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CS-225: Discrete Structures in CS
Homework 9 Part 1
Exercise Set 10.1, Problem# 9, 22, 37.f, 44

9.

i. e1, e2, e7

ii. v1, v2

iii. e2, e7

iv. e1, e3

v. e4 and e5 are parallel

vi. v4

vii. v3 is degree 2

viii. Total degree of the graph is 14

22.

No, a simple graph does not contain any loops or parallel edges. There is no way for one of the vertices to have degree of 5. Since there are 5 vertices, if one vertex connected to all the other vertices it would only have a degree of 4. So it would have to have loop or parallel edges to reach the degree of 5. This would prevent it from being a “simple” graph.

37.f

This is not a bipartite graph. There is no way arrange the vertices into two groups where there is no occurrence of one group member connecting directly to another member of the same group.

44

a) Yes, since this is a simple graph there can not be any loops or parallel edges. Since there can not be any loops or parallel edges, the highest degree any vertex can have is n-1. For example, if we have 5 vertices, if one vertex connects to every other one it will have a degree of 4, or n-1.

b) No, this is not possible. As established before, the highest degree a vertex can have is n-1. In this case, 4 - 1, or 3. So now the vertices can have a degree of 0 – 3. If one of the vertex has a degree 3, it would have to connect to every other vertex in the group. This eliminates the ability of one vertex to have a degree 0. This leaves us with the degree choices of 1-3. Since we have 3 choices for the degree, and 4 vertices, two of the vertices must have the same degree value. (pigeon hole principal)

c) No, this is not possible. It follows the same principal as b). If we have n vertices, then the highest degree value any vertex may have is n-1. If one of the vertices has a degree of n-1, it will have to connect to every other vertex in the group. This eliminates the possibility of one of the vertices to have a degree 0. This leave us with the total number of degree possibilities to n-1. So every vertex can have a different degree until you reach the n^{th} vertex. This last vertex must have the same degree as one of the other vertices. This is by the pigeon hole principal.