CS 5525 - Data Analytics I - Feb 27th

Let consider two random variables X and Y with the following joint probability density function:

$$f_{XY}(\lambda_1, \lambda_2) = \begin{cases} c, & -1 \le \lambda_1 \le 0, 0 \le \lambda_2 \le 1 \\ 0, & \text{Else} \end{cases}$$

2

$\boxed{E[y] > 0}$ ①

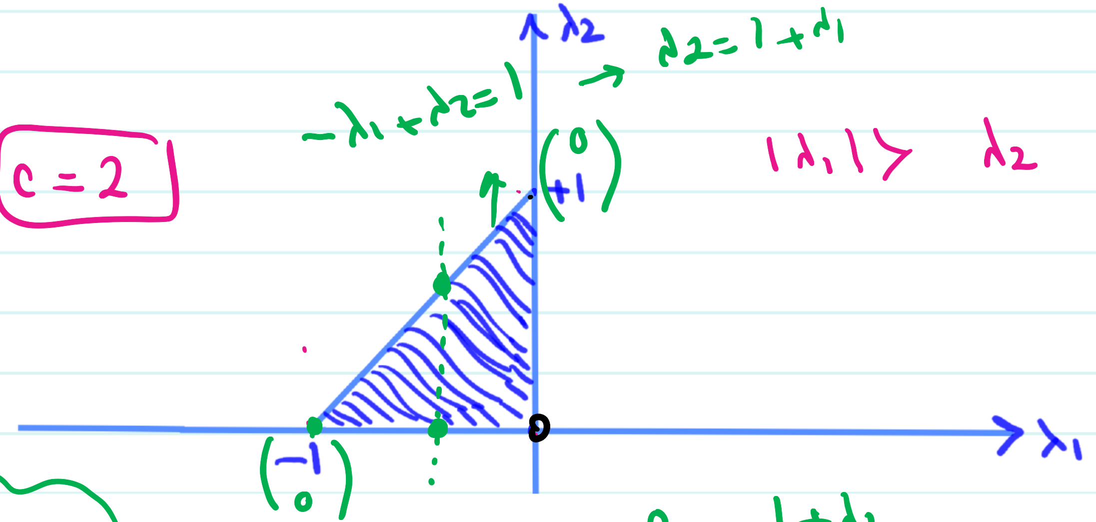
< 0 ②

$= 0$ ③

$\boxed{\text{volume} = 1}$

area * height = 1

$\frac{1}{2} \times c = 1 \rightarrow \boxed{c = 2}$



$E[x] > 0$ ①

$\boxed{< 0}$ ②

$= 0$ ③

$$\int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} c \, d\lambda_2}_{\lambda_2} \, d\lambda_1 = 1$$

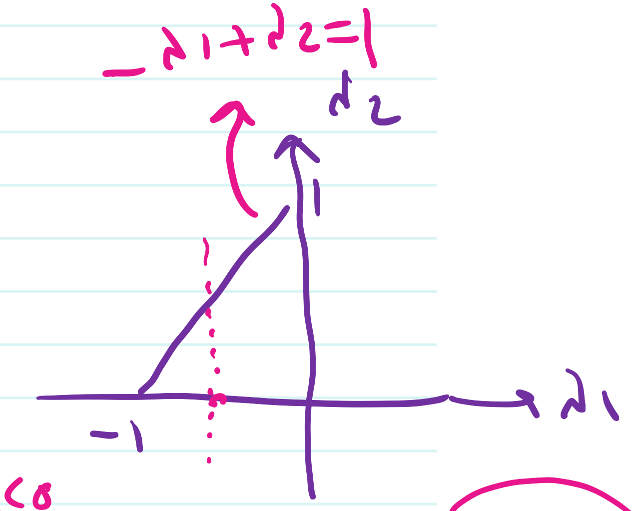
$$\int_{-1}^{0} \int_{0}^{1+\lambda_1} c \, d\lambda_2 \, d\lambda_1 = 1$$

$$\int_{-1}^{0} c(1+\lambda_1)\, d\lambda_1 = 1$$

$$\left(c\lambda_1 + \frac{c\lambda_1^2}{2}\right)\Bigg|_{-1}^{0} = 1$$

$$0 - \Big(\underbrace{-c + \frac{c}{2}}_{-c/2}\Big) = 1 \rightarrow \frac{c}{2} = 1 \rightarrow \boxed{c = 2}$$

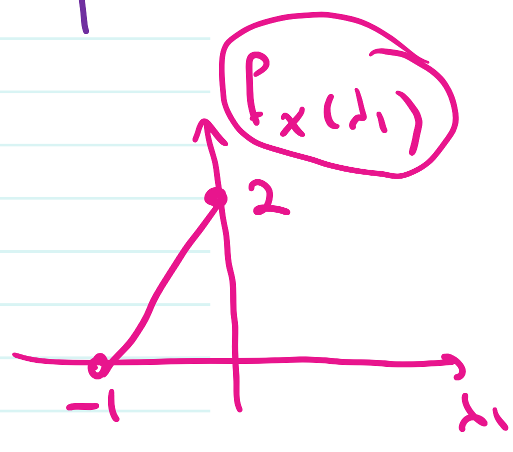
Finding Marginal density function



$$f_X(\lambda_1) = \int f_{X,Y}(\lambda_1, \lambda_2)\, d\lambda_2$$

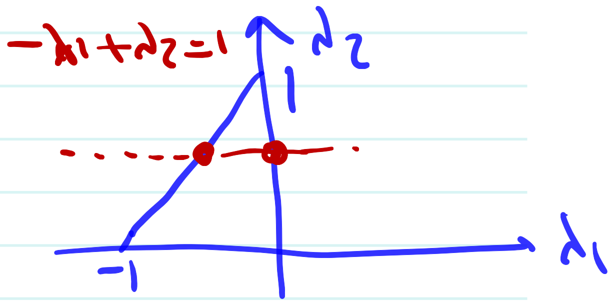
$$= \int_{0}^{1+\lambda_1} 2\, d\lambda_2 = 2(1+\lambda_1) \quad ; \quad -1 < \lambda_1 < 0$$

$$f_X(\lambda_1) = \begin{cases} \underline{2(1+\lambda_1)} & : \quad -1 < \lambda_1 < 0 \\ 0 & : \quad \text{Else} \end{cases}$$



Valid density?

$$\begin{cases} +\text{ive} \\ \text{area} \overset{?}{=} 1 \end{cases}$$

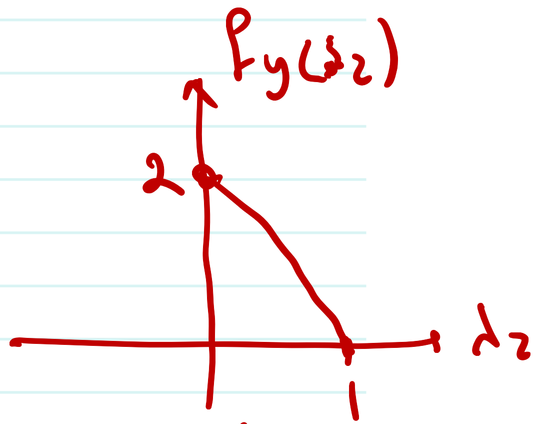


$$f_y(\lambda_2) = \int_{\lambda_2-1}^{0} 2 \, d\lambda_1$$

$$= 2(1-\lambda_2) : \quad 0<\lambda_2<1$$

$\lambda_2 = 0.5$

$$\boxed{f_y(\lambda_2) = \begin{cases} 2(1-\lambda_2) : & 0<\lambda_2<1 \\ 0 : & \text{Else} \end{cases}}$$



Valid $\begin{cases} +\text{ive} \\ \text{area} = 1 \end{cases}$

independency : $f_{x,y}(\lambda_1,\lambda_2) = f_x(\lambda_1) * f_y(\lambda_2)$

$$2 \overset{?}{=} 2(1+\lambda_1) * 2(1-\lambda_2) \quad \times$$

So X and Y are $\begin{cases} \text{not independent} \\ \text{dependent.} \end{cases}$

$$E[x] = \int \lambda_1 f_X(\lambda_1)\, d\lambda_1$$

$$= \int_{-1}^{0} 2\lambda_1 (1+\lambda_1)\, d\lambda_1 = \int_{-1}^{0} (2\lambda_1 + 2\lambda_1^2)\, d\lambda_1$$

$$= \left(\frac{2\lambda_1^2}{2} + \frac{2\lambda_1^3}{3}\right)\Bigg|_{-1}^{0}$$

$$= 0 - \left(1 - \frac{2}{3}\right) = \boxed{\frac{-1}{3} = E[x]}$$

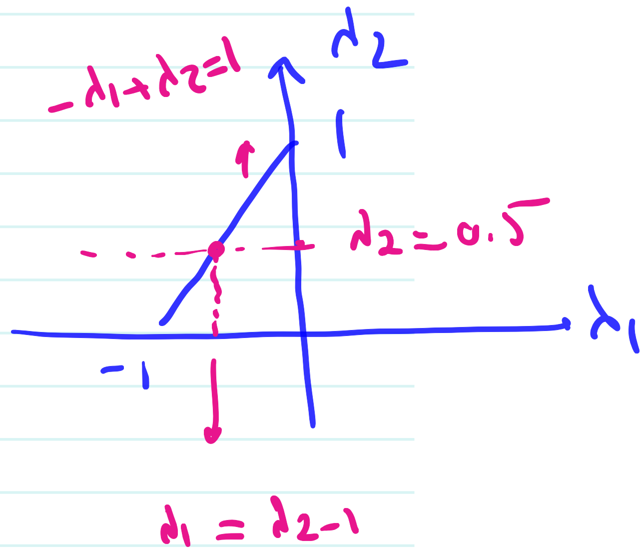
$$E[y] = \int \lambda_2 f_Y(\lambda_2)\, d\lambda_2 = \int_0^1 2\lambda_2 (1-\lambda_2)\, d\lambda_2$$

$$= \left(\frac{2\lambda_2^2}{2} - \frac{2\lambda_2^3}{3}\right)\Big|_0^1 = 1 - \frac{2}{3} = \boxed{1/3}$$

Conditional density function

$$\underbrace{f_{x|y}(\lambda_1 \mid \lambda_2)}_{\text{function of } \lambda_1} = \frac{f_{x,y}(\lambda_1, \lambda_2)}{f_y(\lambda_2)}$$

λ_2 is known

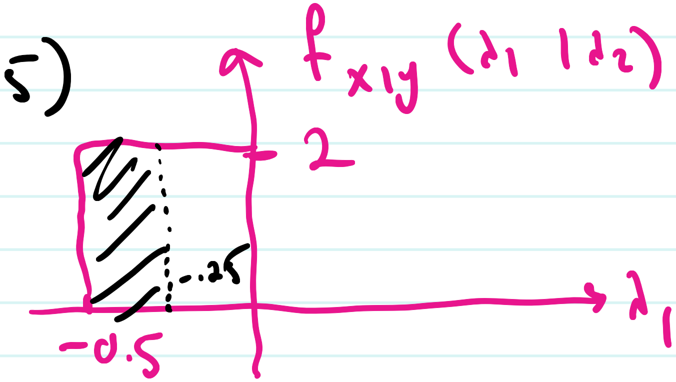


$$f_{x|y}(\lambda_1 \mid \lambda_2 = 0.5) = \frac{2}{f_y(\lambda_2 = 0.5)}$$

$$= \frac{2}{1} = 2$$

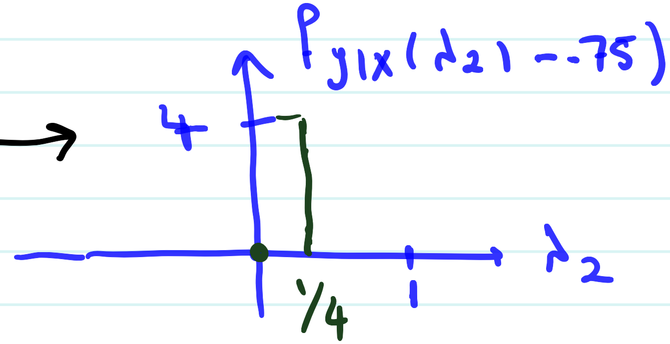
$P(-.5<\lambda_1<-.25 \mid \lambda_2=0.5)$

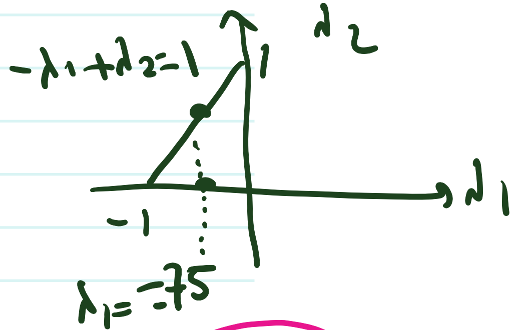
$= 0.5$ or 50%



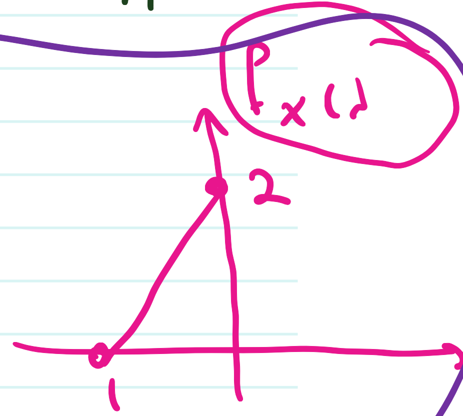
$\lambda_2 = 0.5$

$$f_{y|x}(\lambda_2 \mid \lambda_1=-0.75) = \frac{2}{f_x(\lambda_1=-0.75)} = \frac{\cancel{2}}{\cancel{2}(1-3/4)} = 4$$





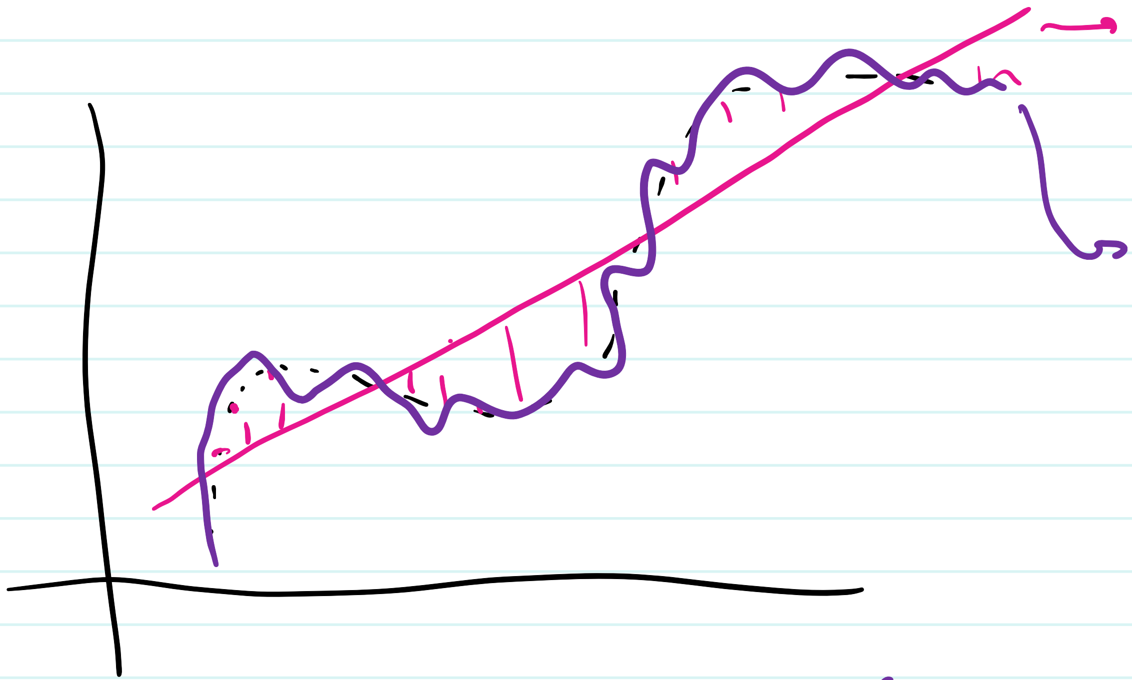
$$f_x(\lambda_1) = \begin{cases} 2(1+\lambda_1) & : -1<\lambda_1<0 \\ 0 & : \text{Else} \end{cases}$$



$P(0.5 < \underline{d_2} < .75 \mid d_1 = -0.75) = 0$

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1}$$

$$\int ce^{ax} \, dx = \frac{ce^{ax}}{a}$$



$x \to \epsilon$

y_c w

x_1

x_2

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & & & \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} \qquad x_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1p} \end{bmatrix}$$