

DEPARTMENT OF  
MATHEMATICS  
UNIVERSITY OF KANSAS

6 (30) \_\_\_\_\_

Practice Final Exam

7 (30) \_\_\_\_\_

MATH 122 Spring 2014

8 (30) \_\_\_\_\_

Your Name: \_\_\_\_\_

9 (30) \_\_\_\_\_

KUID Number: \_\_\_\_\_

10 (30) \_\_\_\_\_

1 (10) \_\_\_\_\_

11 (30) \_\_\_\_\_

2 (10) \_\_\_\_\_

12 (30) \_\_\_\_\_

3 (10) \_\_\_\_\_

13 (30) \_\_\_\_\_

4 (10) \_\_\_\_\_

14 (30) \_\_\_\_\_

5 (10) \_\_\_\_\_

15 (30) \_\_\_\_\_

Total (350) \_\_\_\_\_

## Some Useful Formulas

0.

## Multiple Choice Questions (10 points each)

- (1) Find a unit vector orthogonal to both  $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  and  $-\mathbf{i} + \mathbf{k}$ .
- (A)  $\frac{1}{\sqrt{11}} \langle -1, 3, -1 \rangle$
  - (B)  $\frac{1}{\sqrt{11}} \langle -1, -3, -1 \rangle$
  - (C)  $\frac{1}{\sqrt{11}} \langle 1, 3, -1 \rangle$
  - (D)  $\frac{1}{\sqrt{11}} \langle -1, 3, 1 \rangle$
- (2) Find the volume of the parallelepiped spanned by the vectors  $\langle 1, 1, -2 \rangle$ ,  $\langle 3, -2, 1 \rangle$ ,  $\langle 0, 1, -5 \rangle$ .
- (A) 18
  - (B) 7
  - (C) 4
  - (D) 23
- (3) Find the acceleration vector of a particle with position function  $\vec{r}(t) = \langle t \ln(t), t, e^{-t} \rangle$  at  $t = 1$ .
- (A)  $\langle 0, 0, e^{-1} \rangle$
  - (B)  $\langle 1, 0, e \rangle$
  - (C)  $\langle 1, 0, e^{-1} \rangle$
  - (D)  $\langle 0, 1, e^{-1} \rangle$
- (4) Find the equation of the tangent plane to the surface  $xy + yz + zx = 3$  at the point  $(1, 1, 1)$ .
- (A)  $x + y + z = 1$
  - (B)  $x + yz + z = 3$
  - (C)  $x + y + z = 3$
  - (D)  $2x + 2y + 2z = 3$
- (5) (A)
- (B) 1.07
  - (C)
  - (D) 1.04

**“Show Your Work” Questions (30 points each)**

- (6) Find the plane that contains the line  $x = 1 + t, y = 2 - t, z = 4 - 3t$  and is parallel to the plane  $3x - 7z = 12$ .
- (7) An athlete throws a shot at an angle of  $45^\circ$  at an initial speed of  $v_0 = 43 ft/sec$ . It leaves his hand at  $7 ft$  above the ground. How high does the shot go? How far is the shot landing? Assume that  $g = 33 ft/sec^2$ .

- (8) Let  $C$  be the curve with equations  $x = 2 - t^3, y = 2t - 1, z = \ln(t)$ .
- Find the point where  $C$  intersects the  $xz$  plane.
  - Find the equations of the tangent line at the point  $(1, 1, 0)$ .
  - Find the equation of the normal plane at the point  $(1, 1, 0)$ .
- (9) Find the extreme values of the function  $f(x, y) = 2x^2 + 3y^2 - 4x - 5$  on the region  $D : x^2 + y^2 \leq 16$ .

(10) Let  $u(x, y) = f(x^2 + 1/y)$ . Show that  $u$  satisfies

$$u_x + 2xy^2u_y = 0.$$

(11) For the function  $f(x, y, z) = x^2e^{yz^2}$ , compute the gradient. When is the directional derivative of  $f$  at the point  $\langle 1, 1, 1 \rangle$  a maximum? When is it a minimum?

(12) Compute

$$\int_D \cos(y^2) dx dy,$$

where  $D = \{(x, y) : 0 \leq x \leq 1, x \leq y \leq 1\}$ .

(13) Find the volume of the solid bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $y + z = 3$ .

- (14) Lamina occupies the part of the disk  $x^2 + y^2 \leq 1$  in the first quadrant. Find its center of mass, if the density is proportional to the square of its distance from the origin.

- (15) Evaluate

$$\int \int \int_E x^2 dx dy dz,$$

where  $E$  is the region bounded by the hemispheres  $y = \sqrt{9 - x^2 - z^2}$  and  $y = \sqrt{16 - x^2 - z^2}$ .