Note that there were $44 + 14 + 2 + 0 = 60$ individuals who got five or more correct, so the probability of finding a "gifted" participant each time is about $60/1000 = .06$. In other words, if everyone is equally talented and each guess is correct with probability .3, then there will be five or more correct guesses out of eight tries with probability .06, or about 6% of the time.

7.6 Exercises are on pages 257–258.

THOUGHT QUESTION 7.7 Does using simulation to estimate probabilities rely on the relative frequency or the personal probability interpretation of probability? Explain. Whichever one you chose, could simulation be used to find probabilities for the other interpretation? Explain.*

7.7 Flawed Intuitive Judgments about Probability

People have poor intuition about probability assessments. In Case Study 7.1, Alicia's physician made a common mistake by informing her that the medical test was a good indicator of her disease status because it was 95% accurate as to whether she had the disease or not. As we learned in Example 7.24, the probability that she actually has the disease is small, about 2%. In this section, we explore this phenomenon, called *confusion of the inverse*, and some other ways in which intuitive probability assessments can be seriously flawed.

Confusion of the Inverse

Eddy (1982) posed the following scenario to 100 physicians:

> One of your patients has a lump in her breast. You are almost certain that it is benign; in fact you would say there is only a 1% chance that it is malignant. But just to be sure, you have the patient undergo a mammogram, a breast x-ray designed to detect cancer.
>
> You know from the medical literature that mammograms are 80% accurate for malignant lumps and 90% accurate for benign lumps. In other words, if the lump is truly malignant, the test results will say that it is malignant 80% of the time and will falsely say it is benign 20% of the time. If the lump is truly benign, the test results will say so 90% of the time and will falsely declare that it is malignant only 10% of the time.
>
> Sadly, the mammogram for your patient is returned with the news that the lump is malignant. What are the chances that it is truly malignant?

Most of the physicians to whom Eddy posed this question thought the probability that the lump was truly malignant was about 75% or .75. In truth, given the probabilities described in the scenario, *the probability is only .075*. The physicians' estimates were 10 times too high! When Eddy asked the physicians how they arrived at their answers, he realized they were confusing the answer to the actual question with the answer to a different question. "When asked about this, the erring physicians usually report that they assumed that the probability of cancer given that the patient has a positive x-ray was approximately equal to the probability of a positive x-ray in a patient with cancer" (1982, p. 254).

Robyn Dawes has called this phenomenon **confusion of the inverse** (Plous, 1993, p. 132). The physicians were confusing the conditional probability of cancer *given a positive x-ray* with the inverse, the conditional probability of a positive x-ray *given that the patient has cancer!*

It is not difficult to see that the correct answer is indeed .075. Let's construct a hypothetical table of 100,000 women who fit this scenario. In other words, these are women who would present themselves to the physician with a lump for which the

***HINT:** Could you tell a computer what proportions to use if you and your friend do not agree on what to use?

probability that it was malignant seemed to be about 1%. Thus, of the 100,000 women, about 1%, or 1000 of them, would have a malignant lump. The remaining 99%, or 99,000, would have a benign lump.

Further, given that the test was 80% accurate for malignant lumps, it would show a malignancy for 800 of the 1000 women who actually had one. Given that it was 90% accurate for the 99,000 women with benign lumps, it would show benign for 90%, or 89,100 of them, and malignant for the remaining 10%, or 9900 of them. Table 7.4 shows how the 100,000 women would fall into these possible categories.

Table 7.4 Breakdown of Actual Status versus Test Status for a Rare Disease

	Test Says Malignant	Test Says Benign	Total
Actually Malignant	800	200	1,000
Actually Benign	9,900	89,100	99,000
Total	10,700	89,300	100,000

Let's return to the question of interest. Our patient has just received a positive test for malignancy. Given that her test showed malignancy, what is the actual probability that her lump is malignant? Of the 100,000 women, 10,700 of them would have an x-ray showing malignancy. But of those 10,700 women, only 800 of them actually have a malignant lump! Thus, given that the test showed a malignancy, the probability of malignancy is just $800/10{,}700 = 8/107 = .075$.

Sadly, many physicians are guilty of confusion of the inverse. Remember, in a situation in which the *base rate* for a disease is very low and the test for the disease is less than perfect, there will be a relatively high probability that a positive test result is a false positive. If you ever find yourself in a situation similar to the one just described, you may wish to construct a table like the one above.

To determine the probability of a positive test result being accurate, you need only three pieces of information:

1. What the base rate or probability that you are likely to have the disease is, without any knowledge of your test results.
2. What the **sensitivity** of the test is, which is the proportion of people who correctly test positive when they actually have the disease.
3. What the **specificity** of the test is, which is the proportion of people who correctly test negative when they don't have the disease.

Note that items 2 and 3 are measures of the accuracy of the test. They do not measure the probability that people have the disease when they test positive or the probability that they do not have the disease when they test negative. Those probabilities, which are obviously the ones of interest to the patient, can be computed by constructing a table similar to Table 7.4 or by the other methods shown earlier in this chapter.

Specific People versus Random Individuals

According to the Federal Aviation Administration (http://www.ntsb.gov/aviation/Table6.htm), between 1989 and 2008, there were 64 fatal airline accidents for regularly scheduled flights on U.S. carriers. During that same time period, there were 227,166,913 flight departures. That means that the relative frequency of fatal accidents was about one accident per 3.5 million departures. Based on these kinds of statistics, you will sometimes hear statements like, "The probability that you will be in a fatal plane crash is 1 in 3.5 million" or "The chance that your marriage will end in divorce is 50%."

Do these probability statements really apply to you personally? In an attempt to personalize the information, reporters express probability statements in terms of individuals when they actually apply to the aggregate. Obviously, if you never fly, the probability that you will be in a fatal plane crash is 0. Here are two equivalent, correct ways to restate the aggregate statistics about fatal plane crashes:

- In the long run, about 1 flight departure out of every 3.5 million ends in a fatal crash.