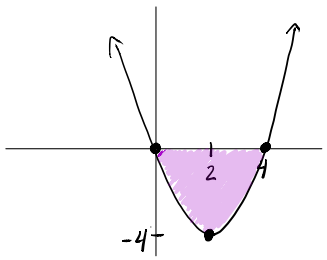
Sols

Find the exact area bounded by the functions f and g. Include a sketch of the graphs with the region shaded. Use algebra to find any intersections of graphs.

1. $f(x) = x^2 - 4x$ and $g(x) = 0$.

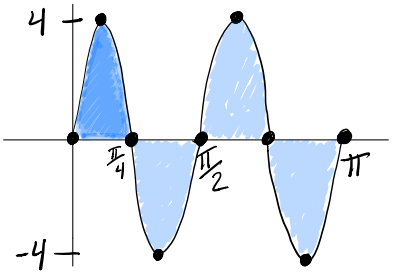


$$\left| \int_0^4 x^2 - 4x \, dx \right| = \left| \left[\frac{x^3}{3} - 2x^2 \right]_0^4 \right|$$

$$= \left| \left(4^3/3 - 2(4)^2 \right) - 0 \right|$$

$$= \boxed{32/3}$$

2. $f(x) = 4\sin(4x)$ and $g(x) = 0$. on $[0, \pi]$



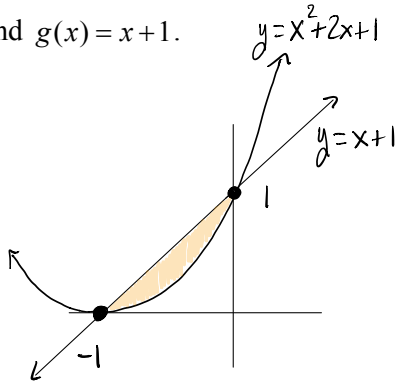
By symmetry,

$$A = 4 \cdot \int_0^{\pi/4} 4\sin(4x)\,dx = 16\left[\frac{1}{4}(-\cos(4x)) \right]_0^{\pi/4}$$

$$= 4\left(-\cos(\pi) + \cos(0) \right)$$

$$= 4(1+1) = \boxed{8}$$

3. $f(x) = x^2 + 2x + 1$ and $g(x) = x + 1$.



$$x^2 + 2x + 1 = x + 1$$

$$x^2 + x = 0$$

$$x(x+1) = 0$$

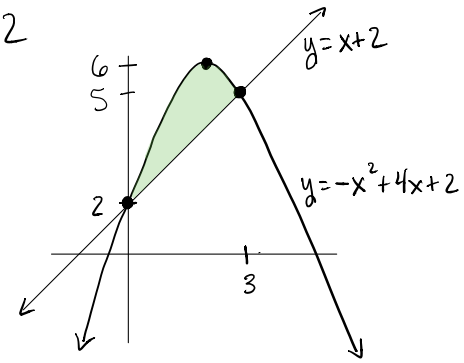
$$x = -1, \; x = 0$$

$$A = \int_{-1}^{0} (x+1) - (x^2+2x+1)\,dx$$

$$= \int_{-1}^{0} (-x^2 - x)\,dx = \left[-\frac{x^3}{3} - \frac{x^2}{2} \right]_{-1}^{0}$$

$$= 0 - (1/3 - 1/2) = \boxed{1/6}$$

4. $f(x) = -x^2 + 4x + 2$ and $g(x) = x + 2$.



$$-x^2 + 4x + 2 = x + 2$$

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$$x = 0, \; x = 3$$

$$A = \int_0^3 (-x^2+4x+2) - (x+2)\,dx$$

$$= \int_0^3 (-x^2 + 3x)\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3 = \left(-\frac{3^3}{3} + \frac{3(3)^2}{2} - 0 \right)$$

$$= -9 + 27/2 = \boxed{9/2}$$