

EXPONENTIAL AND LOGARITHM PROBLEMS

1. A certain menacing biological culture (aka the Blob) grows at a rate proportional to its size. When it arrived unnoticed one Wednesday noon in Chicago's Loop, it weighed just one gram. By 4:00 pm rush hour it weighed 4 grams. The Blob has its "eye" on the Willis Tower (formerly known as the Sears Tower), a tasty morsel weighing around 300000000000 (i. e. 3×10^{12}) grams. The Blob intends to *eat* the Willis Tower as soon as it weighs 1000 times as much (i.e. 3×10^{15} grams). By what time must the Blob be stopped? Will Friday's rush hour commuters be delayed?
2. A bacterial culture is placed in a large glass bottle. Suppose that the volume of the culture doubles every hour, and the bottle is full after one day.
 - (a) If the culture was placed in the bottle at time $t = 0$ hours, when was the bottle half full?
 - (b) Assume the bottle is "almost empty" when the culture occupies less than 1 % of its volume. How long was the bottle "almost empty"?
3. The police guard gave Sara a cold look, but his voice was polite as he directed her to the room she sought. "Don't touch anything, please, Ms Anderson. The Chief said I had to let you in, but he said to tell you to mind your fingers." "Thank you," Sara replied coolly. "The Chief knows he can trust me." The guard opened his mouth as if to speak, but he merely shook his head and withdrew.

Sara was standing in what appeared to be a combination bedroom and laboratory. A relative had found Dr. Howell's body on the floor of this room that morning. By 9:00 am the coroner had completed his examination; he stated that the death was due to a severe blow to the head and that Dr. Howell had been dead between 36 and 40 hours. It seemed critical to Sara to know exactly when Dr. Howell had died so that she could eliminate certain suspects. But how could she possibly discover exactly when he was killed? Puzzled, she wandered around the small, cluttered room, being careful not to touch anything. The old doctor apparently was conducting an experiment when he was killed. Sara absent-mindedly read from the notebook which was lying open on the bench.

The fungus grows at a rate proportional to its current weight.

"Great," she thought. "I am here to investigate a murder and instead I'm getting a biology lesson." At a loss for what else to do, she continued reading.

To exemplify this biological truth, I place the fungus on a scale and record its weight at various times:

10g 5:30 pm

12g 6:15 pm

13g

"Hmmm, the poor guy didn't even get to finish the last entry," thought Sara. Sara suddenly frowned in concentration. She searched her pockets and found a pen and a napkin. When the guard entered the room a few minutes later, Sara had just finished scribbling on the napkin. She smiled as she shoved the napkin back into her pocket. "Don't worry, " Sara said cheerfully, "I'm leaving. I now know exactly when Dr. Howell was killed." The guard looked at her sourly as she left.

When was Dr. Howell killed and how did Sara know?

4. The body concentrates iodine in the thyroid gland. This observation leads to the treatment of thyroid cancer by the injection of radioactive iodine into the bloodstream. One isotope used has a half life of approximately 8 days and decays exponentially in time.
- (a) If 50 micrograms of this isotope are injected, what amount remains in the body after 3 weeks?
 - (b) Suppose it is desired that the amount of iodine in the bloodstream be maintained at no less than 20 micrograms. When should a second injection be scheduled so that it occurs precisely when the original injection has decayed to this amount?
5. When a foreign substance is introduced into the body, the body's defense mechanisms move to break down the substance and excrete it. The rate of excretion is usually proportional to the concentration in the body, and the half-life of the resulting exponential decay is referred to as the *biological half-life* of the substance. Suppose a patient is administered an antibiotic by injection. If after 12 hours, 30% of the antibiotic has been excreted by the body, what is the half-life of the substance? How fast is the body excreting the antibiotic at 24 hours after the injection?