1. (a) $\cos\left(\frac{4\pi}{3}\right) = \frac{-1}{2}$

1. (b) $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

1. (c) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

1. (d) $\cos\left(\arccos\left(\frac{\pi}{2}\right)\right)$ does not exist, because $\frac{\pi}{2}$ is not in the domain of arccos, i.e. $\frac{\pi}{2} \approx 1.57$ is not between -1 and 1.

2. Let $u = \arcsin\left(\frac{5}{x-1}\right)$. Then $\sin u = \frac{5}{x-1}$. Recall that in the right-triangle definition of the trigonometric functions, $\sin u = \frac{\text{opp}}{\text{hyp}}$. So in the right-triangle with acute angle u, we have opp $= 5$ and hyp $= x-1$. We wish to find $\cot\left(\arcsin\left(\frac{5}{x-1}\right)\right) = \cot u = \frac{\text{adj}}{\text{opp}}$, so we will need to know adj in the triangle. By the Pythagorean Theorem,

$$\text{hyp}^2 = \text{adj}^2 + \text{opp}^2 \implies \text{adj} = \sqrt{\text{hyp}^2 - \text{opp}^2} = \sqrt{(x-1)^2 - 5^2} = \sqrt{(x-1)^2 - 25}.$$

Then we conclude $\cot\left(\arcsin\left(\frac{5}{x-1}\right)\right) = \cot u = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{(x-1)^2-25}}{5}$.

3. Since $\sec\alpha = -2$, we immediately know that $\cos\alpha = \frac{1}{\sec\alpha} = -\frac{1}{2}$. Since cosine is negative and tangent is positive, the angle α must be in QIII (i.e. sine is negative). Thus,

$$\cos^2\alpha + \sin^2\alpha = 1 \implies \sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\left(-\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}.$$

The remaining functions are:

$$\begin{aligned}
\csc\alpha &= \frac{1}{\sin\alpha} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}.\\
\tan\alpha &= \frac{\sin\alpha}{\cos\alpha} = \frac{-\sqrt{3}2}{-1/2} = \sqrt{3}.\\
\cot\alpha &= \frac{1}{\tan\alpha} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.
\end{aligned}$$

4. (a)

$$\begin{aligned}
\frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} &= \frac{\sin\theta}{1+\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}\\
&= \frac{\sin^2\theta + \cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}\\
&= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}\\
&= \frac{1+\cos\theta}{\sin\theta(1+\cos\theta)}\\
&= \frac{1}{\sin\theta}\\
&= \csc\theta.
\end{aligned}$$

4. (b)

$$\begin{aligned}\cos x\tan^2 x+\cos x &= \cos x\frac{\sin^2 x}{\cos^2 x}+\cos x\\ &= \frac{\sin^2 x}{\cos x}+\cos x\frac{\cos x}{\cos x}\\ &= \frac{\sin^2 x+\cos^2 x}{\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

5.

$$\begin{aligned}\cos x = 1+\cos(2x) &\implies \cos x = 1+(2\cos^2 x-1)\\ &\implies 2\cos^2 x-\cos x=0\\ &\implies \cos x(2\cos x-1)=0\\ &\implies \cos x=0 \text{ or } 2\cos x-1=0\\ &\implies \cos x=0 \text{ or } \cos x=\frac{1}{2}.\end{aligned}$$

The solutions in the interval $[0,2\pi)$ are $x=\frac{\pi}{2}, x=\frac{3\pi}{2}, x=\frac{\pi}{3}, x=\frac{5\pi}{3}$. We can then write the general solution as: $x=\frac{\pi}{2}+n\pi, x=\frac{\pi}{3}+2n\pi, x=\frac{5\pi}{3}+2n\pi$. (Notice that the first two solutions are combined into one, because $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are halfway around the unit circle from one another).

6. (tip: draw a picture!). If x is the height of the first story of the buliding, and y is the height of the first two stories, then $d=y-x$ is the height of the second story alone. From the information we are given, we know that

$$\begin{aligned}\tan 33^\circ &= \frac{x}{44} \implies x=44\tan 33^\circ.\\ \tan 46^\circ &= \frac{y}{44} \implies y=44\tan 46^\circ.\end{aligned}$$

Thus, the height of the second story of the buliding is $d=y-x=44(\tan 46^\circ-\tan 33^\circ)\approx 16.99$ ft., or we could just say about 17 ft.

Bonus.

(i) $\arcsin\left(\sin\left(\frac{\pi}{4}\right)\right)=\frac{\pi}{4}$, because $\frac{\pi}{4}$ is in the range of arcsin, i.e. it is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

(ii) Vertical shift: up 4; Amplitude: 7; Period: $\frac{2\pi}{2}=\pi$; Phase shift: $\frac{\pi/2}{2}=\frac{\pi}{4}$.

(iii) $\arccos\left(\cos\left(\frac{5\pi}{3}\right)\right)=\arccos\left(\cos\left(2\pi+\frac{\pi}{3}\right)\right)=\arccos\left(\cos\left(\frac{\pi}{3}\right)\right)=\frac{\pi}{3}$, because, although $\frac{5\pi}{3}$ is not in the range of arccos, its coterminal angle $\frac{\pi}{3}$ is.

(iv) First of all, $\sin\beta=\frac{1}{\csc\beta}=\frac{1}{7/3}=\frac{3}{7}$. Then,

$$\cos\left(\beta+\frac{\pi}{2}\right)=\cos\beta\cos\left(\frac{\pi}{2}\right)-\sin\beta\sin\left(\frac{\pi}{2}\right)=\cos\beta\cdot 0-\frac{3}{7}\cdot 1=-\frac{3}{7}.$$