

1. (a) The domain of f is $\mathbb{R} = (-\infty, \infty)$ because it's a polynomial.

(b) The domain of g is $(-\infty, 0) \cup (0, 3/2) \cup (3/2, \infty)$, since setting the denominator equal to zero yields:
 $2x^2 - 3x = 0 \implies x(2x - 3) = 0 \implies x = 0, x = 3/2$.

(c) The domain of h is $(4, \infty)$, because $x + 2 > 0 \implies x > -2$ and $x - 4 > 0 \implies x > 4$ and we take the intersection (common values) of these two restrictions.

2. $(f + g)(x) = (3x^2 + 2x + 1) + (x + 3) = 3x^2 + 3x + 4$.
 $(f - g)(x) = (3x^2 + 2x + 1) - (x + 3) = 3x^2 + x - 2$.
 $(fg)(x) = (3x^2 + 2x + 1)(x + 3) = 3x^3 + 9x^2 + 2x^2 + 6x + x + 3 = 3x^3 + 11x^2 + 7x + 3$.
 $(f/g)(x) = (3x^2 + 2x + 1)/(x + 3)$.
 $(f \circ g)(x) = f(g(x)) = 3(x + 3)^2 + 2(x + 3) + 1 = 3x^2 + 18x + 27 + 2x + 6 + 1 = 3x^2 + 20x + 34$.

3. $m = 24 \cdot 2^{-t/25} \implies \frac{m}{24} = 2^{-t/25} \implies \ln\left(\frac{m}{24}\right) = \ln(2^{-t/25}) \implies \ln(m) - \ln(24) = -\frac{t}{25} \ln(2) \implies t = -\frac{25}{\ln(2)}(\ln(m) - \ln(24)) \implies t = \frac{25}{\ln(2)}(\ln(24) - \ln(m))$.

4. (a) $\lim_{s \rightarrow 0} (2s^2 - 1)(2s + 4) = \left(2\left(\lim_{s \rightarrow 0} s\right)^2 - 1\right)\left(2\left(\lim_{s \rightarrow 0} s\right) + 4\right) = (2(0)^2 - 1)(2(0) + 4) = -4$
 (or just use the DSP, since the function is a polynomial).

(b) $\lim_{x \rightarrow a} \sqrt[3]{5f(x) + 3g(x)} = \sqrt[3]{5\left(\lim_{x \rightarrow a} f(x)\right) + 3\left(\lim_{x \rightarrow a} g(x)\right)} = \sqrt[3]{5(3) + 3(4)} = \sqrt[3]{27} = 3$.

(c) $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x + 5} = \lim_{x \rightarrow -5} \frac{(x - 5)(x + 5)}{x + 5} = \lim_{x \rightarrow -5} (x - 5) = -10$.

(d) $\lim_{x \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right) = \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}}\right) = \lim_{x \rightarrow 0} \left(\frac{1 - \sqrt{1+t}}{t\sqrt{1+t}}\right) \cdot \left(\frac{1 + \sqrt{1+t}}{1 + \sqrt{1+t}}\right) = \lim_{x \rightarrow 0} \frac{1 - (1+t)}{(t\sqrt{1+t})(1 + \sqrt{1+t})} =$
 $\lim_{x \rightarrow 0} \frac{-t}{(t\sqrt{1+t})(1 + \sqrt{1+t})} = \lim_{x \rightarrow 0} \frac{-1}{(\sqrt{1+t})(1 + \sqrt{1+t})} = \frac{-1}{(1)(1+1)} = -\frac{1}{2}$.

(e) $\lim_{x \rightarrow \infty} \frac{2x^2 - 1}{x^3 + x^2 + 1} = \lim_{x \rightarrow \infty} \frac{2x^2 - 1}{x^3 + x^2 + 1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{1}{x^3}}{1 + \frac{1}{x} + \frac{1}{x^3}} = \frac{0 - 0}{1 + 0 + 0} = \frac{0}{1} = 0$
 (or just remember the general rules for the limits at infinity of rational functions).

5. Notice that $\lim_{x \rightarrow 4} (4x - 9) = 4(4) - 9 = 7$ and $\lim_{x \rightarrow 4} (x^2 - 4x + 7) = 4^2 - 4(4) + 7 = 7$. Thus, by the Squeeze Theorem, $\lim_{x \rightarrow 4} f(x) = 7$.

6. (a) The function f is a rational function, so it is continuous except when its denominator is equal to zero. To find these x -values, solve: $x^2 - 2x + 3 = 0$. Using the quadratic formula,

$$x = \frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{4 - 12}}{2} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2i\sqrt{2}}{2} = 1 \pm i\sqrt{2}.$$

Since these are two complex solutions, there are no real numbers that make the denominator zero. Hence, f is continuous on $(-\infty, \infty)$.

(b) Graphically, it is easy to see that: $\lim_{x \rightarrow 3^+} g(x) = 5$, $\lim_{x \rightarrow 3^-} g(x) = 3$, and thus $\lim_{x \rightarrow 3} g(x)$ does not exist.

(c) Each piece of the function is a polynomial, and thus continuous wherever it is defined. We observed that $\lim_{x \rightarrow 3} g(x)$ does not exist, and thus the function g is continuous on the interval $(-\infty, 3) \cup (3, \infty)$.

7. The function f is a polynomial, so it is continuous everywhere, and in particular for all $x \in [-1, 1]$. Notice that $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -4 < 0$ and $f(1) = 1^3 - 2(1)^2 + 3(1) + 2 = 4 > 0$ have different signs. Thus, by the Intermediate Value Theorem, the function f has at least one zero in the interval $(-1, 1)$.

8. (a) By the definition of the derivative:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 + 1) - (2x^2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} \\ &= \lim_{h \rightarrow 0} (4x + 2h) \\ &= 4x. \end{aligned}$$

(b) The slope of the tangent line at the point $(1, 3)$ is given by $m = f'(1) = 4(1) = 4$. We then use point-slope form to find the equation of the tangent line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \implies y - 3 = 4(x - 1) \\ &\implies y - 3 = 4x - 4 \\ &\implies y = 4x - 1. \end{aligned}$$

9. The specified first and second derivative properties of the function determine where it is increasing/decreasing, where it has local minima/maxima, where it is concave up/down, and where it changes concavity. Various graphs are acceptable as long as they are in accordance with these properties.

Bonus. Any number of answers is acceptable, as long as the depicted function is defined everywhere on the interval $[0, 10]$ and satisfies the specified “not continuous” and “not differentiable” requirements. In particular, at $x = 2, 4, 6$ the graph should display either a “jump” discontinuity, a “hole” discontinuity, or an “asymptotic” discontinuity, while at $x = 8$ the function should either have a vertical tangent line (meaning undefined slope) or some sort of kink or corner.