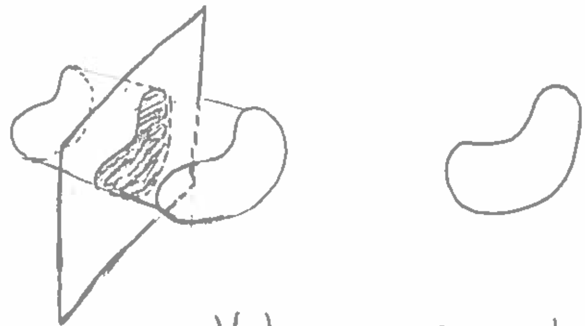
Ch. 6 Applications of Definite Integrals

* Note MML NOT representative of test Qs.

6.1 Volumes Using Cross-Sections

A cross-section of a solid S is the plane region formed by intersecting S with a plane



Volume = area × height

- Slicing Method

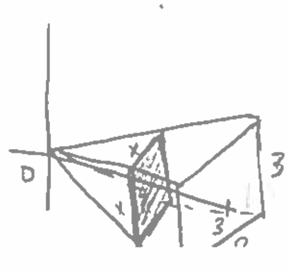
The volume of a solid of integrable cross-sectional area $A(x)$ from $x=a$ to $x=b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\,dx$$

Calculating the Volume of a Solid

1) Sketch the solid and a typical cross-section
2) Find a formula for $A(x)$, the area of a typical cross-section
3) Find the limits of integration
4) Integrate $A(x)$ to find the volume.

Ex:

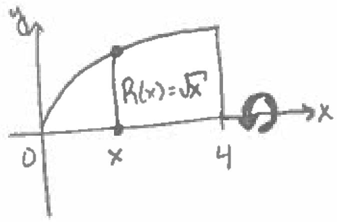


$$A(x) = x^2$$

$$V = \int_0^3 x^2\,dx$$

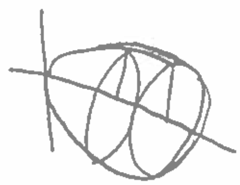
* Disk Method

The solid generated by rotating (or revolving) a plane region about an x-axis in its plane is called a <u>solid of revolution</u>.



The cross-sectional area is the area of a disk of radius $R(x)$

$$A(x) = \pi(\text{radius})^2 = \pi(R(x))^2$$



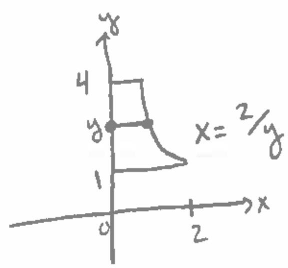
$$V = \int_a^b A(x)\,dx = \int_a^b \pi(R(x))^2\,dx$$

Ex: $V = \int_0^4 \pi(\sqrt{x})^2\,dx = \pi\int_0^4 x\,dx = \pi \frac{x^2}{2}\Big|_0^4 = 8\pi$

Disk method for rotation about y-axis

$$V = \int_c^d \pi(R(y))^2\,dy$$

Ex:

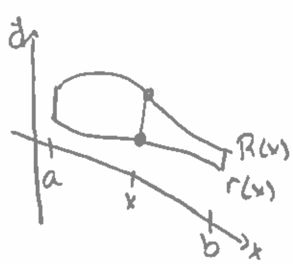


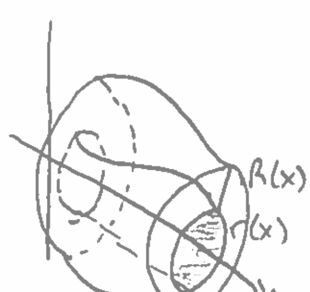
Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = 2/y$, $1 \le y \le 4$, about the y-axis.

$$V = \int_1^4 \pi(2/y)^2\,dy$$

* Washer Method

$$V = \int_a^b \pi\left((R(x))^2 - (r(x))^2\right)dx$$



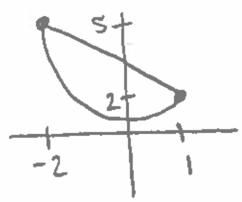


Ex: The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

1) Find limits of intersection by setting the two equal to each other

$$x^2 + 1 = -x + 3$$
$$x^2 + x - 2 = 0$$
$$(x - 1)(x + 2) = 0$$
$$x = 1, -2 \qquad a = -2, \; b = 1$$

2) What is the top or outer radius?
what is the inner radius?



$$R(x) = -x + 3$$
$$r(x) = x^2 + 1$$

3) Find the volume

$$V = \int_{-2}^{1} \pi \left((-x+3)^2 - (x^2+1)^2 \right) dx$$

$$= \pi \int_{-2}^{1} \left(x^2 - 6x + 9 - (x^4 + 2x^2 + 1) \right) dx$$

$$= \pi \int_{-2}^{1} (-x^4 - x^2 - 6x + 8) \, dx$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} - 3x^2 + 8x \right]_{-2}^{1} = \frac{117\pi}{5}$$