1.2 Combining Functions; Shifting & Scaling Graphs

Algebraic Operations on Functions

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$ $(1-x \geq 0)$

$D(f) = [0, \infty)$ $D(g) = (-\infty, 1]$

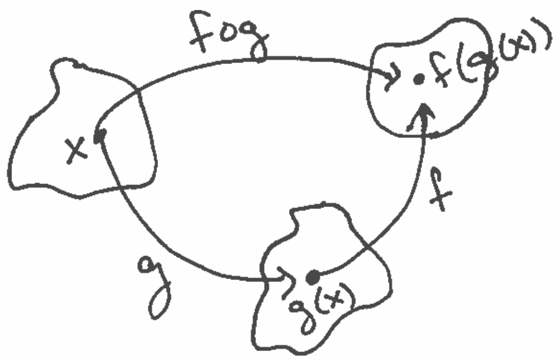
Function	Formula	Domain
$f+g$	$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$	$D(f+g) = [0,1]$ $= D(f) \cap D(g)$
$f-g$	$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$	$D(f-g) = [0,1]$
$f \cdot g$	$(f \cdot g)(x) = \sqrt{x} \cdot \sqrt{1-x} = \sqrt{x(1-x)}$	$D(f \cdot g) = [0,1]$
$\frac{f}{g}$	$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}}$	$D\left(\frac{f}{g}\right) = [0,1)$
$\frac{g}{f}$	$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}}$	$D\left(\frac{g}{f}\right) = (0,1]$

Composite Functions

If f and g are functions, the composite function $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of $f(x)$.



Ex: Let $f(x) = \sqrt{x}$ and $g(x) = x+1$.

$D(f) = [0, \infty)$ $\quad$ $D(g) = (-\infty, \infty)$

Find

(a) $(f \circ g)(x) = f(g(x)) = f(x+1) = \sqrt{x+1}$

$D(f \circ g) = [-1, \infty)$

(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1$

$D(g \circ f) = [0, \infty)$

(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt{x^{1/2}}$

$= (x^{1/2})^{1/2} = x^{1/4} = \sqrt[4]{x}$

$D(f \circ f) = [0, \infty)$

(d) $(g \circ g)(x) = g(g(x)) = g(x+1) = (x+1)+1 = x+2$

$D(g \circ g) = (-\infty, \infty)$

Transformations of Graphs

Given the graph of a function $f(x)$:

Vertical Shift $\quad f(x)+k$

$k>0$ vertical shift up by k

$k<0$ vertical shift down by k

Horizontal Shift $\quad f(x+k)$

$k>0$ horizontal shift left

$k<0$ horizontal shift right

For $c>1$, the graph is scaled:

$y=cf(x)$ stretch vertically by a factor of c

$y=\frac{1}{c}f(x)$ Compresses vertically by a factor of c

$y=f(cx)$ Compresses horizontally by a factor of c

$y=f(\frac{1}{c}x)$ stretch horizontally by a factor of c

For $c=-1$, the graph is reflected:

$y=-f(x)$ over the x-axis

$y=f(-x)$ over the y-axis