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Math 170: Section 2.1, 2.2 Lectures

Section 2.1: Simple Interest

Example 1 When "trading up", homeowners sometimes have to buy a new house before they sell their old house. One way to cover the costs of the new house until they get the proceeds from selling the old house is to take out a short-term *bridge loan.* Suppose a bank charges 12% simple annual interest on such a loan. How much will be owed at the maturation (the end) of a 90-day bridge loan of $90,000?

$$\underbrace{FV}_{\text{future value}} = \underbrace{PV}_{\text{present value}}(1+\underbrace{r}_{\text{rate}}\underbrace{t}_{\text{time}})$$

$$F = P(1+rt)$$

$$t = \frac{90}{365} \qquad P = 90{,}000 \qquad r = .12$$

$$F = 90000\left(1+.12\cdot\frac{90}{365}\right)$$

$$= \$92{,}663.01$$

Side note: Many banks use 360 days for this calculation instead of 365. Why?

$$\text{Amount owed} = F = 90000\left(1+.12\cdot\frac{90}{360}\right) = \$92{,}700$$

Example 2 The Megabucks Corporation is issuing 10-year bonds paying an annual rate of 6.5%. If you buy \$10,000 worth of bonds, how much interest will you earn every 6 months, and how much interest will you earn over the life of the bonds?

Every 6 months:

$t = 1/2$ $\quad I = Prt = 10000 \cdot .065 \cdot 1/2$

$= \$325$

10-year life:

$t = 10$ $\quad I = 10{,}000 \cdot .065 \cdot 10$

$= \$6{,}500$

So, at the end of 10 years, you will have turned the original \$10,000 into \$16,500.

Example 3 U.S. Treasury bills (T-bills) are short-term investments (up to 1 year) that pay you a set amount after a period of time; what you pay to buy a T-bill depends on the interest rate. A U.S. Treasury bill paying $10,000 ~~after~~ 6 months earns 3.67% simple annual interest. How much did it cost to buy?

$F = 10000$

$t = 1/2$

$r = .0367$

$$10000 = P(1 + .0367 \cdot 1/2)$$

$$P = \frac{10000}{(1 + .0367 \cdot 1/2)} = \$9{,}819.81$$

Definition The maturity value of a T-bill is the amount of money it will pay at the end of its life, that is, upon **maturity**. The cost of a T-bill is generally less than its maturity value. (An exception occurred during the financial meltdown of 2008 when T-bills were considered a "safe haven" investment and were sometimes selling at, or even above, their maturity values.) In other words, a T-bill will generally sell at a *discount*, and the discount rate is the *annualized* percentage of this discount; that is, the percentage is adjusted to give an annual percentage.

Examples 1. A 1-year \$10,000 T-bill with a discount rate of 5% will sell for 5% less than its maturity value of \$10,000, that is, for

$$10{,}000 - 10{,}000(.05) = \$9{,}500$$

2. A 6-month \$10,000 T-bill with a discount rate of 5% will sell at an actual discount rate of half of that - 2.5% less than its maturity value - since 6 months is half of a year.

$$10{,}000 - 10{,}000(.025) = \$9{,}750$$

3. A 3-month \$10,000 T-bill with a discount rate of 5% will sell at an actual discount of a fourth of 5%, so 1.25%

$$10{,}000 - 10{,}000(.0125) = \$9{,}875$$

The annual yield of a T-bill is the simple annual interest rate an investor earns when the T-bill matures.

Example 4 A T-bill paying \$10,000 after 6 months sells at a discount rate of 3.6%. What does it sell for? What is the annual yield?

annualized discount rate is 3.6%

So the actual discount is $\frac{3.6}{2}\% = 1.8\%$

So the selling price is

$$\underbrace{10{,}000}_{\text{Maturity Value}} - \underbrace{10{,}000\,(0.018)}_{\text{Discount}} = \boxed{\$9{,}820}$$

To find the annual yield, note the present value of the investment is the price the investor pays and the future value is the maturity value six months later.

$P = \$9{,}820$

$F = \$10{,}000$

$t = \frac{1}{2}$

$r = ?$

$$\frac{10{,}000}{9{,}820} = \frac{9{,}820\,(1 + r \cdot \frac{1}{2})}{9{,}820}$$

$$\frac{10000}{9{,}820} = 1 + \tfrac{1}{2}r$$

$$\frac{10000}{9820} - 1 = \frac{1}{2}r$$

$$r = 2\left(\frac{10000}{9820} - 1\right) \approx .0367 = \boxed{3.67\%}$$

Example 5 You are expecting a tax refund of \$800. Because it may take up to 6 weeks to get the refund, your tax preparation firm offers, for a fee of \$40, to give you an "interest-free" loan of \$800 to be paid back with the refund check. If we think of the fee as interest, what simple annual interest rate is the firm actually charging?

If we think of the fee as interest, then the future value (amount we will owe) is \$840.

$F = 840$

$P = 800$

$t = \frac{6}{52}$ (52 weeks in a year)

$$840 = 800\left(1 + r \cdot \frac{6}{52}\right)$$

$$\frac{840}{800} = 1 + \frac{6}{52} r$$

$$\frac{840}{800} - 1 = \frac{6}{52} r$$

$$\frac{52}{6}\left(\frac{840}{800} - 1\right) = r$$

$$r \approx 43.33\%$$

Moral of the story: SAVE your money! Wait the 6 wks

Recall first example we did Friday, April 13.

Section 2.2: Compound Interest

Example 1 You deposit \$1000 into a savings account. The bank pays you 5% interest, which it deposits into your account, or *reinvests*, at the end of each year. At the end of 5 years, how much money will you have accumulated?

(So, instead of taking our money and going out for half a night one Friday - we put it back in the account)

Time (yrs)	Amount (\$)
0	\$1000
1	$1000(1+.05) = \$1{,}050$
2	$1050(1+.05) = \$1{,}102.50$
	$= [1000(1+.05)]\cdot(1+.05) = 1000(1+.05)^{2}$
3	$1102.50(1+.05) = [1000(1+.05)^2](1+.05)$
	$= 1000(1+.05)^{3} = \$1{,}157.63$
5	$1000(1+.05)^5 = 1{,}276.28$
15	$1000(1+.05)^{15} = \$2{,}078.93$!!!

Simple Interest after 15 years: $1000(1+.05\cdot 15) = \$1750$

Banks often pay interest more often than once a year. Paying interest quarterly (four times per year) or monthly is common. If your bank pays interest monthly, how much will your $1000 deposit be worth after 5 years?

annual interest rate 5%

Monthly interest rate $\frac{.05}{12}$

$$1000\left(1+\frac{.05}{12}\right)^{?} \rightarrow \underbrace{12}_{\text{months}} \cdot \underbrace{5}_{\text{years}} = \text{total \# of times compounded}$$

$$1000\left(1+\frac{.05}{12}\right)^{60} = \$1283.36$$

Future Value for Compound Interest

The future value of an investment of P dollars earning interest at an annual rate of r compounded m times for t years is

$$F = P\left(1+\frac{r}{m}\right)^{mt}$$

annually $m=1$

semi-annually $m=2$

quarterly $m=4$

monthly $m=12$

weekly $m=52$

daily $m=365$

Example 2 In November 2011, the Bank of Montreal was paying 1.30% interest on savings accounts. If the interest is compounded quarterly, find the future value of a \$2000 deposit in 6 years. What is the total interest paid over the period?

$= r$

$= PV$

$= t$

$m = 4$

$r = 0.013$

$P = 2000$

$t = 6$

$$F = 2000\left(1 + \frac{.013}{4}\right)^{4 \cdot 6}$$

$$= 2000\left(1 + \frac{.013}{4}\right)^{24}$$

$$\approx \$2,161.97$$

Total interest is $F - P = 2161.97 - 2000 = \$161.97$

Definitions Example 2 illustrates the concept of the time value of money:
A given amount of money received now will usually be worth a different amount to us than the same amount received some time in the future. In the previous example, we can say that $2000 received now is worth the same as $2,161.97 received 6 years from now. Just as with simple interest, sometimes we want to know how much needs to be invested now to earn a certain amount in the future.

$$F = P\left(1+\frac{r}{m}\right)^{mt}$$

$$P = \frac{F}{\left(1+\frac{r}{m}\right)^{mt}}$$

(Remember, just as with simple interest, you don't need to memorize this formula)

In the preceding section, we mentioned that a bond pays interest until it reaches maturity, at which point it pays you back an amount called its maturity value. The two parts, the interest and maturity value, can be separated and sold and traded by themselves.
A zero coupon bond is a form of corporate bond that pays no interest during its life but, like a U.S. Treasury bill, promises to pay you the maturity value when it reaches maturity.
Zero coupon bonds sell for less than their maturity value, and the return on the investment is the difference between what the investor pays and the maturity value. Although no interest is actually paid, we measure the return on investment by thinking of the interest rate that would make the selling price (present value) grow to become the maturity value (future value).

Example 3 Megabucks Corporation is issuing 10-year zero coupon bonds. How much would you pay for bonds with a maturity value of \$10,000 if you wish to get a return of 6.5% compounded annually?

We think of a zero coupon bond as if it were an account earning compound interest.

Want the amount you pay - Present Value (Principal)

$F = 10{,}000$

$r = .065$

$t = 10$

$m = 1$

$$10{,}000 = P\left(1 + \frac{.065}{1}\right)^{1 \cdot 10}$$

$$10{,}000 = P(1 + .065)^{10}$$

$$P = \frac{10000}{(1+.065)^{10}} \approx \boxed{\$5{,}327.26}$$

Example 4 Inflation in East Avalon is 5% per year. TruVision television sets cost \$200 today. How much will a comparable set cost 2 years from now?

Inflation behaves like compound interest.
If inflation is 5% per year, then prices increase by 5% each year — $P(1+.05)$

$P = 200$
$r = .05$
$m = 1$
$t = 2$

$$F = 200\left(1+\frac{.05}{1}\right)^{1\cdot 2}$$
$$= 200(1+.05)^2$$
$$= \$220.50$$

Example 5 Inflation in North Avalon is 6% per year. Which is really more expensive, a car costing \$20,000 today or one costing \$22,000 in 3 years?

We can not compare the two costs directly because inflation makes \$1 today worth more than \$1 in 3 years. We need the two prices expressed in comparable terms so we convert to <u>constant dollars</u>.
We take the car costing \$22,000 three years from now and ask what it would cost TODAY.
That is, we convert the future value to present value:

$$P = \frac{22{,}000}{(1+.06)^3} \approx \$18{,}471.62$$

So the \$22000 in 3 years is actually LESS after adjusting for inflation.

Example 6 You have just won \$1 million in the lottery and are deciding what to do with it during the next year before you move to the South Pacific. Bank Ten offers 10% interest, compounded annually, while Bank Nine offers 9.8% compounded monthly. In which should you deposit your money?

Bank Ten

$$F = 1(1+0.10)^1 = \$1.1 \text{ million}$$

OR

$$F = 1{,}000{,}000(1+0.10)^1 = \$1{,}100{,}000$$

Bank Nine

$$F = 1\left(1+\frac{.098}{12}\right)^{12\cdot 1} = \$1.1025 \text{ million}$$

Bank Nine is better!

\$102,500 in interest instead of \$100,000

Another way to look at the calculation in Ex. 6 is that Bank 9 gave you a total of 10.25% interest on your investment over the year.

We call 10.25% the <u>effective interest rate</u> of the investment (also referred to a the <u>annual percentage yield</u> or <u>APY</u>); the stated 9.8% is called the <u>nominal</u> interest rate.

In general, to best compare two different investments, it is wisest to compare their effective interest rates.

Notice that we got 10.25% by the following calculation: $\left(1+\frac{.098}{12}\right)^{12}-1=.1025$

In general: The effective interest rate, r_{eff} of an investment paying a nominal interest rate of r_{nom} compounded m times per year is

$$\boxed{r_{eff}=\left(1+\frac{r_{nom}}{m}\right)^{m}-1}$$