Roll two distinguishable dice.

What is the set of outcomes?

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$$

Roll two indistinguishable dice.

Set-builder notation:

Let $B = \{0, 2, 4, 6, 8\}$, B is the set of nonnegative ~~nte~~ even integers less than 10.

$B = \{ n \mid n$ is ~~an~~ a nonnegative even integer less than $10\}$

"such that"

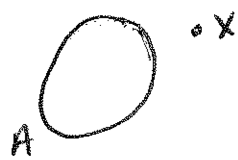
$= \{ n :$ " "$\}$

Venn Diagrams

1) $x \in A$ x is an element of A

2) $x \notin A$ x is not an element of A

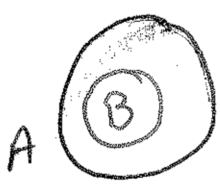
1) 

2) 

3) $B \subseteq A$ B is a subset of A

$B \subset A$ B is a "proper" subset of A

$x \leq 3$

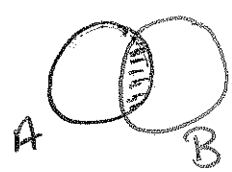


$B \subset A$

4) $B = A$ 

5) Neither A nor B is a subset of the other

disjoint



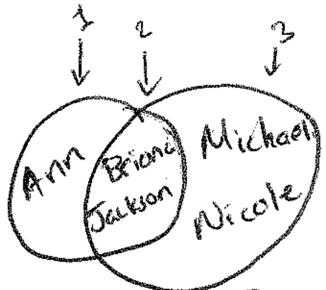
Ex: NobelBooks.com maintains a database of customers and the types of books they purchase. In the database we have this set of customers

$$S = \{\text{Michael}, \text{Jackson}, \text{Briana}, \text{Nicole}, \text{Ann}\}$$

A search for customers who bought cookbooks returns $A = \{\text{Ann}, \text{Jackson}, \text{Briana}\}$.

A search for customers who bought mysteries returns $B = \{\text{Briana}, \text{Michael}, \text{Nicole}, \text{Jackson}\}$

NobelBooks wants to target customers who have bought cookbooks or mysteries or both.



A

B

Set operations

$A \cup B$ - "A union B" is the set of all elements that are either in A or in B or in both

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$A \cup B$

A B

$A \cap B$ is the set of all elements that are in A and B, "A intersect B"

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A \cap B$

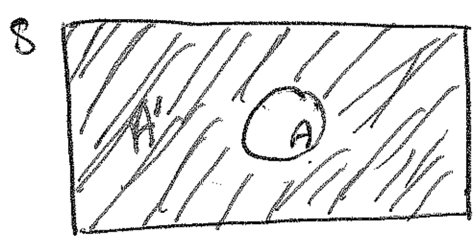
From the NobelBooks example

$A \cup B = \{$Ann, Briana, Jackson, Michael, Nicole$\}$

$A \cap B = \{$Jackson, Briana$\}$

The complement of a set A, is the set of things not in A.

We fix a universal set, S, a set of all objects under discussion.



If S is the universal set and $A \subseteq S$, then A' is the complement of A (in S),

$$\underline{A'} = \{x \in S \mid x \notin A\}$$

Ex: $S = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 3, 5\}$ $B = \{2, 3, 4\}$

$A \cup B = \{1, 3, 5, 2, 4\}$ $A \cap B = \{3\}$ $A' = \{2, 4, 6\}$

1) ⊕ . . . . . . HW, five tries for each

6.1

The <u>Cartesian product</u> of two sets A and B is the set of all ordered pairs (a,b) with $a \in A$ and $b \in B$.

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

<u>Ex</u>: Rolling two distinguishable die, one white one blue.

A = set of outcomes for rolling white die

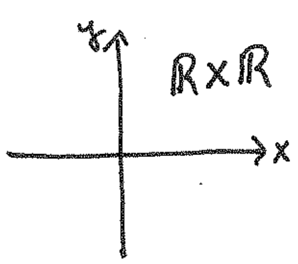
B = " " blue die

$$A \times B = S = \{(1,1), (1,2), \text{etc}\}$$

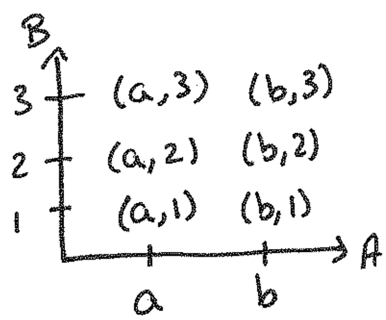
<u>Ex</u>: $A = \{a, b\}$, $B = \{1,2,3\}$

$$A \times B = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$$

Is $(\underset{B}{\underset{\uparrow}{3}}, \underset{A}{\underset{\uparrow}{a}}) \in A \times B$? No, $(3,a) \notin A \times B$.



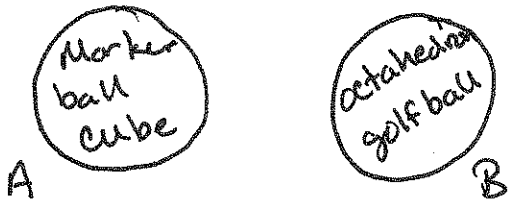
Cartesian plane (Named after Descartes)



Ex: $Y = \{2009, 2010, 2011\}$

$M = \{\text{Acura, Infiniti, Lexus, Mercedes}\}$

	Acura	Infiniti	Lexus	Mercedes
2009	(2009, Acura)			
2010				
2011				



A and B do not have any elements in common so we say A and B are disjoint.

$A \cap B = \emptyset$

$= \{\text{elements in } A \text{ AND in } B\}$

6.2 Cardinality

If A is a finite set, then its cardinality is $n(A) =$ number of elements in A.

Cardinality of $A \cup B$.

$A = \{a, b, c, d\}$ $B = \{c, d, e, f, g\}$

$n(A) = 4$, $n(B) = 5$

$A \cup B = \{a, b, c, d, e, f, g\}$ $n(A \cup B) = 7$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

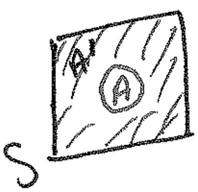
Ex: Given sets A, B, $n(A) = 29$, $n(B) = 17$

$n(A \cup B) = 42$.

Then what is $n(A \cap B)$?

$$42 = 29 + 17 - n(A \cap B)$$
$$42 = 46 - n(A \cap B)$$
$$-46 \quad -46$$
$$-4 = -n(A \cap B)$$
$$4 = n(A \cap B)$$

If S is a finite universal set and A is a subset of S, $A \subseteq S$, then



$$n(A') = n(S) - n(A)$$
$$n(A) = n(S) - n(A')$$

Cardinality of Cartesian Product

Ex: $A = \{H, T\}$ $\quad B = \{1, 2, 3, 4, 5, 6\}$

$n(A) = 2$ $\quad n(B) = 6$

$$A \times B = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$$

$n(A \times B) = 12$

$$n(A \times B) = n(A) \cdot n(B)$$

TEST 1 Monday, Feb. 9

HW 6.3, 6.4
Due Sun. Feb. 8

6.3 Decision Algorithms: The Addition and Multiplication Principles

3 types of house coffee

5 espresso drinks

7 non-coffee options, (tea, fraps)

15 different options = cardinality of disjoint union

Addition Principle

When choosing among r disjoint alternatives, suppose that
alternative 1 has n_1 possible outcomes
" 2 has n_2 " "
⋮
" r has n_r " ",

no 2 outcomes are the same, then there are a total of $n_1 + n_2 + n_3 + \dots + n_r$ possible outcomes.

Must also choose a size for the drink, small, medium or large.

when you order you specify drink and size, (drink, size)

If we let A be possible drinks and B be set of sizes $A \times B$, the cartesian product represents possible coffee orders.

$n(A) = 15 \quad n(B) = 3 \qquad n(A \times B) = n(A) \cdot n(B)$
$= 15 \cdot 3$
$= 45$ possible orders

Multiplication Principle

When making a sequence of choices with r steps, suppose that

step 1 has n_1 possible outcomes

step 2 has n_2 " "

⋮

step r has n_r " "

and each sequence of choices results in a distinct outcome. Then there are

$n_1 \cdot n_2 \cdot \ldots \cdot n_r$ possible outcomes.

Ex: At a restaurant you can choose among 5 appetizers, 34 main dishes, and 10 desserts. Total possible meals $= 5 \cdot 34 \cdot 10 = 1700$ different meals

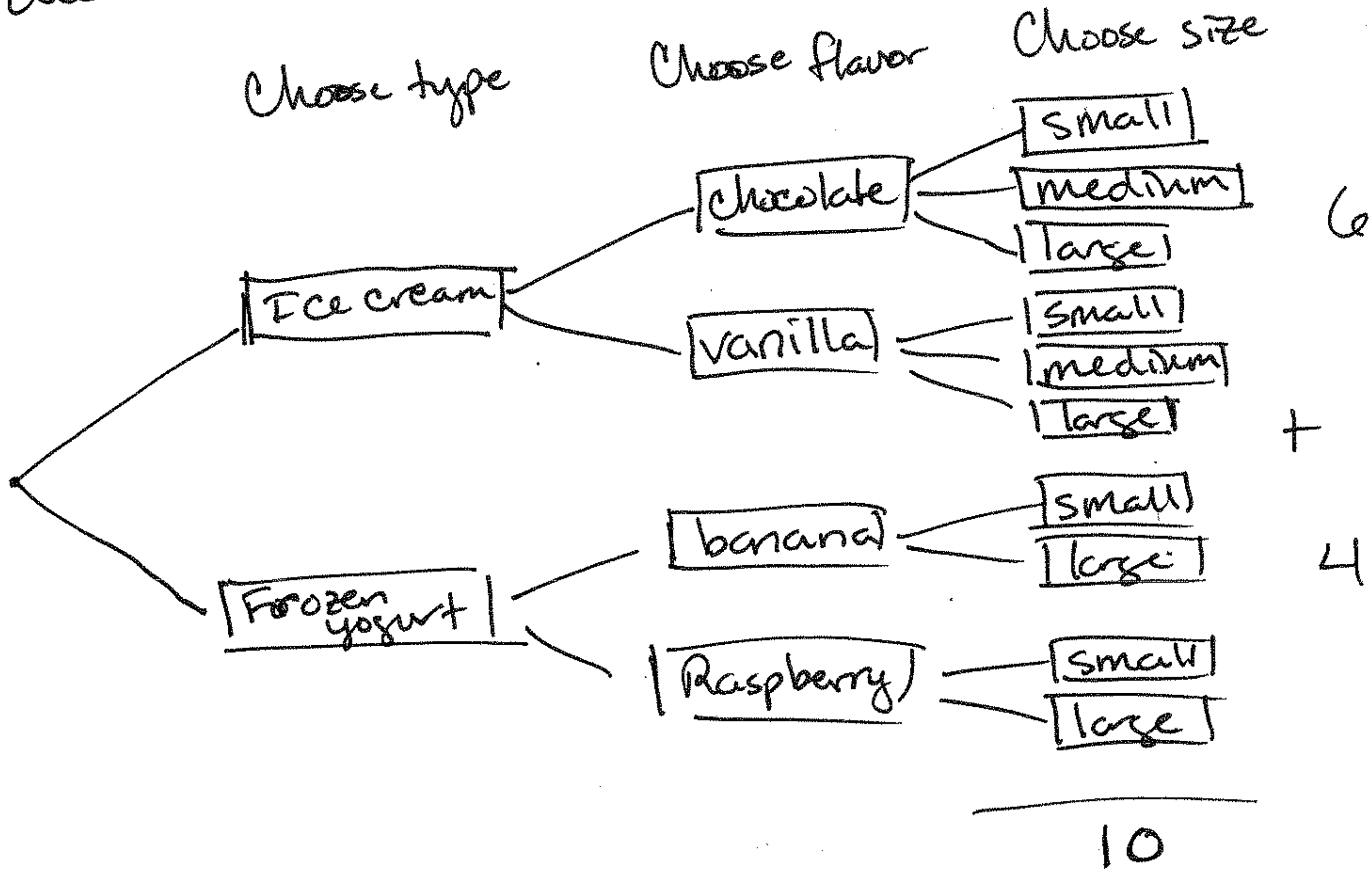
Ex: At an ice cream parlor you can choose between ice cream, of which there are 15 flavors, and frozen yogurt, of which there are 5 flavors. You can then choose 3 different sizes of cones for your ice cream or 2 different sizes of cups for the frozen yogurt.
How many desserts can you choose from?

Alternative 1: ice cream
Step 1 : flavor : 15 choices
Step 2 : cone : 3 choices

Alternative 2: frozen yogurt
Step 1 : flavor : 5 choices
Step 2 : cup : 2 choices

Possible choices for Alternative 1 $= 15 \cdot 3 = 45$
Possible choices for Alternative 2 $= 5 \cdot 2 = 10$
So, there are $45 + 10 = 55$ possible choices of desserts.

We can illustrate decision algorithms with decision trees.



Ex: An exam is broken into two parts, Part A and Part B, both of which you are required to do.
In Part A, you can choose between answering 10 true-false questions or answering 4 multiple-choice questions, each of which has 5 answers to choose from.
In Part B, you can choose between answering 8 true-false questions or 5 multiple choice questions, each of which has 4 answers to choose from.
How many different collections of answers are possible?

Step 1: Do Part A.

Alternative 1: Answer 10 TF questions

Steps 1-10: Choose T or F for each: 2 choices

There are $2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = 2^{10} = 1{,}024$ choices

Alternative 2: Answer 4 multiple-choice

Steps 1-4: Choose one answer for each; 5 choices

There are $5 \cdot 5 \cdot 5 \cdot 5 = 5^4 = 625$ choices

Total choices for Step 1 is $1{,}024 + 625 = \underline{1{,}649}$

Step 2: Do Part B.

Alternative 1: Answer 8 TF questions

Steps 1-8: Choose T or F: 2 choices

There are $2 \cdot 2 \cdot \ldots \cdot 2 = 2^8$ choices $\underline{256}$

Alternative 2: Answer 5 multiple choice

Steps 1-5: Choose one answer: 4 choices

There are $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 = \underline{1{,}024}$ choices

Total choices for Step 2 is $256 + 1024 = \underline{1280}$

How many different collections of answers are possible?

$$1{,}649 \cdot 1{,}280 = \boxed{2{,}110{,}720}$$