13. Find an equation of both lines that are tangent to the curve $y = 1 + x^3$ and parallel to the line $12x - y = 1$.

Solution: In order for the tangent lines to be parallel to $12x - y = 1$, we need to have the derivative of $y = 1 + x^3$ equal to the slope of $12x - y = 1$, which is 12. That is, we need to have

$$3x^2 = 12, \text{ i.e. } x = \pm 2.$$

This means that the points on the curve where the tangent line has slope 12 have x −coordinates 2 and -2. The corresponding y −coordinates can be obtained by substituting these values into the equation $y = 1 + x^3$.

Therefore, the points on the curve where the tangent line has slope 12 are $(2,9)$ and $(-2,-7)$.

Using the point-slope formula the equations of these tangent lines are

$$y - 9 = 12(x - 2) \quad \text{and} \quad y + 7 = 12(x + 2).$$

14. Find equations of both lines that pass through the point $(2,-3)$ that are tangent to the parabola $y = x^2 + x$.

Solution: This is a tricky problem.

Let L be a line that passes through the point $P = (2,-3)$ and is tangent to the curve $y = x^2 + x$ at some point $Q = (a,b)$ on the curve.

Then from the equation of the curve we get $b = a^2 + a$, so that $Q = (a, a^2 + a)$.

We can now calculate the slope m of line L in two ways. Since it is tangent to the curve $y = x^2 + x$, its slope is given by the derivative formula $y' = 2x + 1$. Therefore, its slope is given by

$$m = 2a + 1$$

On the other hand since the line L passes through $P = (2,-3)$ and $Q = (a, a^2 + a)$, its slope is also given by the slope formula for a line. That is,

$$m = \frac{a^2+a+3}{a-2}$$

Comparing these two expressions for the slope, we get

$$\frac{a^2+a+3}{a-2} = 2a + 1$$

Solving for a we get, $a^2 - 4a - 5 = 0$, i.e. $a = 5$ 0r $a = -1$.

Therefore, we have two lines that are tangent to the curve $y = x^2 + x$ and pass through $P = (2, -3)$.

When $a = 5$, we get the tangent line to the curve at the point $(5,30)$ and it has slope $m = 11$ and hence equation, $y - 30 = 11(x - 5)$.

When $a = -1$, we get the tangent line to the curve at the point $(-1,0)$ and it has slope $m = -1$ and hence equation, $y - 0 = -1(x + 1)$.