Math 104 Midterm Review
Fall 2011

Note: The midterm is Thursday, October 13, 5:45-7:45 pm, See your instructor for building and room.

Multiple Choice: Circle the correct answer.

1. The solution set of $|3 - x| > 7$ is

   (a) $(-4, 10)$
   (b) $(-10, 4)$
   (c) $(-\infty, -4) \cup (10, \infty)$
   (d) $(-\infty, -10) \cup (4, \infty)$

2. Let $f(x) = \dfrac{5}{x}$. Then $f^{-1}(x) =$

   (a) $\dfrac{x}{5}$
   (b) $\dfrac{5}{x}$
   (c) $\dfrac{1}{5x}$
   (d) $f^{-1}(x)$ does not exist.

3. The domain of $f(x) = \dfrac{1}{\sqrt{x+3}}$ is:

   (a) $x \neq -3$
   (b) $(-3, \infty)$
   (c) $(-\infty, -3)$
   (d) $(3, \infty)$

4. Which equation represents a transformation of one of our common functions that shifts two units to the right, three units up and reflects over the x-axis?

   (a) $f(x) = -3e^{x+2}$
   (b) $f(x) = \sqrt{-x+2} + 3$
   (c) $f(x) = (-x+2)^3 - 3$
   (d) $f(x) = -(x-2)^2 + 3$

5. Write the exponential equation $5^3 = 125$ in logarithmic form.

   (a) $\log_3 125 = 5$
   (b) $\log_5 3 = 125$
   (c) $\log_5 125 = 3$
   (d) $\log_3 5 = 125$

6. The zero of $f(x) = 5 - e^{x+1}$ is:

   (a) $5 - \frac{1}{e}$

   (b) $5 - e$

   (c) 5

   (d) $\ln 5 - 1$

7. What is $10 + 3i$ multiplied by its conjugate equal to?

   (a) $91 - 60i$

   (b) $100 + 30i$

   (c) 91

   (d) 109

8. How many total asymptotes (including vertical, horizontal, and slant/oblique) does $f(x) = \dfrac{2x^3 + 2x - 19}{x^2 - 2x}$ have?

   (a) 3

   (b) 2

   (c) 1

   (d) 0

9. Which statement is *not* equivalent to the others?

   (a) $x = -1$ is a zero of f

   (b) $(x - 1)$ is a factor of f

   (c) $f(-1) = 0$

   (d) $(-1, 0)$ is an x-intercept of the graph of f

10. Determine whether the function $g(x) = e^{-x^2}$ is even, odd or neither.

    (a) even

    (b) odd

    (c) neither

11. Given that the zeros of a polynomial are $\frac{1}{4}$, -4, and 3, which of the following is a possible factorization of $f(x)$?

    (a) $f(x) = (4x - 1)(x + 4)(x - 3)$

    (b) $f(x) = (8x - 2)(x + 4)(x + 3)$

    (c) $f(x) = (4x + 1)(x + 4)(3x - 9)$

    (d) all the above are possible factorizations of $f(x)$

12. If you invest $\$2200$ compounded continuously at Commerce Bank at an interest rate of 3.6%, how much will your investment be worth after 10 years?

    (a) $3148.25

(b) \$3151.63

(c) \$3153.32

(d) \$8051.61

13. Expressing $\log_{\sqrt{2}} 7$ in terms of the natural logarithm, we get

(a) $\ln \frac{7}{\sqrt{2}}$

(b) $\dfrac{\ln 7}{\ln \sqrt{2}}$

(c) $\ln 7 - \ln \sqrt{2}$

(d) $\dfrac{\ln \sqrt{2}}{\ln 7}$

14. What is the center and radius of the circle with the equation $(x-2)^2 + (y+4)^2 = 9$?

(a) C: $(-2, 4)$ R: 9

(b) C: $(2, -4)$ R: 9

(c) C: $(-2, 4)$ R: 3

(d) C: $(2, -4)$ R: 3

15. The solution(s) to $\dfrac{1}{x-3} + \dfrac{3}{x+3} = \dfrac{18}{x^2-9}$ is (are)

(a) $x = 2, 6$

(b) $x = 6$

(c) $x = 9$

(d) $x = 15$

16. If $f(x) = \dfrac{3x+2}{5x-1}$ and $g(x) = 7(x^2+2x)$, then $(f \circ (f+g))(0) =$

(a) $4/11$

(b) $2x + 5$

(c) 9

(d) 0

17. The solution set for $\dfrac{x+3}{x-5} < 2$ is

(a) $(-3, 2) \cup (2, 5)$

(b) $[-4, 2) \cup (5, \infty)$

(c) $(3, 5)$

(d) $(-\infty, 5) \cup (13, \infty)$

18. The solution(s) to $\ln(3x) + \ln x = 2 + \ln 3$ is (are)

(a) $x = e$

(b) $x = 2, 3$

(c) $x = 5e + 2$

(d) $x = 4$

19. The zeros for $\sqrt{x-1} + \sqrt{x+4} - 5$ is (are)

(a) $x = 5$

(b) $x = 10, 17$

(c) $x = 1$

(d) $x = 4$

20. The graph of the function $f(x) = \dfrac{x^4 + 3x^2}{3x^4 - 2x^2}$ has horizontal asymptote:

(a) $y = 0$

(b) $x = \dfrac{1}{3}$

(c) $y = \dfrac{3}{2}$

(d) $y = \dfrac{1}{3}$

21. The y-intercept of $g(x) = 5 + e^{x-1}$ is

(a) $\dfrac{5e+1}{e}$

(b) 5

(c) e^{-1}

(d) $5 + e$

22. The maximum value of a parabola with $f(x) = ax^2 + bx + c$ is

(a) $\frac{b}{2a}$

(b) $-\frac{b}{2a}$

(c) $f(-\frac{b}{2a})$

(d) $f(\frac{b}{2a})$

23. If $f(3) = 2, f(4) = 3, g(2) = 5$, and $g(3) = 2$, then $(g \circ f^{-1})(2) =$

(a) 5

(b) 4

(c) 3

(d) 2

24. $i^{3527} =$

(a) 1

(b) -1

(c) $-i$

(d) i

25. The distance between the points $(2, 3)$ and $(7, 8)$ is

    (a) $\sqrt{50}$

    (b) $(9/2, 11/2)$

    (c) 5

    (d) 7

26. The vertical asymptote for $g(x) = \ln(3x - 4) + 2$ is

    (a) $x = 4$

    (b) $y = 2$

    (c) $x = 4/3$

    (d) $x = 3/4$

27. Determine whether the statement is true or false.

    (a) If a horizontal line intersects the graph of an equation in more than point, the equation does not define y as a function of x.

    (b) An even function cannot be one-to-one.

    (c) If the remainder obtained by dividing a polynomial $f(x)$ by $x + 3$ is 8, then $f(3) = 8$.

    (d) It is possible that $\frac{1}{3}$ is a zero of a polynomial of the form $f(x) = x^4 + ax^2 + x + 3$, where a is an integer.

    (e) The graph of a rational function may cross its horizontal asymptote but does not cross any of its vertical asymptotes.

    (f) A polynomial of degree 5 with real coefficients must have a least one real zero.

    (g) A polynomial of degree 5 with real coefficients can have at most 4 relative extrema/turning points.

    (h) A rational function can have at most two horizontal asymptotes.

    (i) The logarithm of a negative number is not defined.

    (j) The logarithm of a number cannot be negative.

Long Answer: Show all work. Answers without support will not receive credit. Answers should be exact unless otherwise stated.

28. Solve:

    (a) $\log_4(x + 3) + \log_4(x - 3) = 2$

    (b) $8^{x-1} = 4$

    (c) $e^{2x} - 6e^x = -8$

    (d) $3^x = 2^{x-1}$

    (e) $\ln(\log x) = 0$

    (f) $\ln(x + 3) - \ln x = \ln 6$

29. For $f(x) = 2x^2 + 4x - 7$

    (a) Complete the square to put $f(x)$ in standard form and determine the vertex and axis of symmetry.

    (b) Make an accurate sketch of the function labelling the vertex, all intercepts, the axis of symmetry, and at least 2 additional points.

    (c) Restrict the domain to make $f(x)$ one-to-one and find $f^{-1}(x)$.

30. For $f(x) = -2x(x - 4)(x + 3)^2$

    (a) Find the zeros and multiplicity of each zero.

    (b) Make a rough sketch of $f(x)$ labeling all intercepts.

31. A model rockets height is described by the function $h(t) = -5t^2 + 100t$ where $h(t)$ is the height in meters of the rocket at time t measured in seconds.

    (a) What is the height of the rocket after 6 seconds?

    (b) At what time does the rocket reach a maximum height?

(c) What is the maximum height?

32. Use long division to divide $D(x)$ by $d(x)$ to nd the quotient, $q(x)$, and remainder, $r(x)$. Then verify yourself using synthetic division.
$D(x) = 3x^3 - 7x^2 + 5$
$d(x) = x + 2$

33. Is $(x - 3)$ a factor of the polynomial $P(x) = -x^4 + 8x^2 + 4x - 1$? Show your work to support your answer.

34. For $f(x) = \dfrac{2x^2 - 4x - 6}{x^2 - 2x - 8}$

(a) Find the domain.

(b) Find all intercepts.

(c) Find all asymptotes.

(d) Graph $f(x)$ labeling all information obtained in parts (b) and (c).

(e) Use your graph to solve $f(x) < 0$.

35. Create a rational function that satises the conditions below:
intercept at $(2, 0)$, vertical asymptotes at $x = -3$ and $x = 0$, horizontal asymptote of $y = 0$.

36. Find ALL zeros of

(a) $f(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$

(b) $f(x) = x^3 - 5x^2 - 7x + 51$

(c) $f(x) = x^4 + 34x^2 + 225$

(d) $f(x) = 4x^3 - 11x^2 + 10x - 3$

(e) $f(x) = 2x^3 - 3x^2 - 10x - 4$

37. Write the following in $a + bi$ form:

(a) $(5 + \sqrt{-25}) - (-3 - \sqrt{-9})$

(b) $(3 + 5i)(1 - 2i)$

(c) $\dfrac{2}{1 - i}$

(d) $(-i)^{327}$

38. For $f(x) = 2^{-x} - 3$

(a) Find the domain and range.

(b) Find all intercepts.

(c) Find all asymptotes.

(d) Graph$f(x)$ labelling all the information from parts (b) and (c).

39. $\$7000$ is invested at 6% interest. Find the amount in 10 years if the account is compounded

(a) Quarterly

(b) Monthly

(c) Continuously

40. For $f(x) = \ln(x + 4)$

    (a) Find the domain and range.

    (b) Find all intercepts.

    (c) Find all asymptotes.

    (d) Graph $f(x)$ labelling all the information from parts (b) and (c).

41. Expand the following expression into a sum/difference of logarithms: $\ln \sqrt{\dfrac{x-1}{z^2}}$.

42. The number of bacteria N in a culture is given by the model, $N = 250e^{kt}$, where t is the time (in hours). If $N = 280$ when $t = 10$, estimate the time required for the population to reach 500.

43. Suppose $\$2000$ is invested at interest rate r, compounded continuously, and grows to $\$2983.65$ in 5 years.

    (a) What is the interest rate?

    (b) Find the exponential growth function $A(t)$.

    (c) What will the balance be after 10 years?

    (d) After how long will the $\$2000$ have doubled?

44. The radioactive element carbon-14 has a half-life of 5750 years. Archeologists discovered that the linen wrapping from one of the Dead Sea Scrolls had lost 22.3% of its carbon-14 at the time it was found. How old was the linen wrapping?

45. Students in an accounting class took a nal exam. They took equivalent forms of the exam in monthly intervals thereafter. The average score $S(t)$, in percent, after t months was found to be given by the function : $S(t) = 78 - 15\log(t + 1), t \geq 0$.

    (a) What was the average score when they initially took the test, $t = 0$?

    (b) What was the average score after 9 months?

    (c) After what time t was the average score 50?

46. Is the function $f(x) = 8e^{-0.1x^2}$ even, odd, or neither? (Verify your answer algebraically.)

47. Let $f(x) = x^4 - 8x^3 + 42x^2 - 80x + 125$.

    (a) List all possible rational zeros of $f(x)$.

    (b) Find ALL the zeros of $f(x)$ given that $x = 1 + 2i$ is a zero.

48. Find a polynomial $f(x)$ of least degree and real coefficients with the given zeros: $x = 1$ $(k = 2)$, $x = 13 - i$, and solution point $f(0) = 340$.

49. (a) Solve the system of equations by any algebraic method you see fit.

$$x - 5y = 19$$
$$6x + 5y = 2$$

(b) Are these lines from part (a) parallel, perpendicular, or neither?

50. A ball is thrown upward so its height in feet at time t seconds is $h(t) = 96t - 16t^2$. Algebraically find what time the ball reaches its maximum height. What is the maximum height attained by the ball? When does the ball hit the ground?

51. A hot-air balloon is rising vertically from a point 200 feet from an observer on the ground. Express the height h of the balloon as a function of its distance d from the observer. Find the domain for your function.

52. When Sally takes her antibiotic, the concentration of antibiotic in her blood is 7 mg per ml. To be effective, the concentration needs to be at least 3 mg per ml. The concentration formula is $A(t) = 7e^{-0.14t}$ with t in hours. What is the half-life of this antibiotic? (Round to two decimal places.)

53. A bacterial culture is placed in a large glass bottle. Suppose that the volume of the culture doubles every hour, and the bottle is full after one day. If the culture was placed in the bottle at time $t = 0$ hours, when was the bottle half full? Assume the the bottle is "almost empty" when the culture occupies less than 1% of its volume. How long was the bottle "almost empty"?

54. The atmospheric pressure p on a balloon or an aircraft decreases with increasing height. This pressure, measured in millimeters of mercury, is related to the height h (in kilometers) above sea level by the formula $p = 760e^{-0.145h}$. Find the height of an aircraft if the atmospheric pressure is 340 millimeters in mercury.

55. A box with no top is to be made by cutting squares out of the corners of a 10 in by 5 in rectangular piece of cardboard and then folding up the sides. What size square should be cut out of each corner if the resulting box is to have a volume of 18 cubic inches?

56. John inherited $\$25,000$ and invested part of it in a money market account, part in municipal bonds, and part in a mutual fund. After one year, he received a total of $\$1,620$ in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutually fund paid 8% annually. There was $\$6,000$ more invested in the bonds than the mutual funds. Find the amount John invested in each category.

Math 104 Midterm Review Answers

1. c
2. b
3. b
4. d
5. c
6. d
7. d
8. a
9. b
10. a
11. a
12. c
13. b
14. d
15. b
16. a
17. d
18. a
19. a
20. d
21. a
22. c
23. d
24. c
25. a
26. c
27. (a) False
    (b) True

(c) False

(d) False

(e) True

(f) True

(g) True

(h) False

(i) True

(j) False

28. (a) $x = 5$

(b) $x = 5/3$

(c) $x = \ln 2, \ln 4$

(d) $x = \dfrac{\ln 2}{\ln 2 - \ln 3}$

(e) $x = 10$

(f) $x = 3/5$

29. (a) vertex: $(-1, -9)$; axis of symmetry: $x = -1$

(b) Answers may vary. Check using your graphing calculator.

(c) $x \geq -1$ make $f(x)$ one-to-one; $f^{-1}(x) = \sqrt{\dfrac{x+9}{2}} - 1$

30. (a) $x = 0, k = 1$; $x = 4, k = 1$, $x = -3, k = 2$

(b) Answers may vary. Check using your graphing calculator.

31. (a) 420 meters

(b) 10 seconds

(c) 500 meters

32. $q(x) = 3x^2 - 13x + 26$, $r(x) = -47$

33. No

34. (a) $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

(b) $(0, 3/4), (-1, 0), (3, 0)$

(c) $x = -2, x = 4, y = 2$

(d) Answers may vary. Check using your graphing calculator.

(e) $(-2, -1) \cup (3, 4)$

35. one possible answer: $f(x) = \dfrac{x-2}{x^2 + 3x}$

36. (a) $x = -1/5, 1, \pm 2i$

(b) $x = -3, 4 \pm i$

(c) $x = \pm 3i, \pm 5i$

(d) $x = 3/4, 1 (k = 2)$

(e) $x = -1/2, 1 \pm \sqrt{5}$

37. (a) $8 + 8i$

(b) $13 - i$

(c) $1 + i$

(d) i

38. (a) Domain: $(-\infty, \infty)$ and Range: $(-3, \infty)$

(b) $(0, -2), (-\log_2 3, 0)$

(c) $y = -3$

(d) Answers may vary. Check using your graphing calculator.

39. (a) $\$12698.13$

(b) $\$12735.78$

(c) $\$12754.83$

40. (a) Domain: $(-4, \infty)$ and Range: $(-\infty, \infty)$

(b) $(0, \ln 4), (-3, 0)$

(c) $x = -4$

(d) Answers may vary. Check using your graphing calculator.

41. $\frac{1}{2}\ln(x-1) - \ln z$

42. 61.2 hours

43. (a) r is approximately 8.00%

(b) $A(t) = 2000e^{.08t}$

(c) $\$4451.08$

(d) 8.7 years

44. 2093 years old

45. (a) 78%

(b) 63%

(c) 73 months

46. Even

47. (a) $\pm 1, \pm 5, \pm 25, \pm 125$

(b) $1 \pm 2i, 3 \pm 4i$

48. $f(x) = 2x^4 - 56x^3 + 446x^2 - 732x + 240$

49. (a) $(x, y) = (3, -3.2)$

(b) neither

50. maximum height is 144 ft at $t = 3$ seconds and hits ground when $t = 6$ seconds

51. $h(d) = \sqrt{d^2 - 40,000}$ with $d \geq 200$

52. 4.95 hours

53. half full at $t = 23$ hours, "almost empty" for first 17.356 hours

54. 5.5474 km

55. square should have length 0.5 inch or approximately 1.7 inches

56. $\$15,000$ is invested in the monkey market account, $\$8,000$ is invested in the municipal bonds, and $\$2,000$ is invested in mutual funds.