Math 104. **Unit 2 review worksheet.** **Fall 2011.**

You should attempt the following sample problems. Others may be found on the listed textbook pages.

Since you will not have time to attempt all of these exercises during the workshop, you may like to focus on the following shorter list of topics that I believe were the most challenging for students in Unit 2:

- polynomial division (2.3),
- finding all zeros of a polynomial function (2.5),
- finding asymptotes and understanding the graphs of rational functions by hand (2.7),
- manipulating logarithmic expressions using properties of logarithms (3.3),
- solving exponential and logarithmic equations (3.4),
- working with exponential and logarithmic models (3.5).

Section 2.1.
Exercises: pg's. 99-102.
Topics: Standard form of quadratic functions (vertex, leading coefficient, etc.), maximum/minimum values.
Sample problems:

1. For each of the following quadratic functions, find its vertex and x-intercept(s) algebraically, then sketch its graph by hand (hint: complete the square to put the quadratic equation in standard form).
   (i) $f(x) = x^2 - 7$.
   (ii) $g(x) = x^2 - 8x + 16$.
   (iii) $h(x) = 4x^2 - 4x + 21$.
2. Write the standard form of the equation of the quadratic function that has vertex $(1, -2)$ and passes through the point $(-1, 14)$.
3. Look at word problems 53-62, on pg's. 100-102.

Section 2.2.
Exercises: pg's. 112-115.
Topics: Understanding higher-degree polynomial functions, leading coefficient test, correspondence between zeros and factors (i.e. factor theorem), multiplicity of zeros.
Sample problems:

1. Use the leading coefficient test to describe the right-hand and left-hand behavior of the graph of the polynomial function $f(x) = 5 - \frac{7}{2}x - 3x^2$.
2. Find all real zeros of the following polynomial functions both algebraically and graphically.
   (i) $f(x) = 5x^2 - 10x - 5$.
   (ii) $h(x) = x^3 - 4x^2 - 25x + 100$.
   (ii) $g(t) = t^5 - 6t^3 + 9t$.
3. Find a polynomial with degree 3 and zeros: -2, M2, and -1, M1 (M = multiplicity).
4. Look at word problems 91-94, on pg. 114.

Section 2.3.
Exercises: pg's. 127-130.
Topics: Rational zero test, synthetic division, polynomial long division, remainder theorem.
Sample problems:

1. Use long division to divide $2x^3 - 3x^2 - 50x + 75$ by $2x - 3$.
2. Use synthetic division to divide $5x^3 + 6x + 8)$ by $x + 2$.
3. Use the rational zero test to list all possible rational zeros of $f(x) = 4x^4 - 55x^2 - 45x + 36$, then find all real zeros of f.
4. Verify that $(x + 3)$ is a factor of the polynomial function $f(x) = 3x^3 + 2x^2 - 19x + 6$. Find the remaining factors of f, and use your results to write the complete factorization of f and to list all the real zeros of f.
5. Look at word problems 93-94, on pg. 130.

Section 2.4.
Exercises: pg's. 137-138.
Topics: Intro to complex numbers, complex sums/differences/products/quotients, complex conjugation.
Sample problems:

1. For each of the following, perform the operation and write the result in standard form:
   (i) $\sqrt{-6} \cdot \sqrt{-2}$.
   (ii) $(-1 + \sqrt{-8}) + (8 - \sqrt{-50})$.
   (iii) $(6 - 2i)(2 - 3i)$.
   (iv) $13i - (14 - 7i)$.
   (v) $i/(4 - 5i)^2$.
2. Simplify the complex number $4i^7 - 2i^{12}$ and write in standard form.
3. Write the complex conjugate of $-3 - i\sqrt{2}$. Then multiply by the conjugate and simplify.

Section 2.5.
Exercises: pg's. 144-145
Topics: Fundamental theorem of algebra, linear factorization theorem, finding all zeros of a polynomial function.
Sample problems:

1. Find all zeros of each of the following functions, and write it as a product of linear factors.
   (i) $f(x) = x^3 + 11x^2 + 39x + 29$.
   (ii) $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$.
   (iii) $h(s) = 3s^3 - 4s^2 + 8s + 8$.
   (iv) $f(x) = 2x^3 - 3x^2 + 8x - 12$.
2. Find a polynomial function of degree 4 that has the given zeros: $2, 3, 1 - 5i$.
3. Use the given zero $5 + 2i$ to find all the zeros of the function $g(x) = x^3 - 7x^2 - x + 87$.
4. Look at word problems 63-64, on pg. 145.

Section 2.6.
Exercises: pg's. 152-155.
Topics: Rational functions, finding vertical/horizontal asymptotes, graphing rational functions (look for domain, intercepts, asymptotes, and holes).
Sample problems:

1. Identify any horizontal or vertical asymptotes, or holes, in the graph of the rational function $f(x) = \frac{x^2+2x+1}{2x^2-x-3}$.
2. Find the zeros, if any, of the rational function $g(x) = \frac{x^2-2x-3}{x^2+1}$.
3. Look at word problems 39-40, on pg's. 153-154.

Section 2.7.
Exercises: pg's. 161-164.
Topics: Graphing rational functions by hand, oblique asymptotes.
Sample problems:

1. Sketch the graph of each of the following rational functions. As sketching aids, check for intercepts, asymptotes (horizontal, vertical, and oblique), and holes.
   (i) $f(t) = \frac{1-2t}{t}$.
   (ii) $g(x) = \frac{4(x+1)}{x(x-4)}$.
   (iii) $h(x) = \frac{5(x+4)}{x^2+x-12}$.
   (iv) $f(x) = \frac{2x^2+1}{x}$.
2. Look at word problems 77-84, on pg's. 162-163.

Section 3.1.
Exercises: pg's. 193-195.
Topics: Intro to exponential functions and their graphs.
Sample problems:

1. Graph the exponential function $h(x) = (3/2)^{-x}$ by hand. Identify any asymptotes and intercepts, and determine whether the graph of the function is increasing or decreasing.

2. A principal of \$2500 is invested in an account with a 4% interest rate. How much money is in the account after 20 years if the interest is compounded annually? What if the interest is compounded semiannually? quarterly? monthly? weekly? daily? or continuously?
3. Look at *very important* word problems 66-72, on pg's. 194-195. (see also: Section 3.5).

Section 3.2.
Exercises: pg's. 203-206.
Topics: Intro to logarithmic functions and their graphs, inverse relationship of exponential and log functions.
Sample problems:

1. Write the logarithmic equation in exponential form:
   (i) $\log_{32} 4 = \frac{2}{5}$.
   (ii) $\ln \sqrt{e} = \frac{1}{2}$.
2. Write the exponential equation in logarithmic form:
   (i) $8^2 = 64$.
   (ii) $e^{1.3} = 3.6692\ldots$.
3. Use the properties of logarithms to simplify $3 \log_2 \frac{1}{2}$.
4. Sketch the graph of $f(x) = 3^x$ by hand. Use this to sketch the graph of $g(x) = \log_3(x)$.
5. Find the domain, vertical asymptote, and x-intercept of the function $f(x) = \ln(x-1)$, then sketch its graph by hand.

Section 3.3.
Exercises: pg's. 211-213.
Topics: Properties of logarithms (product property, quotient property, power property).
Sample problems:

1. Evaluate $\log_3(0.015)$ using the change of base formula.
2. Use the properties of logarithms to verify the equation $\log_5 \frac{1}{250} = -3 - \log_5 2$.
3. Use the properties of logarithms to expand each of the following:
   (i) $\log_3 a^2 bc^3$.
   (ii) $\ln \frac{x}{\sqrt{x^2+1}}$.
4. Use the properties of logarithms to condense each of the following to a single quantity:
   (i) $2 \log_2(x+3)$.
   (ii) $\frac{1}{3}\left[2\ln(x+3) + \ln x - \ln(x^2-1)\right]$.
5. Look at word problem 96, on pg. 212.

Section 3.4.
Exercises: pg's. 221-224
Topics: Solving exponential (same-base type, different-base type, and "quadratic" type) and log equations.
Sample problems:

1. Solve each of the following exponential equations:
   (i) $4^x = 16$.
   (ii) $2^{x+3} = 256$.
   (iii) $5(2^{3-x}) - 13 = 100$.
   (iv) $\left(16 + \frac{0.878}{26}\right)^{12t} = 2$.
   (v) $e^{2x} - 4e^x - 5 = 0$.
2. Solve each of the following logarithmic equations:
   (i) $\ln x - \ln 5 = 0$.
   (ii) $\log x = -\frac{1}{2}$.
   (iii) $\ln \sqrt{x+2} = 1$.
   (iv) $\log_3 x + \log_3(x-8) = 2$.
   (v) $\log 8x - \log(1+\sqrt{x}) = 2$.
3. Use properties of logarithms to simplify $3 \log_2 \frac{1}{2}$.
4. Sketch the graph of $f(x) = 3^x$ by hand. Use this to sketch the graph of $g(x) = \log_3(x)$.
5. Find the domain, vertical asymptote, and x-intercept of the function $f(x) = \ln(x-1)$, then sketch its graph by hand.

Section 3.5.
Exercises: pg's. 232-236.
Topics: Exponential and logarithmic models (compound interest, exponential growth and decay).
Sample problems:

1. Compound interest: see problems 7-14, on pg's. 232-233.
2. Radioactive decay: see problems 19-22, on pg. 233, and 31-32, on pg. 234.
3. Population growth: see problems 29-30, on pg. 234.