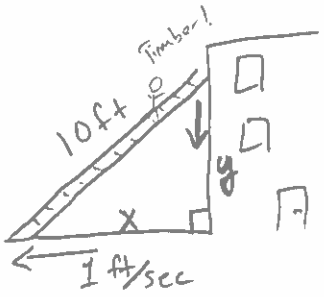
Math 141: Section 3.10 Related Rates Part II - Notes

Example 1 A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down when the bottom is 6 ft from the wall?



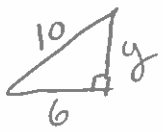
$\frac{dx}{dt} = 1$ ft/sec

$\frac{dy}{dt} = ?$ when $x = 6$ ft

* Chain Rule *

$x^2 + y^2 = 10^2$

$x^2 + y^2 = 100$



$6^2 + y^2 = 10^2$

$y = 8$

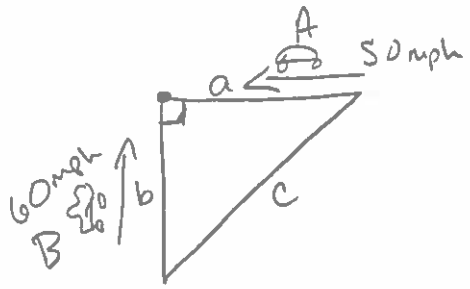
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(6)(1) + 2(8)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{3}{4}$ ft/sec

↳ going down!

Example 2 Car A is traveling west at 50 mph and Car B is traveling north at 60 mph. Both are headed for the intersection of two roads. At what rate are the cars approaching each other when Car A is 0.3 mi and Car B is 0.4 mi from the intersection?



$\frac{da}{dt} = 50 \quad \frac{db}{dt} = 60$

$\frac{dc}{dt} = ?$ when $a = 0.3$

$b = 0.4$

$a^2 + b^2 = c^2$

$(0.3)^2 + (0.4)^2 = c^2$

$c = 0.5$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

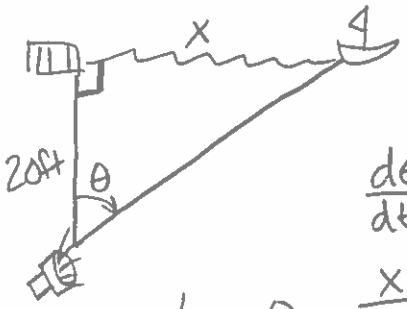
$2(0.3)(-50) + 2(0.4)(-60) = 2(0.5)\frac{dc}{dt}$

$\frac{dc}{dt} = -78$ mph

the cars are getting closer together

20ft

Example 3 A searchlight is following a boat moving east at 4 ft/sec. If the searchlight is located due south of where the boat left dock, how fast is the searchlight rotating when the boat is 15 ft away from the dock?



$\frac{dx}{dt} = 4$ ft/sec

$\frac{d\theta}{dt} = ?$ when $x = 15$ ft

$\tan\theta = \frac{x}{20}$ $\qquad$ $\tan\theta = \frac{15}{20}$

$20\tan\theta = x$ $\qquad$ $\tan\theta = 3/4$

$20\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$ $\qquad$ $\theta = \tan^{-1}(3/4)$

$20\sec^2(\underbrace{\quad}_{\tan^{-1}(3/4)})\frac{d\theta}{dt} = 4$

$20\sec^2(\tan^{-1}(3/4))\frac{d\theta}{dt} = 4$

$$\frac{d\theta}{dt} = \frac{4}{20\sec^2(\tan^{-1}(3/4))}$$

$$\frac{d\theta}{dt} = \frac{1}{5\sec^2(\tan^{-1}(3/4))}$$