Name: _____

Complete the following problems to the best of your ability. Clearly number each question and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

1. (10 pts.) Show that the curve $x = t\cos t$, $y = t\sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$.

2. (10 pts.) Find the tangent line to the curve $x = \ln t$, $y = 2\sqrt{t}$, $z = t^2$ at the point $(0, 2, 1)$.

3. (15 pts.) Let $\vec{r}$ be the vector function defined by $\vec{r}(t) = \langle 2\sin t, 5t, 2\cos t\rangle$.

(a) Find the unit tangent vector $\vec{T}(t)$.

(b) Find the curvature $\kappa(t)$.

4. (10 pts.) Find the position at time t of a particle with initial position $\vec{r}(0) = \langle 1, 0, 0\rangle$ and velocity function $\vec{v}(t) = \langle t, e^t, e^{-t}\rangle$.

5. (20 pts.) Determine whether or not each limit exists. If it does, state its value. Justify your answers.

(a) $\lim\limits_{(x,y,z)\to(0,0,0)} x^2y + e^{x+z} - yz + \cos y.$

(b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2ye^y}{x^4 + 4y^2}.$

(c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^2 + y^2}.$

6. (15 pts.) Let $u = \sqrt{r^2 + s^2}$, where $r = y + x\cos t$ and $s = x + y\sin t$.

(a) Sketch and label a Chain Rule tree diagram for the function u.

(b) Use the Chain Rule to find the partial derivative $\frac{\partial u}{\partial x}$ when $x = 1$, $y = 2$, and $t = 0$.

7. (20 pts.) Let f be the trivariate function defined by $f(x, y, z) = \sqrt{xyz}$.

(a) Find the gradient vector of f as a function of x, y, and z.

(b) Evaluate the gradient at the point $(3, 2, 6)$. What does this vector represent?

(c) Find the directional derivative of f at $(3, 2, 6)$ in the direction of the vector $\vec{v} = \langle -1, -2, 2\rangle$.

E.C. (10 pts.) Find and classify all extrema of the function $f(x, y) = y^2 - 2y\cos x$, where $-1 \le x \le 7$.