**Math 104.** **Unit 3 review worksheet.** **Fall 2011.**

You should attempt the following sample problems. Others may be found on the listed textbook pages.

Since you will not have time to attempt all of these exercises during the workshop, you may like to focus on the following shorter list of topics that I believe were the most challenging for students in Unit 3:

- finding the value of a trigonometric function of any angle (4.4),
- using inverse trigonometric functions (4.7),
- verifying trigonometric identities (5.2),
- solving trigonometric equations (5.3).

**Section 4.1.**
Exercises: pg's. 265-268.
Topics: Radian and degree measure, finding coterminal angles complementary and supplementary angles, converting between radians and degrees.
Sample problems:

1. For each of the following angles given in radian measure, determine the quadrant in which the angle lies. Find (if possible) the complement and supplement of the angle. Determine two coterminal angles. Find the degree measure of the angle.
   (i) $\pi/6$.
   (ii) $7\pi/4$.
   (iii) $2\pi/3$.
   (iv) $8\pi/45$.
2. Rewrite the angle $150^\circ$ in radian measure as a multiple of $\pi$.
3. Look at word problems 91-94, on pg's. 267.

**Section 4.2.**
Exercises: pg's. 274-276.
Topics: Unit circle definitions of trigonometric functions, domain and period of trig functions, which trig functions are even vs. odd.
Sample problems:

1. Evaluate sine, cosine, and tangent of the real number $t = 11\pi/6$.
2. Evaluate $\sin(5\pi)$ exactly using the periodicity of the sine function.

**Section 4.3.**
Exercises: pg's. 284-287.
Topics: Right triangle definitions of trigonometric functions ("sohcahtoa"), trig functions evaluated at common angles, intro to fundamental trig identities.
Sample problems:

1. Sketch a right triangle corresponding to the trigonometric function $\sin\theta = 5/6$. Determine the third side of the triangle, and then find the other five trig functions of $\theta$.
2. Use $\sin 30^\circ = 1/2$ and $\tan 30^\circ = \sqrt{3}/2$ to find the other four trig functions of $\theta = 30^\circ$.
3. Look at word problems 77-82, on pg's. 286-287.

**Section 4.4.**
Exercises: pg's. 294-296.
Topics: Finding trigonometric functions of any angle using reference angles.
Sample problems:

1. Find the values of the six trig functions of $\theta$, given that:
   (i) $\cos\theta = -4/5$, and $\theta$ lies in Quadrant III.
   (ii) $\tan\theta$ is undefined, and $\pi \leq \theta \leq 2\pi$.
   (iii) $\sin\theta = 2/5$ and $\cos\theta < 0$.

**Section 4.5.**
Exercises: pg's. 304-308.
Topics: Graphs of sine and cosine functions, amplitude and period, transformations.
Sample problems:

1. Find the amplitude and period of $y = \frac{2}{3}\sin(\pi x)$.
2. Sketch the graph of $f(x) = 3 - \frac{1}{2}\sin\left(\frac{x}{2}\right)$ by hand (include two full periods).
3. Use a calculator to graph $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$ and identify its amplitude and period.

**Section 4.6.**
Exercises: pg's. 316-319.
Topics: Graphs of other trigonometric functions, period and domain.
Sample problems:

1. You are only required to memorize the graphs of sine and cosine, but you should be able to work with the graphs of the other four trig functions. For example, given the graph of $y = \sec x$, you should be able to sketch the graph of $y = 2\sec(x + \pi)$.

**Section 4.7.**
Exercises: pg's. 327-330.
Topics: Inverse trigonometric functions, domain and range, using inverse properties.
Sample problems:

1. Find the exact value of each of the following expressions without a calculator.
   (i) $\arcsin 1$.
   (ii) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$.
   (iii) $\arctan(-\sqrt{3})$.
2. Use the properties of the inverse functions to find the exact value of each of the following:
   (i) $\tan^{-1}\left(\tan\frac{11\pi}{6}\right)$.
   (ii) $\cos(\arctan(-1))$.
   (iii) $\cos\left(\arcsin\frac{24}{25}\right)$. (hint: make a sketch of a right triangle).
3. Write an algebraic expression that is equivalent to $\csc\left(\arctan\frac{x}{\sqrt{7}}\right)$.

**Section 4.8.**
Exercises: pg's. 337-342.
Topics: Applications of trigonometric functions, angle of depression and angle of elevation.
Sample problems:

1. Look at word problems 17-43, on pg's. 337-340.

**Section 5.1.**
Exercises: pg's. 357-359.
Topics: Using fundamental trigonometric identities.
Sample problems:

1. Use $\sec\phi = -\frac{17}{15}$ and $\sin\phi = \frac{8}{17}$, along with the fundamental trig identities, to evaluate the other four trig functions of $\phi$.
2. Use the fundamental identities to simplify each of the following:
   (i) $\frac{\cos^2 y}{1-\sin y}$.
   (ii) $1 - 2\sin^2 x + \sin^4 x$.
   (iii) $\sec^4 x - \tan^4 x$.
3. Verify the identity $\csc\theta\tan\theta = \sec\theta$ algebraically.
4. Use the substitution $x = 3\sin\theta$ to rewrite $\sqrt{9 - x^2}$ as a trigonometric expression of $\theta$.

**Section 5.2.**
Exercises: pg's. 365-367.
Topics: Verifying trigonometric identities.
Sample problems:

1. Verify each of the following identities:

(i) $\tan^2\theta + 6 = \sec^2\theta + 5$.
(ii) $(1+\sin x)(1-\sin x) = \cos^2 x$.
(iii) $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$.
(iv) $\frac{\sin^3\beta+\cos^3\beta}{\sin\beta+\cos\beta} = 1 - \sin\beta\cos\beta$.

**Section 5.3.**
Exercises: pg's. 376-379.
Topics: Solving trigonometric equations (simple and quadratic types), using inverse functions.
Sample problems:

1. Find all solutions of each of the following equations algebraically.
   (i) $\sqrt{2}\sin x + 1 = 0$.
   (ii) $3\cot^2 x - 1 = 0$.
   (iii) $\sec^2 x - \sec x = 0$.
   (iv) $(\cos(2x))(2\cos x + 1) = 0$.
   (v) $2\sin^2 x = 2 + \cos x$.

**Section 5.4.**
Exercises: pg's. 384-386.
Topics: Sum and difference formulas.
Sample problems:

1. You are not expected to memorize the sum and difference formulas, but you should be able to use them to complete such problems as: 7-30, 35-42, on pg. 384.

**Section 5.5.**
Exercises: pg's. 394-398.
Topics: Double-angle formulas and half-angle formulas.
Sample problems:

1. You are not expected to memorize the double-angle and half-angle formulas, but you should be able to use them to complete such problems as: 1-18, 47-52, on pg's. 394-395.