Date: 11/21/13.

Instructor: Cody Clifton.

Name: ____________________

This 10-point quiz will test you on using integration to find the area between curves and the volume of solids. Read carefully and always show your work. You have 15 minutes... good luck!

(1) Find the area of the region enclosed by the curves $x = 1 - y^2$ and $x = y^2 - 1$.

(Hint: first sketch a graph of the region, then decide whether to integrate with respect to x or y.)

Solution. In this case, it is natural to integrate with respect to y, with $x = 1 - y^2$ as the "upper" curve and $x = y^2 - 1$ as the "lower" curve. The points of intersection are

$$1 - y^2 = y^2 - 1 \implies 2y^2 = 2 \implies y^2 = 1 \implies y = \pm 1.$$

Thus, the area of the enclosed region is given by

$$\begin{aligned} A &= \int_{-1}^{1} \left((1 - y^2) - (y^2 - 1)\right) dy \\ &= \int_{-1}^{1} \left(2 - 2y^2\right) dy \\ &= \left(2y - \frac{2}{3}y^3\right)\Big|_{-1}^{1} \\ &= \left(2(1) - \frac{2}{3}(1)^3\right) - \left(2(-1) - \frac{2}{3}(-1)^3\right) \\ &= \frac{8}{3}. \end{aligned}$$

(2) The region enclosed by the curves $y = 1/x$, $x = 1$, $x = 2$, and $y = 0$ is rotated about the x-axis. Find the volume of the resulting solid of revolution.

Solution. In this case, it is natural to integrate with respect to x. The limits of integration are $x = 1$ and $x = 2$, so the volume of the solid is given by

$$V = \int_1^2 \pi \left(\frac{1}{x}\right)^2 dx = \pi \int_1^2 x^{-2} dx = -2\pi x^{-1}\big|_1^2 = -2\pi\left(2^{-1} - 1^{-1}\right) = \frac{\pi}{2}.$$