

Solutions

MATH 170: EXAM 01A

ANN CLIFTON  
UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. You may only use a four-function calculator. No graphing calculators or cell phones are allowed.

Name: \_\_\_\_\_

1. PROBLEMS

Let  $S = \{\text{USC, Missouri, Georgia, Vanderbilt, Florida, Tennessee, Kentucky}\}$ . Let  $X = \{\text{USC, Kentucky, Florida}\}$ ,  $Y = \{\text{Missouri, Georgia, Tennessee, Vanderbilt, USC}\}$ ,  $Z = \{\text{Kentucky, Missouri, USC}\}$ , and  $W = \{\text{Georgia, Tennessee, Vanderbilt}\}$ .

1 (10 points). Compute

S

(a)  $X \cap Y$

$$X \cap Y = \{\text{USC}\}$$

no brackets -1 point

S

(b) The complement of  $Z$  in  $S$ ,  $Z'$ .

$$Z' = \{\text{Georgia, Vanderbilt, Florida, Tennessee}\}$$

2 (18 points). Use the sets in number 1 to compute the following:

No work! -2pts

No  $n(\rightarrow)$  -1pt

6 (a) What is the cardinality of  $Z \times W$ ?

$$n(Z) = 3$$

$$n(W) = 3$$

$$n(Z \times W) = n(Z) \cdot n(W) = 3 \cdot 3 = 9$$

6 (b) What is the cardinality of  $X \cup Y$ ?

$$X \cup Y = \{ \text{USC, Kentucky, Florida, Missouri, Georgia, Tennessee, Vanderbilt} \}$$

$$n(X \cup Y) = 7$$

6 (c) What is the cardinality of  $X \cap W$ ?

$$X \cap W = \emptyset$$

$$n(X \cap W) = 0$$

$$n(X \cap W) = \emptyset \text{ -3pts}$$

3 (20 points). The "Suggestions for You" section of Netflix has a total of 14 movies, of which 8 have action scenes and 9 have romance. Assuming that all of them have either action scenes or romance or both, how many have both?

$A$  = set of action

$B$  = set of romance

No  $n(\dots)$  -1pt

no or unclear work  
-5pts

$$n(A \cup B) = 14$$

$$n(A \cap B) = ?$$

$$n(A) = 8$$

$$n(B) = 9$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$14 = 8 + 9 - n(A \cap B)$$

$$14 = 17 - n(A \cap B)$$

$$-3 = -n(A \cap B)$$

$$\boxed{n(A \cap B) = 3}$$

4 (12 points). Use a truth table to prove the following logical equivalence. Explain why the equivalence holds.

$$p \Rightarrow q \equiv \sim p \vee q.$$

8 pts

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

\* \*

4 pts

Since the starred columns have the same truth values, the equivalence holds.

5 (12 points). Complete a truth table for the following logical statement. What type of statement is this?

$$(\sim p \Rightarrow \sim q) \vee (\sim q \Rightarrow \sim p)$$

9 pts

P	Q	$\sim P$	$\sim Q$	$\sim P \Rightarrow \sim Q$	$\sim Q \Rightarrow \sim P$	$(\sim P \Rightarrow \sim Q) \vee (\sim Q \Rightarrow \sim P)$
T	T	F	F	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Tautology,  
always true.  
3 pts

6 (8 points). Calculate the following:

4 (a)  $P(4,3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

4 (b)  $C(4,3) = \frac{4!}{3!(4-3)!} = \frac{4!}{3! \cdot 1!} = \frac{4!}{3!} = \frac{4 \cdot 3!}{3!} = 4$

7 (20 points). A bag contains 4 red marbles, 1 green one, 1 lavender one, 3 yellows, and 2 orange marbles. How many sets of four marbles include at least 3 red marbles?

Alternative 1: exactly 3 red marbles

Step 1: Choose 3 red marbles from the 4 total  $C(4,3) = 4$

9 pts

Step 2: choose 1 nonred marble

$$C(7,1) = 7$$

$$\text{Total: } 4 \cdot 7 = 28 \text{ choices}$$

Alternative 2: exactly 4 red marbles + 4 pts

Step 1: choose 4 (all) red marbles + 4 pts

8 pts

$$C(4,4) = 1$$

Total: 1 choice

3 pts Total sets:  $28 + 1 = 29$  possible sets

Know to add alternatives  
~~20 pts~~ 3 pts

Solutions

MATH 170: EXAM 01B

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Name: \_\_\_\_\_

1. PROBLEMS

Let  $S = \{\text{USC, Missouri, Georgia, Vanderbilt, Florida, Tennessee, Kentucky}\}$ . Let  $X = \{\text{USC, Kentucky, Florida}\}$ ,  $Y = \{\text{Missouri, Georgia, Tennessee, Vanderbilt, USC}\}$ ,  $Z = \{\text{Kentucky, Missouri, USC}\}$ , and  $W = \{\text{Georgia, Tennessee, Vanderbilt}\}$ .

1 (10 points). Compute

(a)  $X \cup Y$

No brackets -1pt

S

$$X \cup Y = \{\text{USC, Kentucky, Florida, Missouri, Georgia, Tennessee, Vanderbilt}\}$$

(b) The complement of  $W$  in  $S$ ,  $W'$ .

S

$$W' = \{\text{USC, Missouri, Florida, Kentucky}\}$$

2 (18 points). Use the sets in number 1 to compute the following:

(a) What is the cardinality of  $Z \times W$ ?

No work -2 pts

no  $n(\rightarrow)$  -1 pt

$$n(Z \times W) = n(Z) \cdot n(W) = 3 \cdot 3 = 9$$

(b) What is the cardinality of  $X \cap Y$ ?

$$X \cap Y = \{u, s, c\}$$

$$n(X \cap Y) = 1$$

(c) What is the cardinality of  $X \cap W$ ?

$$X \cap W = \emptyset$$

$$n(X \cap W) = 0$$

$$n(X \cap W) = \emptyset \quad -3 \text{ pts}$$

3 (20 points). The "Suggestions for You" section of Netflix has a total of 16 movies, of which 9 have action scenes and 10 have romance. Assuming that all of them have either action scenes or romance or both, how many have both?

$A$  = set of action

$B$  = set of romance

No  $n(\rightarrow)$  -1 pt

no or unclear work -5 pts

$$n(A \cup B) = 16$$

$$n(A) = 9$$

$$n(B) = 10$$

$$n(A \cap B) = ?$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$16 = 9 + 10 - n(A \cap B)$$

$$16 = 19 - n(A \cap B)$$

$$-3 = -n(A \cap B)$$

$$n(A \cap B) = 3$$

4 (12 points). Use a truth table to prove the following logical equivalence. Explain why the equivalence holds.

8 pts

$$p \Rightarrow q \equiv \sim p \vee q.$$

$p$	$q$	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

\*                      \*

4 pts

Since the starred columns have the same truth values, the equivalence holds.

5 (12 points). Complete a truth table for the following logical statement. What type of statement is this?

9 pts

$$(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$$

$p$	$q$	$\sim q$	$p \Rightarrow \sim q$	$\sim q \Rightarrow p$	$(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T

Tautology,  
always true.  
3 pts

6 (8 points). Calculate the following:

4 (a)  $C(5, 4) = \frac{5!}{4!(5-4)!} = \frac{5!}{4! \cdot 1!} = \frac{5 \cdot 4!}{4!} = 5$

4 (b)  $P(5, 4) = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

7 (20 points). A bag contains 4 red marbles, 1 green one, 1 lavender one, 3 yellows, and 2 orange marbles. How many sets of three marbles include at least 2 yellow marbles?

Alternative 1: Exactly 2 yellow marbles

Step 1: Choose 2 of the 3 yellow marbles

$$C(3, 2) = \frac{3!}{2!(3-2)!} = 3$$

Step 2: Choose 1 non-yellow marble

$$C(8, 1) = 8$$

$$\text{Total: } 3 \cdot 8 = 24$$

Alternative 2: Exactly 3 yellow marbles

Step 1: Choose all 3 yellow marbles

$$C(3, 3) = 1$$

3pts

Total sets:  $24 + 1 = 25$  sets.

(Add  
alternatives)



8 (Bonus (Cube Problem!)) 10 points, No partial credit.). As Product Design Manager for Cerebral Toys, Inc., you are constantly on the lookout for ideas for intellectually stimulating yet inexpensive toys. Your design team recently came up with an idea for a puzzle consisting of a number of plastic cubes. Each cube will have two faces colored red, two white, and two blue, and there will be exactly two cubes with each possible configuration of colors. The goal of the puzzle is to seek out the matching pairs, thereby enhancing a child's geometric intuition and three-dimensional manipulation skills. If the kit is to include every possible configuration of colors, how many cubes will the kit contain? (Hint: The answer is less than 90)

- (1) Alternative 1: Faces with the same color opposite each other. How many choices? 1
- (2) Alternative 2: Red faces opposite each other and the other colors on adjacent pairs of faces. How many choices? 1
- (3) Alternative 3: Blue faces opposite, other colors adjacent. 1
- (4) Alternative 4: White faces opposite, other colors adjacent. 1
- (5) Alternative 5: Faces with the same color adjacent to each other. How many choices? 2

How many cubes total?

$$1 + 1 + 1 + 1 + 2 = 6 \text{ distinct colorings}$$

$$\text{Two of each, } 6 \cdot 2 = \underline{12 \text{ total cubes}}$$

Polya enumeration

