## EXERCISES

Which of Exercises 1–10 are statements? Comment on the ruth values of all the statements you encounter. If a sentence fails to be a statement, explain why. HINT [See Example 1.]

- 1. All swans are white.
- 2. The fat cat sat on the mat.
- 3. Look in thy glass and tell whose face thou viewest.4
- 4. My glass shall not persuade me I am old.5
- 5. There is no largest number.
- **6.** 1,000,000,000 is the largest number.
- 7. Intelligent life abounds in the universe.
- 8. There may or may not be a largest number.
- 9. This is exercise number 9. 10. This sentence no verb.

Let p: "Our mayor is trustworthy," q: "Our mayor is a good speller," and r = "Our mayor is a patriot." Express each of thestatements in Exercises 11–16 in logical form: HINT [See Quick Examples on pages A2, A3, A5.)

- 11. Although our mayor is not trustworthy, he is a good speller.
- 12. Either our mayor is trustworthy, or he is a good speller.
- 13. Our mayor is a trustworthy patriot who spells well.
- 14. While our mayor is both trustworthy and patriotic, he is not a good speller.
- 15. It may or may not be the case that our mayor is trustworthy.
- 16. Our mayor is either not trustworthy or not a patriot, yet he is an excellent speller.

Let p: "Willis is a good teacher," q: "Carla is a good teacher," r: "Willis' students hate math," s: "Carla's students hate math." Express the statements in Exercises 17-24 in words.

17. 
$$p \wedge (\sim r)$$

**18.** 
$$(\sim p) \land (\sim q)$$

**19.** 
$$q \vee (\sim q)$$

**20.** 
$$((\sim p) \land (\sim s)) \lor q$$

**21.** 
$$r \wedge (\sim r)$$

22. 
$$(\sim s) \vee (\sim r)$$

23. 
$$\sim (q \vee s)$$

**24.** 
$$\sim (p \wedge r)$$

Assume that it is true that "Polly sings well," it is false that "Quentin writes well," and it is true that "Rita is good at math." Determine the truth of each of the statements in Exercises 25-32.

- 25. Polly sings well and Quentin writes well.
- 26. Polly sings well or Quentin writes well.
- 27. Polly sings poorly and Quentin writes well.
- 28. Polly sings poorly or Quentin writes poorly.
- 29. Either Polly sings well and Quentin writes poorly, or Rita is good at math.

- 30. Either Polly sings well and Quentin writes poorly, or Rita is not good at math.
- 31. Either Polly sings well or Quentin writes well, or Rita is good at math.
- 32. Either Polly sings well and Quentin writes well, or Rita is bad at math.

Find the truth value of each of the statements in Exercises 33-48. HINT [See Quick Examples on page A6.]

33. "If 
$$I = I$$
, then  $2 = 2$ ."

**34.** "If 
$$1 = 1$$
, then  $2 = 3$ ."

**35.** "If 
$$1 \neq 0$$
, then  $2 \neq 2$ ."

**36.** "If 
$$1 = 0$$
, then  $1 = 1$ ."

37. "A sufficient condition for 1 to equal 2 is 
$$1 = 3$$
."

- **38.** "1 = 1 is a sufficient condition for 1 to equal 0."
- **39.** "1 = 0 is a necessary condition for 1 to equal 1."
- **40.** "1 = 1 is a necessary condition for 1 to equal 2."
- 41. "If I pay homage to the great Den, then the sun will rise in the east."
- 42. "If I fail to pay homage to the great Den, then the sun will still rise in the east."
- 43. "In order for the sun to rise in the east, it is necessary that it sets in the west."
- 44. "In order for the sun to rise in the east, it is sufficient that it sets in the west."
- 45. "The sun rises in the west only if it sets in the west."
- 46. "The sun rises in the east only if it sets in the east."
- 47. "In order for the sun to rise in the east, it is necessary and sufficient that it sets in the west."
- 48. "In order for the sun to rise in the west, it is necessary and sufficient that it sets in the east."

Construct the truth tables for the statements in Exercises 49–62. HINT [See Example 2.]

**49.** 
$$p \wedge (\sim q)$$

**50.** *p* ∨ (
$$\sim q$$
)

**51.** 
$$\sim$$
( $\sim p$ )  $\vee$   $p$ 

**52.** 
$$p \wedge (\sim p)$$

**53.** 
$$(\sim p) \wedge (\sim q)$$

**54.** 
$$(\sim p) \vee (\sim q)$$

**55.** 
$$(p \wedge q) \wedge r$$

**56.** 
$$p \wedge (q \wedge r)$$

**57.** 
$$p \wedge (q \vee r)$$

**58.** 
$$(p \wedge q) \vee (p \wedge r)$$

**59.** 
$$p \rightarrow (q \vee p)$$

**60.** 
$$(p \lor q) \rightarrow \sim p$$

**61.** 
$$p \leftrightarrow (p \lor q)$$

**62.** 
$$(p \land q) \leftrightarrow \sim p$$

Use truth tables to verify the logical equivalences given in Exercises 63-72.

**63.** 
$$p \wedge p \equiv p$$

**64.** 
$$p \lor p \equiv p$$

conjunction)

**65.** 
$$p \lor q \equiv q \lor p$$

**66.** 
$$p \wedge q \equiv q \wedge p$$

(Commutative law for disjunction)

(Commutative law for

<sup>&</sup>lt;sup>4</sup>William Shakespeare Sonnet 3.

<sup>&</sup>lt;sup>5</sup>Ibid., Sonnet 22.

<sup>&</sup>lt;sup>6</sup>From Metamagical Themas: Questing for the Essence of Mind and Pattern by Douglas R. Hofstadter (Bantam Books, New York 1986).