Math 115 - Practice Problems for Test 1

Answer the following true or false. No justification needed.

1. T F If $f(x)$ is not defined at $x = a$, then $\lim_{x\to a} f(x)$ does not exist.

2. T F If $\lim_{x\to a} f(x)$ exists, then both $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist.

3. T F If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists.

4. T F If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist.

5. T F If $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist, then at least one of the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ does not exist.

6. T F If $\lim_{x\to a} f(x)$ exists, then f is continuous at $x = a$.

7. T F If $f(x)$ is continuous at $x = a$, then $\lim_{x\to a} f(x)$ exists.

8. T F If $f(x)$ is continuous at $x = x_0$, then $f(x)$ has a derivative at $x = x_0$.

9. T F If $f(x)$ is differentiable at $x = x_0$, then $f(x)$ is continuous at $x = x_0$.

10. T F If $f(x)$ and $g(x)$ are continuous, then $f(x)g(x)$ is continuous.

Multiple-Choice Problems: no justification needed.

1. The domain of function $f(x) = \dfrac{x+3}{2x^2 - x - 3}$ is

(A) $(-\infty, +\infty)$ (B) $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$ (C) $(-\infty, -1) \cup (-1, +\infty)$

(D) $(-\infty, -1) \cup (-1, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$ (E) None of the above

2. The domain of function $f(x) = \dfrac{2x}{\sqrt{x^2 - 4}}$ is

(A) $(2, +\infty)$ (B) $(-\infty, -2)$ (C) $(-\infty, -2] \cup [2, +\infty)$

(D) $(-\infty, -2) \cup (2, +\infty)$ (E) None of the above

3. Let f be the function defined by $\dfrac{\sqrt{x+1}}{x-2}$. The domain of f is

A) $(-\infty, 2) \cup (2, \infty)$ B) $(-1, 2] \cup [2, \infty)$ C) $[-1, 2) \cup (2, \infty)$ D) $[1, \infty)$ E) None of the above

4. $\lim\limits_{x \to 2} \dfrac{x^2 - 9}{x - 3} =$ (A) 1 (B) does not exist (C) 6 (D) 5 (E) None of the above

5. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x} =$ (A) $\frac{1}{2}$ (B) limit does not exist (C) 0 (D) -1 (E) None of the above

6. $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x + 4}{2x^2 - 3x + 1} =$ (A) 4 (B) $\dfrac{3}{2}$ (C) limit does not exist (D) 0 (E) None of the above

7. $\lim\limits_{x \to -\infty} \dfrac{x^2 + 3}{x + 1} =$ (A) 1 (B) 0 (C) 2 (D) 3 (E) None of the above

8. $\lim\limits_{t \to 5} \dfrac{t - 5}{t^2 + 5} =$ (A) 0 (B) 1 (C) 5 (D) 10 (E) does not exist

9. $\lim\limits_{t \to 5} \dfrac{t^2 + 5}{t - 5} =$ (A) 0 (B) 1 (C) 5 (D) 10 (E) does not exist

10. $\lim\limits_{t \to 5} \dfrac{t^2 - 25}{t - 5} =$ (A) 0 (B) 1 (C) 5 (D) 10 (E) does not exist

11. $\lim\limits_{s \to \infty} \dfrac{5s^{14} - 14s^5}{14s^{13} - 5s^4} =$ (A) 5/14 (B) 14/5 (C) -14/5 (D) 0 (E) does not exist

12. $\lim\limits_{s \to \infty} \dfrac{14s^{13} - 5s^4}{5s^{14} - 14s^5} =$ (A) 5/14 (B) 14/5 (C) -14/5 (D) 0 (E) does not exist

13. $\lim\limits_{s \to \infty} \dfrac{14s^{14} - 5s^5}{5s^{14} - 14s^5} =$ (A) 5/14 (B) 14/5 (C) -14/5 (D) 0 (E) does not exist

14. Let $f(x) = \begin{cases} \sqrt{3x+7} & \text{for } x < 3, \\ 10 - 2x & \text{for } x \geq 3. \end{cases}$

(A) f is not defined at $x = 3$. (B) $\lim\limits_{x \to 3} f(x)$ does not exist. (C) f is continuous at $x = 3$.

(D) f is differentiable at $x = 3$. (E) None of the above

15. Let $f(x) = \begin{cases} x^2 - 2x + 4 & \text{if } x < -1, \\ 0 & \text{if } x = -1, \\ x + 2 & \text{if } x > -1. \end{cases}$

(A) $\lim\limits_{x \to (-1)^-} f(x) = 7$ and $\lim\limits_{x \to -1^+} f(x) = 1$ (B) $\lim\limits_{x \to (-1)^-} f(x) = 0$ and $\lim\limits_{x \to -1^+} f(x) = 0$

(C) $\lim\limits_{x \to -1} f(x)$ exists (D) $\lim\limits_{x \to (-1)^-} f(x) = 1$ and $\lim\limits_{x \to -1^+} f(x) = 1$ (E) None of the above.

16. Let $f(x) = x^2 - 4x$. When simplified, the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ becomes

(A) 2 x + h - 4 (B) 2 (C) 0 (D) 2 x - 4 (E) None of the above

17. Let $f(x) = \frac{1}{x}$. When simplified, the difference quotient $\frac{f(x+h) - f(x)}{h}$ becomes

(A) $\frac{1}{x(x+h)}$ (B) $\frac{1}{x(h-x)}$ (C) $-\frac{1}{x^2}$ (D) $\frac{1}{x(x-h)}$ (E) $\frac{-1}{x(x+h)}$

18. What is the profit in terms of the cost $C(x)$ to produce x units and the unit price $p(x)$ at which x units will sell ?

(A) $x(p(x)-C(x))$ (B) $p(x)-xC(x)$ (C) $p(x)+xC(x)$ (D) $xp(x)-C(x)$ (E) None of the above

19. Given the demand function $p = d(x) = -0.01x^2 - 0.1x + 10$ and the supply function $p = s(x) = 0.01x^2 + 0.2x + 5$, where p is the unit price and x represents the quantity, find the equilibrium quantity and the equilibrium price.

(A) $x = 5$ and $p = 6.25$ (B) $x = 10$ and $p = 8$ (C) $x = 15$ and $p = 10.25$

(D) $x = 20$ and $p = 13$ (E) None of the above

20. It is known that $f(x)$ is continuous in $(-\infty, \infty)$ and $f(-1) = -2$, $f(0) = 2$, and $f(2) = 4$. Which of the following statements is True ?

(A) f must have a zero in (-1,0). (B) f must have a zero in (-2,-1).

(C) f must have a zero in (2,4). (D) f must have a zero in (0,2).

(E) None of the above is true.

21. A ball is thrown straightly up into the air so that its height (in feet) after t seconds is given by $s(t) = -16t^2 + 64t$. The velocity of the ball at time $t = 1$ is

(A) 50 ft/sec (B) 2 ft/sec (C) 32 ft/sec (D) 48 ft/sec (E) None of the above

22. An equation for the line tangent to the curve $y = x^2 - 2x + 5$ at the point (-2, 1) is

(A) $y + 1 = (2x - 2)(x + 2)$ (B) $y - 1 = (2x - 2)(x + 2)$ (C) $y = \frac{-x}{2}$

(D) $y = -6x - 11$ (E) $y = 6x + 11$

23. A manufacturer has monthly fixed costs of \$40,000 and a production cost of \$8 per unit produced. The product sells for \$12 each unit. The profit (or loss) when the production level is 12,000 units is

A) \$8000 B) \$48,000 C) \$40,000 D) -\$48,000 E) None of the above.

24. If $\lim_{x \to 2^+} f(x) = \pi$ and $\lim_{x \to 2^-} f(x) = 3.14$ which of the following is true?

(A) f is continuous at $x = 2$. (B) $\lim_{x \to 2} f(x) = \pi$. (C) $\lim_{x \to 2} f(x)$ does not exist.

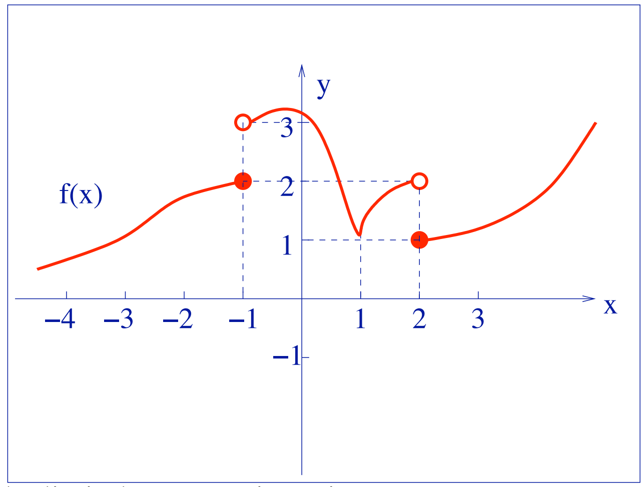
(D) $f(2) = 3.14$. (E) f is differentiable at $x = 2$.

Fill in the Blanks Problems (No justification needed.)

1. The line tangent to the graph of a function f at the point $(1, -3)$ is $y = 3x - 6$.

(a) $f(1) =$ (b) $f'(1) =$

2. Consider the graph of f(x) shown to the right.



(a) $\lim_{x \to (-1)^-} f(x) =$ (b) $\lim_{x \to -1} f(x) =$

(c) $\lim_{x \to 2} f(x) =$ (d) $f(2) =$

(e) $f(1) =$ (f) $\lim_{x \to 2^+} f(x) =$

(f) $f(x)$ is NOT continuous at $x =$

(g) $f(x)$ is NOT differentiable at $x =$

3. Find the following limits if they exist. If the limit does not exist write DNE.

(a) $\lim_{x \to 4} \frac{x^2 - 16}{x^2 - 2x - 8}$ (b) $\lim_{t \to 0} \frac{3t - |t|}{t}$ (c) $\lim_{x \to \infty} \frac{6x^3 + 4x^2 - 17x}{x^4 + 3x}$

(d) $\lim_{x \to \infty} \frac{5x^{22} - 3x^{17} + 99}{10x^{22} - 17}$ (e) $\lim_{x \to 3} 3x^2 - 4$ (f) $\lim_{h \to 0} \frac{(4 + h)^2 - 16}{h}$

(g) $\lim_{x \to -\infty} \frac{2x^3 + 6}{x^2 + 1}$ (h) $\lim_{x \to -3} \frac{x^2 + 5x + 6}{x + 3}$ (i) $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x + 3}$

(j) $\lim_{x \to \infty} \frac{x + 3}{x^2 + 5x + 6}$ (k) $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x^2 + 3}$

Essay Questions - SHOW YOUR WORK!!

1. Let $f(x) = x - x^2$. Find the slope of the line tangent to the graph of f at the point $x = a$ by using the definition of the derivative. Using this answer, find an equation of the line tangent to the graph at $x = 2$.

2. The mathematics office sells sample midterms for 50 cents each. The cost of producing x tests (in cents) is given by $C(x) = 1000 - 60x + x^2$.

(a) Find the equations for the revenue function $R(x)$ and profit function $P(x)$.

(b) For what production levels is $P(x) = 0$? Solve this algebraically.

(c) Find the rate of change of the profit function with respect to the production level x.

(d) Find the rate of change of the profit function when $x = 50$.

(e) Find the production level which yields maximum profit. (Hint: look at the graph.)

3. Let $f(x) = 2^x$. Find the following using your graphing calculator, rounding to 3 decimal places.

(a) An equation of the line tangent to f at $x = 1.7$ (Use zDecimal window)

(b) Find $f(.52)$ and $f(3.9)$ (c) Find $f'(2.7)$.

4. Find the slope of the line tangent to the graph of $f(x) = \dfrac{1}{x+17}$ by using the limit definition of the slope. Using this answer, find the equation of the line tangent to the graph of $f(x) = \dfrac{1}{x+17}$ at x=17.

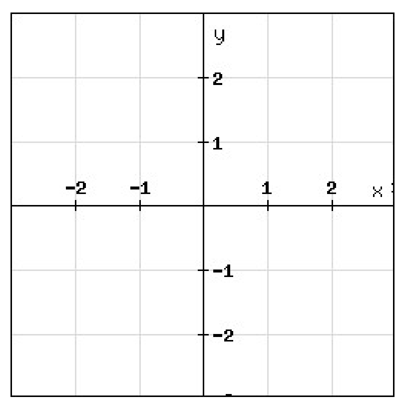
5. A farmer has 120 meters of fencing with which to make a rectangular pen. The pen is to have one internal fence running parallel to the end fence that divides the pen into two sections. The length of the larger section is to be twice the length of the smaller section. Let x represent the length of the smaller section. Find a function in x that gives the total area of the pens.

6. A tool rental company determines that it can achieve 500 daily rentals of jackhammers per year at a daily rental fee of \$30. For each \$1 increase in rental price, 10 fewer jackhammers will be rented. Let x represent the number of \$1 increases. Find the function $R(x)$ giving total revenue from rental of the jackhammers.

7. A homeowner wishes to enclose 800 square feet of garden space to be laid out in the shape of a rectangle. One side of the space is to have a stone wall costing \$50 per running foot. The other three sides are to have cedar fencing costing \$20 per foot. Let x represent the length of the stone wall. Find a function in x giving total cost of the enclosure.

8. Carefully graph

$$f(x) = \begin{cases} \sqrt[3]{x} + 1 & -1 \leq x < 1 \\ 2x - 1 & 1 \leq x \leq 2. \end{cases}$$



9. Show there is a real number that is equal to its cube plus one.

10. Find a function $f(x)$ and a real number a such that $f'(a) = \displaystyle\lim_{h \to 0} \frac{\sqrt{h+9} - 3}{h}$.