Math 141: Section 3.6 The Chain Rule - Notes

Example 1 How do we differentiate the composite $f \circ g$ of two function $y = f(u)$ and $u = g(x)$? The function

$$y = (3x^2 + 1)^2$$

is the composite of

Calculating derivatives, we have

The Chain Rule If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

In Leibniz's notation, if $y = f(u)$ and $u = g(x)$, then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx},$$

where $\frac{dy}{du}$ is evaluated at $u = g(x)$.

Example 2 Differentiate $\sin(x^2 + e^x)$ with respect to x:

Example 3 Differentiate

$$y = e^{\cos x}.$$

Example 4 An object moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = \cos(t^2 + 1)$. Find the velocity of the object as a function of t.

Example 5 Find the derivative of $g(t) = \tan(5 - \sin 2t)$.

Example 6 Compute the following

$$\frac{d}{dx}\left(5x^3 - x^4\right)^7$$

Example 7 Compute the following

$$\frac{d}{dx}\left(\frac{1}{3x-2}\right)$$