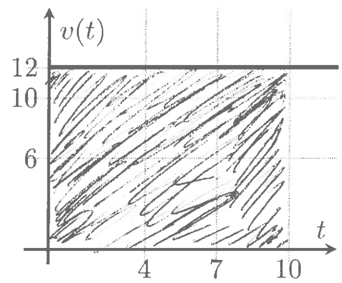
1. A girl is running at a velocity of 12 feet per second for 10 seconds, as shown in the velocity graph below.



How far does she travel during this time?

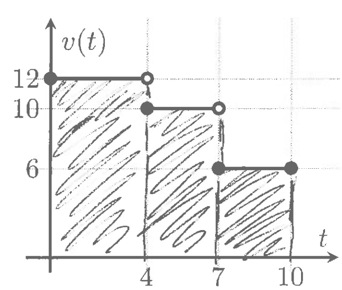
Distance = Rate · Time

$= 12(10) = 120$ ft

This distance can be depicted graphically as a rectangle. Shade such a rectangle and explain why it gives the distance.

The area of the rectangle is found by $v(t) \cdot t =$ Distance

2. Now the girl changes her velocity as she runs. Her velocity graph is approximately as shown:

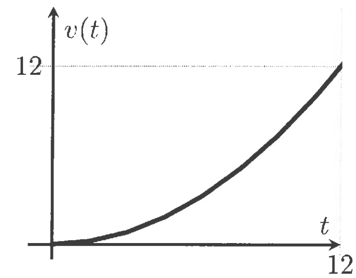


How far does she travel this time?

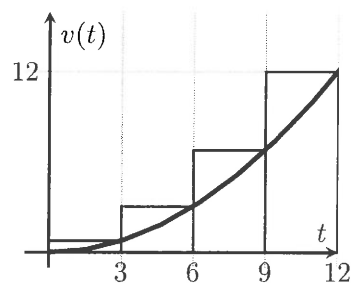
$12(4) + 10(3) + 6(3)$

$= 48 + 30 + 18 = 96$ ft

3. This time she starts off slowly and speeds up.



The velocity is given by $\boxed{v(t) = \frac{t^2}{12}}$ (time in seconds, velocity in ft/sec). We can no longer exactly find the distance travelled using areas of rectangles. But we can estimate it using areas of rectangles.



Find her velocity at time $t = 3, 6, 9, 12$ and use it to estimate her distance travelled in the first 12 seconds.

$$v(3) = \frac{(3)^2}{12} = \frac{9}{12} = \frac{3}{4}$$

$$v(6) = \frac{(6)^2}{12} = \frac{36}{12} = 3$$

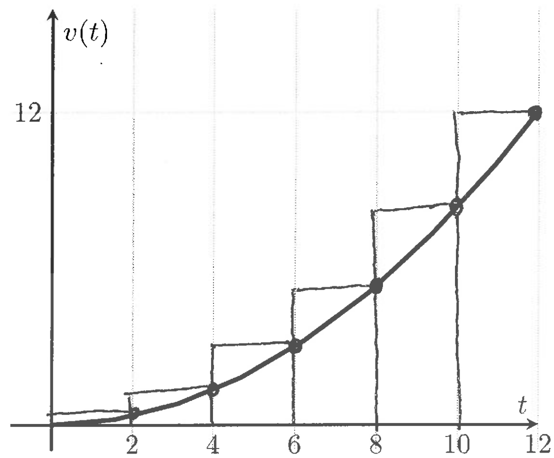
$$v(9) = \frac{(9)^2}{12} = \frac{27}{4}$$

$$v(12) = \frac{(12)^2}{12} = 12$$

$$\text{Distance} \approx \frac{3}{4}(3) + 3(3) + \frac{27}{4}(3) + 12(3) \approx 67.5 \text{ ft}$$

4. Now, for the same velocity function $v(t) = \frac{t^2}{12}$, get a better estimate of how far she travelled using $n = 6$ rectangles. Draw a graph showing the areas, and use their areas to estimate her distance travelled in the first 12 seconds.

$$\Delta t = \frac{12-0}{6} = 2$$



$$\text{Distance} \approx \frac{(2)^2}{12}(2) + \frac{4^2}{12}(2) + \frac{6^2}{12}(2) + \frac{8^2}{12}(2) + \frac{10^2}{12}(2) + \frac{12^2}{12}(2)$$

$$\approx 60.67 \text{ ft}$$

5. Now we will estimate the area when there are $n = 37$ rectangles.

   (a) Width of each rectangle:

   $$\Delta t = \frac{12-0}{37} = \frac{12}{37}$$

   (b) List of right-hand endpoint of each rectangle:

   $\frac{12}{37}$, $\frac{24}{37}$, $\frac{36}{37}$, ..., $\frac{432}{37}$, 12

   (c) List of heights of each rectangle:

   $\frac{(12/37)^2}{12}$, $\frac{(24/37)^2}{12}$, $\frac{(36/37)^2}{12}$, ..., $\frac{(432/37)^2}{12}$, $\frac{(12)^2}{12} = 12$

   (d) List of areas of rectangles:

   $\frac{(12/37)^2}{12}\left(\frac{12}{37}\right)$, $\frac{(24/37)^2}{12}\left(\frac{12}{37}\right)$, $\frac{(36/37)^2}{12}\left(\frac{12}{37}\right)$, ..., $\frac{(432/37)^2}{12}\left(\frac{12}{37}\right)$, $12\left(\frac{12}{37}\right)$

   (e) Sum of all areas:

   ______ + ______ + ______ +...+ ______ + ______