Math 141: Section 1.1 Functions and Their Graphs - Notes

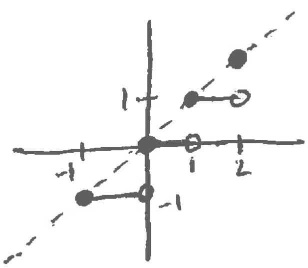
1 Functions; Domain and Range

Natural Domain The **natural domain** is the largest set of real x-values for which a formula gives real y-values. The **domain** of a function is assumed to be the natural domain; if we want to restrict the domain in a certain way, we must say so.

Example 1 Let

$$f(x) = \lfloor x \rfloor.$$ "floor function"

State the domain and the range.



$f(2.75) = 2$

$f(1) = 1$

$f(\pi) = 3$

$D = (-\infty, \infty)$

$Y = \mathbb{Z}$ "The set of all integers"

Example 2 Let

$$f(x) = \sqrt{1 - x^2}.$$

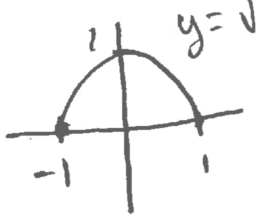
State the domain and range.

$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$



$h(x) = \sqrt{x}$ D $\{x \mid x \geq 0\}$

$[0, \infty)$

$1 - x^2 \geq 0$

$1 \geq x^2$

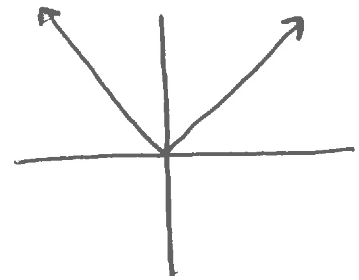
$x^2 \leq 1$

D of $f(x) = \sqrt{1-x^2}$ $[-1, 1]$ Y $[0, 1]$

Piecewise-Defined Functions

Example 3

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

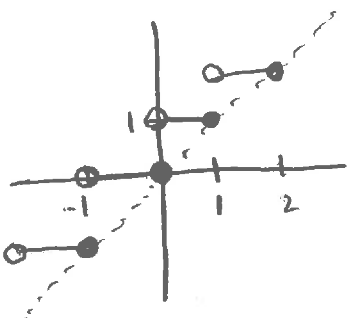


D $(-\infty, \infty)$

Y $[0, \infty)$

Example 4 The **least integer (or ceiling) function** denoted:

$$f(x) = \lceil x \rceil$$



"The smallest integer greater than or equal to x"

D $(-\infty, \infty)$

Y $\mathbb{Z}$

Increasing and Decreasing Functions

Definition : Let f be a function defined on an interval I and x_1, x_2 be two points on I.

1) If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is <u>increasing</u>

2) If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is decreasing

Even and Odd Functions: Symmetry

Definition : A function $y = f(x)$ is an even function of x if

$f(-x) = f(x)$,

odd function of x if

$f(-x) = -f(x)$,

for all x in the domain.

Even: Symmetry wrt y-axis

Odd: Symmetry wrt origin

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$ even

$f(x) = x$

$f(-x) = -x = -f(x)$ odd

$f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

$g(x) = x^4 - 3x$

$g(-x) = (-x)^4 - 3(-x)$

$= x^4 + 3x$ Neither

Examples:

Common Functions

Linear Functions A function of the form $f(x) = mx + b$, for constants m and b, is called a **linear function**. The function $f(x) = x$ where $m = 1$ and $b = 0$ is called the identity.

Definition: Two variables y and x are **proportional** if one is always a constant multiple of the other; that is, if $y = kx$ for some nonzero constant k. If the variable y is proportional to the reciprocal $1/x$, then we say y is **inversely proportional** to x.

Example The variables r and s are inversely proportional, and $r = 6$ when $s = 4$. Determine s when $r = 10$.

$r = k \cdot \frac{1}{s}$

$6 = k \cdot \frac{1}{4}$

$\boxed{k = 24}$

$r = 24 \cdot \frac{1}{s}$

$r = \frac{24}{s}$

$10 = \frac{24}{s}$

$10s = 24$

$\boxed{s = 2.4}$

Power Functions

$$f(x) = x^a,$$

where a is a constant.

1) $a = n$, n is a positive integer

$f(x) = x^2$

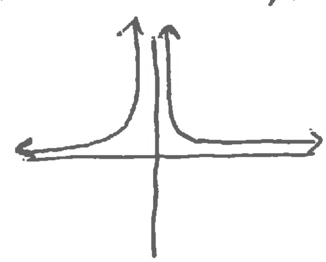
$f(x) = x^3$

2) $a = -1$

$f(x) = x^{-1} = 1/x$

$a = -2$

$f(x) = x^{-2} = \dfrac{1}{x^2}$



3) $a = 1/2,\ 1/3,\ 2/3,\ 3/2$

$f(x) = x^{1/2} = \sqrt{x}$

$f(x) = x^{1/3} = \sqrt[3]{x}$

$f(x) = x^{2/3} = \sqrt[3]{x^2}$
$= (\sqrt[3]{x})^2$

$f(x) = x^{3/2} = \sqrt{x^3} = (\sqrt{x})^3$

Polynomials

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are real constants.

D $(-\infty, \infty)$

If $a_n \neq 0$, then n is the degree of the polynomial

$f(x) = (x-2)^2(x+1)$

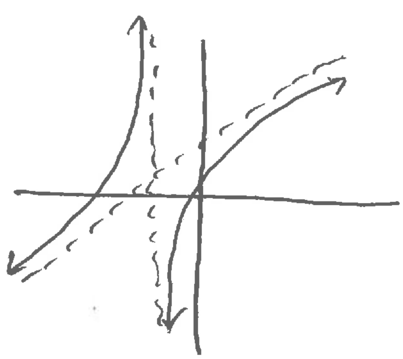
degree: 3

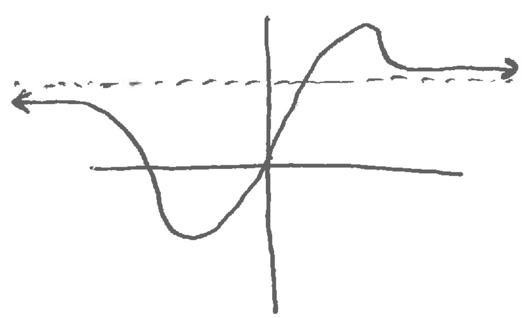
Rational Functions

$$f(x) = \frac{p(x)}{q(x)},$$

where p and q are polynomials.

Domain: All values of x such that $q(x) \neq 0$



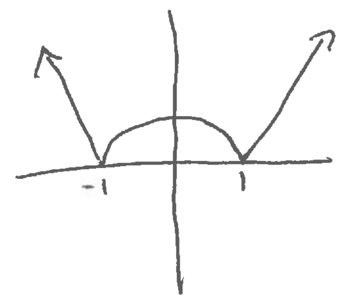


Algebraic Functions Any function constructed from polynomials using algebraic operations.

Desmos (app)

$f(x) = x^{1/3}(x-4)$

$f(x) = \frac{3}{4}(x^2-1)^{2/3}$ —



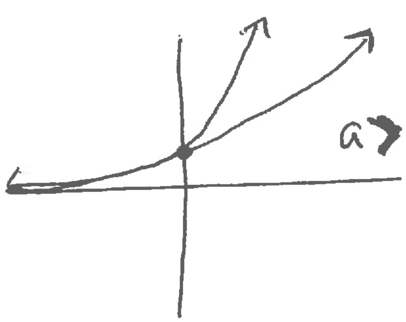
*Trigonometric Functions** We will come back to these in Section 1.3.

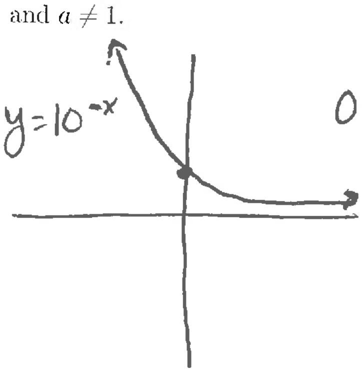
Exponential Functions

$$f(x) = a^x,$$

$f(x) = a^x$, $a > 0$, $a \neq 1$

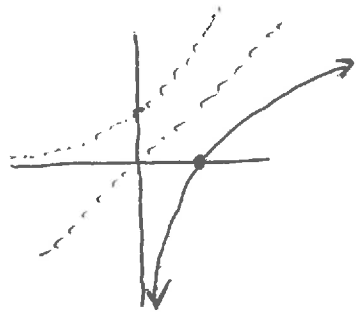
where the base $a > 0$ is a positive constant and $a \neq 1$.





Logarithmic Functions

$f(x) = \log_a x \quad , \quad a \neq 1$



$D: (0, \infty)$