Math 142: Section 10.10 - Notes

1 Applications of Taylor Series

Evaluating Nonelementary Integrals Taylor series can be used to express nonelementary integrals in terms of series. Integrals like the one in the next example arise in the study of the diffraction of light.

Example 1 Express

$$\int \sin x^2 dx$$

as a power series.

Example 2 Estimate

$$\int_0^1 \sin x^2 dx$$

with an error of less than 0.001.

Euler's Identity A complex number is a number of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. If we substitute $x = i\theta$ (where θ is a real number) in the Taylor series for e^x and use the relations

to simplify the result, we obtain

Note: This does not *prove* that $e^{i\theta} = \cos\theta + i\sin\theta$ because we have not yet defined what it means to raise e to an imaginary power. Rather, it says how to define $e^{i\theta}$ to be consistent with other things we know about the exponential function for real numbers.

Definition (Euler's Identity): For any real number θ,

$$e^{i\theta} = \cos\theta + i\sin\theta.$$