Math 170: Section 7.2, 7.3 Lecture

Section 7.2

Definition: When an experiment is performed a number of times, the ____________________________ or ____________________________ of an event E is the fraction of times that the event E occurs. If the experiment is performed N times and the event E occurs $fr(E)$ times, then the relative frequency is given by

$$P(E) = \frac{fr(E)}{N}.$$

The number $fr(E)$ is called the ____________________________ of E. N, the number of times that the experiment is performed, is called the number of **trials** or the **sample size**. If E consists of a single outcomes s, then we refer to $P(E)$ as the relative frequency or estimated probability of the outcomes s, and we write $P(s)$.
The collection of the estimated probabilities of *all* the outcomes is the **relative frequency distribution** or **estimated probability distribution**.

Example 1 In a survey of 250 hybrid vehicles sold in the United States, 125 were Toyota Prii, 30 were Honda Civics, 20 were Toyota Camrys, 15 were Ford Escapes, and the rest were other makes. What is the relative frequency that a hybrid vehicle sold in the United States is not a Toyota Camry?

Bid Price	\$0-\$9.99	\$10-\$49.99	\$50-\$99.99	$\geq$\$100
Relative Frequency	6	23	15	6

Example 2 The above chart shows the results of a survey of the bid prices for 50 paintings on eBay with the highest number of bids.

Consider the experiment in which a painting is chosen and the bid price is observed.
(a) Find the relative frequency distribution.
(b) Find the relative frequency that a painting in the survey had a bid price of less than \$50.

Some Properties of Relative Frequency Distribution Let $S = \{s_1, s_2, \ldots, s_n\}$ be a sample space and let $P(s_i)$ be the relative frequency of the event $\{s_i\}$. Then

1. $0 \le P(s_i) \le 1$

2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$

3. If $E = \{e_1, e_2, \ldots, e_r\}$ then $P(E) = P(e_1) + P(e_2) + \cdots + P(e_r)$.

In words:

1. The relative frequency of each outcome is a number between 0 and 1 (inclusive).

2. The relative frequencies of all the outcomes add up to 1.

3. The relative frequency of an event E is the sum of the relative frequencies of the individual outcomes of E.

Relative Frequency and Increasing Sample Size A "fair" coin is one that is as likely to come up heads as it is to come up tails. In other words, we expect heads to come up 50% of the time if we toss such a coin many times. Put more precisely, we expect the relative frequency to approach .5 as the number of trials gets larger. Let's graph the behavior of the relative frequency for a sequence of coin tosses. For each N we will plot what fraction of times the coin comes up heads in the first N tosses.

Outcome	1	2	3	4	5	6
Probability	.3	.3		.1	.2	

Table 1: Example 3

Section 7.3: Probability and Probability Models

Probability Distribution; Probability

(Compare with the properties of relative frequency.)

A (finite) ______________________________ is an assignment of a number $P(s_i)$, the ______________________________, to each outcome of a finite sample space $S = \{s_1, s_2, \ldots, s_n\}$. The probabilities must satisfy

1. $0 \leq P(s_i) \leq 1$

2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$.

We find the ____, written $P(E)$, by adding up the probabilities of the outcomes in E.

If $P(E) = 0$, we call E an ______________________________. The empty event $\emptyset$ is always impossible, since *something* must happen.

Examples

1. Let us take $S = \{H, T\}$ and make the assignments $P(H) = .5$ and $P(T) =$.

2. We can instead make the assignments $P(H) = .2$ and $P(T) =$.

3. The table at the top of this page gives a probability distribution for the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

It follows that $P(\{1, 6\}) =$

$P(\{2, 3\}) =$

$P(3) =$

Probability Models

A ______________________________ for a particular experiment is a probability distribution that predicts the relative frequency of each outcome if the experiment is performed a ______________________________ number of times. Just as we think of relative frequency as *estimated probability*, we can think of modeled probability as ____.

Examples

1. Fair Coin Model Flip a fair coin and observe the side that faces up. Because we expect that heads is as likely to come up as tails, we model this experiment with the probability distribution specified by:

2. Unfair Coin Model

3. Fair Die Model Roll a fair die and observe the number that faces up. Because we expect to roll each specific number one sixth of the time, we model the experiment with the probability distribution specified by:

4. Roll a pair of fair distinguishable dice

5. In the experiment in Example 4, take E to be the event that the sum of the numbers that face up is 5, so $E = \{(1,4),(2,3),(3,2),(4,1)\}$. By properties of probability distributions,

$$P(E) =$$

Example 1 (Compare to Example 1 in Section 7.2) A total of 1.9 million hybrid vehicles had been sold in the United States through October of 2011. Of these, 955,000 were Toyota Prii, 205,000 were Honda Civics, 170,000 were Toyota Camrys, 105,000 were Ford Escapes, and the rest were other makes.

(a) What is the probability that a randomly selected hybrid vehicle sold in the United States was either a Toyota Prius or a Honda Civic?

(b) What is the probability that a randomly selected hybrid vehicle sold in the United States was not a Toyota Camry?

Example 2 Recall that the sample space when rolling a pair of indistinguishable dice is

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,3),(3,4),(3,5),(3,6),(4,4),(4,5),(4,6),(5,5),(5,6),(6,6)\}$$

Construct a probability model for this experiment.

Example 3 In order to impress your friends with you die-rolling skills, you have surreptitiously weighted your die in such a way that 6 is three times as likely to come up as any one of the other numbers. (All the other outcomes are equally likely.) Obtain a probability distribution for a roll of the die and use it to calculate the probability of an even number coming up.

Addition Principle Consider Example 3 on page 4.

If A and B are any two events, then

$$P(A \cup B) =$$

Example 4 A survey conducted by the Bureau of Labor Statistics found that 68% of the high school graduating class of 2010 went on to college the following year, while 42% of the class was working. Furthermore, 92% were either in college or working, or both.
(a) What percentage went on to college and work at the same time?
(b) What percentage went on to college but not work?

Principles of Probability Distributions The following rules hold for any sample space S and any event A:

$$P(S) =$$

$$P(\emptyset) =$$

$$P(A') =$$

Example 5 A home loan is either current, 30-59 days past due, 60-89 days past due, 90 or more days past due, in foreclosure, or repossessed by the lender. In November 2008, the probability that a randomly selected subprime home mortgage in California was not current was .51. The probability that a mortgage was not current, but neither in foreclosure nor repossessed, was .28. Calculate the probabilities of the following events.
(a) A California home mortgage was current.
(b) A California home mortgage was in foreclosure or repossessed.

Example 6 According to a *New York Times*/CBS poll, 45% agreed that Social Security taxes should be raised if necessary to keep the system afloat, and 35% agreed that it would be a good idea to invest part of their Social Security taxes on their own.

(a) What is the largest percentage of people who could have agreed with at least one of these statements?

(b) What is the smallest percentage of people who could have agreed with at least one of these statements?