$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$$x^T \cdot x = [x_1 \ \ x_2 \ \ \cdots \ \ x_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= x_1^2 + x_2^2 + \cdots + x_n^2$$

$$\underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \rightarrow \quad e^T \cdot e = e_1^2 + e_2^2 + \cdots + e_n^2 = \sum_{i=1}^{n} e_i^2$$

$$= SSE$$

Sales

$$\boldsymbol{X} = \begin{array}{c} \\ \\ \end{array} \begin{array}{ccccc} & \text{TV} & \text{radio} & & \\ & x_1 & x_2 & & \end{array}$$

$$\boldsymbol{X} = \begin{pmatrix} 1 & x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ 1 & x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n,1} & x_{2,T} & \cdots & x_{n,p} \end{pmatrix} \begin{array}{c} x_1^* \\ x_2^* \\ \\ x_n^* \end{array}$$

$n \times (p+1)$

$$(\underbrace{x^T \cdot x}_{})^{-1}$$

$$\frac{1}{0.002}$$

$$\hat{y} = 2.2 + 0.6x$$

$$e = y - \hat{y}$$

X Bill(\$)	Y Tip(\$)	$\hat{y}$	e	e^2
#1 1	2	2.8	−0.8	0.64
#2 2	4	3.4	0.6	0.36
#3 3	5	4	1	1
#4 4	4	4.6	−0.6	0.36
#5 5	5	5.2	−0.2	0.04

$SSE = 0.64 + 0.36 + 0 + 0.36 + .04$

$=$

$SSE = \underline{2.4}$

$R^2 = \text{cor}(y, \hat{y})^2$

$$Y = \begin{bmatrix} 2 \\ 4 \\ 5 \\ 4 \\ 5 \end{bmatrix} \qquad X = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}}_{5\times 2}$$

$$\underset{LSE}{\hat{B}^*} = (X^T \cdot X)^{-1} \cdot X^T \cdot Y \quad 2\times 1$$

$$\bar{X} = \frac{1+2+3+4+5}{5} = 3$$

$$\bar{y} = \frac{2+4+5+4+5}{5} = 4$$

$$\hat{B}^* = \boxed{(X^T \cdot X)^{-1} \cdot X^T \cdot Y}$$

$$\underbrace{(p+1)\times n \qquad n\times(p+1)}$$

$$\underbrace{(p+1)\times(p+1) \qquad (p+1)\times n \qquad n\times 1}_{(p+1)\times 1}$$

$$X^T \cdot X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}$$

$$\hat{B}^* = \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 0.6 \end{bmatrix}$$

$$\boxed{\hat{y} = 2.2 + 0.6X}$$

$$\hat{y} \pm 1.96\hat{\sigma}_e\sqrt{1 + \boldsymbol{x}^*(\boldsymbol{X}^T\boldsymbol{X})^{-1}(\boldsymbol{x}^*)^T}$$

intercept

$$\sigma_e^2 = \frac{SSE}{n-p-1} = \frac{2.4}{5-1-1} = 0.8$$

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{31} \\ 1 & x_{21} & x_{22} & x_{32} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n3} \end{bmatrix}_{n \times 4}$$

prediction interval for y_i

$$-0.21 < \hat{y}_1 = 2.8 < 4.2$$

$$2.00 < \hat{y}_2 = 3.4 < 5.99$$

$$3.07 < \hat{y}_3 = 4 < 6.92$$

$$2.00 < \hat{y}_4 = 4.6 < 5.99$$

$$2.78 < \hat{y}_5 = 5.2 < 7.21$$

$$\widehat{\mathrm{cor}}(X, y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2} \cdot \sqrt{\sum (y-\bar{y})^2}}$$

$$= \frac{\overset{-2}{(1-3)(2-4)} + \overset{-2}{(2-3)(4-4)} + \overset{-1}{(3-3)(5-4)} + \overset{0}{(4-3)(4-4)} + \overset{2}{(5-3)(5-4)}}{\sqrt{(1-3)^2 + (2-3)^2 + (4-3)^2 + (5-3)^2}\sqrt{(2-4)^2 + (5-4)^2 + (5-4)^2}}$$

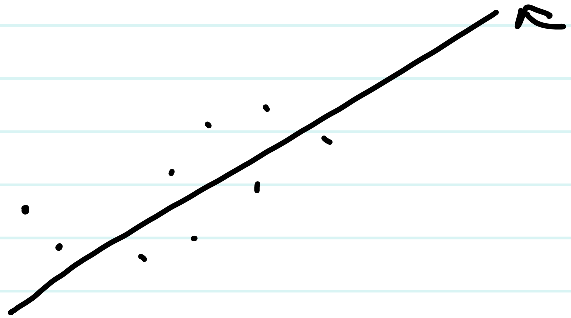
$$= \frac{4+2}{\sqrt{4+1+1+4}\sqrt{4+1+1}} = \frac{6}{\sqrt{10} * \sqrt{6}} = \frac{6}{7.75} = .77$$

$$R^2 = .6 = 60\%$$

$$\hat{\beta} = (\underbrace{x^T . X}_{})^{-1} x^T . y$$

X — sensitive

ill conditioned



$n \times d$

$\uparrow$

$$A = U \Sigma V^T$$

$U \downarrow n \times n$ $\quad \boxed{n \times d}$ $\quad V^T \downarrow d \times d$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

$$y = \beta^T x$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_1^2 \end{bmatrix}$$

polynomial Regression

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_n x^n$$

x Bill	y Tip

min SSE

grid search

for n in range(1, 15)