Name: ______________________

Problem 1. Evaluate $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$.

Solution. Using L'Hôpital's rule (two times), we find

$$\begin{aligned}
\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) &= \lim_{x\to 1}\frac{x\ln x-(x-1)}{(x-1)\ln x}\\
&= \lim_{x\to 1}\frac{x\ln x-x+1}{x\ln x-\ln x}\\
&= \lim_{x\to 1}\frac{(1+\ln x)-1}{(1+\ln x)-1/x}\\
&= \lim_{x\to 1}\frac{\ln x}{1+\ln x-1/x}\\
&= \lim_{x\to 1}\frac{1/x}{1/x+1/x^2}\\
&= \lim_{x\to 1}\frac{1/x}{\left(\frac{x+1}{x^2}\right)}\\
&= \lim_{x\to 1}\frac{1}{x}\cdot\frac{x^2}{x+1}\\
&= \lim_{x\to 1}\frac{x}{x+1}\\
&= \frac{1}{2}.
\end{aligned}$$

Problem 2. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in monthly rent. What rent should the manager charge in order to maximize revenue?

Solution. Let x be the number of occupied apartments. Then the demand function (which represents the cost of rent) is given by

$$p(x) = 800 + 10(100 - x) = 1800 - 10x,$$

and the revenue function is

$$R(x) = x\cdot p(x) = 1800x - 10x^2.$$

It follows that $R'(x) = 1800 - 20x$, so

$$R'(x) = 0 \implies 1800 - 20x = 0 \implies 20x = 1800 \implies x = 90.$$

Thus, the monthly rent should be set at $p(90) = \$900$ in order to maximize revenue.

There are a couple of easy ways to convince yourself that $x = 90$ does, in fact, correspond to a maximum (and not a minimum) of $R(x)$. One way is to show that $R(x) > 0$ for $x < 90$ and $R(x) < 0$ for $x > 90$, and the other is to notice that $R''(x) = -20$ is always negative (i.e. $R(x)$ is always concave down, so any critical number where the derivative is zero must correspond to an absolute maximum; in this case, that critical number is $x = 90$).