Math 141: Section 3.8 Derivatives of Inverse Functions and Logarithms - Notes

The Derivative Rule for Inverses If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is differentiable at every point in its domain. The value of $(f^{-1})'$ at a point b in the domain of f^{-1} is the *reciprocal* of the value of f' at the point $a = f^{-1}(b)$:

$$(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$$

Example 1 Let $f(x) = x^3 - 2x$, $x > 0$. Find the value of df^{-1}/dx at $x = 6 = f(2)$ without finding a formula for $f^{-1}(x)$.

Derivative of the Natural Logarithm Function Since we know the exponential function $f(x) = e^x$ is differentiable everywhere, we can apply the rule to find the derivative of its inverse $f^{-1}(x) = \ln(x)$:

Example 2 Find dy/dx if $y = \ln|x|$.

Derivatives of a^u and $\log_a u$:

1) If $a > 0$ and u is a differentiable function of x, then a^u is a differentiable function of x and

$$\frac{d}{dx}a^u = \ln(a)a^u\frac{du}{dx}.$$

2) For $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}\log_a u = \frac{1}{u\ln(a)}\frac{du}{dx}.$$

Example 3 Find y' if $y = 3^x$.

Logarithmic Differentiation Find dy/dx if

$$y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}, \qquad x > 1.$$

Example 4 Differentiate $y = x^{\sqrt{x}}$.