
Names: _____

Work in groups of five to complete the following problems. Each problem should be solved on a separate sheet of paper and each sheet of paper should have at least one group member's name on it. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

1. (10 pts.) A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ that lies in the **first and second quadrants**. The density at any point is proportional to the **square of its distance** from the y -axis.

(a) Find the moments M_x and M_y . (Hint: can you use symmetry to simplify the problem?)

(b) Find the total mass m and center of mass (\bar{x}, \bar{y}) . (Hint: you may need the identity $\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$.)

2. (5 pts.) Evaluate $\iiint_E yz \cos(x^5) dV$, where $E = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq x, x \leq z \leq 2x\}$.

3. (5 pts.) Evaluate $\iiint_E 1 dV$, where $E = \{(r, \theta, z) : 0 \leq r \leq 4, 0 \leq \theta \leq 2\pi, r \leq z \leq 4\}$.

4. (5 pts.) Evaluate $\iiint_E e^{\sqrt{x^2 + y^2 + z^2}} dV$, where E is enclosed by $x^2 + y^2 + z^2 = 9$ in the **first octant**.

Hint for 4: recall that if f is continuous on a spherical wedge

$$E = \{(\rho, \theta, \phi) : a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\},$$

then

$$\iiint_E f(x, y, z) dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi.$$

The conversion formula $x^2 + y^2 + z^2 = \rho^2$ may also be useful.