Solutions

Math 122: Integration by Substitution Practice

For each problem, identify what (if any) u-substitution needs to be made to evaluate each integral. Make the substitution, simplify, evaluate the integral, and, for indefinite integrals, remember to write your answer in terms of the original variable.

1. $\int x^2 e^{x^3+1} dx$ $= \frac{1}{3}\int e^u du$

   $u = x^3+1$ $= \frac{1}{3}e^u + C$

   $du = 3x^2 dx$ $= \boxed{\frac{1}{3}e^{x^3+1} + C}$

2. $\int 2x(x^2+1)^5 dx$

   $u = x^2+1$ $\int u^5 du = \frac{u^6}{6} + C = \boxed{\frac{(x^2+1)^6}{6} + C}$

   $du = 2x dx$

3. $\int (5x+1)^9 dx$ $= \frac{1}{5}\int u^9 du = \frac{1}{5}\cdot\frac{u^{10}}{10} + C$

   $u = 5x+1$ $= \frac{u^{10}}{50} + C = \boxed{\frac{(5x+1)^{10}}{10} + C}$

   $du = 5dx$

4. $\int \frac{t}{1+3t^2} dt$ $= \frac{1}{6}\int \frac{1}{u} du = \frac{1}{6}\ln|u| + C$

   $u = 1+3t^2$ $= \boxed{\frac{1}{6}\ln|1+3t^2| + C}$

   $du = 6t dt$

5. $\int x\sqrt{3x^2+4} dx$ $= \frac{1}{6}\int \sqrt{u} du = \frac{1}{6}\int u^{1/2} du$

   $u = 3x^2+4$ $= \frac{1}{6}\cdot\frac{u^{3/2}}{3/2} + C$

   $du = 6x dx$ $= \frac{1}{9}u^{3/2} + C = \boxed{\frac{1}{9}(3x^2+4)^{3/2} + C}$

6. $\int \frac{x+1}{x^2+2x+9}dx$ $= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c$

$u = x^2+2x+9$ $= \boxed{\frac{1}{2}\ln|x^2+2x+9| + C}$

$du = (2x+2)dx$
$= 2(x+1)dx$

7. $\int \frac{e^t+1}{e^t+t}dt$ $= \int \frac{1}{u}\,du = \ln|u| + c$

$u = e^t + t$

$du = (e^t+1)dt$ $= \boxed{\ln|e^t+t| + C}$

8. $\int \frac{1}{\sqrt{t+1}}dt$ $= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$

$u = t+1$

$du = dt$ $= \frac{u^{1/2}}{1/2} + c = 2u^{1/2} + c$
$= \boxed{2(t+1)^{1/2} + c}$

9. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$ $= 2\int e^u\,du = 2e^u + c$
$= \boxed{2e^{\sqrt{x}} + c}$

10. $\int \frac{2x}{x^2+1}dx$

$u = x^2+1$ $= \int \frac{1}{u}\,du$

$du = 2x\,dx$ $= \ln|u| + c = \boxed{\ln|x^2+1| + c}$

11. $\int_0^2 x(x^2+1)^2dx$ $= \frac{1}{2}\int_{\square}^{\square} u^2 du = \frac{1}{2}\cdot\frac{u^3}{3}\Big|_{\square}^{\square} = \frac{(x^2+1)}{3}\Big|_0^2$

$u = x^2+1$

$du = 2x\,dx$

$= \frac{(2^2+1)}{3} - \frac{(0^2+1)}{3} = \boxed{4/3}$

12. $\int_0^1 2te^{-t^2}dt$ $-\int_0^{-1} e^u du = -e^u\Big|_0^{-1} = -e^{-1} - -e^0$

$u = -t^2$ $u(1) = -1$

$du = -2t\,dt$ $u(0) = 0$

$= \boxed{1-e^{-1}}$

13. $\int_1^3 \frac{1}{(t+7)^2}dt$ $\int_8^{10} \frac{1}{u^2}du = -u^{-1}\Big|_8^{10}$

$u = t+7$ $u(1) = 8$

$du = dt$ $u(3) = 10$

$= -\frac{1}{10} - -\frac{1}{8} = \boxed{-1/10 + 1/8}$

14. $\int_0^2 \frac{x}{(1+x^2)^2}dx$

$u = 1+x^2$ $u(2) = 5$

$du = 2x\,dx$ $u(0) = 1$

$\frac{1}{2}\int_1^5 \frac{1}{u^2}du = \frac{1}{2}(-u^{-1})\Big|_1^5$

$= \frac{1}{2}\left(-\frac{1}{5} - -\frac{1}{1}\right)$

$= \boxed{-1/10 + 1/2}$

15. $\int_{-1}^{e-2} \frac{1}{t+2}dt$

$u = t+2$ $u(e-2) = e$

$du = dt$ $u(-1) = 1$

$\int_1^e \frac{1}{u}du = \ln|u|\Big|_1^e = \ln(e) - \ln(1)$

$= 1 - 0 = \boxed{1}$