MATH 121, Calculus I — Sample Exam I (Fall 2013)

Name: ____

KU ID No.: ____

Lab Instructor: ____

This exam has total value 200 points. It consists of two parts. The first part contains 14 multiple-choice questions, each worth 10 points. The second part contains 3 long-answer problems, each worth 20 points. There are 17 problems in total to be solved. Additionally the last page of the exam contains an extra-credit problem that is worth 20 points. This is strictly a closed-book exam and the use of calculators is prohibited.

Score

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Problem 11	
Problem 12	
Problem 13	
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Problem 16	
Problem 17	
Extra Credit	
Total	

Multiple-Choice Questions

Instructions: Place the appropriate letter for your answer for each problem in the blank box that is provided. Correct answers do not require work to receive full credit. However, partial credit can be awarded to incorrect answers based on work shown in the adjacent blank space. Hence, you are strongly advised to show work for each problem.

(1) [**10 points**] Find the value of $\lim_{x \to 0} \frac{\sin^2 x}{2x}$.

(A) 1.

(B) 2.

(C) 1/2.

(D) 0.

Answer:

(2) [**10 points**] Let $f(x) = xe^x$. Find the point $(x, f(x))$ at which the second derivative $f''(x)$ equals zero.

(A) $(2, 2e^2)$.

(B) $(0, 0)$.

(C) $(-2, -2e^{-2})$.

(D) $(-4, -4e^{-4})$.

Answer:

(3) [**10 points**] If $y = \tan^{-1}(x^2)$, determine $\dfrac{dy}{dx}$.

(A) $\dfrac{1}{1+x^4}$.

(B) $\dfrac{2x}{1+x^4}$.

(C) $2x \cdot \sec^2(x^2)$.

(D) $2x \cdot \tan^{-1}(x^2)$.

Answer: $\boxed{}$

(4) [**10 points**] What is the slope of the tangent line to the curve

$$x + y = xy$$

at the point $(2, 2)$?

(A) -1.

(B) -2.

(C) -3.

(D) -4.

Answer: $\boxed{}$

(5) [**10 points**] Which of the following statements are true? (Since there may be more than one correct answer, determine all correct answers.)

(A) If $\lim_{x \to a} f(x)$ exists, then f is continuous at a.

(B) If f is continuous at a, then f is differentiable at a.

(C) If f is continuous at a, then $\lim_{x \to a} f(x)$ exists.

(D) If f is differentiable at a, then f is continuous at a.

Answer: $\boxed{}$

(6) [**10 points**] Let f and g be functions that are differentiable everywhere. Suppose that

$$f(2) = 2, \qquad g(2) = 1, \qquad f'(1) = 3, \qquad g'(2) = -2.$$

Use this information to find the value of $(f \circ g)'(2)$.

(A) 6.

(B) 3.

(C) -3.

(D) -6.

Answer: $\boxed{}$

(7) [**10 points**] Suppose that the function g satisfies the following inequality

$$1 + x \leq g(x) \leq e^x$$

for all values of x. Find the value of $\lim_{x \to 0} g(x)$.

(A) 1.

(B) 2.

(C) 3.

(D) None of the above.

Answer: $\boxed{}$

(8) [**10 points**] Find the values of c that make the following piecewise-defined function continuous everywhere:

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2; \\ (c^2 - c)x - 8 & \text{if } x \geq 2. \end{cases}$$

(A) 2 and 4.

(B) -2 and 3.

(C) 0 and 8.

(D) -4 and -8.

Answer: $\boxed{}$

(9) [**10 points**] Find the point on the curve $y = 3x^2 - 4x + 5$ where the tangent line is parallel to the line $y = -22x + 7$.

(A) $(0, 5)$.

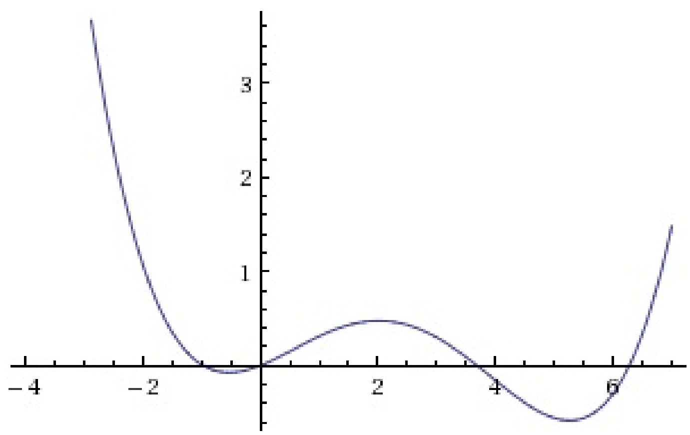
(B) $(-1, 12)$.

(C) $(-2, 25)$.

(D) $(-3, 44)$.

Answer: $\boxed{}$

(10) [**10 points**] Given the graph of the function f below, find the open interval(s) where the second derivative $f''(x) < 0$.



(A) $(4, \infty)$.

(B) $(-\infty, 1)$.

(C) $(1, 4)$.

(D) $(-\infty, 1)$ and $(4, \infty)$.

Answer: $\boxed{}$

(11) [**10 points**] Let $g(t) = \frac{1}{9}(2t+1)^3$. Find an equation of the tangent line to the graph of g at $t = 1$.

(A) $y = 6$.

(B) $y = 6t - 3$.

(C) $y = 3t$.

(D) $y = 6t - 6$.

Answer: $\boxed{}$

(12) [**10 points**] Determine the second derivative of the function $f(x) = x^2 \cdot \ln(2x)$.

(A) $2 \cdot \ln(2x) + 3$.

(B) $2 \cdot \ln(2x) + \frac{3}{2}$.

(C) 0.

(D) $2 + \frac{1}{2x}$.

Answer: $\boxed{}$

(13) [**10 points**] Let $y = \sqrt{x}(x+1)^5 e^{x^2}$. Determine $\dfrac{dy}{dx}$.

(A) $\sqrt{x}(x+1)^5 e^{x^2}\left(\dfrac{1}{2x} + 5x + e^{x^2}\right)$.

(B) $\sqrt{x}(x+1)^5 e^{x^2}\left(\dfrac{1}{2\sqrt{x}} + \dfrac{5}{x+1} + 2x\right)$.

(C) $\sqrt{x}(x+1)^5 e^{x^2}\left(\dfrac{5(x+1)^4}{2\sqrt{x}} + xe^{x^2}\right)$.

(D) $\sqrt{x}(x+1)^5 e^{x^2}\left(\dfrac{1}{2x} + \dfrac{5}{x+1} + 2x\right)$.

Answer: $\boxed{}$

(14) [**10 points**] Find the value of $\displaystyle\lim_{x\to\infty} \frac{3x^2+1}{\sqrt{4x^4+2x^2+1}}$.

(A) 0.

(B) 3.

(C) $\dfrac{3}{2}$.

(D) ∞.

Answer: $\boxed{}$

Long-Answer Problems

Instructions: Please show all necessary work and provide full justification for each answer. Place a box around each answer.

(15) (a) [**5 points**] Use the definition of the derivative to express $g'(3)$ as the limit of the difference quotient of $g(x)$ evaluated at $x = 3$.

(b) [**15 points**] Letting $g(x) = \sqrt{x+1}$, find the value of $g'(3)$ by explicitly evaluating the limit from Part (a).

Caution: Do <u>not</u> use the Power Chain Rule to solve Part (b).

(16) [**20 points**] The position function of a particle on the real line is given, in meters, by

$$s(t) = at^2 + bt + 3.$$

At time $t = 1$, the particle's velocity is 8 m/s and its acceleration is 6 m/s^2. Use this information to find the values of a and b.

(17) [**20 points**] Determine when the function $f(x) = x^3 - 3x + 1$ is increasing or decreasing.

(Extra Credit) [**20 points**] Choose **one** of the following problems.

(i) A problem to be selected from proposed problems by students.

(ii) A spherical balloon has air pumped into it at the rate of 18 in^3 per second. What is the rate at which the radius of the balloon is expanding when its volume is 36π in^3?

Note: The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.