6.1: Introduction to Probability Distributions

Random Variables

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Cedar Crest College

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Sum of the dice (X)	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Probability Distributions

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Dr. Mendoza developed a test to measure boredom tolerance. He administered it to a group of 20,000 adults between the ages of 25 and 35. The possible scores were 0,1,2,3,4,5, and 6, with 6 indicating the highest tolerance for boredom. The test results for this group are shown below. Find the probability distribution for this data.

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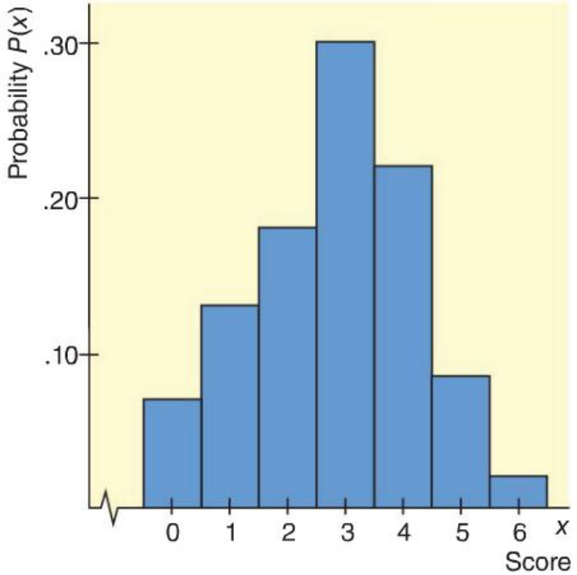
Score (X)	0	1	2	3	4	5	6
$\Pr(X)$	0.07	0.13	0.18	0.30	0.22	0.08	0.02


Probability P(x)
.30
.20
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The mean of a probability distribution is often called the **expected value** of the distribution.

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- Since you earn \$1.00 for each heads, you should expect to win an average of \$1.50 per game. Since the game costs \$2.00 to play, you should expect a net loss of \$0.50 per game.