

Name: \_\_\_\_\_

Complete the following problems to the best of your ability. Clearly number each question, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

(1) (10 pts.) Find the absolute extreme values of  $f(x) = \ln(x^2 + x + 1)$  on the interval  $[-1, 1]$ .

(2) (15 pts.) Evaluate the following limits:

(a)  $\lim_{t \rightarrow 0} \frac{e^{3t} - 1}{t}$ .

(b)  $\lim_{x \rightarrow 0^+} x^{x^2}$ .

(3) (10 pts.) If 1200 cm<sup>2</sup> of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

(4) (15 pts.) A car is braked with constant deceleration of 16 ft/s<sup>2</sup> and skids for 200 ft before coming to a complete stop. How fast was the car traveling when the brakes were first applied?

(5) (10 pts.) Estimate the area under the curve  $f(x) = \frac{2x}{x^2+1}$  between  $x = 1$  and  $x = 3$  using four approximating rectangles and right endpoints (it's fine to leave your answer in correctly-rounded decimal form). Is this an **underestimate** or an **overestimate** of the true area under the curve?

(6) (10 pts.)

(a) Evaluate the definite integral  $\int_1^{e^0} 2 \cos\left(\frac{\sqrt{\sin x}}{\ln(x^6)}\right) e^{12x} dx$ .

(b) If  $\int_0^9 f(x) dx = 17$  and  $\int_0^9 g(x) dx = 8$ , find  $\int_0^9 [2f(x) - 3g(x)] dx$ .

(7) (15 pts.)

(a) If  $f(x) = \int_0^x (1 - t^2)e^{t^2} dt$ , on what interval is  $f$  is increasing?

(b) If  $f(1) = 12$ ,  $f'$  is continuous, and  $\int_1^4 f'(x) dx = 7$ , find  $f(4)$ .

(c) If  $F(x) = \int_x^\pi \sqrt{1 + \sec t} dt$ , what is the formula for the function  $F'$ ?

(8) (15 pts.) Evaluate the following integrals:

(a)  $\int \frac{z^2}{z^3 + 1} dz$ .

(b)  $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ .

Bonus (10 pts.) If  $f(x) = \int_0^{\sin x} \sqrt{1 + t^2} dt$  and  $g(y) = \int_3^y f(x) dx$ , what is the value of  $g''(\pi/6)$ ?