Appendix A Logic

Logic is the underpinning of all reasoned argument.
Greeks recognized role in math & philosophy
Aristotle wrote the first systematic treatise on logic which had a heavy influence in philosophy, science and religion through the Middle Ages.
17th century - Leibniz, want to have a formal and symbolic language for logic
Boole, DeMorgan - Symbolic logic, became recognized as part of math
Logic is a part, but also the language.

A.1 Statements and Logical Operators

Propositional logic: study of propositions.
A statement, or proposition, is any declarative sentence which is either true T or false F. T or F is the truth value of the statement.

Ex: a) "$2+2=4$" T
b) "$1=0$" F
c) "It will rain tomorrow" not a statement
d) "Solve the following for x" not a statement, command not a declaration
e) "The number 5" not even a sentence
f) "This statement is false" liar's paradox

To deal with situations as in (f), self-referential sentences are not allowed to be statements.

We use letters (p, q, r) to denote statements.
Ex: p: "the moon is round"

We can form the negation of of a statement p, denoted $\sim p$. read "not p"
Ex: $\sim p$: "the moon is not round" ($\sim$ is a logical operator)

Observe that if p is true then $\sim p$ is false and vice-versa.

Ex: (a) p: "$2+2=4$" $\sim p$: "$2+2\neq 4$"
(b) q: "$1=0$" $\sim q$: "$1\neq 0$"
(c) r: "All politicians are crooks" $\sim r$: "Not all politicians are crooks"
"Some politicians are not crooks"
(d) Double negation: $\sim(\sim p) = p$
"not(not p)"

If we have two statements, p: "I am wise" and q: "I am strong" we can combine them by saying "I am wise AND I am strong", denoted by $p \wedge q$ ($\wedge$ = and). This is the <u>conjunction</u> of p and q.

The <u>conjunction</u> of p and q is the statement $p \wedge q$, "p and q".
The statement $p \wedge q$ is true only when BOTH p and q are true, otherwise false.

<u>Ex</u>: (a) p: "$2+2=4$", q: "$1=0$" $p \wedge q$: "$2+2=4$ and $1=0$"
T   F   F

(b) p: "The dog is barking" q: "The squirrel is running"

$p \wedge q$: "The dog is barking and the squirrel is running."
"Not only is the dog barking, but the squirrel is running!"

There are lots of ways to phrase statements and still have the same meaning but the point of symbolic logic is to strip away the verbiage and record the underlying logical structure of a statement.

A <u>compound statement</u> is a statement formed from simpler statements via the use of logical operators. Ex: $\sim p$, $(\sim p) \wedge (q \wedge r)$, $p \wedge (\sim p)$.
A statement that cannot be expressed as a compound statement is <u>atomic</u> (Greek for "not divisible", it was believed for a long time that atoms were indivisible)
Ex "I am clever" is an atomic statement.
In a compound statement such as $(\sim p) \wedge (q \wedge r)$, p, q, r are the <u>variables</u>.
So, $\sim p$ is ~~the~~ compound statement in the single variable p.

<u>Truth tables</u>

The <u>truth table</u> for a compound statement shows, for <u>each</u> combination of possible truth values of its variables, the corresponding truth value of the statement.

<u>Ex</u>: (a) Truth table for negation:

p	$\sim p$
T	F
F	T

(b) Truth table for conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: (c) $\sim(p \wedge q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(d) $(\sim p) \wedge q$

p	q	$\sim p$	$(\sim p) \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Last 10 minutes

Fibonacci

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

$F_n = F_{n-1} + F_{n-2}$, $F_1 = 1$, $F_2 = 1$

\# Rabbits come in pairs. Once a pair is 2 months old it bears another pair and from then on bears a pair every month. Starting w/ newborn pair, how many pairs will there be at the end of the year?

Disjunction

Want to go to the movies, rated R.

p: "You are over 18" q: "You are accompanied by an adult"

Don't need $p \wedge q$ so what word could we use to connect?

"You are over 18 OR You are accompanied by an adult"

Do we have to just have one or the other? No, we could be over 18 and be with an adult. So we use the inclusive or: $p \vee q$ means p is true, q is true or both are true. (there is the exclusive or p true, q true but not both, how can we represent that? $(p \vee q) \wedge \sim(p \wedge q)$).

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex: (a) p: "The cook did it" q: "The butler did it"

$p \vee q$: "The cook did it or the butler did it"

(b) p: " " q: " " r: "The lawyer did it"

$(p \vee q) \wedge (\sim r)$: "The cook did it or the butler did it and the lawyer did not do it"

Conditional

Consider "If you earn an A in logic, then I'll buy you a new car"

p: You earn an A in logic q: "I'll buy you a new car"

The original statement says "if p is true, then q is true" or, if p then q.

Also p implies q, $p \Rightarrow q$

Suppose the original is true. This does not mean that you will earn an A in logic. It says IF you do th[illegible]

The only way this can be broken is if you do earn an A and I do NOT buy you a new car.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

" $\Rightarrow$ " conditional operator

p antecedent or hypothesis

q consequent or conclusion

" $p \Rightarrow q$ " also called implication

HW 1 #s 1, 5, 9, 12, 13, 14, 16, 17, 23, 24, 25, 28, 29, 33, 36, 41, 42, 49, 55, 56, 57, 60