67. $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

68. $\sim(p \wedge (\sim q)) \equiv (\sim p) \vee q$

69. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
(Associative law for conjunction)

70. $(p \vee q) \vee r \equiv p \vee (q \vee r)$
(Associative law for disjunction)

71. $p \rightarrow q \equiv (\sim q) \rightarrow (\sim p)$ **72.** $\sim(p \rightarrow q) \equiv p \wedge (\sim q)$

In Exercises 73–78, use truth tables to check whether the given statement is a tautology, a contradiction, or neither. **HINT** [See Quick Examples on page A11.]

73. $p \wedge (\sim p)$ **74.** $p \wedge p$

75. $p \wedge \sim(p \vee q)$ **76.** $p \vee \sim(p \vee q)$

77. $p \vee \sim(p \wedge q)$ **78.** $q \vee \sim(p \wedge (\sim p))$

Apply the stated logical equivalence to the given statement in Exercises 79–84. **HINT** [See Example 5a, b.]

79. $p \vee (\sim p)$; the commutative law

80. $p \wedge (\sim q)$; the commutative law

81. $\sim(p \wedge (\sim q))$; DeMorgan's law

82. $\sim(q \vee (\sim q))$; DeMorgan's law

83. $p \vee ((\sim p) \wedge q)$; the distributive law

84. $(\sim q) \wedge ((\sim p) \vee q)$; the distributive law

In Exercises 85–88, use the given logical equivalence to rewrite the given sentence. **HINT** [See Example 5c.]

85. It is not true that both I am Julius Caesar and you are a fool. DeMorgan's law.

86. It is not true that either I am Julius Caesar or you are a fool. DeMorgan's law.

87. Either it is raining and I have forgotten my umbrella, or it is raining and I have forgotten my hat. The distributive law.

88. I forgot my hat or my umbrella, and I forgot my hat or my glasses. The distributive law.

Give the contrapositive and converse of each of the statements in Exercises 89 and 90, phrasing your answers in words.

89. "If I think, then I am."

90. "If these birds are of a feather, then they flock together."

Exercises 91 and 92 are multiple choice. Indicate which statement is equivalent to the given statement, and say why that statement is equivalent to the given one.

91. "In order for you to worship Den, it is necessary for you to sacrifice beasts of burden."
(A) "If you are not sacrificing beasts of burden, then you are not worshiping Den."
(B) "If you are sacrificing beasts of burden, then you are worshiping Den."
(C) "If you are not worshiping Den, then you are not sacrificing beasts of burden."

92. "In order to read the Tarot, it is necessary for you to consult the Oracle."
(A) "In order to consult the Oracle, it is necessary to read the Tarot."
(B) "In order not to consult the Oracle, it is necessary not to read the Tarot."
(C) "In order not to read the Tarot, it is necessary not to read the Oracle."

In Exercises 93–102, write the given argument in symbolic form (use the underlined letters to represent the statements containing them), then decide whether it is valid or not. If it is valid, name the validating tautology. **HINT** [See Quick Examples on pages A12, A13, A14.]

93. If I am hungry I am also thirsty. I am hungry. Therefore, I am thirsty.

94. If I am not hungry, then I certainly am not thirsty either. I am not thirsty, and so I cannot be hungry.

95. For me to bring my umbrella, it's sufficient that it rain. It is not raining. Therefore, I will not bring my umbrella.

96. For me to bring my umbrella, it's necessary that it rain. But it is not raining. Therefore, I will not bring my umbrella.

97. For me to pass math, it is sufficient that I have a good teacher. I will not pass math. Therefore, I have a bad teacher.

98. For me to pass math, it is necessary that I have a good teacher. I will pass math. Therefore, I have a good teacher.

99. I will either pass math or I have a bad teacher. I have a good teacher. Therefore, I will pass math.

100. Either roses are not red or violets are not blue. But roses are red. Therefore, violets are not blue.

101. I am either smart or athletic, and I am athletic. So I must not be smart.

102. The president is either wise or strong. She is strong. Therefore, she is not wise.

In Exercises 103–108, use the stated tautology to complete the argument.

103. If John is a swan, it is necessary that he is green. John is indeed a swan. Therefore, _____. (*Modus ponens.*)

104. If Jill had been born in Texas, then she would be able to ride horses. But Jill cannot ride horses. Therefore, ___. (*Modus tollens.*)

105. If John is a swan, it is necessary that he is green. But John is not green. Therefore, _____. (*Modus tollens.*)

106. If Jill had been born in Texas, then she would be able to ride horses. Jill was born in Texas. Therefore, ___ (*Modus ponens.*)

107. Peter is either a scholar or a gentleman. He is not, however, a scholar. Therefore, ___. (Disjunctive syllogism.)

108. Pam is either a plumber or an electrician. She is not, however, an electrician. Therefore, ___ (Disjunctive syllogism.)