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CS-225:Discrete Structures for CS

Homework 7 Part 1

Exercise Set 9.2, Problem# 11.c, 14.c, 14.e, 17.a-d; Set 9.3, Problem# 5.a, 24.a, 24.c, 33.e, 33.f

11.c

Bit string has 8 bits that can be 0 or 1. The first and last bit must be 1, and the ones in between have two possibilities, so the total possible combinations is $1*2*2*2*2*2*2*1 = 2^6 = 64$

14.c

License plate has 4 letters followed by 3 numbers. Total possibilities that begin with TGIF is 1(for the letters) multiplied by $10*10*10 = 1000$

14.e

License plate that begins with AB and has the remaining letters/numbers distinct. Since AB is set, that counts as 1. The remaining 2 letters have 24 and 23 possibilities if the letters are to remain distinct. The possibilities of the numbers are $10*9*8$ if they are to remain distinct. So, putting it all together, $1*24*23*10*9*8 = 397,440$

17

a) Since we are counting from 1000 to 9999, the first digit cannot be 0, so there are 9 possibilities for the first digit. The remaining 3 digits each have 10 possibilities. So, the number of integers from 1000 to 9999 is $9*10*10*10 = 9000$

b) Building off part a, the only thing that changes is the possibilities of the units digit. Since we are counting odd numbers, it must be a 1, 3, 5, 7, or 9, so 5 possibilities. Then, $9*10*10*5 = 4500$

c) Building off part a again, the first digit still remains 9, but each succession has one less possibility than before to remain distinct. So, $9*9*8*7 = 4536$

d) Building off part a and b, the units digit still has 5 possibilities to remain odd. Since the units digit is counted for, the first digit has one less possibility than before, so it has 8. The next digit has 2 less than before and the third digit has 3 less than before. So, $8*8*7*5 = 2240$

5.a

In this problem we are looking at integers between 10000 and 99999. To be divisible by 5, we know that the units digit must be a 0 or 5. Also, since we start counting at 10000, the first digit cannot be 0, so it only has 9 possibilities. Then, the number of integers between 10000 and 99999 that are divisible by 5 is $9*10*10*10*2 = 18000$

24.a

We are counting from 1 to 1000, so we know the total number of digits is 1000. To be multiples of 2, then we know it can be written as $2m$ for some integer m . So, $2m = 1000 \implies m = 1000/2 = 500$. To be a multiple of 9, we know it can be written as $9n$ for some integer n . Then $9n = 1000 \implies n = 1000/9 = 111.11$, so if we take the floor value, 111 possibilities. We need to find out how many numbers are both multiples of 2 and multiples of 9 so we don't double count numbers. To be a multiple of both, the number can be written as $2 \cdot 9x$, or $18x$ for some integer x . Then, $18x = 1000 \implies 1000/18 = 55.556$, so we take the floor value, 55. Total number of integers between 1 and 1000 that are multiples of 2 or 9 is $500 + 111 - 55 = 556$

24.c

To find the number of integers between 1 and 1000 that are neither multiples of 2 nor multiples of 9, we subtract the number of integers that are these multiples from the number of total possibilities. So, $1000 - 556 = 444$

33.e

To get the number of students that checked 2 and 3, but not 1

3 checked 2 and 3, but some of those also checked 1, so we need to subtract the students who checked all three. $(\#2 \text{ and } \#3) - (\#1 \text{ and } \#2 \text{ and } \#3)$

So $3 - 2 = 1$ student

33.f

The get the number that checked 2 but neither of the other ones:

$(\#2) - (\#1 \text{ and } \#2) - (\#2 \text{ and } \#3) + (\#1 \text{ and } \#2 \text{ and } \#3)$

$26 - 8 - 3 + 2 = 17$ students