Math 141: More Practice for Exam 1

Section 2.1 Rates of Change and Tangents to Curves

16) Find the average rate of change of $f(x) = x^3+1$ over the interval [-1,1].

17) Find the slope of the curve $y = x^2 - 2x - 3$ at the point $P(2, -3)$ and write an equation of the tangent line at P.

Section 2.2 Limit of a Function and Limit Laws

18) Explain why the limit

$$\lim_{x \to 1} \frac{1}{x-1}$$

does not exist.

19) Find the following limits:
(a)

$$\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - t - 2}$$

(b)

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

20) Evaluate the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for the function $f(x) = x^2$ when $x = 1$.

21) If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

Section 2.3 The Precise Definition of a Limit

22) Prove the following using the precise (ϵ, δ) definition of a limit:

$$\lim_{x \to 4} (9 - x) = 5$$

Section 2.4 One-Sided Limits

23) Know how to calculate one-sided limits given the graph of a function.(See quiz)

24) Find the limit:

$$\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2}$$

25) Find the limit:

$$\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2}$$

Section 2.5 Continuity

True means ALWAYS true.

1) True or False: A function $f(x)$ is continuous at a point $x = c$ if and only if $\lim_{x \to c} f(x) = f(c)$.

2) True or False: If the function f is continuous at $x = c$ and g is a function then $f + g$ is continuous at $x = c$.

3) True or False: If f is continuous at c and g is continuous at $f(c)$, then the composite $f \circ g$ is continuous at c.

4) Use the Intermediate Value Theorem to show that there is a root of the equation $x^3 + 5x^2 + 5x = 0$ between -1/2 and 1.

Section 2.6 Limits Involving Infinity; Asymptotes of Graphs

5) Graph the following function. Include the graphs and equations of the asymptotes. Be sure to label at least four points on the graph.

$$y = \frac{2x}{x+1}$$

6) Graph the following function. Include the graphs and equations of the asymptotes. Be sure to label at least four points on the graph.

$$y = \frac{x^2 - 4}{x - 1}$$

7) Find the limit:

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 25} - \sqrt{x^2 - 1} \right)$$

8) Find the limit:

$$\lim_{x \to 2^+} \frac{1}{x-2}$$

Section 3.1 Tangents and the Derivative at a Point

9) Using the definition of the derivative, find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

$$f(x) = x^3 + 3x, \ (1, 4)$$

Section 3.2 The Derivative as a Function

10) Using the definition, calculate the derivative of the function. Then find the values of $f'(-1)$, $f'(2)$, and $f'(\sqrt{3})$.

$$f(x) = \frac{1}{x^2}$$