Math 141: Section 3.5 Derivatives of Trigonometric Functions - Notes

Derivatives of Sine and Cosine The derivative of the sine function is the cosine function and the derivative of the cosine function is the negative of the sine function:

$$\frac{d}{dx}(\sin x) = \cos x \text{ and } \frac{d}{dx}(\cos x) = -\sin x$$

Example 1 Differentiate:

$$y = \frac{\cos x}{1 - \sin x}$$

Simple Harmonic Motion A weight hanging from a spring is stretched down 5 units beyond its rest position and released at time $t = 0$ to bob up and down. Its position at any later time t is

$$s = 5 \cos t.$$

What are its velocity and acceleration at time t?

Derivatives of Other Basic Trigonometric Functions Because $\sin x$ and $\cos x$ are differentiable functions of x, the related functions $\tan x$, $\cot x$, $\sec x$, and $\csc x$ are differentiable at every value of x at which they are defined. Their derivatives, calculated from the Quotient Rule, are given by the following formulas.

Note: 1) The negatives in the derivative formulas for the cofunctions. 2) I will expect you to know how to derive these formulas on an exam (using the Quotient Rule).

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$