(1) First, we compute $f'(t) = \sqrt{4-t^2} - \frac{t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$. Then

$$f'(t) = 0 \implies 4 - 2t^2 = 0 \implies t^2 = 2 \implies t = \pm\sqrt{2},$$

and

$$f' \text{ is undefined} \implies \sqrt{4-t^2} = 0 \implies 4 - t^2 = 0 \implies t^2 = 4 \implies t = \pm 2,$$

so $x = \sqrt{2}$ is the only critical number of f in the interval $(-1, 2)$. Since

$$f(-1) = -\sqrt{4-(-1)^2} = -\sqrt{3} \approx -1.7321,$$
$$f(\sqrt{2}) = \sqrt{2} \cdot \sqrt{4 - (\sqrt{2})^2} = 2,$$
$$f(2) = 2\sqrt{4-(2)^2} = 0,$$

the Closed Interval Method implies that $f(-1) = -\sqrt{3}$ and $f(\sqrt{2}) = 2$ are the local minimum and local maximum values, respectively, of f on $[-1, 2]$.

(2) (a) By L'Hôpital's Rule, we have

$$\lim_{t\to 0} \frac{e^t - 1}{t^3} = \lim_{t \to 0} \frac{e^t}{3t^2} = \infty.$$

(b) Let $y = (\tan x)^x$, so $\ln y = \ln\left((\tan x)^x\right) = x \ln(\tan x)$. L'Hôpital's Rule (two times!) gives

$$\begin{aligned}
\lim_{t\to 0^+} \ln y &= \lim_{t\to 0^+} x\ln(\tan x) \\
&= \lim_{t\to 0^+} \frac{\ln(\tan x)}{1/x} \\
&= \lim_{t\to 0^+} \frac{\left(\frac{\sec^2 x}{\tan x}\right)}{-1/x^2} \\
&= \lim_{t\to 0^+} \frac{-x^2}{\sin x \cos x} \\
&= \lim_{t\to 0^+} \frac{-2x}{\cos^2 x - \sin^2 x} = 0.
\end{aligned}$$

Thus, $\lim_{t\to 0^+} (\tan x)^x = \lim_{t\to 0^+} y = \lim_{t\to 0^+} e^{\ln y} = e^{\lim_{t\to 0^+} \ln y} = e^0 = 1$.

(3) Let r be the radius length of the cylinder, and let h be its height. Then the volume and surface area of the can are given by $V = \pi r^2 h$ and $S = 2\pi r^2 + 2\pi r h$. Using the fact that $V = 8$, we solve the former equation for $h = \frac{8}{\pi r^2}$. Substituting into the the equation for S, we obtain

$$S = f(r) = 2\pi r^2 + 2\pi r\left(\frac{8}{\pi r^2}\right) = 2\pi r^2 + \frac{16}{r}.$$

Next, we compute $f'(r) = 4\pi r - \frac{16}{r^2} = \frac{4(\pi r^3 - 4)}{r^2}$, and

$$f'(r) = 0 \implies \pi r^3 - 4 = 0 \implies \pi r^3 = 4 \implies r = (4/\pi)^{1/3}.$$

Taking this critical number, we compute the corresponding height $h = \frac{8}{\pi(4/\pi)^{2/3}} = \frac{4}{(2\pi)^{1/3}}$. Thus, the dimensions that will minimize the amount of metal needed to build the can are a radius of $(4/\pi)^{1/3}$ cm and a height of $\frac{4}{(2\pi)^{4/3}}$ cm.

To convince yourself that the critical number $r = (4/\pi)^{1/3}$ corresponds to a minimum (and not a maximum) of $S = f(r)$, simply notice that $f''(r) = 4\pi + 32/r^3$, which is positive for all $r > 0$ (i.e. the surface area function is concave upward, so any critical number where the derivative is zero must correspond to an absolute minimum).

(4) We are given the constant acceleration function $a(t) = -22$, so the general formula for velocity is $v(t) = -22t + C$. Using the fact that $v(0) = 70$, we can rewrite this as $v(t) = -22t + 70$. Thus, the general formula for position is $s(t) = -11t^2 + 70t + D$. Assuming that $s(0) = 0$ (i.e. "position 0" occurs at "time 0"), we can simplify this equation to $s(t) = -11t^2 + 70t$. Now, $v(t) = 0 \implies -22t + 70 = 0 \implies t = 3.182$, so the distance traveled by the car before it comes to a stop is given by the position when $t = 3.182$, i.e. $s(3.182) = -11(3.182)^2 + 70(3.182) = 111.364$ ft.

(5) (a) Splitting $[0, \pi]$ into the four equal-length subintervals $[0, \pi/4]$, $[\pi/4, \pi/2]$, $[\pi/2, 3\pi/4]$, $[3\pi/4, \pi]$, the left endpoint approximation for the area under the curve is

$$\begin{aligned} L_4 &= \frac{\pi}{4} f(0) + \frac{\pi}{4} f(\pi/4) + \frac{\pi}{4} f(\pi/2) + \frac{\pi}{4} f(3\pi/4) \\ &= \frac{\pi}{4}\left[0 + \frac{\pi}{4}\cos\left(\frac{\pi}{4}\right) + \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) + \frac{3\pi}{4}\cos\left(\frac{3\pi}{4}\right)\right] \\ &\approx -0.8724. \end{aligned}$$

(b) Using a calculator to sketch the graph of f, it is easy to see that this approximation is an overestimate (the true area is $\int_0^\pi x \cos x \, dx = -2$, as we will learn how to compute in Sec 5.6).

(6) (a) The limits of integration are both equal to 7, so $\displaystyle\int_7^7 x^{2x^{e^{\sqrt{\ln x}}}} dx = 0$.

(b) $$\int_3^1 f(x)\,dx = -\int_1^3 f(x)\,dx = -\left(\int_1^{12} f(x)\,dx - \int_3^{12} f(x)\,dx\right) = -(-5 - 2) = 7.$$

(7) (a) Letting $u = \tan x$, the Chain Rule and FTC1 give

$$\begin{aligned} f'(x) &= \frac{d}{dx}\int_0^{\tan x} \sqrt{t - \sqrt{t}}\,dt \\ &= \frac{d}{dx}\int_0^{u} \sqrt{t - \sqrt{t}}\,dt \\ &= \frac{d}{du}\left[\int_0^{u} \sqrt{t - \sqrt{t}}\,dt\right] \cdot \frac{du}{dx} \\ &= \sqrt{u - \sqrt{u}} \cdot \sec^2 x \\ &= \sec^2 x \sqrt{\tan x - \sqrt{\tan x}}. \end{aligned}$$

(b) By FTC1, $f'(x) = \frac{x^2}{x^2+x+2}$. It follows that $f''(x) = \frac{x(x+4)}{(x^2+x+2)^2}$. Setting $f''(x) = 0$ yields $x = 0$ or $x = -4$, while setting its denominator equal to zero shows that f'' is never undefined. A sign test of the intervals $(-\infty, -4)$, $(-4, 0)$, and $(0, \infty)$ shows that f'' is only negative on $(-4, 0)$. Hence, the function defined by $f(x) = \int_0^x \frac{t^2}{t^2+t+2}\,dt$ is concave downward on the interval $(-4, 0)$.

(c) By FTC2,

$$\begin{aligned} \int_1^2 \frac{(x-1)^3}{x^2}\,dx &= \int_1^2 \frac{x^3 - 3x^2 + 3x - 1}{x^2}\,dx \\ &= \int_1^2 \left(x - 3 + \frac{3}{x} - \frac{1}{x^2}\right) dx \\ &= \left(\frac{x^2}{2} - 3x + 3\ln|x| + \frac{1}{x}\right)\Bigg|_1^2 \\ &= \left(\frac{2^2}{2} - 3(2) + 3\ln(2) + \frac{1}{2}\right) - \left(\frac{1^2}{2} - 3(1) + 3\ln(1) + \frac{1}{1}\right) \\ &= -2 + 3\ln(2) \approx 0.0794. \end{aligned}$$

(8) (a) Since $a(t) = 2t + 3$, we have $v(t) = t^2 + 3t + C$ for some constant C. Then $v(0) = -4$ implies $C = -4$, so $v(t) = t^2 + 3t - 4$.

(b) Notice that $v(t) = t^2 + 3t - 4 = (t + 4)(t - 1)$, so $v(t) = 0 \implies t = -4, 1$. A sign test shows that $v(t) < 0$ on $(0, 1)$ and $v(t) > 0$ on $(1, 3)$. Thus, the total distance traveled between $t = 0$ and $t = 3$ s is given by

$$\begin{aligned}
\int_0^3 |v(t)|\,dt &= -\int_0^1 v(t)\,dt + \int_1^3 v(t)\,dt \\
&= -\left(\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right)\Bigg|_0^1 + \left(\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right)\Bigg|_1^3 \\
&= -\left(\frac{1^3}{3} + \frac{3(1)^2}{2} - 4(1)\right) + \left(\frac{3^3}{3} + \frac{3(3)^2}{2} - 4(3)\right) - \left(\frac{1^3}{3} + \frac{3(1)^2}{2} - 4(1)\right) \\
&= \frac{89}{6} \approx 14.8333 \text{ m.}
\end{aligned}$$

(9) (a) Let $u = \tan\theta$. Then $du = \sec^2\theta\,d\theta$, and we have

$$\int \tan^2\theta \sec^2\theta\,d\theta = \int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3\theta}{3} + C.$$

(b) Let $u = e^z + z$. Then $du = (e^z + 1)\,dz$, and the new limits of integration are $u = e^0 + 0 = 1$ and $u = e^1 + 1 = e + 1$. Hence,

$$\int_0^1 \frac{e^z + 1}{e^z + z} = \int_{u=1}^{u=e+1} \frac{1}{u}\,du = \ln|u|\Big|_{u=1}^{u=e+1} = \ln(e+1) - \ln(1) = \ln(e+1) \approx 1.3133.$$