Name: ______________________

Complete the following problems to the best of your ability. Clearly number each question, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

1: (10 pts.) A manufacturer has a monthly fixed cost of \$200000, and a production cost of \$9 for each unit produced. The product sells for \$17/unit.

(a): What are the cost, revenue, and profit functions associated with this model?

(b): How many units must the manufacturer sell each month to break even?

2: (15 pts.) For $f(x) = 3x^2 + 2x + 1$ and $g(x) = x + 3$, find and simplify: $f + g$, $f - g$, fg, f/g, $f \circ g$.

3: (15 pts.) Show steps in finding the following.

(a): $\lim_{s\to 0}(2s^2 - 1)(2s + 4)$.

(b): $\lim_{x\to -1} \dfrac{\sqrt{x^2+8}}{2x+4}$.

(c): $\lim_{x\to a} \sqrt[3]{5f(x) + 3g(x)}$, given that $\lim_{x\to a} f(x) = 3$ and $\lim_{x\to a} g(x) = 4$.

4: (20 pts.) Evaluate each of the following limits.

(a): $\lim_{x\to -5} \dfrac{x^2 - 25}{x + 5}$.

(b): $\lim_{x\to \infty} \dfrac{2x^2 - 1}{x^3 + x^2 + 1}$.

5: (15 pts.) Consider the functions $f(x) = \dfrac{x+1}{x^2 - 2x + 3}$ and $g(x) = \begin{cases} (x-3)^2 + 3, & \text{if } x \le 3; \\ -x + 8, & \text{if } x > 3. \end{cases}$

(a): Find the values of x for which the function f is continuous.

(b): Find $\lim_{x\to 3^+} g(x)$, $\lim_{x\to 3^-} g(x)$, and $\lim_{x\to 3} g(x)$.

(c): Use the results of part (b) to find the values of x for which the function g is continuous.

6: (10 pts.) Explain why the function $f(x) = x^3 - 2x^2 + 3x + 2$ is continuous for all values of x in the interval $[-1, 1]$. Then use the Intermediate Value Theorem to show that f must have at least one zero in the interval $(-1, 1)$.

7: (15 pts.) Let $f(x) = 2x^2 + 1$.

(a): Find f', the derivative of f.

(b): Find an equation of the tangent line to the curve at the point $(1, 3)$.

Bonus: (10 pts.) Carefully sketch the graph of a function with all of the following characteristics:

- it is defined at every point in the interval $[0, 10]$;
- it displays a *different* type of discontinuity at each of $x = 2$, $x = 4$, and $x = 6$;
- it is continuous, but not differentiable at the point $x = 8$.