Sols

Math 122 Calculus for Business Admin. and Social Sciences

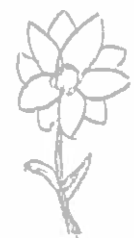
Exam #3 A
August 3, 2017

Instructor: Ann Clifton

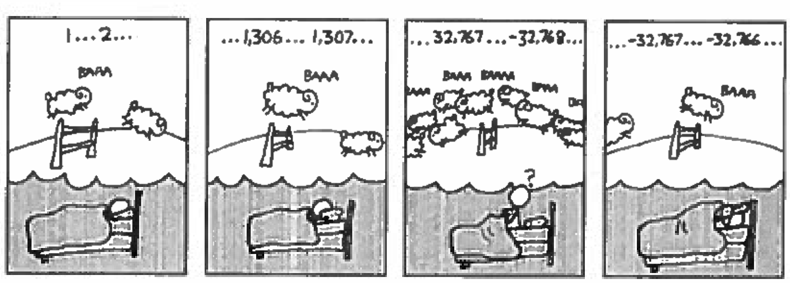
Name: ______________________________

Do not turn this page until told to do so. You will have a total of 1 hour 25 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers will **not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw a flower on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

#	score	out of	#	score	out of
1		3	9		5
2		4	10		6
3		3	11		20
4		3	12		16
5		3	13		15
6		4	14		10
7		4	EC		3
8		4	Total		100



Remember: This exam has no impact on your worth as a human being. You got this!!!



Fill in the blanks.

1. (3 points) If $f(x)$ is a continuous function on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = \underline{F(b) - F(a)}$$

2. (4 points) Assume that $\int f(x)dx$ and $\int g(x)dx$ exist.

(a) $\int f(x) \pm g(x)dx = \underline{\int f(x)dx \pm \int g(x)dx}$

(b) Let a be a number, $\int af(x)dx = \underline{a\int f(x)dx}$

3. (3 points) Let $n \neq -1$ be a fixed number,

$$\int x^n dx = \underline{\frac{x^{n+1}}{n+1} + C}$$

4. (3 points)

$$\int e^x dx = \underline{e^x + C}$$

5. (3 points)

$$\int \frac{1}{x} dx = \underline{\ln|x| + C}$$

Multiple Choice. Choose the best answer. (4 points each.)

6. Find the antiderivative $F(x)$ of the function $f(x) = 3x^2 + e^x$ which satisfies $F(0) = 2$.

   A. $F(x) = x^3 + e^x + 2$ **B.** $F(x) = x^3 + e^x + 1$

   C. $F(x) = x^3 + e^x + c$ **D.** $F(x) = x^3 + e^x + 3$

7. Find the indefinite integral $\int \left(\frac{3}{x} + \frac{1}{\sqrt{x}}\right) dx$.

   A. $2\sqrt{x} + c$ **B.** $3\ln x + \frac{2}{\sqrt{x}} + c$

   C. $3\ln|x| + 2\sqrt{x} + c$ **D.** $3\ln|x| + \frac{2}{\sqrt{x}} + c$

8. Find the definite integral $\int_2^7 \left(\frac{1}{x} - \frac{2}{x^3}\right) dx$. Round your answer to three decimal places.

   A. 1.023 **B.** 0.334

   C. −1.023 **D.** 1.482

Short Answer.

9. (5 points) Explain in words what the definite integral of a function represents and how we *estimate* it.

   Area under the curve on the interval [a,b] ; Riemann sums

10. (6 points) Approximate the area under the curve $y = x^2$ on the interval $[0, 4]$ using $n = 4$ right-endpoint subintervals.

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

$$1(f(1) + f(2) + f(3) + f(4)) = 1 + 4 + 9 + 16$$
$$= 30$$

11. (20 points) Compute the following indefinite integrals.

(a) $\int 7dx$

$7x + C$

(b) $\int (10x + 2)dx$

$5x^2 + 2x + C$

(c) $\int (36x^2 + 26x)dx$

$12x^3 + 13x^2 + C$

(d) $\int x^2 dx$

$\frac{x^3}{3} + C$

(e) $\int \frac{1}{\sqrt{x}}dx$

$\int x^{-1/2} dx = 2x^{1/2} + C$

12. (16 points) Compute the following indefinite integrals.

(a) $\int 25(x+7)^{24}dx$

$u = x+7$

$du = dx$

$\int 25u^{24}dx = u^{25} + c$

$= (x+7)^{25} + c$

(b) $\int (x+2)e^{\frac{1}{2}x^2+2x+1}dx$

$u = \frac{1}{2}x^2 + 2x + 1$

$du = (x+2)dx$

$\int e^u du = e^u + c$

$= e^{\frac{1}{2}x^2+2x+1} + c$

(c) $\int \frac{4x}{2x^2+7}dx$

$u = 2x^2 + 7$

$du = 4x\,dx$

$\int \frac{1}{u} du = \ln|u| + c$

$= \ln|2x^2+7| + c$

$= \ln(2x^2+7) + c$

(d) $\int \frac{x}{\sqrt{x^2+1}}dx$

$u = x^2 + 1$

$du = 2x\,dx$

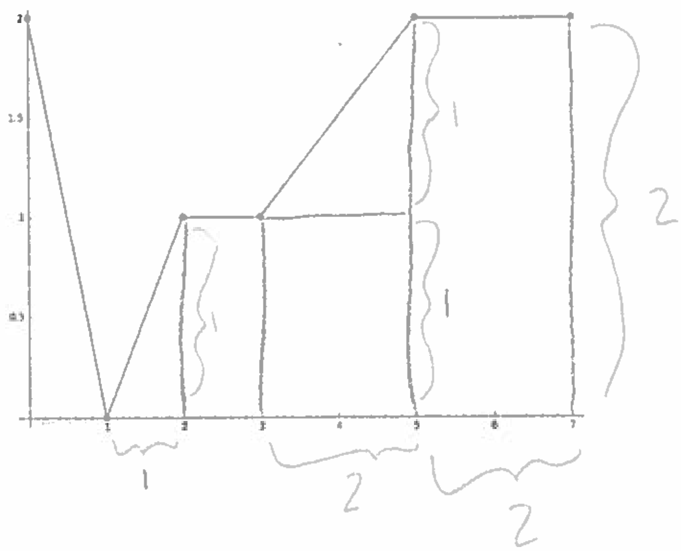
$\frac{1}{2} du = x\,dx$

$\frac{1}{2}\int \frac{1}{\sqrt{u}} du = \frac{1}{2} \cdot 2u^{1/2} + c$

$= u^{1/2} + c$

$= (x^2+1)^{1/2} + c$

13. (15 points) Consider the function f given by the graph:



Compute $\int_0^7 f(x)dx$.

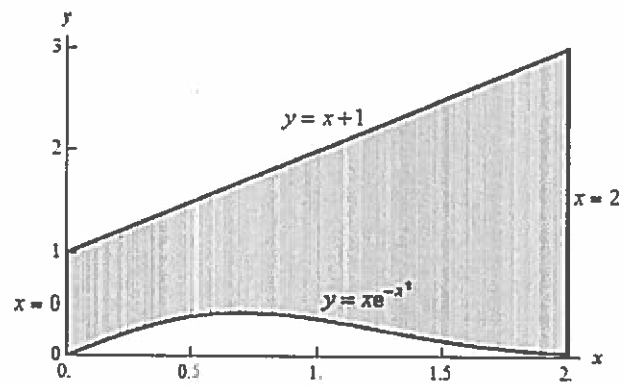
$$A = \int_0^7 f(x)dx$$

$$= \frac{1}{2}(1)(2) + \frac{1}{2}(1)(1) + 1(1) + 2(1) + \frac{1}{2}(2)(1) + 2(2)$$

$$= 1 + \frac{1}{2} + 1 + 2 + 1 + 4$$

$$= 9.5$$

14. (10 points) Find the area of the region bounded by $y = xe^{-x^2}$ and $y = x + 1$ on the interval $[0, 2]$. *Set up but do **not** evaluate the integral.* The graph of the region is given below for reference.



$$A = \int_0^2 (x+1) - (xe^{-x^2})\,dx$$

15. (Extra Credit. 3 points) Evaluate the integral from number 14 (the problem above). Round your answer to four decimal places.

$$A \approx 3.5092$$

Calculator OR $\int_0^2 (x+1) - (xe^{-x^2})\,dx = \int_0^2 (x+1)\,dx - \int_0^2 xe^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u$

$= -\frac{1}{2}e^{-x^2}$

$$= \frac{x^2}{2} + x\Big|_0^2 - -\frac{1}{2}e^{-x^2}\Big|_0^2$$

$$= 2 + 2 - 0 + \frac{1}{2}e^{-4} - \frac{1}{2}$$

$$\approx 3.5092$$