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CS-225: Discrete Math for CS
Homework 5 Part 2
Exercise Set 5.4: Problem# 2, 3, 10

2.

Prove that b_n is divisible by 4 for all integers $n \geq 1$.

<u>Base Case:</u> Show that b_1, b_2 and b_3 are divisible by 4.

$b_1 = 4$ //given and divisible by 4

$b_2 = 12$ //given and divisible by 4

We need to show b_3 is divisible by 4. Using $b_k = b_{k-2} + b_{k-1}$ to find the value,

$P(3) = b_{3-2} + b_{3-1} = b_1 + b_2 = 4 + 12 = 16$ //divisible by 4

All three examples are true, so we assume b_i is divisible by 4 for all integers $1 \leq i \leq k$.

<u>Induction:</u> Show that P(k+1) is divisible by 4.

$P(k+1) = b_{(k+1)-2} + b_{(k+1)-1}$

$= b_{k-1} + b_k$

Since we already assume b_k is divisible by 4, and we assume b_{k-1} is divisible by 4, we can write these as $b_k = 4r$ and $b_{k-1} = 4q$ for some integers r and q. This is by the property of integer division.

$P(k+1) = 4q + 4r$ //using substitution

$= 4(q + r)$ //by distributive law

This proves P(k+1) is divisible by 4 according to the property of integer division. Since both the basis step and the inductive step are proved, b_n is divisible by 4 for all integers $n \geq 1$.

3.

Prove c_n is even for all integers $n \geq 0$.

<u>Base Case:</u> Show c_0 and c_1 and c_2 are even

$c_0 = 2$ even //given

$c_1 = 2$ even //given

$c_2 = 6$ even //given

Assume c_i is even for all integers $0 \leq i \leq k$.

<u>Induction:</u> Show P(k+1) is even

$P(k+1) = 3c_{(k+1)-3} = 3c_{k-2}$

Since we already assume c_{k-2} is even, we can write it as $c_{k-2} = 2a$, for some integer a. Then,

$P(k+1) = 3(2a)$ //using substitution

P(k+1) = 2(3a)   //commutative law

By the definition of even numbers, P(k+1) is even. Since both the basis step and the inductive step have been proved, c_n is even for all integers n ≥ 0

10.

Prove that P(n) for n ≥ 14 can be obtained using a combination of 3 and 5 cent coins.

<u>Base Case:</u> Show P(14), P(15), and P(16) are true.

P(14) = 3(3) + 5(1)   //true

P(15) = 3(0) + 5(3)   //true

P(16) = 3(2) + 5(2)   //true

We assume P(i) is true for 14 ≤ i ≤ k

<u>Induction:</u> Show P(k+1) is true.

P(k+1) = 3(a) + 5(b) for some integers a and b.   //induction hypothesis

If we a look at k+1, we see that it can be written as [(k+1)-3 + 3] without changing the value of k+1. Reducing it further, we get (k-2)+3.

In our assumption we assume P(i), for 14 ≤ i ≤ k, can be obtained using a combination of 3 and 5 cent coins. P(k-2) falls within this assumption, so we know P(k-2) is a combination of 3 and 5 cent coins. To obtain the value of P(k+1), we need to add a 3 cent coin to P(k-2), so P(k+1)= P(k-2)+3. Because P(k-2) is true, and because 3 is the same value of a 3 cent coin, P(k+1) is true and can be represented by a combination of 3 and 5 cent coins.

Since both the basis step and the inductive step have been proven true, P(n) for n ≥ 14 can be obtained using a combination of 2 and 5 cent coins.