Clinton Hawkes
CS-225: Discrete Math for CS
Homework 6
Exercise Set 5.6, Problem# 4, 6; Set 5.7, Problem# 7, 8; Set 5.9, Problem# 15, 18

4.

$d_k = k(d_{k-1})^2$ for all integers $k \geq 1$.

$d_0 = 3$ //given

$d_1 = 1(3)^2 = 1(9) = 9$

$d_2 = 2(9)^2 = 2(81) = 162$

$d_3 = 3(162)^2 = 3(26244) = 78732$

6.

$t_k = t_{k-1} + 2t_{k-2}$ for all integers $k \geq 2$.

$t_0 = -1$ //given

$t_1 = 2$ //given

$t_2 = 2 + 2(-1) = 2 + (-2) = 0$

$t_3 = 0 + 2(2) = 0 + 4 = 4$

7.

$e_k = 4e_{k-1} + 5$ for all integers $k \geq 1$.

$e_0 = 2$ //given

$e_1 = 4(2) + 5 = 8 + 5 = 13$ //2+11

$e_2 = 4(13) + 5 = 52 + 5 = 57$ //2+11+44 <u>or</u> 2+11+4(11)

$e_3 = 4(57) + 5 = 228 + 5 = 233$ //2+11+44+176 <u>or</u> $2+11+4(11)+4^2(11)$

$e_4 = 4(233) + 5 = 932 + 5 = 937$ // 2+11+44+176+704 <u>or</u> $2+11+4(11)+4^2(11)+4^3(11)$

The geometric sequence is $1+r+r^2+r^3+...+r^{n-1}$, so we need to make ours in terms of this sequence.

$e_n = 2+11(1+4+4^2+4^3+...+4^{n-1})$

Now, we can plug our numbers into the summation formula for the geometric sequence.

$$e_n = 2+11* \frac{(4^{((n-1)+1)}-1)}{(4-1)} = 2+11* \frac{(4^n-1)}{(4-1)} = 2+ \frac{(11*4^n-11)}{3} = \frac{2*(3)}{3} + \frac{(11*4^n-11)}{3}$$

$$= \frac{6}{3} + \frac{(11*4^n-11)}{3} = \frac{(11*4^n-5)}{3}$$ for all integers $k \geq 0$.

8.

$f_k = f_{k-1} + 2^k$ for all integers $k \geq 2$.

$f_1 = 1$ //given

$f_2 = 1+2^2 = 1+4 = 5$ //1+4 or $1+2^2$

$f_3 = 5+2^3 = 5+8 = 13$ //1+4+8 or $1+2^2+2^3$

$f_4 = 13+2^4 = 13+16 = 29$ //1+4+8+16 or $1+2^2+2^3+2^4$

$f_5 = 29+2^5 = 29+32 = 61$ //1+4+8+16+32 or $1+2^2+2^3+2^4+2^5$

This looks similar to the geometric sequence, so we need find a way to put this in terms of that sequence. Looks like we are just missing the first "r^1" term in this sequence, so we could write it in the same pattern as the geometric sequence – r, or $(1+2+2^2+2^3+2^4+2^5+...+2^n) – 2$

Plugging these numbers into the summation formula for the geometric sequence we get:

$f_n = \dfrac{(2^{(n+1)}-1)}{(2-1)}-2 = \dfrac{(2^{(n+1)}-1)}{1}-2 = (2^{(n+1)}-1)-2 = 2^{(n+1)}-3$ for all integers $k \geq 1$.

15.

Define the set of all strings of 0's and 1's that have the same number of each.

<u>I. Base</u>: The Ø meets the requirements of having the same number of 0's and 1's, so we can say that $\emptyset \in S$

<u>II. Recursion</u>: if $s \in S$, then the following are also elements of S:

a) 01s b) 10s c) 0s1 d) 1s0 e) s01 f) s10

where these are concatenations of s with 0 and 1.

<u>III. Restrictions</u>: Nothing is in S other than objects defined by I. and II.

18.

Define the set of all strings of a's and b's that contain exactly one a.

<u>I. Base</u>: the string must have one a, and exactly one a, so $a \in S$

<u>II. Recursion</u>: if $s \in S$, then

a) $bs \in S$ b) $sb \in S$

where bs and sb are concatenations of s with b.

<u>III. Restrictions</u>: Nothing is in S other than objects defined by I. and II.