

6.

a) Yes, when we are dividing by 6 there are only 6 distinct remainder possibilities. (0-5) If we have 7 integers that we are dividing by 6, then 1 must share the same remainder value with one of the other integers. If we only had 6 integers that we were dividing by 6, each could have their own distinct remainder.

b) No, when dividing by 8 there are 8 distinct remainder possibilities. (0-7) Since we only have 7 integers that we are dividing by 8, each of those integers can have its own distinct remainder value and not have to share.

7.

Yes, because in set S there are 5 pairings that result in the sum of 15. If we only had to choose 5 integers from S, we could choose only one number from each of the pairings and say that there wasn't a combination that summed to 15. Since we have to choose one more integer to make 6, we have to select the second integer from one of the pairings that sums to 15. This makes the statement true.

16.

To find the number of integers between 1 and 100 that are divisible by 5, we have to count the number of integers in that range that have a 0 or a 5 for the units digit. Then, if we reach 3 digits (ex. 100) there is only one possibility. For the 10's digit there are 10 possibilities and for the units digit there are only 2 possibilities. So, $1 \cdot 10 \cdot 2 = 20$ integers divisible by 5. Since we have 100 total possibilities, $100 - 20 = 80$ gives us the integers that are not divisible by 5. So we could choose 80 integers and still not have one that is divisible by 5. That is why we must choose 81 integers to be certain we have at least one integer that is divisible by 5.

27.

Yes, in a group of 2000 people at least 5 must have the same birthday. Lets assume there to be 365 days in a year. That means there are 365 distinct birthday possibilities. So it takes 366 people to be certain that at least one person shares a birthday with another. If we want the number of people to be certain at least 5 share the same birthday, we have $(365 \cdot 5) + 1 = 1826$. This still leaves room for 174 others people to share the same birthday with 4 others.