Math 122 Calculus for Business Admin. and Social Sciences Exam #1 B

September 25, 2017

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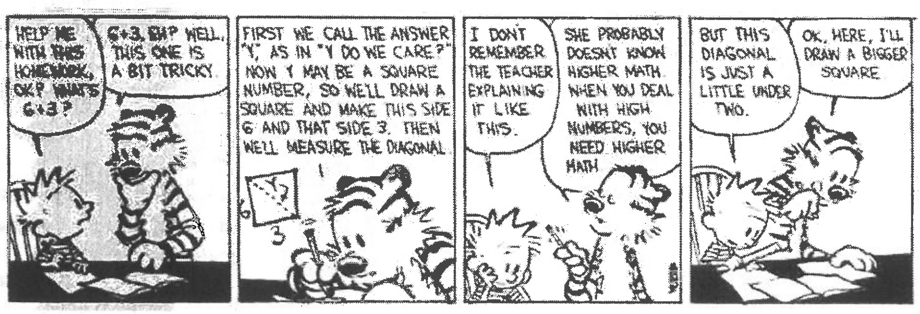
Do not turn this page until told to do so. You will have a total of 50 minutes to complete the exam. Unless otherwise stated, you **must** show all work to receive full credit. Unsupported or otherwise mysterious answers **will not receive credit.** If you require extra space, use the provided scrap paper and indicate that you have done so.
You may use a calculator **without a CAS** if you like, but a calculator is not necessary. NO PHONES ALLOWED.
Draw a fish on this page if you read these directions in full. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.

#	score	out of	#	score	out of
1		2	7		9
2		5	8		12
3		5	9		15
4		5	10		15
5		6	11		20
6		6	Total		100

Remember: This exam has no impact on your worth as a human being. You got this!!!



1. (a) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m.

1

$y - y_0 = m(x - x_0)$

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

1

$y = mx + b$

2. Let f be a function and let $a < b$ be given. State the average rate of change of f on the interval $[a, b]$.

5

$$\frac{f(b) - f(a)}{b - a}$$

3. Given a quantity P, state the relative change of the quantity from P to P'.

5

$$\frac{P' - P}{P}$$

4. (a) State the form of an exponential function of a variable t with initial value P_0 and base a:

$$P(t) = \underline{\quad P_0\, a^t \quad}.$$

(b) The relative rate of change of P is

$$r = \underline{\quad a-1 \quad}.$$

[Hint: If you don't recall the formula, this is just the relative change from $P(t)$ to $P(t+1)$.]

(c) The function P models

(i) exponential growth when r is positive.

(ii) exponential decay when r is negative.

(d) The continuous growth/decay rate is

$$k = \underline{\quad \ln(a) \quad}.$$

5. Let $0 < x$, $0 < y$ be given. Fill in the blanks:

(i) $\ln(1) = 0$

(ii) $\ln(xy) = \ln(x) + \ln(y)$

(iii) $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

(iv) $\ln(x^r) = r\ln(x)$

(v) $\ln(e^x) = x$

(vi) $e^{\ln(x)} = x$

6. (a) Find the slope of the line passing through the points $(\frac{1}{2}, 3)$ and $(1, 2)$.

1

$$m = \frac{3-2}{\frac{1}{2}-1} = \frac{1}{-\frac{1}{2}} = -2$$

(b) Write the equation of this line in Point-Slope Form.

1

$$y - 2 = -2(x-1)$$

OR

$$y - 3 = -2(x - \tfrac{1}{2})$$

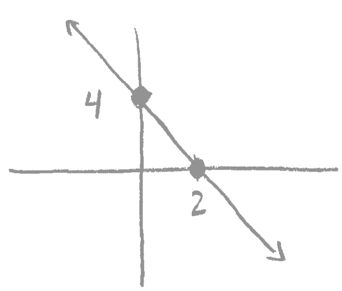
(c) Write the equation of this line in Slope-Intercept Form.

1

$$y - 2 = -2(x-1)$$
$$y - 2 = -2x + 2$$
$$y = -2x + 4$$

(d) Sketch a graph of $f(x)$. Label the x-intercept and the y-intercept.

3



Line with neg. slope +1

x-int +1

y-int +1

7. Let $f(x) = x^2 - 1$.

(a) Compute the average rate of change for f between $x = 3$ and $x = 5$.

3

$$1 \quad \frac{f(5)-f(3)}{5-3} = \frac{(25-1)-(9-1)}{2} = \frac{24-8}{2} = 8$$

1 1

(b) Give the **Point-Slope** form of the line that passes through $(3, f(3))$ and $(5, f(5))$.

3

$m = 8$ from part (a) $(3,8)$ $(5,24)$

$y - 8 = 8(x-3)$ OR $y - 24 = 8(x-5)$

3 (c) Give the **Slope-Intercept** form of the line from part (b).

$y - 8 = 8x - 24$ OR $y - 24 = 8x - 40$

$y = 8x - 16$ $y = 8x - 16$

8. A biologist observes a population with initial size 81. In two years, the biologist returns to observe the population again and finds that only 9 remain.

4 (a) Find an exponential function for the size of the population as a function of t years since the initial observation.

1 $P_0 = 81$ 1 $P(t) = 81(1/3)^t$

$P(2) = 81a^2 = 9$

$a^2 = \frac{1}{9}$

$a = 1/3$ 2

4 (b) Does the function from part (a) model growth or decay?

Decay

4 (c) Use the model from part (a) to determine how many years it will take for the size of the population to reach 1.

1 $1 = 81(1/3)^t$

1 $\frac{1}{81} = (1/3)^t$ $\longrightarrow$

1 $\ln(1/81) = t\ln(1/3)$

$\frac{\ln(1/81)}{\ln(1/3)} = t$

1 $\boxed{t = 4 \text{ years}}$

Without a calculator

$\frac{1}{81} = \left(\frac{1}{3}\right)^t$

$\left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^t$

$t = 4$

9. A bank is offering an account that pays 5% interest compounded continuously. If you decide to invest money in this account, how long will it take for your initial investment to double? Round to the nearest year.

$$P(t) = P_0 e^{rt} \qquad r = .05$$

1 $P(t) = P_0 e^{.05t}$

5 $2 = 1e^{.05t}$

$\ln(2) = .05t$

5

$\frac{\ln(2)}{.05} = t$

$t \approx 13.9$

4 $t = 14$ years

Rule of 70:

15 $\frac{70}{5} = 14$ years

10. Sketch a graph of the function

$$f(x) = -2x^2 + 8x - 6.$$

Find the x-intercepts, the y-intercept, and the vertex. You may use your **calculator** to check your answer but you must show supporting work.

Hint: Given $f(x) = ax^2 + bx + c$, the formula for the vertex of the parabola is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

5

(a) State the x-intercepts: $(3,0)$ $(1,0)$

$0 = -2x^2 + 8x - 6$ $\qquad$ $0 = -2(x-3)(x-1)$

$0 = -2(x^2 - 4x + 3)$ $\qquad$ $x = 3,\ x = 1$

(b) State the y-intercept: $(0,-6)$

2

$y = -2(0)^2 + 8(0) - 6$

$y = -6$

(c) State the vertex: $(2,2)$

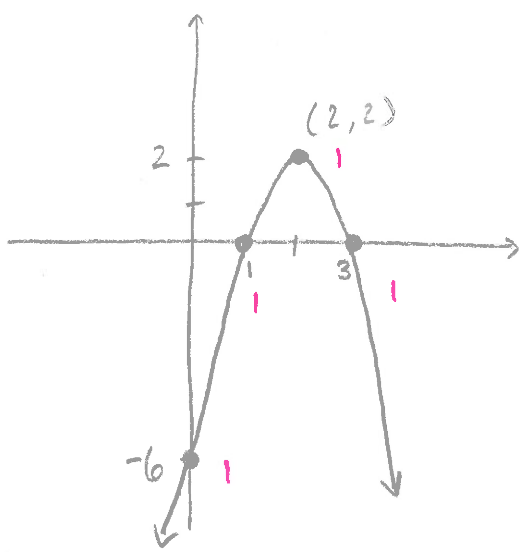
3

$a = -2$ $\quad$ $b = 8$ $\qquad$ $\frac{-b}{2a} = \frac{-8}{2(-2)} = 2$ $\qquad$ $f(2) = -2(2)^2 + 8(2) - 6$

$= -8 + 16 - 6 = 2$

(d) Sketch a graph:

5



Parabola
+1

11. A company hosts a weekly event. They find that 30 people attend at a ticket price of \$25, and 50 people attend at a ticket price of \$15. Assuming this relationship is linear, determine the ticket price that will generate the highest revenue. State the maximum revenue.

Hint: First, find the equation of the line representing the quantity of tickets sold, q, in terms of price, p.

(p,q) $(25,30)$

$(15,50)$

$$m = \frac{50-30}{15-25} = \frac{20}{-10} = -2 \quad 5$$

$$q-30 = -2(p-25) \quad 2$$
$$q-30 = -2p+50$$
$$q = -2p+80 \quad 3$$

Revenue: $R(p) = p \cdot q = p(-2p+80)$

5

$= -2p^2+80p$

Downward facing parabola; Maximum occurs at the vertex!

$$p = \frac{-b}{2a} = \frac{-80}{2(-2)} = 20$$

5

\$20 is the price that maximizes revenue.

The maximum revenue is

$R(20) = -2(20)^2+80(20) = \$800$