Math 142: Section 8.2 - Notes

1 Integration by Parts

A New Technique Integration by parts is a technique used to simplify integrals of the form

$$\int f(x)g(x)dx.$$

It is useful when one of the functions (f or g) can be differentiated repeatedly and the other function can be integrated repeatedly without difficulty. The following are two such integrals:

$$\int x\cos x dx \text{ and } \int x^2 e^x dx.$$

Application of the Product Rule If f and g are differentiable functions of x, the Product Rule says that

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

Integrating both sides and rearranging gives us the the **integration by parts** formula!

Integration by Parts Formula

$$\int udv = uv - \int vdu$$

Remember, all of the techniques that we talk about are supposed to make integrating easier! Even though this formula expresses one integral in terms of a second integral, the idea is that the second integral, $\int vdu$, is easier to evaluate. The key to integration by parts is making the right choice for u and v. Sometimes we may need to try multiple options before we can apply the formula.

Example 1 Find

$$\int x \cos x dx.$$

Example 2 Evaluate

$$\int x^2 e^x dx.$$

Example 3 - Integration by Parts for Definite Integrals Find the area of the region bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = 4$.

Example 4 - "Tabular Integration" Recall Example 2 in which we applied the Integration by Parts Formula multiple times. Creating a table can help organize the steps and simplify the work. Let's look at how we might use a table for Example 2.

Evaluate

$$\int x^2 e^x dx.$$

Example 5 Find the integral

$$\frac{1}{\pi}\int_0^{\pi} x^3 \cos nx dx,$$

on the interval $[0, \pi]$ where n is a positive integer.