

# Ligeti's Combinatorial Tonality



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# Étude 1: Désordre

Musical score for Etude 1: Désordre, page 116. The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. Both staves have a key signature of four sharps. The time signature is 8/8, indicated by a circled '8'. The tempo is marked as '116'. The dynamics are 'f' (fortissimo) and 'p' (pianissimo). The score features six measures of music. Measure 1: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it. Measure 2: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it. Measure 3: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it. Measure 4: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it. Measure 5: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it. Measure 6: Treble staff has a note with a '1' above it, followed by a grace note with a '3' above it. Bass staff has a note with a '1' above it, followed by a grace note with a '2' above it.

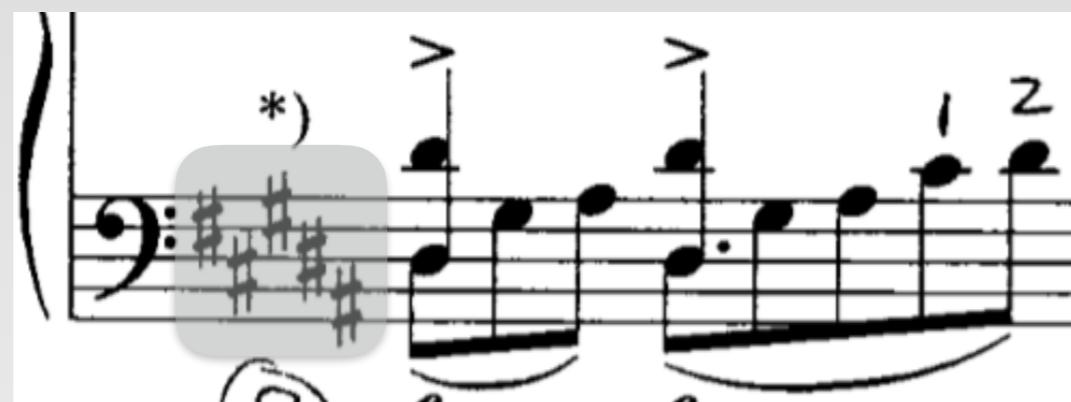
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# Étude 1: Désordre

right hand on white keys



## Étude 1: Désordre



left hand on black keys

# Étude 1: Désordre diatonic

116

\*) > > 3

f p f p

\*) > > 2

f p f p

8 f p f p

f p f p

f p f p

f p f p

1 3 >

1 3 >

1 3 >

1 3 >

?

pentatonic

white key / black key system

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“I am trying to develop a harmony and melody which are no genuine return to tonality, which are neither tonal nor atonal but rather something else, above all in connection with a very high degree of rhythmic and metric complexity.”

—Ligeti, quoted from Bossin 1984



Introduction

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“I am trying to develop a harmony and melody which are no genuine return to tonality, which are neither tonal nor atonal but rather something else, above all in connection with a very high degree of rhythmic and metric complexity.”

—Ligeti, quoted from Bossin 1984

The allocation to each hand of different but complementary scales gives to these pieces what one might call their own '**combinatorial tonality**' (i.e. the illusion of a third or resultant tonality created from the combination of the other two).

—Steinitz (2003)



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# Étude 10: Der Zauberlehrling

15

67

cresc. poco a poco

**pp**

15

poco a poco tre corde

## white key / black key system

## Introduction

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# Piano Concerto I

Musical score for Piano Concerto I, showing two staves in 12/8 time. The top staff is in C major (G clef) and the bottom staff is in G major (C clef). The score features eighth-note patterns with dynamic markings like *mf*, *pp*, and *mf*. Measure 10 is highlighted with a gray oval, and measure 11 is highlighted with a gray circle. The word "sim." appears twice in the score.

Dorian hexachord [023579] system

Close-up of a musical score showing a Dorian hexachord [023579] system. The top staff is in A major (F# clef) and the bottom staff is in E major (C# clef). The score shows six-note chords with dynamic markings like *f* and *p*. Measures 1 through 5 are shown, with measure 1 being the first inversion and measures 2 through 5 being the second inversion of the hexachord.

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# Étude 11: En Suspens

Musical score for Étude 11: En Suspens, page 18. The score consists of two staves. The top staff starts with a dynamic of *pp*. The bottom staff starts with a dynamic of *pp*. Both staves feature various performance instructions: 'non arp.' with a curved arrow above the notes, '*p* *pp*' with a curved arrow below the notes, '*non arp.*' with a curved arrow above the notes, '*< p*' with a curved arrow below the notes, and '*= pp*' with a curved arrow below the notes. The music is written in a treble clef and includes various note heads and stems.

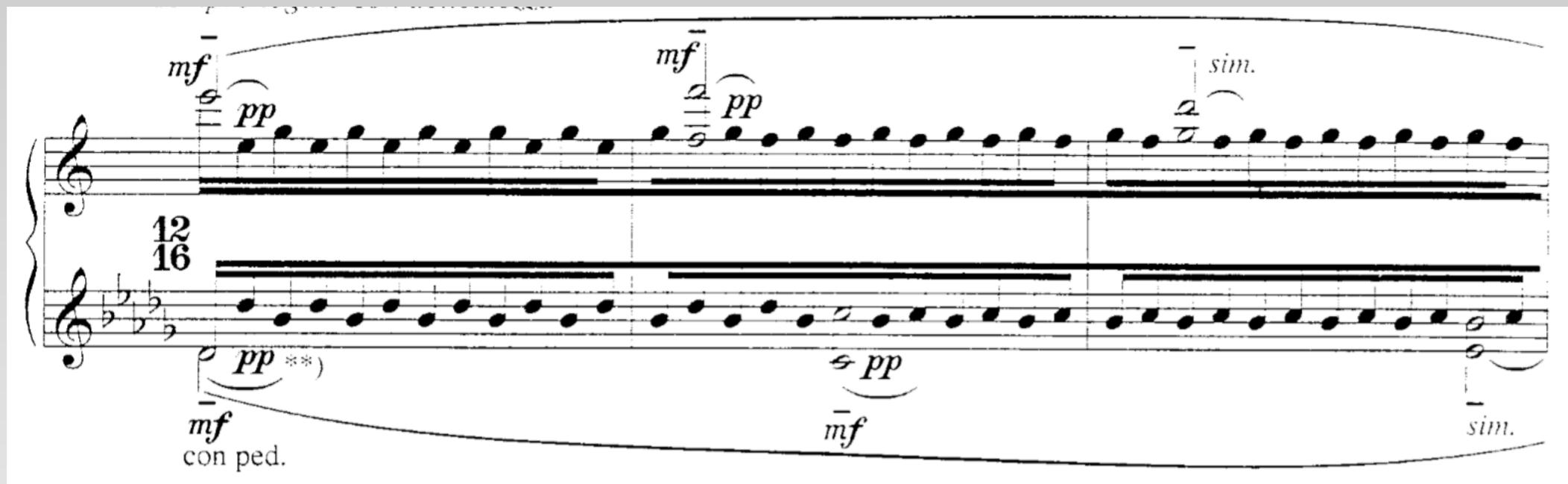
Guidonian system



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Introduction

# Étude 12: Entrelacs



Guidonian system



Introduction

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# Étude 7: Galamb Borong

Musical score for Étude 7: Galamb Borong. The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. Both staves are in B-flat major (two flats). Measure 7 begins with a dynamic of *pochissimo cresc.* followed by a decrescendo (dashed line) leading to a forte dynamic (**p**). The instruction "tre corde" is written below the bass staff. The score features eighth-note patterns and grace notes.

whole-tone system

Musical score for Ligeti's Combinatorial Tonality, titled "Introduction". The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. Both staves are in B-flat major (two flats). The score features complex rhythmic patterns with sixteenth-note figures and grace notes. The instruction "f p" (fortissimo, piano) is repeated multiple times. Measure numbers 1, 2, 3, and 4 are indicated above the staves.

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# Violin Concerto V

**A**

Oboe (Ob.)  
Bassoon (Bass.)  
Violin solo (Vln. solo)  
Viola (Va.)  
Cello (Vcl.)

with fear, as if screaming  
mit Angst, gleichsam schreiend, ferocie

al talone

poco cresc.

poco cresc.

mp espr. dim.

mp espr. dim.

ff (semperf)

ff (semperf)

ff (semperf)

ff (semperf)

separate bowing, legato  
Strich für Strich, legato  
con sord., punta d'arco  
sim.

pp leggiero

whole-tone system

Introduction

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# Properties of intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Lewin's interval function

$$\text{IC}_{A,B}(k) = \text{IFUNC}(A, B)(k) = \#\{(a, b) \in A \times B, b - a = k\}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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Lewin's interval function

$$\text{IC}_{A,B}(k) = \text{IFUNC}(A, B)(k) = \#\{(a, b) \in A \times B, b - a = k\}$$

$$\text{IC}_{\{0,2\}, \{1,3\}} = (0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 1)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Lewin's interval function

$$\text{IC}_{A,B}(k) = \text{IFUNC}(A, B)(k) = \#\{(a, b) \in A \times B, b - a = k\}$$

$$\text{IC}_{\{0,2\},\{1,3\}} = (0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 1)$$

$$\text{IC}_{A,A} = \text{IC}_A$$

$$\text{IC}_{\{0,2,4,5,7,9,11\}} = (7, 2, 5, 4, 3, 6, 2, 6, 3, 4, 5, 2)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

intra-harmonies

$$A_{\text{intra}} = \text{IC}_A + \text{IC}_{\bar{A}}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

intra-harmonies

$$A_{\text{intra}} = \text{IC}_A + \text{IC}_{\bar{A}}$$

$$W = \{0, 2, 4, 5, 7, 9, 11\}$$

$$W_{\text{intra}} = \text{IC}_W + \text{IC}_{\bar{W}}$$

$$W_{\text{intra}} = (7, 2, 5, 4, 3, 6, 2, 6, 3, 4, 5, 2) + (5, 0, 3, 2, 1, 4, 0, 4, 1, 2, 3, 0)$$

$$W_{\text{intra}} = (12, 2, 8, 6, 4, 10, 2, 10, 4, 6, 8, 2)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

inter-harmonies

$$A_{\text{inter}} = \text{IC}_{A,\bar{A}} + \text{IC}_{\bar{A},A}$$

$$A_{\text{inter}} = 2 \cdot \text{IC}_{A,\bar{A}}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

inter-harmonies

$$A_{\text{inter}} = \text{IC}_{A,\bar{A}} + \text{IC}_{\bar{A},A}$$

$$A_{\text{inter}} = 2 \cdot \text{IC}_{A,\bar{A}}$$

$$W = \{0, 2, 4, 5, 7, 9, 11\}$$

$$W_{\text{inter}} = 2 \cdot \text{IC}_{W,\bar{W}}$$

$$W_{\text{inter}} = 2 \cdot (0, 5, 2, 3, 4, 1, 5, 1, 4, 3, 2, 5)$$

$$W_{\text{inter}} = (0, 10, 4, 6, 8, 2, 10, 2, 8, 6, 4, 10)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

The combination of intra-harmonies and inter-harmonies  
yields the interval content of the aggregate.

$$A_{\text{intra}} + A_{\text{inter}} = \text{IC}_{\mathbb{Z}_n} = (n, \dots, n)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

$$A_{\text{intra}} + A_{\text{inter}} = \text{IC}_{\mathbb{Z}_n} = (n, \dots, n)$$

white key/black key system



$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

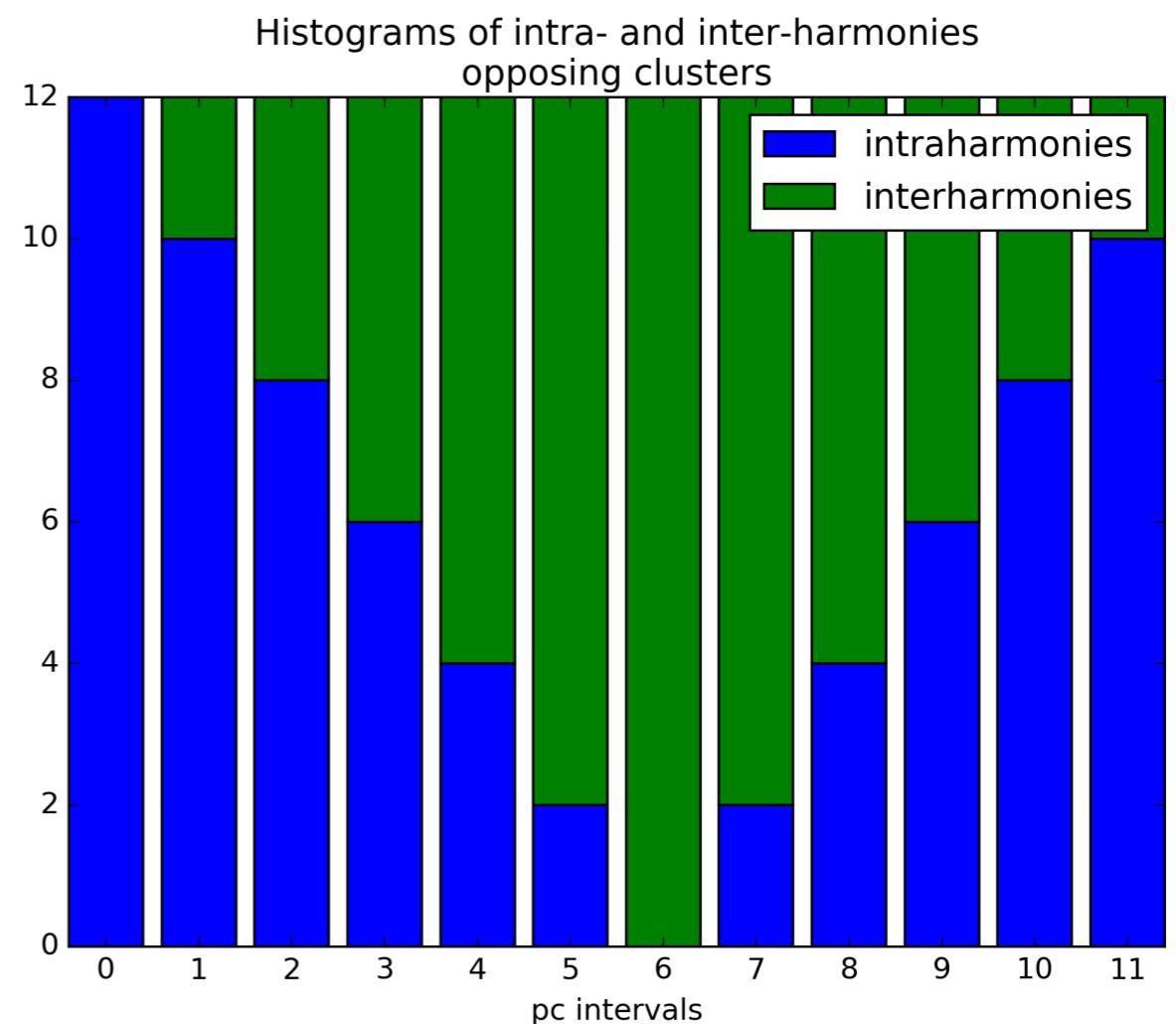
Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

$$A_{\text{intra}} + A_{\text{inter}} = \text{IC}_{\mathbb{Z}_n} = (n, \dots, n)$$

opposing six-note clusters



$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

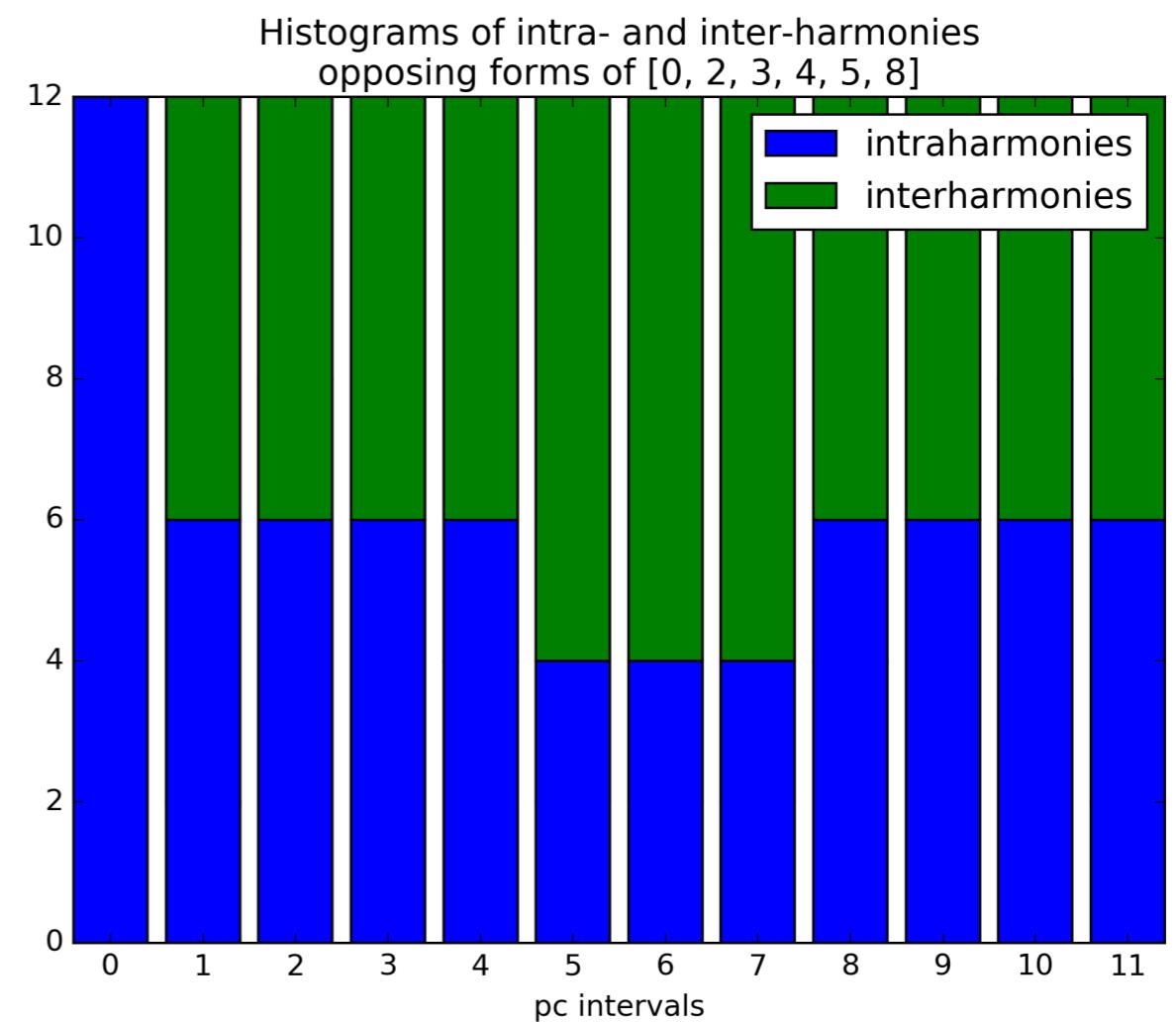
Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

$$A_{\text{intra}} + A_{\text{inter}} = \text{IC}_{\mathbb{Z}_n} = (n, \dots, n)$$

opposing  $[0, 2, 3, 4, 5, 8]$



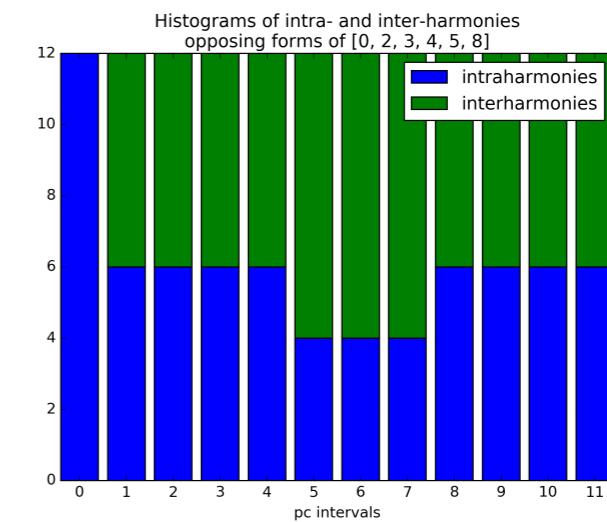
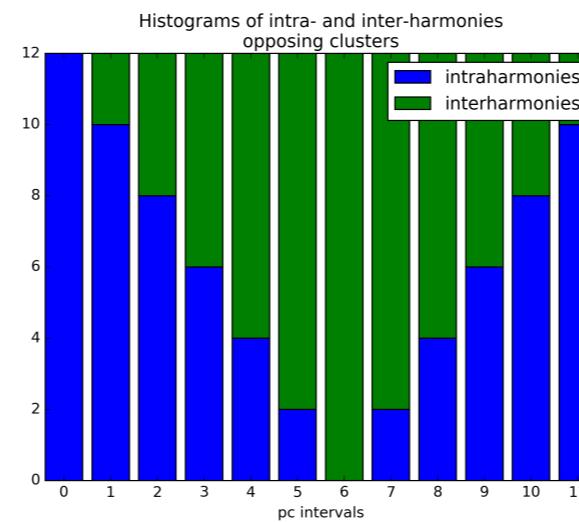
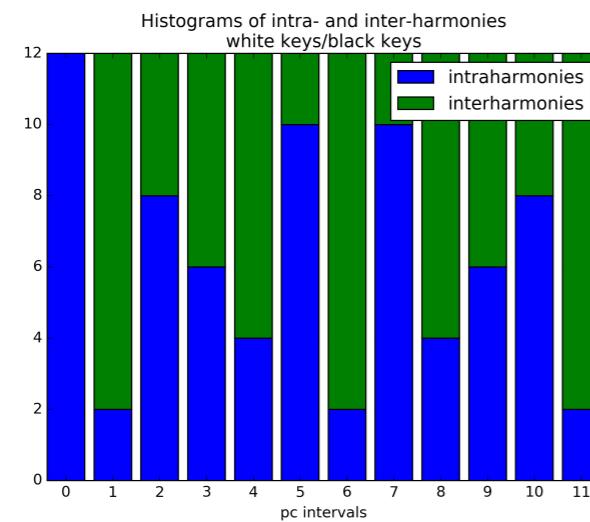
$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content



$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content

Guidonian  
 $\sigma(\text{IC}_{\{0,2,4,5,7,9\}}) = 1.78$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content

Guidonian  
 $\sigma(\text{IC}_{\{0,2,4,5,7,9\}}) = 1.78$

“flat” hexachord  
 $\sigma(\text{IC}_{\{0,2,3,4,5,8\}}) = 1.0$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content

Guidonian  
 $\sigma(\text{IC}_{\{0,2,4,5,7,9\}}) = 1.78$

“flat” hexachord  
 $\sigma(\text{IC}_{\{0,2,3,4,5,8\}}) = 1.0$

whole-tone  
 $\sigma(\text{IC}_{\{0,2,4,6,8,10\}}) = 3.0$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content

$$\sigma(\text{IC}_A) = \sigma(\text{IC}_{\bar{A}})$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring “distinctiveness” or “salience” of the interval content

$$\sigma(\text{IC}_A) = \sigma(\text{IC}_{\bar{A}})$$

$$\sigma(A_{\text{intra}}) = \sigma(A_{\text{inter}})$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring Euclidean distance between intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring Euclidean distance between intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2$$

$$W = \{0, 2, 4, 5, 7, 9, 11\}$$

$$\|W_{\text{inter}} - W_{\text{intra}}\|_2 = 22.98$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring Euclidean distance between intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2$$

$$W = \{0, 2, 4, 5, 7, 9, 11\}$$

$$\|W_{\text{inter}} - W_{\text{intra}}\|_2 = 22.98$$

$$X = \{0, 2, 3, 4, 5, 8\}$$

$$\|X_{\text{inter}} - X_{\text{intra}}\|_2 = 13.86$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Properties of intra- and inter-harmonies

Measuring Euclidean distance between intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2$$

$$\text{if } \#A = \#\bar{A}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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Measuring Euclidean distance between intra- and inter-harmonies

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2$$

$$\text{if } \#A = \#\bar{A}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

$$\text{if } \#A \neq \#\bar{A}$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \underset{\sim}{\propto} \sigma(\text{IC}_A)$$

$$\|A_{\text{inter}} - A_{\text{intra}}\|_2 \propto \sigma(\text{IC}_A)$$

Properties of intra- and inter-harmonies

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# Désordre

*dédicace à Pierre Boulez*  
Étude 1: Désordre

György Ligeti

Molto vivace, vigoroso, molto ritmico,  $\text{c} = 63$

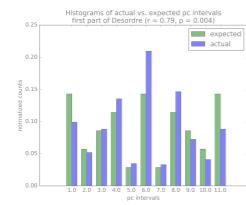
10

(cresc.)  
m.  
(cresc.)  
m.  
(cresc.)  
pianissimo  
(decresc.)  
m.  
(cresc.)  
ff  
(cresc. molto)  
ff

11

12

13



Désordre

\* Gradually use rather more pedal. Dynamic balance: die rechte Hand spielt etwas lebhafter als die linke Hand, so dass die rechte Hand mehr Pedal braucht. Beide Hände sollten gleich laut klingen. Gradual crescendo until the end of the section; the section gradually becomes ff, then ff (the right hand should begin ff earlier than the left). The quarter (ff) figures gradually become esp. then ff.

\* Allmählich etwas mehr Pedal. Dynamische Balance: die rechte Hand spielt etwas lebhafter als die linke Hand, so dass die rechte Hand mehr Pedal braucht. Beide Hände sollten gleich laut klingen. Allmähliches crescendo bis zum Schluß der Etüde; die Alzette werden allmählich lauter. Die rechte Hand sollte schon ff beginnen, während die linke Hand noch esp ist. Die quarto (ff) Figuren sollten allmählich esp. dann ff werden.

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# Désordre

## first part

*dédicace à Pierre Boulez*  
Étude 1: Désordre  
György Ligeti

Molto vivace, vigoroso, molto ritmico,  $\alpha = 63$

\*Use the pedal sparingly throughout.  
Play the melody legato in both hands.

\*Stern opernweise Gehauenes des Pedals  
Die Melodie in beiden Händen legato

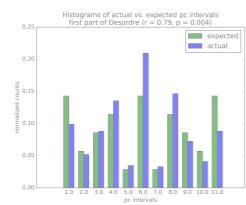
10

ff cresc. molto

13

Duritt ca. 2' 20"

## second part



Désordre

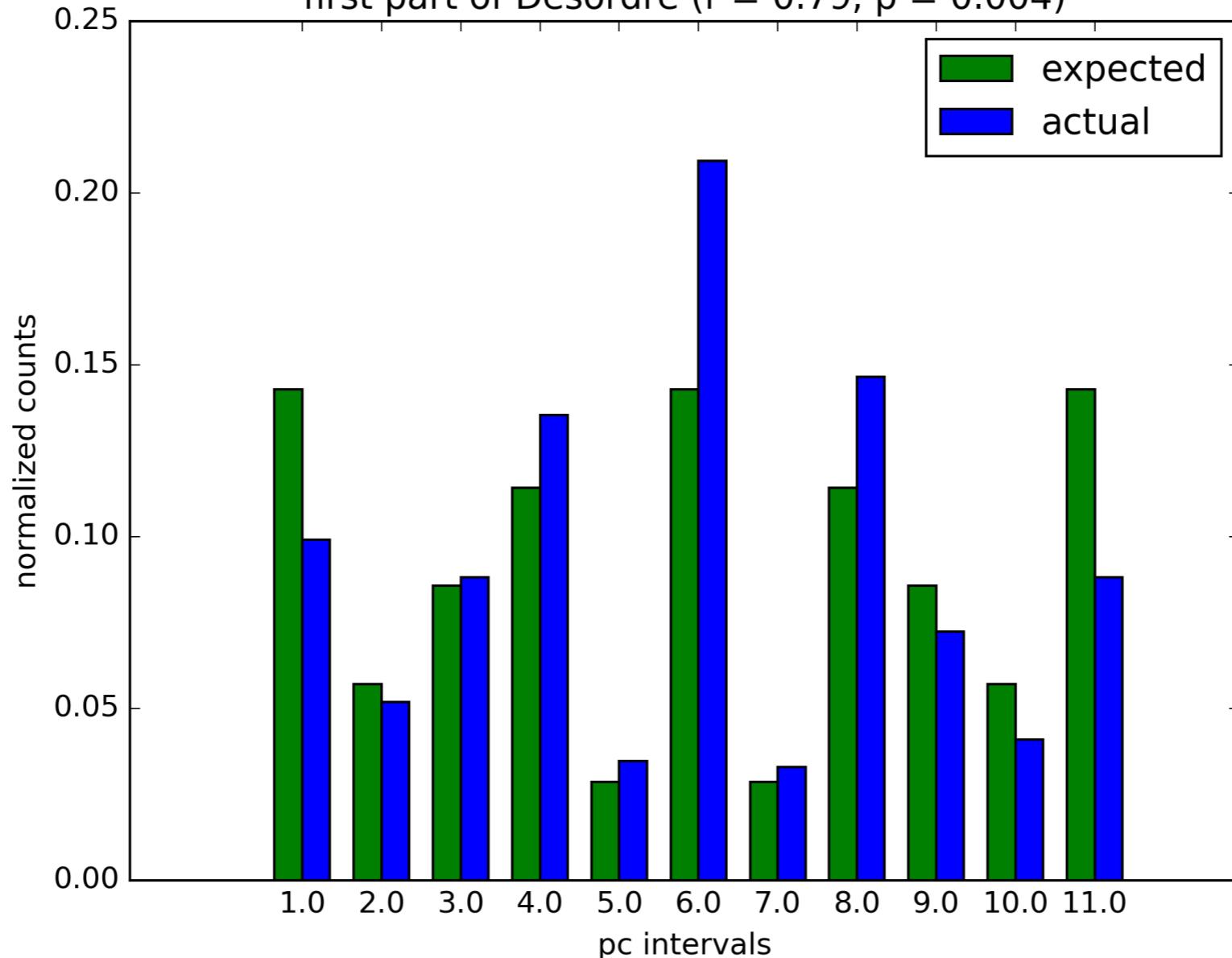
\*Gradually use rather more pedal. Dynamic balance: die rechte Hand spielt etwas lebhafter als die linke Hand, so dass die rechte Hand mehr Pedal braucht. Die Hände sind gleich laut. Gradualmente crescendo wird am Ende der Partie die rechte Hand mehr Pedal braucht. Die Hände sind gleich laut. Allmählich crescendo bis zum Schluß der Etüde; die Alzette werden allmählich gleich laut. Allmählich crescendo wird am Ende der Etüde die rechte Hand mehr Pedal braucht, dann aber wieder gleich laut. Die Hände sind gleich laut.

\*Allmählich etwas mehr Pedal. Dynamische Balance: die rechte Hand spielt etwas lebhafter als die linke Hand, so dass die rechte Hand mehr Pedal braucht. Die Hände sind gleich laut. Gradualmente crescendo wird am Ende der Partie die rechte Hand mehr Pedal braucht. Die Hände sind gleich laut. Allmählich crescendo bis zum Schluß der Etüde; die Alzette werden allmählich gleich laut. Allmählich crescendo wird am Ende der Etüde die rechte Hand mehr Pedal braucht, dann aber wieder gleich laut. Die Hände sind gleich laut.

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# Désordre

Histograms of actual vs. expected pc intervals  
first part of Desordre ( $r = 0.79$ ,  $p = 0.004$ )



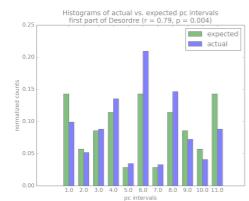
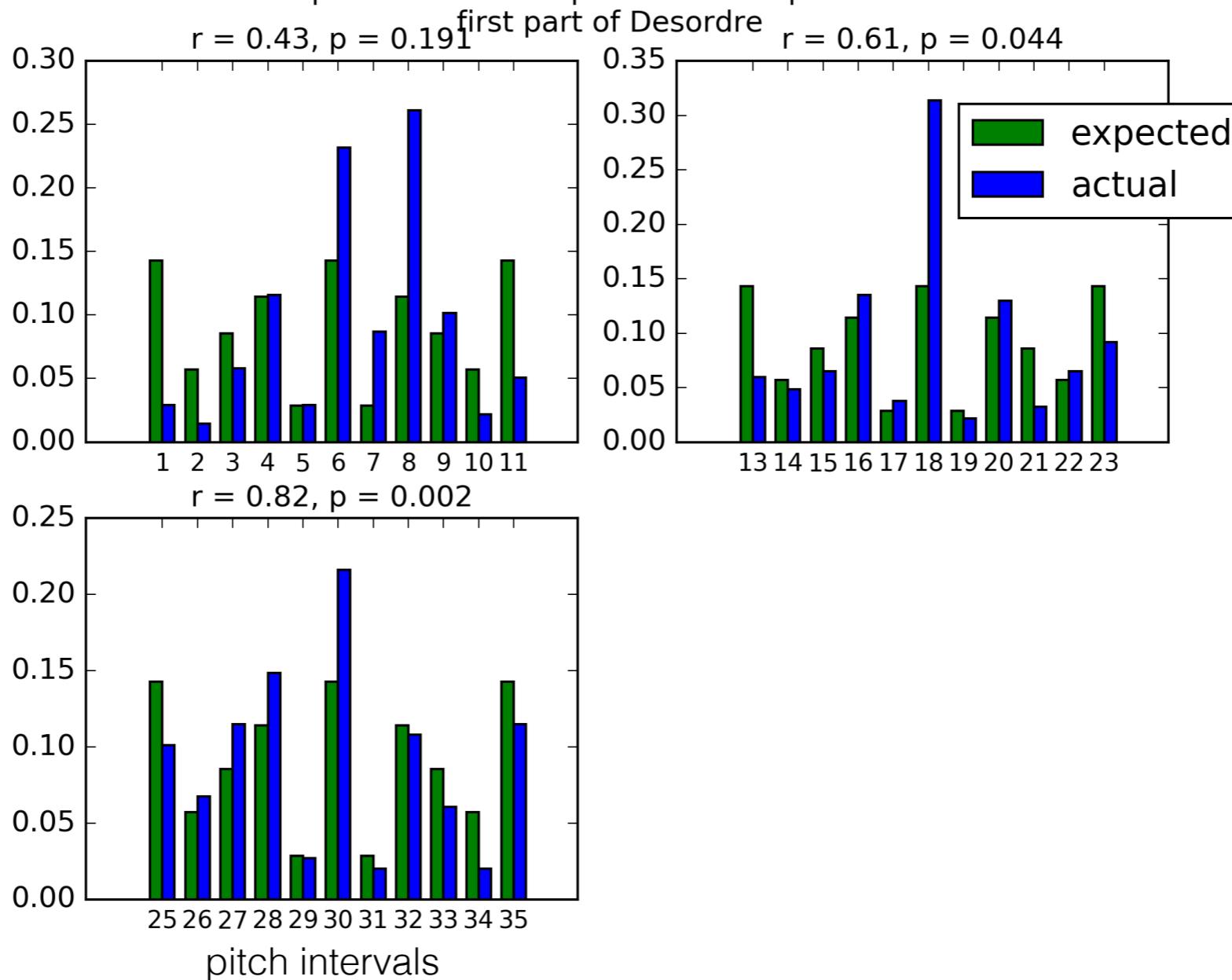
Désordre

(See Quinnett, 2014)

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# Désordre

Expected vs. actual pitch intervals per octave



Désordre

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# Désordre

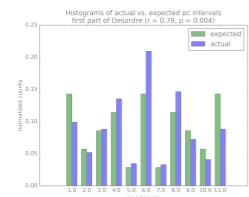
Expected vs. actual pitch intervals by time  
first part of Désordre

correlations

first third:  $r = 0.51, p < 0.11$

second third:  $r = 0.78, p < 0.005$

final third:  $r = 0.95, p < 0.001$



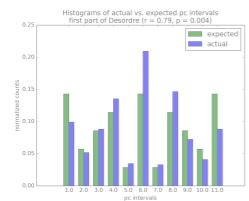
Désordre

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# Désordre

Two explanations:

- 1) pitch proximity
- 2) iso-rhythmic structure



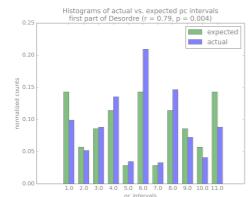
Désordre

Ligeti's Combinatorial Tonality  
Clifton Callender  
2017 JMM

# Désordre

pitch proximity

As distance in pitch space increases, perception of interval quality decreases.



Désordre

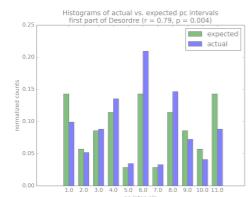
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# Désordre

pitch proximity

As distance in pitch space increases, perception of interval quality decreases.

Distance between left and right hands gradually increases through the first part.



Désordre

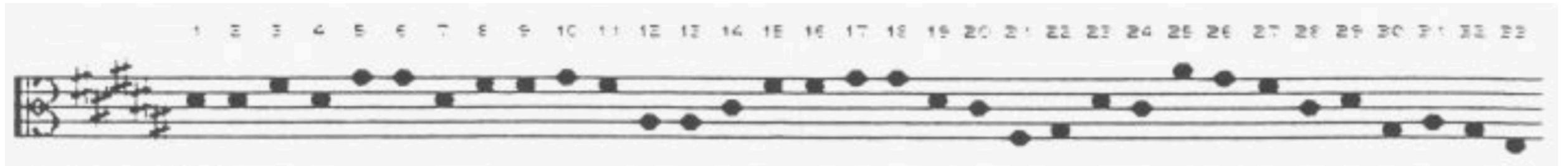
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# Désordre

iso-rhythmic structure

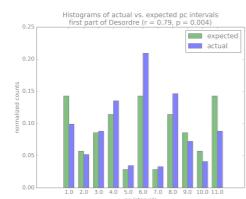


right-hand color



left-hand color

(Taken from Kinzler, 1991)



Désordre

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# Désordre

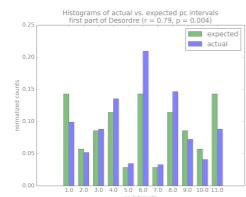
iso-rhythmic structure



beginning of right-hand talea



beginning of left-hand talea



Désordre

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# Désordre

iso-rhythmic structure

progressive diminution of talea

$3 + 5 | 3 + 5 | 5 + 3 | 8 | \dots$

$3 + 4 | 3 + 4 | 4 + 3 | 7 | \dots$

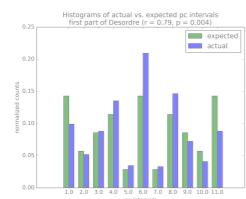
$2 + 4 | 2 + 4 | 4 + 2 | 6 | \dots$

$2 + 3 | 2 + 3 | 3 + 2 | 5 | \dots$

$1 + 3 | 1 + 3 | 3 + 1 | 4 | \dots$

$1 + 2 | 1 + 2 | 2 + 1 | 3 | \dots$

$1 + 1 | 1 + 1 | 1 + 1 | 2 | \dots$



Désordre

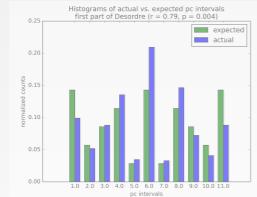
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# Désordre

## iso-rhythmic structure

$$3 + 5 \mid 3 + 5 \mid 5 + 3 \mid 8 \mid \dots$$

The image shows two staves of piano sheet music. The top staff is in treble clef and the bottom is in bass clef. Measure 8 starts with a forte dynamic (f) and a piano dynamic (p). Measures 9 and 10 also begin with f and p dynamics. Measure 11 begins with f and ends with f. Measure 12 begins with f and p. Measure 13 begins with f and p. Measure 14 begins with f and p. Measure 15 begins with f and p. Measure 16 begins with f and p. Measure 17 begins with f and p. Measure 18 begins with f and p. Measure 19 begins with f and p. Measure 20 begins with f and p. Measure 21 begins with f and p. Measure 22 begins with f and p. Measure 23 begins with f and p. Measure 24 begins with f and p. Measure 25 begins with f and p. Measure 26 begins with f and p. Measure 27 begins with f and p. Measure 28 begins with f and p. Measure 29 begins with f and p. Measure 30 begins with f and p. Measure 31 begins with f and p. Measure 32 begins with f and p. Measure 33 begins with f and p. Measure 34 begins with f and p. Measure 35 begins with f and p. Measure 36 begins with f and p. Measure 37 begins with f and p. Measure 38 begins with f and p. Measure 39 begins with f and p. Measure 40 begins with f and p. Measure 41 begins with f and p. Measure 42 begins with f and p. Measure 43 begins with f and p. Measure 44 begins with f and p. Measure 45 begins with f and p. Measure 46 begins with f and p. Measure 47 begins with f and p. Measure 48 begins with f and p. Measure 49 begins with f and p. Measure 50 begins with f and p. Measure 51 begins with f and p. Measure 52 begins with f and p. Measure 53 begins with f and p. Measure 54 begins with f and p. Measure 55 begins with f and p. Measure 56 begins with f and p. Measure 57 begins with f and p. Measure 58 begins with f and p. Measure 59 begins with f and p. Measure 60 begins with f and p. Measure 61 begins with f and p. Measure 62 begins with f and p. Measure 63 begins with f and p. Measure 64 begins with f and p. Measure 65 begins with f and p. Measure 66 begins with f and p. Measure 67 begins with f and p. Measure 68 begins with f and p. Measure 69 begins with f and p. Measure 70 begins with f and p. Measure 71 begins with f and p. Measure 72 begins with f and p. Measure 73 begins with f and p. Measure 74 begins with f and p. Measure 75 begins with f and p. Measure 76 begins with f and p. Measure 77 begins with f and p. Measure 78 begins with f and p. Measure 79 begins with f and p. Measure 80 begins with f and p. Measure 81 begins with f and p. Measure 82 begins with f and p. Measure 83 begins with f and p. Measure 84 begins with f and p. Measure 85 begins with f and p. Measure 86 begins with f and p. Measure 87 begins with f and p. Measure 88 begins with f and p. Measure 89 begins with f and p. Measure 90 begins with f and p. Measure 91 begins with f and p. Measure 92 begins with f and p. Measure 93 begins with f and p. Measure 94 begins with f and p. Measure 95 begins with f and p. Measure 96 begins with f and p. Measure 97 begins with f and p. Measure 98 begins with f and p. Measure 99 begins with f and p. Measure 100 begins with f and p. Measure 101 begins with f and p. Measure 102 begins with f and p. Measure 103 begins with f and p. Measure 104 begins with f and p. Measure 105 begins with f and p. Measure 106 begins with f and p. Measure 107 begins with f and p. Measure 108 begins with f and p. Measure 109 begins with f and p. Measure 110 begins with f and p. Measure 111 begins with f and p. Measure 112 begins with f and p. Measure 113 begins with f and p. Measure 114 begins with f and p. Measure 115 begins with f and p. Measure 116 begins with f and p.



Désordre

# Ligeti's Combinatorial Tonality

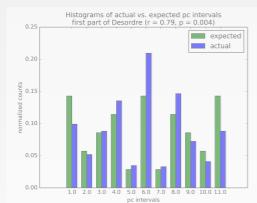
## Clifton Callender

### 2017 JMM

# Désordre

iso-rhythmic structure

$$\begin{array}{c|c|c|c|c} 3+4 & 3+4 & 4+3 & 7 & \dots \\ 2+4 & 2+4 & 4+2 & 6 & \dots \\ 2+3 & 2+3 & 3+2 & 5 & \dots \\ 1+3 & 1+3 & 3+1 & 4 & \dots \end{array}$$



Désordre

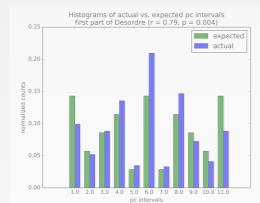
Ligeti's Combinatorial Tonality  
Clifton Callender  
2017 JMM

# Désordre

iso-rhythmic structure

1 + 2 | 1 + 2 | 2 + 1 | 3 | ...

A musical score for two staves. The top staff is in treble clef and the bottom staff is in bass clef. Both staves have a key signature of three sharps. The music consists of various note heads and stems, some with arrows pointing upwards, indicating a specific performance technique or rhythm. The score is divided into measures by vertical bar lines.



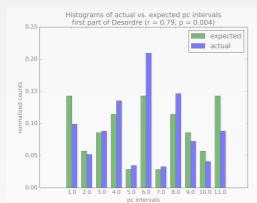
Désordre

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# Désordre

iso-rhythmic structure

$$\begin{aligned} & 1 + 1 | 1 + 1 | 1 + 1 | 2 | \dots \\ & 1 + 1 \dots \end{aligned}$$

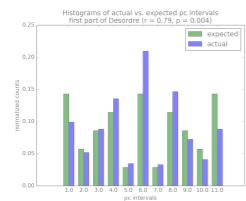
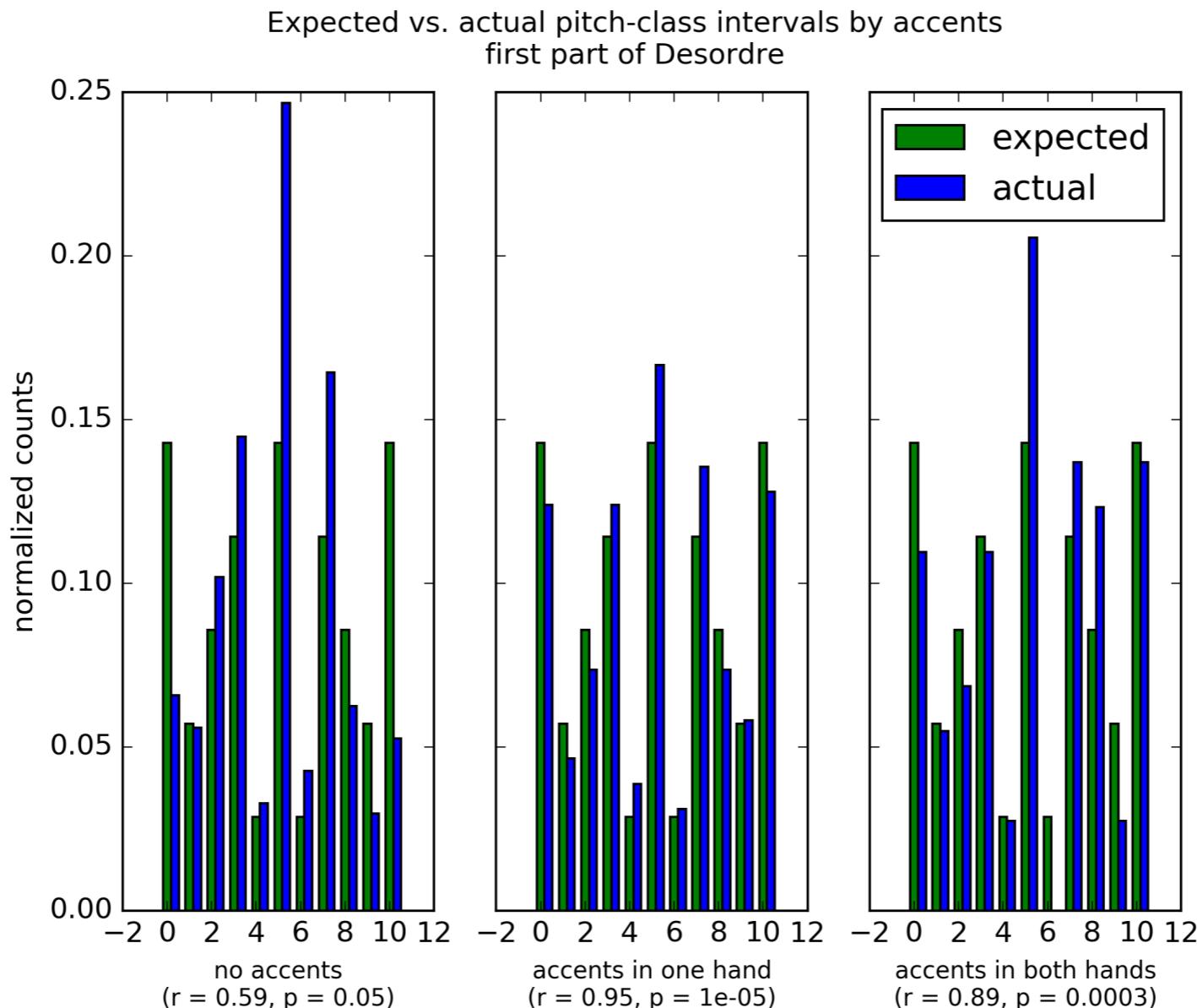


Désordre

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# Désordre

## Expected vs. actual by number of accents



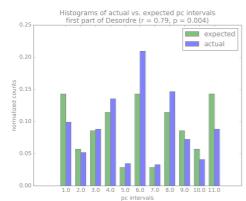
Désordre

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# Désordre

Expected vs. actual by number of accents and by time

	no accents	one accent
first third:	$r = 0.39$	$r = 0.75$
	$p = 0.24$	$p = 0.008$
second third:	$r = 0.65$	$r = 0.81$
	$p = 0.028$	$p = 0.003$
final third:	$r = 0.61$	$r = 0.93$
	$p = 0.048$	$p < 0.001$



Désordre

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# What else?

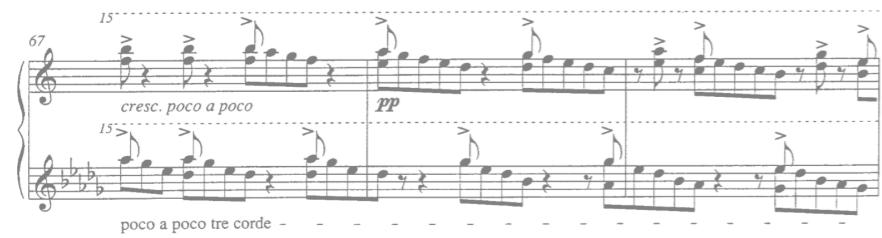


What else?

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We could have talked about:

intra- and inter-trichords, tetrachords, and so on



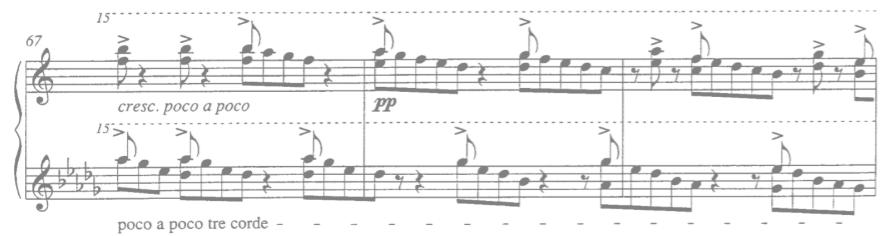
What else?

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We could have talked about:

intra- and inter-trichords, tetrachords, and so on

non-tonal triad motion in *Der Zauberlerling*



What else?

Ligeti's Combinatorial Tonality  
Clifton Callender  
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We could have talked about:

intra- and inter-trichords, tetrachords, and so on

non-tonal triad motion in *Der Zauberlerling*

tertian inter-harmonies in whole-tone systems and Galamb Borong



What else?

Ligeti's Combinatorial Tonality  
Clifton Callender  
2017 JMM

We could have talked about:

intra- and inter-trichords, tetrachords, and so on

non-tonal triad motion in *Der Zauberlerling*

tertian inter-harmonies in whole-tone systems and Galamb Borong

degrees of consonance (*Piano Concerto I*) and dissonance (*En Suspens*)  
in white key / black key and other systems of complementary collections



What else?

Ligeti's Combinatorial Tonality  
Clifton Callender  
2017 JMM

We could have talked about:

intra- and inter-trichords, tetrachords, and so on

non-tonal triad motion in *Der Zauberlerling*

tertian inter-harmonies in whole-tone systems and Galamb Borong

degrees of consonance (*Piano Concerto I*) and dissonance (*En Suspens*)  
in white key / black key and other systems of complementary collections

other melodic/harmonic combinations

major sixths by whole-tone (*Violin Concerto V*)

diatonic sonorities by whole-tone scales

whole-tone sonorities by pentatonic scales



What else?

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# Thank you!

For more information:  
[cliftoncallender.com](http://cliftoncallender.com)

## References

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