* March 13^{th} Exam I : Monday in _ class

on _ paper _

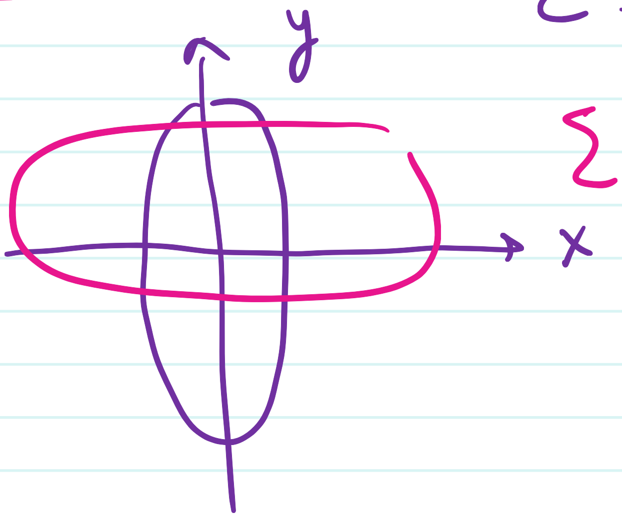
* Term project _

$y = \{1, 2, 3\}$

$\bar{y} = \frac{1+2+3}{3}$

$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\Sigma = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$



$$x = (3, 2, 0, 5, 0, 0, 0, 2, 0, 0)$$
$$y = (1, 0, 0, 0, 0, 0, 0, 1, 0, 2)$$

$\cos(x,y) = x'^T \cdot y'$

$x' = \frac{x}{\|x\|}$, $y' = \frac{y}{\|y\|}$

$$\cos(x,y) = \frac{\langle x, y\rangle}{\|x\| \cdot \|y\|} = \frac{x^T \cdot y}{\sqrt{x^T \cdot x} \cdot \sqrt{y^T \cdot y}} = \frac{3+2}{\sqrt{9+4+25+4} * \sqrt{1+1+4}}$$

$$= \frac{5}{\sqrt{42} * \sqrt{6}} = \boxed{31.4\%}$$

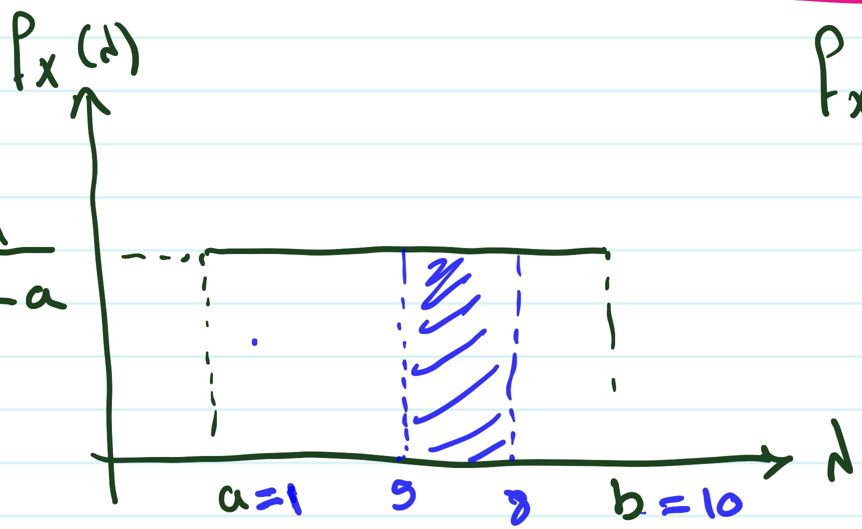
$$\begin{bmatrix} 3, & 2, & 0, & 5, & 0, & 0, & 0, & 2, & 0, & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} =$$

$$EJ(x,y) = \frac{\langle x, y\rangle}{\|x\|^2 + \|y\|^2 - \langle x, y\rangle} = \frac{x^T y}{\|x\|^2 + \|y\|^2 - x^T y}$$

$$= \frac{5}{42 + 6 - 5} = \frac{5}{43} = \boxed{11.6\%}$$

$f_X(\lambda)$ = density function

$$\frac{1}{9} = \frac{1}{b-a}$$



$$P(5 < \lambda < 8) = 3 \times \frac{1}{9} = \frac{1}{3}$$

$$= 33\%$$

Let X be a continuous random variable with the following pdf:

$$f_X(\lambda) = \begin{cases} ce^{-\lambda}, & \lambda \geq 0 \\ 0, & \text{Else} \end{cases}$$

1. Find c.
2. Find the probability distribution function $F_X(\lambda)$ $\lim\limits_{\epsilon \to 0} \int_{2-\epsilon}^{2} f_X(\lambda)d\lambda \to 0$
3. Find $P(1 < X \leq 3)$
4. Find $P(X = 2)$ ≈ 0 $\lim\limits_{\epsilon \to 0} P(2-\epsilon < X < 2)$
5. Find $P(X \in [0, 1] \cup [3, 4])$

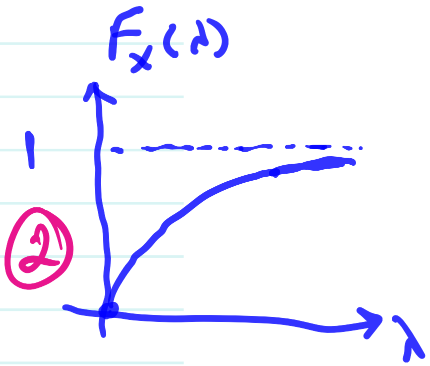
① $\int_{-\infty}^{\infty} f_X(\lambda)d\lambda = 1$ $\int_{0}^{\infty} ce^{-\lambda} d\lambda = 1 \to$

$$\frac{ce^{-\lambda}}{-1}\Big|_0^{\infty} = 1 \rightarrow -ce^{-\infty} + ce^{0} = 1$$

$$0 + \boxed{c=1}$$

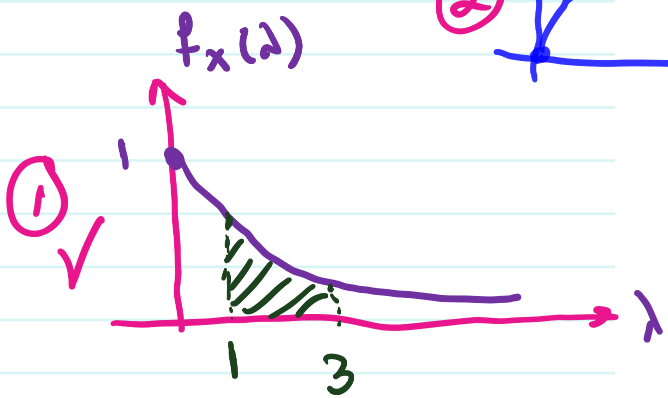
② $$F_x(\lambda) = \int_{-\infty}^{\lambda} f_x(\mu)\,d\mu = \int_0^{\lambda} e^{-\mu}\,d\mu$$

$$= \frac{e^{-\mu}}{-1}\Big|_0^{\lambda} = -e^{-\lambda} + e^{0} = 1 - e^{-\lambda}$$



$$\boxed{F_x(\lambda) = 1 - e^{-\lambda}}$$

$$f_x(\lambda) = \begin{cases} e^{-\lambda} & : \quad \lambda > 0 \\ 0 & : \quad \text{Else} \end{cases}$$



$$P(1 < X < 3) = \int_1^3 f_X(\lambda)\,d\lambda = \int_1^3 e^{-\lambda}\,d\lambda = \frac{e^{-\lambda}}{-1}\Big|_1^3$$

$$= -e^{-3} + e^{-1} = \boxed{e^{-1} - e^{-3}}$$

$$P(a < X < b) = F_X(b) - F_X(a) =$$

$$= 1 - e^{-3} - (1 - e^{-1}) = \boxed{e^{-1} - e^{-3}}$$

$$P(0 < X < 1) \cup P(3 < X < 4)$$

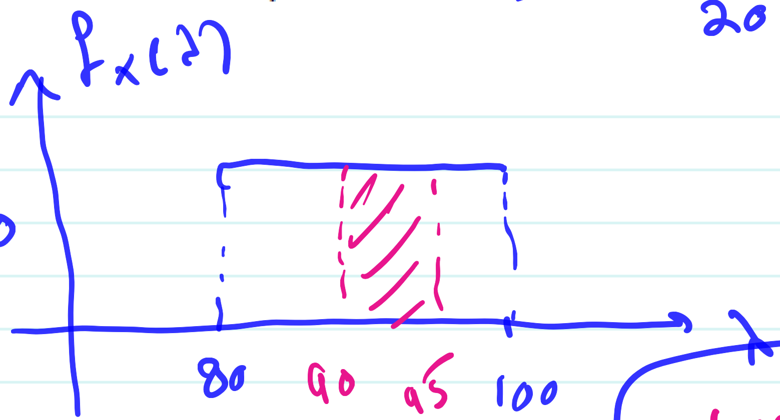
$$= \int_0^1 e^{-\lambda}\,d\lambda + \int_3^4 e^{-\lambda}\,d\lambda = \frac{e^{-\lambda}}{-1}\Big|_0^1 + \frac{e^{-\lambda}}{-1}\Big|_3^4$$

$$= -e^{-1} + e^{0} - e^{-4} + e^{-3} = \boxed{1 - e^{-1} - e^{-4} + e^{-3}}$$

Let X be a continuous random variable that is equally likely to be any valye between 80 and 100.

1. Graph the corresponding probability density function.
2. Find $P(90 < X \leq 95)$ $= \frac{5}{20} = 25\%$

$$f_X(\lambda) = \begin{cases} \frac{1}{20}, & 80 < \lambda < 100 \\ 0 : & \text{Else} \end{cases}$$



$$E[X] = \int_{-\infty}^{\infty} \lambda f_X(\lambda)\, d\lambda$$

$$\boxed{Var(X) = E[(X - \mu_X)^2] = \int_{-\infty}^{\infty} (\lambda - \mu_X)^2 f_X(\lambda)\, d\lambda}$$

$$\boxed{E[X]} = \int_{80}^{100} \frac{\lambda}{20}\, d\lambda = \frac{\lambda^2}{40}\Big|_{80}^{100} = \frac{100^2}{40} - \frac{80^2}{40} = \frac{10000 - 6400}{40} = \frac{3600}{40} = 90$$

$$\boxed{Var[X]} = \int_{80}^{100} \frac{(\lambda - 90)^2}{20}\, d\lambda = \int_{80}^{10} \frac{\lambda^2 - 180\lambda + 8100}{20}\, d\lambda$$

$$= \frac{1}{20}\left(\frac{\lambda^3}{3} - \frac{180\lambda^2}{2} + \frac{8100\lambda}{1} \right)\Big|_{80}^{100}$$

$$= \frac{1}{20}\left(\frac{100^3}{3} - \frac{180 \times 100^2}{2} + 810000 - \frac{80^3}{3} + \frac{180 \times 80^2}{2} - 8100 \times 80\right)$$

$$\boxed{Var(x) = 33.33}$$

$$\int a x^n \, dx = \frac{a x^{n+1}}{n+1}$$

$$\int a e^{-cx} \, dx = \frac{a e^{-cx}}{-c}$$

$$cov = \frac{\sum (x - \bar{x})(y - \bar{y})}{n-1}$$

$$\boxed{Var(x) = \frac{\hat{\sum}_{i=1} (x - \bar{x})^2}{n-1}} \quad , Var(x) = \int_{-\infty}^{\infty} (\lambda - \mu_x)^2 f_x \, d\lambda$$

Let X and Y be two jointly continuous random variables (uniformly distributed) with joint probability density function:

$$f_{XY}(\lambda_1, \lambda_2) = \begin{cases} \lambda_1 + c\lambda_2^2, & 0 \le \lambda_1 \le 1, 0 \le \lambda_2 \le 1 \\ 0, & \text{Else} \end{cases}$$

1. Find c.
2. Find $P(0 < X \le \frac{1}{2}, 0 < Y \le \frac{1}{2})$

λ_2 is changing
λ_1 is constant

λ_1 to change
λ_2 constant

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{xy}(\lambda_1, \lambda_2)\, d\lambda_1\, d\lambda_2 = 1$$

$$\int_0^1 \int_0^1 (\lambda_1 + c\lambda_2^2)\, d\lambda_1\, d\lambda_2 = 1$$

$$\int_0^1 \left(\frac{\lambda_1^2}{2} + c\lambda_2^2 \lambda_1 \right) \Big|_0^1 \, d\lambda_2 = 1$$

$$\int_0^1 \left(\frac{1}{2} + c\lambda_2^2 \right) d\lambda_2 = 1$$

$$\frac{\lambda_2}{2} + \frac{c\lambda_2^3}{3} \Big|_0^1 = 1 \rightarrow \frac{1}{2} + \frac{c}{3} = 1$$

$$\frac{c}{3} = \frac{1}{2} \rightarrow \boxed{c = 3/2}$$

$$P\left(0 < X < \tfrac{1}{2},\, 0 < Y < \tfrac{1}{2}\right) = \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} \left(\lambda_1 + \frac{3}{2}\lambda_2^2\right) d\lambda_1 \, d\lambda_2$$

$$= \int_0^{\frac{1}{2}} \left(\frac{\lambda_1^2}{2} + \frac{3}{2}\lambda_2^2 \lambda_1 \right) \Big|_0^{\frac{1}{2}} d\lambda_2$$

$$= \int_0^{\frac{1}{2}} \left(\frac{1}{8} + \frac{3}{4} \cdot \lambda_2^2\right) d\lambda_2$$

Let X and Y be two jointly continuous random variables with the joint probability density function:

$$f_{XY}(\lambda_1, \lambda_2) = \begin{cases} c\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3}, & 0 \leq \lambda_1 \leq 1, 0 \leq \lambda_2 \leq 2 \\ 0, & \text{Else} \end{cases}$$

1. Find c.
2. Find $P(X + Y \geq 1)$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(c\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3}\right) d\lambda_1 \, d\lambda_2 = 1 \rightarrow \int_0^2 \int_0^1 \left(c\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3}\right) d\lambda_1 \, d\lambda_2 = 1$$

$$= \int_0^2 \left(\frac{c\lambda_1^3}{3} + \frac{\lambda_1^2 \lambda_2}{6}\right)\Big|_0^1 d\lambda_2 = 1$$

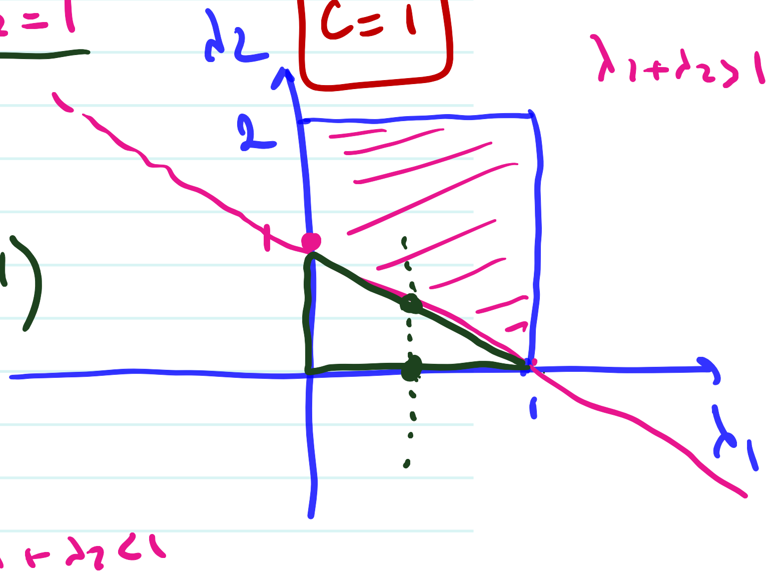
$$\Rightarrow \int_0^2 \left(\frac{c}{3} + \frac{\lambda_2}{6}\right) d\lambda_2 = 1$$

$$\left(\frac{c\lambda_2}{3} + \frac{\lambda_2^2}{12}\right)\Big|_0^2 = 1$$

$$\frac{2c}{3} + \frac{\cancel{4}^{\,1}}{\cancel{12}_{\,3}} = 1 \rightarrow \frac{2c}{3} = \frac{2}{3}$$

$$\boxed{c = 1}$$

$\lambda_1 + \lambda_2 = 1$



$$P(\lambda_1 + \lambda_2 \geq 1) = 1 - P(\lambda_1 + \lambda_2 < 1)$$

$\lambda_1 + \lambda_2 = 1$

$$P(\lambda_1 + \lambda_2 \geq 1) = 1 - \int_0^1 \int_0^{1-\lambda_1} \left(\lambda_1^2 + \frac{\lambda_1 \lambda_2}{3}\right) d\lambda_2$$

$$= 1 - \int_0^1 \left(\lambda_1^2 \lambda_2 + \frac{\lambda_1 \lambda_2^2}{6}\right)\Bigg|_0^{1-\lambda_1} d\lambda_2$$

$1 - 2\lambda_1 + \lambda_1^2$

$$= 1 - \int \left(\lambda_1^2 (1-\lambda_1) + \frac{\lambda_1 (1-\lambda_1)^2}{6}\right) d\lambda_2 \quad \leftarrow$$

$$= 1 - \int_0^1 \left(\lambda_1^2 - \lambda_1^3 + \frac{\lambda_1}{6} - \frac{2\lambda_1^2}{6} + \frac{\lambda_1^3}{6}\right) d\lambda_2$$

$$= 1 - \left(\frac{\lambda_1^3}{3} - \frac{\lambda_1^4}{4} + \frac{\lambda_1^2}{12} - \frac{2\lambda_1^3}{18} + \frac{\lambda_1^4}{24}\right)\Bigg|_0^1$$

$$= 1 - \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{12} - \frac{1}{9} + \frac{1}{24}\right)$$

$$= 1 - \left(\frac{1}{6} - \frac{1}{9} + \frac{1}{24}\right) =$$

$$= 1 - \left(\frac{5}{24} - \frac{1}{9} \right) =$$