Solutions

1.8: Operations on Functions

Basic Operations: Given two functions $f(x)$ and $g(x)$ you can create new functions using the basic operations on real numbers. $(+, -, \cdot, \div)$

Examples: Suppose $f(x) = 5x - 1$, $g(x) = 3x + 2$ and $h(x) = x^2 + 8$. Find the following functions.

- $f + g(x) = 5x - 1 + 3x + 2 = 8x + 1$
- $h - f(x) = x^2 + 8 - (5x - 1) = x^2 - 5x + 9$
- $f \cdot g(x) = (5x-1)(3x+2) = 15x^2 + 10x - 3x - 2 = 15x^2 + 7x - 2$

Composition of Functions: Given two function $f(x)$ and $g(x)$ you can create a new function by putting the output of one function as the input of the second function. This is the idea behind composition of functions.

Definition: For two function $f(x)$ and $g(x)$, the composite function $f \circ g$, also called the composition of f with g is defined by

$$f \circ g(x) = f(g(x)).$$

Example 1: If $f(t) = t^2$ and $g(t) = t + 2$, find

(a) $f(t+1) = (t+1)^2 = t^2 + 2t + 1$

(b) $f(t+h) = (t+h)^2 = t^2 + 2th + h^2$

(c) $f(g(t)) = (t+2)^2 = t^2 + 4t + 4$

(d) $g(f(t)) = t^2 + 2$

Example 2: If $f(x) = e^x$ and $g(x) = 5x + 1$, find

(a) $f \circ g(x) = e^{(5x+1)}$

(b) $g \circ f(x) = 5e^x + 1$

Example 3: Using the following table, find $f \circ g(0)$, $g \circ f(0)$, $f(g(1))$, $f(f(0))$, and $g(f(1))$.

x	0	1	2	3
$f(x)$	3	1	-1	-3
$g(x)$	0	2	4	6

$f \circ g(0) = f(g(0)) = f(0) = 3$

$g \circ f(0) = g(f(0)) = g(3) = 6$

$f(g(1)) = f(2) = -1$

$f(f(0)) = f(3) = -3$

$g(f(1)) = g(1) = 2$

Example 4: Use a new variable u for the inside functions to express each of the following as a composite function:

(a) $y = \ln(3t)$

(b) $w = 5(2x+3)^2$

(c) $P = e^{-0.03t}$

(a) $u = 3t$, $y = \ln u$

(b) $u = 2x+3$, $w = 5u^2$

(c) $u = -0.03t$, $P = e^u$