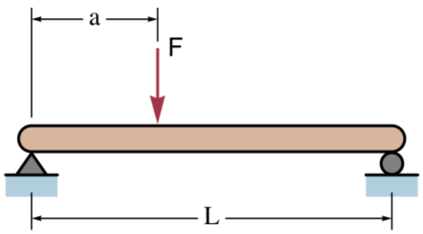
Given:



A simply supported beam of length L subject to a force F. The deflection of the beam y is characterized by the deflection equation

$$y(x) = \begin{cases} -\theta x + \frac{Rx^3}{6EI}, & x \le a \\ -\theta x + \frac{Rx^3}{6EI} - \frac{F}{6EI}(x-a)^3, & x > a \end{cases} \qquad \text{Eq. 1}$$

where I is the moment of inertia of the cross section, R is the reaction force on the beam at the left end, θ is the clockwise rotational angle of the beam at the left end, and E is the Young's modulus of the beam's material. I, R, θ and E are all geometry- or material-based constants. E is found in a table (**MaterialElasticity.mat**), and the other values are found with the following (already derived) equations

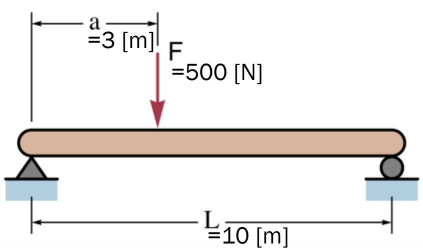
$$I = \frac{wh^3}{12} \qquad \text{Eq. 2}$$

$$R = \frac{F}{L}(L-a) \qquad \text{Eq. 3}$$

$$\theta = \frac{Fa}{6EIL}(2L-a)(L-a) \qquad \text{Eq. 4}$$

Since all the properties are either given or can be directly calculated, to find deflection at a point simply find the coefficients I, R, θ and E and then plug them into Equation 1.

Case 1: Single force



For the beam shown with width w = 0.2 [m], height h = 0.2 [m], and modulus E = 190 * 10^9 [Pa], calculate the beam deflection at x = 2 [m] and x = 5 [m].

First, calculate the constants:

$$I = \frac{wh^3}{12} = \frac{0.2*0.2^3}{12} = 1.33*10^{-4}\ [\text{m}^4]$$

$$R = \frac{F}{L}(L-a) = \frac{500}{10}(10-3) = 350\ [\text{N}]$$

$$\theta = \frac{Fa}{6EIL}(2L-a)(L-a) = \frac{(500)(3)}{6(190*10^9)(1.33*10^{-4})(10)}(17)(7) = 1.682*10^{-5}\ [\text{rad}]$$

For $x = 2$ $(x \le a)$, use the first half of Eq. 1:

$$y(2) = -\theta(2) + \frac{R(2^3)}{6EI} = -(1.682*10^{-5})(2) + \frac{(350)(2^3)}{6(190*10^9)(1.33*10^{-4})} \rightarrow -1.517*10^{-5}[\text{m}]\ or \sim 0.015\ [\text{mm}]$$

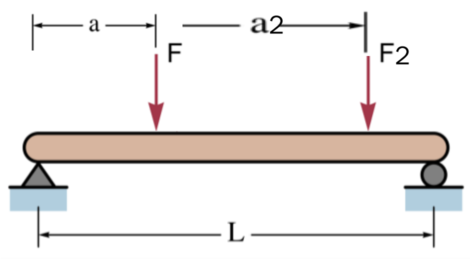
For $x = 5$ $(x > a)$, use the second half of Eq. 1:

$$y(2) = -\theta(5) + \frac{R(5^3)}{6EI} - \frac{F}{6EI}(x-a)^3 = -(1.682*10^{-5})(5) + \frac{(350)(5^3)}{6(190*10^9)(1.33*10^{-4})} - \frac{500}{6(190*10^9)(1.33*10^{-4})}(2)^3 \rightarrow$$

$$-1.781*10^{-4}[\text{m}]\ or \sim 0.18\ [\text{mm}]$$

This application can be generalized with x as a vector instead of a single value to find the deflection at all points on the beam.

Case 2: Multiple forces



Similar case, but with more than one force. Simply treat the problem as two instances of Case 1. Calculate the deflection caused by force F at distance a, then calculate the deflection caused by force $F2$ at distance $a2$. The sum of those deflections will be the total deflection across the beam.