Practice Exam 2 Solutions

1) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$E = \{TTT, TTH, THT, HTT\}$

2) $E = \{(2,2), (3,3), (5,5), (2,3), (2,5), (3,5), (3,2), (5,2), (5,3)\}$

3) (a) $A \cap B$

(b) D'

(c) $D' \cup A$

(d) $B \cup (A \cap D)$

4) $C(4,1) \cdot C(2,1) \cdot C(2,1) = 4 \cdot 2 \cdot 2 = 16$

5) $P(E) = \frac{fr(E)}{N} = \frac{300}{400} = 3/4$

6) $E = \{HH, HT, TH\}$

$$P(E) = \frac{1700 + 1550 + 1800}{6400} = \frac{5050}{6400} = \frac{505}{640} \approx 78.9\%$$

7) Answers may vary.

Properties to check:

$0 \le P(s_i) \le 1$ for all outcomes s_i

$P(s_1) + P(s_2) + \ldots + P(s_r) = 1$ (Rel. frequencies should add to 1).

8) Answers may vary but I want to see at least something to the effect of

"We use relative frequency when we have a sample of outcomes and probability when we can consider the entire sample space or our sample is "large enough"."

Your book describes this more eloquently on p. 482:

"Has the probability been arrived at experimentally, by performing a number of trials and counting the number of times the event occurred? If so, the probability is estimated; that is, relative frequency. If, on the other hand, the probability was computed by analyzing the experiment under consideration rather than by performing actual trials, it is a probability (theoretical) model."

9)

Outcome	a	b	c	d	e
Probability	0.1	0.07	0.4	0.03	0.4

(a) $P(\{a, c, e\}) = 0.1 + 0.4 + 0.4 = 0.9$

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.9 + 0.87 - P(\{c, e\})$

$= 1.77 - 0.8$

$= .97$

(c) $P(E') = P(\{b, d\}) = 0.07 + 0.03 = 0.1$

(d) $P(E \cap F) = 0.8$

10) $n(S) = 5$, $n(E) = 2$

$P(E) = 2/5$

11) See problem #1!

$n(S) = 8$ $n(E) = 7$

$P(E) = 7/8$

12) $n(S) = 36$

$E = \{(4,4), (3,5), (5,3), (2,6), (6,2)\}$

$n(E) = 5$

$P(E) = \frac{5}{36}$

13)

Outcome	1	2	3	4	5	6
Probability	$1/9$	$2/9$	$1/9$	$2/9$	$1/9$	$2/9$

$P(E) = 1/9 + 2/9 + 1/9 = 4/9$

Work: Let x be the probability of the event of rolling a 2, 4, 6

y be the probability of rolling 1, 3, 5

$x = 2y$

$3x + 3y = 1$

$3(2y) + 3y = 1$

$6y + 3y = 1$

$9y = 1$

$y = 1/9 \quad x = 2(1/9) = 2/9$

14) $A \cap B = \emptyset$ implies $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.8 = P(A) + .8 - 0$

$0 = P(A)$

15) $P(A') = 1 - P(A)$

$P(A') = 1 - 0.6$

$= 0.4$

16) 4 red 3 green 4 white 1 purple, 12 total

(a) $n(S) = C(12,8) = 495$

At least one green

Alt. 1 : Exactly 1 green
Step 1: Choose 1 green, $C(3,1) = 3$
Step 2: Choose 7 more, $C(9,7) = 36$

Total: $3 \cdot 36 = 108$

Alt. 2 : Exactly 2 green
Step 1: Choose 2 green, $C(3,2) = 3$
Step 2: Choose 6 nongreen, $C(9,6) = 84$

Total: $3 \cdot 84 = 252$

Alt. 3: All 3 green
Step 1: Choose all green, $C(3,3) = 1$
Step 2: Choose 5 nongreen, $C(9,5) = 126$

Total: $108 + 252 + 126 = 486$ $P(E) = \frac{n(E)}{n(S)} = \frac{486}{495} = \boxed{\frac{54}{55}}$

(b) All the green

Step 1: Choose all the green

$C(3,3) = 1$

Step 2: Choose 5 nongreen

$C(9,5) = 126$

$$P(E) = \frac{n(E)}{n(S)} = \frac{126}{495} = 14/55$$

(c) 2 red, one of each of the other colors

*** As stated, this question is vague and the solution is more difficult than I anticipated. The question should read: "She chooses **5** marbles, 2 red and one of each of the other colors." ***

Step 1: Choose 2 red.

$C(4,2) = 6$

Step 2: Choose 1 green.

$C(3,1) = 3$

Step 3: Choose 1 white.

$C(4,1) = 4$

Step 4: Choose 1 purple.

$C(1,1) = 1$

Total: $6 \cdot 3 \cdot 4 \cdot 1 = 72$

$$P(E) = \frac{n(E)}{n(S)} = \frac{72}{792} = 1/11$$

$n(S) = C(12,5) = 792$