1. (a) $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

1. (b) $\cot(0) = \frac{\cos(0)}{\sin(0)} = \frac{1}{0}$ is undefined.

1. (c) $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

1. (d) $\tan(\arctan(2011)) = 2011$, because the domain of arctan is all real numbers.

2. We are given $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{4}{5}$. To use the sum formula, we need to also find $\sin v$ and $\cos u$, which must both be negative, since both u and v are in QIII. In particular, by the Pythagorean identity:

$$\sin v = -\sqrt{1-\cos^2 v} = -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\frac{3}{5}$$
$$\cos u = -\sqrt{1-\sin^2 u} = -\sqrt{1-\left(-\frac{8}{17}\right)^2} = -\frac{15}{17}.$$

Then, $\cos(u+v) = \cos u \cos v - \sin u \sin v = \left(-\frac{15}{17}\right)\left(-\frac{4}{5}\right) - \left(-\frac{8}{17}\right)\left(-\frac{3}{5}\right) = \frac{36}{85}$.

3. (a)

$$\begin{aligned} 2\sin x - 1 = 0 &\implies \sin x = \frac{1}{2} \\ &\implies x = \frac{\pi}{6}, x = \frac{5\pi}{6}. \end{aligned}$$

3. (b)

$$\begin{aligned} \cos^2 x = \cos x &\implies \cos x(\cos x - 1) = 0 \\ &\implies \cos x = 0 \text{ or } \cos x - 1 = 0 \\ &\implies \cos x = 0 \text{ or } \cos x = 1 \\ &\implies x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x = 0. \end{aligned}$$

4. (a) (tip: draw a picture!). If y is the horizontal distance from the lighthouse to the first (farther off shore) ship, and x is the horizontal distance from the lighthouse to the second ship, then $d = y - x$ is the distance between the two ships. We can use the complementary angles to the given angles of depression (i.e. $90° - 4° = 86°$ for the first ship, and $90° - 6.5° = 83.5°$ for the second ship) to conclude that

$$\tan 86° = \frac{y}{350} \implies y = 350\tan 86°.$$
$$\tan 83.5° = \frac{x}{350} \implies x = 350\tan 83.5°.$$

Thus, the distance between the two ships is $d = y - x = 350(\tan 86° - \tan 83.5°) \approx 1933$ ft.

5. Since $\csc\theta = -\frac{3}{2}$, we immediately know that $\sin\theta = \frac{1}{\sec\theta} = -\frac{2}{3}$. Since sine is thus negative and we are told tangent is positive, the angle θ must be in QIV (i.e. cosine is negative). Thus,

$$\cos^2\theta + \sin^2\theta = 1 \implies \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}.$$

The remaining functions are:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{5}/3} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}.$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/3}{\sqrt{5}/3} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2/\sqrt{5}} = -\frac{\sqrt{5}}{2}.$$

6. (a)

$$\frac{\sin^2 x}{\tan^2 x} = \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} = \cos^2 x.$$

6. (b)

$$\begin{aligned}\sec^4\theta - \tan^4\theta &= \left(\sec^2\theta\right)^2 - \tan^4\theta\\ &= \left(1+\tan^2\theta\right)^2 - \tan^4\theta\\ &= 1 + 2\tan^2\theta + \tan^4\theta - \tan^4\theta\\ &= 1 + 2\tan^2\theta.\end{aligned}$$

Bonus.

(i)

$$\begin{aligned}\cos x + \sin x\tan x = 2 &\implies \cos x + \frac{\sin x}{\cos x} = 2\\ &\implies \cos^2 x + \sin^2 x = 2\cos x\\ &\implies 1 = 2\cos x\\ &\implies \cos x = \frac{1}{2}\\ &\implies x = \frac{\pi}{3} + 2n\pi, x = \frac{5\pi}{3} + 2n\pi.\end{aligned}$$

(iii) Let $u = \arctan x$. Then $\tan u = x$. Recall that in the right-triangle definition of the trigonometric functions, $\tan u = \frac{\text{opp}}{\text{adj}}$. So in the right-triangle with acute angle u, we have opp $= x$ and adj $= 1$. We wish to find $\sin(\arctan x) = \sin u = \frac{\text{pop}}{\text{hyp}}$, so we will need to know hyp in the triangle. By the Pythagorean Theorem,

$$\text{hyp}^2 = \text{adj}^2 + \text{opp}^2 \implies \text{hyp} = \sqrt{\text{opp}^2 + \text{adj}^2} = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}.$$

Then we conclude $\sin(\arctan x) = \sin u = \frac{\text{pop}}{\text{hyp}} = \frac{x}{\sqrt{x^2+1}}$.

(ii) Vertical shift: down 3; Amplitude: 3; Period: $\frac{2\pi}{\pi/2} = 4$; Phase shift: $\frac{\pi/2}{\pi/2} = 1$.