Math 170: Section 6.3, 6.4 Lecture

Section 6.3

Example 3 You are playing Scrabble and have the following letters to work with: k, e^1, r, e^2. Because you are losing, you want to use all your letters to make a single word, but you can't think of any four-letter words using all these letters. In desperation, you decide to list *all* the four-letter sequences possible to see if there are any valid words among them. How large is your list?

$$\underline{r}\quad \underline{\overset{e_2}{e_1}}\quad \underline{k}\quad \underline{\overset{e_1}{e_2}}$$

Mult. Principle

Step 1: choose 1 of 4 letters: 4 choices

Step 2: 3 choices

Step 3: 2 choices

Step 4: 1 choices

$4 \cdot 3 \cdot 2 \cdot 1 = 24$ options

reke

$\boxed{e}\ \boxed{k}\ \boxed{r}\ \boxed{e}$

Step 1: Choose a slot for "r": 4 choices

Step 2: Choose a slot for "k": 3 choices

Step 3: Choose slots for "e"s: 1 choice

List is $4 \cdot 3 \cdot 1 = 12$ "words" long

Section 6.4 Permutations and Combinations

Example 1 Ms. Birkitt, the English teacher at Barkpan Girls High School, wants to stage a production of R.B. Sheridan's play, *The School for Scandal.* The casting was going well until she realized she still has five unfilled characters and five seniors yet to be assigned roles. The characters are Lady Sneerwell, Lady Teazle, Mrs. Candour, Maria and Snake; while the unassigned seniors were April, May, June, Julia, and Augusta. How many possible assignments are there?

1) Lady Sneerwell — 5 choices
2) Lady Teazle — 4 "
3) Candour — 3 "
4) Maria — 2 "
5) Snake — 1 choice

\# of possible castings is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$= 5!$

Definition: A *permutation of n items* is $n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$,

an ordered list of those items.

for n a positive integer and $0!=1$.

The number of possible permutations of n items is given by n factorial, $n! = n \cdot (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$

Example 2 At the end of 2008, the 10 largest companies (by market capitalization) listed on the New York Stock Exchange were, in alphabetical order, AT&T Inc., Berkshire Hathaway Inc., Chevron Corporation, China Mobile Ltd. (ADR), Exxon Mobile Corporation, Johnson & Johnson, PetroChina Company Limited (ADR), Royal Dutch Shell plc (ADR), The Procter & Gamble Company, and Wal-Mart Stores, Inc. You would like to apply to six of these companies for a job and you would like to list them in order of job preference. How many such ordered lists are possible?

1) 10 choices
2) 9 "
3) 8 "
4) 7 "
5) 6 "
6) 5 "

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151{,}200$ possible lists of 6

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{4!}$$

Definition: A *permutation of n items taken r at a time* is an ordered list of r items chosen from a set of n items. The number of permutations of n items taken r at a time is given by

$$P(n,r) = n(n-1)(n-2)\dots(n-r+1)$$

(there are r terms multiplied together). We can also write

$$P(n.r) = \frac{n!}{(n-r)!}.$$

Example 3 Suppose we simply wanted to pick two of the 10 companies listed in Example 2 to apply to, without regard to order. How many possible choices do we have? What if we wanted to choose six to apply to, without regard to order?

$$P(10,2) = \frac{10!}{(10-2)!} = \frac{10!}{8!} = 10 \cdot 9 = 90$$

1. AT&T 2. J&J

1. J&J 2. AT&T

Every set is counted twice, so there are only $90/2 = 45$

$$P(10,6) = \frac{10!}{(10-6)!} = \frac{10!}{4!} = 151{,}200$$

$$\frac{151{,}200}{6!} = \frac{151{,}200}{720} = 210$$

Definition: A *permutation* of n items taken r at a time is an *ordered list* of r items chosen from n. A *combination* of n items taken r at a time is an *unordered set* of r items chosen from n.

How do we count the number of possible combinations of n items taken r at a time? We generalize the calculation done in Example 3. The number of permutations is $P(n.r)$, but each set of r items occurs $r!$ times because this is the number of ways in which those r items can be ordered. So, the number of combinations is $P(n,r)/r!$.

Definition: The number of *combinations of n items taken r at a time* is given by

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}.$$

We can also write

$$C(n,r) = \frac{n!}{r!(n-r)!}.$$

Example 4 Calculate **a.** $C(11,3)$ and **b.** $C(11,8)$.

$$a)\ C(\underset{n}{11},\underset{r}{3}) = \frac{11!}{3!(11-3)!} = \frac{11!}{3!(8)!}$$

$$= \frac{11\cdot 10\cdot 9\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{3!\cdot \cancel{8!}}$$

$$= \frac{11\cdot 10\cdot 9}{3\cdot 2\cdot 1} = \frac{11}{1}\cdot\frac{10}{2}\cdot\frac{9}{3} = 11\cdot 5\cdot 3 = 165$$

$$b)\ C(11,8) = \frac{11!}{8!(11-8)!} = \frac{11!}{8!\cdot 3!} = 165$$

$$C(n,r) = C(n,n-r)$$

Example 5 Calculate **a.** $C(11, 11)$ and **b.** $C(11, 0)$.

a) $C(11,11) = \frac{11!}{11!\,(11-11)!} = \frac{11!}{11!\cdot 0!} = 1$

b) $C(11,0) = \frac{11!}{0!\,(11-0)!} = 1$

Example 6 In the betting game Lotto, used in many state lotteries, you choose six different numbers in the range 1-55 (the upper number varies). The order in which you choose them is irrelevant. If your six numbers match the six numbers chosen in the "official drawing," you win the top prize. If Lotto tickets cost \$1 for two sets of numbers and you decide to buy tickets that cover every possible combination, thereby guaranteeing that you will win the prize, how much money will you have to spend?

Example 7 A bag contains three red marbles, three blue ones, three green ones, and two yellow ones (all distinguishable from one another).

a. How many sets of four marbles are possible?

b. How many sets of four are there such that each one is a different color?

c. How many sets of four are there in which at least two are red?

d. How many sets of four are there in which none are red, but at least one is green?

Solution (continued)

Step 1: Choose slots for "e"s : 6 choices

Step 2: choose slot for "r" : 2 choices

Step 3: choose slot for "k" : 1 choice

Total : $6 \cdot 2 \cdot 1 = \underline{12}$

1	2	3	4
e	e		
e		e	
e			e
	e	e	
	e		e
		e	e

Jami's question about example 3 section 6.3, "What if we place the "e"'s first?"

Solution (continued)