Sols

Math 141 Calculus I

Exam #3 A
November 29, 2017

Instructor: Ann Clifton

Name: ______________________________

Do not turn this page until told to do so. You will have a total of 1 hour and 15 minutes to complete the exam. Draw a snowflake on the last page of the test if you read these directions in full. You **must** show all work to receive full credit. NO CALCULATOR/PHONE ALLOWED. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

#	score	out of	#	score	out of
1		4	10		6
2		4	11		15
3		4	12		5
4		4	13		5
5		4	14		15
6		6	15		10
7		6	Bonus		5
8		6			
9		6	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!

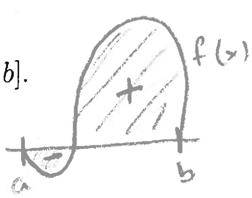
True or False. No work/explanation required. True means always true. 4pts each.

1. If f is a continuous, increasing function on the interval $[a, b]$ then the right-hand Riemann sum with $n = 3$ subintervals gives an over estimate of the area under the curve.

True

2. If $\int_a^b f(x)dx > 0$ then $f(x) > 0$ on $[a, b]$.

False



$\int_a^b f(x)dx > 0$ but $f(x) < 0$ for some portion

3. L'Hôpital's Rule states that if $f(a) = g(a) = 0$ and f and g are differentiable on an open interval I containing a, and $g'(x) \neq 0$ on I if $x \neq a$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \left(\frac{f(x)}{g(x)} \right)'.$$

False, $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)}$

4. $$\left(\sum_{k=1}^{20} \frac{1}{k+1} \right) \left(\sum_{k=1}^{20} k^3 \right) = \sum_{k=1}^{20} \frac{k^3}{k+1}$$

False

5. $$\int f(x)g(x)dx = \int f(x)dx \int g(x)dx$$

False

Multiple Choice. No work required. 6pts each. Choose the best answer. There is only one correct answer but you may choose up to *two*. One correct choice will receive full credit. If you choose two and one of the answers is correct, you will receive half the points. If both choices are incorrect, you will receive zero points. If you choose more than two answers, you will receive zero points.

C

6. Find the general indefinite integral

$$\int \left(3x^2 - 7\sqrt{x} + \frac{3}{x^2 + 1} \right) dx = \frac{3x^3}{3} - 7 \cdot \frac{2}{3} x^{3/2} + 3 \arctan x + c$$

A. $x^3 - \frac{21}{2}x^{3/2} + 3\arcsin x + c$

B. $x^3 - \frac{21}{2}x^{3/2} + 3\arctan x + c$

C. $x^3 - \frac{14}{3}x^{3/2} + 3\arctan x + c$

D. $3x^3 - \frac{14}{3}x^{3/2} + 3\arccos x + c$

B 7. Evaluate the integral.

$$\int_0^{\frac{\pi}{4}} \cos(t)dt$$

A. $\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{2}}{2}$

C. $-\frac{\sqrt{3}}{2}$ D. $-\frac{\sqrt{2}}{2}$

$= \sin(t) \Big|_0^{\pi/4} = \sin(\pi/4) - \sin(0)$

$= \sqrt{2}/2 - 0$

D 8. Evaluate

$$\int_0^{\sqrt{\pi}} x \sin(x^2)dx$$

A. 2 B. 0

C. -1 D. 1

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int_0^{\pi} \sin(u)\,du = \frac{1}{2}(-\cos(\pi) - -\cos(0))$

$= \frac{1}{2}(1+1) = 1$

A 9. If $\sum_{k=1}^{n} a_k = 6$ and $\sum_{k=1}^{n} b_k = 20$, what is $\sum_{k=1}^{n}(a_k + b_k)$?

A. 26 B. 14

C. 120 D. Not enough information

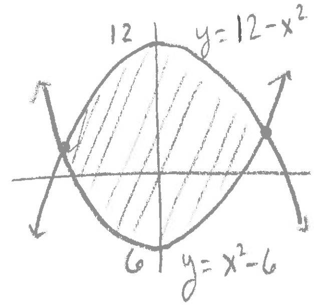
$\sum_{k=1}^{n}(a_k+b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k = 6+20$

10. Find the area of the region enclosed by the curves $y = 12 - x^2$ and $y = x^2 - 6$.

B

A. 36 B. 72

C. -72 D. -36



$12 - x^2 = x^2 - 6$

$18 = 2x^2$

$9 = x^2$

$x = 3, -3$

$\int_{-3}^{3} (12-x^2) - (x^2-6)\,dx$

$= \int_{-3}^{3} (18 - 2x^2)dx = 18x - \frac{2x^3}{3}\Big|_{-3}^{3}$

$= 18(3) - \frac{2(3)^3}{3} - \left(18(-3) - \frac{2(-3)^3}{3}\right)$

$= 2(18(3)) - 2(2(3)^2)$

$= 108 - 36 = 72$

Short answer. You must show ALL work/explain your answer to receive full credit.

11. (15 points) If f is integrable on $[a, b]$, then the following equation is correct.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x.$$

(a) (10pts) Use the given form of the definition to evaluate the integral

$$\int_0^2 (7 - x^2)dx.$$

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}$$

$$x_k = k \cdot \frac{2}{n}$$

$$\sum_{k=1}^{n} f\left(k\cdot\frac{2}{n}\right)\frac{2}{n} = \frac{2}{n}\sum_{k=1}^{n}\left(7-\left(k\cdot\frac{2}{n}\right)^2\right) = \frac{2}{n}\left[\sum_{k=1}^{n} 7 - \sum_{k=1}^{n} k^2\cdot\frac{4}{n^2}\right]$$

$$= \frac{2}{n}\left[7n - \frac{4}{n^2}\sum_{k=1}^{n} k^2\right] = \frac{2}{n}\left[7n - \frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= 14 - \frac{8n(n+1)(2n+1)}{6n^3}$$

$$\lim_{n\to\infty}\left(14 - \frac{8n(n+1)(2n+1)}{6n^3}\right) = 14 - \frac{16}{6} = 14 - \frac{8}{3} = \frac{34}{3}$$

(b) (5pts) Use the properties of integrals and the Fundamental Theorem of Calculus to evaluate the integral in part (a).

$$\int_0^2 (7-x^2)dx = 7x - \frac{x^3}{3}\Big|_0^2 = 7(2) - \frac{2^3}{3} - (0-0)$$

$$= 14 - \frac{8}{3}$$

$$= \frac{34}{3}$$

12. (5 points) Find $\int_5^{10} f(x)dx$ if $\int_0^5 f(x)dx = -12$ and $\int_0^{10} f(x)dx = 2$. Show your work!

$$\int_0^5 f(x)\,dx + \int_5^{10} f(x)\,dx = \int_0^{10} f(x)\,dx$$

$$-12 + \int_5^{10} f(x)\,dx = 2$$

$$\int_5^{10} f(x)\,dx = 14$$

13. (5 points) A honeybee population increases at a rate of $n'(t)$ bees per week. What does $\int_0^{12} n'(t)dt$ represent? Be specific!

$\int_0^{12} n'(t)\,dt = n(12) - n(0)$ which represents the total change in the bee population at 12 weeks.

14. (15 points) Evaluate

$$\int 2x\sqrt{x-5}dx$$

$u = x-5 \rightarrow x = u+5$

$du = dx$

$$\int 2(u+5)\sqrt{u}\,du$$

$$= 2\int u^{3/2} + 5u^{1/2}\,du$$

$$= 2\left(\frac{2}{5}u^{5/2} + \frac{10}{3}u^{3/2}\right) + C$$

$$= \frac{4}{5}(x-5)^{5/2} + \frac{20}{3}(x-5)^{3/2} + C$$

15. (10 points) Evaluate the indefinite integral.

$$\int \frac{\sin x}{\cos^2 x}\,dx$$

$u = \cos x$

$du = -\sin x \, dx$

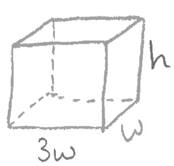
$$-\int \frac{1}{u^2}\,du$$

$$= -\int u^{-2}\,du$$

$$= \frac{1}{u} + C$$

$$= \frac{1}{\cos x} + C \quad \text{or } (\cos x)^{-1} + C$$

Bonus: No partial credit will be given for this problem.
We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10 per square foot and the material used to build the sides cost \$6 per square foot. If the box must have a volume of $50 ft^3$ determine the dimensions that will minimize the cost to build the box. (From Paul's Online Calculus I Notes, Example 2)



Area of top: $3w^2$

bottom: $3w^2$

Cost: $10(3w^2+3w^2)$

$= 60w^2$

Area of sides:

$2\cdot w\cdot h + 2\cdot 3w\cdot h$

$2wh + 6wh$

$8wh$

Cost: $6\cdot 8wh$

$= 48wh$

Total Cost: $60w^2 + 72wh$

Want to minimize but need h in terms of w.

$V = l\cdot w\cdot h$

$= 3w^2 h$

$50 = 3w^2 h$

$h = \frac{50}{3w^2}$

$60w^2 + 48w \cdot \frac{50}{3w^2}$

$C(w) = 60w^2 + \frac{800}{w}$

$C'(w) = 120w - \frac{800}{w^2}$

$= \frac{120w^3 - 800}{w^2}$

$w \neq 0$

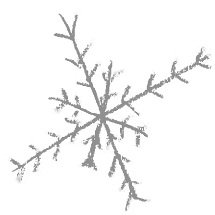
$\boxed{w = \sqrt[3]{\frac{800}{120}} = \sqrt[3]{\frac{20}{3}}}$ $\boxed{l = 3\sqrt[3]{\frac{20}{3}}}$ $\boxed{h = \frac{50}{3\left(\sqrt[3]{\frac{20}{3}}\right)^2}}$

Useful Facts

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$



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