Math 104 - Precalculus Midterm Fall 2011

Name Key
KUID
Instructor

Part 1: Multiple Choice (120 points). Show your work in the space provided. Circle the correct answer. Partial credit will be given only if work is shown. Each problem is worth 10 points.

1. The solutions to the equation $x^2 - 8x + 32 = 0$ are

   (a) $3 \pm 3i$

   (b) $2 \pm 4i$

   (c) $1 \pm 2i$

   (d) $4 \pm 4i$

$$x = \left(8 \pm \sqrt{64 - 4(1)(32)}\right) / 2(1)$$
$$= \left(8 \pm \sqrt{-64}\right) / 2$$
$$= \left(8 \pm 8i\right)/2 \quad \boxed{= 4 \pm 4i}$$

2. The domain of $f(x) = \dfrac{1}{\sqrt{-5x+10}}$ is:

   (a) $x \neq 2$

   (b) $(2, \infty)$

   (c) $(-\infty, 2)$

   (d) $(0, \infty)$

$-5x + 10 > 0$

$-5x > -10$

$x < 2$

if don't "<" flip when solve

3. List all of the asymptotes of the function $g(x) = \dfrac{2x^2 - x}{x^2 - 9} = \dfrac{x(2x-1)}{(x+3)(x-3)}$

   (a) Vertical: $x = 9$; Horizontal: $y = 2$

   (b) Vertical: none; Horizontal: $y = 0$

   (c) Vertical: $x = 3, x = -3$; Horizontal: $y = 2$

   (d) Vertical: $x = 3, x = -3$; Horizontal: none

$\Rightarrow$ V.a : $x = -3, 3$

H.a : $y = \frac{2}{1} = 2$

If factor & just get v.asy's (& show work) +6

4. Suppose $h = (f \circ g^{-1})(x)$ and $f(1) = 2, f(-3) = 5, g(-3) = 1, g(1) = 10$. Find $h(1)$.

   (a) -6

   (b) 2

   (c) 5

   (d) 10

$h(1) = (f \circ g^{-1})(1) = f\left(g^{-1}(1)\right)$ 3

$= f(-3) = 5$

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5. Let $p(x) = x^3 + 2x^2 - 11x - 12$. Which of the following statements is not true?

   (a) $p(3) = 0$ ✓ $x = 3$ a zero $\Leftrightarrow$ $(x-3)$ a factor

   (b) $(x + 3)$ is a factor of $p(x)$

   (c) $(3, 0)$ is an x-intercept of the graph of $p(x)$ ✓

   (d) p is a polynomial function ✓

6. Using properties of logarithms, which of the following is equivalent to

   $3\log_5(x-1) - \left[\frac{1}{2}\log_5(y) + \log_5(3z)\right]$ $= \underbrace{\log_5 (x-1)^3 - [\log_5\sqrt{y} + \log_5 3z]}$

   $\log_5(x-1)^3 - \log_5\left((\sqrt{y})3z\right)$

   $= \log_5\left(\frac{(x-1)^3}{\sqrt{y}\,3z}\right)$

   (a) $\log_5\left(\frac{x-1}{y^{\frac{1}{2}}z}\right)$ (b) $\log_5\left(\frac{(x-1)^3}{3z\sqrt{y}}\right)$

   (c) $\log_5\left(\frac{3z(x-1)^3}{\sqrt{y}}\right)$ (d) $\log_5\left(\frac{3(x-1)}{y^{\frac{1}{2}}z^3}\right)$

7. Find the solution set of the inequality $|2x - 5| + 2 < 7$

   solve 1 sided inequ. for $x < 5$ (+3)

   (a) $(5, \infty)$

   (b) $(0, 5)$

   (c) $(-2, 5)$

   (d) No solution

   $|2x-5| < 5$

   $-5 < 2x - 5 < 5$

   $0 < 2x < 10$

   $0 < x < 5$

   if do: $-7 < 2x - 5 + 2 < 7$ ⋮

   $\Rightarrow$ (+3)

8. Evaluate the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for the function $f(x) = 3x - x^2$

   (a) $-3x + 2$

   (b) $3 - 2x - h$

   (c) $-2x + 3 + 3h$

   (d) 1

   $= \left[3(x+h) - (x+h)^2 - (3x - x^2)\right] / h$

   $= \left[\cancel{3x} + 3h - (x^2 + 2xh + h^2) - \cancel{3x} + x^2\right] / h$

   $= \left[3h \cancel{- x^2} - 2xh - h^2 \cancel{+ x^2}\right] / h$

   $= (3h - 2xh - h^2) / h$

   $(h \neq 0)$ $= \boxed{3 - 2x - h}$

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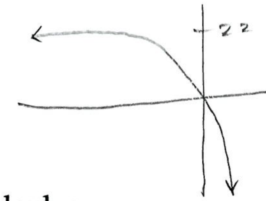
Actual Graph:



∴ dom: $(-\infty, \infty)$
range: $(-\infty, 22)$
HA: $y = 22$

9. List the properties of the function $q(x) = 22 - e^{x+3}$

   (a) Domain: $(-\infty, \infty)$; Range: $(22, \infty)$ HA: $y = 22$

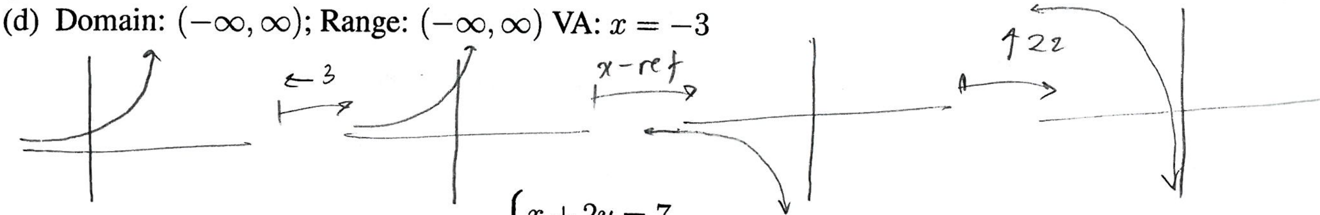
   (b) Domain: $(-3, \infty)$; Range: $(-\infty, \infty)$ VA: $x = -3$

   (c) Domain: $(-\infty, \infty)$; Range: $(-\infty, 22)$ HA: $y = 22$

   (d) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$ VA: $x = -3$

• H sh.
• H st/comp y-ref.
• V st/comp x-ref
• V sh.

$3 \leftarrow$ ①
x-refl ②
$\uparrow 22$ ③



10. The solution to the system of equations: $\begin{cases} x + 2y = 7 \\ 3x - y = 1 \end{cases}$

   (a) $(\frac{7}{9}, \frac{28}{9})$ cons.

   (b) $(\frac{-1}{2}, 3)$

   (c) $(\frac{7}{2}, -1)$

   (d) $(\frac{9}{7}, \frac{20}{7})$

+6 if wrong algebra

$2R_2 \rightarrow \begin{cases} x + 2y = 7 \\ 6x - 2y = 2 \end{cases}$

$7x = 9$

$\boxed{x = 9/7}$

$R1: \; 2y = 7 - \frac{9}{7}$

$\Rightarrow 2y = \frac{49}{7} - \frac{9}{7} = \frac{40}{7} \Rightarrow \boxed{y = \frac{20}{7}}$

Note: Alternative
R_1: $2y = 7 - x$
$y = \frac{7}{2} - \frac{1}{2}x$
R_2: $-y = 1 - 3x$
$y = 3x - 1$
Now intersect to solve

11. What is the slope of any line perpendicular to the line $2x - y = 8$?

   (a) 2

   (b) -2

   (c) $\frac{1}{2}$

   (d) $-\frac{1}{2}$

+1/6 pts for solve for y & pick m = 2

$2x - y = 8$

$-y = 8 - 2x$

$y = 2x - 8 \Rightarrow m_1 = 2$

$\Rightarrow m_\perp = -\left(\frac{1}{m_1}\right) = -\frac{1}{2}$

12. What is $8 + 6i$ multiplied by its conjugate equal to?

   (a) 100

   (b) $100 + 96i$

   (c) $36 + 96i$

   (d) 28

$(8 + 6i)(8 - 6i)$

$= 64 - 6^2 i^2$

$= 64 - 6^2(-1) = 64 + 36 = 100$

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Part 2: Long answer (80 points). Show all your work. Each question will be graded based on the accuracy of work shown. Answers should be exact unless otherwise stated. Each problem is worth 20 points.

13. In a nuclear power plant leak, radioactive Iodine-131 is leaked into the air. It has a half life of only 8 days. (Leave answers in exact form.)

10 (a) Assume an exponential model and find the decay constant for the model.

3 $Q(t) = Q_0 e^{kt}$

2 $Q(8) = Q_0 e^{8k} = \frac{1}{2} Q_0$

$e^{8k} = 1/2$

5 $\ln(e^{8k}) = \ln(1/2)$

$8k \ln(e) = \ln(1/2)$

$8k = \ln(1/2)$

$\Rightarrow \boxed{k = \frac{\ln(1/2)}{8}}$

$(\approx -.0866)$

10 (b) If this element was released today, how long would it be until exactly 4% of the original amount leaked remained in the air?

When will $Q(t) = .04 Q_0$? 3

$\cancel{Q_0} e^{kt} = .04 \cancel{Q_0}$

$e^{kt} = .04$

7 $\ln(e^{kt}) = \ln(.04)$

$kt \ln(e) = \ln(.04)$

$t = \frac{\ln(.04)}{k} = \frac{\ln(.04)}{\frac{\ln(1/2)}{8}}$

$\Rightarrow \boxed{t = \frac{8\ln(.04)}{\ln(.5)}}$ (≈ 37.1508)

20/20

14. Solve the following equations for x.

6 (a) $\log_2(x^2 - 6x) - 6 = -2$

2 $\log_2(x^2 - 6x) = 4$

2 $\Updownarrow$ $2^4 = x^2 - 6x$

$0 = x^2 - 6x - 16$

2 $0 = (x-8)(x+2)$

$\boxed{x = 8}$ check ✓ $\boxed{x = -2}$ check ✓

6 (b) $e^{3x}e^5 = (e^x)^2$

$e^{3x+5} = e^{2x}$ 2

(1 to 1) $3x + 5 = 2x$ 2 6

$x + 5 = 0$ 2

$\boxed{x = -5}$

check ✓

8 (c) $3^x = 2^{x-1}$

2 $\ln(3^x) = \ln(2^{x-1})$

2 $x\ln 3 = (x-1)\ln(2)$

$x\ln 3 = x\ln(2) - \ln(2)$

4 $x\ln 3 - x\ln(2) = -\ln(2)$

$x(\ln(3) - \ln(2)) = -\ln(2)$

$\boxed{x = \dfrac{-\ln(2)}{(\ln 3 - \ln 2)}} \approx \dfrac{-.6931}{.4055} = 1.7095$. Check ✓

/20

15. Consider the function $f(x) = -\sqrt{x+3} - 2$

4 (a) Identify the parent function. Describe the transformations of f from its parent function.

parent: $\sqrt{x}$:

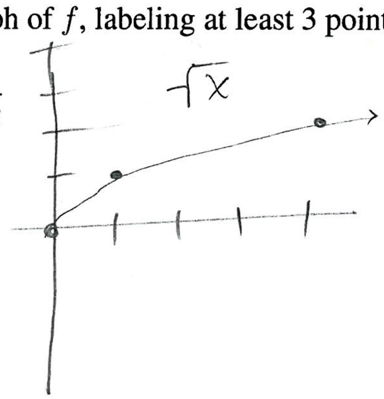
- $3 \leftarrow$
- x-ref.
- $\downarrow 2$

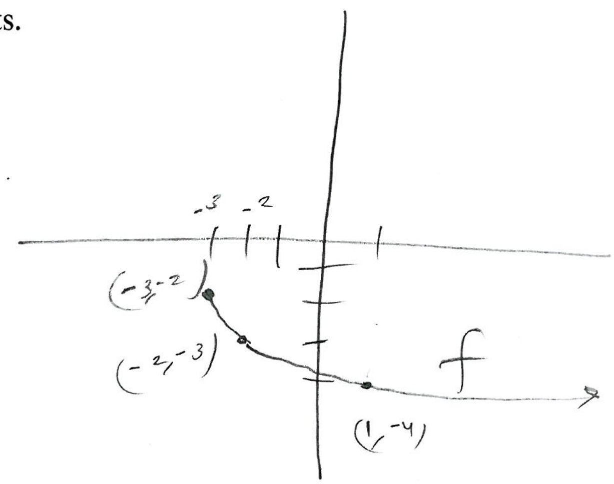
ORDER matters!

4 (b) Sketch a graph of f, labeling at least 3 points.

$-\sqrt{x+3} - 2$

x	$\sqrt{x}$	$\sqrt{x+3}$	$-\sqrt{x+3}$
-3	0	0 0	-2
-2	1	1 1	-3
1	4	2 2	-4





8 (c) Find $f^{-1}(x)$ algebraically.

$$y = -\sqrt{x+3} - 2, \quad x \geq -3$$
$$x = -\sqrt{y+3} - 2, \quad y \geq -3$$
$$x + 2 = -\sqrt{y+3}, \quad y \geq -3$$
$$(x+2)^2 = y + 3, \quad y \geq -3$$
$$(x+2)^2 - 3 = y$$
$$\therefore \boxed{f^{-1}(x) = (x+2)^2 - 3} = x^2 + 4x + 4 - 3 = x^2 + 4x + 1$$

4 (d) What are the domain and range of f^{-1}?

dom f^{-1} = range f = $(-\infty, -2]$ = dom f^{-1}

range f^{-1} = dom f = $[-3, \infty)$ = range f^{-1}

/20

16. Consider the polynomial $p(x) = x^4 - 10x^3 - 9x^2 - 20x - 22$.

5 (a) List all possible rational zeros of $p(x)$.

$$\pm \left\{ \frac{1,\ 2,\ 11,\ 22}{1} \right\}$$

11 (b) Algebraically find all zeros of p.

Note: from graph $x = -1$ a zero & $p(-1) = 0$

$\Rightarrow (x - -1)$ a factor 2

$$\begin{array}{r|rrrrr} -1 & 1 & -10 & -9 & -20 & -22 \\ & & -1 & +11 & -2 & 22 \\ \hline & 1 & -11 & 2 & -22 & 0 \end{array}$$ 1

$\Rightarrow p(x) = (x+1)(x^3 - 11x^2 + 2x - 22)$ 1

grading Note: If do $(x-11)$ 1st:

$$\begin{array}{r|rrrrr} 11 & 1 & -10 & -9 & -20 & -22 \\ & & 11 & 11 & 22 & 22 \\ \hline & 1 & 1 & 2 & 2 & 0 \end{array}$$

$\Rightarrow p(x) = (x-11)(x^3 + x^2 + 2x + 2)$

Note: from graph $x = 11$ a zero & $p(11) = 0$ $\Rightarrow$ $(x - 11)$ factor 2

$$\begin{array}{r|rrrr} 11 & 1 & -11 & 2 & -22 \\ & & 11 & 0 & 22 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$ 1

$\Rightarrow p(x) = (x+1)(x-11)(x^2+2)$ 1

2

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm\sqrt{-2}$

$x = \pm\sqrt{2}\, i$

are zeros

$\Rightarrow \boxed{p(x) = (x+1)(x-11)(x-\sqrt{2}\, i)(x+\sqrt{2}\, i)}$

$\boxed{\text{Zeros are } x = -1,\ 11,\ \sqrt{2}\, i,\ -\sqrt{2}\, i}$ 1

4 (c) Use the zeros to write p as a product of linear factors.

$p(x) = (x+1)(x-11)(x-\sqrt{2}\, i)(x+\sqrt{2}\, i)$

/20

Bonus Answer any of the following questions.

1. (10 pts) What positive value of k would make the given lines parallel?
$kx + 2y = 8$, and $18x + ky = 12$

$$ky = 12 - 18x$$
$$y = \frac{12}{k} - \frac{18}{k}x \qquad 2$$

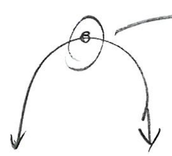
$$2y = 8 - kx$$
$$y = 4 - \frac{k}{2}x \qquad 2$$

$$\Rightarrow \left(-\frac{k}{2}\right) = -\frac{18}{k} \iff \frac{k}{2} = \frac{18}{k} \qquad 3$$

$$k^2 = 36$$
$$k = + 6 \quad \cancel{-} \quad = \boxed{6 = k} \qquad 3$$

2. (10 pts) A ball is thrown upward so its height in feet at time t seconds is $h(t) = 96t - 16t^2$. **Algebraically** find the following: What time the ball reaches its maximum height. What is the maximum height attained by the ball? When does the ball hit the ground?

$$h(t) = -16t^2 + 96t \Rightarrow$$



local max at vertex.

vertex at $t = \frac{-b}{2a} = \frac{-96}{2(-16)} = \frac{-96}{-32} = 3$

$\therefore$ ball reaches max ht. at $\boxed{t = 3}$ 4

$\Rightarrow$ max ht is $\boxed{h(3) = 144}$ 1

hits ground when $h(t) = 0$

$$96t - 16t^2 = 0$$
$$16t(6 - t) = 0$$
$$t = 0 \qquad \boxed{t = 6} \qquad 5$$