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CS-225: Discrete Math for CS
Homework 3 Part 1
Set 4.6 Problem # 12, 28 (contradiction) Set 4.7 Problem # 8, 16.c (contradiction)

12.

Proposition: If a and b are rational and r is irrational, then a + br is irrational.

Contradiction: If a and b are rational and r is irrational, then a + br is rational.

For sake of contradiction, suppose that a, b and (a +br) are rational numbers and r is an irrational number. By the properties of rational numbers, let $a=\frac{m}{n}$, $b=\frac{p}{q}$, and (a + br) = $\frac{t}{s}$ for any integers m, n, p, q, t and s. Then,

(a + br) = $\frac{m}{n}+\frac{p}{q}r=\frac{t}{s}$ Using substitution

$\frac{p}{q}r=\frac{t}{s}-\frac{m}{n}$ Using algebra

$\frac{p}{q}r=\frac{nt}{ns}-\frac{ms}{ns}$

$r=\frac{q}{p}(\frac{nt}{ns}-\frac{ms}{ns})$

$r=\frac{(qnt-qms)}{pns}$ Distributive law

By multiplication and subtraction of integers, (qnt – qms) and pns are integers, and that makes r a rational number. This is a contradiction to the supposition.

28.

Proposition: For all integers m and n, if mn is even, then m is even or n is even.

Contradiction: For all integers m and n, if mn is even, then m is odd and n is odd.

For sake of contradiction, let m and n be odd numbers. By the definition of odd numbers, let m=2a+1 and n=2b+1 for any integers a and b. Then,

(2a+1)(2b+1) = mn Using substitution

4ab+2a+2b+1 = Using algebra

2(2ab+a+b)+1 = Distributive Law

But (2ab+a+b) is and integer by multiplication and addition of integers, so by the definition of odd numbers, mn is odd. This is a contradiction to the supposition.

8.

False

Proposition: If a and b are irrational numbers, then a - b is irrational.

Let a and b be irrational numbers and have the values $a=\sqrt{2}$ and $b=\sqrt{2}$. Then,

a - b=

$\sqrt{2}-\sqrt{2}=0$ Using substitution

But 0 is a rational number and can be represented as $\frac{0}{1}$. So it is not always true that the difference of any two irrational numbers is irrational.

16.c

Proposition: $\sqrt{3}$ is irrational.

Contradiction: $\sqrt{3}$ is rational.

For the sake of contradiction, suppose $\sqrt{3}$ is rational. By the property of rational numbers, $\sqrt{3}=\frac{r}{s}$. Then,

$(\sqrt{3})^2=\left(\frac{r}{s}\right)^2$ Using algebra

$3=\frac{r^2}{s^2}$

$r^2=3s^2$

Since $3s^2$ is divisible by 3, we know that r^2 is divisible by 3. So r=3k for some integer k. Using substitution, $(3k)^2=3s^2$. Then,

$9k^2=3s^2$ Using algebra

$\frac{(9k^2)}{3}=\frac{(3s^2)}{3}$

$3k^2=s^2$

Since $3k^2$ is divisible by 3, we know that s^2 is divisible by three. This shows that the integers in the rational number of $\sqrt{3}$ have an additional common factor of 3. This contradicts the supposition that 1 is the only common factor, therefore, the original proposition is true.