Math 142: Section 8.4 - Notes

1 Trigonometric Substitution

Motivation If we want to find the area of a circle or ellipse, we have an integral of the form

$$\int \sqrt{a^2 - x^2}dx,\ a > 0.$$

Note that u substitution will not work here:

Workaround: Parametrize! We change x to a function of θ by letting $x = a\sin\theta$ so,

$$\sqrt{a^2 - x^2} =$$

Generally, we use an injective (one-to-one) function (so it has an inverse) to simplify calculations. Above, we ensure $a\sin\theta$ is invertible by restricting the domain to $[-\pi/2, \pi/2]$.

Common Trig Substitutions The following is a summary of when to use each trig substitution.

Integral has	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta, \theta \in [-\pi/2, \pi/2]$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta, \theta \in [-\pi/2, \pi/2]$	$1 - \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta, \theta \in [0, \pi/2)$	$\sec^2\theta - 1 = \tan^2\theta$

Example 1 Evaluate

$$\int \frac{\sqrt{9-x^2}}{x^2} dx$$

Example 2 Find

$$\int \frac{dx}{x^2\sqrt{x^2+4}}$$

Example 3 Evaluate

$$\int \frac{dx}{\sqrt{25x^2 - 4}},\ x > \frac{2}{5}.$$