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CS-225: Discrete Structures in CS
Homework 8 Part 1
Set 9.2, Problem# 32c, 33, 39b, 39d Set 9.5, Problem# 7b, 14, 20

32.

c) GOR is now considered the same as a single letter, so we continue as if there are 7 letters to consider instead of 9.   7! = 7*6*5*4*3*2*1 = 5040

33.

a) 6! = 6*5*4*3*2*1 = 720

b) Doctor sits on the left aisle, so the orders of the remaining 5 are calculated. 5! = 5*4*3*2*1 = 120. Since there is an aisle on the right side as well, we need to take that into consideration. If the doctor sits on the right side, the remaining 5 people can sit in the same number of orders as was calculated before. So the total number of ways people can be seated with the doctor sitting at either aisle is 120*2 = 240

c) The couples are considered one unit each, so the number of ways they can sit is 3! = 3*2*1 = 6

39.

b) P(9,6) = $\frac{(9!)}{((9-6)!)} = \frac{9!}{3!}$ = 9*8*7*6*5*4 = 60480

d) First two letters never change and are removed from group of possible letters to choose from, so

P(7,4) = $\frac{7!}{(7-4)!} = \frac{7!}{3!}$ = 7*6*5*4 = 840

7b.

I) $\binom{7}{4}*\binom{6}{3} = \frac{7!}{4!(7-4)!}*\frac{6!}{3!(6-3)!} = \frac{7!}{4!(3)!}*\frac{6!}{3!(3)!} = \frac{7*6*5}{3!}*\frac{6*5*4}{3!} =$

$\frac{210}{6}*\frac{120}{6}$ =700

II) $\binom{13}{7}$ - $\binom{6}{0}*\binom{7}{7} = \frac{13!}{7!(13-7)!}-1 = \frac{13!}{7!6!}-1 = \frac{13*12*11*10*9*8}{6!}-1 =$

$\frac{13*12*11*10*9*8}{6*5*4*3*2*1}-1 = \frac{1235520}{720}-1$ =1715

III) $\binom{7}{3}\binom{6}{4}+\binom{7}{2}\binom{6}{5}+\binom{7}{1}\binom{6}{6} = \frac{7!}{3!4!}\frac{6!}{4!2!}+\frac{7!}{2!5!}\frac{6!}{5!1!}+\frac{7!}{1!6!}\frac{6!}{6!0!} =$

$\frac{7*6*5}{3!}\frac{6*5}{2!}+\frac{7*6}{2!}\frac{6}{1!}+\frac{7}{1!}\frac{1}{1} = \frac{210}{6}\frac{30}{2}+\frac{42}{2}\frac{6}{1}+\frac{7}{1}\frac{1}{1}$ = 658

14.

a) $\binom{16}{7} = \frac{16!}{7!9!} = \frac{16*15*14*13*12*11*10}{7!} = \frac{16*15*14*13*12*11*10}{7*6*5*4*3*2*1} = 11440$

b) $\binom{16}{13}+\binom{16}{14}+\binom{16}{15}+\binom{16}{16} = \frac{16!}{13!3!}+\frac{16!}{14!2!}+\frac{16!}{15!1!}+1 = \frac{16*15*14}{6}+\frac{16*15}{2}+\frac{16}{1}+1 =$

$\frac{3360}{6}+\frac{240}{2}+\frac{16}{1}+1 = 560+120+16+1 = 697$

c) $2^{16}-1 = 65535$

d) $1+\binom{16}{1} = 1+\frac{16!}{1!15!} = 1+\frac{16}{1} = 17$

20.

a) $\binom{11}{3}\binom{8}{2}\binom{6}{2}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1} = \frac{11!}{3!8!}\frac{8!}{2!6!}\frac{6!}{2!4!}\frac{4!}{1!3!}\frac{3!}{1!2!}\frac{2!}{1!1!}1 =$

$\frac{11*10*9}{3*2*1}\frac{8*7}{2*1}\frac{6*5}{2*1}\frac{4}{1}\frac{3}{1}\frac{2}{1}1 = \frac{990}{6}\frac{56}{2}\frac{30}{2}\frac{4}{1}\frac{3}{1}\frac{2}{1}1 = 1,663,200$

b) $\binom{9}{3}\binom{6}{2}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1} = \frac{9!}{3!6!}\frac{6!}{2!4!}\frac{4!}{1!3!}\frac{3!}{1!2!}\frac{2!}{1!1!}1 = \frac{9*8*7}{3*2*1}\frac{6*5}{2}\frac{4}{1}\frac{3}{1}\frac{2}{1}1 = 30,240$

c) $\binom{9}{3}\binom{6}{2}\binom{4}{2}\binom{2}{1}\binom{1}{1} = \frac{9!}{3!6!}\frac{6!}{2!4!}\frac{4!}{2!2!}\frac{2!}{1!1!}1 = \frac{9*8*7}{3*2*1}\frac{6*5}{2}\frac{4*3}{2}\frac{2}{1}1 = 15,120$