Name: ______________________________

Complete the following problems to the best of your ability. Clearly number each question, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

(1) (5 pts.) State the domain of each of the following functions:

(a) $f(x) = x^7 - 5x^2 + 1$.

(b) $g(x) = \dfrac{7x}{2x^2 - 3x}$.

(c) $h(x) = \dfrac{\ln(x+2)}{\sqrt{x-4}}$.

(2) (10 pts.) For $f(x) = 3x^2 + 2x + 1$ and $g(x) = x + 3$, find and simplify: $f + g$, $f - g$, fg, f/g, $f \circ g$.

(3) (5 pts.) Solve $m = 24 \cdot 2^{-t/25}$ for t.

(4) (20 pts.) Evaluate each of the following limits:

(a) $\lim\limits_{s \to 0} (2s^2 - 1)(2s + 4)$.

(b) $\lim\limits_{x \to a} \sqrt[3]{5f(x) + 3g(x)}$, given that $\lim\limits_{x \to a} f(x) = 3$ and $\lim\limits_{x \to a} g(x) = 4$.

(c) $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x + 5}$.

(d) $\lim\limits_{t \to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} \right)$.

(e) $\lim\limits_{x \to \infty} \dfrac{2x^2 - 1}{x^3 + x^2 + 1}$.

(5) (10 pts.) If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$, find $\lim\limits_{x \to 4} f(x)$.

(6) (15 pts.) Consider the functions $f(x) = \dfrac{x+1}{x^2 - 2x + 3}$ and $g(x) = \begin{cases} (x-3)^2 + 3, & \text{if } x \leq 3; \\ -x + 8, & \text{if } x > 3. \end{cases}$

(a) Find the values of x for which the function f is continuous.

(b) Find $\lim\limits_{x \to 3^+} g(x)$, $\lim\limits_{x \to 3^-} g(x)$, and $\lim\limits_{x \to 3} g(x)$.

(c) Use the results of part (b) to find the values of x for which the function g is continuous.

(7) (10 pts.) Explain why the function $f(x) = x^3 - 2x^2 + 3x + 2$ is continuous for all values of x in the interval $[-1, 1]$. Then use the Intermediate Value Theorem to show that f must have at least one zero in the interval $(-1, 1)$.

(8) (15 pts.) Let $f(x) = 2x^2 + 1$.

(a) <u>Use a limit definition</u> to find f', the derivative of f.

(b) Find an equation of the tangent line to the curve at the point $(1, 3)$.

(9) (15 pts.) Sketch the graph of a function that satisfies all of the given conditions:

$$f'(1) = f'(-1) = 0,\ f'(x) < 0 \text{ if } |x| < 1,$$
$$f'(x) > 0 \text{ if } 1 < |x| < 2,\ f'(x) = -1 \text{ if } |x| > 2,$$
$$f''(x) < 0 \text{ if } -2 < x < 0,\ \text{inflection point at } (0,1).$$

Bonus (10 pts.) Carefully sketch the graph of a function with all of the following characteristics:

- it is defined at every point in the interval $[0, 10]$;
- it displays a *different* type of discontinuity at each of $x = 2$, $x = 4$, and $x = 6$;
- it is continuous, but not differentiable at the point $x = 8$.