1. a) Domain : R

Continuity : f is continuous on R

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}(x-1)^2 \lim_{x\to-\infty}(x-4) = +\infty\cdot(-\infty) = -\infty$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}(x-1)^2 \lim_{x\to+\infty}(x-4) = +\infty\cdot(+\infty) = +\infty$

$\lim_{x\to0} f(x) = \lim_{x\to0}(x-1)^2 \cdot \lim_{x\to0}(x-4) = 1\times(-4) = -4$

b) No

c). $f'(x) = (x-1)^2 + 2(x-1)(x-4) = 3(x-1)(x-3)$

when $x = 1, 3$, $f'(x) = 0$

when $x \le 1$, $f'(x) \geqslant 0$, so $f(x)$ is increasing on $(-\infty, 1]$

when $1 \le x \le 3$, $f'(x) \le 0$, so $f(x)$ is decreasing on $[1, 3]$.

when $x \geqslant 3$. $f'(x) \geqslant 0$, so $f(x)$ is increasing on $[3, +\infty)$

d) Local maxima: $f(1) = 0$

Local minima: $f(3) = -4$

Since $\lim_{x\to-\infty} f(x) = -\infty$ & $\lim_{x\to+\infty} f(x) = +\infty$, None of these two are absolute extrema.

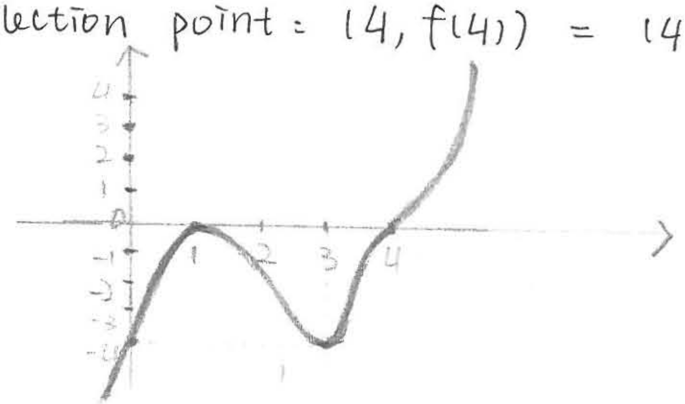
e) $f''(x) = 3(x-1) + 3(x-3) = 6x-24$

when $x \le 4$, $f''(x) \le 0$, so $f(x)$ is concave down on $(-\infty, 4]$

when $x \geqslant 4$, $f''(x) \geqslant 0$, so $f(x)$ is concave up on $[4, +\infty)$

Inflection point: $(4, f(4)) = (4, 0)$

f)



2. a) F b) T c) F d) T e) F f) F g) F

3. (a) $\bar{C} = \dfrac{C(x)}{x} = \dfrac{2}{\sqrt{x}} + \dfrac{x}{8000}$

(b) $\dfrac{d\,C(x)}{dx} = 2 \times \dfrac{1}{2} \times \dfrac{1}{\sqrt{x}} + \dfrac{2x}{8000} = \dfrac{1}{\sqrt{x}} + \dfrac{x}{4000}$

(c) $\dfrac{d\,\bar{C}(x)}{dx} = 2 \times (-\dfrac{1}{2}) \times x^{-\frac{3}{2}} + \dfrac{1}{8000} = -x^{-\frac{3}{2}} + \dfrac{1}{8000} = 0.$

$x = 400$

check: when $0 < x \leq 400$, $\dfrac{d\bar{C}(x)}{dx} \leq 0$, $\bar{C}(x)$ is decreasing

when $x \geq 400$, $\dfrac{d\bar{C}(x)}{dx} \geq 0$. $\bar{C}(x)$ is increasing

So $x = 400$ is the minimum point.

(d) $\bar{C}(400) = \dfrac{2}{\sqrt{400}} + \dfrac{400}{8000} = \dfrac{2}{20} + \dfrac{1}{20} = \dfrac{3}{20}$

4.(a) $R(x) = x\,p(x) = 1700x - 7x^2$

$R'(x) = 1700 - 14x$

when $R'(x) = 0$. $x = \dfrac{1700}{14}$

when $0 \leq x \leq \dfrac{1700}{14}$, $R'(x) \geq 0$, $R(x)$ is increasing

when $x \geq \dfrac{1700}{14}$, $R'(x) \leq 0$, $R(x)$ is decreasing.

so at $x = \dfrac{1700}{14}$ $R(x)$ takes the maximum value.

Because x is the production level, we should choose an integer.

$x = \dfrac{1700}{14} \approx 121.4.$ So we choose either 121 or 122

$R(121) = 103213$ $R(122) = 103212$

So $x = 121$ is the production level that maximizes revenue.

$R(121) = 103213.$

b) $P(x) = R(x) - C(x) = 1700x - 7x^2 - (16000 + 500x - 1.6x^2 + 0.004x^3)$

$= -0.004x^3 - 5.4x^2 + 1200x - 16000$

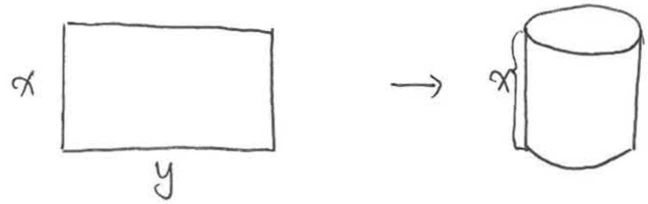
$P'(x) = -0.012x^2 - 10.8x + 1200 = -0.012(x - 100)(x + 1000)$

when $0 \le x \le 100$, $P'(x) \ge 0$, so $P(x)$ is increasing

when $x \ge 100$, $P'(x) \le 0$, So $P(x)$ is decreasing.

$P(x)$ takes the maximum at $x = 100$; $P(100) = 46000$

5.



$2(x + y) = 100$

V = Volume of Cylinder $= x \cdot \dfrac{y^2}{4\pi}$

$= x \cdot \dfrac{(50-x)^2}{4\pi}$

$V' = \frac{1}{4\pi}[(50-x)^2 + 2(50-x)\times(-1)x] = \frac{1}{4\pi}(50-x)(50-3x)$

When $x = 50$, $y = 0$, there doesn't exist such a cylinder.

when $x = \frac{50}{3}$, $V'(x) = 0$.

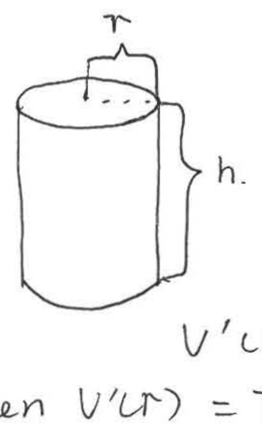
when $x \le \frac{50}{3}$ $V'(x) \ge 0$, $V(x)$ is increasing

when $x \ge \frac{50}{3}$, $V'(x) \le 0$, $V(x)$ is decreasing

So $V(x)$ takes the maximum at $x = \frac{50}{3}$

i.e. the rectangle has a length of $\frac{100}{3}$, wide of $\frac{50}{3}$.

6.



Surface area $= 2\pi r^2 + 2\pi r \cdot h = 150\pi$

So we have $h = \frac{150}{2r} - r$

$V = \pi r^2 \cdot h = \pi r^2 (\frac{150}{2r} - r) = 75\pi r - \pi r^3$

$V'(r) = 75\pi - 3\pi r^2$

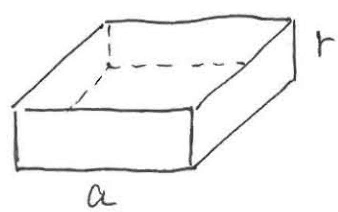
when $V'(r) = 75\pi - 3\pi r^2 = 0$, $r = 5$ (since r is positive, it can't be -5)

when $0 < r \le 5$, $V'(r) \ge 0$, $V(r)$ is increasing

when $r \ge 5$, $V'(r) \le 0$, $V(r)$ is decreasing

So $V(r)$ takes maximum at $r = 5$. $h = \frac{150}{2\times5} - 5 = 10$

7.



$a^2 r = 42875 \qquad r = \dfrac{42875}{a^2}$

C = Costs to build the box

$= 0.06 \cdot a^2 + 4ar \times 0.03 = 0.06a^2 + \dfrac{0.12\times 42875}{a}$

when ~~$\frac{dC}{da}$~~ $\dfrac{dC(a)}{da} = 0.12a - \dfrac{0.12\times 42875}{a^2} = 0$, $a = 42875^{\frac{1}{3}} = 35$

when $0<a\leq 35$, $C'(a) \leq 0$, $C(a)$ is decreasing

when $a \geq 35$, $C'(a) \geq 0$, $C(a)$ is increasing.

So $C(a)$ takes the ~~maxi~~ minimum at $a = 35$.

8. (b)

9. $f'(x) = 1 - 2\sin^2 x - \sin x = -(2\sin x - 1)(\sin x + 1)$

$\sin x + 1 \geq 0$

so when $2\sin x - 1 < 0$, $f'(x) > 0 \Rightarrow \sin x < \frac{1}{2}$, $x \in (0, \frac{\pi}{6}) \cup (\frac{5}{6}\pi, 2\pi)$

10. (a) $f'(x) = 3x^2 - 6x = 3x(x-2)$; when $f'(x)=0$, $x = 0$ & 2

when $-2 \leq x \leq 0$, $f'(x) \geq 0$, $f(x) \uparrow$

when $0 \leq x \leq 2$, $f'(x) \leq 0$, $f(x) \downarrow$

when $x \geq 2$, $f'(x) \geq 0$, $f(x) \uparrow$

$f(0) = 2$, $f(2) = -2$. Check the endpoints: $f(-2) = -18$

$f(3) = 2$

So the absolute maximum value of $f(x)$ on $[-2,3]$ is 2, corresponding x are 0 and 3.

Absolute minimum value of $f(x)$ on $[-2,3]$ is -18, corresponding x is -2.

(b) Absolute Max: $f(2) = \frac{1}{2}$. Absolute Min: $f(0) = 0$.

(c) Absolute Max: $f(\pi) = \pi$ Absolute Min: $f(\frac{\pi}{3}) = \frac{\pi}{3} - \sqrt{3}$

(d). Absolute Max: $f(\frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}} e^{-\frac{1}{2}}$ Absolute Min: $f(-\frac{1}{\sqrt{2}}) = -\frac{1}{\sqrt{2}} e^{-\frac{1}{2}}$

(When checking endpoints, $\lim\limits_{x\to+\infty} xe^{-x^2} = \lim\limits_{x\to+\infty} \dfrac{x}{e^{x^2}} = \lim\limits_{x\to+\infty} \dfrac{1}{2xe^{x^2}} = 0$.

The same for $x \to -\infty$)

11. C

12. $f'(x) = 3 - \frac{1}{3}x^{-\frac{2}{3}}$ $\Rightarrow$ when $x = \pm\frac{1}{27}$, $f'(x) = 0$

when $x \le -\frac{1}{27}$, $f'(x) \ge 0$, $f(x) \uparrow$

when $-\frac{1}{27} \le x \le \frac{1}{27}$, $f'(x) \le 0$, $f(x) \downarrow$.

when $x \ge \frac{1}{27}$, $f'(x) \ge 0$, $f(x) \uparrow$

So at $x = -\frac{1}{27}$, $f(x)$ changes from increasing to decreasing.

13. $g'(t) = \dfrac{t^2+1-2t^2}{(t^2+1)^2} = \dfrac{1-t^2}{(t^2+1)^2}$

when $t^2 \ge 1$, $g'(t) \le 0$

So on $(-\infty, -1]$ and $[1, +\infty)$, $g(t)$ is decreasing

14. two.

15. a) $f'(x) = 4x^3 - 8x$. $f'(x) = 0 \Rightarrow 4x(x-\sqrt{2})(x+\sqrt{2}) = 0$

$\Rightarrow x = 0, \sqrt{2}, -\sqrt{2}$

So critical numbers are $0, \sqrt{2}, -\sqrt{2}$

when $x \le -\sqrt{2}$, $f'(x) \le 0$, $f(x) \downarrow$

when $-\sqrt{2} \le x \le 0$, $f'(x) \ge 0$, $f(x) \uparrow$

when $\sqrt{2} \ge x \ge 0$, $f'(x) \le 0$, $f(x) \downarrow$

when $x \ge \sqrt{2}$, $f'(x) \ge 0$, $f(x) \uparrow$

So f is increasing on $[-\sqrt{2}, 0] \cup [\sqrt{2}, +\infty)$.

f is decreasing on $(-\infty, -\sqrt{2}] \cup [0, \sqrt{2}]$

Local Maximum takes at $x = 0$, $f(0) = 0$. So $(0,0)$ is a local Maximum.

Local minimum takes at $x = -\sqrt{2}$ & $\sqrt{2}$, $f(-\sqrt{2}) = -4$

$f(\sqrt{2}) = -4$. So $(-\sqrt{2}, -4)$ and $(+\sqrt{2}, -4)$ are two local minimums.

b) $f''(x) = 12x^2 - 8$. $f''(x) = 0 \Rightarrow x = \pm \frac{\sqrt{6}}{3}$.

Inflection points: $\left(-\frac{\sqrt{6}}{3}, -\frac{20}{9}\right)$, $\left(\frac{\sqrt{6}}{3}, -\frac{20}{9}\right)$

when $x \le -\frac{\sqrt{6}}{3}$, $f''(x) \geq 0$. $\Rightarrow$ concave up on $(-\infty, -\frac{\sqrt{6}}{3}]$

when $-\frac{\sqrt{6}}{3} \le x \le \frac{\sqrt{6}}{3}$, $f''(x) \le 0$. $\Rightarrow$ concave down on $[-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}]$

when $x \geq \frac{\sqrt{6}}{3}$, $f''(x) \geq 0 \Rightarrow$ concave up on $[\frac{\sqrt{6}}{3}, +\infty)$

16. c)

17. a) $-\sqrt{17}$ b). 0. c). $-e$ ($f(x)$ is not continuous at $x=\pi$, So $x=\pi$ is not a inflection point).

d). $y = 2.3$. e) $x = \pi$

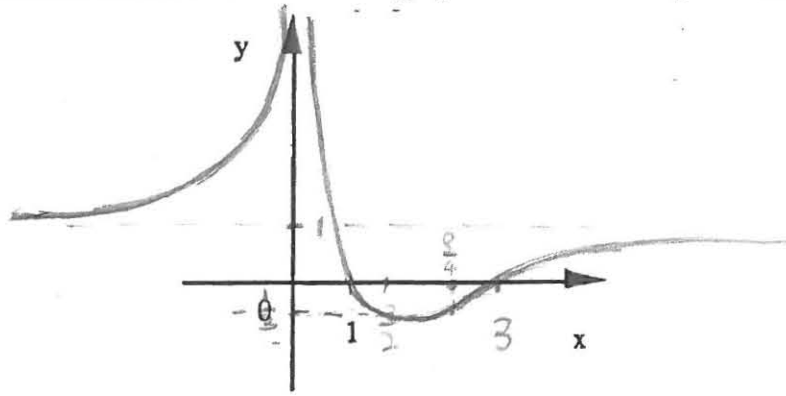
18. a) $(1, 0)$, $(3, 0)$ b) None c) $(\frac{4}{3}, 1)$

d) None e). $(\frac{3}{2}, -\frac{1}{3})$ f) $(\frac{9}{4}, -\frac{5}{27})$

g). Vertical: $x = 0$

Horizontal: $y = 1$

h) Use the information above to sketch the graph of f. Label the points listed above.



19. D 20. B.

21. $\int \sin^2 x \cos x \, dx = \int \sin^2 x \, d(\sin x) = \int u^2 du$ (where $u = \sin x$)

$= \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$\int x e^{-x^2} dx = \frac{1}{2}\int e^{-x^2} dx^2 = \frac{1}{2}\int e^{-u} du$ (where $u = x^2$)

$= -\frac{1}{2} e^{-u} + C = -\frac{1}{2} e^{-x^2} + C.$

So $\int_0^1 x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} \Big|_0^1 = -\frac{1}{2} e^{-1} + \frac{1}{2}.$

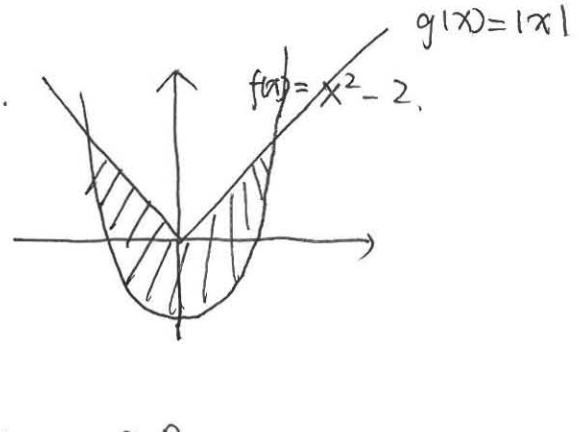
22. D.

23. Divide $[1,3]$ into n subintervals, each of which is of length $\frac{2}{n}$. choose the right endpoint of each subinterval and use Riemann Sum:

$$\int_1^3 2x\,dx = \lim_{n \to +\infty} \sum_{k=1}^{n} \frac{2}{n} \times 2\left(1 + \frac{2k}{n}\right) = \lim_{n\to+\infty} \frac{4}{n^2} \sum_{k=1}^{n} (n+2k)$$

$$= \lim_{n\to+\infty} \frac{4}{n^2}\left(n^2 + \sum_{k=1}^{n} 2k\right) = \lim_{n\to+\infty} \frac{4}{n^2}\left(n^2 + 2\times\frac{n(n+1)}{2}\right) = 8$$

24.



$x^2 - 2 = x$ & $x \geq 0$.

$\Rightarrow x = 2.$

$x^2 - 2 = -x$ & $x \leq 0.$

$\Rightarrow x = -2.$

$$A = \int_{-2}^{0} (-x - (x^2 - 2))\,dx + \int_0^2 (x - (x^2-2))\,dx$$

$$= \left(-\frac{x^2}{2} - \frac{x^3}{3} + 2x\right)\Big|_{-2}^{0} + \left(\frac{x^2}{2} - \frac{x^3}{3} + 2x\right)\Big|_0^2 = \frac{20}{3}$$

25. Average value of f on the interval $2 \le x \le 5$ is 4

$$\Rightarrow \frac{\int_2^5 f(x)\,dx}{5-2} = 4 \Rightarrow \int_2^5 f(x)\,dx = 12.$$

So $\int_2^5 (3f(x)+2)\,dx = 3\int_2^5 f(x)\,dx + \cancel{x^2\Big|_2^5}\ 2x\Big|_2^5$

$\cancel{= 36 + 25 - 4 = 57.}$

$= 36 + 10 - 4$

$= 42.$