Sols

PRACTICE WITH RATES

1. The volume of a tree is given by $V = \frac{1}{12\pi} C^2 h$ where C is the circumference of the tree in meters at ground level and h is the height of the tree in meters. Both C and h are functions of time t in years.

A. Find a formula for $\frac{dV}{dt}$. What does it represent in practical terms?

$$\frac{dV}{dt} = \frac{1}{6\pi} C \frac{dC}{dt} h + \frac{1}{12\pi} C^2 \frac{dh}{dt}$$

$\frac{dV}{dt}$ represents how the volume of the tree is changing with respect to time (t) in years

B. Suppose the circumference grows at a rate of 0.2 meters/year and the height grows at a rate of 4 meters/year. How fast is the volume of the tree growing when the circumference is 5 meters and the height is 22 meters?

$\frac{dC}{dt} = 0.2 \text{ m/yr}$ $\quad C = 5$

$\frac{dh}{dt} = 4 \text{ m/yr}$ $\quad h = 22$

$$\frac{dV}{dt} = \frac{1}{6\pi}(5)(0.2)(22) + \frac{1}{12\pi}(5)^2(4)$$

$$= \frac{11}{3\pi} + \frac{25}{3\pi} = \boxed{\frac{12}{\pi} \text{ m/yr}}$$

2. A. When the radius of a spherical balloon is 10 cm, how fast is the volume of the balloon changing with respect to change in its radius?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$r = 10$

$$\boxed{\frac{dV}{dt} = 400\pi \frac{dr}{dt}}$$

B. If the radius of the balloon is increasing by 0.5 cm/sec, at what rate is the air being blown into the balloon when the radius is 6 cm?

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 0.5$

$r = 6$

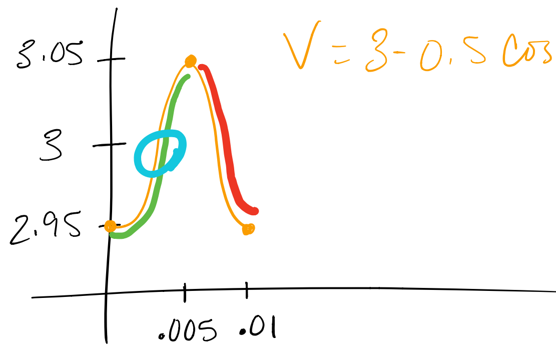
$$\frac{dV}{dt} = 4\pi(6)^2(0.5)$$

$$= \boxed{72\pi \text{ cm/sec}}$$

~~C. When the volume of the balloon is 50 cubic cm, at what rate is the radius of the balloon changing?~~

3. When hyperventilating, a person breathes in and out very rapidly. A spirogram is a machine that draws a graph of the volume of air in a person's lungs as a function of time. During hyperventilation, the person's spirogram trace might be represented by $V = 3 - 0.05\cos(200\pi t)$ where V is the volume of air in liters in the lungs at time t minutes.

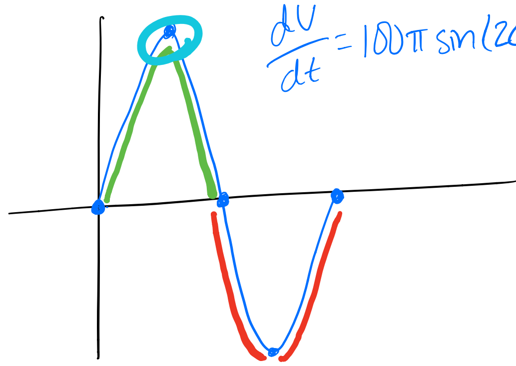
A. Sketch a graph of one period of this function.



$V = 3 - 0.5\cos(200\pi t)$

B. What is the rate of flow of air in liters/minute? Sketch a graph of this function.

$$\frac{dV}{dt} = 0.5\sin(200\pi t)\cdot 200\pi$$
$$= 100\pi\sin(200\pi t)$$



$\frac{dV}{dt} = 100\pi\sin(200\pi t)$

C. Mark the following on each of the graphs above.

i) the interval when the person is breathing in

ii) the interval when the person is breathing out

iii) the time when the rate of flow of air is a maximum when the person is breathing in