Sols

Math 122 Calculus for Business Admin. and Social Sciences

Exam #2 A
October 25, 2017

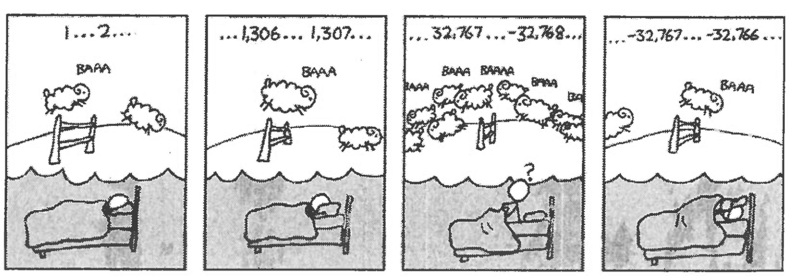
Instructor: Ann Clifton

Name: ____________________

Do not turn this page until told to do so. You will have a total of 50 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers will **not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw a dinosaur on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

#	score	out of	#	score	out of
1		3	9		10
2		3	10		12
3		9	11		12
4		10	12		10
5		4	13		16
6		4	EC		5
7		2	Total		100
8		5			

Remember: This exam has no impact on your worth as a human being. You got this!!!



Throughout this section, let f and g be differentiable functions. Fill in the blanks.

1. (3 points)

   (a) Let a be a constant; $\frac{d}{dx}(af(x)) =$ $a \frac{d}{dx} f(x)$ or $a f'(x)$

   (b) $\frac{d}{dx}(f(x) + g(x)) =$ $f'(x) + g'(x)$

   (c) $\frac{d}{dx}(f(x) - g(x)) =$ $f'(x) - g'(x)$

2. (3 points)

   (a) For n a number, $\frac{d}{dx}(x^n) =$ $n x^{n-1}$

   (b) $\frac{d}{dx}(\ln(x)) =$ $\frac{1}{x}$

   (c) $\frac{d}{dx}(e^x) =$ e^x

3. (9 points) Write the formula for each of the following derivatives.

(a)

$$\frac{d}{dx}(f(x)g(x))$$

$f'(x)g(x) + f(x)g'(x)$

(b)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

(c)

$$\frac{d}{dx}(f \circ g(x))$$

$f'(g(x)) \cdot g'(x)$

Multiple Choice For each of the following questions, circle the correct answer.

4. (10 points) Assume that f is a function such that $f'(x)$ and $f''(x)$ are defined for all x.

A

(a) A point p is a critical point of f if

A. $f'(p) = 0$ **B.** $f'(p) < 0$

C. $f'(p) > 0$ **D.** $f(p) = 0$

B

(b) f is increasing on an interval if

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

A

(c) f is decreasing on an interval if

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

C

(d) f is concave down on an interval if

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

B

(e) f is concave up on an interval if

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

5. (4 points) The **first** derivative test says that a critical point, p, of f is a

(a) local maximum if

B

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

(b) local minimum if

A

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

6. (4 points) The **second** derivative test says that a critical point, p, of f is a

(a) local maximum if

C

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C. $f'' < 0$

D. $f'' > 0$

(b) local minimum if

D

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C. $f'' < 0$

D. $f'' > 0$

7. (2 points) Suppose that $f''(p) = 0$. We say that p is an inflection point of f if

A

A. f'' changes sign at p

B. f changes sign at p

C. $f'(p) = 0$

D. $f(p) = 0$

8. (5 points) Find the derivative of the following functions.

(a) $f(x) = 3x + 7$

3

(b) $g(x) = 5x^2 + 2x + 1$

$10x + 2$

(c) $h(x) = 12x^3 + 13x^2$

$36x^2 + 26x$

(d) $r(x) = \frac{1}{3}x^3 + 2$

x^2

(e) $s(x) = \sqrt{x} + 3$

$\frac{1}{2}x^{-1/2}$

9. (10 points) Find the derivative of the following functions.

2 (a) $(x+7)^{25}$

$25(x+7)^{24}$

2 (b) $e^{\frac{1}{2}x^2+2x+1}$

$e^{\frac{1}{2}x^2+2x+1}(x+2)$

2 (c) $\ln(2x^2+7)$

$$\frac{4x}{2x^2+7}$$

2 (d) $\sqrt{x^2+1}$

$\frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2}$

2 (e) $6e^{5x}+e^{-x^2}$

$30e^{5x} - 2xe^{-x^2}$

10. (12 points) Differentiate the following functions.

4 (a) xe^{-2x}

$e^{-2x} - 2xe^{-2x}$

4 (b) $x \ln(x)$

$\ln x + 1$

4 (c) $(x^2+3)e^x$

$2xe^x + (x^2+3)e^x$

11. (12 points) Differentiate the following functions.

4 (a) $\dfrac{x+1}{x-1}$

$$\frac{(x-1)-(x+1)}{(x-1)^2}$$

.1 $$\frac{-2}{(x-1)^2}$$

4 (b) $\dfrac{x}{e^x}$

$$\frac{e^x - xe^x}{e^{2x}}$$

1 $$\frac{1-x}{e^x}$$

4 (c) $\dfrac{x}{\ln(x)}$

$$\frac{\ln(x)-1}{(\ln(x))^2}$$

12. (10 Points) Let $f(x) = 10x^4 - 4x^5$.

(a) Find the derivative of f.

$$f'(x) = 40x^3 - 20x^4 = 20x^3(2-x)$$

(b) Find the critical points of f. [Hint: Factoring after taking the derivative will make this much easier.]

$$20x^3(2-x) = 0$$

$$x = 0 \,,\ x = 2$$

(c) Find any local maxima and local minima of f. Clearly indicate whether a point is a maximum or a minimum.

$x = 0$ Local minimum

$x = 2$ Local maximum

$f'(-1) < 0 \quad f'(3) < 0$

$f'(1) > 0$

(d) Find the global maximum and global minimum on the interval $[1, 3]$.

$$f(1) = 10(1)^4 - 4(1)^5 = 6$$

$f(2) = 32$ Global max

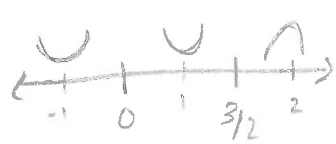
$f(3) = -162$ Global min

(e) Find any inflection points of f.

$$f''(x) = 120x^2 - 80x^3$$

$$= 40x^2(3-2x)$$

$$x = 0 \,,\ \boxed{x = 3/2}$$



$f''(-1) > 0$

$f''(1) > 0$

$f''(2) < 0$

13. (16 points) A company sells a product for \$30 each and the manufacturing costs can be modeled by the function

$$C(q) = q^3 - 9q^2 + 45q + 15$$

of q units produced. For each of the quantities below, determine whether the company should increase, decrease, or not change the production levels in order to maximize profit. Justify your answers using calculus. **You will not receive credit for guess and check solutions.**

(a) $q = 2$ Increase 1 (b) $q = 6$ Decrease 1 (c) $q = 7$ Decrease 1

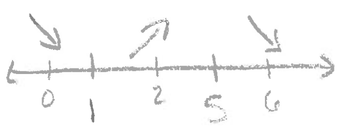
2 $R(q) = p \cdot q = 30q$

3 $\pi(q) = R(q) - C(q) = 30q - (q^3 - 9q^2 + 45q + 15)$

$= -q^3 + 9q^2 - 15q - 15$

4 $\pi'(q) = -3q^2 + 18q - 15 = -3(q^2 - 6q + 5) = -3(q-5)(q-1)$

Critical points: $q = 5$ 1, $q = 1$ 1



$\pi'(0) < 0$

$\pi'(2) > 0$

$\pi'(6) < 0$

2 $q = 5$ is the quantity that maximizes profit

MC < MR Increase

MR < MC Decrease

Extra Credit No partial credit will be given for this problem.

If a person is lost in the wilderness, the search and rescue team identifies the boundaries of the search area and then uses probabilities to help optimize the chances of finding the person, assuming the subject is immobile. The probability, O, of the person being outside the search area after the search has begun and the person has not been found is given by

$$O = \frac{S}{1-(1-S)E},$$

where S is the probability of the person being outside the search area at the start of the search and E is the search effort, a measure of how well the search area has been covered by the resources in the field.

Evaluate $O'(E)$. Is it positive or negative? What does that tell you about O as E increases?

$$O = S(1-(1-S)E)^{-1}$$

$$O'(E) = -S(1-(1-S)E)^{-2}(-(1-S))$$

$$= \frac{S(1-S)}{(1-(1-S)E)^2}$$

$O'(E)$ is positive. This means O is an increasing function of E. That is, as E increases, O also increases.

In practical terms, the longer the search goes on, the more likely it is that the person is outside the search area.