MATH 186

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Applied Statistics

OUTLINE

Random Variables

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Answer: This variable is continuous.

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• Pick a random sample of 50 registered voters in a district and find the number who voted in the last county election.

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• Measure the amount of gasoline needed to drive your car 200 miles.

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Sum of the dice(X)	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Probability Distributions

Example 11

Dr. Mendoza developed a test to measure boredom tolerance. He administered it to a group of 20,000 adults between the ages of 25 and 35. The possible scores were 0,1,2,3,4,5, and 6, with 6 indicating the highest tolerance for boredom. The test results for this group are shown below. Find the probability distribution for this data.

Score	0	1	2	3	4	5	6
# of subjects	1400	2600	3600	6000	4400	1600	400

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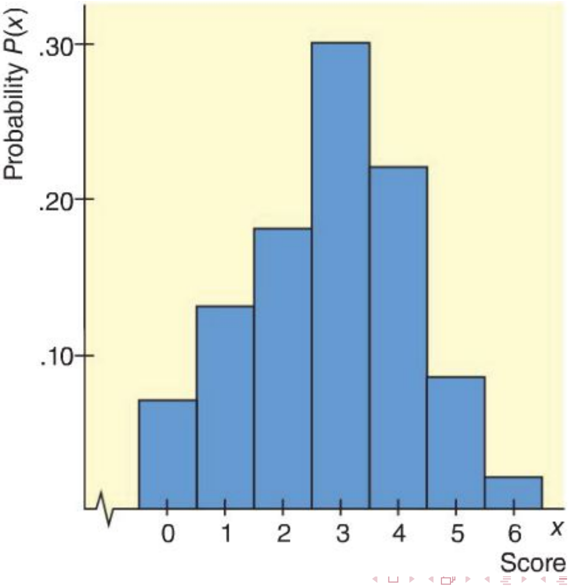
Score (X)	0	1	2	3	4	5	6
$\Pr(X)$	0.07	0.13	0.18	0.30	0.22	0.08	0.02

Probability Distributions



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- The **standard deviation** of a discrete population distribution is found by the formula

$$\sigma = \sqrt{\sum (X - \mu)^2 \Pr(X)}$$

Definition 12

The mean of a probability distribution is often called the **expected value** of the distribution.

Expected Value

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Example 13

At a carnival, you pay \$2.00 to play a coin-flipping game with three fair coins. You flip three coins at one time and you win \$1.00 for every head that appears. Should your expect to win more money than you pay to play?

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$$0 \cdot \Pr(0) = 0 \qquad 1 \cdot \Pr(1) = \frac{3}{8}$$

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$$0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8} = \frac{3}{2}$$

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- Since you earn \$1.00 for each heads, you should expect to win an average of \$1.50 per game. Since the game costs \$2.00 to play, you should expect a net loss of \$0.50 per game.

BINOMIAL EXPERIMENTS

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- There are exactly two possible outcomes for each trial. These outcomes can be considered <u>success</u> and <u>failure</u>.
- For each trial, the probability of success is the same. We denote the probability of success by p and the probability of failure by q. Because each trial results in either success or failure, $p + q = 1$.

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The central problem of a binomial experiment is to find the probability of r successes out of n trials.

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Example 17

Determine if the following experiment is a binomial experiment. If it is not a binomial experiment, explain why.

- Selecting 20 university students and recording their class rank.

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This is not a binomial experiment because there are more than two outcomes for the variable.

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Determine if the following experiment is a binomial experiment. If it is not a binomial experiment, explain why.

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Determine if the following experiment is a binomial experiment. If it is not a binomial experiment, explain why.

- Drawing five cards from a standard deck of cards without replacement and recording whether they are red or black.

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Determine if the following experiment is a binomial experiment. If it is not a binomial experiment, explain why.

- Drawing five cards from a standard deck of cards without replacement and recording whether they are red or black.

This is not a binomial experiment because the probability of success will change with each draw.

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Example 20

A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. We select 10 teenagers at random to determine the probability that exactly 4 of them will have part-time jobs. Find the values p, q, n, and r.

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A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. We select 10 teenagers at random to determine the probability that exactly 4 of them will have part-time jobs. Find the values p, q, n, and r.

- We will consider having a part-time job a success.

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- Since p is the probability of success, the example states that $p = 0.3$.
- We can compute $q = 1 - p = 0.7$. Recall that q is the probability of failure.

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- We can compute $q = 1 - p = 0.7$. Recall that q is the probability of failure.
- We consider each selected teenager a trial. So $n = 10$.
- Since we want to consider the probability that exactly 4 of the selected teenagers will have a part-time job, $r = 4$.

Binomial Probability Distribution Formula

In a binomial experiment, the probability of r successes out of n trials is given by the formula

$$\Pr(r) = \frac{n!}{r!(n-r)!} p^r \cdot q^{n-r} = (C_{n,r}) \cdot p^r \cdot q^{n-r}$$

where p is the probability of success in each trial and q is the probability of failure in each trial.

Binomial Probability Distribution Formula

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Example 21

A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. If we select 10 teenagers at random, what is the probability that exactly 4 of them will have part-time jobs?

- In the previous example we found the following values

$$p = 0.3 \qquad q = 0.7$$

$$n = 10 \qquad r = 4$$

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- In the previous example we found the following values

$$p = 0.3 \qquad q = 0.7$$
$$n = 10 \qquad r = 4$$

- Using the binomial probability distribution formula

$$\Pr(4) = \frac{10!}{4!(10-4)!}(0.3)^4(0.7)^{10-4}$$
$$\approx 0.2$$

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If a die is rolled 20 times, what is the probability that exactly half of the rolls will land on 3?

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- Next we identify $n = 20$, $r = 10$, $p = 1/6$ and $q = 5/6$.

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$$\Pr(\text{Ten 3s}) = \frac{20!}{10!(20-10)!}\left(\frac{1}{6}\right)^{10} \cdot \left(\frac{5}{6}\right)^{20-10}$$

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$$\begin{aligned} \Pr(\text{Ten 3s}) &= \frac{20!}{10!(20-10)!}\left(\frac{1}{6}\right)^{10} \cdot \left(\frac{5}{6}\right)^{20-10} \\ &\approx 0.00049 \end{aligned}$$

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- $\mu = np$
- $\sigma = \sqrt{npq}$.

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The mean value μ can be thought of as the **expected number of successes** in the experiment.

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If we roll a single die 20 times, how many times can we expect 3 to roll?

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If we roll a single die 20 times, how many times can we expect 3 to roll?

- Using the binomial experiment formula for μ, we can expect the number of 3s rolled to be

$$\mu = 20 \cdot \left(\frac{1}{6}\right) = 3.\overline{3}$$

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If we roll a single die 20 times, how many times can we expect 3 to roll? Find the standard deviation for the number of 3s rolled.

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If we roll a single die 20 times, how many times can we expect 3 to roll? Find the standard deviation for the number of 3s rolled.

- Using the binomial experiment formula for σ, find the standard deviation for the number of 3s rolled to be

$$\sigma = \sqrt{20 \cdot \left(\frac{1}{6}\right) \cdot \left(\frac{5}{6}\right)} = 1.\overline{6}$$