Math 141 Calculus I

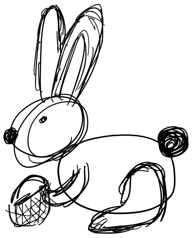
Exam #2

March 28, 2018

Instructor: Ann Clifton

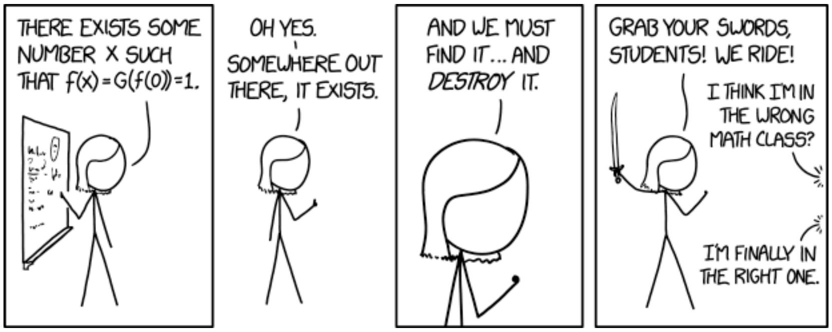
Name: Sols

Do not turn this page until told to do so. You will have a total of 1 hour and 15 minutes to complete the exam. You **must** show all work to receive full credit unless otherwise noted.
NO CALCULATOR/PHONE ALLOWED.
Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%. Draw a bunny on this page if you read these directions in full.



#	score	out of	#	score	out of
1		4	8		8
2		4	9		8
3		4	10		10
4		4	11		10
5		4	12		10
6		8	13		18
7		8	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



True or False. No work/explanation required. 4pts each. True means always true.

1. *If a function is continuous, it is always differentiable.*

False

2. *A critical point c is only where* $f'(c) = 0$.

False

3. *If f and g are differentiable functions of x, then* $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$.

True

4. *If* $f''(c) = 0$, *then* $x = c$ *is an inflection point of f.*

False

5. *The absolute value function,* $f(x) = |x|$, *is differentiable at* $x = 0$.

False

Multiple Choice. No work required. 8pts each. Choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive half the points.

B

6. *Find* $\frac{dy}{dx}$ *(Hint: Use trig identities to simplify):*

$$y^{\cot x} = 6$$

A. $\frac{dy}{dx} = -y \ln y \csc^2 x$

[✓ **B.** $\frac{dy}{dx} = y \ln y \csc x \sec x$]

C. $\frac{dy}{dx} = y \ln y \cot x$

D. $\frac{dy}{dx} = \cot x$

$$\cot x \ln y = \ln(6)$$

$$-\csc^2 x \ln y + \cot x \frac{1}{y}\frac{dy}{dx} = 0$$

$$\frac{\cot x}{y}\frac{dy}{dx} = \csc^2 x \ln y$$

$$\frac{dy}{dx} = \frac{y \ln y \csc^2 x}{\cot x}$$

$$\frac{dy}{dx} = y \ln y \csc x \sec x$$

C

7. *Find $h'(2)$, given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$, if $h(x) = \dfrac{g(x)}{1+f(x)}$.*

A. $h'(2) = -7/2$ **B.** $h'(2) = -11/2$

[✓ **C.**] $h'(2) = -3/2$ **D.** $h'(2) = -1/2$

$$h'(2) = \frac{g'(2)(1+f(2))-g(2)f'(2)}{(1+f(2))^2}$$

$$= \frac{7(1+-3)-4(-2)}{(1+-3)^2}$$

B

8. *Find the derivative, y':*

$$y = \arctan\left(4x^2\right)$$

A. $y' = \frac{1}{1+2x^3}$ [✓ **B.**] $y' = \frac{8x}{1+16x^4}$

C. $y' = \frac{1}{\sqrt{1+16x^4}}$ **D.** $y' = \frac{1}{1+16x^4}$

$$y' = \frac{1}{1+(4x^2)^2}\cdot(4x^2)' = \frac{8x}{1+16x^4}$$

D

9. *Find the derivative, y':*

$$y = \frac{-2x^3 - 5x + \sqrt{x}}{x^2} = -2x - 5x^{-1} + x^{-3/2}$$

A. $y' = \frac{-6x^2-5+\frac{1}{2}x^{-1/2}}{2x}$ **B.** $y' = -7 + \frac{1}{2}x^{-1/2}$

C. $y' = (-6x^2 - 5 + \frac{1}{2}x^{-1/2})x^2 - 2x(-2x^3 - 5x + \sqrt{x})$ [✓ **D.**] $y' = -2 + \frac{5}{x^2} - \frac{3}{2x^{5/2}}$

Short Answer. You must show all work to receive full credit. Simplify your answers.

10 (10 points)**.** *A student turns in the incorrect solution to the problem below. Explain the student's mistake in words, using complete sentences. Then work out the correct solution.*

$$\frac{d}{d\theta}(\theta^2 \tan\theta) = 2\theta \sec^2\theta$$

The student did not use the product rule.

$$\frac{d}{d\theta}(\theta^2\tan\theta) = 2\theta\tan\theta + \theta^2\sec^2\theta$$

11 (10 points). *Find the value or values of c that satisfy the equation*

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$ *on* [0.1].

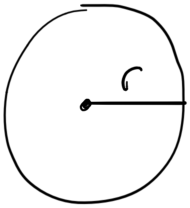
$$\frac{f(1)-f(0)}{1-0} = \frac{2--1}{1} = 3$$

$$f'(c) = 2c+2$$

$$3 = 2c+2$$

$$\boxed{c = 1/2}$$

12 (10 points). *When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50cm? (Recall the area of a circle is given by* $A = \pi r^2$*.)*



$$\frac{dr}{dt} = 0.01 \text{ cm/min}$$

$$\frac{dA}{dt} = ? \text{ when } r = 50\text{cm}$$

$$A = \pi r^2 \qquad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(50)(0.01)$$

$$= \boxed{\pi \text{ cm}^2/\text{min}}$$

13. *(18 pts) Sketch the curve*

$$y = \frac{x^2 - 4}{2x}$$

(a) State the domain.

$(-\infty, 0) \cup (0, \infty)$

(b) Find the intercepts. Enter NONE if there are none.

x-intercepts: $(2,0), (-2,0)$

y-intercept: None

$$y = \frac{(x+2)(x-2)}{2x}$$

$y=0$ when $(x+2)(x-2)=0$

$x=-2, x=2$

$x=0$ is not in the domain

(c) Is the function even, odd, or neither? What type of symmetry does the function have?

$$f(-x) = \frac{(-x)^2-4}{2(-x)} = \frac{x^2-4}{-2x} = -\frac{x^2-4}{2x} = -f(x)$$

Odd, origin

(d) Find the asymptotes. Enter NONE if there are none.

Horizontal: None

Oblique: $y = \frac{1}{2}x$

Vertical: $x=0$

O: $2x \overline{\big) x^2-4}$ with quotient $\frac{1}{2}x$

$-x^2$

-4

V: $\lim_{x \to 0^-} \frac{x^2-4}{2x} = \infty \quad \lim_{x \to 0^+} \frac{x^2-4}{2x} = -\infty$

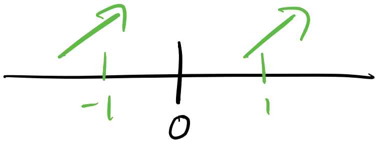
(e) Find the intervals where the function is increasing and decreasing. Enter NONE if not applicable.

Increasing: $(-\infty, 0) \cup (0, \infty)$

Decreasing: None

$$y' = \frac{2x(2x) - 2(x^2-4)}{(2x)^2} = \frac{4x^2 - 2x^2 + 8}{4x^2} = \frac{2x^2+8}{4x^2} = \frac{x^2+4}{2x^2}$$

y' is never 0 but is undefined at $x=0$



(f) State the local maximum and local minimum value(s). Enter NONE if not applicable.

Local maximum value(s): None

Local minimum value(s): None

(g) Find the intervals on which the function is concave up and concave down. State the inflection points. Enter NONE if not applicable.

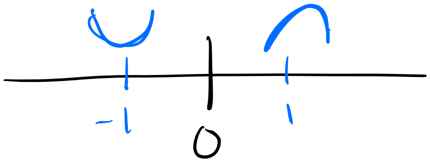
Concave Up: $(-\infty, 0)$

Concave Down: $(0, \infty)$

Inflection Points: None ($x=0$ is not in the domain)

$$y'' = \frac{2x(2x^2) - 4x(x^2+4)}{4x^4} = \frac{\cancel{4x^3} - \cancel{4x^3} - 16x}{4x^4} = \frac{-4}{x^3}$$

y'' is never 0 but is undefined at $x=0$



(h) Use parts (a)-(g) to sketch the curve. Be sure that your graph is labeled and neat. Messy/incoherent graphs will receive zero points.