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CS-225: Discrete Structures in CS
Homework 2, Part 2
Set 4.1: Question # 32, 58; Set 4.2: Question # 20, 25; Set 4.6 Question # 28 (contraposition only)

32.

Suppose a is any odd integer and b is any even integer. By definition of odd and even, a=2k+1 and b=2m for any integer k and m. Then,

2a+3b = 2(2k+1)+3(2m)

= 4k + 2 + 6m

= 4k + 6m + 2

= 2(2k + 3m + 1)

But (2k + 3m + 1) is an integer by multiplication and addition of integers, so 2a+3b is equal to 2 multiplied by some integer. This is the definition of an even number, and therefore, proves 2a+3b is even.

58.

This is true. Suppose a and b are any consecutive integers, such that b=a+1. Then,

$b^2 – a^2 = (a+1)^2 - a^2$

$= (a^2 + 2a +1) - a^2$

$= a^2 – a^2 + 2a + 1$

$= 2a + 1$

By definition of odd numbers, the difference of the squares of any two consecutive integers is odd.

20.

Suppose r and s are any two rational numbers, such that r < s. Then

(r+r) < (r+s) < (s+s)

2r < (r+s) < 2s

$r < \frac{(r+s)}{2} < s$ by rational definition, $r = \frac{j}{k}$ $s = \frac{m}{n}$ for any integers j, k, m, or n.

$\frac{(r+s)}{2} = \frac{((\frac{j}{k})+(\frac{m}{n}))}{2}$ by substitution

$= \frac{((\frac{jn}{kn})+(\frac{km}{kn}))}{2}$ using algebra on middle term

$= \left(\frac{1}{2}\right)\left(\left(\frac{jn}{kn}\right)+\left(\frac{km}{kn}\right)\right)$

$= \frac{(jn+km)}{(2kn)}$

r < $\frac{(jn+km)}{(2kn)}$ < s

By integer multiplication and addition, (jn+km) and (2kn) are both integers, thus making $\frac{(jn+km)}{(2kn)}$ a rational number. Therefore, given any two rational numbers r and s where r < s, there exists a rational number between them.

25.

Suppose r is any rational number. By theorem 4.2.1, all integers are rational numbers, so 3 and 2 and 4 are rational numbers. From exercise 12, we learn that the square of a rational number is rational, so r^2 is rational. From exercise 15 we learn that the product of any two rational numbers is rational, so $3r^2$ and 2r are rational. Finally, theorem 4.2.2 states the sum of any two rational numbers is rational, so $3r^2 - 2r + 4$ is rational. This proves that if r is any rational number, then $3r^2 - 2r + 4$ is rational.

28.

Original Exercise: $\forall\, integers\, m \text{ and } n, if\ E(mn) \rightarrow (E(m) \vee E(n))$ where E() is even.

Contrapositive: $\forall\, integers\, m \text{ and } n, if\ (\neg(E(m)) \wedge \neg(E(n))) \rightarrow \neg E(mn)$

Suppose m and n are any odd integers. By definition of odd numbers, m=2a+1 and n=2b+1 for any integers a and b. Then

(2a+1)(2b+1)=mn using substitution

4ab+2a+2b+1=mn

2(2ab+a+b)+1=mn

But (2ab+a+b) is an integer by multiplication and addition of integers, so mn is equal to 2 multiplied by an integer plus 1. By the definition of odd numbers, mn is odd. This proves the contrapositive to be true, and in turn, the original exercise to be true.