Instructor: Ann Clifton   Name: _____

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Give exact answers. Indicate your final answer with a $\boxed{\text{box}}$.

1. State the Quadratic Formula.
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. State the formula for the amount of money after t years if P dollars is invested at a rate of r compounded n times per year.
$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

3. State the formula for the amount of money after t years if P dollars is invested at a rate of r compounded continuously.
$$A(t) = Pe^{rt}$$

4. State the definition of the logarithmic function.
$$\log_a x = y \text{ if and only if } a^y = x$$

5. State the Change of Base Formula.
$$\log_a x = \frac{\log_b x}{\log_b a}$$

6. A function is one-to-one if different inputs produce **different** **outputs**. You can tell from the graph that a function is one-to-one by using the **horizontal** **line** Test.

7. For a function to have an inverse, it must be **one-to-one**.

8. Let $f(x)$ be a function and assume it has an inverse, $f^{-1}(x)$. Then $f(f^{-1}(x)) = f^{-1}(f(x)) = $**x**, the **identity** function.

9. State the General Form of a quadratic function: $\underline{f(x) = ax^2 + bx + c}$. State the Standard Form of a quadratic function: $\underline{f(x) = a(x - h)^2 + k}$.

10. The bacteria population is a certain culture grows exponentially. **(a)** Find the one-hour growth rate if the 20-minute growth rate is 0.08. **(b)** Find the 10-minute growth rate if the 20-minute growth rate is 0.08. *Give an exact answer. Do not use your calculator to approximate.*

$$(a) \ (0.08)^3$$

$$(b) \ (0.08)^{1/2}$$

11. If \$10,000 is invested at an interest rate of 8% per year, compounded monthly, find the value of the investment after 4 years. *Give an exact answer.*

$$A(t) = 10000\left(1 + \frac{0.08}{12}\right)^{12\cdot4}$$

12. Find the function $f(x) = Ca^x$ given the following two points on the graph: $(2, \frac{13}{4})$ and $(0, 13)$.

$$f(x) = 13\left(\frac{1}{2}\right)^x$$

13. Evaluate the logarithm:

   (a) $\log_8 64 = 2$
   (b) $\log_2 2 = 1$
   (c) $\log 1 = 0$
   (d) $3^{\log_3 x} = x$
   (e) $\ln e^5 = 5$

14. Solve for x: $\log_4 x = 3$

$$x = 64$$

15. Solve for x: $\log_x 9 + \log_x 3 + 1 = 4$

$$x = 3$$

16. Use the Laws/Properties of Logarithms to expand the given expression:

$$\log\left(\frac{x^3\sqrt{y}}{z^4}\right)$$

$$= 3\log x + \frac{1}{2}\log y - 4\log z$$

17. Use the Laws/Properties of Logarithms to combine the given expression (*Be sure to simplify your answer completely!*:

$$\frac{1}{3}\log x + \log(x^2 - 4) - \log(x + 2)$$

$$= \log(x^{1/3}(x - 2))$$

18. Find a model for the amount $A(t)$ after t years if \$5,000 is invested at a rate of 5% compounded continuously.

$$A(t) = 5000e^{.05t}$$

19. Find $f^{-1}(x)$ if $f(x) = \ln x + 6$. Check to make sure your answer is correct (evaluate $f(f^{-1}(x))$ and $f^{-1}(f(x))$).

$$f^{-1}(x) = e^{x-6}$$

20. Let $f(x) = 2x^2 - 4x + 2$.

(a) Write the function in Standard Form by completing the square.
$$f(x) = 2(x-1)^2$$

(b) What is the vertex of the parabola?
$$(1, 0)$$

(c) Does the function have a maximum or a minimum value? What is the maximum/minimum value?

The function has minimum value of 0 when x=1.

(d) Calculate the discriminant. How many x-intercepts does the function have?

D=0; f(x) has exactly one x-intercept.

(e) Find the x- and y-intercepts. If there are none, write NONE.
x-intercepts: **(1,0)**
y-intercept: **(0,2)**

(f) Using parts (a)-(e), sketch a graph of the function. Be sure to label the vertex and intercept(s).