Math 141: Section 3.7 Implicit Differentiation - Notes

What to do when there are multiple variables? Most of the functions we have dealt with so far have been described by an equation of the form $y = f(x)$ that expresses y explicitly in terms of the variable x. Another situation occurs when we encounter equations like

$$x^3 + y^3 - 9xy = 0.$$

This equation defines an *implicit* relation between the variables x and y. Sometimes, we can solve such an equation for y and differentiate using the rules we already know. Other times, we use a technique called *implicit differentiation.*

Example 1 Find dy/dx if $y^2 = x$.

Example 2 Find the slope of the circle $x^2 + y^2 = 25$ at the point $(3, -4)$.

Implicit Differentiation Steps:

1) Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

2) Collect the terms with dy/dx on one side of the equation and solve for dy/dx.

Example 3 Implicit differentiation can also be used to find higher order derivatives:

Find d^2y/dx^2 if $2x^3 - 3y^2 = 8$.

Lenses, Tangents, and Normal Lines In the law that describes how light changes direction as it enters a lens, the important angles are the angles the light makes with the line perpendicular to the surface of the lens at the point of entry. This line is called the **normal** to the surface at the point of entry.

In a profile view, the **normal** is the line perpendicular (or **orthogonal**) to the tangent of the profile curve at the point of entry.

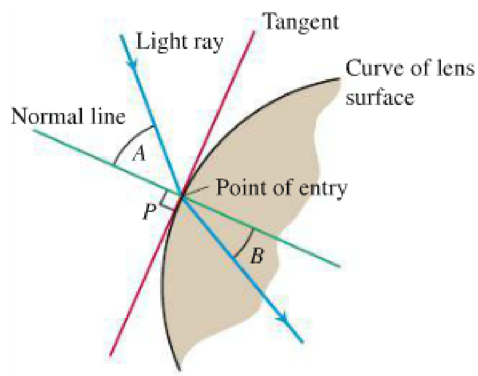

FIGURE 3.32 The profile of a lens, showing the bending (refraction) of a ray of light as it passes through the lens surface.