Date: 10/25/13.

Instructor: Cody Clifton.

Name: _____

This 10-point quiz will test you on finding extrema and using l'Hôpital's Rule. Read carefully and always show your work. You have 15 minutes... good luck!

(1) Find the absolute maximum value and absolute minimum value of $f(x) = 12 + 4x - x^2$ on the closed interval $[0, 5]$.

Solution. First, we find the critical number(s) of f by setting its derivative equal to zero.

$$f'(x) = 4 - 2x = 0 \implies 4 = 2x \implies x = 2.$$

Then, we compare:

$$f(0) = 12 + 4(0) - (0)^2 = 12,$$
$$f(2) = 12 + 4(2) - (2)^2 = 16,$$
$$f(5) = 12 + 4(5) - (5)^2 = 7.$$

The conclusion is that $f(2) = 16$ and $f(5) = 7$ are the absolute maximum and absolute minimum values, respectively, of the function f.

(2) Find each of the following limits. Use l'Hôpital's Rule as many times as needed, but note that it may not be needed at all...

(a) $\lim_{x \to 0^+} \dfrac{\ln x}{x}$.

Solution. Since $\lim_{x \to 0^+} \ln x = -\infty$ and $\lim_{x \to 0^+} x = 0$, l'Hôpital's Rule does not apply. However, since a very large (negative) number divided by a very small (positive) number is an even larger (negative) number, we just have

$$\lim_{x \to 0^+} \frac{\ln x}{x} = -\infty.$$

(b) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

Solution. Using l'Hôpital's Rule two times (check that it is really needed!), we find that

$$\begin{aligned} \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} &= \lim_{x \to 0} \frac{e^x - 1}{2x} \\ &= \lim_{x \to 0} \frac{e^x}{2} \\ &= \frac{1}{2}. \end{aligned}$$