Math 141: Section 2.6 Limits Involving Infinity; Asymptotes of Graphs - Notes

Example 1 Consider the function $f(x) = \frac{1}{x}$.

Theorem All the Limit Laws in Theorem 1 are true when we replace $\lim_{x \to c}$ by $\lim_{x \to \infty}$ or $\lim_{x \to -\infty}$. That is, the variable x may approach a finite number c or $\pm\infty$.

Example 2 We can use limit laws to calculate limits in the same was as when x approaches a finite number c.

a)

$$\lim_{x \to \infty} \left(5 + \frac{1}{x} \right)$$

b)

$$\lim_{x \to -\infty} \frac{\pi\sqrt{3}}{x^2}$$

Limits at Infinity of Rational Functions Various things can happen when we consider the limit of a rational function as $x \to \pm\infty$. The next example considers when the degree of the numerator is less than or equal to the degree of the denominator.

Example 3 Evaluate the following limits:

a)
$$\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$$

b)
$$\lim_{x \to -\infty} \frac{11x + 2}{2x^3 - 1}$$

Definition: A line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = b \text{ or } \lim_{x \to -\infty} f(x) = b.$$

Example 4 Find the horizontal asymptotes of the graph of

$$f(x) = \frac{x^3 - 2}{|x|^3 + 1}.$$

Example 5 Find

$$\lim_{x \to 0^-} e^{1/x}.$$

3

Example 6 Find

$$\lim_{x \to \infty} (x - \sqrt{x^2 + 16}).$$

Oblique Asymptotes What if the degree of the numerator is exactly one more than the degree of the denominator?

Example 7 Consider the function

$$f(x) = \frac{x^2 - 3}{2x - 4}.$$

Oblique Asymptotes If the degree of the numerator of a rational function is 1 greater than the degree of the denominator, the graph has an **oblique** or **slant** asymptote. We find an equation for the asymptote by dividing the numerator by the denominator to express f as a linear function plus a remainder that goes to zero as $x \to \pm\infty$.

Example 8: Infinite Limits Find

$$\lim_{x \to 1^+} \frac{1}{x - 1}.$$

Vertical Asymptotes A line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty \text{ or } \lim_{x \to a^-} f(x) = \pm\infty.$$

Example 9 Find the horizontal and vertical asymptotes of the curve

$$y = \frac{x + 3}{x + 2}.$$

Example 9 (cont.)