Name: ______________________________

Complete the following problems to the best of your ability. Clearly number each question and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

1. At what points does the curve $\vec{r}(t) = t\hat{i} + (2t - t^2)\hat{k}$ intersect the paraboloid $z = x^2 + y^2$?

2. Find parametric equations for the tangent line to the curve $x = t$, $y = e^{-t}$, $z = 2t - t^2$ at the point $(0, 1, 0)$.

3. Find the length of the curve $\vec{r}(t) = \langle 2t, t^2, t^3/3 \rangle$ for $0 \le t \le 1$.

4. Find the curvature of $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.

5. The position function of a particle is given by $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed of the particle at its minimum?

6. If $x - z = \arctan(yz)$, use implicit differentiation (or the nice formula from the Chain Rule) to find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

7. Find the limit or conclude that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2 + y^2}$.

(b) $\displaystyle\lim_{(x,y,z)\to(1,0,0)} \frac{yz}{x^2 + 4y^2 + 9z^2}$.

8. Let $z = x^2 + xy^3$, where $x = uv^2 + w^3$ and $y = u + ve^w$. Use the Chain Rule to find the partial derivatives $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$, and $\frac{\partial z}{\partial w}$ when $u = 2$, $v = 1$, and $w = 0$.

9. Consider the function $g(p, q) = p^4 - p^2 q^3$.

(a) Find the gradient vector of g as a function of p and q.

(b) Evaluate the gradient at the point $(2, 1)$.

(c) Find the directional derivative of g at $(2, 1)$ in the direction of $\vec{v} = \hat{i} + 3\hat{j}$.

E.C. (Problem will be related to those in Section 11.7 on extrema of multivariate functions.)