Name: ______________________________

Complete the following problems to the best of your ability. Clearly number each question, and write your name on each sheet of paper you turn in. Algebraic support must be shown to receive full credit (i.e. show work!). Answers should be exact unless otherwise specified.

(1) (5 pts.) State the domain of $f(x) = \dfrac{x^2 - 3x + 2}{|x-1|}$ and $g(x) = \ln\left(\sqrt{x+\pi}\right)$.

(2) (5 pts.) Let $f(x) = \frac{x}{x+1}$ and $g(x) = x^{10}$. Find the composite functions $f \circ g$ and $g \circ f$.

(3) (5 pts.) Solve the equation $\ln(8x) - \ln(1+x) = 2$.

(4) (20 pts.) Evaluate each of the following limits (hint: check (b) - (d) graphically on your calculator).

(a) $\displaystyle\lim_{t \to ☺} \frac{t^3 + 2t^2 - 1}{5 - 3t}$, assuming $☺ \neq \dfrac{5}{3}$ is a real number.

(b) $\displaystyle\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$.

(c) $\displaystyle\lim_{u \to \infty} \frac{4u^5 + 5}{(u^2 - 2)(2u^2 - 1)}$.

(d) $\displaystyle\lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x\right)$.

(5) (10 pts.) Given that $-|x| \leq x\cos\left(\dfrac{1}{x}\right) \leq |x|$ for all x, find $\displaystyle\lim_{x \to 0}\left[x\cos\left(\frac{1}{x}\right)\right]$.

(6) (15 pts.) Consider the functions $f(x) = \ln(2x+5)$ and $g(x) = \begin{cases} \frac{1}{x-2}, & \text{if } x \neq 2; \\ 1, & \text{if } x = 2. \end{cases}$

(a) Find the intervals of x on which the function f is continuous.

(b) Find $\displaystyle\lim_{x \to 2^+} g(x)$, $\displaystyle\lim_{x \to 2^-} g(x)$, and $\displaystyle\lim_{x \to 2} g(x)$.

(c) Use the results of part (b) to find the intervals of x on which the function g is continuous.

(7) (10 pts.) Explain why the function $f(x) = e^x - 3x$ is continuous for all $x \in [0,1]$. Then use the Intermediate Value Theorem to show that f must have at least one zero between $x = 0$ and $x = 1$.

(8) (15 pts.) Let $f(x) = x + \frac{1}{x}$.

(a) <u>Use a limit definition</u> to find f', the derivative of f.

(b) Find an equation of the tangent line to the curve at the point $(1, 2)$.

(9) (15 pts.) Let $f'(x) = e^{-x^2}$ be the derivative of a function f. Use the graph of f' to find:

(a) the interval(s) where f is increasing/decreasing.

(b) the interval(s) where f is concave upward/downward.

(c) any local minima, local maxima, and inflection points of f.

Bonus (10 pts.) We have seen that the function $f(x) = |x|$ is continuous, but not differentiable, at $x = 0$. As unbelievable as it may seem, there exist functions that are continuous at every real number, but are not differentiable at any real number. Defining such a function algebraically is very complicated, and sketching its graph is impossible. However, we can grasp the general concept as follows.

(a) First, sketch the graph of $f(x) = |x|$. Notice that f is continuous everywhere, but that the "corner" at the point $x = 0$ prevents differentiability (i.e. no matter how much we zoom in around $x = 0$, the curve will never look like a straight line).

(b) Now use the idea from part (a) to sketch the graph of a function that is continuous everywhere, but is not differentiable every integer.

(c) Explain briefly how you might extend this method to construct an everywhere-continuous function that is not differentiable at any real number. The concept of limits is vital to understanding how there can be an infinite number of infinitesimal "corners" filling up the infinitely long real number line...