Solutions

Math 111I-006 Basic College Mathematics

Practice Exam #1
September 16, 2014

Instructor: Ann Clifton

Name: _____

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Give exact answers. Indicate your final answer with a box.

Factor completely. Indicate if prime.

1. $18x^9y^7 + 90x^7y^5 - 72x^3y^3$

$18x^3y^3(x^6y^4 + 5x^4y^2 - 4)$

2. $m^2(n-14) - (n-14)$

$(n-14)(m^2-1)$

$(n-14)(m+1)(m-1)$

3. $15x^2 + 8y^2 - 10xy - 12xy$

$15x^2 - 10xy + 8y^2 - 12xy$

$5x(3x-2y) - 4y(-2y+3x)$

$(3x-2y)(5x-4y)$

4. $121k^2 + 16m^2$

Prime

(sum of two squares)

5. $3x^2 - 3x - 18$

$3(x^2 - x - 6)$

$3(x-3)(x+2)$

6. $6y^2 + 17y + 12$

$(3y+4)(2y+3)$

7. $x^2 - x - 56$

$(x-8)(x+7)$

8. $x^4 + 7x^2 + 12$

$(x^2+3)(x^2+4)$

9. $49x^2 - 16$

$(7x+4)(7x-4)$

10. $216p^3 - 1$

$(6p-1)(36p^2+6p+1)$

(Difference of two cubes)

11. $64x^3 + 1$

$(4x+1)(16x^2-4x+1)$

(Sum of two cubes)

12. $z^2 + 10z + 25$

$(z+5)(z+5)$

$(z+5)^2$

13. $x^3 + 4x^2 - 9x - 36$

$x^2(x+4)-9(x+4)$

$(x+4)(x^2-9)$

$(x+4)(x+3)(x-3)$

14. $25x^2 + 16$

Prime

(Sum of two squares)

Simplify using exponent rules.

15. $x^{5/4}x^{11/4}$

$x^{5/4 + 11/4}$

$x^{16/4}$

$\boxed{x^4}$

16. $(4a^{5/6})^4$

$4^4 a^{\frac{5}{6}\cdot 4}$

$\boxed{256a^{10/3}}$

17. $\left(\frac{9}{k^2}\right)^{-1/2}$

$\left(\frac{k^2}{9}\right)^{1/2} = \frac{k^{2\cdot 1/2}}{9^{1/2}} = \frac{k}{\sqrt{9}} = \boxed{\frac{k}{3}}$

18. $\sqrt[5]{27u^3v^2}\sqrt[5]{9u^12v^3}$

$(27u^3v^2)^{1/5}(9u2v^3)^{1/5} = (27\cdot 9\cdot 2\, u^3 u v^2 v^3)^{1/5} = (243\cdot 2\, u^4 v^5)^{1/5}$

$= \sqrt[5]{243}\cdot\sqrt[5]{2}\cdot u^{4/5}v = \boxed{3u^{4/5}v\sqrt[5]{2}}$

Perform the indicated operation and simplify.

19. $\dfrac{k^2+15k+54}{k^2+12k+27}\cdot\dfrac{k^2+8k+15}{k^2+11k+30}$

$\dfrac{(k+9)(k+6)}{(k+9)(k+3)}\cdot\dfrac{(k+5)(k+3)}{(k+5)(k+6)} = \boxed{1}$

20. $\dfrac{k^2+7k+10}{k^2+14k+45} \div \dfrac{k^2+6k+8}{k^2+9k}$

$$\frac{k^2+7k+10}{k^2+14k+45} \cdot \frac{k^2+9k}{k^2+6k+8}$$

$$\frac{(k+5)(k+2)}{(k+5)(k+9)} \cdot \frac{k(k+9)}{(k+2)(k+4)}$$

$$\boxed{\frac{k}{k+4}}$$

Note! We can NOT simplify this solution further, that is, do NOT cancel the "k"s!

21. $\dfrac{2}{y^2-3y+2} + \dfrac{6}{y^2-1}$

$$\frac{(y+1)}{(y+1)} \cdot \frac{2}{(y-2)(y-1)} + \frac{6}{(y+1)(y-1)} \cdot \frac{(y-2)}{(y-2)} \quad LCD = (y-2)(y-1)(y+1)$$

$$= \frac{2(y+1)}{(y-2)(y-1)(y+1)} + \frac{6(y-2)}{(y-2)(y-1)(y+1)}$$

$$= \frac{2(y+1)+6(y-2)}{(y-2)(y-1)(y+1)} = \frac{2y+2+6y-12}{(y-2)(y-1)(y+1)} = \frac{8y-10}{(y-2)(y-1)(y+1)} = \boxed{\frac{2(4y-5)}{(y-2)(y-1)(y+1)}}$$

22. $\dfrac{6}{x+3} - \dfrac{2}{x-3}$

$$LCD = (x+3)(x-3)$$

$$\frac{(x-3)}{(x-3)} \cdot \frac{6}{x+3} - \frac{2}{x-3} \cdot \frac{(x+3)}{(x+3)}$$

$$\frac{6(x-3) - 2(x+3)}{(x+3)(x-3)} = \frac{6x-18-2x-6}{(x+3)(x-3)} = \frac{4x-24}{(x+3)(x-3)} = \boxed{\frac{4(x-6)}{(x+3)(x-3)}}$$

Solve the quadratic equation by factoring.

23. $\frac{7x^2 - 18}{3} = -13x$

$3 \cdot \left(\frac{7x^2-18}{3}\right) = (-13x) \cdot 3$

$7x^2 - 18 = -39x$

$7x^2 + 39x - 18 = 0$

$(7x - 3)(x + 6) = 0$

$7x - 3 = 0 \quad x + 6 = 0$

$7x = 3$

$\boxed{x = 3/7 \quad x = -6}$

24. $5x^2 - 30x + 40 = 0$

$(5x - 20)(x - 2) = 0$

$5x - 20 = 0 \quad x - 2 = 0$

$5x = 20$

$\boxed{x = 4 \quad x = 2}$

Solve the quadratic equation using any method learned in class.

25. $\frac{1}{2}x^2 - x - 2 = 0$

$2 \cdot \left(\frac{1}{2}x^2 - x - 2\right) = (0) \cdot 2$

$x^2 - 2x - 4 = 0$

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 1$
$b = -2$
$c = -4$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$= \frac{2 \pm \sqrt{4 + 16}}{2}$

$= \frac{2 \pm \sqrt{20}}{2}$

$= \frac{2 \pm \sqrt{4 \cdot 5}}{2}$

$= \frac{2 \pm 2\sqrt{5}}{2}$

$\boxed{x = 1 \pm \sqrt{5}}$

Since each term has a factor of 2, we can reduce.

26. $x^2 + 5x + 1 = 0$

$a = 1$
$b = 5$
$c = 1$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(1)}}{2(1)}$

$= \frac{-5 \pm \sqrt{25 - 4}}{2}$

$= \frac{-5 \pm \sqrt{21}}{2}$

$\boxed{x = \frac{-5 \pm \sqrt{21}}{2}}$

$21 = 3 \cdot 7$
neither 3 nor 7 is a perfect square so we can't simplify further.

27. $3x^2 + 6x - 1 = 0$

$a = 3$
$b = 6$
$c = -1$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{36 + 12}}{6}$$

$$= \frac{-6 \pm \sqrt{48}}{6}$$

$$= \frac{-6 \pm \sqrt{16 \cdot 3}}{6}$$

$$= \frac{-6 \pm 4\sqrt{3}}{6}$$

-6, 4, and 6 all have a common factor of 2.

$$x = \frac{-3 \pm 2\sqrt{3}}{3}$$

← This is our final answer since we cannot simplify under the $\sqrt{}$.

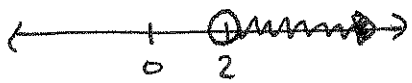
Solve the inequality. Write your solution in interval notation and graph on the real number line.

28. $3x - 5 > 1$

$3x > 6$

$x > 2$

$(2, \infty)$

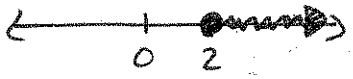


29. $1 - 3x \leq -5$

$-3x \leq -6$

$x \geq 2$

$[2, \infty)$



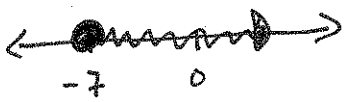
30. $2x + 8 \leq 7x + 43$

$8 \leq 5x + 43$

$-35 \leq 5x$

$-7 \leq x$

$[-7, \infty)$

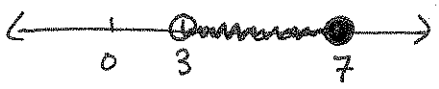


31. $3 < 2x - 3 \leq 11$

$6 < 2x \leq 14$

$3 < x \leq 7$

$(3, 7]$

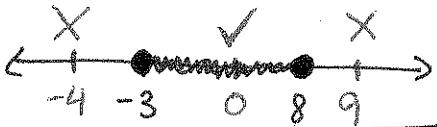


32. $x^2 - 5x - 24 \leq 0$

$(x-8)(x+3) \leq 0$

$x-8=0 \quad x+3=0$

$x=8 \quad x=-3$



$[-3, 8]$

$x=-4$

$(-4-8)(-4+3) = (-12)(-1) > 0$

$x=0$

$(0-8)(0+3) = (-8)(3) < 0$

$x=9$

$(9-8)(9+3) = (1)(12) > 0$

33. $x^2 \leq 64$

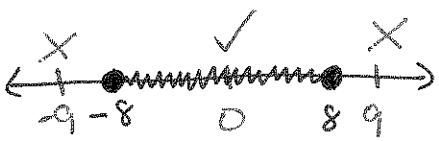
$x^2 \leq 64$

$x^2 - 64 \leq 0$

$(x+8)(x-8) \leq 0$

$x+8=0 \quad x-8=0$

$x=-8 \quad x=8$



$[-8, 8]$

$x=-9$

$(-9+8)(-9-8) = (-1)(-17) > 0$

$x=0$

$(0+8)(0-8) = (8)(-8) < 0$

$x=9$

$(9+8)(9-8) = (17)(1) > 0$

34. Let $P(3,7)$ and $Q(-2,1)$ be two points in the coordinate plane.

(a) Find the distance between P and Q.

(x_1, y_1) (x_2, y_2)

$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$P(3,7)$ $Q(-2,1)$

x_1 y_1 x_2 y_2

$d_{P,Q} = \sqrt{(3-{-2})^2+(7-1)^2} = \sqrt{(5)^2+(6)^2} = \sqrt{25+36} = \boxed{\sqrt{61}}$

(b) Find the midpoint between P and Q.

$(x_1, y_1), (x_2, y_2)$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M_{P,Q} = \left(\frac{3+-2}{2}, \frac{7+1}{2}\right) = \boxed{\left(\frac{1}{2}, 4\right)}$

$10.00 plus $.75

35. A taxi service in NYC charges a flat fee of $10.00 *plus* .75 per mile. Find an equation that models the total cost C for travelling x miles. How much would it cost to travel from Times Square to LaGuardia Airport (10 miles)?

x	C
0	10
1	10.75
2	11.50
3	12.25
⋮	⋮

$\boxed{C = 10 + .75x}$

When $x = 10$, $C = 10 + .75(10)$

$= 10 + 7.5$

$= 17.5$

So to travel 10 miles, it will cost $\boxed{\$17.50}$