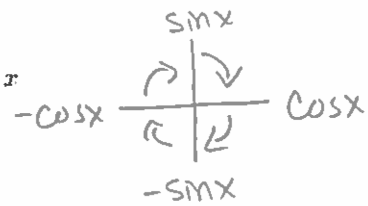
Math 141: Section 3.5 Derivatives of Trigonometric Functions - Notes

Derivatives of Sine and Cosine The derivative of the sine function is the cosine function and the derivative of the cosine function is the negative of the sine function:

$$\frac{d}{dx}(\sin x) = \cos x \text{ and } \frac{d}{dx}(\cos x) = -\sin x$$



Example 1 Differentiate:

$$y = \frac{\cos x}{1 - \sin x}$$

$$y' = \frac{(1-\sin x)(\cos x)' - \cos x\,(1-\sin x)'}{(1-\sin x)^2}$$

$$= \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$

$\sin^2 x + \cos^2 x = 1$

$$= \frac{-\sin x + 1}{(1-\sin x)^2}$$

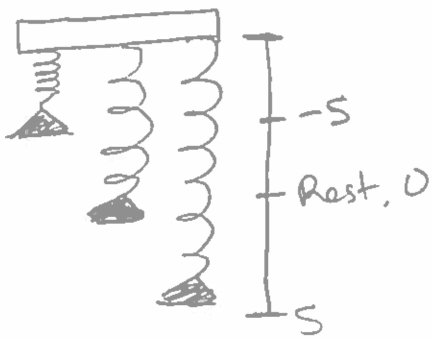
$$= \frac{1-\sin x}{(1-\sin x)^2}$$

$$= \boxed{\frac{1}{1-\sin x}}$$

Simple Harmonic Motion A weight hanging from a spring is stretched down 5 units beyond its rest position and released at time $t = 0$ to bob up and down. Its position at any later time t is

$$s = 5\cos t.$$

What are its velocity and acceleration at time t?



Position: $s = 5\cos t$

Velocity: $v = \frac{ds}{dt} = \frac{d}{dt}(5\cos t) = -5\sin t$

Acceleration: $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{d}{dt}(-5\sin t)$

$= -5\cos t$

Observations:

1) As time passes, the weight moves up and down between $s = 5$ and $s = -5$. Amplitude = 5
Period = 2π

2) The velocity gains its greatest magnitude, 5, when $\cos t = 0$. Hence, the speed of the weight $|v| = 5|\sin t|$ is greatest when $\cos t = 0$, ie when $s = 0$ (the rest position).

3) The jerk, $j = \frac{da}{dt} = \frac{d^3s}{dt^3} = \frac{d}{dt}(-5\cos t)$

$= 5\sin t$

Derivatives of Other Basic Trigonometric Functions Because $\sin x$ and $\cos x$ are differentiable functions of x, the related functions $\tan x$, $\cot x$, $\sec x$, and $\csc x$ are differentiable at every value of x at which they are defined. Their derivatives, calculated from the Quotient Rule, are given by the following formulas.

Note: 1) The negatives in the derivative formulas for the cofunctions. 2) I will expect you to know how to derive these formulas on an exam (using the Quotient Rule).

$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$
$$= \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x}$$
$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$
$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$1 + \frac{\sin^2 x}{\cos^2 x}$$
$$= 1 + \tan^2 x$$
$$= \sec^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$
$$\tan^2 x + 1 = \sec^2 x$$

$f(x) = e^x \qquad f'(x) = e^x$

$g(x) = \cos x \qquad g'(x) = -\sin x$

$e^{\cos x} \cdot -\sin x$