EXAMPLE 5 **Exponential functions get big fast** Use a graphing device to find the values of x for which $e^x > 1{,}000{,}000$.

SOLUTION In Figure 16 we graph both the function $y = e^x$ and the horizontal line $y = 1{,}000{,}000$. We see that these curves intersect when $x \approx 13.8$. Thus $e^x > 10^6$ when $x > 13.8$. It is perhaps surprising that the values of the exponential function have already surpassed a million when x is only 14.
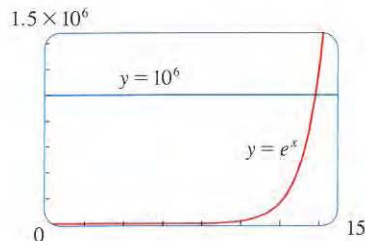

FIGURE 16

1.5 Exercises

1–4 Use the Law of Exponents to rewrite and simplify the expression.

1. (a) $\dfrac{4^{-3}}{2^{-8}}$
   (b) $\dfrac{1}{\sqrt[3]{x^4}}$

2. (a) $8^{4/3}$
   (b) $x(3x^2)^3$

3. (a) $b^8(2b)^4$
   (b) $\dfrac{(6y^3)^4}{2y^5}$

4. (a) $\dfrac{x^{2n} \cdot x^{3n-1}}{x^{n+2}}$
   (b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

5. (a) Write an equation that defines the exponential function with base $a > 0$.
   (b) What is the domain of this function?
   (c) If $a \neq 1$, what is the range of this function?
   (d) Sketch the general shape of the graph of the exponential function for each of the following cases.
       (i) $a > 1$ (ii) $a = 1$ (iii) $0 < a < 1$

6. (a) How is the number e defined?
   (b) What is an approximate value for e?
   (c) What is the natural exponential function?

7–10 Graph the given functions on a common screen. How are these graphs related?

7. $y = 2^x$, $y = e^x$, $y = 5^x$, $y = 20^x$

8. $y = e^x$, $y = e^{-x}$, $y = 8^x$, $y = 8^{-x}$

9. $y = 3^x$, $y = 10^x$, $y = \left(\tfrac{1}{3}\right)^x$, $y = \left(\tfrac{1}{10}\right)^x$

10. $y = 0.9^x$, $y = 0.6^x$, $y = 0.3^x$, $y = 0.1^x$

11–16 Make a rough sketch of the graph of the function. Do not use a calculator. Just use the graphs given in Figures 3 and 13 and, if necessary, the transformations of Section 1.3.

11. $y = 10^{x+2}$

12. $y = (0.5)^x - 2$

13. $y = -2^{-x}$

14. $y = e^{|x|}$

15. $y = 1 - \tfrac{1}{2}e^{-x}$

16. $y = 2(1 - e^x)$

17. Starting with the graph of $y = e^x$, write the equation of the graph that results from
   (a) shifting 2 units downward
   (b) shifting 2 units to the right
   (c) reflecting about the x-axis
   (d) reflecting about the y-axis
   (e) reflecting about the x-axis and then about the y-axis

18. Starting with the graph of $y = e^x$, find the equation of the graph that results from
   (a) reflecting about the line $y = 4$
   (b) reflecting about the line $x = 2$

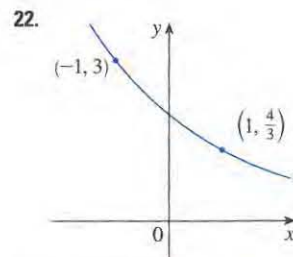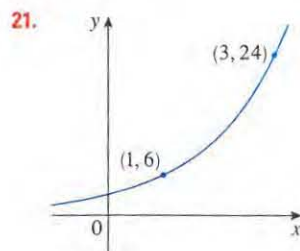**19–20** Find the domain of each function.

19. (a) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$
   (b) $f(x) = \dfrac{1 + x}{e^{\cos x}}$

20. (a) $g(t) = \sin(e^{-t})$
   (b) $g(t) = \sqrt{1 - 2^t}$

Graphing calculator or computer with graphing software required   **1.** Homework Hints available in TEC

21–22 Find the exponential function $f(x) = Ca^x$ whose graph is given.

21.

22.

23. If $f(x) = 5^x$, show that

$$\frac{f(x + h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

24. Suppose you are offered a job that lasts one month. Which of the following methods of payment do you prefer?
  I. One million dollars at the end of the month.
  II. One cent on the first day of the month, two cents on the second day, four cents on the third day, and, in general, 2^{n-1} cents on the nth day.

25. Suppose the graphs of $f(x) = x^2$ and $g(x) = 2^x$ are drawn on a coordinate grid where the unit of measurement is 1 inch. Show that, at a distance 2 ft to the right of the origin, the height of the graph of f is 48 ft but the height of the graph of g is about 265 mi.

26. Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place. Which function grows more rapidly when x is large?

27. Compare the functions $f(x) = x^{10}$ and $g(x) = e^x$ by graphing both f and g in several viewing rectangles. When does the graph of g finally surpass the graph of f?

28. Use a graph to estimate the values of x such that $e^x > 1,000,000,000$.

29. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria.
  (a) What is the size of the population after 15 hours?
  (b) What is the size of the population after t hours?
  (c) Estimate the size of the population after 20 hours.

  (d) Graph the population function and estimate the time for the population to reach 50,000.

30. A bacterial culture starts with 500 bacteria and doubles in size every half hour.
  (a) How many bacteria are there after 3 hours?
  (b) How many bacteria are there after t hours?
  (c) How many bacteria are there after 40 minutes?
  (d) Graph the population function and estimate the time for the population to reach 100,000.

31. The half-life of bismuth-210, ^{210}Bi, is 5 days.
  (a) If a sample has a mass of 200 mg, find the amount remaining after 15 days.
  (b) Find the amount remaining after t days.
  (c) Estimate the amount remaining after 3 weeks.
  (d) Use a graph to estimate the time required for the mass to be reduced to 1 mg.

32. An isotope of sodium, ^{24}Na, has a half-life of 15 hours. A sample of this isotope has mass 2 g.
  (a) Find the amount remaining after 60 hours.
  (b) Find the amount remaining after t hours.
  (c) Estimate the amount remaining after 4 days.
  (d) Use a graph to estimate the time required for the mass to be reduced to 0.01 g.

33. Use a graphing calculator with exponential regression capability to model the population of the world with the data from 1950 to 2000 in Table 1 on page 56. Use the model to estimate the population in 1993 and to predict the population in the year 2010.

34. The table gives the population of the United States, in millions, for the years 1900–2000. Use a graphing calculator with exponential regression capability to model the US population since 1900. Use the model to estimate the population in 1925 and to predict the population in the years 2010 and 2020.

Year	Population	Year	Population
1900	76	1960	179
1910	92	1970	203
1920	106	1980	227
1930	123	1990	250
1940	131	2000	281
1950	150		

35. If you graph the function

$$f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$$

you'll see that f appears to be an odd function. Prove it.

36. Graph several members of the family of functions

$$f(x) = \frac{1}{1 + ae^{bx}}$$

where $a > 0$. How does the graph change when b changes? How does it change when a changes?