m\mathbf{a}$

Let  $x(t)$  be time dependent location. And  $x_0 = L$  be the equilibrium point. Then

$$m\ddot{x} = -k(x - L) \quad (1)$$

Letting  $x' = x - L$ , then

$$m\ddot{x}' = -kx' \quad (2)$$

The solution is

$$x(t) = A \cos(\omega t + \phi_0) \quad (3)$$

### Using energy conservation

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x - L)^2 = c \quad (4)$$

$$\partial_t E = m\ddot{x}\dot{x} + k(x - L)\dot{x} = 0 \quad (5)$$

Thus

$$m\ddot{x} = -k(x - L) \quad (6)$$

## Energy of Oscillations

$$U = \frac{1}{2}ky^2 \iff \frac{1}{2}kC^2 \cos^2(\omega t - \phi) \quad (7)$$

Non negative.

$$T = \frac{m}{2}C^2\omega^2 \sin^2(\omega t - \phi) \quad (8)$$

Thus total energy is

$$E = \frac{kC^2}{2} \quad (9)$$