MATH 170: GRAPH THEORY CASE STUDY

In reading maps, it is helpful to be able to distinguish countries or states by means of their colors. States that touch one another should not have the same color as it would be hard to find their boundaries. When coloring maps, it is optimal to use as few colors as possible. How does one determine how many different colors are needed to color any map?

This question was first posed by Francis Guthrie in 1852. He was a student of famous mathematician De Morgan (remember De Morgan's laws from logic?). He believed that it was possible to color any map with at most four colors so that no two countries that share a border had the same color. Some people say he recruited his entire family to color all sorts of maps trying to use only four colors. Once he was convinced, he worked tirelessly trying to find a proof of his conjecture. He finally turned the question over to De Morgan who was also unable to come up with a sufficient proof.

In 1879 a mathematician named Alfred Bray Kempe announced that he had a proof of the Four Color Conjecture. He received great acclaim but in 1890, Percy John Heawood published a paper which showed a flaw in Kempe's proof. Heawood was able to prove a weaker theorem, the Five Color Theorem: Any map may be colored with at most 5 colors so that no two countries sharing a border have the same color.

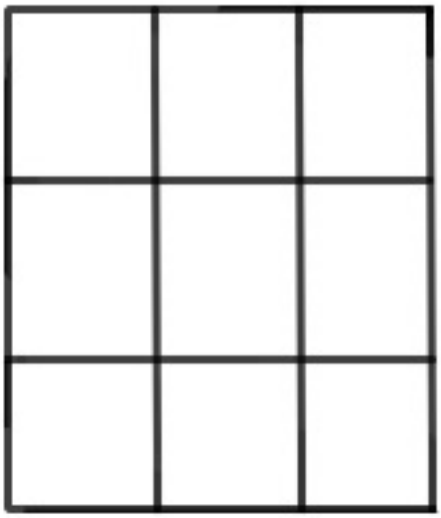
Not until 1976, over 120 years later, was the Four Color Conjecture finally the Four Color Theorem. Mathematicians Appel and Haken provided a computer assisted proof. The Four Color Theorem was the first major theorem to be proved using a computer and having a proof that could not be verified directly by humans. There is some debate amongst mathematicians about whether using computers is "cheating."

Sources:
https://www.mathsisfun.com/activity/coloring.html
http://www.colorado.edu/education/DMP/activities/graph/ddgact03.html

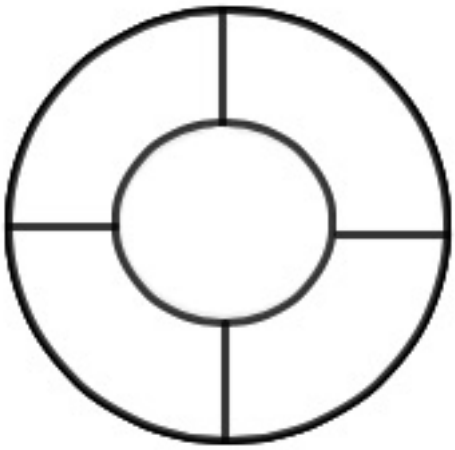
For questions 1-3, color in the patterns so that no two sections that share an edge have the same color. Having a common corner is OK, just not an edge.

If you do not have access to colored pencils, pens, or markers, you may represent colors by numbers. So if you need colors red, blue, and green, you can instead say you need colors 1, 2, and 3 where 1 represents red, 2 represents blue, and 3 represents green.

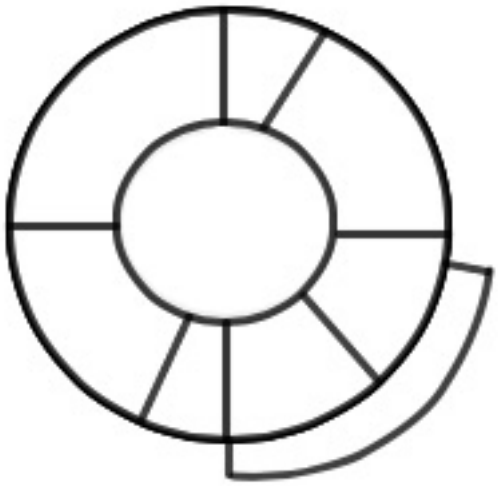
1. *What is the least number of colors you need to color the following image? What is the maximum number of colors you could use?*



2. *What is the least number of colors you need to color the following image? What is the maximum number of colors you could use?*



3. *What is the least number of colors you need to color the following image? What is the maximum number of colors you could use?*



4. *Consider the following map of 9 European countries. Find the fewest number of colors you need to color the map so that no two countries which share a border have the same color.*

5. *Describe your method for coloring the different maps in problems 1-4 when finding the least number of colors needed. Write down an explicit set of instructions (an algorithm) that any person could follow with any map and get the fewest number of colors. You may want to let a friend try your algorithm on a map and see if you get the same numbers. If they do not get the same answer as you, you may want to modify your algorithm.*

6. *Draw a graph (vertices and edges) which represents the map in problem number 4. Label your vertices clearly and explain what the edges between vertices represent. Are there any advantages to representing the map in this way? Explain your reasoning.*

7. *Some mathematicians believe that only the mind should be used to develop a proof of a theorem. Do you think using a computer to prove a mathematical theorem should be considered "cheating"? Why or why not?*