Math 141: Section 2.1 Rates of Change and Tangents to Curves - Notes

Average and Instantaneous Speed If y denotes the distance fallen in feet after t seconds, then Galileo's law is

$$y = 16t^2$$

, where 16 is the (approximate) constant of proportionality. (If y is measured in meters, the constant is 4.9).
A moving object's **average speed** during an interval of time is found by dividing the distance covered by the time elapsed.

Example 1 A rock breaks loose from the top of a tall cliff. What is its average speed
(a) during the first 2 sec of fall?
(b) during the 1-second interval between second 1 and second 2?

Example 2 Find the speed of the falling rock in Example 1 at t=1 sec.

The average speed on the interval starting at $t_0 = 1$ seems to approach a limiting value of 32 as the length of the interval decreases. This suggests that the rock is falling at a speed of 32 ft/sec at $t_0 = 1$ sec. We can confirm this algebraically:

Average Rate of Change The **average rate of change** of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} =$$

Slope of a Curve To define tangency for general curves, we need an approach that takes into account the behavior of the secants through P and nearby points Q as Q moves toward P along the curve.

1) Start with what we can calculate, namely, the slope of the secant PQ.
2) Investigate the limiting value of the secant slope as Q approaches P along the curve.
3) If the *limit* exists, take it to be the slope of the curve at P and *define* the tangent to the curve P to be the line through P with this slope.

Example 3 Find the slope of the parabola $y = x^2$ at the point $P(2, 4)$. Write an equation for the tangent to the parabola at this point.

Instantaneous Rate of Change and Tangent Lines The rates at which the rock in Example 2 was falling at the instant $t = 1$ is called the **instantaneous rate of change**. Instantaneous rates and slopes of tangent lines are closely connected.

The instantaneous rate is the value the average rate approaches as the length h of the interval over which the change occurs approaches zero.

The average rate of change corresponds to the slope of a secant line.

The instantaneous rate of change corresponds to the slope of the tangent line as the independent variable approaches a fixed value.