Sols

Math 141 Calculus I

Exam #2 B
November 1, 2017

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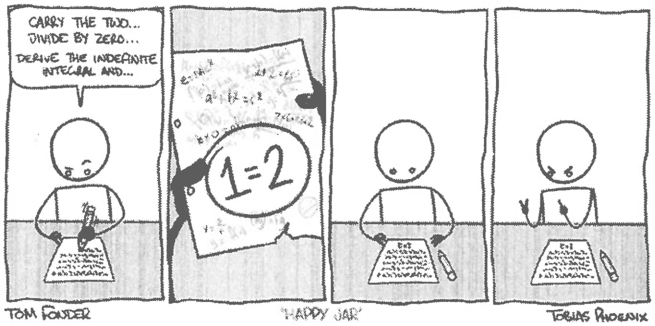
Name: ______________________________

Do not turn this page until told to do so. You will have a total of 1 hour and 15 minutes to complete the exam. You **must** show all work to receive full credit. NO CALCULATOR/PHONE ALLOWED. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%. Draw a ghost on this page if you read this.

#	score	out of	#	score	out of
1		4	8		8
2		4	9		8
3		4	10		10
4		4	11		10
5		4	12		10
6		8	13		18
7		8	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



True or False. No work/explanation required. 4pts each. True means always true.

1. *If a function is continuous, it is always differentiable.*

False

2. *A critical point c is only where $f'(c) = 0$.*

False

3. *If f and g are differentiable functions of x, then $(fg)'(x) = f'(x)g(x) - f(x)g'(x)$.*

False

4. *If $f''(c) = 0$, then $x = c$ is an inflection point of f.*

False

5. *The absolute value function, $f(x) = |x|$, is differentiable at $x = 0$.*

False

Multiple Choice. No work required. 8pts each. Choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive half the points.

6. *Find $\frac{dy}{dx}$ (Hint: Use trig identities to simplify):*

B

$$y^{\tan x} = 6$$

A. $\frac{dy}{dx} = y \ln y \sec^2 x$ [✓ **B.** $\frac{dy}{dx} = -y \ln y \sec x \csc x$]

C. $\frac{dy}{dx} = -y \ln y \tan x$ **D.** $\frac{dy}{dx} = \tan x$

$\tan x \ln y = \ln(6)$

$\sec^2 x \ln y + \tan x \frac{1}{y} \frac{dy}{dx} = 0$

$\frac{\tan x}{y} \frac{dy}{dx} = -\sec^2 x \ln y$

$\frac{dy}{dx} = -y \ln y \sec^2 x \cdot \frac{1}{\tan x}$

$= -y \ln y \sec x \csc x$

C

7. *Find $h'(2)$, given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$, if $h(x) = \dfrac{g(x)}{1+f(x)}$.*

A. $h'(2) = -7/2$ **B.** $h'(2) = -11/2$

[✓ **C.** $h'(2) = -3/2$] **D.** $h'(2) = -1/2$

$$h'(x) = \frac{g'(x)(1+f(x)) - g(x)\,f'(x)}{(1+f(x))^2}$$

$$h'(2) = \frac{7(1+-3) - 4(-2)}{(1+-3)^2} = \frac{-6}{4} = -\frac{3}{2}$$

8. *Find the derivative, y':*

$$y = \arctan\left(4x^2\right)$$

B

A. $y' = \frac{1}{1+2x^3}$ [✓ **B.** $y' = \frac{8x}{1+16x^4}$]

C. $y' = \frac{1}{1+16x^4}$ **D.** $y' = \frac{8x}{1+4x^4}$

$$\frac{1}{1+(4x^2)^2}\,(4x^2)'$$

9. *Find the derivative, y':*

$$y = \frac{-2x^3 - 5x + \sqrt{x}}{x^2}$$

D

A. $y' = \frac{-6x^2-5+\frac{1}{2}x^{-1/2}}{2x}$ **B.** $y' = -7 + \frac{1}{2}x^{-1/2}$

C. $y' = (-6x^2 - 5 + \frac{1}{2}x^{-1/2})x^2 - 2x(-2x^3 - 5x + \sqrt{x})$ [✓ **D.** $y' = -2 + \frac{5}{x^2} - \frac{3}{2x^{5/2}}$]

$$y = -2x - 5x^{-1} + x^{-3/2}$$

$$y' = -2 + 5x^{-2} - \frac{3}{2}x^{-5/2}$$

Short Answer. You must show all work to receive full credit.

10 (10 points). *A student turns in the incorrect solution to the problem below. Explain the student's mistake in words, using complete sentences. Then work out the correct solution.*

$$\frac{d}{d\theta}(\theta^2 \tan\theta) = 2\theta \sec^2\theta$$

The student did not use the product rule.

$$\frac{d}{d\theta}(\theta^2 \tan\theta) = (\theta^2)' \tan\theta + \theta^2 (\tan\theta)'$$

$$= 2\theta \tan\theta + \theta^2 \sec^2\theta$$

11 (10 points). *If $x^2y = 5y + x - 2$, find $\frac{dy}{dx}$ by implicit differentiation.*

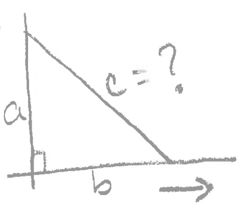
$$2xy + x^2 \frac{dy}{dx} = 5\frac{dy}{dx} + 1$$

$$x^2 \frac{dy}{dx} - 5\frac{dy}{dx} = 1 - 2xy$$

$$\frac{dy}{dx} = \frac{1-2xy}{x^2-5}$$

12 (10 points). *The top of a ladder slides down a vertical wall at a rate of* 0.15m/s. *At the moment when the bottom of the ladder is* 3m *from the wall, it slides away from the wall at a rate of* 0.2m/s. *How long is the ladder?*

$\frac{da}{dt} = 0.15$



$b = 3 \quad \frac{db}{dt} = 0.2$

$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$\frac{dc}{dt} = 0$ since the ladder's height doesn't change

$$2a(-0.15) + 2(3)(0.2) = 0$$

$$-.3a + 1.2 = 0$$

$$-.3a = -1.2$$

$$a = 4$$

$$4^2 + 3^2 = c^2 \Rightarrow \boxed{c = 5m}$$

13. *(18 pts) Sketch the curve*

$$y = \frac{x^2 - 4}{2x}$$

(a) State the domain.

$(-\infty, 0) \cup (0, \infty)$

(b) Find the intercepts. Enter NONE if there are none.

x-intercepts: (-2, 0) , (2, 0)

y-intercept: None

$$0 = \frac{x^2-4}{2x} = \frac{(x+2)(x-2)}{2x}$$

$$x = -2, \; x = 2$$

(c) Is the function even, odd, or neither? What type of symmetry does the function have?

$$f(-x) = \frac{(-x)^2-4}{2(-x)} = \frac{x^2-4}{-2x} = -\frac{x^2-4}{2x} = -f(x)$$

Odd, origin

(d) Find the asymptotes. Enter NONE if there are none.

Horizontal: NONE

Oblique: $y = \frac{1}{2}x$

Vertical: $x=0$

$$\begin{array}{r} \frac{1}{2}x \\ 2x\overline{)\,x^2-4} \\ \underline{x^2} \\ -4 \end{array}$$

$$\lim_{x\to 0^+} \frac{x^2-4}{2x} = -\infty \qquad \lim_{x\to 0^-} \frac{x^2-4}{2x} = \infty$$

(e) Find the intervals where the function is increasing and decreasing. Enter NONE if not applicable.

Increasing: $(-\infty,0)\cup(0,\infty)$

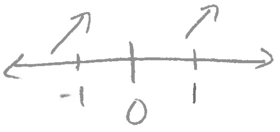
Decreasing: NONE

$$y' = \frac{2x(2x)-(x^2-4)(2)}{(2x)^2}$$

$$= \frac{4x^2-2x^2+8}{4x^2}$$

$$= \frac{2x^2+8}{4x^2}$$

$x=0$ CP



(f) State the local maximum and local minimum value(s). Enter NONE if not applicable.

Local maximum value(s): NONE

Local minimum value(s): NONE

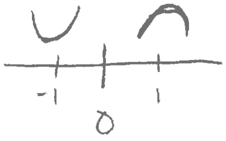
(g) Find the intervals on which the function is concave up and concave down. State the inflection points. Enter NONE if not applicable.

Concave Up: $(-\infty, 0)$

Concave Down: $(0, \infty)$

Inflection Points: $x = 0$

$$y'' = \frac{4x(4x^2) - (2x^2+8)(8x)}{(4x^2)^2} = \frac{-64x}{16x^4} = \frac{-4}{x^3}$$



(h) Use parts (a)-(g) to sketch the curve. Be sure that your graph is labeled and neat. Messy/incoherent graphs will receive zero points.