L'HOPITAL'S RULE (4.5) NAME Sols

In each problem determine if L'Hopital's Rule applies. If so, use the rule to find the limit. If not, find the limit numerically. Express your final answers in exact form.

1. $\lim_{x\to\pi}\dfrac{\sin(3x)}{x-\pi}$

$\dfrac{\sin(3\pi)}{\pi-\pi} \to \dfrac{0}{0}$ ✓

$\lim_{x\to\pi}\dfrac{3\cos(3x)}{-\pi}$

$= \dfrac{3(-1)}{-\pi}$

$= 3/\pi$

2. $\lim_{t\to0}\dfrac{e^{2t}-1}{e^{t}}$

$= \dfrac{e^0-1}{e^0}$

$= \dfrac{1-1}{1}$

$= 0$

3. $\lim_{\theta\to0}\dfrac{\arctan\theta}{2\theta}$

$\dfrac{\arctan(0)}{2(0)} \to \dfrac{0}{0}$ ✓

$\lim_{\theta\to0}\dfrac{\frac{1}{1+\theta^2}}{2} = 1/2$

4. $\lim_{x\to\infty}\dfrac{e^{-x}}{1+\ln x}$

$\to \dfrac{0}{\infty}$

$= 0$

5. $\lim_{x\to\infty}\dfrac{(\ln x)^2}{x}$

$\to \dfrac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty}\dfrac{2\ln x\cdot\frac{1}{x}}{1}$

$= \lim_{x\to\infty}\dfrac{2\ln x}{x} \to \dfrac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty}\dfrac{\frac{2}{x}}{1} = 0$

6. $\lim_{u\to\infty}\dfrac{\sqrt{u^2+1}}{u}$

$= 1$

(Degree top = Degree bottom)

7. $\lim_{y\to0}\dfrac{2^{y}}{y^2}$

$\to \dfrac{1}{\text{small positive}}$

$= \infty$

8. $\lim_{x\to1^+}\left(\dfrac{1}{\ln(1+x)}-\dfrac{1}{x}\right)$

$= \dfrac{1}{\ln 2} - 1$

9. $\lim_{\theta\to\infty}\theta\sin\left(\dfrac{1}{\theta}\right)$

To apply L'H, rewrite:

$\lim_{\theta\to\infty}\dfrac{\sin(1/\theta)}{\frac{1}{\theta}} \to \dfrac{0}{0}$

$\overset{L'H}{=} \lim_{\theta\to\infty}\dfrac{-\frac{1}{\theta^2}\cos(1/\theta)}{-1/\theta^2}$

$= \lim_{\theta\to\infty}\cos(1/\theta) = \cos(0)$

$= 1$

10. $\lim_{z\to 0^+} \cos\left(\frac{1}{z}\right)$

DNE

11. $\lim_{t\to\infty} \cos^2\left(\frac{1}{t}\right)$

$= \cos^2(0)$

$= 1$

12. $\cancel{\lim_{x\to 0} \frac{x^2+3x}{\sinh x}}$

13. $\lim_{y\to 0} \frac{y}{\sqrt[3]{\sin y}} \to \frac{0}{0}$

$\overset{L'H}{=} \lim_{y\to 0} \frac{1}{\frac{1}{3}(\sin y)^{-2/3}\cos y}$

$= \lim_{y\to 0} \frac{3(\sin y)^{2/3}}{\cos y}$

$= \frac{3(0)^{2/3}}{1}$

$= 0$

14. $\lim_{x\to 0^+} \frac{\cot x}{\ln x} \to \frac{\infty}{-\infty} \to -\frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-\csc^2 x}{1/x} \to \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-2\csc x \csc x \cot x}{-1/x^2}$

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Let's be clever:

$\lim_{x\to 0^+} \frac{\cot x}{\ln x}$

$= \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{1}{\ln x}$

$= \lim_{x\to 0^+} \cos x \cdot \frac{1}{\sin x} \cdot \frac{1}{\ln x}$

$= \lim_{x\to 0^+} \cos x \cdot \frac{x}{\sin x} \cdot \frac{1}{x\ln x}$

$= 1 \cdot 1 \cdot \frac{1}{\text{small, negative}}$

$= -\infty$

15. $\lim_{x\to\infty} \frac{x+\sin(2x)}{x}$

$= \lim_{x\to\infty} 1 + \frac{\sin(2x)}{x}$

$= 1 + \underbrace{\lim_{x\to\infty} \frac{\sin(2x)}{x}}_{\text{Squeeze Theorem}}$

$-1 \le \sin(2x) \le 1$

$-\frac{1}{x} \le \frac{\sin(2x)}{x} \le \frac{1}{x}$

$= 1 + 0$

$= 1$