2023 Math Camp

One-Dimensional Heat Transfer

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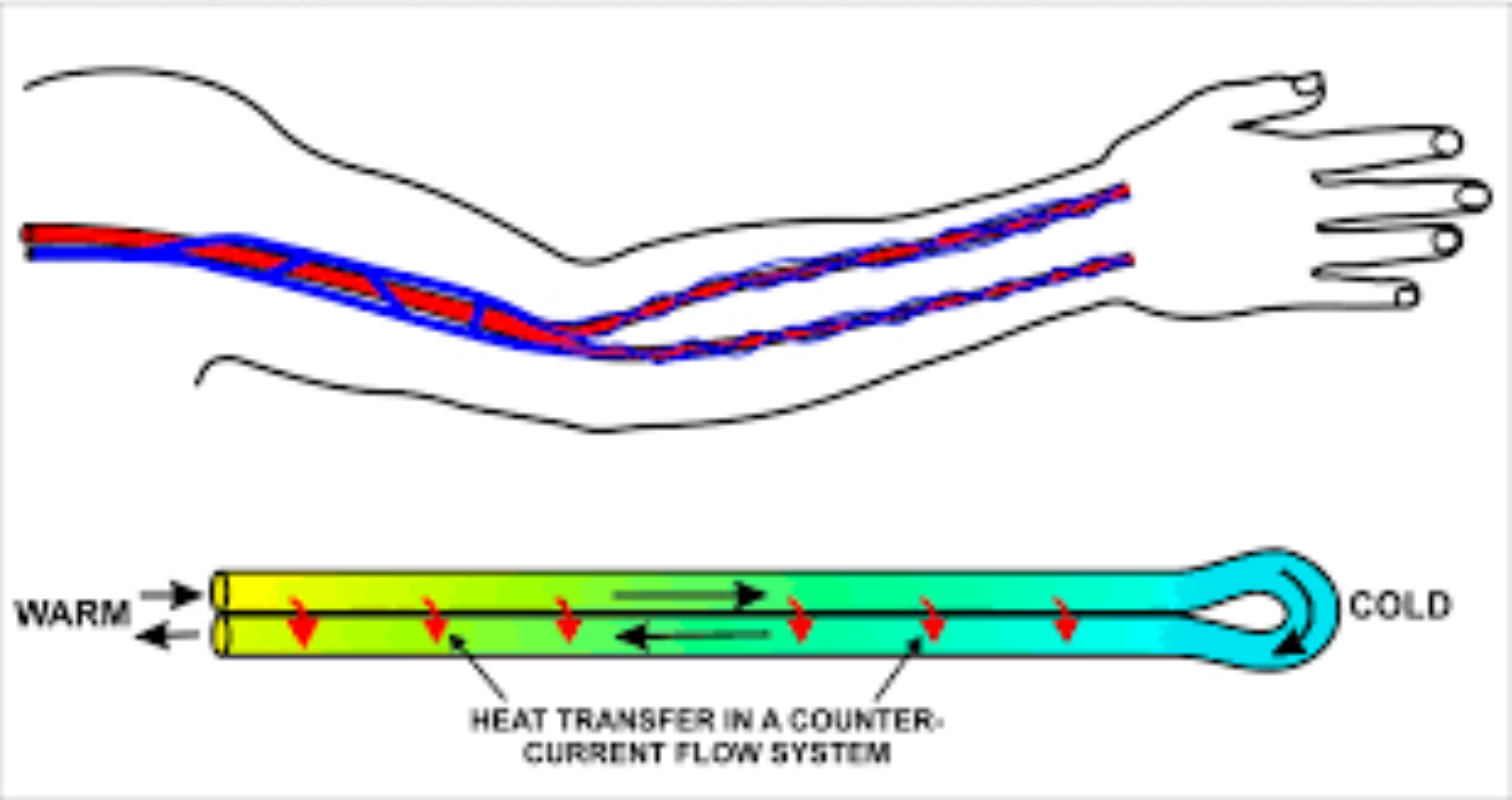
Who We Are

Motivation

- Understanding of Heat Transfer Details is Critical in Many Applications



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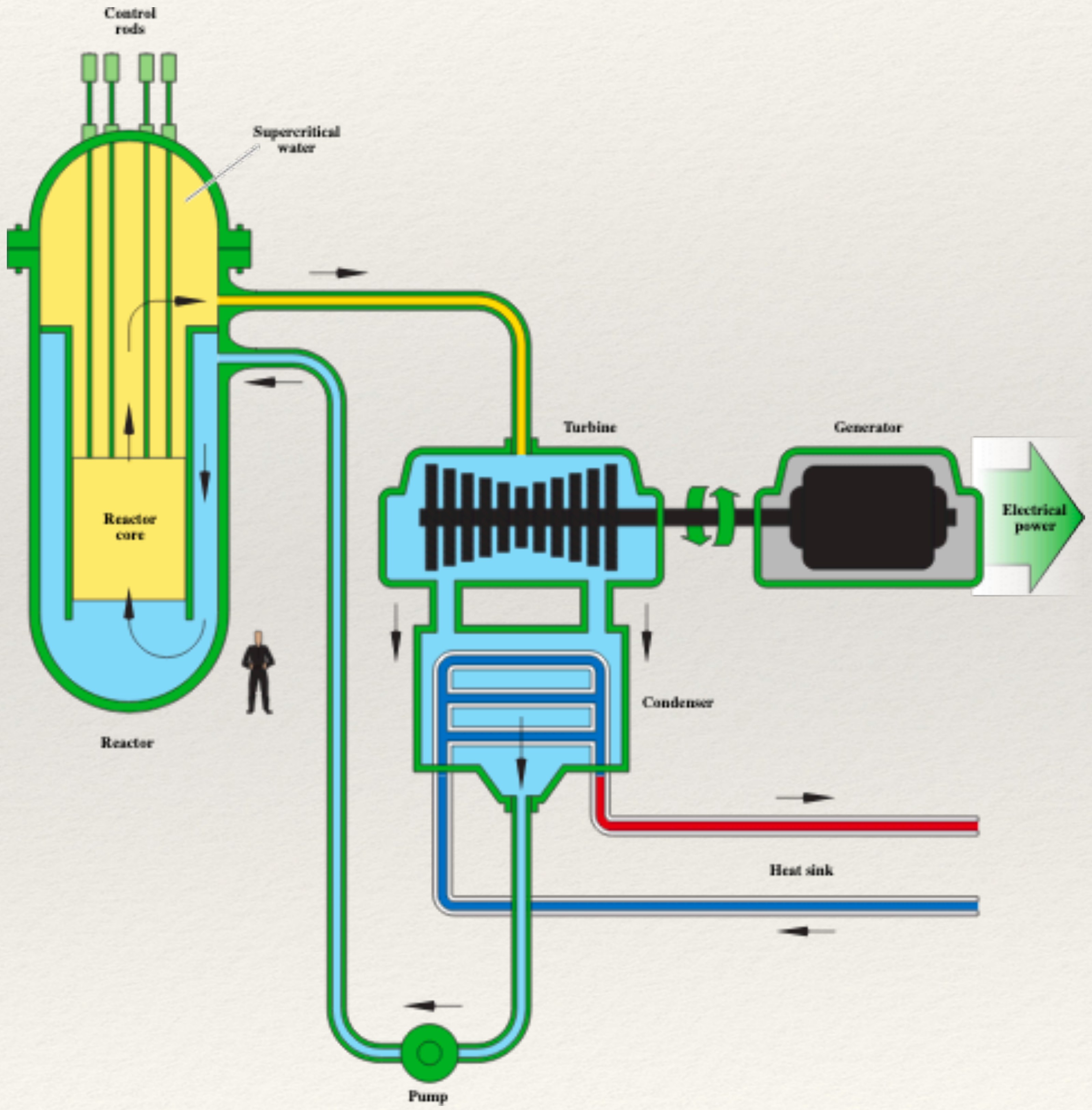

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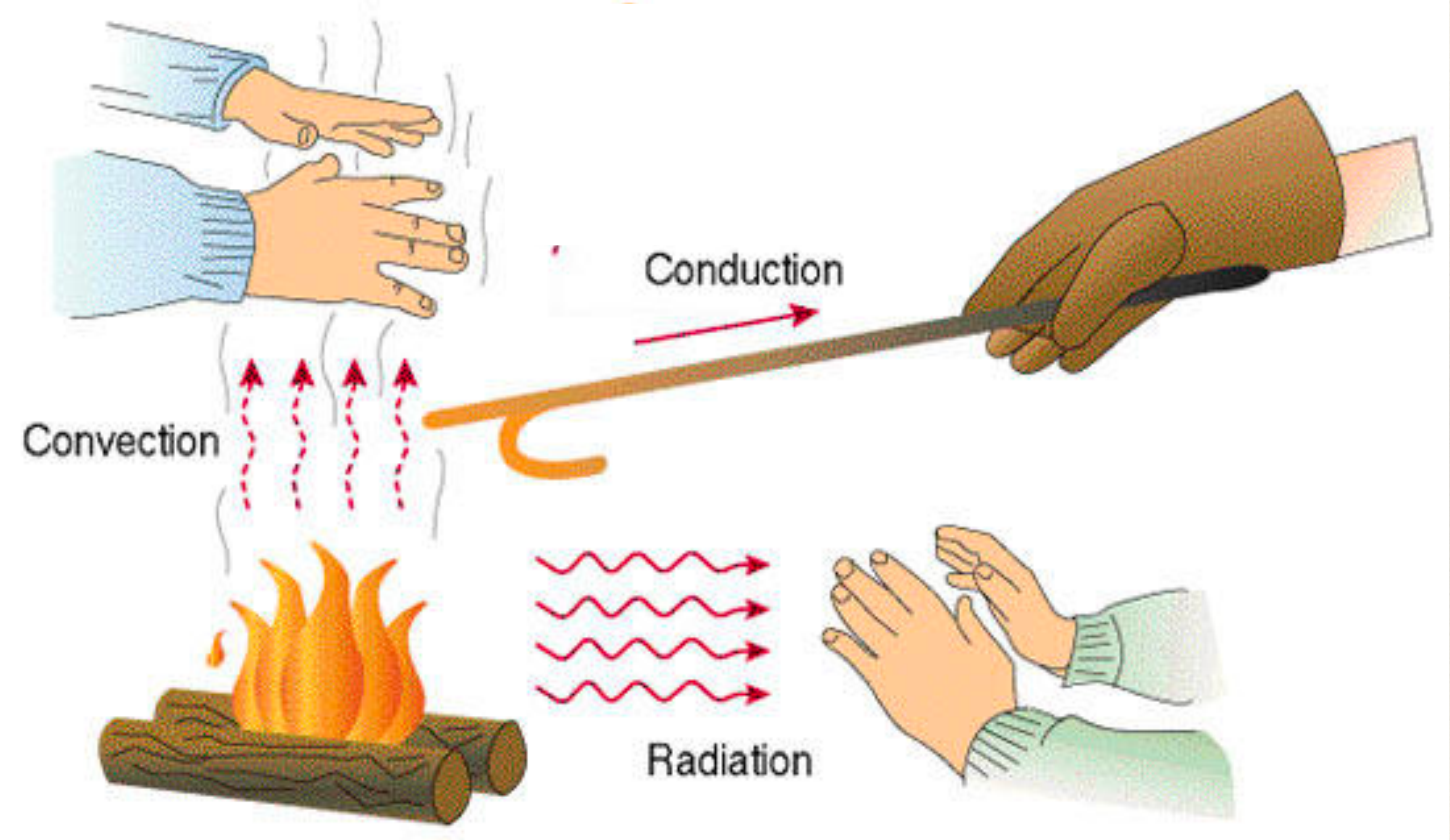
Types of Heat Transfer

- Conduction, Convection, Radiation



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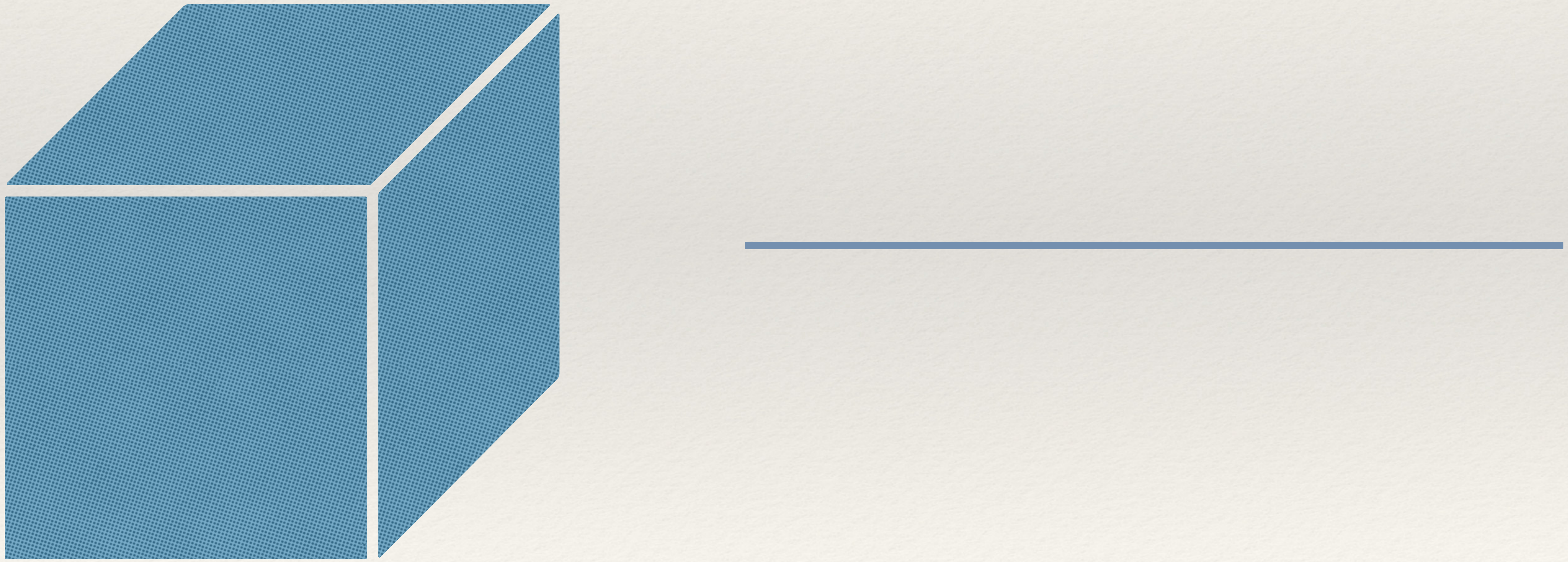
3-D vs 1-D Conduction

- One dimensional, 1-D, heat transfer is dominated by one direction
- 3-D it's important to include all three dimensions



Transient vs. Steady State

- Transient indicates conditions change with time
- Steady state indicates that the conditions are static
    - Car engine that is warmed up
    - Other scenarios after an initial startup period

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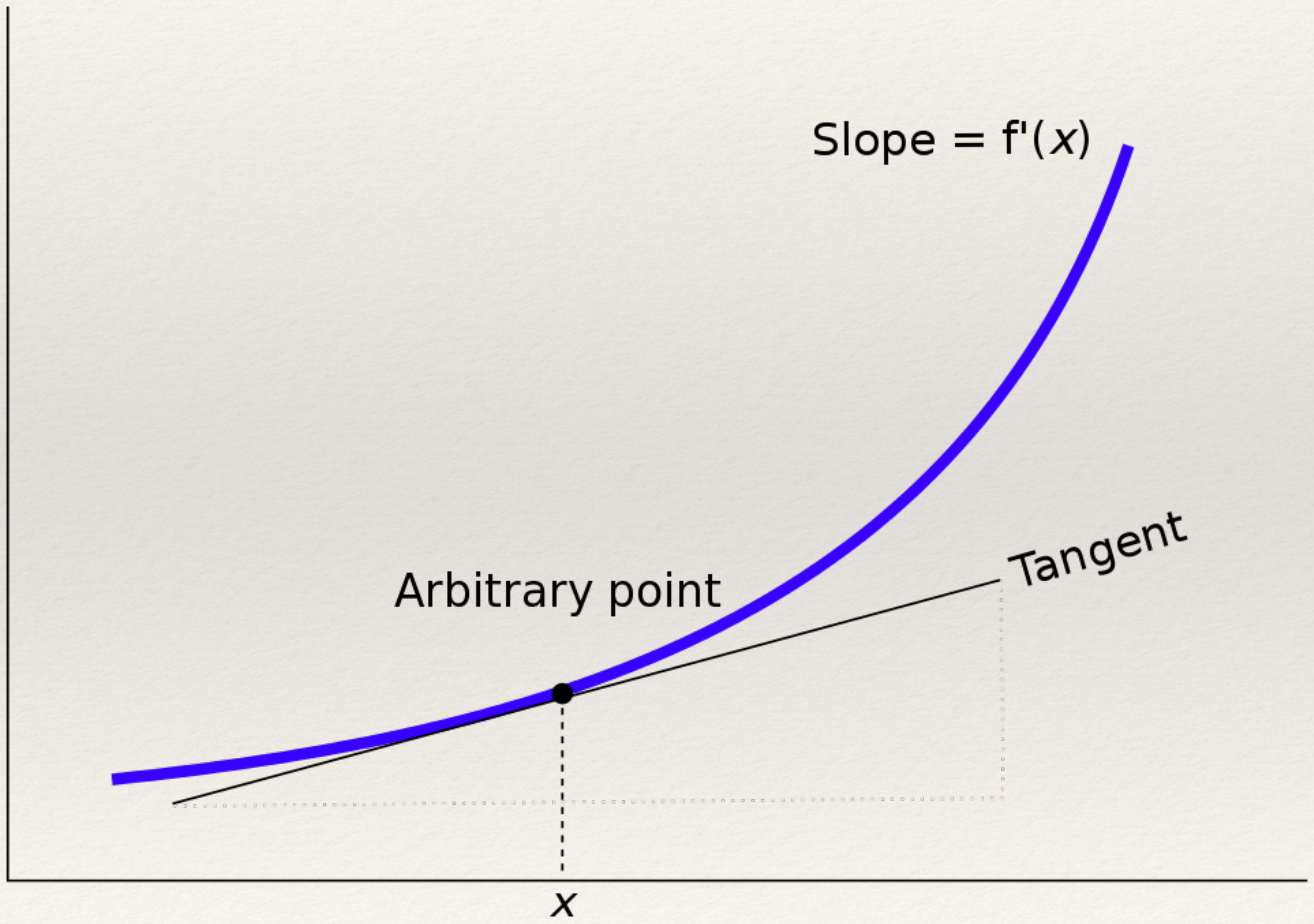
Application Example

- 1-D Transient Heat Conduction Down a Rod
  - We'll solve this heat equation three different ways
    - Analytically
    - Numerically
    - Experimentally

Derivatives

- How things change (temperature) with respect to a variable like position or time



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Analytical Solution

- Transient Energy Balance:

$$\frac{dE}{dt} = -\frac{\partial q}{\partial x}$$

$$q = -k\frac{\partial T}{\partial x}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$$

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}}$$

Boundary Condition:
Constant Heat Flux

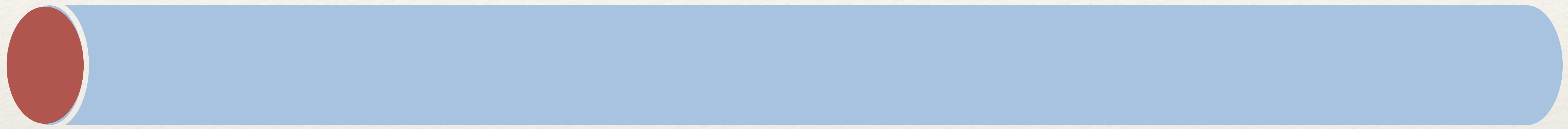
$$T(x,t) = T_0 + \frac{2q_o''(\alpha t/\pi)^{0.5}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k} erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Boundary Condition:
Constant End Surface Temperature

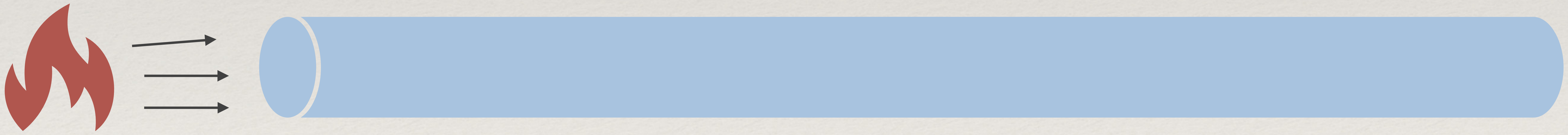
$$T(x,t) - T_i = (T_s - T_i)\cdot\left[1 - erf\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

Boundary Conditions

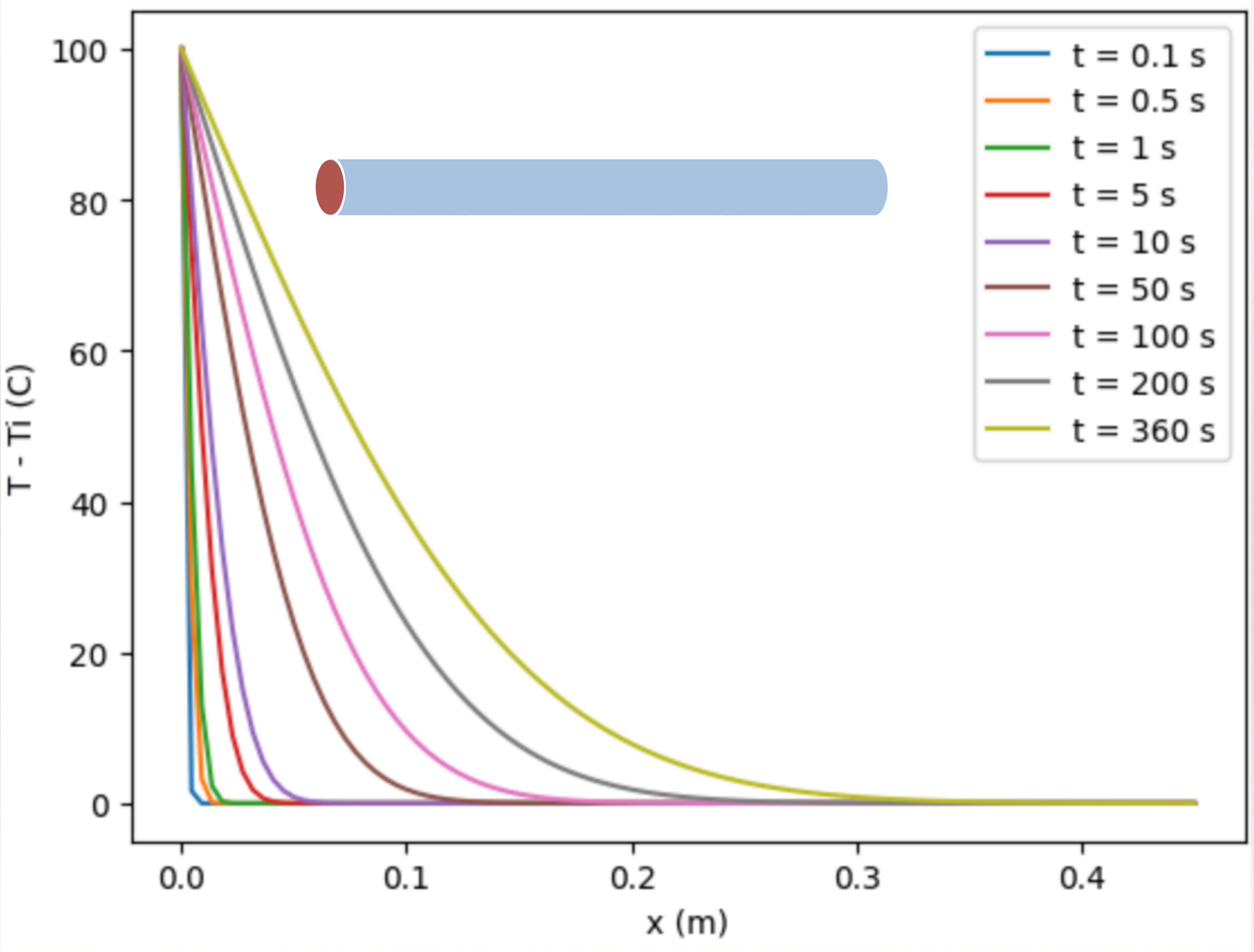
Constant End Surface Temperature



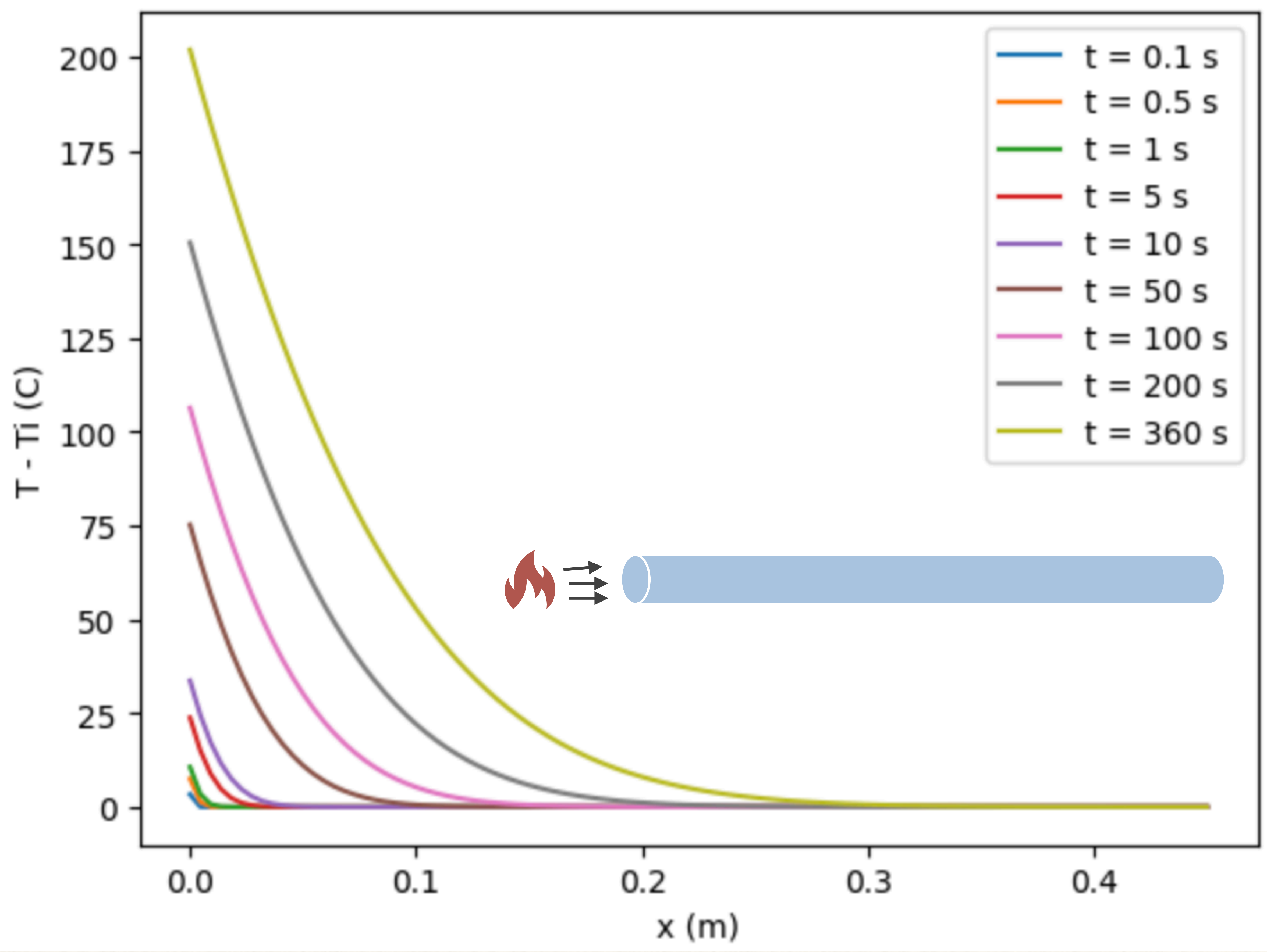
Constant Heat Flux



Analytical Solution Plots

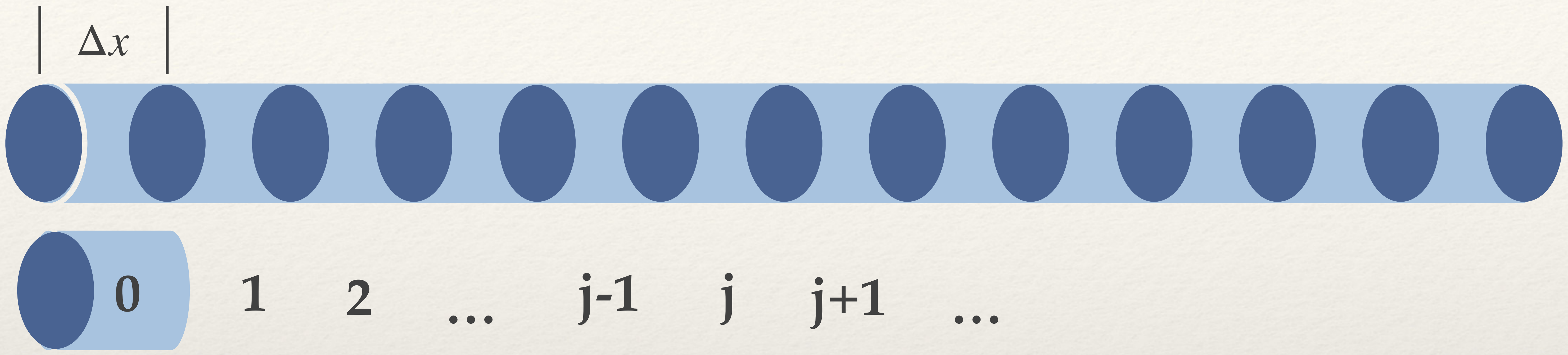


Constant Surface Temperature



Constant Heat Flux

Numerical Solution



- Process is the same where the energy balance for each segment is found and the time and x-coordinate are discretized or broken up into small pieces

$$T[i+1,j] = T[i,j] + \alpha \cdot \Delta t/\Delta x^2 \cdot \left(T[i,j+1] - 2 \cdot T[i,j] + T[i,j-1]\right)$$ at time = i

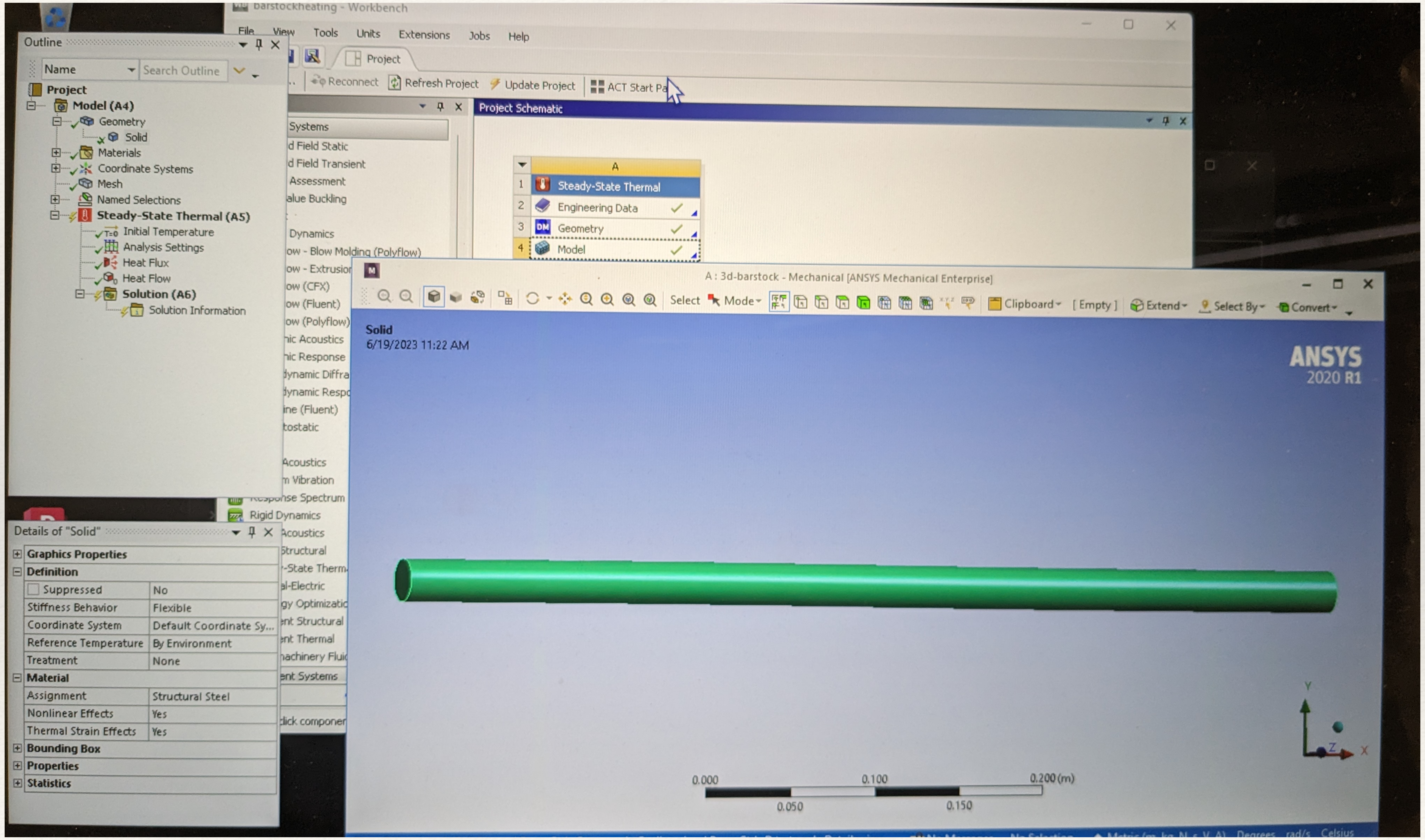
Constant surface temperature

$$T[i+1,0] = T[i,0] + \alpha \cdot \Delta t/\Delta x^2 \cdot (T[i,1] - T[i,0]) + \alpha \cdot q_o'' \cdot \Delta t/(\Delta x \cdot k)$$ For 0th node

Constant heat flux

Other Numerical Options

- Ansys Workbench
- COMSOL
- Etc.



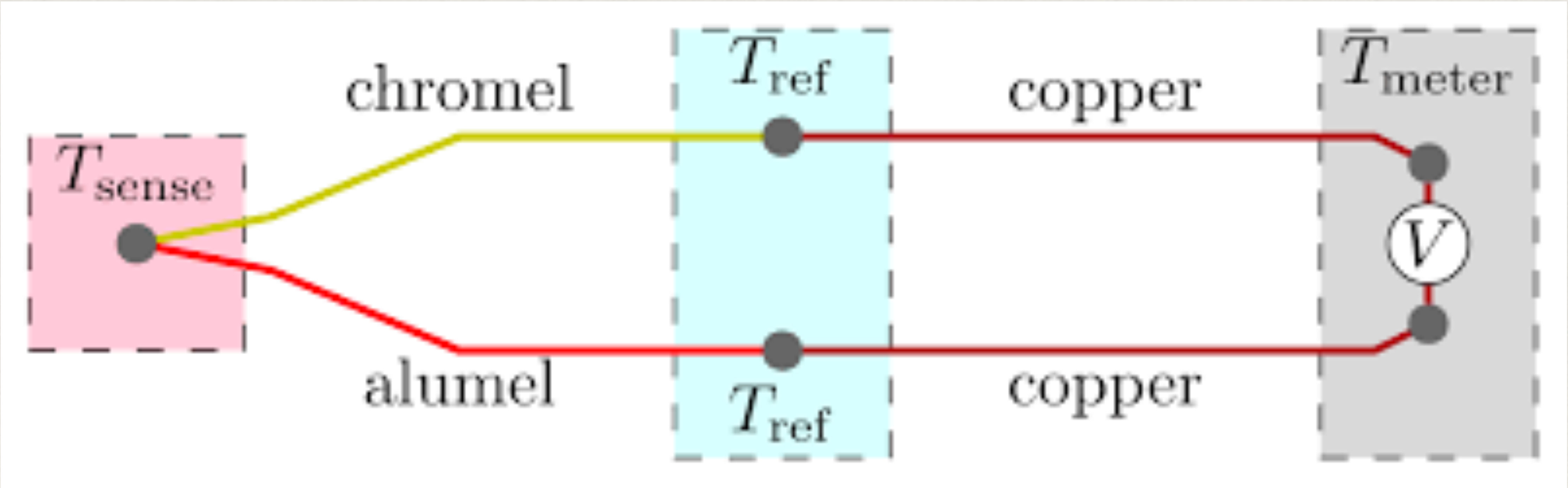
Ansys Workbench

Experimental Option

- Use a K-type thermocouple
- Use a low-cost data collection device with an amplifier: ESP32 and MAX6675
- Add code to allow the ESP32 to connect to your phone via bluetooth
  - App for Android:
- Collect the Data

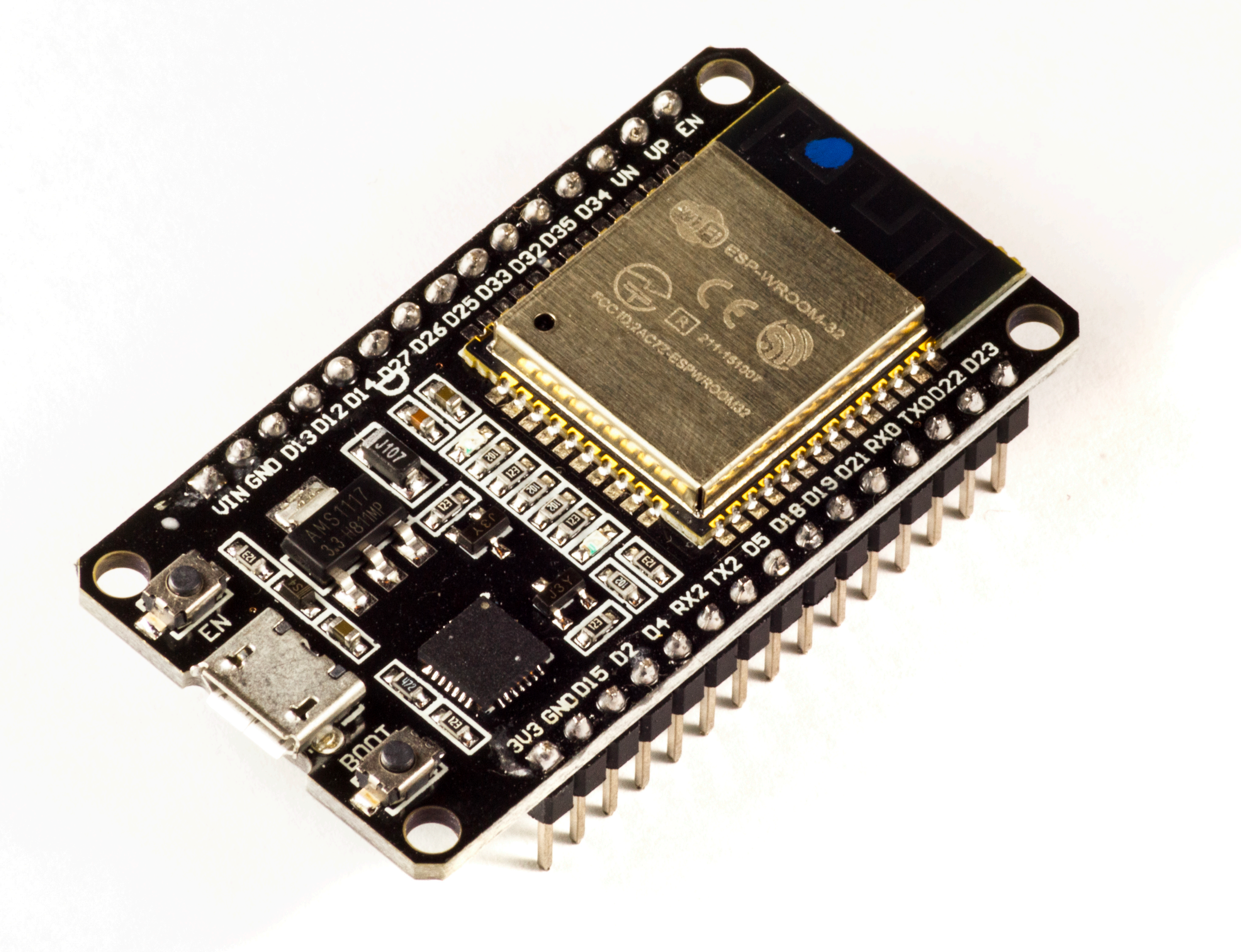
K-type Thermocouple

- Used to measure temperature
- Two dissimilar metals connect and the voltage difference indicates the temperature

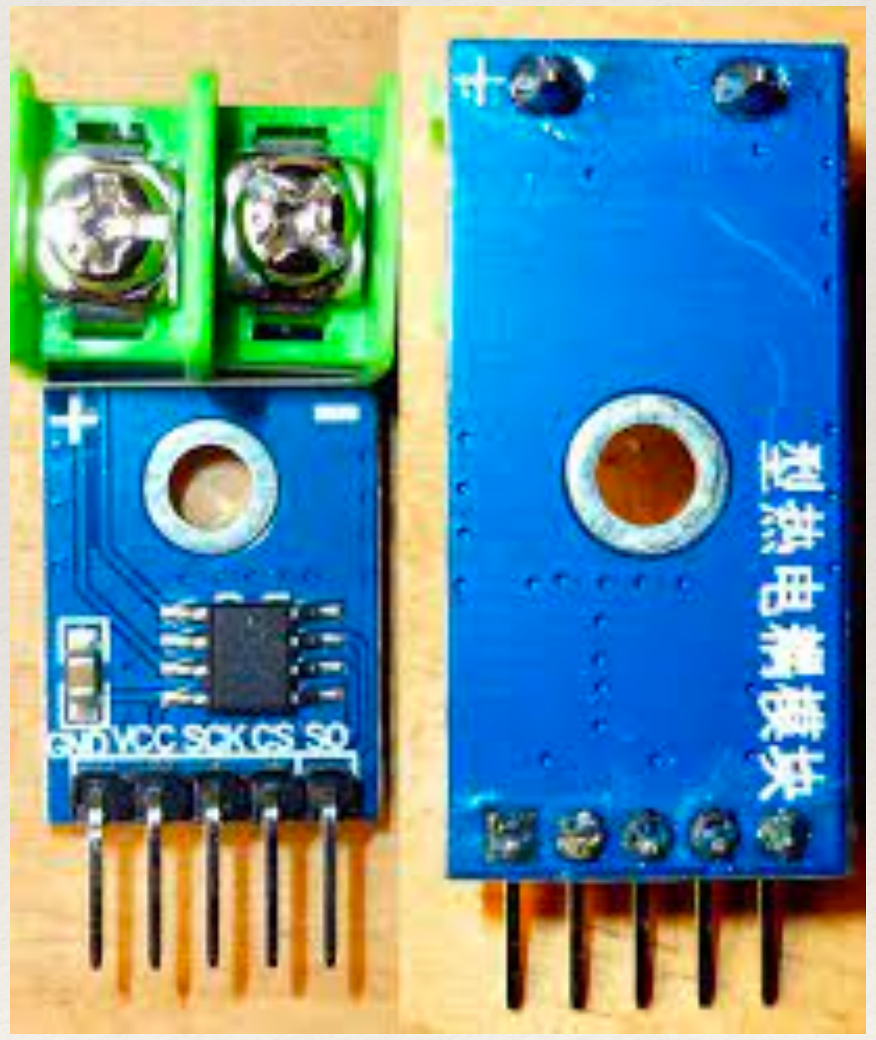


ESP32 & MAX6675

- A low cost data collection option
- Similar to an Arduino device



- A low cost thermocouple amplifier



Data Interface and Collection Code

Arduino IDE File Edit Sketch Tools Help

Final_ESP32_2Temp_Pressure_c1 | Arduino IDE 2.1.0

DOIT ESP32 DEVKIT V1

Final_ESP32_2Temp_Pressure_c1.ino

```
#include "max6675.h"
#include "BluetoothSerial.h"

int thermoDO_small = 19;
int thermoCS_small= 23;
int thermoCLK_small= 5;

MAX6675 thermocouple_small(thermoCLK_small, thermoCS_small, thermoDO_small);

BluetoothSerial SerialBT;
bool isBluetoothConnected = false;

unsigned long previousMillis = 0;
const unsigned long interval = 200; // Time interval between readings in milliseconds

void setup() {
  Serial.begin(9600);

  Serial.println("MAX6675 test");
  // Wait for MAX chip to stabilize
  delay(200);

  SerialBT.begin("ESP32_C1"); // Set the Bluetooth name of your ESP32
  SerialBT.register_callback(handleBluetoothEvent);
}

void loop() {
  unsigned long currentMillis = millis();

  if (currentMillis - previousMillis >= interval) {
    previousMillis = currentMillis;

```

Output

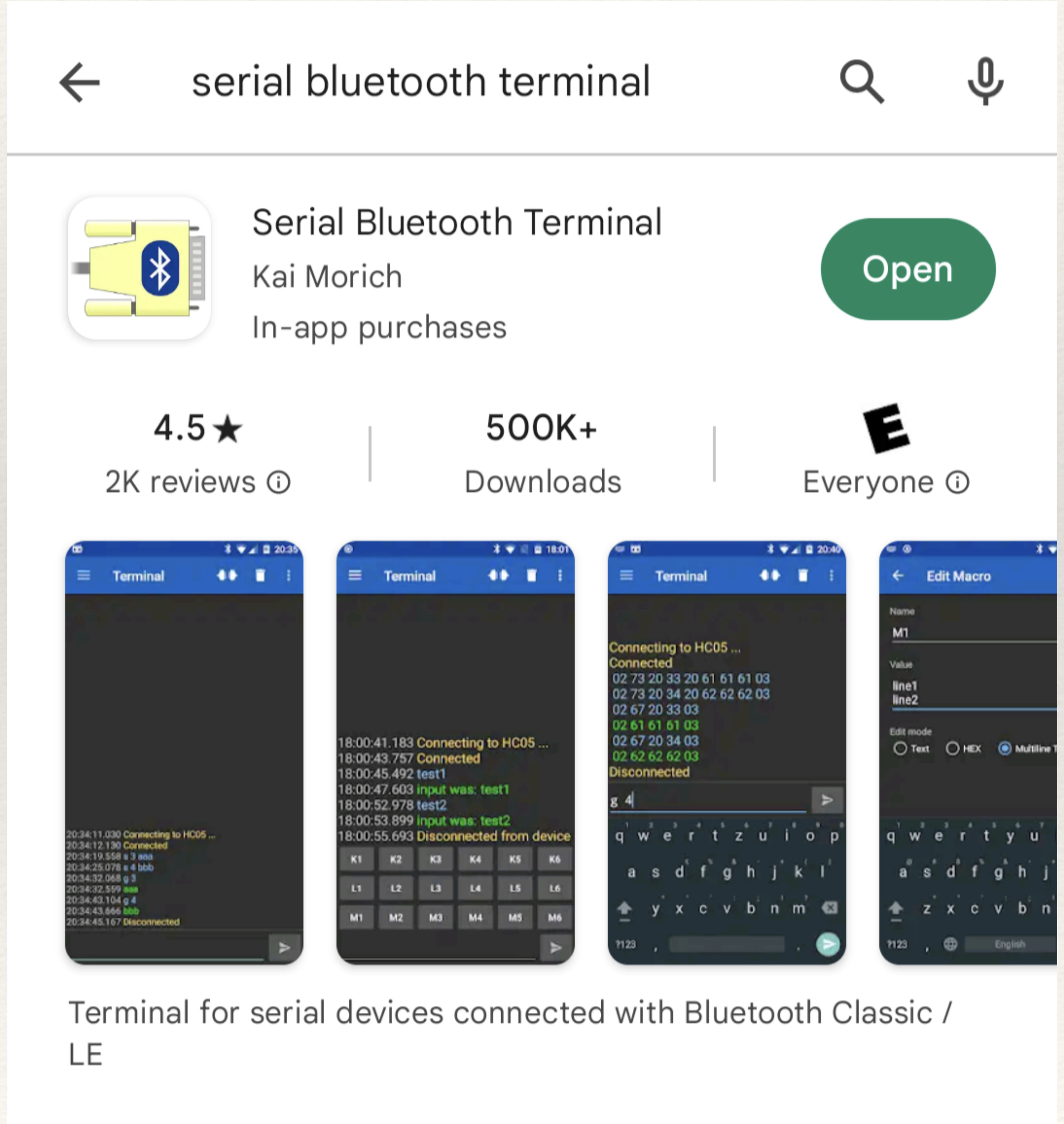
Final_ESP32_2Temp_Pressure_c1.ino

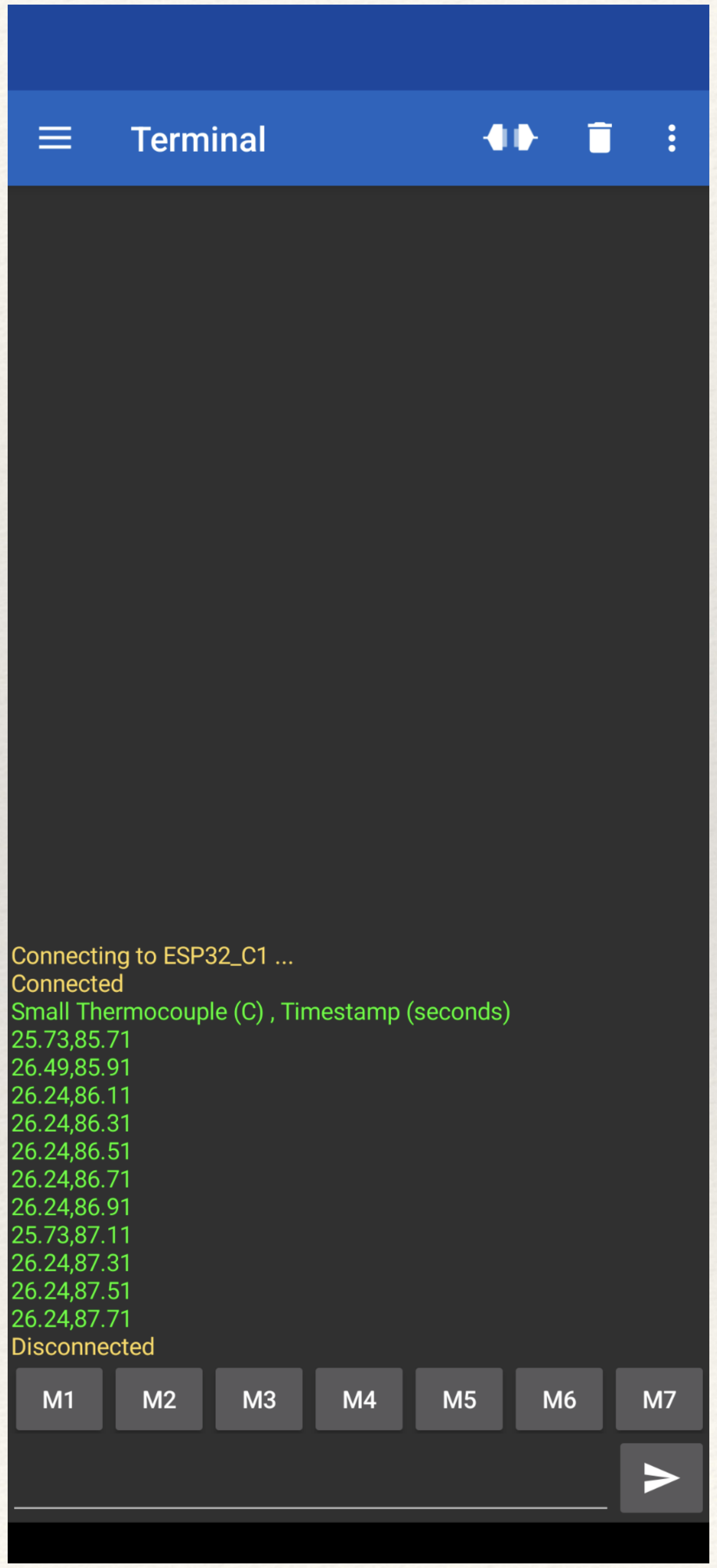
```

    // Basic readout test, just print the current temperature
    float temperature_small = (thermocouple_small.readCelsius()-16.0*0)*1.019;
    Serial.print(temperature_small);
    Serial.print(" C\t");

    Serial.print(currentMillis / 1000); // Convert timestamp to seconds

    // Display the hundredths place for seconds
    int hundredths = (currentMillis / 10) % 100;
    if (hundredths < 10) {
      Serial.print(".0");
    } else {
      Serial.print(".");
    }
    Serial.print(hundredths);

    Serial.println(" seconds");

    // Send temperature, pressure, and timestamp via Bluetooth when connected
    if (isBluetoothConnected) {
      String data = String(temperature_small) + "," + String(currentMillis / 1000) + "." + String(hundredths);
      SerialBT.println(data);
    }
  }

  // Adjust the delay to achieve temperature readings every half second
  delay(200);
}

void handleBluetoothEvent(esp_spp_cb_event_t event, esp_spp_cb_param_t *param) {
  if (event == ESP_SPP_SRV_OPEN_EVT) {
    isBluetoothConnected = true;
    SerialBT.println("Small Thermocouple (C) , Timestamp (seconds)"); // Send units when connected
  } else if (event == ESP_SPP_CLOSE_EVT) {
    isBluetoothConnected = false;
  }
}
```

Output

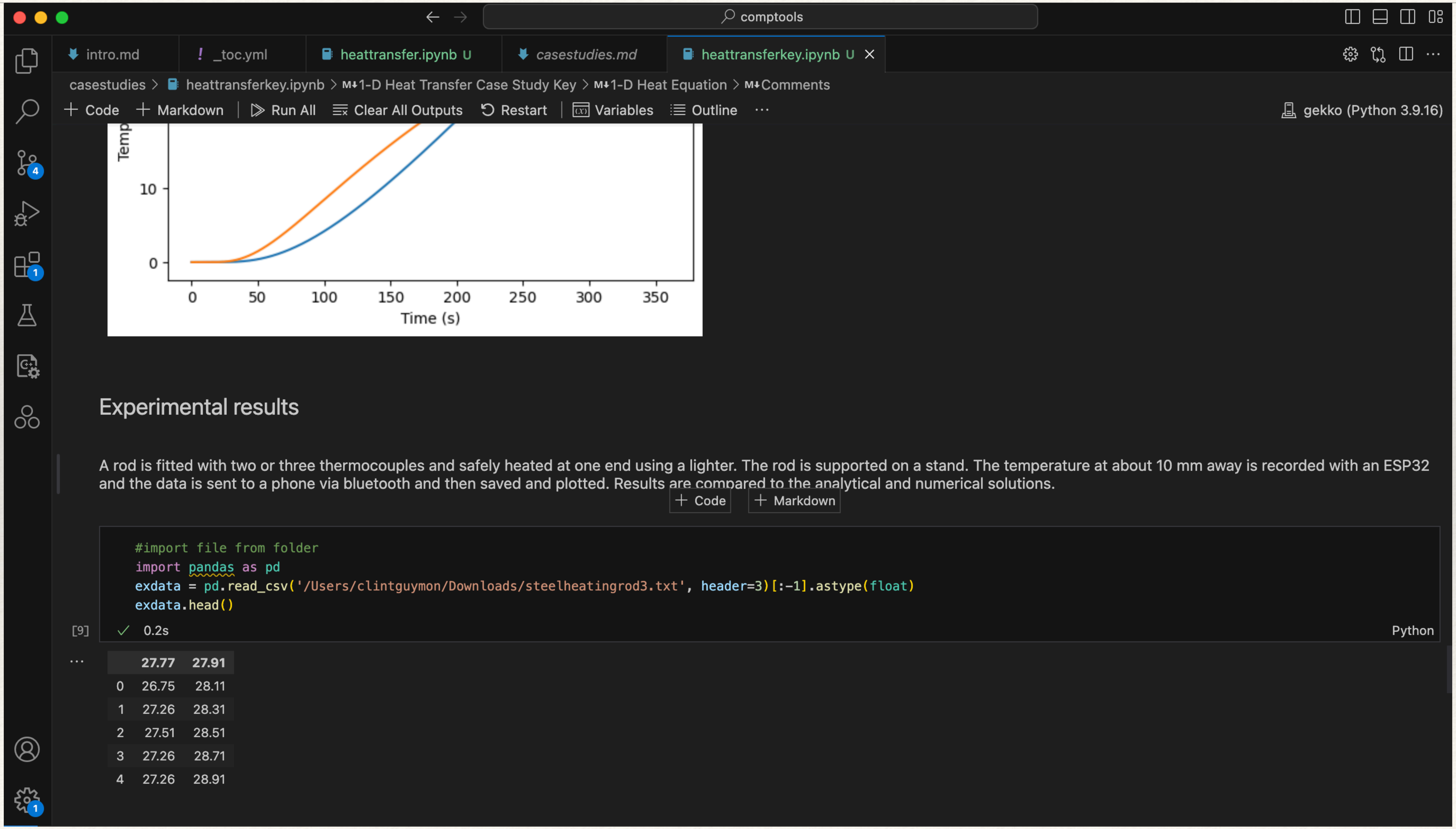
Android App

- Data is collected with the Serial Bluetooth Terminal App
- Temperature and Time is recorded.

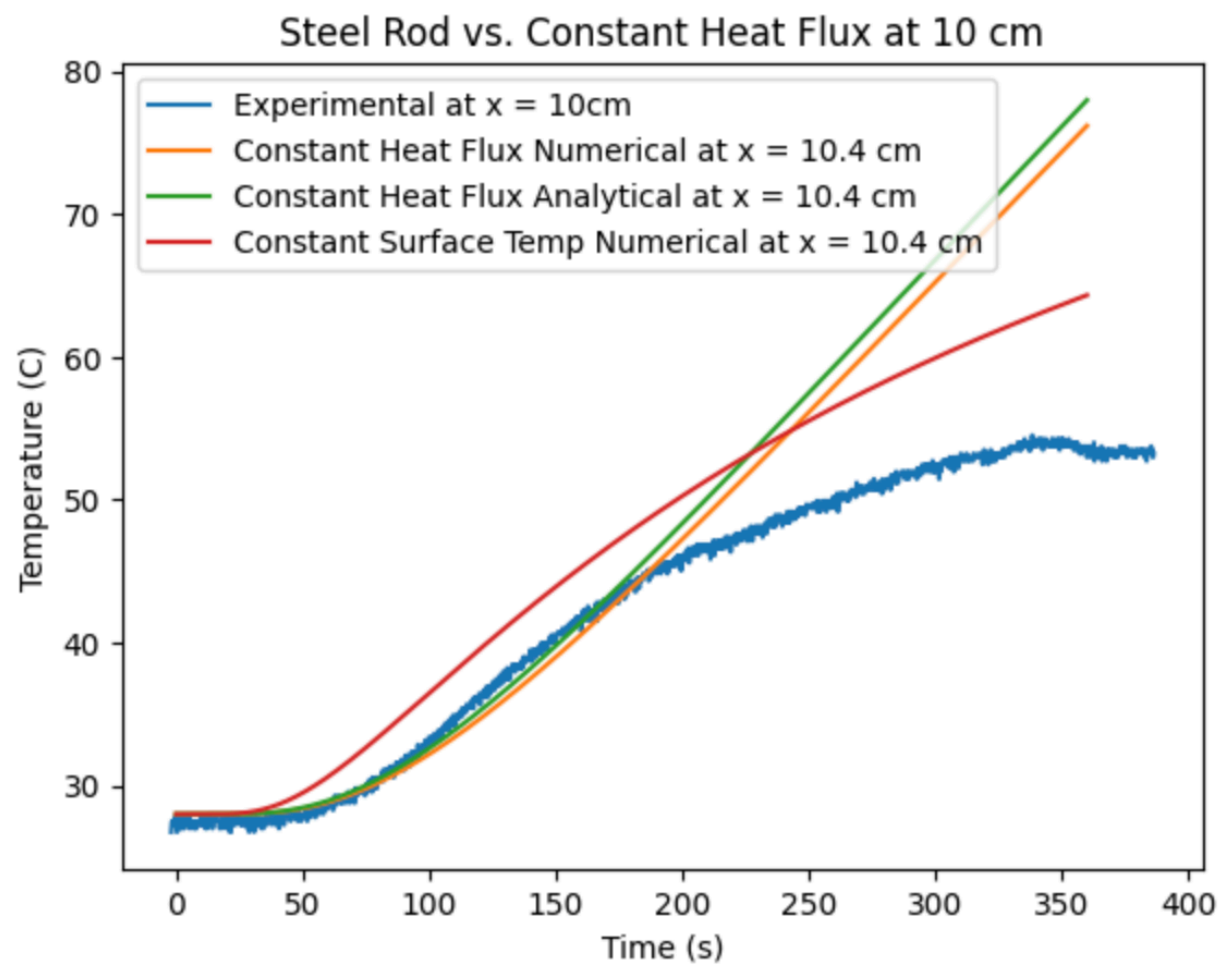




Visual Studio Code Example



Experimental Data vs. Theory



Review: What I Hope You Got

- One application of math is to heat transfer which is used all around us
- You can describe heat transfer using an energy balance as an equation
- You can solve that equation many different ways:
  - Analytically, Numerically, Experimentally
  - There are very affordable options for your to collect data
- Your assumptions can affect your estimate (vs. real life)
- God lives and you are a child of God

This Presentation Location

https://github.com/clint-bg/tools