Math 104. **Unit 1 review worksheet.** **Fall 2011.**

You should attempt the following sample problems. Others may be found in the textbook as listed.

Since you will not have time to attempt all of these exercises during the workshop, you may like to focus on the following shorter list of topics that I believe were the most challenging for students in Unit 1:

- solving rational equations (B.2),
- solving absolute value and quadratic inequalities (B.4),
- working with matrices (7.4),
- determining even/odd functions (1.3),
- finding inverse functions (1.6).
- understanding domain and range (all of Chapter 1).

Section B.1.
Exercises: pg's. A32-A35.
Topics: Distance formula, midpoint formula, equations of circles.
Sample problems:

1. Plot the points $(8, 5)$ and $(0, 20)$ in the Cartesian plane. Algebraically find the exact distance between the points. Find the midpoint of the line segment joining the points.
2. Write the standard form of the equation of the circle with center $(-1, 2)$ and with $(0, 0)$ as a solution point.
3. Look at word problems 86-89, on pg. A35.

Section B.2.
Exercises: pg's. A43-A46.
Topics: Graphing equations by plotting points and on the calculator.
Sample problems:

1. Sketch the graph of the equation $y = x^3 - 3$ both by plotting points and with a calculator.
2. Describe the graph of the equation $(x-1)^2 + (y-2)^2 = 4$. Now solve for y and graph with a calculator.
3. Look at word problems 71-72, on pg. A45.

Section B.3.
Exercises: pg's. A59-A62.
Topics: Solving linear equations algebraically and graphically (two graphical methods: intercept, intersect), identifying types of equations (identity, conditional, contradiction), solving rational equations, solving quadratic equations (factoring, extracting square roots, completing the square, quadratic formula), solving higher-degree polynomial equations.
Sample problems:

1. Determine whether the equation $-7(x-3)+4x = 3(7-x)$ is an identity, a contradiction, or a conditional equation.
2. Solve the equation $4y - 2 - 5y = 7 - 6y$ both algebraically and graphically.
3. Solve the equation $\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6}$.
4. Find the x- and y-intercepts of the equation $y = 4-x^2$ algebraically, and check graphically.
5. Solve the following quadratic equations with the indicated algebraic method:
   (i) $x^2 - 2x - 8 = 0$, by factoring.
   (ii) $(2x+3)^2 + 25 = 0$, by extracting square roots.
   (iii) $9x^2 - 12x - 14 = 0$, by completing the square.
   (iv) $x^2 + 16 = -5x$, using the Quadratic Formula.
6. Solve the equation $x^3 - 4x^2 - x + 3 = 0$.
7. Solve the equation $4x^4 - 65x^2 + 16 = 0$.
8. Solve the equation $\sqrt{x+1} - 3x = 1$.
9. Solve the equation $|3x + 2| = 7$.
10. Look at word problems 195-198, on pg. A62.

Section B.4.
Exercises: pg's. A72-A75.
Topics: Solving simple inequalities algebraically and graphically, solving absolute-value inequalities, solving polynomial inequalities.
Sample problems:

1. Solve the following linear and absolute value inequalities algebraically and graphically:
   (i) $\frac{3}{4}x - 6 \leq x - 7$.
   (ii) $0 \leq 2 - 3(x + 1) < 20$.
   (iii) $|x + 14| + 3 > 17$.
   (iv) $|(x - 3)/2| \geq 5$.
2. Solve the following quadratic inequalities algebraically and graphically:
   (i) $(x + 2)^2 < 25$.
   (ii) $3x^2 - 11x + 16 \leq 0$.
   (iii) $\frac{1}{x} - 4 < 0$.
3. Look at word problems 79-83, on pg. A74.

Section 7.1.
Exercises: pg's. 481-484
Topics: Solving systems of equations by substitution and by graphing.
Sample problems:

1. Solve the following system by substitution and check by graphing: $\begin{cases} 2x - y + 2 = 0 \\ 4x + y - 5 = 0 \end{cases}$
2. Look at word problems 69=74, on pg's. 482-483.

Section 7.2.
Exercises: pg's. 491-494
Topics: Solving systems of equations by elimination.
Sample problems:

1. Solve the following system by elimination and check by graphing: $\begin{cases} \frac{x+3}{4} + \frac{y-1}{3} = 1 \\ 2x - y = 12 \end{cases}$
2. Solve the following system by elimination and check by graphing: $\begin{cases} 6.3x + 7.2y = 5.4 \\ 5.6x + 6.4y = 4.8 \end{cases}$
3. Look at word problems 71-78, on pg's. 492-493.

Section 7.4.
Exercises: pg's. 521-525
Topics: Solving systems of equations with matrices.
Sample problems:

1. Solve the following system by Gaussian elimination with back-substitution: $\begin{cases} 2x + 6y = 16 \\ 2x + 3y + 7 \end{cases}$
2. Solve the following system by Gauss-Jordan elimination: $\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$
3. Look at word problems 83-84, on pg. 525.

Section 1.1.
Exercises: pg's. 11-15.
Topics: Slope, types of equations of lines (general form, slope-intercept form, point-slope form, vertical, horizontal), parallel and perpendicular lines.
Sample problems:

1. Find the slope of the line passing through the points $(-3, -2)$ and $(1, 6)$.
2. Find the slope-intercept form of the equation of the line that satisfies the following:
   (i) slope: $m = -2$; passes through the point: $(-3, 6)$.
   (ii) passes through the points: $(1, 1)$, $(6, -2/3)$.

3. Find the slope-intercept form of the equation of the line that passes through the point $(2, 1)$ and is (a) parallel, and (b) perpendicular, to the line $4x - 2y = 3$.
4. Look at word problems 45-46, on pg. 12, and 83-87, on pg's. 14-15.

Section 1.2.
Exercises: pg's. 24-29.
Topics: Identifying functions algebraically, piecewise defined functions.
Sample problems:

1. Which of the following equations represent y as a function of x? Justify your answer algebraically.
   (i) $x^2 + (y-2)^2 = 16$.
   (ii) $y = \sqrt{x^2 - 1}$.
   (iii) $y = |4 - x|$.
   (iv) $x + y^2 = 3$.
2. Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = 2x$. Simplify your answer.
3. Look at word problems 74-86, on pg's. 27-29.

Section 1.3.
Exercises: pg's. 38-41.
Topics: Domain and range, identifying functions graphically (vertical line test), even and odd functions.
Sample problems:

1. Use a calculator to graph each of the following functions and estimate its domain and range. Then find the domain and range algebraically.
   (i) $f(x) = 2x^2 + 3$.
   (ii) $h(t) = \sqrt{4 - t^2}$.
   (iii) $g(x) = |x + 3|$.
2. Use a calculator to graph each of the following functions and determine the intervals on which it is increasing, decreasing, and constant.
   (i) $f(x) = 1$.
   (ii) $g(t) = t^{2/3}$.
   (iii) $h(y) = -|x + 4| - |x + 1|$.
3. Algebraically determine whether each of the following functions is even, odd, or neither.
   (i) $f(x) = x^3 - 5x$.
   (ii) $g(s) = s\sqrt{1 - s^2}$.
   (iii) $f(x) = x^2 - 2$.
4. Sketch by hand the graph of the piecewise function $f(x) = \begin{cases} 2x + 3, & x < 0; \\ 3 - x, & x \geq 0. \end{cases}$
5. Look at problems 11-14, on pg. 38.

Section 1.4.
Exercises: pg's. 48-50.
Topics: Transformations (shifting up/down/left/right, reflecting across x-/y- axis, horizontal/vertical shrink/stretch).
Sample problems:

1. For each of the following transformed functions, first identify the parent function f, then describe the sequence of transformations from f to h. Sketch a graph of both f and h.
   (i) $h(x) = 2 - (x + 5)^2$.
   (ii) $h(x) = -\frac{1}{2}(x + 1)^3$.
   (iii) $h(x) = 2|x - 3| - 4$.
2. Look at problems 13-14, on pg. 48, and word problem 58, on pg. 50.

Section 1.5.
Exercises: pg's. 58-61.
Topics: Arithmetic combinations of functions (sum, difference, product, quotient), composition of functions.
Sample problems:

1. For each of the following pairs of functions $f(x)$ and $g(x)$, find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, and $(f/g)(x)$. What is the domain of $(f/g)(x)$?
   (i) $f(x) = x + 3$, $g(x) = x - 3$.
   (ii) $f(x) = 2x - 5$, $g(x) = 4$.
   (iii) $f(x) = \sqrt{x^2 - 4}$, $g(x) = \frac{x^2}{x^2+1}$.
2. Evaluate the indicated function for $f(x) = x^2 - 1$ and $g(x) = x - 2$ algebraically.
   (i) $(f+g)(3)$.
   (ii) $(fg)(-6)$.
   (iii) $(f-g)(-2)$.
   (iv) $(f/g)(-5)$.
   (v) $(fg)(3t^2)$.
3. For each of the following pairs of functions $f(x)$ and $g(x)$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Algebraically determine the domain of each of f, g, $f \circ g$, and $g \circ f$, then check graphically.
   (i) $f(x) = x^2$, $g(x) = x - 1$.
   (ii) $f(x) = \sqrt{x+4}$, $g(x) = x^2$.
4. Find two functions f and g such that $(f \circ g)(x) = h(x)$ for $h(x) = \frac{4}{(5x-2)^2}$.
5. Look at word problems 73-84, on pg's. 59-61.

Section 1.6.
Exercises: pg's. 69-72.
Topics: Verifying inverse functions, one-to-one functions (horizontal line test!) and the existence of inverses, finding inverses algebraically.
Sample problems:

1. For each of the following: (a) determine algebraically if the function is one-to-one; (b) determine graphically if the function is one-to-one (recall: horizontal line test); (c) find the inverse of the function algebraically (if the function is not one-to-one, you must restrict the domain so that it becomes one-to-one).
   (i) $f(x) = 3 - x/2$.
   (ii) $f(x) = \frac{1}{4}(x+2)^2 - 1$.
   (iii) $f(x) = x^5 - 7$.
   (iv) $f(x) = \frac{3x+5}{4}$.
   (v) $f(x) = |x - 2|$.
2. Look at word problems 103-106, on pg's. 71-72.