

Date: 9/27/13.

Instructor: Cody Clifton.

Name: _____

This 10-point quiz will test your knowledge of the chain rule and implicit differentiation. Read carefully and always show your work. You have 10 minutes... good luck!

Note: for this quiz, it's *not* necessary to simplify your answers.

- (1) Differentiate $f(x) = (x^2 + 1)^{-10}$.

Solution. By the chain rule:

$$f'(x) = -10(x^2 + 1)^{-11} \cdot 2x = -20x(x^2 + 1)^{-11}.$$

- (2) Differentiate $k(s) = \cos(\pi \sin(1 - s^2))$.

Solution. By two applications of the chain rule:

$$k'(s) = -\sin(\pi \sin(1 - s^2)) \cdot \pi \cos(1 - s^2) \cdot (-2s) = 2\pi s \sin(\pi \sin(1 - s^2)) \cos(1 - s^2).$$

- (3) Use implicit differentiation to find y' for the equation $e^y \cos x = 1 + \sin(xy)$.

Solution. By implicit differentiation:

$$\begin{aligned} e^y \cos x = 1 + \sin(xy) &\implies \frac{d}{dx} (e^y \cos x) = \frac{d}{dx} (1 + \sin(xy)) \\ &\implies -e^y \sin x + y' e^y \cos x = \cos(xy) (y + xy') \\ &\implies y' (e^y \cos x - x \cos(xy)) = e^y \sin x + y \cos(xy) \\ &\implies y' = \frac{e^y \sin x + y \cos(xy)}{e^y \cos x - x \cos(xy)}. \end{aligned}$$