Introduction to Probability: Sols

Suppose you roll a die twice, and are interested in the following events:

- A_1 = roll two evens
- A_2 = roll two odds
- A_3 = roll one odd and one even (in either order)
- A_4 = roll a sum of 7
- A_5 = roll doubles

(1) Compute the probability of each of the five events. (Hint: It may help you to make a table of all of the possible outcomes in your sample space)

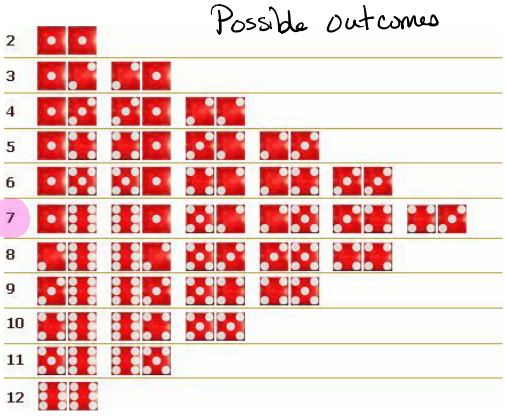
A_1: 2,2 4,2 6,2 / 2,4 4,4 6,4 / 2,6 4,6 6,6 $P(A_1) = 9/36 = 1/4$

A_2: 1,1 3,1 5,1 / 1,3 3,3 5,3 / 1,5 3,5 5,5 $P(A_2) = 1/4$

A_3: $P(A_3) = 1 - (P(A_1) + P(A_2)) = 1/2$

A_4: $P(A_4) = \frac{6}{36} = 1/6$

A_5: $P(A_5) = \frac{6}{36} = 1/6$



(2) Are there any disjoint events whose probabilities add up to 1?

A_1, A_2, A_3

(3) Give me an example of two events that are mutually exclusive.

A_1, A_2 A_2, A_3 A_2, A_4 A_4, A_5

A_1, A_3 A_1, A_4 A_3, A_5

(4) Compute $P(A_1 \mid \text{first roll is even})$

$P(B) = \frac{18}{36}$ $P(A_1 \mid B) = \frac{9}{18} = 1/2$ | $\frac{P(A_1 \text{ and } B)}{P(B)} = \frac{1/4}{1/2} = 1/2$

(5) Compute $P(A_4 \mid \text{first roll is a 3})$

$P(B) = \frac{6}{36}$ $P(A_4 \mid B) = 1/6$ | $\frac{P(A_4 \text{ and } B)}{P(B)} = \frac{1/36}{6/36} = 1/6$

Suppose you roll two separate dice; one red and one green. For each of the following, determine whether events A and B are mutually exclusive, independent, both, or neither.

a) A = red die is a 3; B = red die is a 6.

Mutually exclusive

b) A = red die is a 3; B = green die is a 6.

Independent

c) A = red die and green die sum to 4; B = red die is a 3.

Neither

d) A = red die and green die sum to 4; B = red die is a 4.

Mutually Exclusive

Compute P(red die is a 3). Then compute P(red die is a 3 | green die is a 6). What do you notice and why do you think it happens? (Hint: look back at (**b**); what is true about these two events?)

$P(\text{red die is a 3}) = 1/6$

$P(\text{red 3} \mid \text{green 6}) = 1/6$ The events are independent!

Compute P(red die and green die sum to 4). Then compute P(red die and green die sum to 4 | red die is a 3). What do you notice, and why do you think this happens? (Hint: look back at (c); what is true about these two events?)

$P(A) = 3/36 = 1/12$ (1,3; 2,2; 3,1)

$P(B) = 1/6$ $\quad P(A|B) = 1/6 = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{1/36}{1/6} = 1/6$

The events are neither mutually exclusive nor independent; the red being a 3 affects the probability they sum to 4.