Math 141 Calculus I

Exam #3
April 23, 2018

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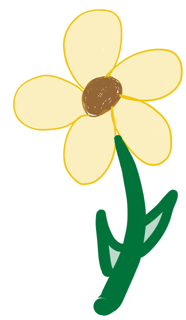
Name: Sols

Do not turn this page until told to do so. You will have a total of 1 hour and 15 minutes to complete the exam. You **must** show all work to receive full credit unless otherwise noted.
NO CALCULATOR/PHONE ALLOWED.
Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%. Draw a flower on this page if you read these directions in full.

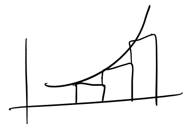
#	score	out of	#	score	out of
1		5	9		8
2		5	10		8
3		5	11		5
4		5	12		10
5		5	13		11
6		5	14		12
7		8	EC		5
8		8	Total		100



Remember: This exam has no impact on your worth as a human being. You got this!!!

True or False. No work/explanation required. 5pts each. True means always true.

1. If $f(x)$ is a continuous, increasing function on the interval $[a, b]$, then a left-hand Riemann sum gives an overestimate of the area under the curve.

False, 

2. l'Hôpital's Rule states that if $f(a) = g(a) = 0$ and f and g are differentiable on an open interval I containing a, and $g'(x) \neq 0$ on I if $x \neq a$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \left(\frac{f(x)}{g(x)} \right)'.$$

False, $\left(\frac{f(x)}{g(x)} \right)' \neq \frac{f'(x)}{g'(x)}$

3. When estimating the area under a curve using Riemann Sums, the more subintervals (rectangles) are used, the more accurate the estimate will be.

True

4. If $f(x)$ and $g(x)$ are two integrable functions over the interval $[a, b]$ and $f(x) \geq g(x)$ for all x in $[a, b]$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

True

5. If $f(x)$ and $g(x)$ are continuous with $\underbrace{f(x)}_{\text{upper}} \geq \underbrace{g(x)}_{\text{lower}}$ throughout $[a, b]$, then the area of the region between the curves from a to b is

$$\int_a^b [g(x) - f(x)]dx.$$

False

6. If $f(x)$ is integrable over the interval $[a, b]$, then

$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx.$$

True (may assume $a<c<b$)

Multiple Choice. No work required. 8pts each. Choose the best answer. There is only one correct answer but you may choose up to *two*. One correct choice will receive full credit. If you choose two and one of the answers is correct, you will receive half the points. If both choices are incorrect, you will receive zero points. If you choose more than two answers, you will receive zero points.

C

7. Evaluate the limit

$$\lim_{x\to\infty}(\ln 3x - \ln(x+1))$$

A. ∞ **B.** 0

C. $\ln 3$ **D.** 3

$$= \lim_{x\to\infty} \ln\left(\frac{3x}{x+1}\right) = \ln\left(\lim_{x\to\infty}\frac{3x}{x+1}\right)$$

D

8. Evaluate the limit

$$\lim_{x\to\infty}\sqrt[x]{x} = e^0$$

A. ∞ **B.** 0

C. e **D.** 1

$$\ln(x^{1/x}) = \frac{\ln(x)}{x}$$

$$\lim_{x\to\infty}\frac{\ln(x)}{x} \overset{L'H}{=} \lim_{x\to\infty}\frac{1}{x} = 0$$

D

9. Evaluate the integral.

$$\int_0^{\frac{\pi}{4}}\cos(t)dt$$

A. $\frac{\sqrt{3}}{2}$ **B.** $-\frac{\sqrt{2}}{2}$

C. $-\frac{\sqrt{3}}{2}$ **D.** $\frac{\sqrt{2}}{2}$

$$= \sin(\pi/4) - \sin(0)$$

A

10. Evaluate using the Fundamental Theorem of Calculus, Part I:

$$\frac{d}{dx}\int_1^{x^2}\csc(t)dt$$

A. $2x\csc(x^2)$ **B.** $\csc(x^2)$

C. $\csc(t)$ **D.** $\csc(t)+C$

$$= 2x\csc(x^2)$$

Short answer. You must show ALL work/explain your answer to receive full credit.

11. (5 pts) If $\int_1^7 f(x)dx = 10$ and $\int_3^7 f(x)dx = 6$, find $\int_1^3 f(x)dx$.

$$\int_1^3 f(x)dx + \int_3^7 f(x)dx = \int_1^7 f(x)dx$$

$$\int_1^3 f(x)dx + 6 = 10$$

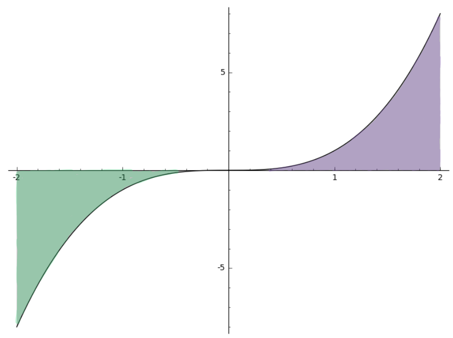
$$\boxed{\int_1^3 f(x)dx = 4}$$

12. (10 pts) Find the indefinite integral

$$\int \left(3x^2 - 7\sqrt{x} + \frac{3}{x^2+1}\right) dx$$

$$= \boxed{x^3 - \frac{14}{3}x^{3/2} + 3\arctan(x) + C}$$

13. (11 pts) Find the **total** area between the graph of $f(x) = x^3$ and the $x-$axis, between $x = -2$ and $x = 2$. That is, find the area of the shaded region below:



$$A = 2\int_0^2 x^3\,dx \quad \left(\text{or } \left|\int_{-2}^{0} x^3\,dx\right| + \int_0^2 x^3\,dx \right)$$

$$= 2\left[\frac{x^4}{4}\right]_0^2$$

$$= 2\left(\frac{2^4}{4} - 0\right) = \boxed{8}$$

14. (12 pts) Find the indefinite integral:

$$\int \frac{\sqrt{\ln(x)}}{x} dx$$

$u = \ln(x)$

$du = \frac{dx}{x}$

$$\int \sqrt{u}\,du = \int u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{3}(\ln(x))^{3/2} + C}$$

Extra Credit. (5 pts) No partial credit will be given for this problem.

Evaluate the integral

$$\int_0^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta} d\theta$$

$$= \int_0^{\pi/3} \frac{\sin\theta(1+\tan^2\theta)}{\sec^2\theta} d\theta$$

$$= \int_0^{\pi/3} \frac{\sin\theta(\sec^2\theta)}{\sec^2\theta} d\theta$$

$$= \int_0^{\pi/3} \sin\theta \, d\theta$$

$$= -\cos(\pi/3) + \cos(0)$$

$$= -\frac{1}{2} + 1 = \boxed{1/2}$$