Balances Part 2

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Learning Outcomes

- Understand and set up fundamental mass and energy balance equations for control volumes.
- Review principles of vapor-liquid equilibrium (VLE) and mass transfer at chemical surfaces.
- Analyze how these balances apply to environmental remediation, such as estimating transfer rates for spills.

Reading

- Mass and energy balances examples here

In previous discussions on environmental remediation, we have discussed ways in which material can be transferred from one location to another or that it can be broken down and then transferred. The rate of that transfer depends on the concentration immediately next to the surface and out in the middle of the fluid. Principles of vapor-liquid equilibrium (concentration next to the surface) and mass transfer are reviewed here.

1 Heat and Mass Transfer Preview

1.1 Heat Transfer: Convection

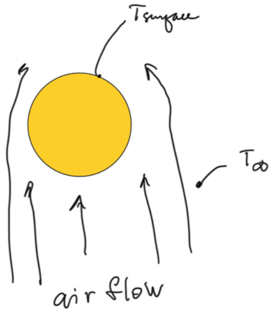
1.2 Mass Transfer: Convection

2 Vapor Liquid Equilibrium Preview

2.1 Raoult's Law

2.2 Henry's Law

Action Items



Heat transfer for cooling of sphere

$$q = hA(T_s - T_\infty) \; [=] \; \frac{W}{\cancel{m^2} \cdot \cancel{K}} \cdot \cancel{m^2} \cdot \cancel{K}$$

Newton's Law of Cooling [heat transfer is proportional to temperature difference]

h - heat transfer coefficient

A - Area

T - temperature

Also from an energy balance:

$$\left.\frac{dnU}{dt}\right|_{sphere} = \cancel{\dot{n}_{in} H_{in}}^{\,0} - \cancel{\dot{n}_{out} H_{out}}^{\,0} + q + \cancel{\dot{W}_s}^{\,0}$$

$$\left.\frac{dnU}{dt}\right|_{sphere} = n\frac{dU}{dt} + \cancel{U\frac{dn}{dt}}^{\,0} = n C_p \frac{dT}{dt} = q \; [=] \; \cancel{mol} \cdot \frac{\cancel{J}}{\cancel{mol}\,\cancel{K}} \cdot \frac{\cancel{K}}{\cancel{s}} \cdot \frac{W}{\cancel{J/s}}$$

$$\rightarrow \frac{dT}{dt} \approx \frac{\Delta T}{\Delta t}$$

Solving those equations:

$$m C_p \frac{dT}{dt} = hA(T - T_\infty) \quad (\text{Assuming } T_s \sim T)$$

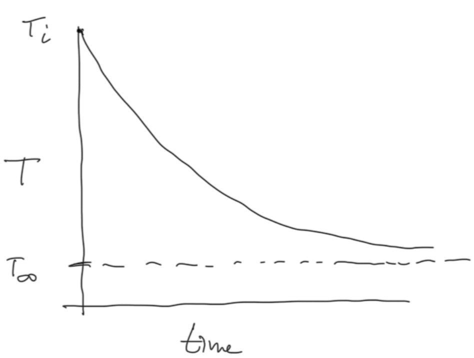
$$\frac{dT}{T - T_\infty} = \frac{hA}{mC_p} dt$$

$$\ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right] = -\frac{hA}{mC_p} t$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{mC_p} t}$$

$$T = T_\infty + (T_i - T_\infty) e^{-\frac{hA}{mC_p} t}$$

What is h?



The heat transfer coefficient h can be found from correlations

- First its best to get a Dimensionless equivalent

$$\frac{Lh}{k} = Nu \quad \text{- The Nusselt \#}$$

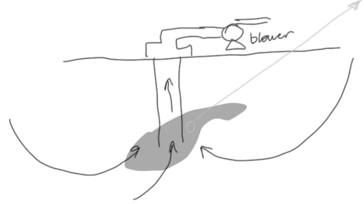
L - characteristic length, m
k - thermal conductivity $\frac{W}{m \cdot K}$

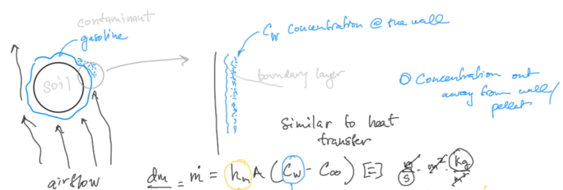
One such correlation for a sphere is from the NLEES PE handbook on page 148

$$Nu = 2 + 0.6\,Re^{0.5}\,Pr^{1/3}$$

Reynolds ↑ ↑ Prandtle $= \frac{C_p \mu}{k}$

Soil Remediation





$$\frac{dm}{dt} = \dot{m} = h_m A (C_w - C_\infty) \quad [=] \quad \frac{\cancel{m}}{s} \cdot \cancel{m^2} \cdot \frac{kg}{\cancel{m^3}}$$

h_m → mass transfer coefficient
C_w → concentration @ the surface

How to get this (same way as heat transfer coefficient)

$$Sh = 2 + 0.6\,Re^{0.5}\,Sc^{1/3}$$

Sc - Schmidt # $= \frac{\mu}{\rho D}$ ← viscosity, ← Diffusivity

$$Sh = \frac{h_m L}{D}$$

L - characteristic length, m
D - diffusivity, $\frac{m^2}{s}$

How do you get C_w? VLE (Vapor Liquid Equilibrium)

Raoult's Law (Ideal Soln) $P_i = x_i P_i^{sat}$ if $x_i = 1$ $P_i = P_i^{sat}$ ⇒ concentration @ surface

Henry's Law $P_i = C_i H_i$ ← Henry's Coefficient

For dissolved gases C_i is low

How long will it take to remove gasoline?

$$\frac{dm}{dt} = h_m A (C_w - C_\infty)$$

$$0 = m = m_0 - h_m A (C_w - C_\infty) \cdot t$$

$$t \cong \frac{m_0}{h_m A (C_w - C_\infty)}$$

Well, heat is required for vaporization...
so Energy balance w/ mass balance required.

1. For a reaction A ->B in a 100-gallon reactor, calculate the amount of chilled cooling water needed to keep the reactor at a constant 300 K given the provided reaction rate and heat of reaction:

   r = k·C$_A$; C$_A$ = 1 mol/L

   k = Aexp(-Ea/RT); T = 325K

   A = 1.30 x 101̂6 min-1

   E = 100. kJ/mol

   ΔHrxn = -85.0 kJ/mol

2. Compare the values of the rate of depressurization for a relief valve positioned either to vent the gas or liquid of a tank of ammonia. See Example 3 on page 177 in Section VI.3.2(Guymon, 2025) for help. Use the same conditions cited there but instead of CO2, use Ammonia.

References

C. Guymon. *Foundations of Spiritual and Physical Safety: with Chemical Processes*. 2025.