Math 142: Section 10.4 - Notes

1 Comparison Tests

The Comparison Test

Let $\sum a_n$, $\sum c_n$, and $\sum d_n$ be series with nonnegative terms. Suppose that for some integer N

$$d_n \leq a_n \leq c_n \text{ for all } n > N.$$

a) If $\sum c_n$ converges, then $\sum a_n$ also converges.
b) If $\sum d_n$ diverges, then $\sum a_n$ also diverges.

Example 1 Determine if the series converges or diverges using the Comparison Test.

a)

$$\sum_{n=1}^{\infty} \frac{5}{5n-1}$$

b)

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

c)

$$5+\frac{2}{3}+\frac{1}{7}+1+\frac{1}{2+\sqrt{1}}+\frac{1}{4+\sqrt{2}}+\frac{1}{8+\sqrt{3}}+\cdots+\frac{1}{2^n+\sqrt{n}}+\cdots$$

The Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

1) If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

2) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

3) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Example 2 Determine if the following series converge or diverge using the Limit Comparison Test.

a)

$$\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\frac{9}{25}+\cdots=\sum_{n=1}^{\infty}\frac{2n+1}{(n+1)^2}$$

b)

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{7}+\frac{1}{15}+\cdots=\sum_{n=1}^{\infty}\frac{1}{2^n-1}$$

c)

$$\frac{1+2\ln 2}{9}+\frac{1+3\ln 3}{14}+\frac{1+4\ln 4}{21}+\cdots=\sum_{n=2}^{\infty}\frac{1+n\ln n}{n^2+5}$$