$\{y\}_{i=1}^{N} \xrightarrow{f(\ )} \{z\}_{i=1}^{N}$ $\boxed{z_i = \sqrt{y_i}}$

prediction

even n

$f(x) = \sqrt[n]{x}$ $x \geq 0$

$\sqrt[n]{y_i}$ n is odd $\forall y_i$

reverse transformation

time ⟶ Freq. domain

$\{y\} \xrightarrow{\log(\ )} \{z\}_{i=1}^{N}$ $z_i = \log(y_i)$

$y_i > 0$

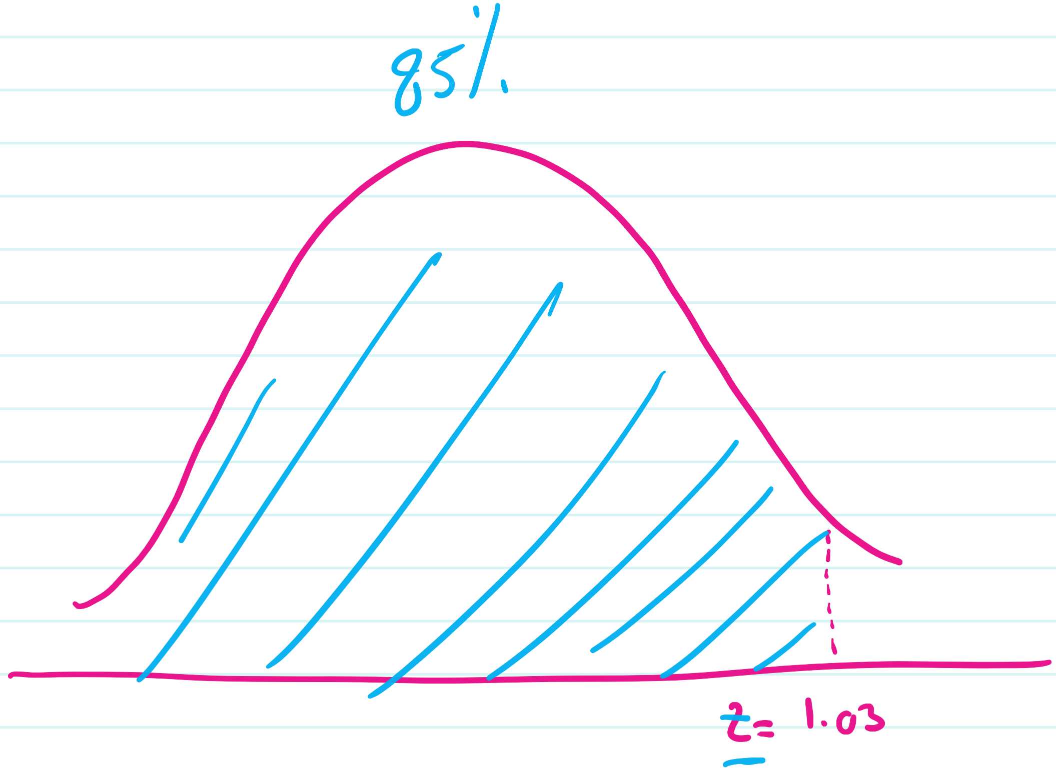
$\{y\} \xrightarrow{\ln(y)} \{z\}_{i=1}^{N}$ $z_i = \ln y_i$ $\forall y_i > 0$

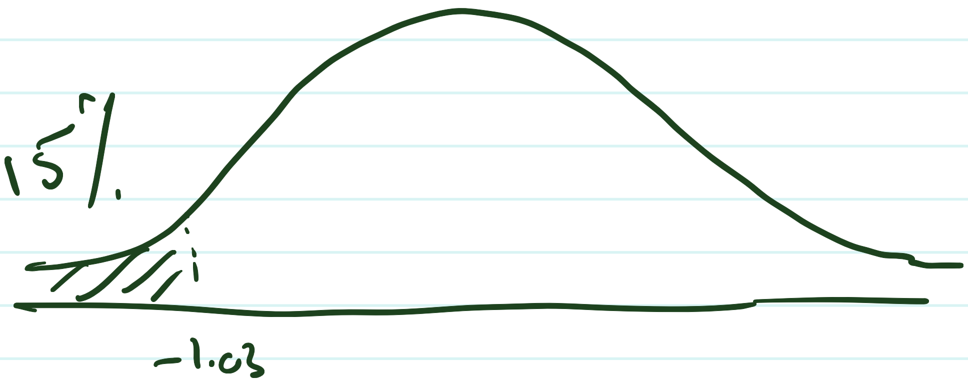
$$\hat{y}_{T+1} = e^{\hat{z}_{T+1}} \longleftarrow \hat{z}_{T+1}$$

$$\hat{y}_{T+2} = e^{\hat{z}_{T+2}} \longleftarrow \hat{z}_{T+2}$$

$$\vdots$$

$$\hat{y}_{T+h} = e^{\hat{z}_{T+h}} \longleftarrow \hat{z}_{T+h}$$





mean = 194

std = 11.2

Low = 175

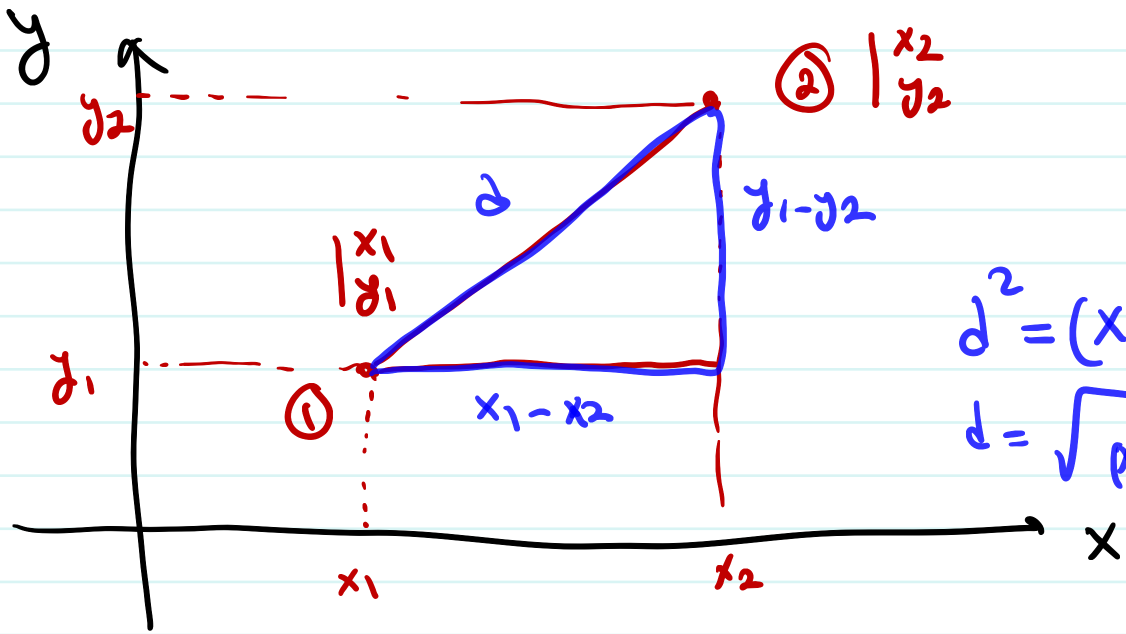
High = 225

$P(175 < W < 225) = 99.7\% - 4.5\%$

$= 95.2\%$

$$z_{low} = \frac{175 - 194}{11.2} = -1.69$$

$$z_{high} = \frac{225 - 194}{11.2} = 2.76$$



$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times1} \qquad y = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 5 \end{bmatrix}_{4\times1}$$

$$d = \sqrt{(1+1)^2 + (2-2)^2 + (3)^2 + (4-5)^2}$$

$$= \sqrt{4+9+1} = \sqrt{14}$$

Edu.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times1} \qquad \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n\times1}$$

$$\text{distance} = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + \cdots + (x_n-y_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

dot product

$$x^T \cdot y = [x_1 \;\; x_2 \;\; \cdots \;\; x_n] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \boxed{x_1 y_1 + x_2 y_2 + \cdots + x_n y_n}$$

$\rightarrow$ $\underline{x^T \cdot x} = x_1^2 + x_2^2 + \cdots + x_n^2 = ||x||^2$

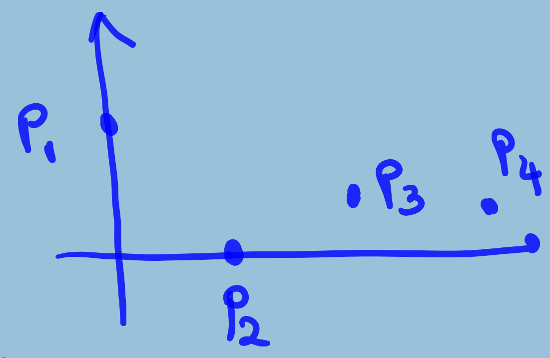
$\rightarrow$ $y^T \cdot y = y_1^2 + y_2^2 + \cdots + y_n^2 = ||y||^2$

$$\boxed{x^T \cdot y = 0 \rightarrow x \perp y}$$

- Let two vectors x & y to be defined as below.

point	x	y
p_1	0	2
p_2	2	0
p_3	3	1
p_4	5	1

4x2



- Calculate L_1, L_2 & L_∞ distances.

L_1	p_1	p_2	p_3	p_4
p_1	0	4	4	6
p_2	4	0	2	4
p_3	4	2	0	2
p_4	6	4	2	0

L_2	p_1	p_2	p_3	p_4
p_1	0	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{26}$
p_2	$\sqrt{8}$	0	$\sqrt{2}$	$\sqrt{10}$
p_3	$\sqrt{10}$	$\sqrt{2}$	0	2
p_4	$\sqrt{26}$	$\sqrt{10}$	2	0

L_∞	p_1	p_2	p_3	p_4
p_1	0	2	3	5
p_2	2	0	1	3
p_3	3	1	0	2
p_4	5	3	2	0

$L_1 = \sum |x_k - y_k|$

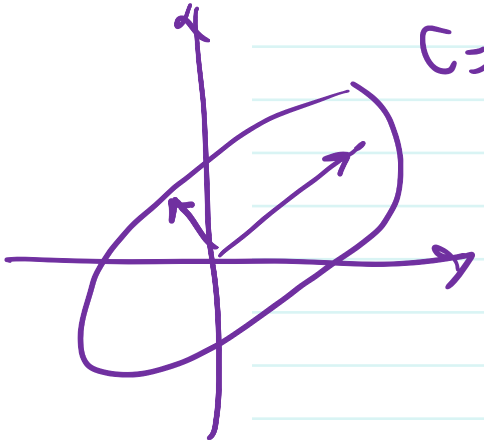
$L_2 = \sqrt{\sum |x_k - y_k|^2}$

$$\boxed{L_\infty = \max(|x_1 - y_1|, |x_2 - y_2|, \ldots, |x_k - y_k|)}$$

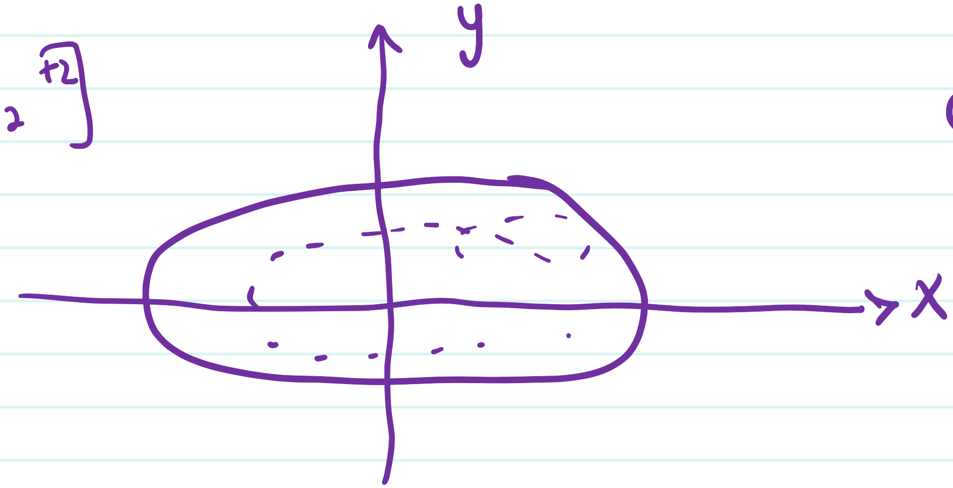
L_∞ for $P_1 - P_2$: $\max(|0-2|, |2-0|)$

$\max(2, 2) = 2$

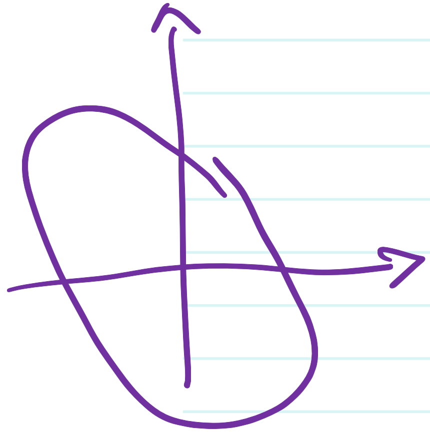
$P_1 - P_3$: $\max(|0-3|, |2-1|) = 3$

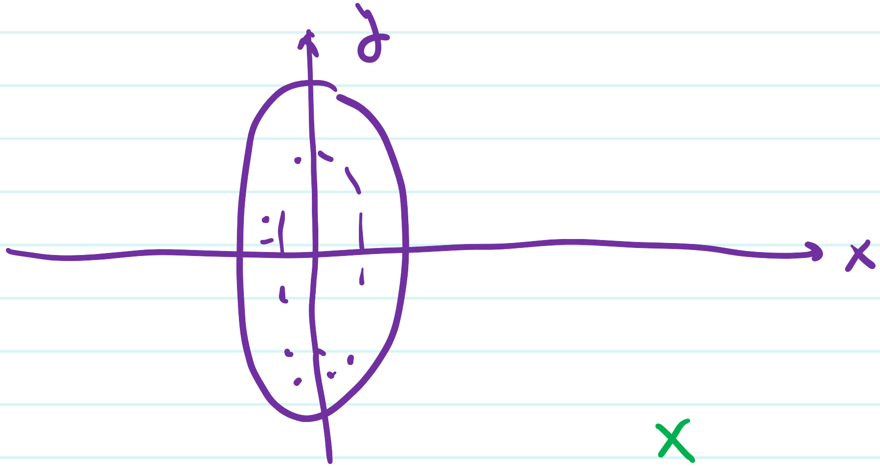


$C = \begin{bmatrix} & +2 \\ +2 & \end{bmatrix}$



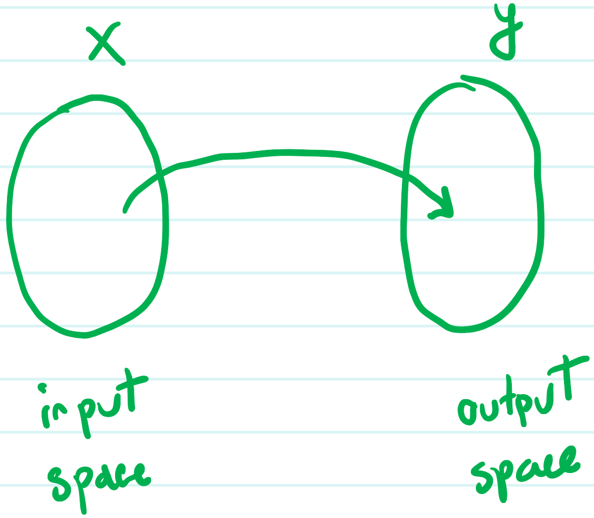
$C = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$





$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

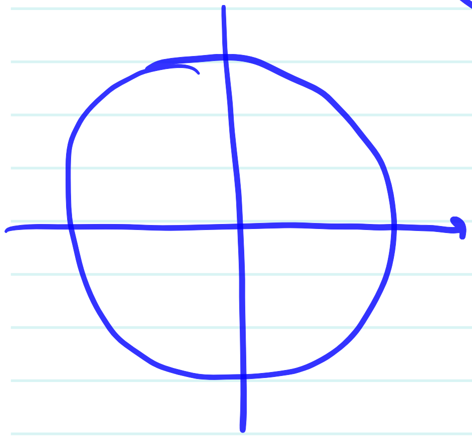
$y = f(x)$



$$y = \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_d x_d$$

$$\underset{n\times 1}{=} \underset{n\times d}{\begin{bmatrix} \beta_1 & \cdots & \beta_d \end{bmatrix}} \underset{d\times 1}{\begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}}$$

$$= A \cdot X$$



$$\xrightarrow{Ax}$$

$\Sigma_{d\times d}$ eigen space decompusion:

$$\Sigma \cdot v = \lambda v \rightarrow$$

$$\Sigma \cdot v - \lambda v = 0$$

$$(\Sigma - \lambda I)\, v = 0 \rightarrow |\Sigma - \lambda I| = 0$$

$$\Sigma = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \rightarrow \text{Find eigen values, eign vectors:}$$

$$|\Sigma - \lambda I| = 0 \rightarrow \begin{vmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{vmatrix} = 0$$

$$(-6-\lambda)(5-\lambda) - 12 = 0$$

$$(\lambda+6)(\lambda-5) - 12 = 0$$

$$\lambda^2 + \lambda - 30 - 12 = 0$$

$$\rightarrow \lambda^2 + \lambda - 42 = 0 \rightarrow (\lambda - 6)(\lambda + 7) = 0$$

$\lambda_1 = 6 \rightarrow$ eign vector $\begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$

$\lambda_2 = -7 \rightarrow \begin{bmatrix} -1 \\ 1/3 \end{bmatrix}$

$$\Sigma v = 6v$$

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6x \\ 6y \end{bmatrix}$$

$$\begin{cases} -6x + 3y = 6x \\ 4x + 5y = 6y \end{cases} \rightarrow 12x = 3y \rightarrow y = 4x$$

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7x \\ -7y \end{bmatrix}$$

$$\begin{cases} -6x + 3y = -7x \\ 4x + 5y = -7y \end{cases} \rightarrow \begin{array}{l} -x = 3y \\ 4x = -12y \rightarrow \end{array}$$

$$\boxed{x = -3y}$$

X $\quad \Sigma x = y$