Solutions

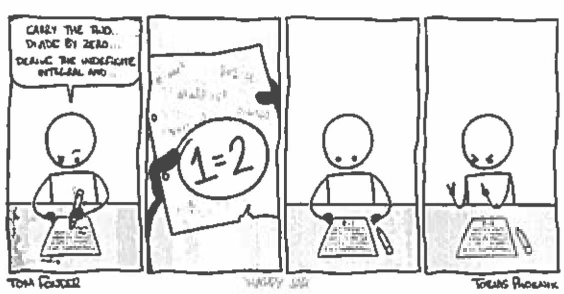
Math 122 Calculus for Business Admin. and Social Sciences | Exam #2 A
July 20, 2017

Instructor: Ann Clifton | Name: ______________________

Do not turn this page until told to do so. You will have a total of 1 hour 25 minutes to complete the exam. **Unless otherwise stated**, you **must** show all work to receive full credit. Unsupported or otherwise **mysterious** answers will **not receive credit.** If you require extra space, use the provided scrap paper and **indicate that** you have done so.
You may use a **calculator without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw a sun on this **page if** you read these directions in full. Cheating of any kind on the exam will not be tolerated and **will result** in a grade of 0%.

#	score	out of	#	score	out of
1		3	9		10
2		3	10		12
3		3	11		12
4		5	12		10
5		2	13		12
6		2	14		20
7		1	EC		6
8		5	Total		100

Remember: **This exam** has no impact on your worth as a human being. **You got this!!!**



Throughout this section, let f and g be differentiable functions. Fill in the blanks.

1. (3 points)

   (a) Let a be a constant; $\frac{d}{dx}(af(x)) =$ $af'(x)$

   (b) $\frac{d}{dx}(f(x) + g(x)) =$ $f'(x) + g'(x)$

   (c) $\frac{d}{dx}(f(x) - g(x)) =$ $f'(x) - g'(x)$

2. (3 points)

   (a) For n a number, $\frac{d}{dx}(x^n) =$ nx^{n-1}

   (b) $\frac{d}{dx}(\ln(x)) =$ $1/x$

   (c) $\frac{d}{dx}(e^x) =$ e^x

3. (3 points) Write the formula for each of the following derivatives.

(a)

$$\frac{d}{dx}(f(x)g(x))$$

$f'(x)g(x) + f(x)g'(x)$

(b)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

(c)

$$\frac{d}{dx}(f \circ g(x))$$

$f'(g(x)) \cdot g'(x)$

For each of the following questions, circle the correct answer.

4. (5 points) Assume that f is a function such that $f'(x)$ and $f''(x)$ are defined for all x.

(a) A point p is a critical point of f if

A. $f'(p) = 0$ **B.** $f'(p) < 0$

C. $f'(p) > 0$ **D.** $f(p) = 0$

(b) f is increasing on an interval if

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

(c) f is decreasing on an interval if

A. $f' < 0$ on that interval **B.** $f' > 0$ on that interval

C. $f > 0$ on that interval **D.** $f < 0$ on that interval

(d) f is concave down on an interval if

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

(e) f is concave up on an interval if

A. $f'' = 0$ on that interval **B.** $f'' > 0$ on that interval

C. $f'' < 0$ on that interval **D.** $f' = 0$ on that interval

5. (2 points) The first derivative test says that a critical point, p, of f is a

(a) local maximum if

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

(b) local minimum if

A. f' changes from negative to positive at p

B. f' changes from positive to negative at p

C. f changes from positive to negative at p

D. f changes from negative to positive at p

6. (2 points) The second derivative test says that a critical point, p, of f is a

(a) local maximum if

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C. $f'' < 0$

D. $f'' > 0$

(b) local minimum if

A. f'' changes from negative to positive at p

B. f'' changes from positive to negative at p

C. $f'' < 0$

D. $f'' > 0$

7. (1 point) Suppose that $f''(p) = 0$. We say that p is an inflection point of f if

A. f'' changes sign at p

B. f changes sign at p

C. $f'(p) = 0$

D. $f(p) = 0$

8. (5 points) Find the derivative of the following functions.

(a) $f(x) = 3x + 7$

$f'(x) = 3$

(b) $g(x) = 5x^2 + 2x + 1$

$g'(x) = 10x + 2$

(c) $h(x) = 12x^3 + 13x^2$

$h'(x) = 36x^2 + 26x$

(d) $r(x) = \frac{1}{3}x^3 + 2$

$r'(x) = x^2$

(e) $s(x) = \sqrt{x} + 3$

$s'(x) = \frac{1}{2}x^{-1/2}$

9. (10 points) **Find the derivative of the following functions.**

(a) $(x+7)^{25}$

$25(x+7)^{24}$

(b) $e^{\frac{1}{2}x^2+2x+1}$

$e^{\frac{1}{2}x^2+2x+1}(x+2)$

(c) $\ln(2x^2+7)$

$\frac{4x}{2x^2+7}$

(d) $\sqrt{x^2+1}$

$x(x^2+1)^{-1/2}$

(e) $6e^{5x}+e^{-x^2}$

$30e^{5x}-2xe^{-x^2}$

10. (12 points) Differentiate the following functions.

(a) xe^{-2x}

$e^{-2x} - 2xe^{-2x}$

(b) $x\ln(x)$

$\ln x + 1$

(c) $(x^2+3)e^x$

$2xe^{x} + (x^2+3)e^{x}$

11. (12 points) Differentiate the following functions.

(a) $\frac{x+1}{x-1}$

$$\frac{-2}{(x-1)^2}$$

(b) $\frac{x}{e^x}$

$$\frac{e^x - xe^x}{e^{2x}}$$

(c) $\frac{x}{\ln(x)}$

$$\frac{\ln x - 1}{(\ln x)^2}$$

12. (10 Points) Let $f(x) = 10x^4 - 4x^5$.

1 (a) Find the derivative of f.

$f'(x) = 40x^3 - 20x^4$

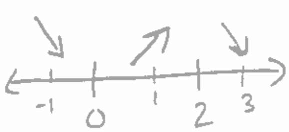
1 (b) Find the critical points of f.
[Hint: Factoring after taking the derivative will make this much easier.]

$40x^3 - 20x^4 = 20x^3(2-x)$

$20x^3(2-x) = 0$

$x = 0$ or $x = 2$

4 (c) Find any local maxima and local minima of f. Clearly indicate whether a point is a maximum or a minimum.



$f'(-1) < 0$

$f'(1) > 0$

$f'(3) < 0$

OR $f''(x) = 120x^2 - 80x^3$

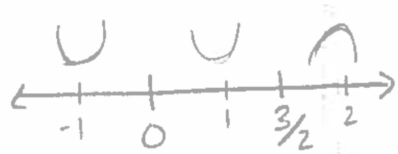
$f''(0) = 0$ Inconclusive!
(Back to first der. test)

$f''(2) < 0$ $x = 2$ local max

By the first derivative test,
$x = 0$ is a local min, $x = 2$ is a local max

4 (d) Find any inflection points of f.

$f''(x) = 120x^2 - 80x^3$
$= 40x^2(3-2x)$
$x = 0$, $x = 3/2$



$f''(-1) > 0$
$f''(1) > 0$
$f''(2) < 0$

By second derivative test,
$x = 3/2$ is an inflection point

13. (12 points) Find the global maximum and global minimum of $f(x) = 10x^4 - 4x^5$ on the interval $[1, 3]$. [Hint: This is the same function as the previous problem.]

$f'(x) = 40x^3 - 20x^4$

$x = 0$ and $x = 2$ are critical points

Since only $x = 2$ is in the interval, $[1,3]$, we only need to check

$f(1) = 6$

$f(2) = 32$

$f(3) = -162$

So, the global minimum occurs at $x = 3$ and the global maximum occurs at $x = 2$.

14. (20 points) A company sells a product for \$21 each and the manufacturing costs can be modeled by the function

$$C(q) = \frac{1}{3}q^3 - 2q^2 + 100$$

of q units produced. For each of the quantities below, determine whether the company should increase, decrease, or not change the production levels in order to maximize profit. Justify your answers using calculus. **You will not receive credit for guess and check solutions.**

(a) $q = 3$ — 1 Increase

(b) $q = 7$ — 1 No Change

(c) $q = 9$ — 1 Decrease

2 $R(q) = 21q$

4 $\pi(q) = 21q - (\frac{1}{3}q^3 - 2q^2 + 100)$

$= 21q - \frac{1}{3}q^3 + 2q^2 - 100$

4 $\pi'(q) = 21 - q^2 + 4q$

$= -q^2 + 4q + 21$

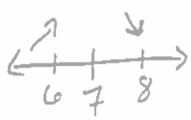
$-q^2 - 4q + 21 = 0$

$-(q^2 + 4q - 21) = 0$

$-(q-7)(q+3) = 0$

4 $q = 7, q = -3$

Since we can't produce negative items, we only consider $q = 7$.

3 

$\pi'(6) > 0$

$\pi'(8) < 0$

So, $q = 7$ is a local max and since $\pi(q)$ is decreasing after $q = 7$, this is also the global max on the relevant domain, $[0, \infty)$.