

Clinton Hawkes

CS-225: Discrete Structures in CS

Homework 1, Part 2

Exercise Set 2.1 # 22, 42, 45; Exercise Set 2.2 # 11, 13.b, 15, 20, 38, 41, 43, 45

22.

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

These two statement forms produce the same truth values, so this means they are logically equivalent.
(Also, this is an example of the distributive law)

42.

p	q	r	$\neg p$	$(q \wedge r)$	$(\neg p \wedge q)$	$\neg q$	$((\neg p \wedge q) \wedge (q \wedge r))$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
T	T	T	F	T	F	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	F	F	T	F	F
T	F	F	F	F	F	T	F	F
F	T	T	T	T	T	F	T	F
F	T	F	T	F	T	F	F	F
F	F	T	T	F	F	T	F	F
F	F	F	T	F	F	T	F	F

This is a contradiction, because the truth values for the statement form are false for every possible input.

45.

Let p = Bob Double Major, q = Ann Double Major and r = Ann Math Major

p	q	r	$(p \wedge r)$	$(p \wedge q)$	$\neg q$	$\neg(p \wedge q)$	$(p \wedge r) \wedge \neg q$	$\neg(p \wedge q) \wedge (p \wedge r)$
T	T	T	T	T	F	F	F	F
T	T	F	F	T	F	F	F	F
T	F	T	T	F	T	T	T	T
T	F	F	F	F	T	T	F	F
F	T	T	F	F	F	T	F	F
F	T	F	F	F	F	T	F	F
F	F	T	F	F	T	T	F	F
F	F	F	F	F	T	T	F	F

Because the statement forms of these two statements produce the same truth values, we conclude that the statements are logically equivalent.

11.

p	q	r	$(q \rightarrow r)$	$(p \wedge q)$	$(p \rightarrow (q \rightarrow r))$	$((p \wedge q) \rightarrow r)$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

13. b)

p	q	$\neg q$	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$p \wedge \neg q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Both statement forms produce the same truth values, so these statement forms are logically equivalent.

15.

p	q	r	$(q \rightarrow r)$	$(p \rightarrow q)$	$(p \rightarrow (q \rightarrow r))$	$((p \rightarrow q) \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T

We know that both statement forms will be true if their first component is false, and the first component will be false if p is false, so we only need to look at the table where p is true. Since both statement forms produce the same truth values, they are logically equivalent.

20.

- a) P is square and P is not a rectangle.
- b) Today is New Years Eve and tomorrow is not January.
- c) The decimal expansion of r is terminating and r is not rational.
- d) n is prime and n is neither odd nor 2.
- e) x is non-negative and x is neither positive nor 0.
- f) Tom is Ann's father and either Jim is not her uncle or Sue is not her aunt.
- g) n is divisible by 6 and either n is not divisible by 2 or n is not divisible by 3.

38. If it doesn't rain, then Ann will go.

41. If I have two 45 degree angles, then this triangle is a right triangle.

43. If Jim passes the course, then Jim does his homework regularly.

45. If the computer program is correct, then it did not produce error messages during translation.