BIOS 705 HW 1

Q2 Show that $\sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})(Y_i - \hat{Y}_i) = 0$

(1) $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$ $\qquad \bar{Y} = \frac{1}{n} \sum Y_i$

$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \varepsilon_i$ $\qquad \bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X}$

(2) from Q1, we know that

$Q = \sum_i \varepsilon_i^2$, $\frac{\partial Q}{\partial \beta_0} = 0$, $\frac{\partial Q}{\partial \beta_1}$

$$\Rightarrow \begin{cases} \frac{\partial Q}{\partial \beta_0} = 0 \rightarrow \sum \varepsilon_i = 0 \\ \frac{\partial Q}{\partial \beta_1} = 0 \rightarrow \sum \varepsilon_i X_i = 0 \end{cases}$$

(3)

$$\begin{aligned} \sum_i (\hat{Y}_i - \bar{Y})(Y_i - \hat{Y}_i) &= \sum_{i=1}^{n} \left[(\hat{\beta}_0 + \hat{\beta}_1 X_i) - (\hat{\beta}_0 + \hat{\beta}_1 \bar{X}) \right] \varepsilon_i \\ &= \sum_i \hat{\beta}_1 (X_i - \bar{X}) \varepsilon_i \\ &= \hat{\beta}_1 \sum X_i \varepsilon_i - \hat{\beta}_1 \bar{X} \sum \varepsilon_i \\ &= 0 \end{aligned}$$