Selected Chapter 6 solutions.

6.1 # 92. A pilot lands a fighter aircraft on an aircraft carrier. At the moment of touchdown, the speed of the aircraft is 160 mph. If the aircraft is brought to a complete stop in 1 sec and the decceleration is assumed to be constant, find the number of g's the pilot is subjected to during landing (1 g = 32 ft/sec^2).

Solution. Recall that the antiderivative of acceleration (or decceleration, which is just negative acceleration) is velocity. Since decceleration is assumed to be constant, the velocity of the aircraft as it is stopping must be linear (to justify this, simply think about the rule for finding the antiderivative of a constant). Since velocity is given in mph and time is in seconds, and the question we are trying to answer involves g's (ft/sec^2), we should convert the initial velocity of 160 mph to ft/sec as follows

$$160\frac{\text{miles}}{\text{hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \approx 235\frac{\text{feet}}{\text{second}}.$$

Now, assuming that time $t = 0$ corresponds to the moment of touchdown, we know two points on the velocity curve: $(0, 235)$ and $(1, 0)$. Since the curve is linear, its slope at any point is easily computed to be $m = (0 - 235)/(1 - 0) = -235$, and this is precisely the acceleration of the aircraft. That is, the aircraft deccelerates at a rate of 235 ft/sec^2, which corresponds to $235/32 \approx 7.3$ g's.

6.1. # 96. A fighter aircraft is launched from the deck of a Nimitz-class aircraft carrier with the help of a steam catapult. If the aircraft is to attain a takeoff speed of at least 240 ft/sec after traveling 800 ft along the flight deck, find the minimum acceleration it must be subjected to, assuming it is constant.

Solution. Since acceleration is constant, say $a(t) = k$. We wish to determine this k. Recall that the antiderivative of acceleration is velocity (say $v(t)$), and the antiderivative of velocity is position (say $s(t)$). By the power rule for indefinite integration, we find

$$v(t) = \int a(t)\,dt = \int k\,dt = kt + C = kt \qquad \text{(it is reasonable to assume } v(0) = 0, \text{ which makes } C = 0).$$

$$s(t) = \int v(t)\,dt = \int kt\,dt = \frac{1}{2}kt^2 + C = \frac{1}{2}kt^2 \qquad \text{(it is also reasonable to assume } s(0) = 0, \text{ so again } C = 0).$$

The aircraft much reach a takeoff speed of 240 ft/sec (velocity) by the time it has traveled 800 ft (position). That is, by some time t_0 (the unknown "takeoff" time), we must have both $v(t_0) = kt_0 = 240$ and $s(t_0) = (1/2)kt_0^2 = 800$. Solving the first equation for t_0 yields $t_0 = 240/k$. Substituting this into the second equation, we can find k as follows

$$\begin{aligned}\frac{1}{2}kt_0^2 = 800 &\implies \frac{1}{2}k\left(\frac{240}{k}\right)^2 = 800 \\ &\implies \frac{1}{2}\cdot\frac{240^2}{k} = 800 \\ &\implies 240^2 = 1600k \\ &\implies k = 36.\end{aligned}$$

Thus, the minimum acceleration the aircraft must have in order to safely depart the carrier is 36 ft/sec^2.