

1. Let R be the set of positive real numbers and define addition, denoted by \oplus , and multiplication, denoted by \otimes , as follows. For every $a, b \in R$, $a \oplus b = ab$, and $a \otimes b = a^{\log b}$. Please prove or disprove (R, \oplus, \otimes) .

Since R contains only real positive numbers, both a and b are positive real numbers. axiom 4 states that for any number there must be some element that when the addition operator is applied, it results in 0. In range R , there cannot exist a number, b , in R where $a \oplus b = 0$. Therefore (R, \oplus, \otimes) is not a field

2. Denote the set $0,1,2,3$ in Z_4 and define addition, denoted by $+$, and multiplication, denoted by \cdot or juxtaposition, via the following tables:

axiom 8 states that for every number in Z_4 there is a multiplicative inverse where $a \cdot a^{-1} = 1$. 2 from the range Z_4 has no multiplicative inverse because there exists no other number, b , in Z_4 where $a \cdot b = 1$.

3. Suppose x is a positive integer with n digits, say $x = d_1d_2d_3\dots d_n$ in other words, $d_i \in [0, 1, 2, \dots, 9]$ for $1 \leq i \leq n$, but $d_1 \neq 0$. Please prove or disprove the following. Recall that, for $a, b \in Z$, a is a divisor of b if $b = ak$, for some $k \in Z$.

(a) If 9 is a divisor of $d_1 + d_2 + d_3 + \dots + d_n$, then 9 is a divisor of x .

Given that for $a, b \in Z$, a is a divisor of b if $b = ak$, for some $k \in Z$ the only time where 9 is a divisor of $d_1 + d_2 + d_3 + \dots + d_n$ is when it is also a value of $9 * k$.

(b) If $d_n = 0$ or $d_n = 5$, then 5 is a divisor of x .

solution(b)

4. Please prove or disprove: if $n \in Z^+$, then $n^2 + n + 41$ is prime:

question 4 asserts that if $n \in Z^+$, then for all values n , $n^2 + n + 41$ will be prime. In order to disprove this, there must be some value for n where the output isn't prime. take $n = 40$, this results in $40^2 + 40 + 41 = 1681$. 1681 is not prime because 41 is a factor of 1681. Therefore $n^2 + n + 41$ is not prime for all $n \in Z^+$.