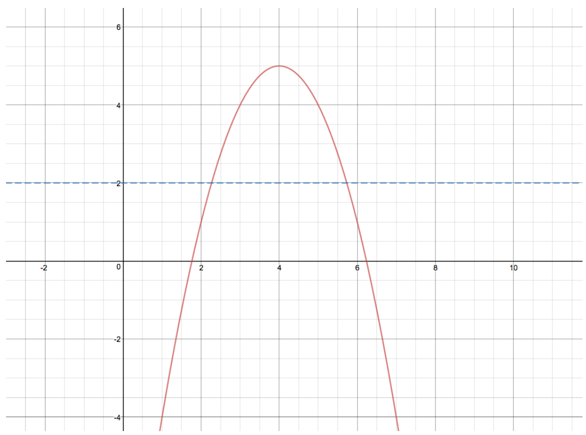
Math 141: Section 4.2 The Mean Value Theorem - Notes

Rolle's Theorem

Consider the following graph:



Rolle's Theorem Suppose that $y = f(x)$ is continuous over the closed interval $[a, b]$ and differentiable at every point of its interior (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) at which $f'(c) = 0$.

Example 1 Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution.

The Mean Value Theorem Suppose $y = f(x)$ is continuous over a closed interval $[a, b]$ and differentiable on the interval's interior, (a, b). Then there is at least one point c in (a, b) at which

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Example 2 If a car accelerating from zero takes 8 sec to go 352 ft, its average velocity for the 8-sec interval is 352/8=44 ft/sec. The Mean Value Theorem says that at some point during the acceleration the speedometer must read exactly 30 mph (44 ft/sec).

Corollary 1 If $f'(x) = 0$ at each point x of an open interval (a, b), then $f(x) = C$ for all $x \in (a, b)$, where C is a constant.

Corollary 2 If $f'(x) = g'(x)$ at each point x in an open interval (a, b), then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a, b)$. That is, $f - g$ is a constant function on (a, b).

Example 3 Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0, 2)$.