Math 141: Section 3.1 Tangents and the Derivative at a Point - Notes

Finding a Tangent to the Graph of a Function To find a tangent to an arbitrary curve $y = f(x)$ at a point $P(x_0, f(x_0))$, we use the ideas introduced in Section 2.1:

Definition: The **slope of the curve** $y = f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}, \text{ (provided the limit exists).}$$

The **tangent line** to the curve at P is the line through P with this slope.

Example 1 a) Find the slope of the curve $y = 1/x$ at any point $x = a \neq 0$. What is the slope at the point $x = -1$?

b) Where does the slope equal $-1/4$?

c) What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes?

Definition The expression

$$\frac{f(x_0+h)-f(x)}{h)}, \; h \neq 0,$$

is called the **difference quotient of f at x_0 with increment h.**

The **derivative of a function f at a point** x_0, denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

provided this limit exists.

Example 2 We previously looked at the speed of a freely falling rock near the surface of the earth. We knew that the rock fell $y = 16t^2$ feet during the first t sec, and we used a sequence of average rates over increasingly short intervals to estimate the rock's speed at the instant $x = 1$. What was the rock's *exact* speed at this time?

Summary The following are all interpretations for the limit of the difference quotient,

$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}.$$

1) The slope of the graph of $y=f(x)$ at $x=x_0$.
2) The slope of the tangent to the curve $y=f(x)$ at $x=x_0$.
3) The rate of change of $f(x)$ with respect to x at $x=x_0$.
4) The derivative $f'(x_0)$ at a point.