(1) The domain of f is $(-\infty, 1) \cup (1, \infty)$, i.e. all real numbers for which the denominator is nonzero.

Notice that g is a composite function. The inside function $\sqrt{x+\pi}$ gives the domain restriction $x + \pi \geq 0 \implies x \geq -\pi$. The outside function $\ln(\cdot)$ gives the additional restriction $\sqrt{x+\pi} > 0 \implies x > -\pi$. Thus, the domain of g is $(-\pi, \infty)$.

(2) $(f \circ g)(x) = f(g(x)) = \dfrac{x^{10}}{x^{10}+1}$

$(g \circ f)(x) = g(f(x)) = \left(\dfrac{x}{x+1}\right)^{10} = \dfrac{x^{10}}{(x+1)^{10}}.$

(3) $\ln(8x) - \ln(1+x) = 2 \implies \ln\left(\dfrac{8x}{1+x}\right) = 2 \implies \dfrac{8x}{1+x} = e^2 \implies 8x = e^2 + xe^2 \implies x = \dfrac{e^2}{8-e^2}.$

(4) (a) Since ☺ is in the domain of the rational function, the DSP gives: $\lim\limits_{t \to ☺} \dfrac{t^3+2t^2-1}{5-3t} = \dfrac{☺^3+2☺^2-1}{5-3☺}.$

(b) $\lim\limits_{x \to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim\limits_{x \to -4} \dfrac{\frac{x+4}{4x}}{4+x} = \lim\limits_{x \to -4} \dfrac{1}{4x} = -\dfrac{1}{16}.$

(c) $\lim\limits_{u \to \infty} \dfrac{4u^5+5}{(u^2-2)(2u^2-2)} = \infty$, since the numerator has greater degree than the denominator.

(d) Since direct substitution does not apply (i.e. $\infty - \infty$ is undefined), we instead compute:

$$\begin{aligned}
\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right) &= \lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)\cdot\left(\frac{\sqrt{x^2+x+1}+x}{\sqrt{x^2+x+1}+x}\right)\\
&= \lim_{x\to\infty}\left(\frac{(x^2+x+1)-x^2}{\sqrt{x^2+x+1}+x}\right)\\
&= \lim_{x\to\infty}\left(\frac{x+1}{\sqrt{x^2+x+1}+x}\right)\\
&= \lim_{x\to\infty}\left(\frac{x+1}{\sqrt{x^2+x+1}+x}\right)\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\\
&= \lim_{x\to\infty}\left(\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}\right)\\
&= \frac{1}{2}.
\end{aligned}$$

(5) Notice that $\lim_{x\to 0} -|x| = \lim_{x\to 0} |x| = 0$. Thus, by the Squeeze Theorem, $\lim_{x\to 0}\left[x\cos\left(\dfrac{1}{x}\right)\right] = 0.$

(6) (a) The function f is continuous wherever it is defined, i.e. $2x+5 > 0 \implies (-5/2, \infty)$.

(b) It can be seen that $\lim\limits_{x\to 2^+} g(x) = \infty$ and $\lim\limits_{x\to 2^-} g(x) = -\infty$, and thus $\lim\limits_{x\to 2} g(x)$ does not exist.

(c) Away from $x = 2$, the function g is a rational function and thus is continuous everywhere it is defined. As we have seen in part (b), $\lim_{x\to 2} g(x)$ DNE. Hence, g is continuous on $(-\infty, 2)\cup(2,\infty)$.

(7) The function f is the sum of two continuous functions (an exponential function and a polynomial function) so it is continuous everywhere, and in particular for all $x \in [0, 1]$. Notice that $f(0) = e^0 - 3(0) = 1 > 0$ and $f(1) = e^1 - 3(1) \approx -0.2817 < 0$ have different signs. Thus, by the Intermediate Value Theorem, the function f has at least one zero between $x = 0$ and $x = 1$.

(8) (a) By the definition of the derivative:

$$\begin{aligned}
f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} [f(x+h)-f(x)]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\left[\left((x+h)+\frac{1}{x+h}\right)-\left(x+\frac{1}{x}\right)\right]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\left[\frac{(x+h)^2+1}{x+h}-\frac{x^2+1}{x}\right]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\left[\frac{x((x^2+2xh+h^2)+1)-(x+h)(x^2+1)}{x(x+h)}\right]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\left[\frac{x^3+2x^2h+xh^2+x-x^3-x^2h-x-h}{x(x+h)}\right]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\left[\frac{x^2h+xh^2-h}{x(x+h)}\right]\cdot\frac{1}{h} \\
&= \lim_{h\to 0}\frac{x^2+xh-1}{x(x+h)} \\
&= \frac{x^2-1}{x^2}, \quad \text{or} \quad = 1-\frac{1}{x^2}
\end{aligned}$$

(b) The slope of the tangent line at the point $(1, 2)$ is given by $m = f'(1) = 1 - \frac{1}{1^2} = 0$. Hence, the equation of the tangent line is $y - 2 = 0(x - 1) \implies y - 2 = 0 \implies y = 2$.

(9) (a) Since $f'(x) > 0$ for all x, the function f is increasing on $\mathbb{R} = (-\infty,\infty)$.

(b) Since f' is increasing on $(-\infty, 0)$, the function f is concave upward on $(-\infty, 0)$. Likewise, since f' is decreasing on $(0,\infty)$, the function f is concave downward on $(0,\infty)$.

(c) Since f' is an exponential function (which has not been vertically shifted), it is strictly positive (i.e. never zero) for all x. Hence the function f does not have any local maxima nor local minima. On the other hand, the function f changes concavity at $x = 0$ (as seen from the answer to part (b)), so $(0, f(0))$ is an inflection point of f.

Bonus. Explicitly defining an everywhere-continuous, nowhere-differentiable function is a complicated procedure that is far beyond the scope of this course. The Weierstrass function is an example of one such function (see the graph below). The purpose of this bonus question was to inspire you to consider more abstract generalizations of the basic limit / continuity / differentiability concepts we have studied so far. In response to the various parts of the question, I expected to see: (a) a simple graph of $y = |x|$, (b) some sort of "zig-zag" graph that has a "corner" at every integer, and (c) a brief explanation of your best understanding of how the concept could be generalized. For parts (b) and (c), many different answers are acceptable.

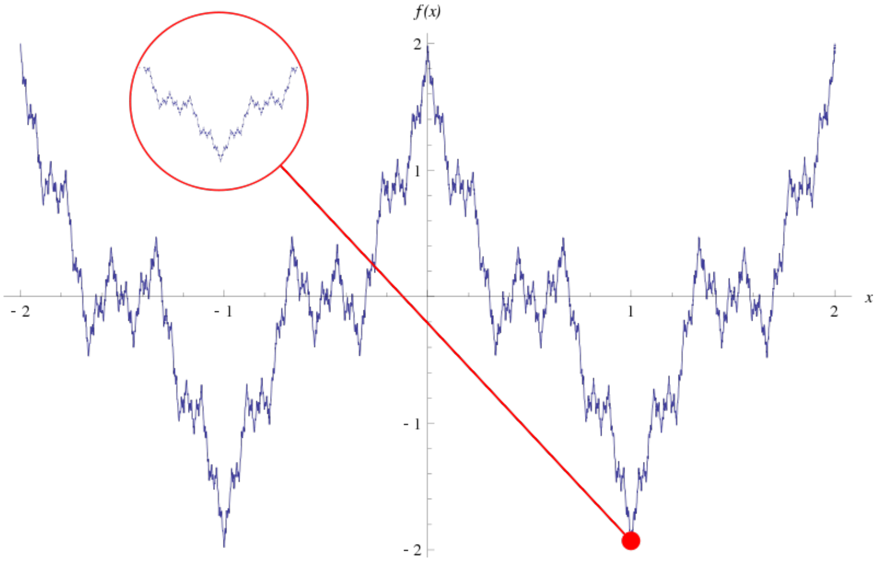

FIGURE 1. Weierstrass function: everywhere-continuous, nowhere-differentiable.