Sols

Math 141 Calculus I

Exam #1
February 12, 2018

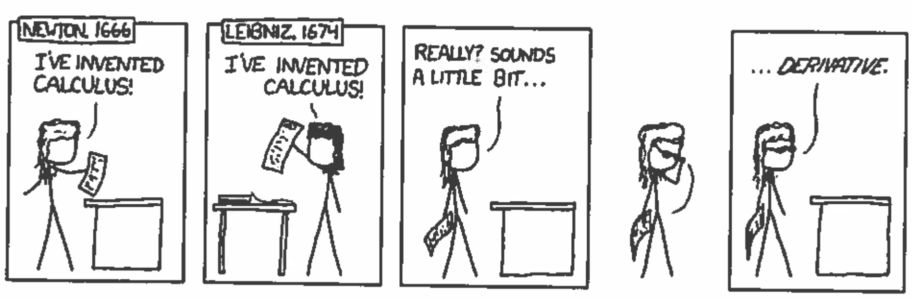
Instructor: Ann Clifton

Name: ______________________________

Do not turn this page until told to do so.
You will have a total of 1 hour and 15 minutes to complete the exam. When specified, you **must** show all work to receive full credit. NO CALCULATOR/PHONE ALLOWED. Draw a pumpkin on this page if you read this.
Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

#	score	out of	#	score	out of
1		4	9		6
2		4	10		6
3		4	11		14
4		4	12		20
5		4	13		16
6		6			
7		6	EC		5
8		6	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



True or False. No work/explanation required. True means ALWAYS true. 4pts each.

1. If $f(x) \leq g(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself, and the limits of f and g both exist as x approaches c, then $\lim_{x\to c} f(x) \leq \lim_{x\to c} g(x)$.

   True

2. If the function f is continuous at $x = c$ and g is a function of x, then $f + g$ is continuous at $x = c$.

   False

3. If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

   $g(f(c))$

   True

4. If $P(x)$ and $Q(x)$ are polynomials, $Q(c) \neq 0$, then $\lim_{x\to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$.

   True

5. If L and c are real numbers and $\lim_{x\to c} f(x) = L$, then $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer.

   True

Multiple Choice. No work required. 6 points each. Choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive half the points.

6. Evaluate the given limit:

$$\lim_{x\to\frac{\pi}{2}} \cos\left(2x + \sin\left(\frac{3\pi}{2} + x\right)\right)$$

$\cos\left(2(\pi/2) + \sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)\right)$

$= \cos(\pi + \sin(2\pi))$

$= \cos(\pi + 0)$

$= \cos(\pi) = -1$

A. -1 B. $\frac{-1}{2}$

C. 0   D. $\frac{\sqrt{3}}{2}$

7. Find the limit:

$$\lim_{y\to 4} \frac{y^2 - 4y}{y^2 - y - 12}$$

$\frac{y(\cancel{y-4})}{(\cancel{y-4})(y+3)} = \frac{y}{y+3}$

A. 0   B. $\frac{-3}{7}$

C. $\frac{4}{7}$ D. Does Not Exist

8. Find the limit:

$$\lim_{x \to -\infty} \left(\frac{x^2 + x - 1}{27x^2 - 3} \right)^{1/3}$$

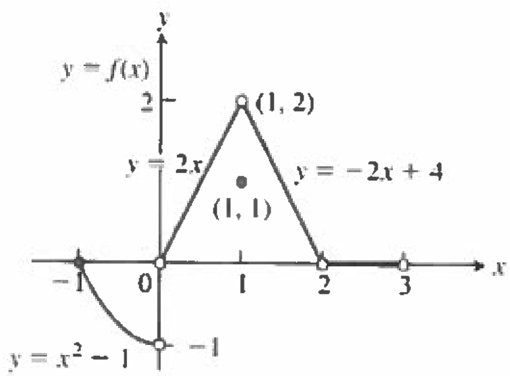
$$\left(\lim_{x \to -\infty} \frac{x^2+x-1}{27x^2-3} \right)^{1/3}$$

$$\left(\frac{1}{27}\right)^{1/3} = \sqrt[3]{\frac{1}{27}}$$

A. $\frac{1}{27}$ B. $\frac{1}{3}$

C. 0 D. Does Not Exist

Use the graph below for questions 9 and 10.



9. Using the given graph, find $\lim_{x \to 0^+} f(x)$.

A. −1 B. 0

C. 2 D. Does Not Exist

10. Using the given graph, list the points where $f(x)$ is not continuous.

A. $x = -1, 0, 1, 2, 3$ B. $x = 0, 1, 2$

C. $x = 0, 2, 3$ D. $x = 0, 1, 2, 3$

Short Answer. You must show all work to receive full credit. If you need more space, use the provided scrap paper and write a note indicating where to find your work.

11. (14 points) Evaluate the following limit:

$$\lim_{x \to 16} \frac{4 - \sqrt{x}}{16x - x^2}$$

$$\frac{4-\sqrt{x}}{x(16-x)} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} = \frac{\cancel{16-x}}{x(\cancel{16-x})(4+\sqrt{x})} = \frac{1}{x(4+\sqrt{x})}$$

$$\lim_{x \to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+\sqrt{16})} = \frac{1}{16(8)} = \boxed{\frac{1}{128}}$$

12. (18 points) Find the derivative, $f'(x)$, using the limit definition, of the function $f(x) = x^2 + x$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \to 0} 2x + h + 1$$

$$= \boxed{2x+1}$$

13. (18 points) Let $f(x) = \dfrac{x^2 - 4}{x - 1}$.

(a) Find $\lim_{x \to 1^+} f(x)$

$$\lim_{x \to 1^+} \frac{x^2 - 4}{x - 1} = \frac{\lim_{x \to 1^+} x^2 - 4 \rightarrow -3}{\lim_{x \to 1^+} x - 1 \rightarrow \text{small positive}}$$

$$= \boxed{-\infty}$$

(b) Find $\lim_{x \to 1^-} f(x)$

$$\lim_{x \to 1^-} \frac{x^2 - 4}{x - 1} = \frac{\lim_{x \to 1^-} x^2 - 4 \rightarrow -3}{\lim_{x \to 1^-} x - 1 \rightarrow \text{small negative}}$$

$$= \boxed{\infty}$$

(c) Find the oblique asymptote of the graph of $f(x)$. That is, find $\lim_{x \to \pm\infty} f(x)$.

$$\begin{array}{r|l} & x + 1 \\ x - 1 & x^2 - 4 \\ & -x^2 + x \\ \hline & x - 4 \\ & -x + 1 \\ \hline & -3 \end{array}$$

$$\frac{x^2 - 4}{x - 1} = x + 1 + \frac{-3}{x - 1}$$

$$\boxed{y = x + 1}$$

Extra Credit (5 points) No partial credit will be given for this problem.

For the given function $f(x)$ and values of L, c, and $\epsilon > 0$ determine the largest value for $\delta > 0$ such that $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$.

$$f(x) = 6x + 4, \qquad L = 34, \qquad c = 5, \qquad \epsilon = 0.6$$

$$|6x + 4 - 34| < 0.6$$

$$|6x - 30| < 0.6$$

$$|6(x - 5)| < 0.6$$

$$6|x - 5| < 0.6$$

$$|x - 5| < 0.1$$

Let $\delta = 0.1$.