

May 29, 2024

CHEN — Mass and Energy Balances (Draft)

Intro

Mass and energy balances are keystone principles of chemical engineering. Some students struggle completing those balances to solve problems. In an effort to help with that and facilitate consistency, it's recommended that in addition to the specific format and notation with which you teach mass and energy balances, you would also include the below format and notation as you see fit.

Control Volume Inlets and Outlets

Some students don't fully grasp the principle of a control volume and that that control volume may be any number of operational units.

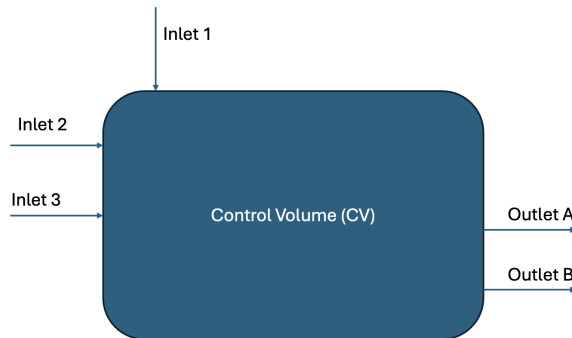


Figure 1: Image showing the control volume and inlets and outlets

Of course the discussion on conservation of mass and energy in a system helps to ingrain the below equation.

$$\boxed{\text{accumulation}_{(CV)} = \text{in} - \text{out} + \text{generation}_{(CV)} - \text{consumption}_{(CV)}} \quad (1-1)$$

Mass (or Mole) Total Balance

$$\frac{dm_{(CV)}}{dt} = \sum_{\text{inlets}} \dot{m}_{in} - \sum_{\text{outlets}} \dot{m}_{out} + \cancel{\text{gen}_{(CV)}} - \cancel{\text{cons}_{(CV)}} \quad (1-2)$$

m is mass and that mass balance can be replaced by a total mole balance n . Assuming no nuclear conversion of mass to energy or vice-versa so total mass (or total moles) is not generated or consumed.

Species balances

$$\begin{aligned}
 \frac{dn_{(CV)}^i}{dt} &= \left(\sum_{inlets} \dot{n}_{in}^i - \sum_{outlets} \dot{n}_{out}^i \right) + gen_{(CV)}^i - cons_{(CV)}^i \\
 \frac{dn_{(CV)}^j}{dt} &= \left(\sum_{inlets} \dot{n}_{in}^j - \sum_{outlets} \dot{n}_{out}^j \right) + gen_{(CV)}^j - cons_{(CV)}^j \\
 \frac{dn_{(CV)}^k}{dt} &= \left(\sum_{inlets} \dot{n}_{in}^k - \sum_{outlets} \dot{n}_{out}^k \right) + gen_{(CV)}^k - cons_{(CV)}^k
 \end{aligned} \tag{1-3}$$

et cetera for other species like i,j,k or l,m,n

where n is moles, i, j, k are species, and generation (gen) and consumption ($cons$) terms inside the control volume are typically due to reactions.

Energy balance

$$\begin{aligned}
 \frac{dE_{(CV)}}{dt} &= \frac{d(U + K + P)_{(CV)}}{dt} = \frac{dU_{(CV)}}{dt} \\
 &= \left(\sum_{inlets} \dot{m} \cdot \left(h + \frac{v^2}{2} + gz \right) - \sum_{outlets} \dot{m} \cdot \left(h + \frac{v^2}{2} + gz \right) \right) + Q_{(CV)} + W_{(CV)}^s
 \end{aligned} \tag{1-4}$$

where U is internal energy, K is kinetic energy (assumed zero for the control volume), P is potential energy of the control volume (also assumed to be zero). h is enthalpy (includes flow work), v is the fluid velocity, g is the gravitational constant, z is the height of the inlet or outlet, Q is the heat added (+) or removed (-), and W^s is the shaft work done on (+) or by (-) the control volume.