Name: _____

Answer the following questions on a separate sheet of paper. Clearly number each problem, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise stated.

1: (20 pts.) Determine exact value (no decimal approximations!) of each of the following, if possible.

(a): $\sin\left(\frac{-9\pi}{4}\right)$.

(b): $\cot(0)$.

(c): $\arcsin\left(\frac{\sqrt{2}}{2}\right)$.

(d): $\tan(\arctan(2011))$. Briefly justify your answer to receive full credit.

2: (15 pts.) Recall the sum formula $\cos(u+v) = \cos u \cos v - \sin u \sin v$. Given $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{4}{5}$, where both u and v are in Quadrant III, find the exact value of $\cos(u+v)$.

3: (15 pts.) Find all solutions of the following equations in the interval $[0, 2\pi)$ algebraically.

(a): $2\sin x - 1 = 0$.

(b): $\cos^2 x = \cos x$. (hint: set equal to zero, then factor).

4: (15 pts.) An observer in a lighthouse 350 feet above sea level sees two ships directly offshore. The angles of depression to the ships are 4° and 6.5°. How far apart are the ships? (Hint: first draw a picture and think about the right triangles formed by the positions of the described objects).

5: (20 pts.) Evaluate all six trigonometric functions at θ, given that $\csc\theta = -\frac{3}{2}$ and $\tan\theta < 0$.

6: (15 pts.) Prove the following trigonometric identities.

(a): $\frac{\sin^2 x}{\tan^2 x} = \cos^2 x$.

(b): $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$.

Bonus: Attempt any or all of the following, as time permits (5 pts. each).

(i): Solve the equation $\cos x + \sin x \tan x = 2$.

(ii): Write an algebraic expression (i.e. without trig functions) that is equivalent to $\sin(\arctan x)$.

(iii): Identify the amplitude, period, vertical shift, and phase shift of the graph of the function

$$h(x) = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 3.$$