6.2: Binomial Probabilities

Binomial Experiments

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- For each trial, the probability of success is the same. We denote the probability of success by p and the probability of failure by q. Because each trial results in either success or failure, $p + q = 1$.

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The central problem of a binomial experiment is to find the probability of r successes out of n trials.

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- Drawing five cards from a standard deck of cards without replacement and recording whether they are red or black.

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- Drawing five cards from a standard deck of cards without replacement and recording whether they are red or black.

This is not a binomial experiment because the probability of success will change with each draw.

Binomial Experiments

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- We can compute $q = 1 - p = 0.7$. Recall that q is the probability of failure.
- We consider each selected teenager a trial. So $n = 10$.
- Since we want to consider the probability that exactly 4 of the selected teenagers will have a part-time job, $r = 4$.

Binomial Probability Distribution Formula

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Formula

In a binomial experiment, the probability of r successes out of n trials is given by the formula

$$\Pr(r) = \frac{n!}{r!(n-r)!} p^r \cdot q^{n-r} = (C_{n,r}) \cdot p^r \cdot q^{n-r}$$

where p is the probability of success in each trial and q is the probability of failure in each trial.

Example

A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. If we select 10 teenagers at random, what is the probability that exactly 4 of them will have part-time jobs?

- In the previous example we found the following values

$$p = 0.3 \qquad q = 0.7$$

$$n = 10 \qquad r = 4$$

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$$n = 10 \qquad r = 4$$

- Using the binomial probability distribution formula

$$\begin{aligned}\Pr(4) &= \frac{10!}{4!(10-4)!}(0.3)^4(0.7)^{10-4} \\ &\approx 0.2\end{aligned}$$

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- We begin by noticing that this is a binomial experiment. Although there are six possible values on the die, we consider landing on a 3 a success and anything else a failure.
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$$\Pr(\text{Ten 3s}) = \frac{20!}{10!(20-10)!}\left(\frac{1}{6}\right)^{10} \cdot \left(\frac{5}{6}\right)^{20-10}$$

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$$\begin{aligned}\Pr(\text{Ten 3s}) &= \frac{20!}{10!(20-10)!}\left(\frac{1}{6}\right)^{10}\cdot\left(\frac{5}{6}\right)^{20-10} \\ &\approx 0.00049\end{aligned}$$

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If we roll a single die 20 times, how many times can we expect 3 to roll?

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Example

If we roll a single die 20 times, how many times can we expect 3 to roll?

- Using the binomial experiment formula for μ, we can expect the number of 3s rolled to be

$$\mu = 20 \cdot \left(\frac{1}{6}\right) = 3.\overline{3}$$

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In a binomial experiment

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If we roll a single die 20 times, how many times can we expect 3 to roll? Find the standard deviation for the number of 3s rolled.

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If we roll a single die 20 times, how many times can we expect 3 to roll? Find the standard deviation for the number of 3s rolled.

- Using the binomial experiment formula for σ, find the standard deviation for the number of 3s rolled to be

$$\sigma = \sqrt{20 \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)} = 1.\overline{6}$$