Math 141: Section 4.1 Extreme Values of Functions - Notes

Definition: Let f be a function with domain D. Then f has an **absolute (global) maximum** value on D at a point c if

$$f(x) \leq f(c) \text{ for all } x \text{ in } D$$

and an **absolute (global) minimum** value on D at c if

$$f(x) \geq f(c) \text{ for all } x \text{ in } D.$$

Example 1 Consider the function $y = x^2$ on the domains $(-\infty, \infty)$, $[0, 2]$, $(0, 2]$, and $(0, 2)$.

Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a, b]$. That is, there are numbers x_1 and x_2 in $[a, b]$ with $f(x_1) = m$, $f(x_2) = M$, and $m \leq f(x) \leq M$ for every other x in $[a, b]$.

Local Extreme Values; Definition A function f has a **local maximum** value at a point c within its domain D if $f(x) \leq f(c)$ for all $x \in D$ lying in some open interval containing c.
A function f has a **local minimum** value at a point c within its domain D if $f(x) \geq f(c)$ for all $x \in D$ lying in some open interval containing c.

Example 2 Consider the following graph:

The First Derivative Theorem for Local Extreme Values If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then

$$f'(c) = 0.$$

Definition: An interior point of the domain of a function f where f' is zero or undefined is a **critical point** of f.

How to Find the Absolute Extrema of a Continuous Function f on a Finite Closed Interval

1) Evaluate f at all critical points and endpoints.
2) Take the largest and smallest of these values.

Example 3 Find the absolute maximum and minimum values of

$$f(x) = 10x(2 - \ln x)$$

on the interval $[1, e^2]$.