Math 141: Calc I

Sols For extra practice,
      evaluate the integrals.
      *Don't forget "+C"!!!
Identifying Integral Substitutions

Goal: To identify what (if any) u-substitutions are necessary to compute an integral and to practice making such substitutions.

For each problem, identify what (if any) u-substitution(s) need to be made to evaluate each integral. Make the substitution and simplify, but **do not** evaluate the integral.

1. $\int x \sin(x^2)\, dx$

$u = x^2$

$du = 2x\, dx$

$\frac{1}{2} \int \sin(u)\, du$

2. $\int \sqrt{x}(x + 3)\, dx$

$= \int x^{1/2}(x+3)\, dx$

$= \int (x^{3/2} + 3x^{1/2})\, dx$

3. $\int x\sqrt{x+3}\, dx$

$u = x+3 \rightarrow x = u-3$

$du = dx$

$\int (u-3)\sqrt{u}\, du$

$= \int (u^{3/2} - 3u^{1/2})\, du$

4. $\int \frac{\sqrt{\ln(x)}}{x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$\int \sqrt{u}\, du = \int u^{1/2}\, du$

1

5. $\int \frac{x+4}{x}\, dx$

$= \int \left(1 + \frac{4}{x}\right) dx$

6. $\int \frac{x}{x+4}\, dx$

$u = x+4 \rightarrow x = u-4$ $\int \frac{u-4}{u}\, du = \int 1 - \frac{4}{u}\, du$

$du = dx$

7. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx$

$u = e^x$ $\int \frac{1}{\sqrt{1-u^2}}\, du$

$du = e^x dx$

8. $\int \frac{\arctan(x)}{1+x^2}\, dx$

$u = \arctan(x)$ $\int u\, du$

$du = \frac{1}{1+x^2}\, dx$

2

9. $\int \frac{x^3}{(1+x^2)^2} \, dx$ $= \int \frac{x^2 \cdot x}{(1+x^2)^2} dx$ $\rightarrow \frac{1}{2} \int \frac{1}{u} - \frac{1}{u^2} \, du$

$u = 1 + x^2$

$du = 2x \, dx$ $\quad = \frac{1}{2} \int \frac{(u-1)}{u^2} du =$ ⟍

10. $\int \frac{x^2}{\sqrt{1-x^3}} \, dx$

$\quad\quad\quad\quad\quad -\frac{1}{3} \int \frac{1}{\sqrt{u}} \, du = -\frac{1}{3} \int u^{-1/2} \, du$

$u = 1 - x^3$

$du = -3x^2 \, dx$

11. $\int \sin(x)(3\cos^4(x) + 4\cos^3(x) - 9) \, dx$

$u = \cos(x)$ $\quad\quad\quad -\int (3u^4 + 4u^3 - 9) \, du$

$du = -\sin(x) \, dx$

12. $\int x^3 + e^{3-x} \, dx$

$= \int x^3 \, dx + \int e^{3-x} \, dx$

$\vdots \quad\quad\quad\quad u = 3-x \quad\quad -\int e^u \, du$

$\quad\quad\quad\quad du = -dx$

13. $\int \frac{\sin(\sqrt{x}+1)}{\sqrt{x}} + \frac{1}{x^2+1} \, dx$

$= \int \frac{\sin(\sqrt{x}+1)}{\sqrt{x}} \, dx + \int \frac{1}{x^2+1} \, dx$

$u = \sqrt{x} + 1 \quad\quad\quad\quad\quad \vdots$

$du = \frac{1}{2\sqrt{x}} \, dx$

$\quad\quad 2 \int \sin(u) \, du$

3