Sols

Math 141 - Spring 2018
Practice Problems for Exam 1

On the exam, you must show all work to receive full credit. No calculators or other technology will be permitted.

1. Evaluate the given limit.

$$\lim_{x\to 4} \frac{2-\sqrt{x}}{4x-x^2} \rightarrow \frac{2-\sqrt{4}}{4(4)-4^2} \rightarrow 0 \,\ddot{\smile}$$

$$\frac{2-\sqrt{x}}{4x-x^2}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}} = \frac{4-x}{(4x-x^2)(2+\sqrt{x})} = \frac{\cancel{4-x}}{x(\cancel{4-x})(2+\sqrt{x})} = \frac{1}{x(2+\sqrt{x})}$$

$$\lim_{x\to 4}\frac{2-\sqrt{x}}{4x-x^2} = \lim_{x\to 4}\frac{1}{x(2+\sqrt{x})} = \frac{1}{4(2+\sqrt{4})} = \boxed{\frac{1}{16}}$$

2. Evaluate the given limit.

$$\lim_{x\to \frac{\pi}{2}} \sin(x+\cos x)$$

$$= \sin\left(\lim_{x\to \pi/2}(x+\cos x)\right)$$

$$= \sin\left(\pi/2 + \cos(\pi/2)\right)$$

$$= \sin(\pi/2 + 0)$$

$$= \sin(\pi/2)$$

$$= \boxed{1}$$

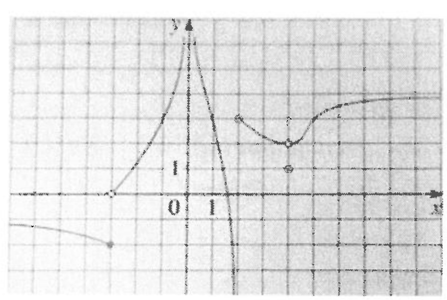
3. Evaluate the given limit.

$$\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12}$$ $\rightarrow 0$:/

$$\frac{x^2+3x}{x^2-x-12} = \frac{x\cancel{(x+3)}}{(x-4)\cancel{(x+3)}} = \frac{x}{x-4}$$

$$\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to -3} \frac{x}{x-4} = \frac{-3}{-3-4} = \boxed{3/7}$$

4. Does the function graphed below have any discontinuities? If so, where?



Yes : x = -3, 0, 2, 4

5. Consider the function

$$f(x) = \begin{cases} \sqrt{-x} & x < 0 \\ 3 - x & 0 \le x < 3 \\ (x-3)^2 & x \ge 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

$\lim_{x\to 0^-} f(x) = 0$

$\lim_{x\to 3^-} f(x) = 0$

$\lim_{x\to 0^+} f(x) = 3$

$\lim_{x\to 3^+} f(x) = 0$

$\lim_{x\to 0} f(x)$ DNE

$\lim_{x\to 3} f(x) = 0$

(b) Use interval notation to state where $f(x)$ is **continuous**.

$(-\infty, 0) \cup (0, \infty)$

6. Evaluate the different quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - 3x + 1$. Simply your answer.

$$\frac{(x+h)^2 - 3(x+h) + 1 - (x^2 - 3x + 1)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{x^2} + \cancel{3x} - \cancel{1}}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \boxed{2x + h - 3}$$

7. Evaluate the following limits

$$\lim_{x\to -3^+} \frac{x+2}{x+3} \to \frac{\text{negative}}{\text{small positive}}$$

$$= -\infty$$

$$\lim_{x\to -3^-} \frac{x+2}{x+3} \to \frac{\text{negative}}{\text{small negative}}$$

$$= \infty$$

$$\lim_{x\to -3} \frac{x+2}{x+3}$$

DNE

(vertical asymptote at $x=-3$)

8. $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5} g(x) = 1$ evaluate the following.

(a) $\lim_{x\to 5} [3f(x) - 5g(x)]$

$$= 3\cdot \lim_{x\to 5} f(x) - 5\cdot \lim_{x\to 5} g(x) = 3(2) - 5(1) = \boxed{1}$$

(b) $\lim_{x\to 5} \frac{f(x)}{g(x)}$

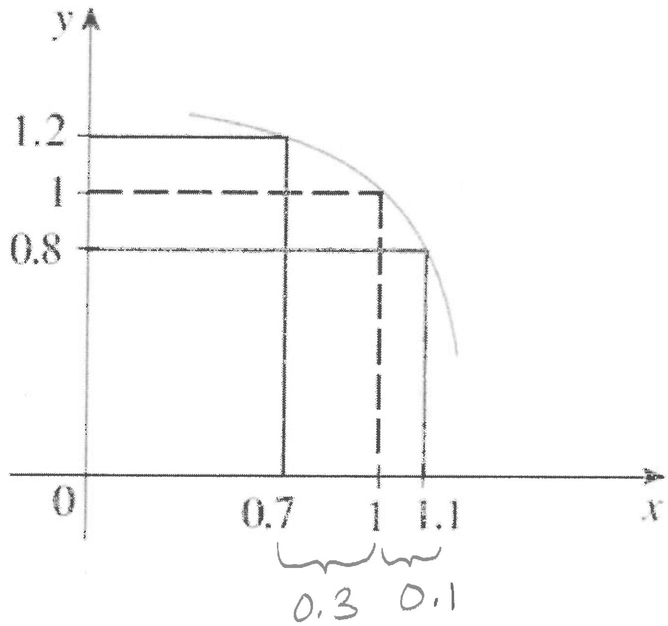
$$= \frac{\lim_{x\to 5} f(x)}{\lim_{x\to 5} g(x)} = \frac{2}{1} = \boxed{2}$$

(c) $\lim_{x\to 5} [f(x)^4 + g(x)]$

$$= \left(\lim_{x\to 5} f(x)\right)^4 + \lim_{x\to 5} g(x)$$

$$= (2)^4 + 1 = \boxed{17}$$

9. Use the given graph of f to find a number δ such that if $|x-1|<\delta$ then $|f(x)-1|<0.2$.



Let $\delta = \min\{0.3, 0.1\} = 0.1$

10. Suppose f and g are continuous functions such that $g(1)=4$ and $\lim_{x\to 1}[2f(x)+f(x)g(x)] = -24$. Find $f(1)$.

$$2\cdot\lim_{x\to 1} f(x) + \lim_{x\to 1} f(x)\cdot g(x) = -24$$

$$2f(1) + f(1)\cdot g(1) = -24$$

$$2f(1) + 4f(1) = -24$$

$$6f(1) = -24$$

$$\boxed{f(1) = -4}$$

11. Write out the proper mathematical notation associated with the sentence below.

"The limit of $f(x)$ as x approaches 2 is equal to 10."

$$\lim_{x \to 2} f(x) = 10$$

12. True or False.
Review limit laws, properties of continuous functions, how a function can fail to be differentiable, and any theorems we discussed.

Also see "More Practice for Exam 1" on the course webpage.