Solutions

Math 122 Calculus for Business Admin. and Social Sciences

Exam #1 A
July 6, 2017

Instructor: Ann Clifton

Name: ______________________________

Do not turn this page until told to do so. You will have a total of 1 hour 25 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers will **not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.

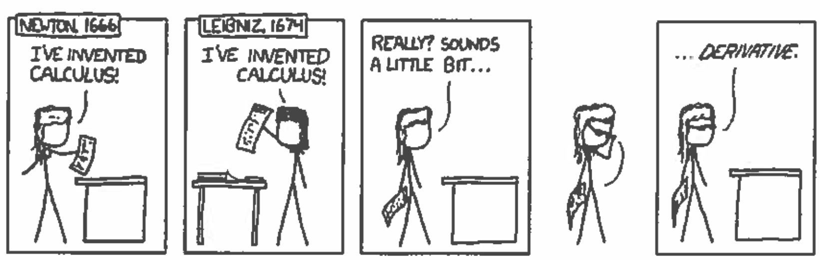
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.

Draw a fish on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.


+1

#	score	out of	#	score	out of
1		2	7		9
2		5	8		12
3		5	9		15
4		5	10		15
5		6	11		20
6		6	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



1. (a) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m.

$$y - y_0 = m(x - x_0)$$

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

$$y = mx + b$$

2. Let f be a function and let $a < b$ be given. State the average rate of change of f on the interval $[a, b]$.

$$\frac{f(b) - f(a)}{b - a}$$

3. Given a quantity P, state the relative change of the quantity from P to P'.

$$\frac{P' - P}{P}$$

4. (a) State the form of an exponential function of a variable t with initial value P_0 and base a:

$P(t) =$ $P_0 a^t$.

(b) The relative rate of change of P is

$r =$ $a-1$.

[Hint: If you don't recall the formula, this is just the relative change from $P(t)$ to $P(t+1)$.]

(c) The function P models

(i) exponential growth when r is positive.

(ii) exponential decay when r is negative.

(d) The continuous growth/decay rate is

$k =$ $\ln(a)$.

5. Let $0 < x$, $0 < y$ be given. Fill in the blanks:

(i) $\ln(1) =$ 0

(ii) $\ln(xy) =$ $\ln(x) + \ln(y)$

(iii) $\ln\left(\frac{x}{y}\right) =$ $\ln(x) - \ln(y)$

(iv) $\ln(x^r) =$ $r\ln(x)$

(v) $\ln(e^x) =$ x

(vi) $e^{\ln(x)} =$ x

6. (a) Find the slope of the line passing through the points $(3, \frac{1}{2})$ and $(2, 1)$.

$$m = \frac{1 - 1/2}{2 - 3} = \frac{1/2}{-1} = -1/2$$

(b) Write the equation of this line in Point-Slope Form.

$$y - 1 = -1/2\,(x - 2)$$

OR

$$y - 1/2 = -1/2\,(x - 3)$$

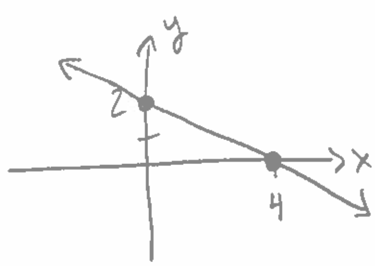
(c) Write the equation of this line in Slope-Intercept Form.

$$y - 1 = -1/2\,(x - 2)$$

$$y - 1 = -1/2\,x + 1$$

$$y = -1/2\,x + 2$$

(d) Sketch a graph of $f(x)$. Label the x-intercept and the y-intercept.



7. Let $f(x) = -x^2 + 1$.

(a) Compute the average rate of change for f between $x = 3$ and $x = 5$.

$$\frac{f(5)-f(3)}{5-3} = \frac{-(5)^2+1-(-(3)^2+1)}{2}$$

$$= \frac{-25+1+9-1}{2}$$

$$= -16/2 = -8$$

(b) Give the Point-Slope form of the line that passes through $(3, f(3))$ and $(5, f(5))$.

$m = -8$ (from part (a))

$f(3) = -9+1 = -8$

$f(5) = -25+1 = -24$

$y - -8 = -8(x-3)$

$y+8 = -8(x-3)$ OR

$y - -24 = -8(x-5)$

$y+24 = -8(x-5)$

(c) Give the Slope-Intercept form of the line from part (b).

$y+8 = -8x+24$

$y = -8x+16$

OR $y+24 = -8x+40$

$y = -8x+16$

8. A biologist observes a population with initial size 9. In two years, the biologist returns to observe the population again and finds that there are 81.

(a) Find an exponential function for the size of the population as a function of t years since the initial observation.

$P_0 = 9$

$P(2) = 9a^2 = 81$

$a^2 = 9$

$a = 3$

$\boxed{P(t) = 9(3)^t}$

(b) Does the function from part (a) model growth or decay?

Growth

(c) Use the model from part (a) to determine how many years it will take for the size of the population to reach 243.

$243 = 9(3)^t$

$\frac{243}{9} = 3^t$

$27 = 3^t$

$\ln(27) = t\ln(3)$

$\frac{\ln(27)}{\ln(3)} = t$

$\boxed{t = 3 \text{ years}}$

Without Calculator:

$27 = 3^t$

$3^3 = 3^t$

$\boxed{3 = t}$

9. A bank is offering an account that pays 7% interest compounded continuously. If you decide to invest money in this account, how long will it take for your initial investment to double? Round to the nearest year.

$$P(t) = P_0 e^{rt}$$

$$P(t) = P_0 e^{.07t}$$

$$2 = 1e^{.07t}$$

$$\ln(2) = \ln(e^{.07t})$$

$$\ln(2) = .07t$$

$$\frac{\ln(2)}{.07} = t$$

$$t \approx 9.9 \text{ or } \boxed{10 \text{ years}}$$

Rule of 70:

$$\frac{\ln(2)}{.07} = \frac{100\ln(2)}{7} \approx \frac{70}{7} = 10 \text{ years}$$

10. Sketch a graph of the function

$$f(x) = 2x^2 - 8x + 6.$$

Find the x-intercepts, the y-intercept, and the vertex. You may use your calculator to check your answer but you must show supporting work.

Hint: Given $f(x) = ax^2 + bx + c$, the formula for the vertex of the parabola is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

X-intercepts

$0 = 2x^2 - 8x + 6$

$0 = 2(x^2 - 4x + 3)$

$0 = 2(x-3)(x-1)$

$x = 3, \quad x = 1$

y-intercept

$y = 2(0)^2 - 8(0) + 6 = 6$

vertex

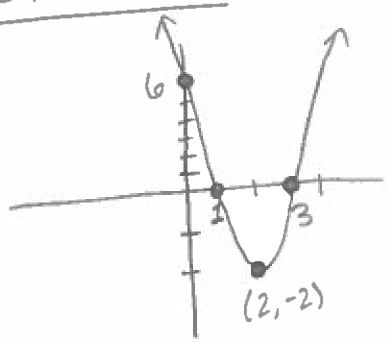
$b = -8$ $\quad \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2$, $f(2) = 2(2)^2 - 8(2) + 6$

$a = 2$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = 2(4) - 16 + 6$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = 8 - 16 + 6$

$(2, -2)$ $\quad\quad\quad\quad\quad\quad\quad\quad = -2$

Sketch



11. A company hosts a weekly event. They find that 25 people attend at a ticket price of \$30, and 15 people attend at a ticket price of \$50. Assuming this relationship is linear, determine the ticket price that will generate the highest revenue. State the maximum revenue.

Hint: First, find the equation of the line representing the quantity of tickets sold, q, in terms of price, p.

(p, q) $\quad (30, 25)$

$(50, 15)$ $\quad m = \dfrac{25-15}{30-50} = \dfrac{10}{-20} = -\tfrac{1}{2}$

$q - 25 = -\tfrac{1}{2}(p - 30)$

$q - 25 = -\tfrac{1}{2}p + 15$

$q = -\tfrac{1}{2}p + 40$

Revenue

$R(p) = p \cdot q = p(-\tfrac{1}{2}p + 40)$

$= -\tfrac{1}{2}p^2 + 40p$

$R(p) = -\tfrac{1}{2}p^2 + 40p$ is a downward facing parabola. So the maximum occurs at the vertex.

$p = \dfrac{-b}{2a} = -\dfrac{40}{2(-\tfrac{1}{2})} = 40$

\$40 is the price which maximizes the revenue

$R(40) = -\tfrac{1}{2}(40)^2 + 40(40) = \$800$ is the maximum revenue