Common Confusion 14
Newton Raphson Method
Matthew Woodring

Every semester, many students are confused by what the Newton Raphson Method is and why it is useful. This paper will provide a derivation of the method along with some of its advantages and disadvantages to help students better understand it.

Suppose that we have the below function:

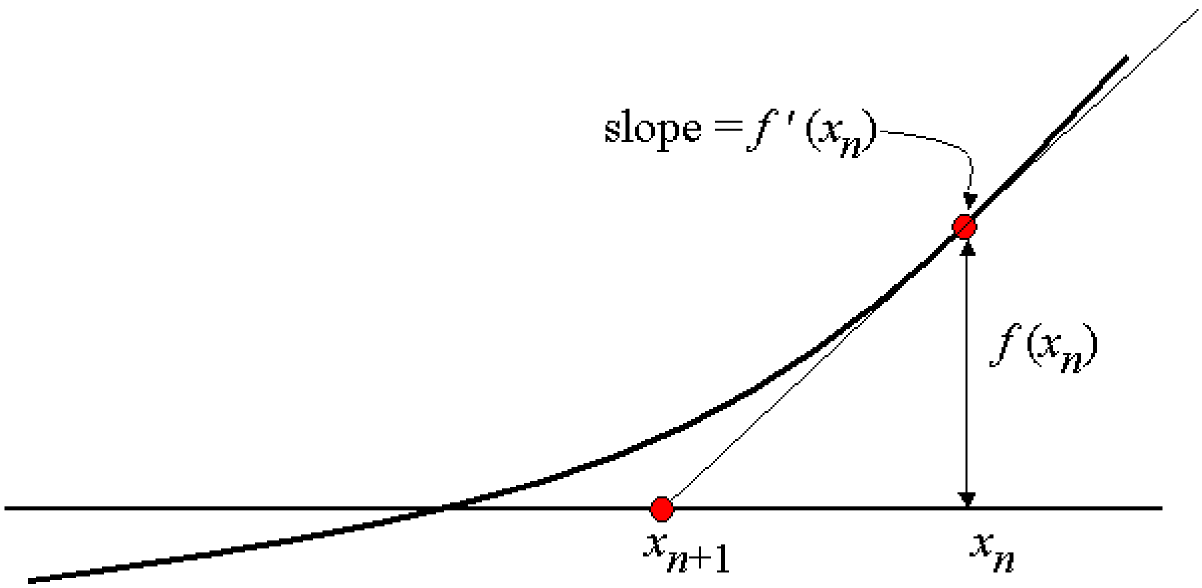

Figure 01: F(x) with Labeled Key Features

The bold line is F(x), the line running tangent to F(x) is T(x) and the points are labeled as appropriate. Remember, $F(x_n)$ is the same thing as "the y-value at the x-value n".

Since the goal of the Newton Raphson Method is to get closer to the root value of a function, we can first start by just finding the point x_{n+1}, which is the second approximation after the initial guess. To do this, we can write the following equation:

$$T(x) = f'(x_n) * (x - x_n) + f(x_n) \leftarrow \text{Eq.}(1)$$

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The first part of the right-side of the equation is $f'(x_n)$. This is the value of the derivative of the function at the point x_n. The second part of the equation is $(x - x_n)$, which is being multiplied by $f'(x_n)$. The reason for this is because we need to shift the tangent line value horizontally to make it line up with the plotted tangent line shown above. If we did not include this part, the tangent line value would be to the left of the actual value. Finally, the third part of the equation is $f(x_n)$. Similar to how we shifted the tangent line value horizontally, we must also shift it vertically to make it align with the actual value of the tangent line.

Next, we can note that the tangent line at the point x_{n+1} is 0:

$$T(x_{n+1}) = 0 \leftarrow \text{Eq.(2)}$$

This is obvious and can be seen in the above plot. Next, we can plug in x_{n+1} to the first equation:

$$T(x_{n+1}) = f'(x_n) * (x_{n+1} - x_n) + f(x_n) \leftarrow \text{Eq.(3)}$$

In the above function, we simply replaced every instance of x with x_{n+1}. Finally, we can set Eq.(3) equal to Eq.(2):

$$0 = f'(x_n) * (x_{n+1} - x_n) + f(x_n) \leftarrow \text{Eq.(4)}$$

Rearranging for x_{n+1} gives us:

$$-f(x_n) / f'(x_n) = (x_{n+1} - x_n) \leftarrow \text{Eq.(5)}$$

Which is equivalent to:

$$x_{n+1} = x_n - [f(x_n) / f'(x_n)] \leftarrow \text{Eq.(6)}$$

Eq.(6) is the standard way to write the formula of the Newton Raphson Method, but I will shift all of the xn values by -1 to match the MA4 prompt:

$$x_n = x_{n-1} - [f(x_{n-1}) / f'(x_{n-1})] \leftarrow \text{Eq.(7)}$$

Eq.(6) and Eq.(7) are equivalent.

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Hopefully this derivation makes sense. Essentially, the Newton Raphson Method is just drawing a tangent line to a function, finding the intersection of the tangent line along the x-axis, and making this intersection point your new starting point. Following this, you can repeat the Newton Raphson Method until you reach an answer that is close enough to the true root of the function.

Some advantages of the Newton Raphson Method include it having a quadratic convergence and being simple enough to easily implement on a computer.

Some disadvantages of the Newton Raphson Method include it getting complicated if a simple derivative is not possible, it sometimes not converging due to poor assumptions, and the method terminating if a stationary point is encountered.