Math 115 - Practice Problems for Test 3

1. Let $f(x) = \ln(3x)$. Determine whether each of the following statements is true or false.

   (a) T F.....$f(x) = (\ln 3)(\ln x)$.

   (b) T F.....The domain of f is $(0, \infty)$.

   (c) T F.....f is increasing on $(0, \infty)$.

   (d) T F.....f is CCU on $(0, \infty)$

   (e) T F.....For all $x > 0$, $f(x) > 0$.

   (f) T F.....$f'(x) = \dfrac{1}{x}$ for all x in the domain of f.

   (g) T F.....$f(e^x) = \ln 3 + x$.

2. Find the absolute maximum and minimum values of $f(x) = x^3 - 4x^2 + 12$ on the interval $[-3, 1]$. Justify your answer.

3. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

   (A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

4. Identify the intervals where $f(x) = e^x - 2e^{-x}$ is concave upward.

5. Identify the intervals where $g(x) = e^x + e^{-2x}$ is increasing.

6. Suppose that a rectangular box with open top and square base is to be made using two different materials. The material for the base cost \$2 per square foot and the material for the four sides costs \$1 per square foot. Find the dimensions of the box of maximum value if the total cost for materials is to be \$96. What is the maximum volume? Justify your answer.

7. If $f(x) = \dfrac{e^{2x}}{2x}$ then $f'(x) =$

   (A) 1 (B) $\dfrac{e^{2x}(1-2x)}{2x^2}$ (C) e^{2x} (D) $\dfrac{e^{2x}(2x+1)}{x^2}$ (E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

8. Find $f'(x)$ when $f(x) = x^4 + 4^x$.

9. Find $f'(x)$ when $f(x) = \ln \dfrac{1}{x} - \dfrac{1}{\ln x}$.

10. Find the absolute maximum and minimum values of $f(x) = x^3 - 6x$ on $[0, 3]$. Justify your answer.

11. Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 3$ on $[0, 2]$. Justify your answers.

12. JHTS Company can produce Jayhawk T-shirts for \$5 each. Market research suggests that 20,000 shirts can be sold at \$12 each and sales can be increased by 2000 shirts with each \$1 reduction in price. Let x be the number of \$1 price increases. What shirt price maximizes profit? Justify your answer.

13. Consider a function f whose domain is all real numbers. Suppose that f has first and second derivatives on its entire domain. Circle T if the statement is always true, otherwise circle F.

    (a) T F.....f has an absolute maximum for some x in its domain.

    (b) T F.....f has a relative minimum for some x in its domain.

    (c) T F.....If $f'' > 0$ on $0 < x < 17$, then f' is increasing on (0, 17).

    (d) T F.....If $f'' > 0$ on $0 < x < 17$, then f is concave up on (0,17).

    (e) T F.....If $f' > 0$ on $0 < x < 17$, then f is concave up on (0, 17).

14. How many points of inflections does the graph of $y = -x^5 - 2x^3 + 10x - 1$ have?

    (A) None (B) One (C) Two (D) Three (E) Five

15. If g is continuous on $[a, b]$, which one of the following must be true?

    (a) g' exists on (a, b).

    (b) If $a < c < b$ and $g(c)$ is a relative maximum of g, then $g'(c) = 0$.

    (c) $\lim_{x \to c} g(x) = g(c)$ for $a < c < b$.

    (d) For some $c \in (a, b)$, $g'(c) = 0$.

    (e) The graph of g' is a straight line.

16. $\dfrac{d}{dx}\left(x^{\sqrt{x}}\right)=$

    (A) $(\sqrt{x})x^{\sqrt{x}-1}$ (B) $\dfrac{x^{\sqrt{x}}}{\sqrt{x}}(1+\ln(\sqrt{x}))$ (C) $.5x^{\sqrt{x}-1}$ (D) $\left(x^{\sqrt{x}}\right)(.5+\ln(\sqrt{x}))$ (E) $x^{\sqrt{x}-1}$

17. Let f be a function such that $f(x)$ is defined for all $x \neq -\pi$. Also, suppose that f has a continuous second derivative for $x \neq -\pi$.
    $f(\sqrt{3}) = e,\ \ f(\sqrt{10}) = \dfrac{\pi}{2}$
    $\lim_{x \to +\infty} f(x) = 0, \quad \lim_{x \to -\infty} f(x) = 2.1$
    $\lim_{x \to (-\pi)^+} f(x) = -\infty, \quad \lim_{x \to (-\pi)^-} f(x) = -\infty$
    $f' > 0$ on $(-\pi, \sqrt{3})$
    $f' < 0$ on $(-\infty, -\pi)$ and $(\sqrt{3}, \infty)$
    $f'' < 0$ on $(-\infty, -\pi)$ and $(-\pi, \sqrt{10})$
    $f'' > 0$ on $(\sqrt{10}, \infty)$

    (a) Find the x coordinates of all relative maxima.

(b) Find the x coordinates of all relative minima.

(c) Find the x coordinates of all inflection points.

(d) Find the equations of all horizontal asymptotes.

(e) Find the equations of all vertical asymptotes.

(f) Carefully sketch a graph of f.

18. The maximum possible area of a rectangle of perimeter $200m$ is

(A) $2000m^2$ (B) $2500m^2$ (C) $3500m^2$ (D) $2400m^2$ (E) None of the above.

19. Evaluate $\lim\limits_{x \to \infty} \dfrac{e^x - x}{e^x + x}$.

20. Let f be the function defined by $f(x) = 2xe^{-3x}$ for all real numbers x. Identify the domain, intercepts, intervals where f is increasing and decreasing, extrema, intervals of concavity, inflection points and asymptotes. (Give all coordinates in exact form.) Sketch the graph of $f(x)$, showing all this information and labeling important points.

21. Let $f(x) = x^2 - \ln(x^2)$.

(a) What is the domain of f?

(b) Find the intervals where f is increasing and decreasing.

(c) Find the intervals where f is concave up and concave down.

(d) Find the absolute maximum and minimum points of f on $[\frac{1}{2}, 3]$.

(e) Sketch a graph of f.

22. Find the vertical asymptotes of the function $f(x) = \dfrac{2+x}{(1-x)^2}$.

23. Find an equation of the tangent line to the graph of $y = e^{2x-3}$ at $x = \dfrac{3}{2}$.

24. Find the horizontal asymptotes of the function $f(x) = \dfrac{x^2}{1+4x^2}$.

25. Find the absolute maximum and absolute minimum of the function $f(t) = \dfrac{\ln t}{t}$ on [1,2].

26. An open box is to be made from a square sheet of cardboard by cutting out squares of equal size from the corners and bending up the flaps. The sheet of cardboard measures 20 cm on each side. Find the dimensions of the box of maximum volume that can be made this way.

27. The half life of radium is approximately 1600 years. How long will it take 10 grams of radium to decay to 6 grams?

28. When a foreign substance is introduced into the body, the body's defense mechanisms move to break down the substance. The rate of excretion is proportional to the concentration in the body. The half life of the substance is called the biological half life. If after 12 hours 30% of a massive dose of a drug has been excreted by the body, what is the biological half-life of the drug?

29. The number of bacteria grows rapidly in a rich culture, at a rate proportional to its current population. If the number of bacteria grows from 100,000 to 150,000 bacteria in 2 hours, how many bacteria will be present after 5 hours?

30. The population of a certain city is projected to be $P(t) = 100,000(1 + 5t)e^{-0.05t}$ in t years. When will the city's population be at a maximum?

31. A certain menacing biological culture (aka the Blob) grows at a rate proportional to its size. When it arrived unnoticed one Wednesday noon in Chicago's Loop, it weighed just one gram. By 4:00 pm rush hour it weighed 4 grams. The Blob has its "eye" on the Willis Tower (formerly known as the Sears Tower), a tasty morsel weighing around 3000000000000 (i. e. 3×10^{12}) grams. The Blob intends to *eat* the Willis Tower as soon as it weighs 1000 times as much (i.e. 3×10^{15} grams). By what time must the Blob be stopped? Will Friday's rush hour commuters be delayed?

32. A bacterial culture is placed in a large glass bottle. Suppose that the volume of the culture doubles every hour, and the bottle is full after one day.

    (a) If the culture was placed in the bottle at time $t = 0$ hours, when was the bottle half full?

    (b) Assume the bottle is "almost empty" when the culture occupies less than 1 % of its volume. How long was the bottle "almost empty"?

Answers - Practice Problems for Test 3

1. (a) $\boxed{F}$ (b) $\boxed{T}$ (C) $\boxed{T}$ (D) $\boxed{F}$ (E) $\boxed{F}$ (F) $\boxed{T}$ (G) $\boxed{T}$

2. Abs max = 12, abs min = -51 3. (C) 4. $(\frac{\ln 2}{2}, \infty)$ 5. $(\frac{\ln 2}{3}, \infty)$

6. Max volume of 64 ft^3 when box size is 4 ft $\times$ 4 ft $\times$ 4 ft

7. None of the choices are correct; correct answer is $\frac{e^{2x}(2x-1)}{2x^2}$.

8. $f'(x) = 4x^3 + (\ln 4)4^x$ 9. $f'(x) = \frac{1}{x}((\ln x)^{-2} - 1)$

10. Abs max = 9 @ $x = 3$, abs min = $-4\sqrt{2}$ @ $x = \sqrt{2}$

11. Abs max = 3 @ $x = 0$, abs min = -18 @ $x = 2$

12. max profit of \$72.25 @\$13.50 for each shirt

13. (a) $\boxed{F}$ (b) $\boxed{F}$ (C) $\boxed{F}$ (D) $\boxed{T}$ (E) $\boxed{F}$ 14. (B) 15. (c) 16. (B)

17. (a) $x = \sqrt{3}$ (b) none (c) $x = \sqrt{10}$ (d) $y = 0, y = 2.1$ (e) $x = -\pi$

18. (B) 19. 1

20. dom = $(-\infty, \infty)$, intercept = $(0, 0)$, incr on $(-\infty, \frac{1}{3})$, decr on $(\frac{1}{3}, \infty)$, CCU on $(\frac{2}{3}, \infty)$, CCD on $(-\infty, \frac{2}{3})$, rel max $(\frac{1}{3}, \frac{2}{3}e^{-1})$, no rel min, HA: $y = 0$, no VA

21. (a) dom = $(-\infty, 0) \cup (0, \infty)$

(b) incr on $(-1, 0), (1, \infty)$; decr on $(-\infty, -1), (0, 1)$,

(c) CCU on $(-\infty, 0), (0, \infty)$, not CCD

(d) abs max value of $9 - \ln 9$ @ $x = 3$, abs min value of 1 @ $x = 1$

22. $x = 1$ 23. $y = 2x - 2$ 24. $y = \frac{1}{4}$

25. Abs max value of $\frac{\ln 2}{2}$ @ $x = 2$, abs min value of 0 @ $x = 1$

26. Max vol of $4(\frac{10}{3})^3$ cm^3. 27. ≈ 1179.14 years 28. ≈ 6.914 hours

29. $\approx 275{,}568$ bacteria 30. 19.8 years 31. ≈ 102.8 hours = Sunday $\approx$ 6:48 pm

32. (a) 23 hours (b) 17.36 hours