Name: ________________________

Complete the following problems to the best of your ability. Clearly number each question, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

1: (15 pts.) Determine the intervals of concavity of the function $f(x) = \frac{1}{x^2+2}$.

2: (15 pts.) Carefully sketch the graph of a function with all of the following features. You must label intercepts, asymptotes, extrema, and inflection points on the graph to receive full credit.

Domain: $(-\infty, 0) \cup (0, \infty)$.
Intercept at $x = 1$.
Asymptotes: x-axis and y-axis.
Increasing on $(0, 2)$. Decreasing on $(-\infty, 0)$, $(2, \infty)$.
Relative maximum at $(2, 1)$.
Concave downward on $(-\infty, 0)$ and $(0, 3)$. Concave upward on $(3, \infty)$.
Inflection point at $(3, 8/9)$.

3: (15 pts.) The speed of a vehicle on a stretch of highway between 6 a.m. and 10 a.m. on a typical weekday is approximated by the function $f(t) = 20t - 40\sqrt{t} + 50$, $0 \leq t \leq 4$, where $f(t)$ is measured in miles per hour and t is measured in hours with $t = 0$ corresponding to 6 a.m. At what time of the morning is the traffic moving at the slowest rate? What is the speed of a vehicle at that time?

4: (10 pts.) Solve the equation $\frac{200}{1+3e^{-0.3t}} = 100$ for t. Round your answer to 4 decimal places.

5: (15 pts.) Find the equation of the tangent line of $f(x) = e^{-x^2}$ at its positive inflection point.

6: (10 pts.) Find the derivative of the function $f(x) = \ln\sqrt{x^2 - 4}$.

7: (20 pts.) A quantity of 5 mg of sodium-25 (^{25}Na) is needed for an experiment. Since the half-life of ^{25}Na is only 60 seconds, it will decay substantially during the 2 minute setup of the experiment. What initial quantity of ^{25}Na should be selected to ensure that exactly 5 mg remains by the time the experiment begins?

Bonus: (5 pts. each) Complete one or both of the following problems as time permits.

(i): Find the derivative of $f(x) = x^{\ln x}$.

(ii): On the basis of data collected during an experiment, a biologist found that the population growth of a fruit fly with a limited food supply could be approximated by the logistic model $N(t) = \frac{400}{1+39e^{-0.16t}}$, where t denotes the number of days since the beginning of the experiment. What was the initial fruit fly population in the experiment? What was the maximum fruit fly population that could be expected under this laboratory condition?