Math 141: Section 4.3 Monotonic Functions and the First Derivative Test - Notes

Increasing and Decreasing Functions As another corollary to the Mean Value Theorem, we can show that function with positive derivatives are increasing functions and functions with negative derivatives are decreasing functions.

Definition: A function that is increasing or decreasing on an interval is said to be **monotonic** on the interval.

Corollary 3: Suppose that f is continuous on $[a.b]$ and differentiable on (a, b).
If $f'(x) > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.
If $f'(x) < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Example 1 Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the open intervals on which f is increasing and on which f is decreasing.

First Derivative Test for Local Extrema

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across the interval from left to right,
1. If f' changes from negative to positive at c, then f has a local minimum at $x = c$;
2. If f' changes from positive to negative at c, then f has a local maximum at $x = c$;
3. If f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local extremum at $x = c$.

Example 2 Find the critical points of

$$f(x) = x^{1/3}(x - 4).$$

Identify the open intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.