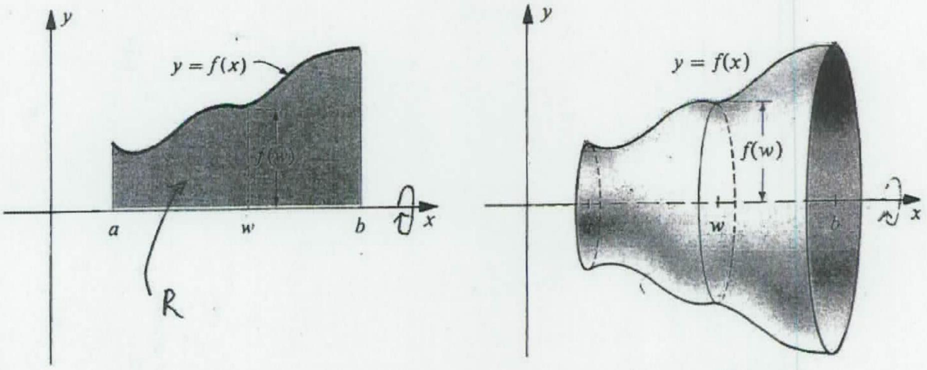
§ 6.2 & 6.3 Volume of Solids of Revolution

(i) Start out with a region R in the xy-plane

(ii) Revolve (i.e. spin) the region R around a horizontal or vertical line to form your "Solid of Revolution"



(i) (ii)

(iii) Find the volume V of this Solid of Revolution by expressing V as a definite integral and integrating it.

Game Plan

① partition the appropriate axis (either the x-axis or y-axis)
If you partition the z-axis, then $V = \int_{z=\#}^{z=\#} (\text{some function of } z)\, dz$.

see next+2 pages: (i) → ② form Riemann typical rectangles as if you were looking for the Area of R.

(ii) → ③ revolve the typical rectangles to get "typical elements",
here an element will be: disk or washer or shell.

④ Find the volume $\tilde{V}$ of a typical element. $\tilde{V}$ will look like
$\tilde{V} = (\text{some function of } z)\, \Delta z$.

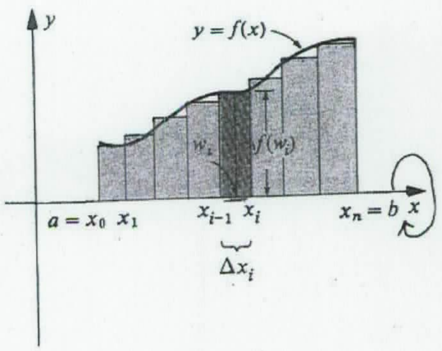
⑤ Sum the volume of all the typical elements resulting from your partition

⑥ take the limit as $\Delta z \to 0$ to get

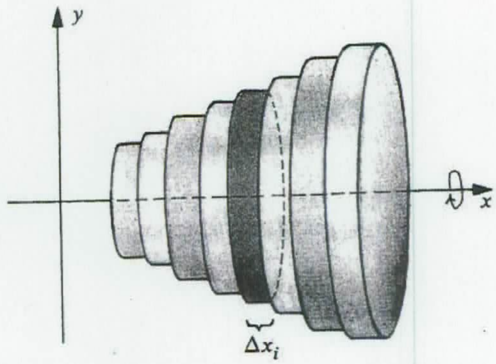
$$V = \int_{z=\#}^{z=\#} (\text{that some function of } z)\, dz.$$

the same

§6.2 Disk Method   revolve abt x-axis & partition x-axis



(i)



(ii)

no hole

⇓

disk

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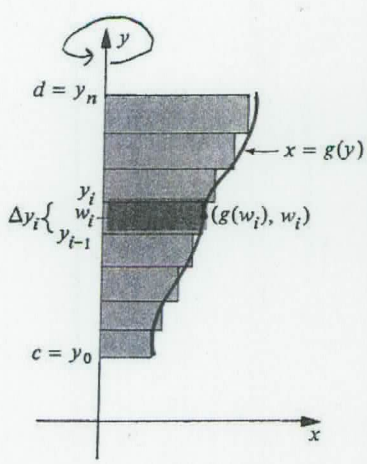
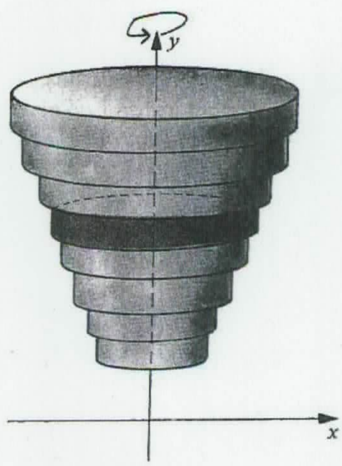
§6.2 Disk Method   revolve abt y-axis & partition y-axis



(i)



(ii)

no hole

⇓

disk

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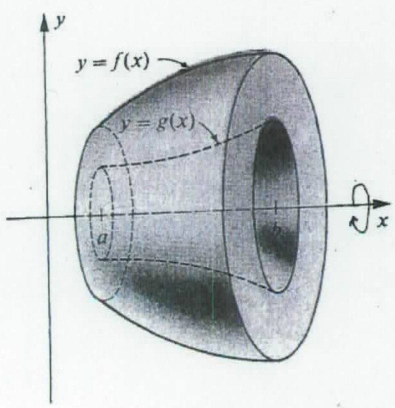
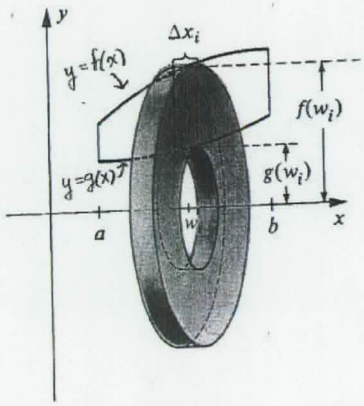
§6.2 Washer Method   revolve abt x-axis
partitioned x-axis



(i)



(ii)

has hole

⇓

washer

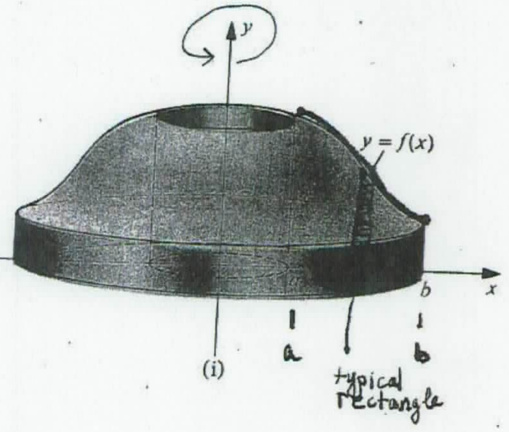
§6.3 Shell Method

Picture • our textbook pages 433 - 434

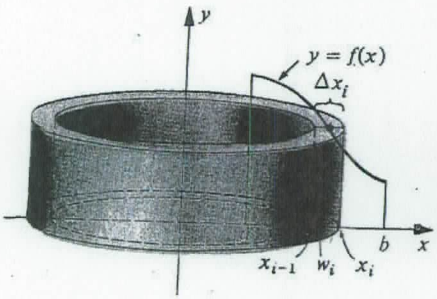
or

• Swoko p 290 (below)

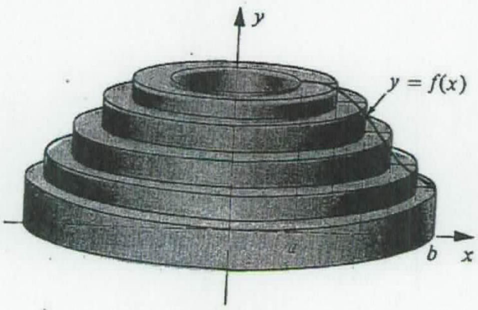
Revolving about y-axis → partition x-axis



(i)



(ii)



(iii)

- Let's say partitioned z-axis (where $z = x$ or $z = y$)
- So $V = \int_{z=?}^{z=?}$ (some function of z) dz

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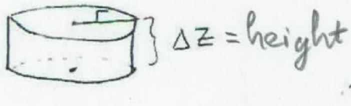
§6.2 $\boxed{\text{Disk/Washer Method : partition } (\parallel\text{-to}) \text{ axis of revolution}}$

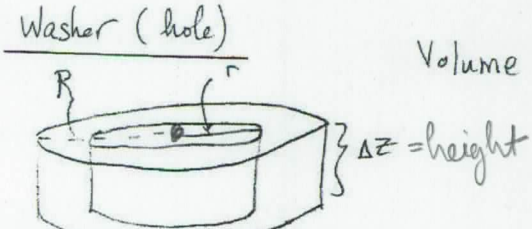
- typical element = disk or washer
- Disk (no hole)



Volume = (area of base) (height)

$= (\pi r^2) \quad (\Delta z)$

- Washer (hole)



Volume = (Volume of big) − (Volume of little)

$= \pi R^2 \Delta z - \pi r^2 \Delta z$

$= \pi (R^2 - r^2) \Delta z$

$\neq \pi (R - r)^2 \Delta z$

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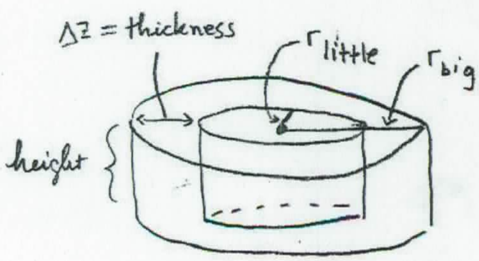
§6.3 $\boxed{\text{Shell Method : partition } \perp \text{ to axis of revolution}}$

- typical element = Shell.



Volume of typical shell

$= 2\pi$ (average radius) (height) (thickness)

$= 2\pi \left(\frac{r_{big} + r_{little}}{2}\right) (\text{height}) \underbrace{(r_{big} - r_{little})}_{(\Delta z)}$

$= 2\pi$ ((avg) radius) (height) (Δz)