Review Problems – Math115 Final Exam
(Final covers Sec 2.1-5.2, 5.4-6.5)

Final Exam, Monday, May 7, 4:30 - 7:00pm.

The final exam is comprehensive and will consist of 12 *True or False* problems and 25 *Multiple-Choice* problems. The practice problems below are in addition to the practice problems on the Midterm review (http://www.math.ku.edu/ porter/MidtermReview.pdf).

True or False Problems

1. T F If $f'(c) = 0$, then $x = c$ is a critical number of f.

2. T F If $f(c)$ is an absolute extrema of $f(x)$, then $x = c$ is a critical number of $f(x)$.

3. T F If $x = 1$ is a critical number of $f(x)$, then $f(1)$ is either a relative maximum or a relative minimum of $f(x)$.

4. T F Any continuous function must have absolute extrema.

5. T F Any continuous function on $[a, b]$ has absolute extrema on $[a, b]$.

6. T F No continuous function on (a, b) can have absolute extrema on (a, b).

7. T F If f is continuous on $[a, b]$, then $\displaystyle\int_a^b f(x)dx$ exists.

8. T F The area of the region bounded by the graph of a nonnegative, continuous function f and above the x-axis over the interval $[0, 1]$ is $\displaystyle\int_0^1 f(x)dx$.

9. T F If $f' > 0$ on $[2, 4]$, then f is concave up on $[2, 4]$.

10. T F If f and g are both positive and increasing on (a, b), then fg is increasing on (a, b).

11. T F If f is concave up on (a, b), then the graph of $-f$ is concave down on (a, b).

12. T F If c is a critical number of f where $a < c < b$ and $f'' < 0$ on (a, b), then f has a relative maximum at $x = c$.

13. T F $e^x e^y = e^{xy}$

14. T F If $x < y$, then $e^x < e^y$.

15. T F If $0 < b < 1$ and $x < y$, then $b^x > b^y$.

16. T F $(\ln x)^3 = 3 \ln x$ for all $x > 0$.

17. T F $\ln a - \ln b = \ln(a - b)$ for all positive real numbers a and b.

18. T F $\ln(x^2 e^{x^2}) = \ln x^2 + x^2$ for all $x > \pi$.

19. T F The function $f(x) = \ln |x|$ is continuous for all $x \neq 0$.

20. T F If $f(x) = 3^x$, then $f'(x) = x \cdot 3^{x-1}$.

21. T F If $f(x) = e^\pi$, then $f'(x) = e^\pi$.

22. T F If f has a derivative for all x and $h(x) = f(\pi x)$, then $h'(x) = \pi f'(\pi x)$.

23. T F If $f(x) = \pi^2$, then $f'(x) = 2\pi$.

24. T F If $f(x) = \ln 5$, then $f'(x) = \frac{25}{5}$.

25. T F If $f(x) = \ln a^x$, then $f'(x) = \ln a$.

26. T F If $f(x) = \dfrac{1}{\ln 2x}$, then $f'(x) = \dfrac{-1}{2x \ln 2x}$.

27. T F If $f(x) = \dfrac{1}{e^{2x}}$, then $f'(x) = \dfrac{-1}{2xe^{2x}}$.

28. T F If $f(x) = \dfrac{1}{\sqrt{2x}}$, then $f'(x) = \dfrac{-1}{2x\sqrt{2x}}$.

29. T F If F and G are antiderivatives of f on an interval I, then $F(x) = G(x) + C$ on I.

30. T F If f and g are both integrable, then $\displaystyle\int [2f(x) - 3g(x)]dx = 2\int f(x)dx - 3\int g(x)dx$.

31. T F If f and g are both integrable, then $\displaystyle\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$.

32. T F $\displaystyle\int_0^2 (1-x)dx$, gives the area of the region under the graph of $f(x) = 1 - x$ and above the x-axis on the interval $[0, 2]$.

33. T F The total revenue realized in selling the first 5000 units of a product is given by $\displaystyle\int_0^{5000} R'(x)\, dx = R(5000) - R(0)$ with $R(x)$ the total revenue.

34. T F $\displaystyle\int_2^2 \frac{e^x}{\sqrt{1+x}}\, dx = 0$

35. T F $\displaystyle\int_1^3 \frac{dx}{x - \frac{1}{2}} = -\int_3^1 \frac{dx}{x - \frac{1}{2}}$

36. T F $\displaystyle\int_0^1 x\sqrt{x+1}\, dx = \sqrt{x+1}\int_0^1 x\, dx = \frac{\sqrt{2}}{2}$

37. T F If $f''(x)$ is continuous on $[0, 2]$, then $\displaystyle\int_0^2 f''(x)\, dx = f'(2) - f'(0)$.

38. T F If f is continuous on $[a, b]$ and $a < c < b$, then $\displaystyle\int_b^c f(x)\, dx = \int_a^c f(x)\, dx - \int_a^b f(x)\, dx$.

Multiple-Choice Problems

1. Let $f(x) = \ln(2-x)$. The domain of f is

   (A) $(-\infty, +\infty)$ (B) $(-2, +\infty)$ (C) $(-\infty, -2)$ (D) $(-\infty, 2)$ (E) $(2, \infty)$

2. Find the vertical asymptotes of function $f(x) = \dfrac{2+x}{(1-x)^2}$.

   (A) $x=-2$ (B) $x=1$ (C) $y=0$ (D) $y=-2$ (E) $y=1$

3. Find an equation of the tangent line to the graph of $y = x\ln x$ at the point (1,0).

   (A) $y = x+1$ (B) $y = x-1$ (C) $y = (x+1)e$ (D) $y = (x-1)e$ (E) $y-1 = x+1$

4. Find an equation of the tangent line to the graph of $y = \ln(x^2)$ at the point $(2, \ln 4)$.

   (A) $y = x+2-\ln 4$ (B) $y = 2(x-2)-\ln 4$ (C) $y = 2(x-2)+\ln 4$

   (D) $y = x-2+\ln 4$ (E) $y = x-2-\ln 4$

5. Find an equation of the tangent line to the graph of $y = e^{2x-3}$ at the point $(\frac{3}{2}, 1)$.

   (A) $y = 2e^{2x-3}$ (B) $y = 2x-4$ (C) $y = 2x-2$ (D) $y = 2x-3$ (E) $y = x - \dfrac{1}{2}$

6. Find an equation of the tangent line to the graph of $y = e^{-x^2}$ at the point $(1, 1/e)$.

   (A) $y = -\dfrac{2}{e}(x+1) + \dfrac{1}{e}$ (B) $y = -\dfrac{2}{e}(x-1) - \dfrac{1}{e}$ (C) $y = -\dfrac{2}{e}(x-1) + \dfrac{1}{e}$

   (D) $y = \dfrac{1}{e}(x-1) + \dfrac{1}{e}$ (E) $y = \dfrac{1}{e}(x+1) + \dfrac{1}{e}$

7. The absolute minimum value of the function $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}$ on [0, 3] is:

   (A) $-\dfrac{3}{2}$ (B) 0 (C) $\dfrac{9}{2} - 2\sqrt{3}$ (D) 1 (E) 2

8. The absolute maximum value of the function $f(x) = \dfrac{1}{1+x^2}$ is:

   (A) 0 (B) 1 (C) 2 (D) e (E) $\ln 2$

9. The absolute maximum value of the function $f(t) = te^{-t}$ is:

   (A) 0 (B) 1 (C) $\dfrac{1}{e}$ (D) e (E) -1

10. The absolute minimum value of the function $f(t) = \dfrac{1}{t}\ln(\dfrac{1}{t})$ on [1, 2] is:

    (A) $\dfrac{-\ln 2}{2}$ (B) $\dfrac{-1}{2\ln 2}$ (C) $-2\ln 2$ (D) -2 (E) $-\dfrac{1}{2}$

11. Let $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. Find the interval(s) where f is concave downward.

    (A) $(-\infty, 1), (1, 2)$ (B) $(-\infty, 1), (2, \infty)$ (C) $(-2, 1), (2, \infty)$ (D) $(-\infty, -2), (1, 2)$ (E) $(-\infty, 1)$

12. Let $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. Find the inflection points.

(A) $(1, f(1))$ (B) $(\frac{1}{2}, f(\frac{1}{2}))$ (C) $(0, f(0))$ (D) $(2, f(2))$ (E) $(3, f(3))$

13. Let $f(x) = x \ln x$. Determine the intervals where the function is decreasing.

(A) $(-\infty, \frac{1}{e}), (\frac{1}{e}, e)$ (B) $(-\infty, \frac{1}{e}), (\frac{1}{e}, \infty)$ (C) $(0, \frac{1}{e}), (\frac{1}{e}, e)$ (D) $(0, \frac{1}{e}), (e, \infty)$ (E) $(0, \frac{1}{e})$

14. Find the derivative of function $y = x^{\ln x}$.

(A) $y' = (\ln x)^2$ (B) $y' = \frac{2 \ln x}{x} x^{\ln x}$ (C) $y' = x^{\ln x}$

(D) $y' = \ln x(x^{\ln x - 1})$ (E) $y' = 2 \ln x(x^{\ln x})$

15. Find the derivative of function $y = 10^x$.

(A) $y' = 10^x \ln 10$ (B) $y' = 10^x$ (C) $y' = 10^{10} \ln 10^x$

(D) $y' = x10^{x-1}$ (E) $y' = \frac{10^x}{10}$

16. An open box is to be made from a square sheet of tin measuring 12 in.$\times$12 in. by cutting out a square of side x inches from each corner of the sheet and folding up the four resulting flaps. To maximize the volume of the box, x should be

(A) 1 in. (B) 2 in. (C) 3 in. (D) 4 in. (E) 5 in.

17. A rectangular box is to have a square base and a volume of 20 ft^3. The material for the base costs 30 cents/square foot, the material for the four sides costs 10 cents/square foot, and the material for the top costs 20 cents/square foot. What are the dimensions of the box that can be constructed at minimum cost ?

(A) $1 \times 1 \times 20$ (B) $2 \times 2 \times 5$ (C) $2.5 \times 2.5 \times 3.2$

(D) $3 \times 3 \times 2.22$ (E) $4 \times 4 \times 1.25$

18. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time $t > 0$ its velocity v(t) is given by $v(t) = t/(1 + t^2)$. Determine the maximum velocity attained by the particle.

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{10}$ (D) $\frac{1}{2}$ (E) $\frac{4}{17}$

19. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time $t > 0$ its velocity v(t) is given by $v(t) = t/(1 + t^2)$. Determine the position of the particle at t=6.

(A) $\ln \sqrt{5} + 5$ (B) $\ln \sqrt{2} + 5$ (C) $\ln \sqrt{10} + 5$ (D) $\ln \sqrt{17} + 5$ (E) $\ln \sqrt{37} + 5$

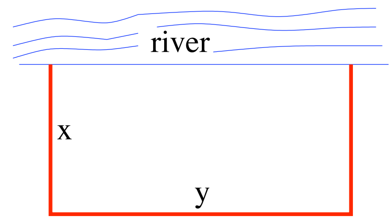
20. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time $t > 0$ its velocity v(t) is given by $v(t) = t/(1 + t^2)$. Find the limiting value of the velocity as t increases without bound.

(A) 0 (B) 2 (C) 5 (D) 10 (E) 17

21. A farmer wishes to enclose a rectangular piece of land along the straight part of a river. No fencing is needed along the river bank. The farmer has 2000 meters of fencing. What is the largest area that can be enclosed?

(A) $50,000$ m^2 (B) $1,000^2$ m^2 (C) $2(500^2)$ m^2

(D) $2(1,000)^2$ m^2 (E) $6(500^2)$ m^2



22. The number of lamps a company sells is a function of the price charged, and it can be approximated by the function $N(x) = 200 + 50x + 36.5x^2 - x^3$, where x is the price (in dollars) charged for each lamp and $0 \leq x \leq 37$. The price to maximize the number of lamps sold should be

(A) \$13 (B) \$20 (C) \$25 (D) \$37 (E) \$30

23. Use differentials to estimate the change in $\sqrt{x^2+5}$ when x increases from 2 to 2.123.

(A) 0.083 (B) 0.082 (C) 0.081 (D) 0.080 (E) 0.084

24. The velocity of a car (in feet/second) t seconds after starting from rest is given by the function $v(t) = 2\sqrt{t}$ $(0 \leq t \leq 30)$. Find the car's position at any time t.

(A) $\frac{2}{3}t^{3/2} + C$ (B) $\frac{4}{3}t^{3/2}$ (C) $\frac{4}{3}t^{1/2} + \frac{2}{3}$ (D) $\frac{4}{3}t^{1/2}$ (E) $\frac{2}{3}t^{3/2} + \frac{2}{3}$

25. Evaluate $\int (\sqrt{x} - 2e^x)\, dx$.

(A) $\frac{2}{3}x^{3/2} - 2e^x$ (B) $\frac{2}{3}x^{3/2} - 2e^x + C$ (C) $\frac{3}{2}x^{2/3} - 2e^x$

(D) $\frac{3}{2}x^{2/3} - 2e^x + C$ (E) $\frac{2}{3}x^{3/2} - \frac{2e^{x+1}}{x+1} + C$

26. Evaluate $\int xe^{-x^2} dx$.

(A) $-e^{-x^2} + C$ (B) $-\frac{1}{2}e^{-x} + C$ (C) $(1-2x^2)e^{-x^2} + C$ (D) $-\frac{1}{2}e^{-x^2} + C$

(E) $-2e^{-x^2} + C$

27. If $F'(x) = \sqrt{x}$ and $F(1) = 1$, then $F(4) =$

(A) 2/5 (B) 2 (C) 17/3 (D) 1/5 (E) 23/2

28. Calculate $\int_1^8 \left(4x^{1/3} + \frac{8}{x^2}\right) dx$.

(A) 49 (B) 50 (C) 51 (D) 52 (E) 54

29. Evaluate $\int_0^3 |1-x|\,dx$.

(A) $3/2$ (B) $5/2$ (C) $7/2$ (D) $9/2$ (E) $11/2$

30. Calculate $\int_1^5 x\sqrt{x-1}\,dx$.

(A) $\dfrac{272}{15}$ (B) 80 (C) $5\sqrt{5}-5$ (D) $\dfrac{40}{3}\sqrt{5}-\dfrac{16}{15}$ (E) $5\sqrt{5}$

31. Let R be the region under the graph of $f(x)=\dfrac{1}{x}$ on $[1,5]$. Find an approximation of the area using Riemann sums for 4 equal subintervals and left hand endpoints.

(A) $\ln(5)$ (B) $\dfrac{77}{60}$ (C) $\dfrac{25}{12}$ (D) $\dfrac{496}{315}$ (E) $\dfrac{58}{45}$

32. Let R be the region under the graph of $f(x)=\ln(x)$ on $[1,5]$. Find an approximation of the area using Riemann sums for 4 equal subintervals and right hand endpoints.

(A) $\ln(120)$ (B) $\ln(5)$ (C) $5\ln(5)-5$ (D) $5\ln(5)-4$ (E) $5\ln(5)-5$

33. Let $f(x)=x^2+1$. If you correctly compute the Riemann sum of f over the interval $[0,1]$ using two subintervals of equal length and choosing the midpoint of the interval, what answer do you get?

(A) 1.3125 (B) 1.5 (C) 1.625 (D) 2.625 (E) 3

34. If you want to calculate the Riemann sum of x^5 over the interval [2,5] using six subintervals of equal length and choosing the right endpoints, which of the following should you calculate?

(A) $2^5+(2.5)^5+3^5+(3.5)^5+4^5+(4.5)^5$

(B) $(2.5)^5+3^5+(3.5)^5+4^5+(4.5)^5+5^5$

(C) $\dfrac{1}{2}(2^5+(2.5)^5+3^5+(3.5)^5+4^5+(4.5)^5)$

(D) $\dfrac{1}{2}(2^5+(2.5)^5+3^5+(3.5)^5+4^5+(4.5)^5+5^5)$

(E) $\dfrac{1}{2}((2.5)^5+3^5+(3.5)^5+4^5+(4.5)^5+5^5)$

35. The area under the curve $y=\pi x^3$ and above the x-axis on the interval [1,2] is

(A) 2π (B) $\dfrac{7\pi}{4}$ (C) $\dfrac{15\pi}{4}$ (D) 4π (E) 7π

36. The area under the curve $y = 16 - x^2$ and above the x-axis is

(A) $\frac{64}{3}$ (B) 64 (C) $\frac{128}{3}$ (D) $\frac{256}{3}$ (E) $\frac{16}{3}$

37. Find the area of the region under the graph of $y = x(x^2-1)^3 + 1$ and above the x-axis on the interval [0, 2] is

(A) 2 (B) 4 (C) 8 (D) 10 (E) 12

38. Let $f(x) = x + 2\ln x$ and $g(x) = e^x$. Then, $(g \circ f)(x)$ is

(A) xe^{x^2} (B) x^2e^x (C) $e^x + x^2$ (D) $e^x + 2x$ (E) xe^{2x}

39. Find the vertical asymptotes of function $f(x) = \dfrac{2-x}{(1+x)^2}$.

(A) $x = -1$ (B) $x = 2$ (C) $y = -1$ (D) $y = 2$ (E) $x = 0$

40. The horizontal asymptote of graph of the function $g(x) = \dfrac{e^2x^3+1}{ex^3-e^4}$ is the line

(A) $y = -e^4$ (B) $y = -1/e^2$ (C) $y = 0$ (D) $y = e$ (E) $y = e + 1$

41. How many vertical asymptotes does the graph of $y = \dfrac{x^{17}}{(x-1)(x-2)(x-3)}$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

42. If $\lim\limits_{x\to-\infty} f(x) = 17$ then what sort of asymptote(s) must the graph of $y = f(x)$ have?

(A) a horizontal asymptote of $y = 17$

(B) a vertical asymptote at $x = 17$

(C) no vertical asymptote

(D) no horizontal asymptote

43. Find $\dfrac{dy}{dx}$ in terms of x and y when x and y are related by the equation $x + y + e^y = 3$.

(A) $-\dfrac{2}{e^y}$ (B) $-\dfrac{1}{e^y}$ (C) $-(1+e^y)$ (D) $-\dfrac{1}{1+e^y}$ (E) $2 - e^y$

44. Find an equation of the tangent line to the graph of $y = e^{2x} - \ln(x^2+1)$ at the point $(0, 1)$.

(A) $y = 2x + 1$ (B) $y = 2x - 1$ (C) $y - 1 = 2e^{2x} - \dfrac{1}{x^2+1}x$

(D) $y - 1 = 2e^{2x} - \dfrac{2x}{x^2+1}x$ (E) $y = x + 1$

45. Let $f(x) = 10x^9 - 9x^{10}$. Then f has inflection point(s) at $x =$:

(A) $0, 1$ (B) $0, \frac{8}{9}$ (C) $1, \frac{9}{8}$ (D) $0, \frac{8}{9}, 1$ (E) $0, 1, \frac{9}{8}$

46. Find the derivative of function $f(x) = 5^{x-3}$.

(A) $f'(x) = (x-3)5^{x-4}$ (B) $f'(x) = (\ln 5)5^{x-3}$ (C) $f'(x) = (\ln 5 + x - 3)5x^{x-3}$

(D) $f'(x) = (x-3)\ln 5$ (E) $f'(x) = (x-3)\ln 5^x$

47. The velocity (in feet/second) of a hawk flying straight down towards its prey is $v(t) = -3t$, where t is the number of seconds since it started flying straight down. If the hawk starts flying from 200 feet, i.e. $h(0) = 200$, what is its height h at t seconds?

(A) $1.5t + C$ (B) $200 + 1.5t$ (C) $-1.5t + 200$ (D) $200 - 1.5t$ (E) $-1.5t^2 + 200$

48. Evaluate $\int (3x^2 + 2)e^{x^3+2x} dx$.

(A) $e^{x^3+2x} + C$ (B) $(x^3 + 2x)e^{x^3+2x} + C$ (C) $e^{3x^2+2} + C$ (D) $(x^3 + 2x)e^{\frac{x^4}{4}+x^2} + C$

(E) $3x^2 e^{x^3+2x} + C$

49. If $F'(x) = e^x + x$ and $F(0) = 3$, then $F(2)$ equals

(A) $e^2 + 2$ (B) $e^2 + 4$ (C) $e^2 + 5$ (D) $e^2 + 6$ (E) $e^2 + 8$

50. Calculate $\int_1^4 \left(1 + \frac{1}{x^2}\right) dx$.

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$ (E) $\frac{21}{4}$

51. At 8:00 am a wading pool is filled with water. The temperature (in Farenheit) of the water in the pool t hours after it is filled is given by $g(t) = t^3 - 3t^2 + 58$ for $0 \le t \le 5$. What is the average temperature of the water between 9:00 am and noon?

(A) 43.5265 (B) 58 (C) 58.25 (D) 174.75 (E) 232

52. The area of the region bounded by the graph of $y = \frac{x+1}{x}$ and the x-axis from $x = 1$ to $x = e$ is

(A) 1 (B) e (C) $e + 1$ (D) $e - 1$ (E) $e + 2$

53. Snow is falling so that t hours after noon the rate at which it is falling is $\sqrt{t}$ cm per hour. How much snow (in cm) fell between 1 p.m. and 9 p.m.?

(A) 8 (B) $\frac{26}{3}$ (C) $\frac{52}{3}$ (D) 26 (E) 39

54. Bacteria in refrigerated milk grows exponential by doubling every 24 hours. The bacteria count in milk starts at 500 and milks spoils when it reaches a count of 4,000,000. How many days will it take for milk to spoil?

(A) $\ln(4000)$ (B) $\dfrac{\ln(8000)}{\ln(2)}$ (C) $\dfrac{\ln(4,000,000)}{\ln(500)}$ (D) 12 (E) 9.5

55. The half life of polonium-210 is 140 days. How long will it take 300 micrograms of polonium- 210 to decay to 80 micrograms?

(A) $140\dfrac{\ln 2}{\ln 5}$ (B) $80\dfrac{\ln 2}{\ln 5}$ (C) $\dfrac{-\ln 2}{140}$ (D) $80\dfrac{\ln 5}{\ln 2}$ (E) $140\dfrac{\ln 5}{\ln 2}$

56. A riverbank is eroding exponentially so that every year it loses 10% of its soil. How much of its soil will it have in 20 years?

(A) about 20% (B) about 12% (C) about 10% (D) about 6.5% (E) about 5.2%

57. A riverbank is eroding exponentially so that every year it loses 10% of its soil. How long will it take for it to lose half its soil?

(A) about 20 years (B) about 12 years (C) about 10 years (D) about 6.5 years

(E) about 5.2 years

58. A population of bacteria is growing exponentially so that it triples every 20 hours. How long does it take to double? (Pick the closest approximation.)

(A) 5.7 hours (B) 6.3 hours (C) 8.9 hours (D) 10 hours (E) 12.6 hours