Math 141: Section 4.8 Antiderivatives - Notes

Definition: A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example 1 Find an antiderivative for each of the following functions:

(a) $f(x) = 2x$

(b) $g(x) = \cos x$

(c) $h(x) = \frac{1}{x} + 2e^{2x}$

Theorem If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Example 2 Find an antiderivative of $f(x) = 3x^2$ that satisfies $F(1) = -1$.

Example 3 Find the general antiderivative of each of the following functions:

(a) $f(x) = x^5$

(b) $g(x) = \frac{1}{\sqrt{x}}$

(c) $h(x) = \sin x$

(d) $k(x) = e^{-3x}$

General Formulas The following table includes a list of general formulas:

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative	Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$	**8.** e^{kx}	$\frac{1}{k}e^{kx} + C$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$	**9.** $\frac{1}{x}$	$\ln\lvert x\rvert + C, \quad x \neq 0$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$	**10.** $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$	**11.** $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$	**12.** $\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, kx > 1$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$	**13.** a^{kx}	$\left(\frac{1}{k\ln a}\right)a^{kx} + C, a > 0, a \neq 1$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$		

Antiderivative Linearity Rules If a function is being multiplied by a constant or combined with another function, the following rules apply:

Example 4 Find the general antiderivative of

$$f(x) = \frac{3}{\sqrt{x}} + \sin 2x$$

Example 5 - Differential Equations A hot-air balloon ascending at the rate of 12 ft/sec is at a height of 80 ft above the ground when a package is dropped. How long does it take the package to reach the ground?

Indefinite Integrals - Definition: The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by

$$\int f(x)dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

We can restate each of the previous examples as finding the indefinite integral. Antiderivatives play a key role in computing limits of certain infinite sums, an unexpected and wonderfully useful role that is described in a central result of Chapter 5, called the *Fundamental Theorem of Calculus.*

Example 6 Evaluate

$$\int (x^2 - 2x + 5)dx.$$