SUBSTITUTION (5.5) NAME Sols

For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int (x^3+1)\,dx$

Nothing

$= \frac{1}{4}x^4 + x + C$

$\int x^2(x^3+1)^4\,dx$

u-sub

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{1}{3}\int u^4\,du$

$= \frac{(x^3+1)^5}{15} + C$

$\int \sqrt{x^3+1}\,dx$

Need more tools

$\int (x^3+1)^2\,dx$

Nothing

$\int (x^6+2x^3+1)\,dx$

$= \frac{x^7}{7} + \frac{x^4}{2} + x + C$

B. $\int \sqrt{x}(1-x^2)\,dx$

Nothing

$\int (x^{1/2} - x^{5/2})\,dx$

$= \frac{2}{3}x^{3/2} - \frac{2}{7}x^{7/2} + C$

$\int \sqrt{1-x^2}\,dx$

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$\int \frac{1}{\sqrt{1-x^2}}\,dx$

Nothing

$= \arcsin(x) + C$

$\int \frac{x\,dx}{\sqrt{1-x^2}}$

u-sub

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int \frac{1}{u^{1/2}}\,du$

$= -\frac{1}{2}\int u^{-1/2}\,du$

$= -\sqrt{1-x^2} + C$

C. $\int \cos^2 x \sin^3 x\,dx$

$= \int \cos^2 x \sin^2 x \sin x\,dx$

Trig Identity: $\sin^2 x = 1-\cos^2 x$

$= \int \cos^2 x (1-\cos^2 x)\sin x\,dx$

u-sub

$u = \cos x$

$du = -\sin x$

$= -\int u^2(1-u^2)\,du$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

$\int \sqrt{1-\cos^2 x}\,dx$

Trig Identity

$= \int \sqrt{\sin^2 x}\,dx$

$= \int \sin x\,dx$

$= -\cos x + C$

$\int \frac{dx}{\cos^2 x}$

Trig Identity

$= \int \sec^2 x\,dx$

$= \tan x + C$

$\int \frac{dx}{\cos x\sqrt{\sin x}}$

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D. $\int \tan x \sec x\,dx$

$= \sec x + C$

$\int \tan x \cos x\,dx$

$= \int \frac{\sin x}{\cos x}\cdot \cos x\,dx$

$= \int \sin x\,dx$

$= -\cos x + C$

$\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

u-sub

$u = \tan x$

$du = \sec^2 x$

$= \int \frac{1}{u^{1/2}}\,du$

$= 2\sqrt{\tan x} + C$

$\int \frac{dx}{\tan x + 1}$

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E. $\int e^{-x^2}\,dx$

Need more tools

$\int \frac{e^x}{3+e^x}\,dx$

u-sub

$u = 3+e^x$

$du = e^x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|3+e^x| + C$

$\int (e^x+3)\,dx$

Nothing

$= e^x + 3x + C$

$\int \frac{\ln(e^{2x})}{x^2}\,dx$

Nothing

$= \int \frac{2x}{x^2}\,dx = 2\int \frac{1}{x}\,dx$

$= 2\ln|x| + C$