Math 141: Section 1.6 Inverse Functions and Logarithms - Notes

Inverse Functions Suppose that f is a one-to-one function on a domain D with range R. The **inverse function** f^{-1} is defined by

$$f^{-1}(b) = a \text{ if } f(a) = b.$$

The domain of f^{-1} is R and the range of f^{-1} is D.
The graph of $y = f^{-1}$ is obtained by reflecting the graph of $y = f(x)$ about the line $y = x$.

Given a function $y = f(x)$ we can find it's inverse by taking two steps:
Step 1: Interchange x and y.
Step 2: Solve for the new y.

Example 1 Find the inverse of the function $y = x^2$, $x \geq 0$. expressed as a function of x.

$y = x^2$

Step1: $x = y^2$

Step2: $\sqrt{x} = y$

Domain of $f(x) = x^2$, $[0, \infty)$

Range of $f^{-1}(x) = \sqrt{x}$, $[0, \infty)$

Logarithmic Functions If a is any positive real number other than 1, the base a exponential function $f(x) = a^x$ is one-to-one. It therefore has an inverse. Its inverse is called the *logarithmic function with base a*.
The **logarithm function with base a,** $y = \log_a(x)$, is the inverse of the base a exponential function $y = a^x$, $(a > 0, a \neq 1)$.

If $f(x) = a^x$, $a > 0$, $a \neq 1$

then $f^{-1}(x) = \log_a x$

- $e^y = x \overset{\text{if and only if}}{\Longleftrightarrow} \ln x = y$
- $\ln(e) = 1$

Properties of Logarithms For any numbers $b > 0$ and $x > 0$, the base a logarithm satisfies the following rules:

1) $\log_a(bx) = \log_a(b) + \log_a(x)$

2) $\log_a\left(\frac{b}{x}\right) = \log_a(b) - \log_a(x)$

3) $\log_a\left(\frac{1}{x}\right) = \log_a(1) - \log_a(x) = -\log_a(x)$

4) $\log_a(x^r) = r \cdot \log_a x$ ($10 = 5(1.13)^t$ use log properties to solve.)

5) $\log_a(a^x) = x$ 6) $a^{\log_a x} = x$

We can rewrite any exponential function as a power of the natural exponential function:

$$a^x = e^{x \ln a}$$

That is, a^x is the same as e^x raised to the power $\ln a$: $a^x = e^{kx}$, where $k = \ln a$.

Inverse Trigonometric Functions Recall the six basic trig functions reviewed in Section 1.3. These functions were not one-to-one but we can restrict their domains to intervals on which they are one-to-one:

Function	Domain	Range
$y = \sin x$	$[-\pi/2, \pi/2]$	$[-1, 1]$
$y = \cos x$	$[0, \pi]$	$[-1, 1]$
$y = \tan x$	$(-\pi/2, \pi/2)$	$(-\infty, \infty)$
$y = \cot x$	$(0, \pi)$	$(-\infty, \infty)$
$y = \sec x$	$[0, \pi/2) \cup (\pi/2, \pi]$	$(-\infty, -1] \cup [1, \infty)$
$y = \csc x$	$[-\pi/2, 0) \cup (0, \pi/2]$	$(-\infty, -1] \cup [1, \infty)$

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Since these functions are now one-to-one, they have inverses denoted by:

$y = \sin^{-1} x = \arcsin x$

$y = \cos^{-1} x = \arccos x$

$y = \tan^{-1} x = \arctan x$

$y = \cot^{-1} x = \operatorname{arccot} x$

$y = \sec^{-1} x = \operatorname{arcsec} x$

$y = \csc^{-1} x = \operatorname{arccsc} x$

The Arcsine and Arccosine Functions $y = \sin^{-1} x$ is the number in $[-\pi/2, \pi/2]$ for which $\sin y = x$

$y = \cos^{-1} x$ is the number in $[0, \pi]$ for which $\cos y = x$. The graph of $y = \sin^{-1} x$ is symmetric about the origin and is hence an odd function. The graph of $y = \cos^{-1} x$ has no such symmetry.