More Practice Exam 1 Sols

16) $\dfrac{f(1)-f(-1)}{1--1} = \dfrac{1^3+1-((-1)^3+1)}{2} = \dfrac{2-0}{2} = \boxed{1}$

17) $\dfrac{f(2+h)-f(2)}{h} = \dfrac{(2+h)^2-2(2+h)-3-(-3)}{h} = \dfrac{\cancel{4}+4h+h^2-\cancel{4}-2h}{h}$

$= 4+h-2 = 2+h$ As $h \to 0$, $2+h \to 2$ so $m=2$.

$\boxed{y+3=2(x-2)}$

18) $\lim\limits_{x\to 1^+} \dfrac{1}{x-1} = \infty$ $\lim\limits_{x\to 1^-} \dfrac{1}{x-1} = -\infty$ The one-sided limits do not agree and are unbounded.

19) (a) $\lim\limits_{t\to -1} \dfrac{t^2+3t+2}{t^2-t-2} = \lim\limits_{t\to -1} \dfrac{(t+2)\cancel{(t+1)}}{(t-2)\cancel{(t+1)}} = \lim\limits_{t\to -1} \dfrac{t+2}{t-2} = \boxed{-\frac{1}{3}}$

(b) $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} \cdot \dfrac{\sqrt{x}+3}{\sqrt{x}+3} = \lim\limits_{x\to 9} \dfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = \lim\limits_{x\to 9} \dfrac{1}{\sqrt{x}+3} = \boxed{\frac{1}{6}}$

20) $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0} \dfrac{(1+h)^2-1}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \lim\limits_{h\to 0} 2+h = \boxed{2}$

21) Use the Squeeze Theorem:

$\lim\limits_{x\to 0} 2-x^2 = 2 = \lim\limits_{x\to 0} 2\cos(x)$ so $\lim\limits_{x\to 0} g(x) = 2$

22) $0<|x-c|<\delta \Rightarrow |f(x)-L|<\varepsilon$.

Find δ by working backwards: $|9-x-5|<\varepsilon$

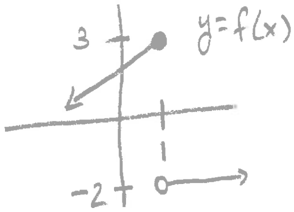
$|-x+4|<\varepsilon$

$|-1(x-4)|<\varepsilon$

$|-1||x-4|<\varepsilon$

$|x-4|<\varepsilon$

Let $\delta=\varepsilon$. Then $|x-4|<\delta=\varepsilon \Rightarrow |f(x)-5|<\varepsilon$.

23) Ex: 

$\lim_{x\to 1^+} f(x) = -2$ $\lim_{x\to 1} f(x)$ DNE

$\lim_{x\to 1^-} f(x) = 3$ (So, f is discontinuous at $x=1$)

24) Consider $\frac{|x+2|}{x+2}$. For values larger than (to the right of)
25)
-2, $\frac{|x+2|}{x+2} = 1$. For values less than -2 (to the left of -2), $\frac{|x+2|}{x+2} = -1$.

Thus, $\lim_{x\to -2^+} (x+3)\frac{|x+2|}{x+2} = (-2+3)(1) = \boxed{1}$

and $\lim_{x\to -2^-} (x+3)\frac{|x+2|}{x+2} = (-2+3)(-1) = \boxed{-1.}$

Section 2.5

1) True (assuming $\lim_{x\to c} f(x)$ and $f(c)$ exist)

2) False (we were not given that g is also continuous)

3) False ($g\circ f$, $g(f(c))$, is continuous at c)

4) $f(x) = x^3+5x^2+5x$

$f(-\frac{1}{2}) = (-\frac{1}{2})^3 + 5(-\frac{1}{2})^2 + 5(-\frac{1}{2})$

$= -\frac{1}{8} + \frac{5}{4} - \frac{5}{2}$

$= -\frac{11}{8} < 0$

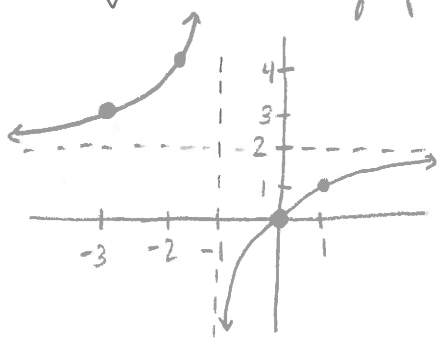
$f(1) = (1)^3 + 5(1)^2 + 5(1)$

$= 11 > 0$

By the IVT, f has a root between $-\frac{1}{2}$ and 1.

5) Horizontal Asymptote(s): $\lim_{x \to \pm\infty} \frac{2x}{x+1} = 2$, $y = 2$

Vertical Asymptote: $\lim_{x \to -1^-} \frac{2x}{x+1} = \infty$ $\lim_{x \to -1^+} \frac{2x}{x+1} = -\infty$



$x = -1$

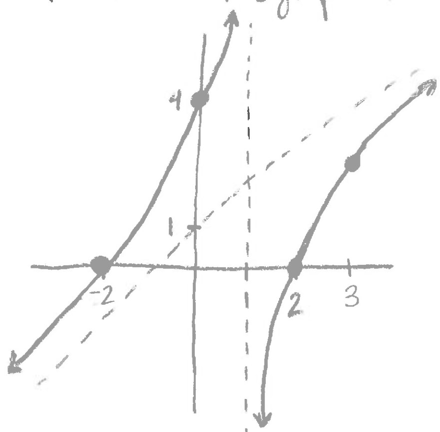
6). Oblique Asymptote:

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2 - 4} \\ \underline{-x^2 + x} \\ x - 4 \\ \underline{-x + 1} \\ -3 \end{array}$$

$\frac{x^2-4}{x-1} = x+1 + \frac{-3}{x-1}$

$y = x + 1$

Vertical Asymptote: $\lim_{x \to 1^+} \frac{x^2-4}{x-1} = -\infty$ $\lim_{x \to 1^-} \frac{x^2-4}{x-1} = \infty$



$x = 1$

7) $\lim_{x \to \infty} \frac{\sqrt{x^2+25} - \sqrt{x^2-1}}{1} \cdot \frac{\sqrt{x^2+25} + \sqrt{x^2-1}}{\sqrt{x^2+25} + \sqrt{x^2-1}}$

$= \lim_{x \to \infty} \frac{x^2 + 25 - (x^2 - 1)}{\sqrt{x^2+25} + \sqrt{x^2-1}}$

$= \lim_{x \to \infty} \frac{26}{\sqrt{x^2+25} + \sqrt{x^2-1}} = \boxed{0}$

8) $\lim\limits_{x\to 2^+} \frac{1}{x-2} = \boxed{\infty}$

9) $m = f'(1) = \lim\limits_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0} \frac{(1+h)^3+3(1+h)-4}{h}$

$= \lim\limits_{h\to 0} \frac{(1+h)(1+2h+h^2)+3+3h-4}{h}$

$= \lim\limits_{h\to 0} \frac{\cancel{1}+3\cancel{h}+3h^{\cancel{2}}+h^{\cancel{3}\,2}+\cancel{3}+3\cancel{h}-\cancel{4}}{\cancel{h}}$

$= \lim\limits_{h\to 0} (3+3h+h^2+3) = 6$, $\boxed{y-4 = 6(x-1)}$

10) $f'(x) = \lim\limits_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim\limits_{h\to 0} \frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$

$= \lim\limits_{h\to 0} \frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2} = \lim\limits_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2x\cancel{h}-h^{\cancel{2}}}{\cancel{h}x^2(x+h)^2}$

$= \lim\limits_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^2\cdot x^2} = \frac{-2x}{x^4} = \boxed{-\frac{2}{x^3}}$

$f'(-1) = -\frac{2}{(-1)^3} = \boxed{2}$

$f'(2) = -\frac{2}{2^3} = \boxed{-\frac{1}{4}}$

$f'(\sqrt{3}) = -\frac{2}{(\sqrt{3})^3} = \boxed{-\frac{2}{3\sqrt{3}}}$