• More Practice Problems for Exam II - Math 121

1. Let $f(x) = (x - 1)^2 (x - 4)$

   a) Determine the domain of f, the continuity of f, the limits at negative and positive infinity, and the zeros of f.
   b) Does the function have any asymptotes?
   c) Find the intervals on which f is increasing or decreasing.
   d) Find the local maxima and minima of f. Which of these are also absolute extrema?
   e) Find the intervals of concavity and the inflection points.
   f) Use the above information to graph the function.

2. A function f defined on the set of all real numbers has the following properties. If $x < 2$ or $3 < x$, then $f'(x) > 0$. If $2 < x < 3$, then $f'(x) < 0$. $f'(3) = 0$ but $f'(2)$ is undefined. If $x < 0$, then $f''(x) < 0$. If $0 < x < 2$ or $x > 2$, then $f''(x) > 0$. Decide whether each of the following statements is true or false.

   T F (a) f is increasing on the interval $(2, 3)$.

   T F (b) f is increasing on the interval $(-\infty, 2)$.

   T F (c) f is concave down on the interval $(2, \infty)$.

   T F (d) f is concave down on the interval $(-\infty, 0)$.

   T F (e) f has a relative maximum at $x = 3$.

   T F (f) f has a relative maximum at $x = 2$.

   T F (g) f has a relative maximum at $x = -3$.

3. Given the cost function (in dollars)

$$C(x) = 2\sqrt{x} + \frac{x^2}{8000},$$

find

(a) the average cost function,

(b) the marginal cost function,

(c) the production level that minimizes the average cost,

(d) the minimum average cost.

4. Given the cost function and the demand function

$$C(x) = 16,000 + 500x - 1.6x^2 + 0.004x^3, \qquad p(x) = 1700 - 7x,$$

find

(a) the production level that maximize the revenue and the maximum revenue,

(b) the production level that maximize the profit and the maximum profit,

5. A rectangle of perimeter 100 inches is rotated about one of its sides so as to form a cylinder. What are the dimensions of the rectangle which generates a cylinder of maximum volume¿? Justify your answer.

6. A right circular cylinder is to have surface area 150π square inches including both ends and is to enclose the maximum possible volume. What is the height of the cylinder? Justify your answer.

7. A rectangular box with a square base and open top is constructed to have volume of 42,875 cubic inches. The material used to make the bottom costs \$0.06 per square inch and the material used to make the sides costs \$0.03 per square inch. Find the dimensions of the box that minimizes the total costs. Justify your answer.

8. The maximum possible area of a rectangle of perimeter $200m$ is

(a) $2000\,m^2$ (b) $2500\,m^2$ (c) $3500\,m^2$ (d) $2400\,m^2$ (e) $1600\,m^2$

9. Given $f'(x) = \cos(2x) - \sin x, \quad 0 < x < 2\pi$. On which open intervals is the function f increasing?

10. Find the absolute maximum and minimum values of $f(x)$ and the corresponding x-values on the given interval.

(a) $f(x) = x^3 - 3x^2 + 2, \quad [-2, 3]$ (b) $f(x) = \dfrac{2x}{x^2+4}, \quad [0, 3]$

(c) $f(x) = x - 2\sin x, \quad [0, \pi]$ (d) $f(x) = xe^{-x^2}, \quad (-\infty, \infty)$

11. The absolute maximum and minimum values of $y = x^3 - 9x + 8$ on the interval $[-3, 1]$ are

(A) $8+6\sqrt{3},\ 8$ (B) $8+6\sqrt{3},\ 0$ (C) $8,\ 0$ (D) $8-6\sqrt{3},\ 0$ (E) None of these.

12. At what value of x does the function $f(x) = 3x - x^{1/3}$ change from increasing to decreasing?

13. On what interval(s) is the function $g(t) = \dfrac{t}{t^2+1}$ decreasing?

14. How many points of inflection does $h(x) = x^3e^{-x}$ have?

15. Let $f(x) = x^4 - 4x^2$.

(a) Find the critical numbers of f, the intervals on which f is increasing or decreasing. Find the (x, y) coordinates of any local extrema.

(b) Find the inflection points of f and the intervals on which f is concave upward or concave downward.

(c) Sketch the graph of f by using the information obtained in (a) and (b).

16 If $f(3) = 0$, $f'(x) \geq 2$ for $2 < x < 4$, and $f''(x) \leq -2$ for $2 < x < 4$, then

(a) f is increasing and concave upward at $x = 3$.

(b) f is decreasing and concave upward at $x = 3$.

(c) f is increasing and concave downward at $x = 3$.

(d) f is decreasing and concave downward at $x = 3$.

(e) None of the above.

$$\lim_{x\to+\infty} g(x) = 2.3, \ \lim_{x\to-\infty} g(x) = -\infty, \ \lim_{x\to\pi^+} g(x) = -\infty, \ \lim_{x\to\pi^-} g(x) = +\infty.$$

The following information is also known about g' and g''.

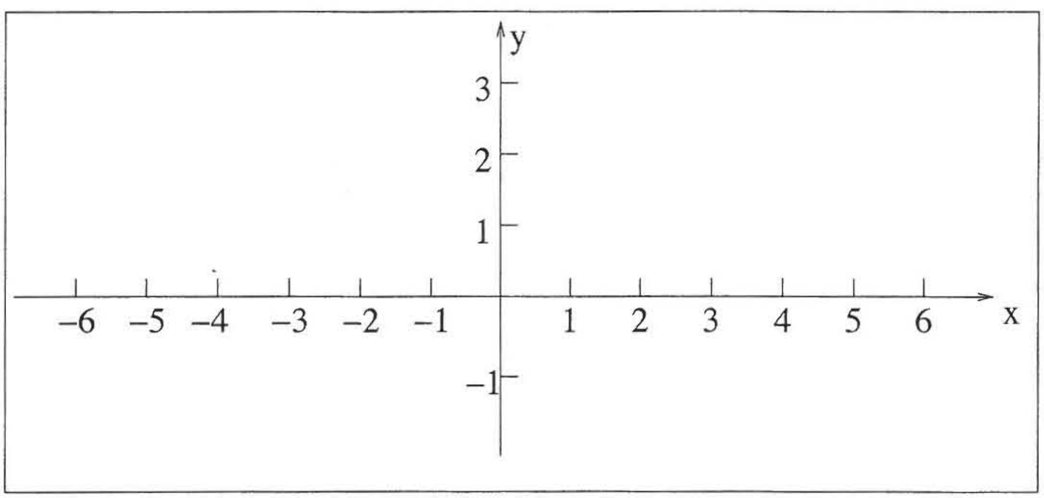
x	$x < -\sqrt{17}$	$-\sqrt{17} < x < -e$	$-e < x < 0$	$0 < x < \pi$	$\pi < x$
$g'(x)$	+	−	−	+	+
$g''(x)$	−	−	+	+	−

17. Find (no justification needed):

(a) The x coordinates of all local maxima. (b) The x coordinates of all local minima.

(c) The x coordinates of all inflection points. (d) The equations of all horizontal asymptotes.

(e) The equations of all vertical asymptotes. (f) Carefully sketch a graph of g below:



18. If $f(x) = \dfrac{x^2 - 4x + 3}{x^2}$ then $f'(x) = \dfrac{4x - 6}{x^3}$ and $f''(x) = \dfrac{-8x + 18}{x^4}$. Give the coordinates of all the points as described. If none, write "None."

a) Where the graph crosses the x-axis

b) Where the graph crosses the y-axis

c) Where the graph crosses the line y = 1

d) Relative maxima

e) Relative minima

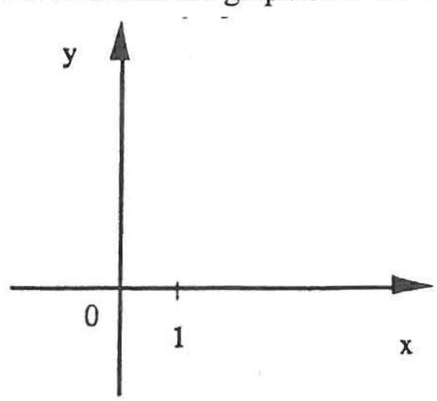
f) Inflection points

g) Give the equations of the asymptotes of various kinds. If none, write "None."

Vertical

Horizontal

h) Use the information above to sketch the graph of f. Label the points listed above.



19. Let $s(t)$ be the displacement function of a mouse moving along the x-axis. Let $v(t)$ and $a(t)$ be its velocity and acceleration functions respectively. If

$$a(t) = 2 + 4e^{2t}, \quad v(0) = 1 \quad \text{and} \quad s(0) = 4,$$

determine which of the following expressions represents $s(t)$.

(A) $8e^{2t}$.

(B) $t^2 + e^{2t}$.

(C) $t^2 + 8e^{2t} - 3t - 4$.

(D) $t^2 + e^{2t} - t + 3$.

20. Determine which of the following equals $\int x\sqrt{x^2+1}\, dx$.

(A) $\frac{1}{3}x^2(x^2+1)^{3/2} + c$.

(B) $\frac{1}{3}(x^2+1)^{3/2} + c$.

(C) $\frac{1}{2}x^2(x^2+1)^{3/2} + c$.

(D) $\frac{1}{2}(x^2+1)^{3/2} + c$.

21. Evaluate the following integrals:

$\int \sin^2 x \cos x \, dx$

$\int_0^1 xe^{-x^2}\, dx$

22. Let f be a differentiable function defined on $[a, b]$ whose derivative f' is continuous on $[a, b]$. Let F be an antiderivative of f, i.e., $F'(x) = f(x)$ for all x in $[a, b]$. Determine which of the following statements is false in general.

(A) $\int_a^b f'(t)\, dt = f(b) - f(a)$.

(B) $\int_a^b f(t)\, dt = F(b) - F(a)$.

(C) $\int_a^b F''(t)\, dt = f(b) - f(a)$.

(D) $\int_a^b F'(t)\, dt = f(b) - f(a)$.

Extra Credit Problems.

23. Choose only one problem from the following six problems. Provide a very well done presentation of your solution. Circle clearly which problem you selected.

(i) Evaluate $\int_1^3 2x\,dx$ directly by calculating a limit of Riemann sums. Use partitions consisting of subintervals of equal lengths, and augment the partitions by using the left or right endpoints.

(Hint: $\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$)

Integral as Riemann Sum

24. Find the area A of the region R bounded by the graphs of $f(x) = x^2 - 2$ and $g(x) = |x|$.

Area between curves

25. If the average value of f on the interval $2 \le x \le 5$ is 4, then $\int_2^5 (3f(x) + 2)\,dx$ is

Average Value