

1. (a) $C(x) = 9x + 200000$, $R(x) = 17x$, $P(x) = R(x) - C(x) = 17x - (9x + 200000) = 8x - 200000$.

1. (b) Breaking even occurs when $R(x) = C(x)$ or equivalently $P(x) = 0$. Solving the latter, we find:

$$P(x) = 0 \implies 8x - 200000 = 0 \implies 8x = 200000 \implies x = 25000.$$

So the manufacturer must produce 25000 units/month to break even.

2. $(f + g)(x) = (3x^2 + 2x + 1) + (x + 3) = 3x^2 + 3x + 4$.
 $(f - g)(x) = (3x^2 + 2x + 1) - (x + 3) = 3x^2 + x - 2$.
 $(fg)(x) = (3x^2 + 2x + 1)(x + 3) = 3x^3 + 9x^2 + 2x^2 + 6x + x + 3 = 3x^3 + 11x^2 + 7x + 3$.
 $(f/g)(x) = (3x^2 + 2x + 1)/(x + 3)$.
 $(f \circ g)(x) = f(g(x)) = 3(x + 3)^2 + 2(x + 3) + 1 = 3x^2 + 18x + 27 + 2x + 6 + 1 = 3x^2 + 20x + 34$.

3. (a) $\lim_{s \rightarrow 0} (2s^2 - 1)(2s + 4) = \left(2 \left(\lim_{s \rightarrow 0} s\right)^2 - 1\right) \left(2 \left(\lim_{s \rightarrow 0} s\right) + 4\right) = (2(0)^2 - 1)(2(0) + 4) = -4$.

3. (b) $\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8}}{2x + 4} = \frac{\sqrt{\left(\lim_{x \rightarrow -1} x\right)^2 + 8}}{2 \left(\lim_{x \rightarrow -1} x\right) + 4} = \frac{\sqrt{(-1)^2 + 8}}{2(-1) + 4} = \frac{3}{2}$.

3. (c) $\lim_{x \rightarrow a} \sqrt[3]{5f(x) + 3g(x)} = \sqrt[3]{5 \left(\lim_{x \rightarrow a} f(x)\right) + 3 \left(\lim_{x \rightarrow a} g(x)\right)} = \sqrt[3]{5(3) + 3(4)} = \sqrt[3]{27} = 3$.

4. (a) $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x + 5} = \lim_{x \rightarrow -5} \frac{(x - 5)(x + 5)}{x + 5} = \lim_{x \rightarrow -5} (x - 5) = -10$.

4. (b) $\lim_{x \rightarrow \infty} \frac{2x^2 - 1}{x^3 + x^2 + 1} = \lim_{x \rightarrow \infty} \frac{2x^2 - 1}{x^3 + x^2 + 1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{1}{x^3}}{1 + \frac{1}{x} + \frac{1}{x^3}} = \frac{0 - 0}{1 + 0 + 0} = \frac{0}{1} = 0$.

5. (a) The function f is a rational function, so it is continuous except when its denominator is equal to zero. To find these x -values, solve: $x^2 - 2x + 3 = 0$. Using the quadratic formula,

$$x = \frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{4 - 12}}{2} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2i\sqrt{2}}{2} = 1 \pm i\sqrt{2}.$$

Since these are two complex solutions, there are no real numbers that make the denominator zero. Hence, f is continuous on $(-\infty, \infty)$.

5. (b) Graphically, it is easy to see that: $\lim_{x \rightarrow 3^+} g(x) = 5$, $\lim_{x \rightarrow 3^-} g(x) = 3$, and thus $\lim_{x \rightarrow 3} g(x)$ does not exist.

5. (c) Each piece of the function is a polynomial, and thus continuous wherever it is defined. We observed that $\lim_{x \rightarrow 3} g(x)$ does not exist, and thus the function g is continuous on the interval $(-\infty, 3) \cup (3, \infty)$.

6. The function f is a polynomial, so it is continuous everywhere, and in particular for all $x \in [-1, 1]$. Notice that $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -4 < 0$ and $f(1) = 1^3 - 2(1)^2 + 3(1) + 2 = 4 > 0$ have different signs. Thus, by the Intermediate Value Theorem, the function f has at least one zero in the interval $(-1, 1)$.

7. (a) By the definition of the derivative:

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 + 1) - (2x^2 + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 + 1 - 2x^2 - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} \\
 &= \lim_{h \rightarrow 0} (4x + 2h) \\
 &= 4x.
 \end{aligned}$$

7. (b) The slope of the tangent line at the point $(1, 3)$ is given by $m = f'(1) = 4(1) = 4$. We then use point-slope form to find the equation of the tangent line.

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \implies y - 3 = 4(x - 1) \\
 &\implies y - 3 = 4x - 4 \\
 &\implies y = 4x - 1.
 \end{aligned}$$

Bonus. Any number of answers is acceptable, as long as the depicted function is defined everywhere on the interval $[0, 10]$ and satisfies the specified “not continuous” and “not differentiable” requirements. In particular, at $x = 2, 4, 6$ the graph should display either a “jump” discontinuity, a “hole” discontinuity, or an “asymptotic” discontinuity, while at $x = 8$ the function should either have a vertical tangent line (meaning undefined slope) or some sort of kink or corner.