Math 121

Fall 2013

Practice Problems for Exam 2

Many of the following practice problems will be useful in preparing for Exam 2.

Focus on Problems 1 – 19, 22 – 27, 29 – 32.

Problems 20 – 21 are on approximation of area under a curve via Riemann sums, a topic that will not be emphasized on Exam 2, but may be a candidate for extra credit.

Problems 28 and 33 are on the "average value of a function" and the "area between two curves," respectively, both of which are topics that will be discussed more after Exam 2. However, they are intuitively related to concepts that have already been covered and are thus also potential candidates for extra credit.

The remaining problems may be disregarded at this time, but will be relevant to preparing for the Final Exam.

Note that an answer key for all problems is provided on the final page of this document.

Math 121 Final Exam A Spring 2012 Instructor: Jeff Lang

Be Sure to Bubble In Your Name And ID Number On The Scantron.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine all critical points for the function.

1) $f(x) = x^3 - 9x^2 + 3$ 1) ________

A) $x = 0$ and $x = 3$
B) $x = 0$ and $x = 6$
C) $x = 0$
D) $x = -3$ and $x = 3$

Find the absolute extreme values of the function on the interval.

2) $g(x) = -x^2 + 13x - 42,\ 6 \le x \le 7$ 2) ________

A) absolute maximum is $\frac{5}{4}$ at $x = \frac{15}{2}$; absolute minimum is 0 at 7 and 0 at $x = 6$

B) absolute maximum is $\frac{1}{4}$ at $x = \frac{13}{2}$; absolute minimum is 0 at 7 and 0 at $x = 6$

C) absolute maximum is $\frac{337}{4}$ at $x = \frac{13}{2}$; absolute minimum is 0 at 7 and 0 at $x = 6$

D) absolute maximum is $\frac{1}{4}$ at $x = \frac{15}{2}$; absolute minimum is 0 at 7 and 0 at $x = 6$

Find the extreme values of the function and where they occur.

3) $y = x^3 - 12x + 2$ 3) ________

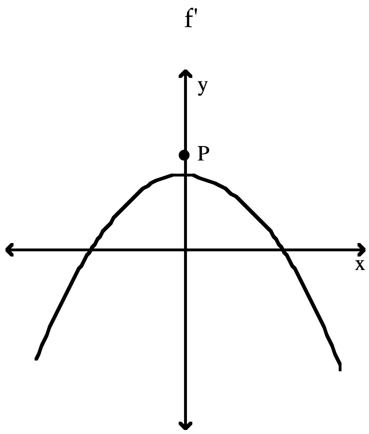
A) Local maximum at (0, 0).
B) None
C) Local maximum at (–2, 18), local minimum at (2, –14).
D) Local maximum at (2, –14), local minimum at (–2, 18).

Find the largest open interval where the function is changing as requested.

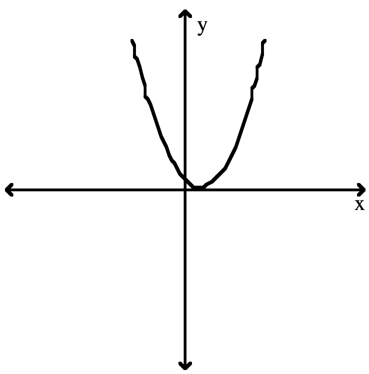
4) Increasing $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x$ 4) ________

A) $(1, \infty)$
B) $(-\infty, -1)$
C) $(-1, 1)$
D) $(-\infty, \infty)$

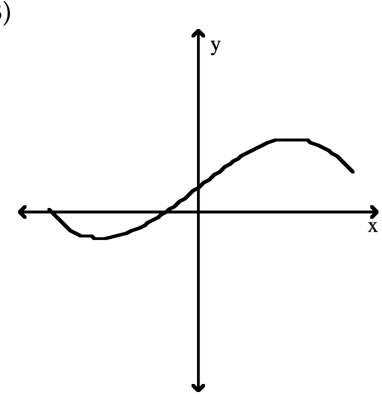
5) The graph below shows the first derivative of a function y = f(x). Select a possible graph for f from among the choices. 5) ________



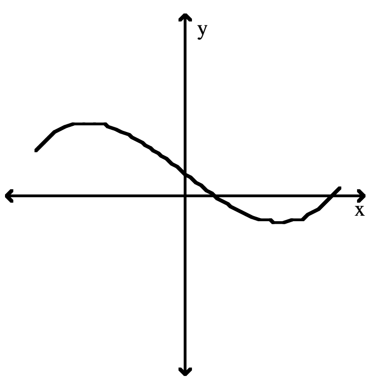
A)



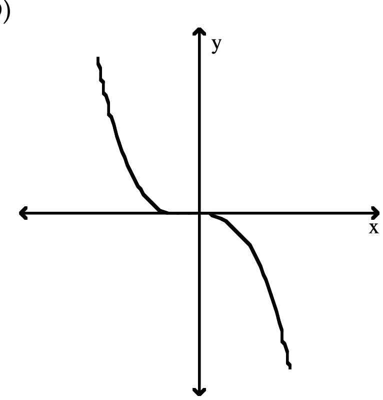
B)



C)

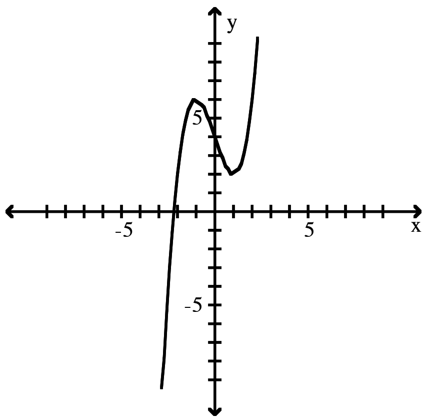


D)



Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.

6) 6) ________



A) Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(0, \infty)$; concave up on $(-\infty, 0)$

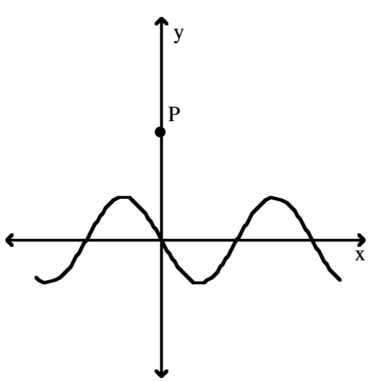
B) Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(-\infty, \infty)$

C) Local minimum at $x = 1$; local maximum at $x = -1$; concave down on $(-\infty, \infty)$

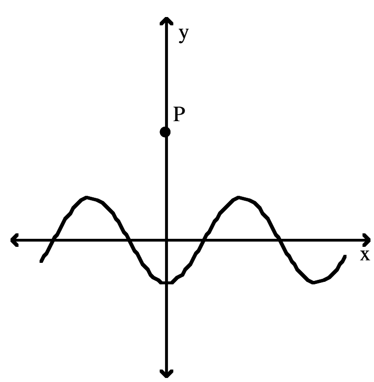
D) Local minimum at $x = 1$; local maximum at $x = -1$; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

7) The graphs below show the first and second derivatives of a function $y = f(x)$. Select a possible graph for f. The choices are continued on the next page. 7) ________

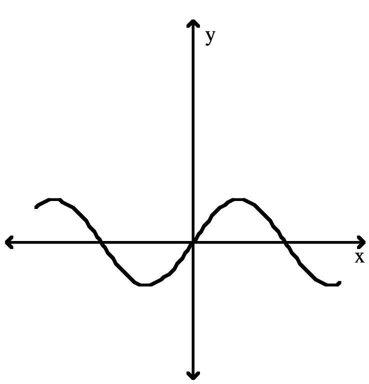
f'



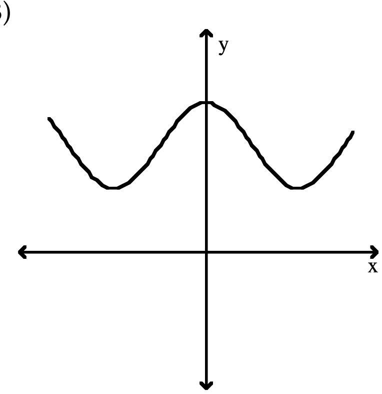
f''



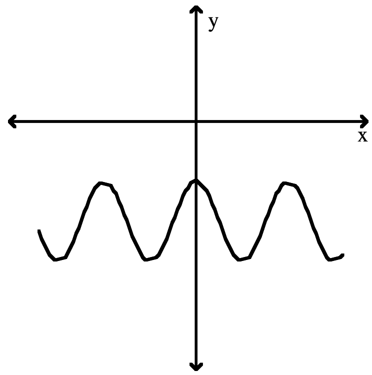
A)



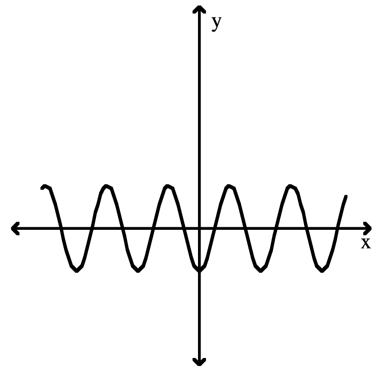
B)



C)



D)

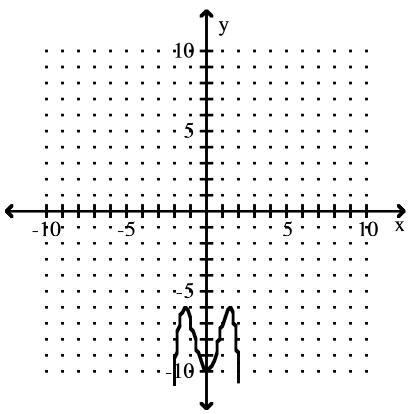


8) **Sketch the graph of** $y = -x^4 + 4x^2 - 10$ **and show all extrema and inflection poins. The choices are continued on the next page.** 8) ________

A) Local maxima: $(-\sqrt{2}, -6)$, $(\sqrt{2}, -6)$

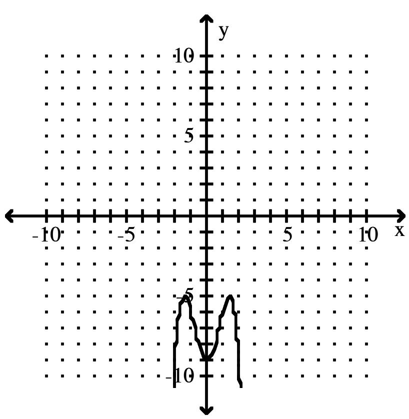
Local minimum: $(0, -10)$

Inflection points: $\left(-\sqrt{\frac{2}{3}}, \frac{2}{3}\right)$, $\left(\sqrt{\frac{2}{3}}, \frac{2}{3}\right)$

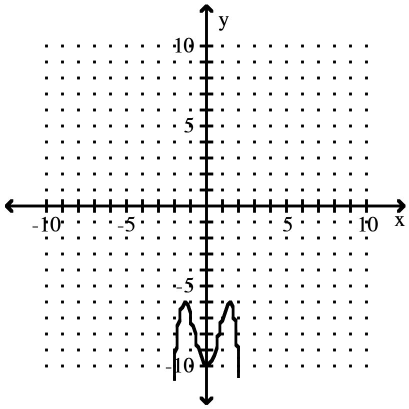


B) Local maxima: $(-\sqrt{2}, -6)$, $(\sqrt{2}, -6)$

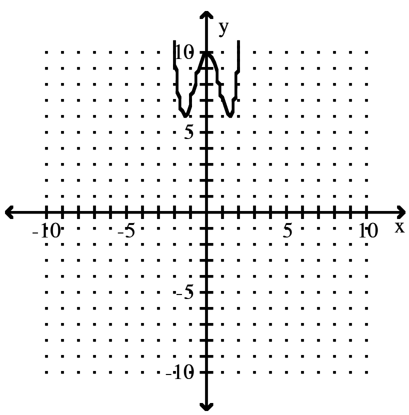
Inflection points: $\left(-\sqrt{\frac{2}{3}}, \frac{2}{3}\right)$, $\left(\sqrt{\frac{2}{3}}, \frac{2}{3}\right)$



C) Local maxima: $(-\sqrt{2}, -6)$, $(\sqrt{2}, -6)$
Local minimum: $(0, -10)$
No inflection points



D) Local minima: $(-\sqrt{2}, 6)$, $(\sqrt{2}, 6)$
Local maximum: $(0, 10)$
Inflection point: $\left(-\sqrt{\frac{2}{3}}, \frac{70}{9}\right), \left(\sqrt{\frac{2}{3}}, \frac{70}{9}\right)$

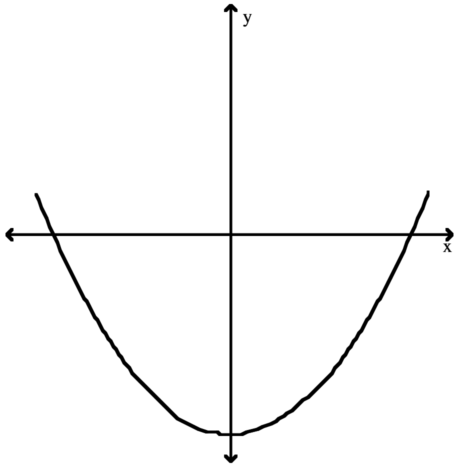


Solve the problem.

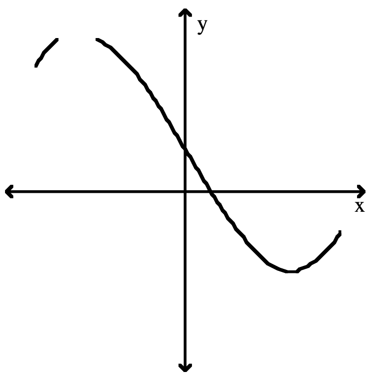
9) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary. 9) ________

A) 6.7 in. × 6.7 in. × 3.3 in.; 148.1 in^3
B) 5 in. × 5 in. × 2.5 in.; 62.5 in^3
C) 3.3 in. × 3.3 in. × 3.3 in.; 37 in^3
D) 6.7 in. × 6.7 in. × 1.7 in.; 74.1 in^3

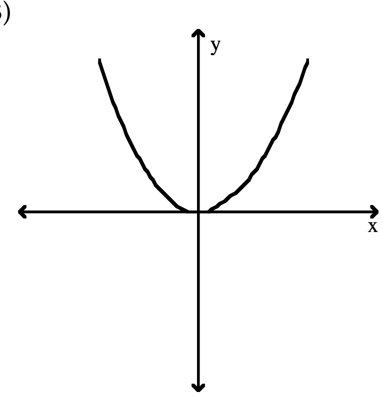
10) **If $y' = \frac{x^2}{4} - 5$, find y'' and sketch the general shape of the graph of y = f(x).** 10) _______



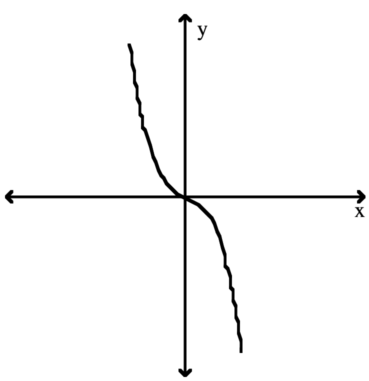
A)



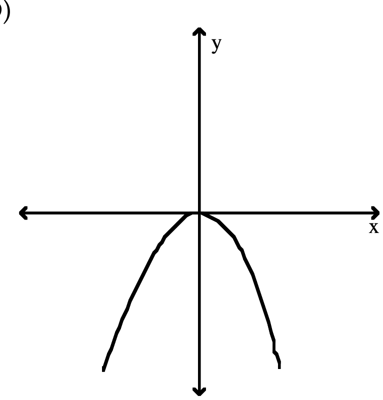
B)



C)



D)



Solve the problem.

11) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 101 - \frac{x}{24}$. How many candy bars must be sold to maximize revenue? 11) _______

A) 2424 thousand candy bars
B) 1212 candy bars
C) 1212 thousand candy bars
D) 2424 candy bars

Use l'Hopital's Rule to evaluate the limit.

12) $\lim\limits_{x \to 0} \dfrac{\sin 5x}{\sin x}$ 12) _______

A) 1 B) 0 C) –5 D) 5

13) $\lim\limits_{x \to \infty} \dfrac{x^2 + 3x + 3}{x^3 - 2x^2 + 2}$ 13) _______

A) –1 B) 0 C) ∞ D) 1

L'Hopital's rule does not help with the given limit. Find the limit some other way.

14) $\lim\limits_{x \to \infty} \dfrac{\sqrt{36x + 1}}{\sqrt{x + 8}}$ 14) _______

A) 36 B) 6 C) 0 D) ∞

Find an antiderivative of the given function.

15) $x^6 - \dfrac{1}{x^6}$ 15) _______

A) $\dfrac{x^7}{6} - \dfrac{1}{6x^5}$ B) $\dfrac{x^7}{7} - \dfrac{1}{7x^7}$ C) $6x^5 + \dfrac{1}{6x^5}$ D) $\dfrac{x^7}{7} + \dfrac{1}{5x^5}$

Find the most general antiderivative.

16) $\displaystyle\int \frac{x\sqrt{x} + \sqrt{x}}{x^2}\, dx$ 16) _______

A) $\dfrac{2}{\sqrt{x}} - 2\sqrt{x} + C$ B) C

C) $-\dfrac{\sqrt{x}}{2} - \dfrac{3\sqrt{x}}{2} + C$ D) $2\sqrt{x} - \dfrac{2}{\sqrt{x}} + C$

17) $\displaystyle\int (2e^{4x} - 5e^{-x})\, dx$ 17) _______

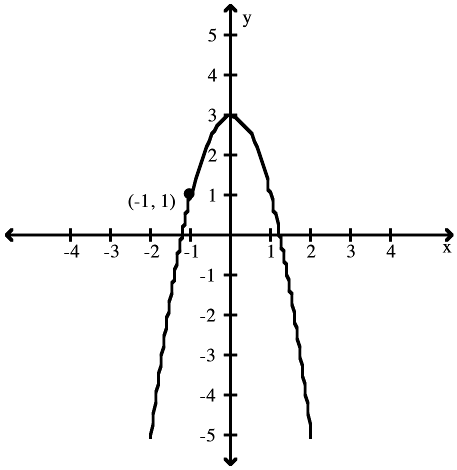
A) $\dfrac{1}{2}e^{4x} + \dfrac{1}{5}e^{-x} + C$ B) $\dfrac{1}{2}e^{4x} - 5e^{-x} + C$

C) $2e^{4x} + 5e^{-x} + C$ D) $\dfrac{1}{2}e^{4x} + 5e^{-x} + C$

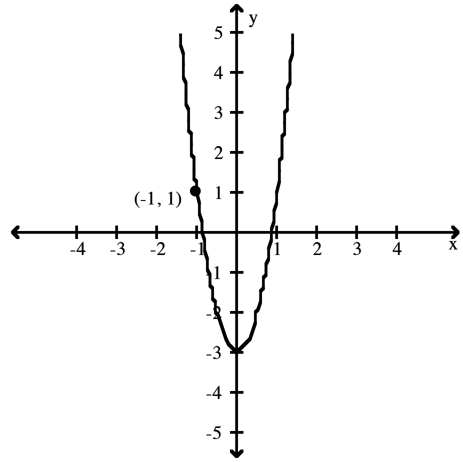
Which of the graphs shows the solution of the given initial value problem?

18) $\frac{dy}{dx} = 4x$, y = 1 when x = −1 18) _______

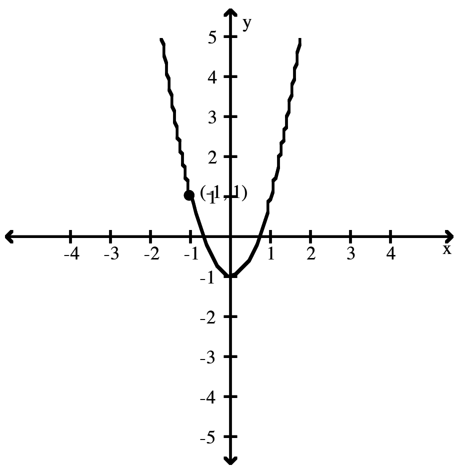
A)



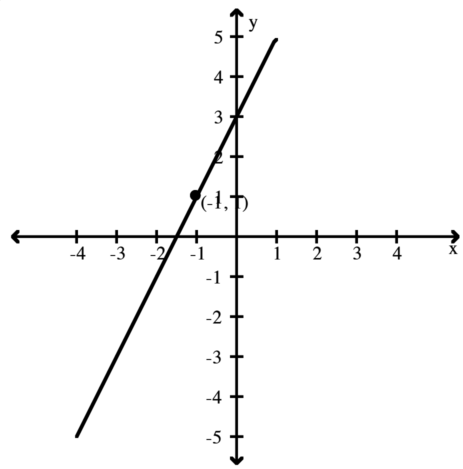
B)



C)



D)



Solve the problem.

19) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 19) _______

$v = -9t + 5,\ s(0) = 9$

A) $s = \frac{9}{2}t^2 + 5t - 9$

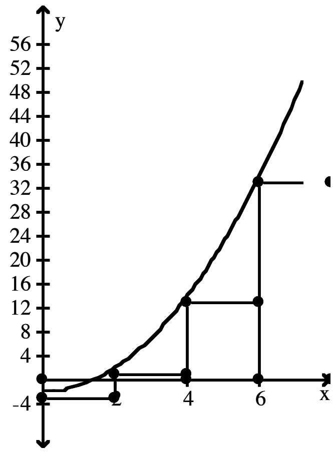
B) $s = -\frac{9}{2}t^2 + 5t - 9$

C) $s = -\frac{9}{2}t^2 + 5t + 9$

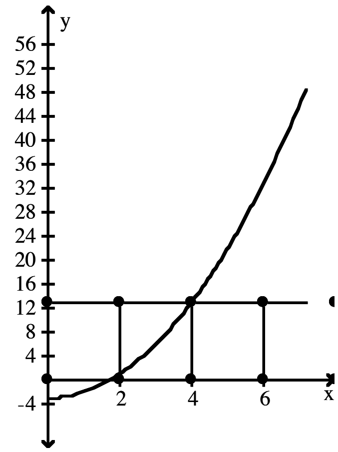
D) $s = -9t^2 + 5t + 9$

20) **Graph $f(x) = x^2 - 3$ over the interval [0, 8]. Partition the interval into 4 subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum f(x_k) \triangle x$, using the midpoint method.** 20) _______

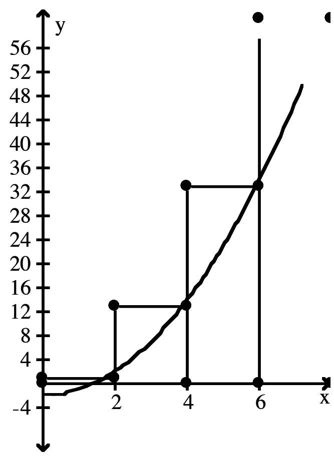
A)



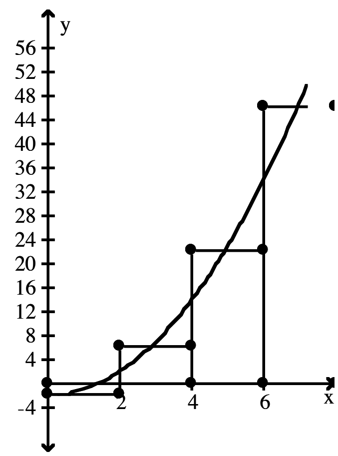
B)



C)



D)



Estimate the value of the quantity.

21) The table shows the velocity of a remote controlled race car moving along a dirt path for 8 seconds. Estimate the distance traveled by the car using 8 subintervals of length 1 with right–end point values. 21) _______

Time (sec)	Velocity (in./sec)
0	0
1	8
2	23
3	30
4	29
5	27
6	30
7	25
8	4

A) 182 in. B) 172 in. C) 176 in. D) 166 in.

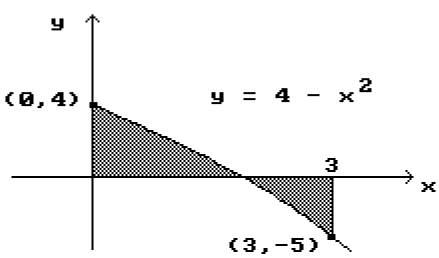
Solve the problem.

22) Suppose that g is continuous and that $\int_1^7 g(x)\,dx = 9$ and $\int_1^{10} g(x)\,dx = 16$. Find $\int_{10}^7 g(x)\,dx$. 22) _______

A) –25 B) 7 C) –7 D) 25

Find the area of the shaded region.

23) 23) _______



A) $\frac{5}{3}$ B) 5 C) $\frac{23}{3}$ D) 3

Evaluate the integral.

24) $\int_1^2 \left(t + \frac{1}{t}\right)^2 dt$ 24) _______

A) $\frac{15}{2}$ B) $\frac{29}{6}$ C) $\frac{37}{6}$ D) $\frac{5}{6}$

Find the total area of the region between the curve and the x–axis.

25) $y = \frac{1}{\sqrt{x}}$; $1 \le x \le 4$ 25) ______

A) $\frac{1}{2}$
B) 2
C) 4
D) $\frac{1}{4}$

Evaluate the integral.

26) $\int \csc^2 (5\theta + 9)\, d\theta$ 26) ______

A) $5 \cot (5\theta + 9) + C$
B) $10 \csc (5\theta + 9) \cot (5\theta + 9) + C$
C) $-\cot (5\theta + 9) + C$
D) $-\frac{1}{5} \cot (5\theta + 9) + C$

Find the derivative.

27) $\frac{d}{dx} \int_0^{x^3} \sin t\, dt$ 27) ______

A) $\sin (x^3)$
B) $-\cos (x^3) - 1$
C) $\frac{1}{4} x^4 \sin (x^3)$
D) $3x^2 \sin (x^3)$

Find the average value of the function over the given interval.

28) $f(x) = 2x + 12$ on $[-6, 6]$ 28) ______

A) 12
B) 24
C) 6
D) 144

Evaluate the integral.

29) $\int \frac{\sin t}{(3 + \cos t)^4}\, dt$ 29) ______

A) $\frac{1}{(3 + \cos t)^3} + C$
B) $\frac{3}{(3 + \cos t)^3} + C$
C) $\frac{1}{3(3 + \cos t)^3} + C$
D) $\frac{1}{5(3 + \cos t)^5} + C$

Use the substitution formula to evaluate the integral.

30) $\int_{\pi}^{3\pi/2} \frac{\sin \theta\, d\theta}{2 + \cos \theta}$ 30) ______

A) $-\ln 2$
B) 0
C) $-\ln 3$
D) $\ln 3$

31) $\int_0^{\pi} (1 + \cos 5t)^2 \sin 5t\, dt$ 31) ______

A) $\frac{8}{15}$
B) $\frac{8}{3}$
C) $\frac{1}{5}$
D) $\frac{1}{15}$

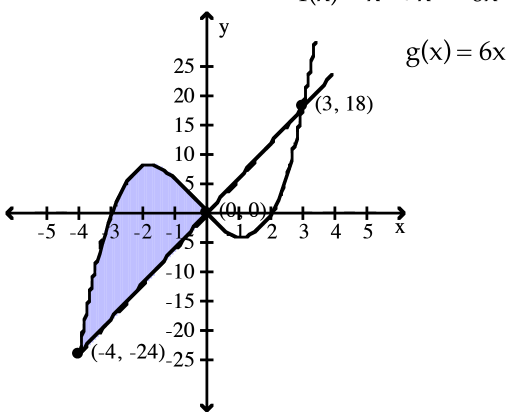
Solve the problem.

32) After a new firm starts in business, it finds that its rate of profits (in hundreds of dollars per year) after t years of operation is given by $\frac{dP}{dt} = 3t^2 + 2t + 2$. Find the profit in year 5 of the operation. 32) _______

A) $15,000 B) $12,400 C) $7200 D) $8800

Find the area of the shaded region.

33) $f(x) = x^3 + x^2 - 6x$ 33) _______



A) $\frac{160}{3}$ B) $\frac{343}{12}$ C) $\frac{81}{12}$ D) $\frac{768}{12}$

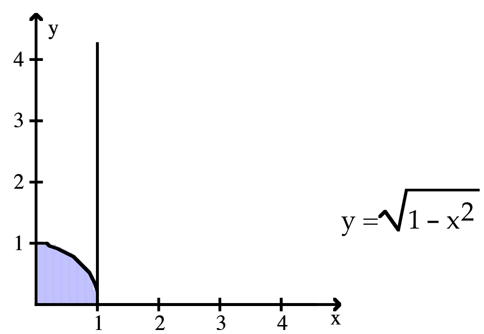
Find the area enclosed by the given curves.

34) $y = \frac{1}{2}x^2,\ y = -x^2 + 6$ 34) _______

A) 4 B) 8 C) 16 D) 32

Use the disc method to find the volume of the solid generated by revolving the shaded region about the indicated axis.

35) About the x–axis 35) _______



A) $\frac{2}{3}\pi$ B) 1π C) $\frac{3}{2}\pi$ D) $\frac{1}{3}\pi$

Evaluate the integral.

36) $\int \frac{\cos x \, dx}{1 + 4 \sin x}$ 36) ______

A) $\frac{1}{4} \ln | 1 + 4 \sin x | + C$

B) $\ln | 1 + 4 \sin x | + C$

C) $4 \sin x + C$

D) $4 \ln | 1 + 4 \sin x | + C$

37) **Use integration by parts to evaluate** $\int 11x \sin x \, dx$ 37) ______

A) $11 \sin x + 11x \cos x + C$

B) $11 \sin x - 11 \cos x + C$

C) $11 \sin x - 11x \cos x + C$

D) $11 \sin x - x \cos x + C$

Evaluate the improper integral or state that it is divergent.

38) $\int_0^{\infty} 21e^{-21x} \, dx$ 38) ______

A) 1 B) –1 C) Divergent D) 0

Express the integrand as a sum of partial fractions and evaluate the integral.

39) $\int \frac{144 \, dx}{x^3 - 36x}$ 39) ______

A) $-\frac{4}{x} + 2\ln|x - 6| + 2\ln|x + 6| + C$

B) $-4 \ln|x| + \frac{1}{6}\tan^{-1} \frac{x}{6} + C$

C) $4 \ln|x| - 2\ln|x - 6| - 2\ln|x + 6| + C$

D) $-4 \ln|x| + 2\ln|x - 6| + 2\ln|x + 6| + C$

Answer Key
Testname: 121 FINAL A

1) B
2) B
3) C
4) A
5) B
6) D
7) B
8) A
9) D
10) A
11) C
12) D
13) B
14) B
15) D
16) D
17) D
18) C
19) C
20) D
21) C
22) C
23) C
24) B
25) B
26) D
27) D
28) A
29) C
30) A
31) A
32) C
33) A
34) C
35) A
36) A
37) C
38) A
39) D