Testing the series $\sum a_n$ for convergence or divergence.

Divergence Test
Does $\lim\limits_{n\to\infty} a_n = 0$?

— no → $\sum a_n$ **diverges**

↓ yes

Geometric Series
Does $a_n = ar^{n-1}$?

— yes → Is $|r| < 1$? — yes →

$\qquad$ no

↓ no

p-Series
Does $a_n = \frac{1}{n^p}$?

— yes → Is $p > 1$? — yes →

$\qquad$ no

↓ no

Is $a_n \geq 0$ for all n? — yes →

↓ no

Does $\sum |a_n|$ converge? — yes →

↓ no or ?

Alternating Series
Does $a_n = (-1)^n b_n$
or $a_n = (-1)^{n-1} b_n$
for $b_n = |a_n| \geq 0$?

— yes → Is $b_{n+1} \leq b_n$ and
does $\lim\limits_{n\to\infty} b_n = 0$? — yes →

$\qquad$ no

↓ no

Ratio Test
Does $\left\{ \frac{a_{n+1}}{a_n} \right\}$ converge to a real number other than 1?

— yes → Is $\lim\limits_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$? — yes →

$\qquad$ no

↓ no

Root Test
Does $\left\{ \sqrt[n]{|a_n|} \right\}$ converge to a real number other than 1?

— yes → Is $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} < 1$? — yes →

$\qquad$ no

↓ no

???

Integral Test
Does $a_n = f(n)$ for some
continuous, positive,
decreasing function f?

— yes → Does $\displaystyle\int_N^\infty f(x)\,dx$ converge for some $N \geq 1$? — yes →

$\qquad$ no

↓ no

Comparison Test or **Limit Comparison Test**

(a) $\sum b_n$ converges, where $0 \leq a_n \leq b_n$.

(b) $\sum b_n$ diverges, where $0 \leq b_n \leq a_n$.

(c) $\sum b_n$ converges, where $b_n \geq 0$ and $0 < \lim\limits_{n\to\infty} \frac{a_n}{b_n} < \infty$.

(a) or (c) →

none ; (b)

$\sum a_n$ **converges**