1. $f(x) = \frac{1}{x^2+2} = (x^2+2)^{-1}$.

$$f'(x) = -(x^2+2)^{-2} \cdot 2x,$$
$$f''(x) = -(x^2+2)^{-2} \cdot 2 + 2x(2(x^2+2)^{-3} \cdot 2x) = \frac{2}{(x^2+2)^2} + \frac{8x^2}{(x^2+2)^3} = \frac{2(3x^2-2)}{(x^2+2)^3}.$$

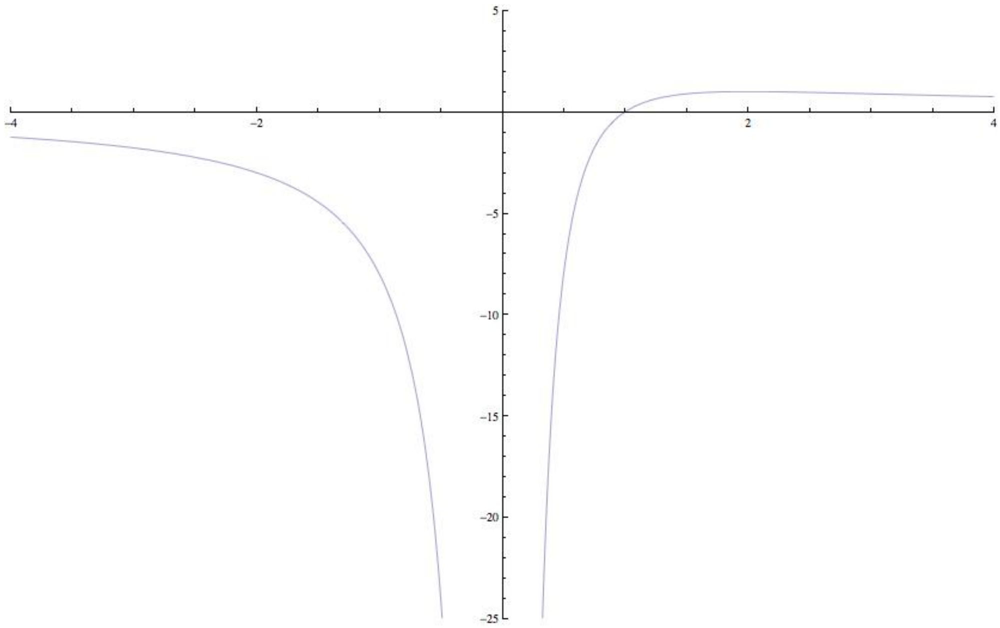
Setting $f''(x) = 0$ yields $3x^2 - 2 = 0 \implies x^2 = 2/3 \implies x = \pm\sqrt{2/3}$. Note that setting the denominator of f'' equal to zero results in the nonreal solutions $x = \pm\sqrt{-2}$.

Sign test:

interval	test pt. c	sign of $f''(c)$
$(-\infty, -\sqrt{2/3})$	-1	$+$
$(-\sqrt{2/3}, \sqrt{2/3})$	0	$-$
$(\sqrt{2/3}, \infty)$	1	$+$

Thus, the function f is concave up on $(-\infty, -\sqrt{2/3})$ and $(\sqrt{2/3}, \infty)$, and concave down on $(-\sqrt{2/3}, \sqrt{2/3})$.

2. One possible graph is for the function $f(x) = (4x-4)/x^2$. Note that labels are required for full credit, as specified in the problem statement. Thus, the following is simply a sketch of the shape of the curve, but not a complete solution to this problem.



3. $f(t) = 20t - 40\sqrt{t} + 50 = 20t - 40t^{1/2} + 50$, $0 \le t \le 4$.

$$f'(t) = 20 - 40\left(\frac{1}{2}t^{-1/2}\right) = 20 - \frac{20}{\sqrt{t}} = 20\left[\frac{\sqrt{t}-1}{\sqrt{t}}\right].$$

Setting $f'(t) = 0$ yields $\sqrt{t} - 1 = 0 \implies \sqrt{t} = 1 \implies t = 1$. Note that setting the denominator of f' equal to zero results in the solution $t = 0$, which is already accounted for as one endpoint of the domain of f.

$$\left.\begin{array}{l} f(0) = 20(0) - 40\sqrt{0} + 50 = 50 \\ f(1) = 20(1) - 40\sqrt{1} + 50 = 30 \\ f(4) = 20(4) - 40\sqrt{4} + 50 = 50 \end{array}\right\} \implies \text{The absolute minimum rate of 30mph occurs at 7 a.m. } (t = 1).$$

4.

$$\begin{aligned}
\frac{200}{1+3e^{-0.3t}} = 100 &\implies 200 = 100(1+3e^{-0.3t}) \\
&\implies 200 = 100 + 300e^{-0.3t} \\
&\implies \frac{200-100}{300} = e^{-0.3t} \\
&\implies -0.3t = \ln(1/3) \\
&\implies t = -\ln(1/3)/0.3 \approx 3.6620.
\end{aligned}$$

5. $f(x) = e^{-x^2}$.

$$\begin{aligned}
f'(x) &= -2xe^{-x^2}, \\
f''(x) &= -2e^{-x^2} + (-2x)(-2xe^{-x^2}) = 2e^{-x^2}[-1+2x^2].
\end{aligned}$$

Setting $f''(x) = 0$ yields $-1+2x^2 = 0 \implies x^2 = 1/2 \implies x = \pm 1/\sqrt{2}$. Thus, the only positive number that could be an inflection point is $x = 1/\sqrt{2} \approx 0.7071$. With test points $x = 0$ and $x = 1$, it is easy to see that f'' changes sign as we move across $x = 1/\sqrt{2}$. Furthermore, $f(1/\sqrt{2}) = e^{-1/2}$ and $f'(1/\sqrt{2}) = (-2/\sqrt{2})e^{-1/2}$ are defined, so $x = 1/\sqrt{2}$ is indeed the positive inflection point of f.

The slope of the tangent line at this point is given by the derivative, i.e. $m = f'(1/\sqrt{2}) = (-2/\sqrt{2})e^{-1/2}$. Then, using the point $(x_1, y_1) = (1/\sqrt{2}, e^{-1/2})$, we obtain the equation for the tangent line as follows.

$$y - y_1 = m(x - x_1) \implies y - e^{-1/2} = (-2/\sqrt{2})e^{-1/2}(x - 1/\sqrt{2}) \implies y = (-2/\sqrt{2})e^{-1/2}x + 2e^{-1/2}.$$

6. $f(x) = \ln\sqrt{x^2-4} = \ln(x^2-4)^{1/2} = (1/2)\ln(x^2-4) \implies f'(x) = \left(\frac{1}{2}\right)\left(\frac{2x}{x^2-4}\right) = \frac{x}{x^2-4}$.

7. General formula: $Q(t) = Q_0 e^{-kt}$, where t is in seconds. Half-life is $t = 60$, so

$$1/2 = e^{-60k} \implies k = \ln(1/2)/(-60).$$

Now that we have found k, we need to find Q_0 such that $Q(120) = 5$ (i.e. the quantity remaining after $t = 120$ seconds is 5 mg).

$$5 = Q_0 e^{\frac{\ln(1/2)}{60}\cdot 120} \implies 5 = Q_0 e^{2\ln(1/2)} \implies 5 = Q_0 e^{\ln(1/2)^2} \implies 5 = Q_0(1/4) \implies Q_0 = 5\cdot 4 = 20.$$

Thus, 20 mg should be selected in order to ensure that 5 mg remain after 2 minutes.

Bonus (i). Using the $a = e^{\ln a}$ trick, and the chain rule for exponential functions, we obtain

$$\begin{aligned}
f(x) &= x^{\ln x} = e^{\ln(x^{\ln x})} = e^{\ln x\cdot\ln x} = e^{(\ln x)^2} \\
f'(x) &= e^{(\ln x)^2}\left(2\ln x\cdot\frac{1}{x}\right) = x^{\ln x}\left(\frac{2\ln x}{x}\right) = 2\ln x\cdot x^{\ln x - 1}
\end{aligned}$$

Bonus (ii). $N(t) = \frac{400}{1+39e^{-0.16t}}$

The initial number of flies is: $N(0) = \frac{400}{1+39} = 10$. The maximum number is: $\lim_{t\to\infty} N(t) = \frac{400}{1+0} = 400$ flies.