Solutions

1

a

$y = x^2 + xy$

$$\frac{dy}{dx} = 2x + y + x\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2x+y}{1-x}.$$

b

$x^2y + y = 3$

$$2xy + x^2\frac{dy}{dx} + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2+1}.$$

c

$x^{1/4} + y^{1/4} = 2$

$$\frac{1}{4x^{3/4}} + \frac{1}{4y^{3/4}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\left(\frac{y^{1/4}}{x^{1/4}}\right)^3 \Rightarrow \frac{dy}{dx} = \left(\frac{x^{1/4}-2}{x^{1/4}}\right)^3.$$

d

$\sqrt{x} + \sqrt{y} = 25$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{x}-25}{\sqrt{x}}.$$

e

$x + \sin y = y + 1$

$$1 + \cos y\frac{dy}{dx} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{1}{\cos y - 1} \Rightarrow \frac{dy}{dx} = \frac{1}{1-\cos y}.$$

f

$\sin xy = 2x + 5$

$$\left(y + x\frac{dy}{dx}\right)\cos xy = 2 \Rightarrow \frac{dy}{dx} = \frac{2 - y\cos xy}{x\cos xy}.$$

g

$x^2 + xy - y^3 = xy^2$

$$2x + y + x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y^2 - 2x - y}{x - 3y^2 - 2xy}.$$

h

$e^{\cos y} = x^3 \sin y$

$$-(\sin y)e^{\cos y}\frac{dy}{dx} = 3x^2\sin y + x^3\cos y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{3x^2\sin y}{x^3\cos y + (\sin y)e^{\cos y}}.$$

i

$\sin(2x^2y^3) = 3x^3 + 1$

$$\left(4xy^3 + 6(xy)^2\frac{dy}{dx}\right)\cos(2x^2y^3) = 9x^2 \Rightarrow \frac{dy}{dx} = \frac{9x^2 - 4xy^3\cos(2x^2y^3)}{6(xy)^2\cos(2x^2y^3} \Rightarrow \frac{dy}{dx} = \frac{9x - 4y^3\cos(2x^2y^3)}{6xy^2\cos(2x^2y^3}.$$

j

$4y^2 + 2 = 3x^2$

$$8y\frac{dy}{dx} = 6x \Rightarrow \frac{dy}{dx} = \frac{3x}{4y}.$$

2

a

$1 - xy = x - y^2$

$$-y - x\frac{dy}{dx} = 1 - 2y\frac{dy}{dx} \left(\Rightarrow \frac{dy}{dx} = \frac{1+y}{2y-x}\right)$$

$$-\frac{dy}{dx} - \frac{dy}{dx} = -2\left(\frac{dy}{dx}\right)^2 - 2y\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{2\frac{dy}{dx}\left(1 - \frac{dy}{dx}\right)}{2y-x} \Rightarrow \frac{d^2y}{dx^2} = \frac{2\frac{1+y}{2y-x}\left(1 - \frac{1+y}{2y-x}\right)}{2y-x} \Rightarrow \frac{d^2y}{dx^2} = \frac{2(1+y)(y-x-1)}{(2y-x)^3}.$$

b

$x - y = (x+y)^2$

$$1 - \frac{dy}{dx} = 2(x+y) + 2(x+y)\frac{dy}{dx} \left(\Rightarrow \frac{dy}{dx} = \frac{1-2(x+y)}{1+2(x+y)}\right)$$

$$-\frac{d^2y}{dx^2} = 2 + 2\frac{dy}{dx} + 2\frac{dy}{dx} + 2x\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2\left(1 + \frac{dy}{dx}\right)}{1+2(x+y)} \Rightarrow \frac{d^2y}{dx^2} = \frac{-8}{(1+2(x+y))^3}$$

c

$\sin x - 4\cos y = y$

$$\cos x + 4\sin y\frac{dy}{dx} = \frac{dy}{dx} \left(\Rightarrow \frac{dy}{dx} = \frac{\cos x}{1 - 4\sin y}\right)$$

$$-\sin x + 4\cos y\frac{dy}{dx} + 4\sin y\frac{d^2y}{dx^2} = \frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{4\cos y\frac{dy}{dx} - \sin x}{1 - 4\sin y} \Rightarrow \frac{d^2y}{dx^2} = \frac{\frac{4\cos x\cos y}{1-4\sin y} - \sin x}{1 - 4\sin y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{4(\cos x\cos y + \sin x\sin y) - \sin x}{(1-4\sin y)^2} = \frac{4\cos(x-y) - \sin x}{(1-4\sin y)^2}.$$

3

$x^2 + y^2 - xy + 3x - 9 = 0$

a

$$2x + 2y\frac{dy}{dx} - y - x\frac{dy}{dx} + 3 = 0 \Rightarrow \frac{dy}{dx} = \frac{2x - y + 3}{x - 2y}.$$

b

$y = 2x + 3$

c

$y = \frac{x}{2}$

d

$$2 + 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} - \frac{dy}{dx} - \frac{dy}{dx} - x\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{2 - 2\frac{dy}{dx} + 2\left(\frac{dy}{dx}\right)^2}{x - 2y} \Rightarrow \frac{d^2y}{dx^2} = \frac{2\left(\left(\frac{dy}{dx} - \frac{1}{2}\right)^2 + \frac{3}{4}\right)}{x - 2y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{6x^2 + (18-6y)x + 6y^2 + 18}{(x-2y)^3}.$$

4

$x^2 + xy + y^2 = 5$

a

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$$

b

$y = -2x$

c

$y = -\frac{x}{2}$

d

$$2 + \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{-(x^2+y^2}{y^3}$$

5

$y^2\cos(x) + (3\sin(x) - 1)y + 7x - 2 = 0$

a

$$2y\frac{dy}{dx}\cos(x) - y^2\sin(x) + 3\cos(x)y + (3\sin(x) - 1)\frac{dy}{dx} + 7 = 0 \Rightarrow \frac{dy}{dx} = \frac{y^2\sin(x) - 3\cos(x)y - 7}{2y\cos(x) + 3\sin(x) - 1}$$

At $(0, 2)$, $\frac{dy}{dx} = \frac{-13}{3}$.

b

lol jk.

c

lol jk.