DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
Pretest on Calc I
MATH 122 Spring 2014

Your Name: ______________________________

| | | |
|---|---|---|
| 1 | (5) | ________ |
| 2 | (5) | ________ |
| 3 | (5) | ________ |
| 4 | (5) | ________ |
| 5 | (5) | ________ |
| 6 | (5) | ________ |
| Total | (30) | ________ |

(1) Which integral matches the Riemann sum?

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(\frac{2n+i}{n}\right)^2$$

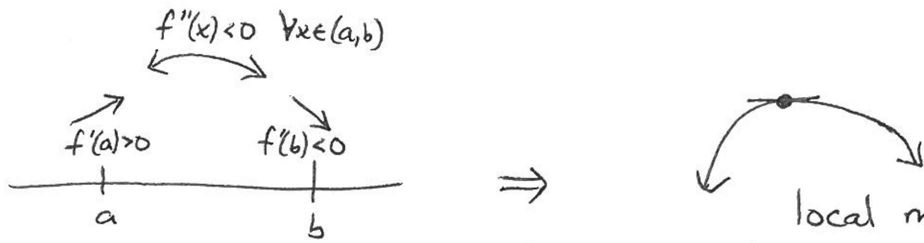
(a) $\int_0^1 x\,dx$ (b) $\int_2^3 (2x+1)^2\,dx$ (c) $\int_0^1 \frac{dx}{x^3}$ (d) $\int_0^1 (2+x)^2\,dx$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(\frac{2n+i}{n}\right)^2 = \lim_{n\to\infty}\sum_{i=1}^{n}\underbrace{\frac{1}{n}}_{\Delta x}\cdot\underbrace{\left(2+\frac{i}{n}\right)^2}_{(2+x)^2}$$

$$= \int_0^1 (2+x)^2\,dx.$$

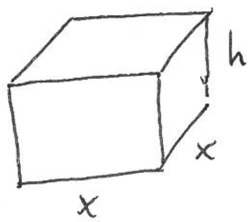
(2) If $f$, $f'$ and $f''$ are continuous functions on the interval [a,b], such that $f'(a) > 0$, $f'(b) < 0$, and $f''(x) < 0$ for all x in (a,b), then which of the following must be true:

(a) There exist a number c, in (a,b), such that $f(c) = 0$.
(b) There exist a number c in (a,b) such that $f(c) = \frac{a+b}{2}$.
(c) $f(x)$ has a local minimum in the interval (a,b).
(d) $f(x)$ has a local maximum in the interval (a,b).
(e) None of the above.




local max.
between a and b.

(3) A rectangular box with a square base and open top is constructed to have volume of 625 cubic inches. The material used to make the bottom costs 4 cents per square inch and the material used to make the sides costs 2 cents per square inch. Find the dimensions of the box that minimizes the total costs. Justify your answer.




$$V = x^2 h = 625 \Rightarrow h = \frac{625}{x^2}.$$

$$C = 4(x^2) + 2(4xh) = 4x^2 + 8xh$$

$$\Rightarrow C(x) = 4x^2 + 8x\left(\frac{625}{x^2}\right) = 4x^2 + \frac{5000}{x}.$$

$$C'(x) = 8x - \frac{5000}{x^2} = \frac{8x^3 - 5000}{x^3} \Rightarrow \text{critical \#s: } \underline{x=0},$$

$$8x^3 - 5000 = 0$$

$$\Rightarrow x = \sqrt[3]{625}$$

$$C''(x) = 8 + \frac{10000}{x^3} > 0 \text{ for all } x, \text{ so}$$

$x = \sqrt[3]{625}$ corresponds to a min. cost. $\Rightarrow$ dimensions are: $\boxed{\sqrt[3]{625} \times \sqrt[3]{625} \times \sqrt[3]{625} \text{ in.}}$

(4) Compute the limit

$$\lim_{x\to 1+} \left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right)$$

$$\lim_{x\to 1^+}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1^+} \frac{x\ln x - (x-1)}{(x-1)\ln x} \quad \text{(common denom.)}$$

$$= \lim_{x\to 1^+} \frac{x\ln x - x + 1}{x\ln x - \ln x} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$= \lim_{x\to 1^+} \frac{x\cdot\frac{1}{x} + \ln x - 1}{x\cdot\frac{1}{x} + \ln x - \frac{1}{x}} \quad \text{(L'Hôpital's rule)}$$

$$= \lim_{x\to 1^+} \frac{\ln x}{\ln x - \frac{1}{x} + 1} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$= \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{x^2}} \quad \text{(L'Hôpital's rule again)}$$

$$= \lim_{x\to 1^+} \frac{x}{x+1} = \frac{1}{2}$$

(5) For the implicitly defined curve

$$x^y = y^x,$$

write the equation of the tangent line at the point $P(2,2)$.

$$x^y = y^x \Rightarrow \ln x^y = \ln y^x \Rightarrow y \ln x = x \ln y$$

$$\frac{d}{dx}: \quad y \cdot \frac{1}{x} + \ln x \cdot y' = x \cdot \frac{1}{y} \cdot y' + \ln y$$

$$\Rightarrow y'\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$$

$$\Rightarrow y' = \left(\ln y - \frac{y}{x}\right) \Big/ \left(\ln x - \frac{x}{y}\right).$$

$$y'\Big|_{(x,y)=(2,2)} = \left(\ln(2) - \frac{2}{2}\right) \Big/ \left(\ln(2) - \frac{2}{2}\right) = 1.$$

$\left(y - y_1 = m(x - x_1),\right.$ but $\left. m = 1. \right)$

Equation of tangent line: $\boxed{y - 2 = x - 2}$ $\Leftrightarrow$ or just $\boxed{y = x}$

(6) Evaluate the improper integral or otherwise show it is divergent

$$\int_0^\infty x^2 e^{-x} dx.$$

Integration by parts: $u = x^2$, $dv = e^{-x} dx$,

| | $d/dx$ | $\int \cdot \, dx$ |
|---|---|---|
| $+$ | $x^2$ | $e^{-x}$ |
| $-$ | $2x$ | $-e^{-x}$ |
| $+$ | $2$ | $e^{-x}$ |
| $-$ | $0$ | $-e^{-x}$ |

$$\Rightarrow \int_0^\infty x^2 e^{-x} dx = \left(-x^2 e^{-x} - 2x e^{-x} - 2e^{-x}\right)\Big|_0^\infty$$

$$= \lim_{t \to \infty} \left( \left(-e^{-x}(x^2 + 2x + 2)\right)\Big|_0^t \right)$$

$$= \lim_{t \to \infty} \left( \underbrace{-e^{-t}(t^2 + 2t + 2)}_{\to 0} + \underbrace{e^{-0}(\cancel{0^2} + \cancel{2(0)} + 2)}_{=2} \right)$$

$$= 0 + 2$$

$$= 2$$