Sec. 4.4 notes

Example 6 Sketch the graph of

$$f(x) = \frac{x^2+4}{2x}.$$

1) Domain: $(-\infty, 0) \cup (0, \infty)$

Symmetry: $f(-x) = \frac{(-x)^2+4}{2(-x)} = -\frac{x^2+4}{2x} = -f(x)$

$f(-x) = -f(x)$ means f is an odd function so it has symmetry wrt the origin

2) $f'(x) = \frac{2x(2x) - (x^2+4)(2)}{(2x)^2}$

$= \frac{4x^2 - 2x^2 - 8}{4x^2}$

$= \frac{2x^2-8}{4x^2}$

$f'(x) = \frac{x^2-4}{2x^2}$

$f''(x) = \frac{2x^2(2x) - (x^2-4)(4x)}{(2x^2)^2}$

$= \frac{\cancel{4x^3} - \cancel{4x^3} + 16x}{4x^4}$

$f''(x) = \frac{4}{x^3}$

3) $f'(x) = 0$ when $\boxed{x = -2, x = 2}$

$f'(x)$ undefined $\boxed{x = 0}$

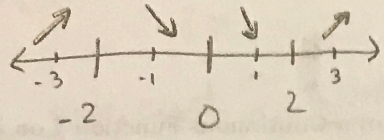
Since f is not defined at 0, it is neither a max nor a min.

$f''(-2) < 0$ so $x = -2$ is a local max

$f''(2) > 0$ so $x = 2$ local min

$f'(x) = \dfrac{x^2-4}{2x^2}$ $\qquad$ $f''(x) = \dfrac{4}{x^3}$

4) Intervals of Increase / Decrease



$f'(-3) > 0$ $\qquad$ $f'(1) < 0$

$f'(-1) < 0$ $\qquad$ $f'(3) > 0$

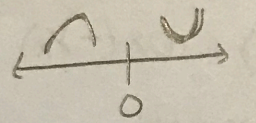
Increasing : $(-\infty, -2) \cup (2, \infty)$

Decreasing : $(-2, 0) \cup (0, 2)$

5) Inflection Points:

$f''(x)$ is undefined at $x = 0$

Test! (Even though f is not defined at $x = 0$, could still be an IP?)



$f''(-1) < 0$ $\qquad$ $x = 0$ is an IP!

$f''(1) > 0$

f is Concave down on $(-\infty, 0)$

concave up on $(0, \infty)$

(6) Asymptotes: $f(x) = \dfrac{x^2+4}{2x}$

Horizontal Asymptotes: NA

Oblique Asymptote: Since the degree of the numerator is EXACTLY one more than the degree of f has an OA. Long division!

$$\begin{array}{r} \tfrac{1}{2}x \\ 2x \overline{)\, x^2+4} \\ -x^2 \quad \\ \hline 4 \end{array}$$

$\boxed{y = \frac{1}{2}x}$

Vertical Asymptote(s): $\lim\limits_{x \to 0^-} \dfrac{x^2+4}{2(x)} = -\infty$ $\quad$ $\lim\limits_{x \to 0^+} \dfrac{x^2+4}{2x} = \infty$

$\boxed{x = 0}$

7) Plot key points and graph!

$$f(x) = \frac{x^2+4}{2x}$$

X, y intercepts? <u>None</u>

Local max: $(-2, -2)$

Local min: $(2, 2)$

$\Big\}$ (symmetric about origin! ✔)

Asymptotes: $y = \frac{1}{2}x$ $\quad x = 0$