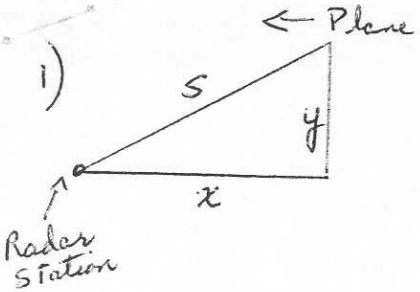
# Related Rates

1)




$\frac{dy}{dt} = 0 \quad \frac{ds}{dt} = -400 km/hr \quad \frac{dx}{dt} = ?$

$x^2 + y^2 = s^2$

$x^2 + 36 = 100$

$x = 8$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$8\frac{dx}{dt} + 0 = (10)(-400)$

$\frac{dx}{dt} = -500 km/hr$

The horizontal speed of the plane toward the station is 500 km/hr

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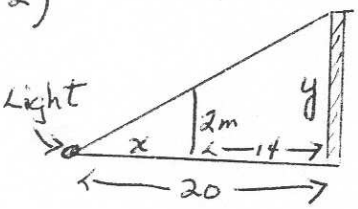
2)




$\frac{x}{20} = \frac{2}{y} \quad \frac{dx}{dt} = 1 m/s \quad \frac{dy}{dt} = ? \Big|_{x=6}$

$xy = 40$

$\frac{dx}{dt}y + x\frac{dy}{dt} = 0$

$(1)(y) + (6)\frac{dy}{dt} = 0$

$-\frac{20}{3} = 6\frac{dy}{dt}$

$\frac{dy}{dt} = -\frac{10}{9} m/s$

$xy = 40$

$6y = 40$

$y = \frac{20}{3}$

The length of the shadow is changing by $-\frac{10}{9}$ m/s

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3)

$V = \frac{1}{3}\pi r^2 h$ diameter = 4 r = radius = 2 h = height = 6 $\frac{r}{h} = \frac{1}{3}$ $r = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = -2 cm^3/s$

$\frac{dh}{dt} = ? \Big|_{h=3}$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \frac{dh}{dt}$

$-2 = \pi\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{2}{\pi} cm/s$

Height dropping at $\frac{2}{\pi}$ cm/s

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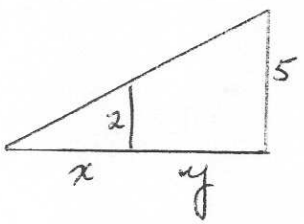
4.




$\frac{dx}{dt} =$ m/s $\quad \frac{dy}{dt} = -.5 m/s$

$\frac{2}{5} = \frac{x}{x+y}$

$2x + 2y = 5x$

$2y = 3x$

$2\frac{dy}{dt} = 3\frac{dx}{dt}$

$2(-.5) = 3\frac{dx}{dt}$

$-\frac{1}{3} m/s = \frac{dx}{dt}$

The shadow is decreasing at a rate of $\frac{1}{3}$ m/s

5. $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = -2\,cm^3/min$ $\quad \frac{dS}{dt} = ?$ $\quad r = 1$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$-2 = 4\pi(1)^2 \frac{dr}{dt}$

$\frac{-1}{2\pi} = \frac{dr}{dt}$ cm/min

$S = 4\pi r^2$

$\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt}$

$\frac{dS}{dt} = 4\pi \cdot 2(1)\left(\frac{-1}{2\pi}\right)$

$= -4\,cm^2/min$

Surface area is shrinking by $4\,cm^2/min$

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6. $V = \frac{1}{3}\pi r^2 h$

diameter = 20 cm

radius = 10 cm

$h = 30$ cm

$\frac{r}{h} = \frac{1}{3}$

$3r = h$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^3}{9}$

$\frac{dV}{dt} = -12\,cm^3/s$ $\quad \frac{dh}{dt} = ?\Big|_{h=20}$

$\frac{dV}{dt} = \pi \cdot \frac{h^2}{9} \frac{dh}{dt}$

$-12 = \pi \frac{(20)^2}{9} \frac{dh}{dt}$

$\frac{(-12)(9)}{400\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{27}{100\pi}$ cm/s

Liquid decreasing at a rate of $\frac{27}{100\pi}$ cm/s

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7) $A = \pi r^2$ $\quad C = 2\pi r$ $\quad \frac{C}{2\pi} = r$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{C^2}{4\pi}$

$\frac{dA}{dC} = \frac{2C}{4\pi} \cdot \frac{dC}{dC}$ ← with respect to circumference

$\frac{dA}{dC} = \frac{C}{2\pi}$ units

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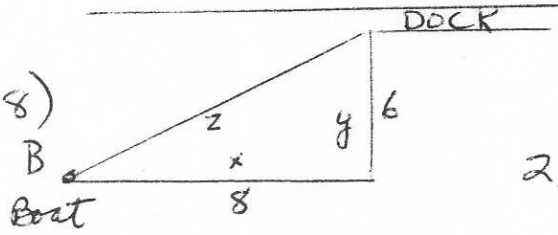
8)




$x^2 + y^2 = z^2$

$z = 10$

$\frac{dx}{dt} = ?$ $\quad \frac{dy}{dt} = 0$ $\quad \frac{dz}{dt} = -3\,m/s$

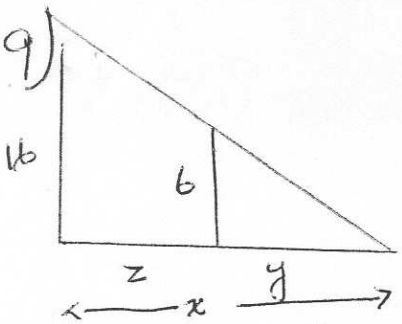
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$8\frac{dx}{dt} = 10(-3)$

$\frac{dx}{dt} = \frac{-30}{8}\,m/s$

Boat being pulled in at a rate of $\frac{30}{8}\,m/s$

9)




@ Tip of shadow

$\frac{6}{y} = \frac{16}{x}$ $\quad \frac{dx}{dt} = ?$ $\quad \frac{dy}{dt} =$ ft/s

$6x = 16y$

$6\frac{dx}{dt} = 16\frac{dy}{dt}$

$\frac{dz}{dt} = -5$ ft/s $\quad \frac{dy}{dt} = ?$

$\frac{dx}{dt} = \frac{8}{3}(-3) = -8$ ft/s

Tip of shadow decreasing at $\underline{\underline{-8 \text{ ft/s}}}$

(b) Length of shadow $\frac{dy}{dt}$ must do (b) first

$\frac{6}{y} = \frac{16}{y+z}$

$6y + 6z = 16y$

$6z = 10y$

$6\frac{dz}{dt} = 10\frac{dy}{dt}$

$-\frac{30}{10} = \frac{dy}{dt}$

$-3$ ft/s

Length of shadow decreasing by 3 ft/s

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10) $V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{5h}{4}\right)^2 h$

$\frac{dV}{dt} = \frac{\pi}{3}\frac{25}{16} \cdot 3h^2\frac{dh}{dt}$

$2 = \frac{\pi}{16}25(8)^2\frac{dh}{dt}$

radius $r = 15$

$h = 12$

$\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$

$\frac{dh}{dt} = ? \Big|_{h=8}$

$\frac{r}{h} = \frac{5}{4}$ $\quad 4r = 5h$ $\quad r = \frac{5}{4}h$

$\frac{dh}{dt} = \frac{1}{50\pi}$ m/s

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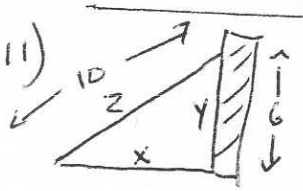
11)




$\frac{dz}{dt} = 0$ $\quad \frac{dx}{dt} = ?$ $\quad \frac{dy}{dt} = -2$ m/s $\quad y = 8$ m

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(8)\frac{dx}{dt} + 6(-2) = 0$

$\frac{dx}{dt} = \frac{3}{2}$ m/s

Bottom moving away from wall at rate of $\frac{3}{2}$ m/s

12. $\frac{dV}{dt} = -4\,cm^3/m$ $\quad \frac{ds}{dt} = ?$ $\quad r = 24$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$-4 = 4\pi(24)^2 \frac{dr}{dt}$

$\frac{-1}{24^2\pi} = \frac{dr}{dt}$

$S = 4\pi r^2$

$\frac{ds}{dt} = 4\pi \cdot 2r \frac{dr}{dt}$

$= 8\pi(24)\left(\frac{-1}{24^2\pi}\right)$

$= -\frac{1}{3}\,cm^2/min$

Surface area decreasing at rate of $\frac{1}{3}\,cm^2/min$

13.

$\frac{dr}{dt} = 4\,cm/s$ $\quad \frac{dh}{dt} = ?$ $\quad \frac{ds}{dt} = 0$ $\quad r = 10$

Surface Area $= S = 2\pi r^2 + 2\pi r h$

$\frac{ds}{dt} = 4\pi r \frac{dr}{dt} + 2\pi r \frac{dh}{dt} + 2\pi h \frac{dr}{dt}$

$0 = 4\pi(10)(4) + 2\pi(10)\frac{dh}{dt} + 2\pi(20)(4)$

$= 160\pi + 20\pi \frac{dh}{dt} + 160\pi$

$-320\pi = 20\pi \frac{dh}{dt}$

$-16\,cm/s = \frac{dh}{dt}$

$S = 600\pi =$

$2\pi(100) + 20\pi h$

$400\pi = 20\pi h$

$20 = h$

The height is changing at a rate of $-16\,cm/s$

14. $V = \frac{\pi}{3} r^2 h$ $\quad \frac{dh}{dt} = -1\,cm/hr$ $\quad \frac{dr}{dt} = -1\,cm/hr$

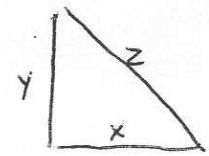
$\frac{dV}{dt} = \frac{\pi}{3} r^2 \frac{dh}{dt} + \frac{\pi}{3} h \cdot 2r \frac{dr}{dt}$

$= \frac{\pi}{3}(100)(-1) + \frac{\pi}{3}(10)(2)(10)(-1)$

$= -100\pi$

Volume is decreasing at the rate of $100\pi\,cm/hr$

15)




$\frac{dx}{dt} = 2\ m/s \quad \frac{dy}{dt} = \text{-ve} \quad \frac{dz}{dt} = 0$

$A = 6 \qquad x = 3,\ y = 4,\ z = 5$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$A = \frac{1}{2}x \cdot y$

$6 = \frac{1}{2}(3)y$

$12 = 3y$

$4 = y$

$xy = 12$

$y\frac{dx}{dt} + x\frac{dy}{dt} = 0$

$4(2) + 3\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = -8/3$

$3(2) + 4\left(-\frac{8}{3}\right) = 5\frac{dz}{dt}$

$6 - \frac{32}{3} = 5\frac{dz}{dt}$

$-\frac{14}{15} = \frac{dz}{dt}$ m/s

The side y is decreasing at a rate of $\frac{14}{15}$ m/s

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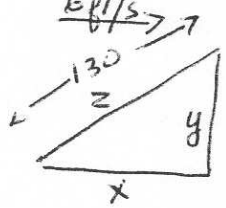
16.




$\frac{dx}{dt} = 6\ ft/s \quad \frac{dy}{dt} = 0 \quad \frac{dz}{dt} = ?$

$y = 120, \quad z = 130 \qquad x = 50$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$50(6) + 0 = 130\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{300}{130}$ ft/s

String is being let out at a rate of $\frac{30}{13}$ ft/s.

17) $A = \pi r^2$ $\quad \frac{dr}{dt} = .03$ mm/s $\quad \frac{dA}{dt} = ?$

$$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$$
$$= \pi(2)(200)(.03)$$
$$= 12\pi \text{ m/s}$$

Area changing at rate $12\pi$ mm/s

18) 

$V = \pi r^2 h$ $\quad r = 3,\ h = 4$ $\quad \frac{dh}{dt} = .2$ cm/s $\quad \frac{dV}{dt} = 0$ $\quad \frac{dr}{dt} = ?$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + \pi h \cdot 2r \frac{dr}{dt}$$
$$0 = \pi(3)^2(.2) + \pi(4)(2)(3)\left(\frac{dr}{dt}\right)$$
$$0 = 1.8\pi + 24\pi \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{-1.8\pi}{24\pi} = \frac{3}{40} \text{ cm/s}$$

Rate of change of radius $\frac{3}{40}$ cm/s.

19) $V = \frac{\pi}{3} r^2 h$

$h$ = diameter

$h = 2r$

$\frac{h}{2} = r$

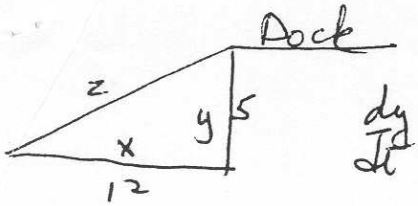
$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h$$
$$= \frac{\pi}{3}\frac{h^3}{4} = \frac{\pi h^3}{12}$$
$$\frac{dV}{dt} = \frac{\pi \cdot 3h^2}{12} \cdot \frac{dh}{dt}$$
$$2 = \frac{\pi \cdot 3(8)^2}{12} \frac{dh}{dt}$$
$$\frac{24}{\pi(3)(8)^2} = \frac{1}{8\pi} = \frac{dh}{dt}$$

$\frac{dV}{dt} = 2 \text{ m}^3/\text{s}$ $\quad \frac{dh}{dt} = ?$

when base is 8, $h = 8$ m

height increasing at a rate of $\frac{1}{8\pi}$ m/s

20)




$\frac{dy}{dt} = 0 \quad \frac{dz}{dt} = -2 \text{ m/s} \quad \frac{dx}{dt} = ?$

$x^2 + y^2 = z^2 \qquad z = 13$

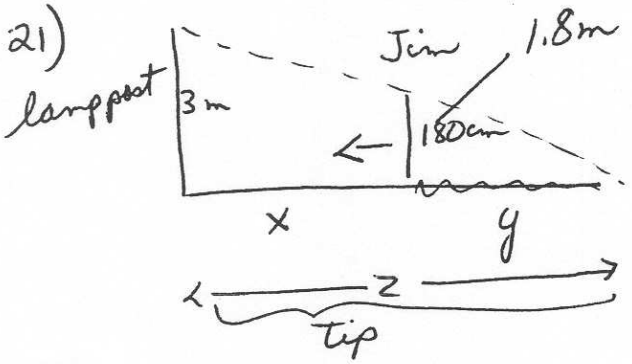
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(12)\left(\frac{dx}{dt}\right) + 0 = 13(-2)$

$\frac{dx}{dt} = -\frac{26}{12} \text{ m/s}$

Pulled in at rate of $\frac{26}{12}$ or $\underline{\frac{13}{6} \text{ m/s}}$.

21)




$3 \text{ m} = 300 \text{ cm}$

$\frac{dx}{dt} = -2.4 \text{ m/s}$

$\frac{dy}{dt} = ?$

$\frac{dz}{dt}_{(tip)} = ?$

$\frac{y}{x+y} = \frac{1.8}{3}$

$3y = 1.8x + 1.8y$

$1.2y = 1.8x$

$y = \frac{3}{2}x$

$\frac{dy}{dt} = \frac{3}{2}\frac{dx}{dt}$

$= \frac{3}{2}(-2.4)$

$= -3.6 \text{ m/s}$

Shadow decreasing by 3.6 m/s

(b) $\frac{y}{z} = \frac{1.8}{3}$

$3y = 1.8z$

$\frac{3}{1.8}y = z$

$\frac{3}{1.8}\frac{dy}{dt} = \frac{dz}{dt}$

$\frac{3}{1.8}(-3.6) = \frac{dz}{dt}$

$-6 \text{ m/s}$

Tip of shadow decreasing by 6 m/s