Math 141: Section 2.4 One-Sided Limits - Notes

Approaching a Limit from One Side To have a limit L as x approaches c, a function f must be defined on *both sides* of c and its values $f(x)$ must approach L as x approaches c from either side. Ordinary limits are two-sided!

If f fails to have a two-sided limit at c, it may still have a one-sided limit. If the approach is from the right, the limit is a **right-hand limit**. From the left, it is a **left-hand limit**.

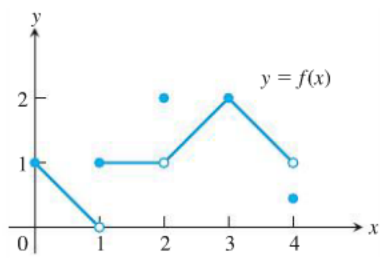
Example 1 Consider the function $f(x) = \dfrac{x}{|x|}$.

Example 2 The domain of $f(x) = \sqrt{4 - x^2}$ is [-2,2]; its graph is the semicircle centered at the origin with radius 2.

Theorem 6 A function $f(x)$ has a limit as x approaches c if and only if it has left-hand and right-hand limits there and these one-sided limits are equal:

$$\lim_{x \to c} f(x) = L \iff \lim_{x \to c^-} f(x) = L \text{ and } \lim_{x \to c^+} f(x) = L.$$

Example 3 Consider the graph of the function:



Example 4 Show that $y = \sin(1/x)$ has no limit as x approaches zero from either side.