Sols

Math 141: Practice Problems for Exam 3
Sections 4.5-5.6

See section:

Questions to Guide Your Review:

4.5
1. Describe l'Hôpital's Rule. How do you know when to use it and when to stop? Give an example.
2. How can you sometimes handle limits that lead to indeterminate forms ∞/∞, $\infty \cdot 0$, and $\infty - \infty$? Given examples.
3. How can you sometimes handle limits that lead to indeterminate forms 1^∞, 0^0, and ∞^∞? Give examples.

4.8
4. Can a function have more than one antiderivative? If so, how are the antiderivatives related? Explain.
5. What is an indefinite integral? How do you evaluate one? What general formulas do you know for finding indefinite integrals?
6. What is an initial value problem? How do you solve one? Give an example.
7. If you know the acceleration of a body moving along a coordinate line as a function of time, what more do you need to know to find the body's position function? Give an example.

5.1/5.2
8. How can you sometimes estimate quantities like distance traveled, area, and average value with finite sums? Why might you want to do so?
9. What is sigma notation? What advantage does it offer? Give examples.

5.3
10. What is a Riemann sum? Why might you want to consider such a sum?
11. What is the norm of a partition of a closed interval?
12. What is the definite integral of a function f over a closed interval $[a, b]$? When can you be sure it exists?

5.4
13. What is the Fundamental Theorem of Calculus? Why is it so important? Illustrate each part of the theorem with an example.

5.5
14. How is integration by substitution related to the Chain Rule?
15. How does the method of substitution work for definite integrals?

5.6
16. How do you define and calculate the area of the region between the graphs of two continuous functions? Give an example.

See next page for detailed solutions

Practice Exercises:

17. Evaluate the limit: $\lim_{x\to\infty} \sqrt{4x^2+3x} - 2x$. $3/4$

18. A student turns in the incorrect solution to the problem below. Explain the student's mistake in words, using complete sentences. Then work out the correct solution.

$$\lim_{x\to-\infty} x^2 + 5x = (-\infty)^2 + 5(-\infty) = \infty - \infty = 0$$

Can't do arithmetic with "∞"! $\lim_{x\to-\infty} x^2+5x = \infty$

19. If 30,000 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box. $500{,}000\ cm^3$

20. Find $f(x)$ if $f'(x) = 12x^2 + 6x - 4$ and $f(1) = 1$. $f(x) = 4x^3 + 3x^2 - 4x - 2$

21. Evaluate using the Fundamental Theorem of Calculus, Part I.

$$\frac{d}{dx}\int_1^{x^2} \sec(t)dt = 2x\sec(x^2)$$

22. Evaluate the following definite integral using **(a)** the limit (Riemann sum) definition and **(b)** the Fundamental Theorem of Calculus.

$$\int_0^2 (7 - x^2)dx = 34/3$$

23. Evaluate the indefinite integral.

$$\int \csc\theta(\sin\theta - \csc\theta)d\theta = \theta + \cot\theta + C$$

24. Find $\int_5^{10} f(x)dx$ if $\int_0^5 f(x)dx = -12$ and $\int_0^{10} f(x)dx = 2$. 14

25. Evaluate the indefinite integral.

$$\int \frac{\sin x}{\cos^2 x}dx = \sec x + C$$

26. The velocity function (in meters per second) for a particle moving along a line is given by $v(t) = t^2 - 2t - 24$, $1 \le t \le 3$. Find the displacement of the particle and the total distance traveled by the particle. $-142/3$, $142/3$

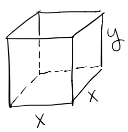
27. Find the area bounded by the curves $y = 2 - x^2$ and $y = -x$. $9/2$

17) $$\lim_{x\to\infty} \sqrt{4x^2+3x} - 2x = \lim_{x\to\infty} \frac{\sqrt{4x^2+3x}-2x}{1}\cdot\frac{\sqrt{4x^2+3x}+2x}{\sqrt{4x^2+3x}+2x}$$
$$= \lim_{x\to\infty} \frac{4x^2+3x-4x^2}{\sqrt{4x^2+3x}+2x}$$
$$= \lim_{x\to\infty} \frac{3x}{\sqrt{4x^2+3x}+2x}$$
$$= \lim_{x\to\infty} \frac{3}{\sqrt{4+3/x}+2} = \boxed{\frac{3}{4}}$$

18) $\lim_{x\to -\infty} x^2-5x$; $y=x^2-5x$ is a concave up parabola so as x decreases (approaches $-\infty$), $y=x^2-5x$ approaches ∞.

Alternatively, $\lim_{x\to-\infty} x^2-5x = \lim_{x\to-\infty} x(x-5) \to$ (large negative)(large negative) $\to$ large positive

19) $V = x^2 y$



$SA = x^2+4xy$

$30000 = x^2+4xy$

$$y = \frac{30000-x^2}{4x}$$

$$V(x) = x^2\left(\frac{30000-x^2}{4x}\right)$$
$$= \frac{1}{4}(30000x - x^3)$$
$$V'(x) = \frac{30000}{4} - \frac{3}{4}x^2 = 0$$
$x = 100$

$y = 50$

$V = (100)^2(50)$

$= 500{,}000 \text{ cm}^3$

20) $f(x) = 4x^3+3x^2-4x+C$

$f(1)=1 \Rightarrow 4+3-4+C=1$

$3+C=1$

$C=-2$

$f(x) = 4x^3+3x^2-4x-2$

21) $$\frac{d}{dx}\int_1^{x^2} \sec t\,dt = \sec(x^2)(x^2)' = 2x\sec(x^2)$$

22) $$\int_0^2 (7-x^2)dx$$

(a) $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$c_k = \frac{2k}{n}$

$$\lim_{n\to\infty}\sum_{k=0}^{n} f(2k/n)\tfrac{2}{n} = \lim_{n\to\infty}\sum_{k=0}^{\infty}\frac{2}{n}\left(7-\frac{4k^2}{n^2}\right) = \lim_{n\to\infty}\sum_{k=0}^{\infty}\left(\frac{14}{n}-\frac{8k^2}{n^3}\right) = \lim_{n\to\infty}\left(\sum_{k=0}^{n}\frac{14}{n} - \frac{8}{n^3}\sum_{k=0}^{n}k^2\right)$$
$$= \lim_{n\to\infty}\left(14 - \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right) = 14 - 8/3 = 34/3$$

(b) $$\int_0^2 (7-x^2)dx = 7x - \frac{x^3}{3}\Big|_0^2 = 14 - 8/3 = 34/3$$

23) $$\int \csc\theta(\sin\theta - \csc\theta)d\theta$$
$$= \int (1-\csc^2\theta)d\theta \quad \left(\csc\theta = \frac{1}{\sin\theta}\right)$$
$$= \theta + \cot\theta + C$$

24) $$\int_0^5 f(x)dx + \int_5^{10} f(x)dx = \int_0^{10} f(x)dx$$
$$-12 + \int_5^{10} f(x)dx = 2$$
$$\int_5^{10} f(x)dx = 14$$

25) $$\int \frac{\sin x}{\cos^2 x}dx$$
$$= \int \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}dx$$
$$= \int \sec x\tan x\,dx$$
$$= \sec x + C$$

26) $$\int_1^3 t^2-2t-24\,dt$$
$$= \frac{t^3}{3} - t^2 - 24t\Big|_1^3$$
$$= \left(\frac{3^3}{3}-3^2-24(3)\right) - \left(\frac{1^3}{3}-1^2-24(1)\right)$$
$$= -72 + 74/3 = -142/3$$

Since $y = t^2-2t-24 < 0$ on $[1,3]$, total distance traveled is
$$\left|-\frac{142}{3}\right| = \frac{142}{3}$$

27) $2-x^2 = -x$

$0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x=-1,\ x=2$

$$A = \int_{-1}^2 (2-x^2)-(-x)dx$$
$$= \int_{-1}^2 (-x^2+x+2)dx$$
$$= -\frac{x^3}{3}+\frac{x^2}{2}+2x\Big|_{-1}^2$$
$$= \left(-8/3+2+4\right) - \left(1/3+\tfrac{1}{2}-2\right)$$
$$= 9/2$$