Sols

Math 141 Calculus I

Exam #1 B
October 4, 2017

Instructor: Ann Clifton

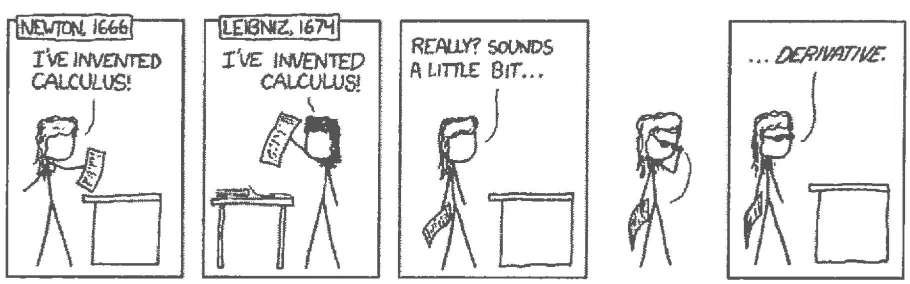
Name: ______________________________

Do not turn this page until told to do so.
You will have a total of 1 hour and 15 minutes to complete the exam. When specified, you **must** show all work to receive full credit. NO CALCULATOR/PHONE ALLOWED. Draw a pumpkin on this page if you read this.
Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

+1

#	score	out of	#	score	out of
1		4	9		6
2		4	10		6
3		4	11		14
4		4	12		20
5		4	13		16
6		6			
7		6	EC		5
8		6	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



True or False. No work/explanation required. True means ALWAYS true. 4pts each.

1. If L and c are real numbers and $\lim_{x\to c} f(x) = L$, then $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer.

True

2. If $P(x)$ and $Q(x)$ are polynomials, $Q(c) \neq 0$, then $\lim_{x\to c} \dfrac{P(x)}{Q(x)} = \dfrac{P(c)}{Q(c)}$.

True

3. If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

True

4. If $f(x) \leq g(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself, and the limits of f and g both exist as x approaches c, then $\lim_{x\to c} f(x) \leq \lim_{x\to c} g(x)$.

True

5. If the function f is continuous at $x = c$ and g is a function of x, then $f + g$ is continuous at $x = c$.

False

Multiple Choice. No work required. 6 points each. Choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive half the points.

6. Find the limit:

$$\lim_{y\to 2} \frac{y+2}{y^2+5y+6} = \frac{\lim_{y\to 2}(y+2)}{\lim_{y\to 2}(y^2+5y+6)} = \frac{4}{20} = \frac{1}{5}$$

A. 0 **B.** 1/16

C. 1/5 **D.** Does Not Exist

7. Find the average rate of change of the function over the given interval:

$$P(\theta) = \theta^2 - 4\theta + 5, \ [1, 2]$$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(2)-f(1)}{2-1}$$

$$= \frac{(2^2-4(2)+5)-(1^2-4(1)+5)}{1}$$

$$= 1-2 = -1$$

A. −1/2 **B.** −1

C. 3 **D.** 3/2

8. Find the limit:

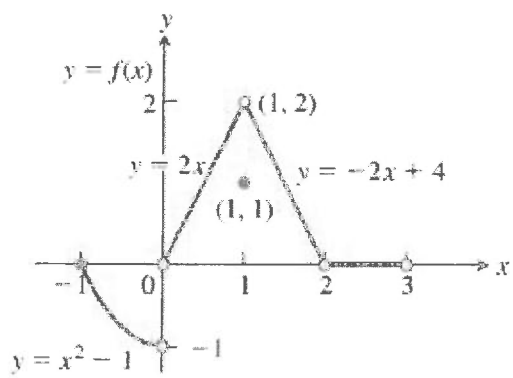
$$\lim_{x\to-\infty}\left(\frac{x^2+x-1}{8x^2-3}\right)^{1/3} = \left(\lim_{x\to-\infty}\frac{x^2+x-1}{8x^2-3}\right)^{1/3}$$

$$= \left(\tfrac{1}{8}\right)^{1/3} = \tfrac{1}{2}$$

A. 0 B. 1/2

C. 1/9 D. Does Not Exist

Use the graph below for questions 9 and 10.



9. Using the given graph, find $\lim_{x\to 0^+} f(x)$.

A. 0 B. 1

C. 2 D. Does Not Exist

10. Using the given graph, determine whether the function $f(x)$ is continuous at the point $x = 2$. Explain why or why not.

A. $f(x)$ is NOT continuous at $x = 2$ as $f(2) \neq \lim_{x\to 2} f(x)$.

B. $f(x)$ is NOT continuous at $x = 2$ as there is a jump discontinuity there.

C. $f(x)$ IS continuous at $x = 2$ as $\lim_{x\to 2} f(x) = f(2)$.

D. $f(x)$ IS continuous at $x = 2$ as $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.

Short Answer. You must show all work to receive full credit. If you need more space, use the provided scrap paper and write a note indicating where to find your work.

11. (14 points) Let

$$f(x) = \frac{x^3 + x^2 - 56x}{x + 8}.$$

(a) Does $f(x)$ have a discontinuity? If so, is it removable?

Yes, at $x = -8$.

$$\frac{x^3 + x^2 - 56x}{x+8} = \frac{x(x^2 + x - 56)}{x+8} = \frac{x\cancel{(x+8)}(x-7)}{\cancel{x+8}}$$

$$= x(x-7)$$

yes, it is removable

(b) Use limit laws to evaluate

$$\lim_{x \to -8} \frac{x^3 + x^2 - 56x}{x + 8}.$$

$$= \lim_{x \to -8} x(x-7)$$

$$= -8(-8-7)$$

$$= -8(-15)$$

$$= 120$$

12. (20 points) Find the derivative, $f'(x)$, using the limit definition, for the function $f(x) = x^2 - x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-(x+h)-(x^2-x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2x\cancel{h}+h^2-\cancel{x}-h-\cancel{x^2}+\cancel{x}}{\cancel{h}}$$

$$= \lim_{h\to 0} 2x+h-1 = \boxed{2x-1}$$

13. (16 points) If

$$\lim_{x\to -2} \frac{f(x)}{x^2} = 1, \quad \Rightarrow \frac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} x^2} = 1$$

$$\Rightarrow \frac{\lim_{x\to -2} f(x)}{4} = 1$$

find

(a) $\lim_{x\to -2} f(x)$

$= 4$

(b) $\lim_{x\to -2} \frac{f(x)}{x}$ $= \dfrac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} x} = \dfrac{4}{-2} = -2$

Extra Credit (5 points) No partial credit will be given for this problem. Give exact answers.

For the given function $f(x)$ and values of L, c, and $\epsilon > 0$ determine the largest value for $\delta > 0$ such that $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$.

$$f(x) = 6x + 4, \qquad L = 34, \qquad c = 5, \qquad \epsilon = 0.6$$

$|f(x) - L| < \varepsilon$

$|6x + 4 - 34| < 0.6$

$-0.6 < 6x - 30 < 0.6$

$-0.6 < 6(x-5) < 0.6$

$-0.1 < x - 5 < 0.1$

$|x - 5| < 0.1$ $\boxed{\delta = 0.1}$