MATH 170: PRACTICE EXAM 01

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. You may only use a four-function calculator. No graphing calculators or cell phones are allowed.

Name: Solutions

1. Problems

Let $S = \{$Burton, Ride, Forum, Völkl, LibTech, Gnu, Rome$\}$. Let $X = \{$Burton, Ride, Forum$\}$, $Y = \{$Forum, Völkl, LibTech, Gnu, Rome$\}$, $Z = \{$Ride, Forum, Völkl$\}$, and $W = \{$Burton, Gnu, LibTech$\}$.

1. *Compute*

(a) $X \cap Y$,

$X \cap Y = \{$Forum$\}$

(b) $X \cup Z$,

$X \cup Z = \{$Burton, Ride, Forum, Völkl$\}$

(c) The complement of Z *in* S, Z'. (everything in S that is NOT in Z)

$Z' = \{$Burton, LibTech, Gnu, Rome$\}$

Date: February 4, 2015.

2. *Use the sets in number 1 to compute the following:*

(a) What is the cardinality of $X \times Z$*?*

$$n(X \times Z) = n(X) \cdot n(Z) = 3 \cdot 3 = 9$$

(b) What is the cardinality of $Y \cup Z$*?*

$Y \cup Z = \{$Forum, Völkl, LibTech, Gnu, Rome, Ride$\}$

$$n(Y \cup Z) = 6$$

(c) What is the cardinality of $X \cap Y$*?*

$X \cap Y = \{$Forum$\}$

$$n(X \cap Y) = 1$$

(d) What is the cardinality of $X \cap Y \cap Z$*?*

$X \cap Y \cap Z = \{$Forum$\}$

$$n(X \cap Y \cap Z) = 1$$

(e) What is the cardinality of $Z \cap W$*?*

$$Z \cap W = \emptyset$$

$$n(Z \cap W) = 0$$

3. *If* $n(A) = 45$, $n(B) = 18$, *and* $n(A \cap B) = 5$, *find* $n(A \cup B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 45 + 18 - 5 = \boxed{58}$$

4. *Your favorite restaurant offers a total of 13 desserts, of which 9 have ice cream as a main ingredient and 9 have fruit as a main ingredient. Assuming that all of them have either ice cream or fruit or both as a main ingredient, how many have both?*

Let A be the set of desserts with ice cream and let B be the set of desserts with fruit. Then,

$n(A \cup B) = 13$ since there are 13 total desserts

$n(A) = 9$ since 9 have ice cream

$n(B) = 9$ since 9 have fruit. We want to find $n(A \cap B)$.

Since $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, $\underline{n(A \cap B) = 5}$

5. *Use a truth table to prove the following logical equivalences. Explain why the equivalence holds.*

(a)

$$\sim(p \vee q) \equiv \sim p \wedge \sim q.$$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Because the last two columns, which correspond to the two statements, are the same (ie, have the same truth values) the equivalence holds.

(b)

$$\sim(p \wedge q) \equiv \sim p \vee \sim q.$$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Because the outlined columns, corresponding to the two statements, have the same truth values, they are equivalent.

(c)

$$p \Rightarrow q \equiv \sim p \vee q.$$

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Because the outlined columns, corresponding to the statements, are the same $p \Rightarrow q \equiv \sim p \vee q$.

6. *Use a truth table to prove the following is a tautology. Explain why it is a tautology.*

$$(p \Rightarrow q) \vee (q \Rightarrow p)$$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \vee (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

A tautology is a statement that is true no matter the truth values of it's parts. So in any situation, it is a true statement. (Ex. "My hair is blue or my hair is not blue") Since all the truth values are T, this statement is a tautology.

7. *How many different five-letter sequences can be formed with the letters a, a, a, b, c?*

This is similar to the scrabble example. So we should think about filling five slots with letters.

□ □ □ □ □

Step 1: Choose a slot for b *
5 choices

Step 2: Choose a slot for c *
4 choices

Step 3: Fill the rest with a's
1 choice (since we want DISTINCT words)

Total: $5 \cdot 4 \cdot 1 = \underline{20}$ words

* I think it's easier to start with the letters you have the least of

8. *Professor Easy's final exam has 12 true-false questions followed by 3 multiple-choice questions. In the multiple-choice questions, you must select the correct answer from 5 choices. How many answer sheets are possible?*

Steps 1–12: answer the T/F
2 choices for each
so 2^{12} possibilities

Total: $\underline{2^{12} \cdot 5^3}$

Steps 13–15: answer the multiple choice; 5 choices for each so
5^3 possibilities

(On your homework you have to multiply this out to get an exact number. <u>On the test I will accept this form.</u>)

9. *Caculate the following:*

(a) $7!/5!$

$$\frac{7!}{5!} = \frac{7\cdot 6\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 7\cdot 6 = 42$$

(b) $P(2,2)$

$$P(2,2) = \frac{2!}{(2-2)!} = \frac{2!}{0!} = \frac{2!}{1} = \frac{2\cdot 1}{1} = 2$$

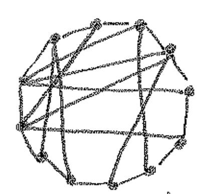
(c) $P(9,4)$

$$P(9,4) = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9\cdot 8\cdot 7\cdot 6 = 4{,}698$$

(d) $C(6,5)$

$$C(6,5) = \frac{6!}{5!(6-5)!} = \frac{6!}{5!\,1!} = \frac{6!}{5!} = 6$$

10. *If 12 businesspeople have a meeting and each pair exchanges business cards, how many business cards, total, get exchanged?*



Imagine the 12 people stand in a circle and if they exchange cards there is a line between them. Since everyone exchanges cards there will be a line between each pair. So, if we count the number of lines, we are counting how many ways there are to make a pair out of 12 people, ie, $C(12,2)$.

<u>But</u> $C(12,2) = \frac{12!}{2!(12-2)!} = \frac{12!}{2!\,10!} = \frac{12\cdot 11}{2} = 66$ is NOT the final answer. Why? What do we want? We want the number of cards exchanged. How many cards are exchanged in a single pair? Two! So the answer is $2\cdot C(12,2) = 2\cdot \frac{12\cdot 11}{2} = 12\cdot 11 = 132$

11. *A bag contains 4 red marbles, 1 green one, 1 lavender one, 3 yellows, and 2 orange marbles. How many sets of four marbles include one of each color other than lavender?*

Want a set of four with each color except lavender.

Step 1: Choose a red marble
$C(4,1) = 4$ choices

Step 2: choose a green marble
$C(1,1) = 1$ choices

Step 3: Choose a yellow marble
$C(3,1) = 3$ choices

Step 4: Choose an orange marble
$C(2,1) = 2$ choices

Total: $4 \cdot 1 \cdot 3 \cdot 2 = 24$ possible sets

We do not have to worry about alternatives in this case since we wanted 4 marbles and there are 4 colors other than lavender.