1. Let R be the set of positive real numbers and define addition, denoted by $\oplus$, and multiplication, denoted by $\otimes$, as follows. For every $a, b \in R$, $a \oplus b = ab$, and $a \otimes b = a^{\log b}$. Please prove or disprove $(R, \oplus, \otimes)$.

axiom -1: since $R \in R^+$, $\oplus$ is closed in R. There is no positive real number for which $ab \in R^+$

axiom 0: since $R \in R^+$ and $\otimes$ is defined as $a \otimes b = a^{\log b}$, starting with the lowest possible value, $a, b = 1$, $1^{log 1}$ results in 1, which is included in R^+. as both values approach infinity the result also approaches infinity which is a real number.

axiom 1: addition is commutative, for all values a, b, $a \cdot b = b \cdot a$

axiom 2: addition is associative, for all values a, b, c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

axiom 3: there exists an additive identity. since the $\otimes$ is defined as $a \oplus b = ab$, the addtive identity is 1. $1 \cdot x = x \cdot 1 = x$.

axiom 4: There is an additive inverse. Since the additive identity is 1, there must be a value for every $x \in R^+$ where $x \cdot y = 1$. This would be the inverse of of the number x. For example: $x = 3$, $x \cdot x^{-1} = 3 \cdot 1/3 = 1$

axiom 5: multiplication is commutative. $a \otimes bi = a^{\log b}$. This is commutative because if you take log_a of both sides it resutls in $logb = log_a b^{loga}$. Breaking apart the right side, $\frac{loga \cdot logb}{loga}$ in which both $loga$ cancel each other out and you're left with $logb = logb$. Since they are equal, $\otimes$ is commutative.

axiom 6: Multiplication is associative. Given a, b, c: $(a \otimes b) \otimes c = a \otimes (b \otimes c)$ on the left, the resultant is $(a^{logb})^{logc}$, and on the right side, $(a^{logb})^{logc}$. Since these are equal, multiplication is associative

axiom 7: there exists a multiplicatave identity. This axiom holds true for the value 10. $(a^{log10} = 10^{loga} = a)$

axiom 8: there exists a multiplicatave inverse. since the multiplicatave identity is 10, the multiplicatave inverse will be a number where $a^{logb} = 10$ solving for be, $logb = log_a 10$, and solving for b, $b = 10^{log_a 10}$. Since $a \in R^+$, $log_a 10$ is positive for all values a. This means that 10^{R^+} and will result in a positive real number meaning that for all values a, there exists another number $b \in R^+$ for the multiplicatave inverse.

Since R contains only real positive numbers, both a and b are positive real numbers. axiom 4 states that for any number there must be some element that when the addition operator is applied, it results in 0. In range R, there cannot exist a number, b, in R where $a \oplus b = 0$. Therefore $(R, \oplus, \otimes)$ is not a field

2. Denote the set 0,1,2,3 in Z_4 and define addition, denoted by $+$, and multiplication, denoted by $\cdot$ or juxtapostion, via the following tables:

axiom 8 states that for every number in Z_4 there is a multiplicative inverse where $a \cdot a^{-1} = 1$. 2 from the range Z_4 has no multiplicatave inverse because there exists no other number, b, in Z_4 where $a \cdot b = 1$.

3. Suppose x is apositive integer with n digits, say $x = d_1 d_2 d_3 ... d_n$ in other words, $d_i \in [0, 1, 2..., 9]$ for $1 \leq i \leq n$, but $d_1 \neq 0$. Please prove or disprove the following. Recall that, for $a, b \in Z$, a is a divisor of b if $b = ak$, for some $k \in Z$.

(a)If 9 is a divisor of $d_1 + d_2 + d_3 + ... + d_n$, then 9 is a divisor of x.

Given that for $a, b \in Z$, a is a divisor of b if $b = ak$, for some $k \in Z$ the only time where 9 is a divisor of $d_1 + d_2 + d_3 + ... + d_n$ is when it is also a value of $9 * k$.

(b)If $d_n = 0$ or $d_n = 5$, then 5 is a divisor of x.

to begin consider all of the values in $x = d_1 d_2 d_3 ... d_n$ where $d_n = 0$. this can be re-written as $x = d_1(10^n) + d_2(10^{n-1}) + d_3(10^{n-2}) + ... + d_{n-1}(10^1)$ now we can factor out a 10 from the entire thing resulting in $x = 10(d_1(10^{n-1}) + d_2(10^{n-2}) + d_3(10^{n-3}) + ... + d_{n-1})$. Since the 10 is divisible by 5, for all values of x where $d_n = 0$, it will be divisible by 5. Now to consider when $d_n = 5$ given that any number where $d_n = 0$ is divisible by 5, let us consider the following $x = 10(d_1(10^{n-1}) + d_2(10^{n-2}) + d_3(10^{n-3}) + ... + d_{n-1}) + 5$ now if we were to factor out a five from the equation, it would resutl in $x = 5(2(d_1(10^{n-1}) + d_2(10^{n-2}) + d_3(10^{n-3}) + ... + d_{n-1}) + 1)$ which will be divisible by 5 as well. Thus all numbers where $d_n = 0, 5$ are divisible 5.

4. Please prove or disprove: if $n \in Z^+$, then $n^2 + n + 41$ is prime:

question 4 asserts that if $n \in Z^+$, then for all values n, $n^2 + n + 41$ will be prime. In order to disprove this, there must be some value for n where the output isn't prime. take $n = 40$, this results in $40^2 + 40 + 41 = 1681$. 1681 is not prime because 41 is a factor of 1681. Therefore $n^2 + n + 41$ is not prime for all $n \in Z^+$.