Math 142: Section 11.1 - Notes

1 Parametrization of Plane Curves

Parametric Equations

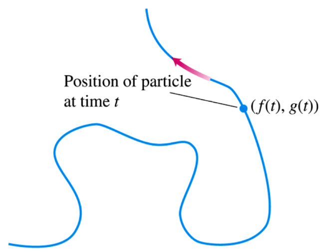

FIGURE 11.1 The curve or path traced by a particle moving in the xy-plane is not always the graph of a function or single equation.

Figure 11.1 shows the path of a moving particle in the xy-plane. We can sometimes describe such a path by a pair of equations, $x = f(t)$ and $y = g(t)$, where f and g are continuous functions. Equations like these describe more general curves than those described by a single function, and they provide not only the graph of the path traced out but also the location of the particle $(x, y) = (f(t), g(t))$ at any time t.

Definition: If x and y are given as functions

$$x = f(t), \qquad y = g(t)$$

over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$

defined by these equations is a ______________________________.

The equations are ______________________________ for the curve.

The variable t is a ______________________________ for the curve, and

its domain I is the ______________________________.

If I is a closed interval, $a \le t \le b$, the point $(f(a), g(a))$ is the ______________________________

of the curve and $(f(b), g(b))$ is the ______________________________.

Example 1 Sketch the curve defined by the parametric equations

$$x = t^2, \qquad y = t + 1, \qquad -\infty < t < \infty$$

Example 2 Identify geometrically the curve in Example 1 by eliminating the parameter t and obtaining an algebraic equation in x and y.

Example 3 Graph the parametric curves

a) $x = \cos t, \qquad y = \sin t, \qquad 0 \leq t \leq 2\pi$

b) $x = a\cos t, \qquad y = a\sin t, \qquad 0 \leq t \leq 2\pi$

Example 4 The position $P(x, y)$ of a particle moving in the xy-plane is given by the equations and parameter interval

$$x = \sqrt{t}, \qquad y = t, \qquad t \geq 0$$

Identify the path traced by the particle and describe the motion.

Example 5: Natural Parametrization A parametrization of the graph of the function $f(x) = x^2$ is given by

Example 6 Find a parametrization for the line through the point (a, b) having slope m.

Example 7 Sketch and identify the path traced by the point $P(x, y)$ if

$$x = t + \frac{1}{t}, \qquad y = t - \frac{1}{t}, \qquad t > 0.$$