$Sol's$

MATH 122: IDENTIFYING INTEGRAL SUBSTITUTIONS

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. Goal: To identify what (if any) u-substitutions are necessary to compute an integral and to practice making such substitutions.

For each problem, identify what (if any) u-substitution(s) need to be made to evaluate each integral. Make the substitution and simplify, but **do not** evaluate the integral.

1: $\int x \sin(x^2) dx$

$$u = x^2 \qquad \frac{1}{2}\int \sin(u) du$$

$$du = 2x\, dx$$

2: $\int \sqrt{x}(x+3) dx$

Simplify and use "anti"-power rule

$$\int (x^{3/2} + 3x^{1/2})\, dx$$

3: $\int \frac{\sqrt{\ln(x)}}{x} dx$

$$= \int \sqrt{\ln x} \cdot \frac{1}{x}\, dx$$

$$u = \ln x$$
$$du = \frac{1}{x}\, dx \qquad \int \sqrt{u}\, du = \int u^{1/2} du$$

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4: $\int \frac{x+4}{x} dx$

Simplify $\int (1 + 4/x)\, dx$

Typo   5: $\int \frac{\not{x} e^{2x}}{\sqrt{1-e^{2x}}} dx$

$u = 1 - e^{2x}$ $-\frac{1}{2} \int \frac{1}{\sqrt{u}} du = -\frac{1}{2} \int u^{-1/2} du$
$du = -2e^{2x} dx$

6: $\int \frac{3x^2}{\sqrt{1-x^3}} dx$

$u = 1 - x^3$ $-\int \frac{1}{\sqrt{u}} du = -\int u^{-1/2} du$
$du = -3x^2 dx$

7: $\int (4 - 2x)^3 dx$

Could FOIL but easier to use u-sub!

$u = 4 - 2x$ $du = -2dx$ $-\frac{1}{2} \int u^3 du$

8: $\int 4x \tan(x^2) dx$

$u = x^2$ $\frac{1}{2} \int 4 \tan(u) du = \int 2 \tan(u) du$
$du = 2x dx$

9: $\int \frac{dx}{x(\ln x)^2} = \int \frac{1}{x} \cdot \frac{1}{(\ln x)^2} dx$

$u = \ln x$ $\int \frac{1}{u^2} du = \int u^{-2} du$
$du = \frac{1}{x} dx$

10: $\int x 4^{x^2} dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int 4^u\,du$

11: Show the following two integrals are equivalent by identifying the correct u-substitution:

$$\int 3x\sqrt{9+x^2}\,dx = \int \frac{3\sqrt{u}}{2}\,du$$

$u = 9 + x^2$

$du = 2x\,dx$

$\frac{1}{2}\int 3\sqrt{u}\,du$

$= \int \frac{3\sqrt{u}}{2}\,du$