Sols

Math 141: u-Substitution Practice

Warm-Up/Review Problems

1. Evaluate the following integrals

(a) $\int x^2(\sqrt{x}+5)+e^2dx$

$= \int x^{5/2} + 5x^2 + e^2\,dx$

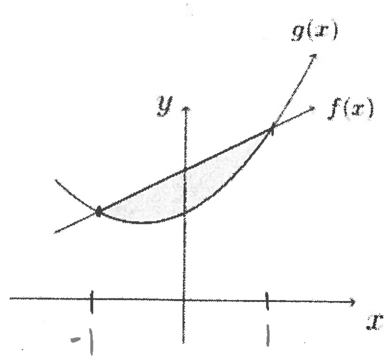
$= \boxed{\frac{2}{7}x^{7/2} + \frac{5}{3}x^3 + e^2x + C}$

(b) $\int_1^2 \frac{3x^3+1}{4x}dx$

$= \int_1^2 \frac{3}{4}x^2 + \frac{1}{4x}\,dx = \left[\frac{1}{4}x^3 + \frac{1}{4}\ln|x|\right]_1^2$

$= \frac{1}{4}(2)^3 + \frac{1}{4}\ln(2) - \left(\frac{1}{4}(1)^3 + \frac{1}{4}\ln(1)\right) = \boxed{\frac{7}{4} + \frac{1}{4}\ln(2}$

2. Find the area bounded between the line $f(x) = x + 3$ and the parabola $g(x) = x^2 + x + 2$.



(a) Find where $f(x)$ and $g(x)$ intersect by setting them equal and solving for x.

$x+3 = x^2+x+2 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$

(b) Set up the integral and evaluate to find the area bounded by $f(x)$ and $g(x)$.

$\int_{-1}^{1} (x+3) - (x^2+x+2)\,dx = \int_{-1}^{1} -x^2+1\,dx$

$= -\frac{x^3}{3} + x \Big|_{-1}^{1} = -\frac{1}{3} + 1 - \left(\frac{1}{3} - 1\right)$

$= -\frac{2}{3} + 2 = \boxed{4/3}$

Practicing u/du Substitution

3. Find the following indefinite integrals using substitution.

(a) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$2 \int \cos(u)\, du$

$= 2\sin(u) + C = \boxed{2\sin(\sqrt{x}) + C}$

(b) $\int \frac{e^x}{e^x + 1} dx$

$u = e^x + 1$

$du = e^x dx$

$\int \frac{1}{u} du$

$= \ln|u| + C = \ln|e^x + 1| + C$

$= \boxed{\ln(e^x + 1) + C}$

4. Evaluate the following definite integrals using substitution.

(a) $\int_2^3 \frac{xe^{x^2}}{3} dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{6} \int_4^9 e^u\, du$

$= \frac{1}{6} [e^u]_4^9 = \boxed{\frac{1}{6}(e^9 - e^4)}$

(b) $\int_0^1 \frac{x}{1+3x^2} dx$

$u = 1 + 3x^2$

$du = 6x\,dx$

$\frac{1}{6} \int_1^4 \frac{1}{u} du$

$= \frac{1}{6} [\ln|u|]_1^4 = \frac{1}{6}(\ln(4) - \ln(1))$

$= \boxed{\frac{1}{6}\ln(4)}$

5. Show the following two integrals are equivalent:

$$\int_0^2 3x\sqrt{9 - x^2}\, dx = \int_5^9 \frac{3\sqrt{u}}{2} du.$$

$u = 9 - x^2$

$du = -2x\,dx$

Lower $x = 0 \Rightarrow u = 9$

Upper $x = 2 \Rightarrow u = 5$

$-\frac{3}{2} \int_9^5 \sqrt{u}\, du = \frac{3}{2} \int_5^9 \sqrt{u}\, du$

2

Integral Properties $\rightarrow$ $= \int_5^9 \frac{3\sqrt{u}}{2} du$