Planar Configuration Spaces
of Disk Arrangements and Hinged Polygons

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December 6, 2016

Motivation: Hinged Dissection

* *Dudeney Problem*: Can a square
  and an equilateral triangle of
  the same area have a common
  dissection into four pieces?

* Can finite collection of polygons
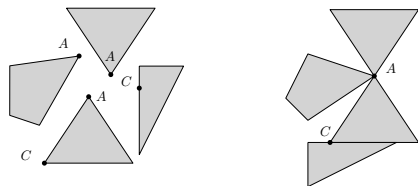 of equal area has a common
  hinged dissection?

Figure: A collection of polygons with
hinges.

* *Dudeney Problem*: Can a square and an equilateral triangle of the same area have a common dissection into four pieces?

* Can finite collection of polygons of equal area has a common hinged dissection?

Figure: blah

Motivation: Protein Folding

Protein folding is the process in which a protein chain acquires its 3-dimensional structure.

* Proteins in an organism fold into a specific geometric pattern (sometimes referred as its *native state*).

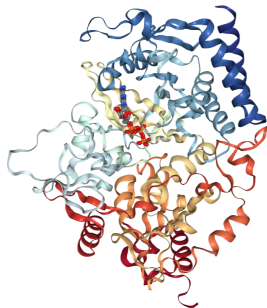* Geometric patterns can determine a protein's function and behavior.

Figure: The structure of rat cytosolic PEPCK variant E89A in complex with oxalic acid and GTP [**?**].

Contribution

It is strongly NP-hard to decide whether a polygonal linkage whose inge graph is a *tree* can be realized.

It is strongly NP-hard to decide whether a polygonal linkage whose inge graph is a *tree* can be realized with fixed orientation.

It is NP-Hard to decide whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with pecified radii.

Logic Engine Realized as Hinged Polygons.pdf

* Suppose we are given an
  Boolean formula with m clauses
  and n variables in 3-CNF form,
  Φ, we construct the polygonal
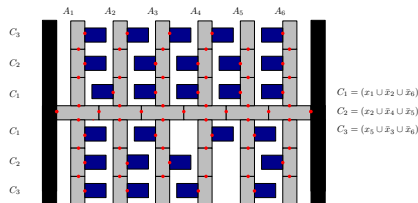 linkage similarly to the logic
  engine.

Figure: Caption Text

* Breu and Kirkpatrick [**?**] proved
  that it is NP-hard to decide
  whether a graph G is the
  contact graph of unit disks in
  the plane, i.e., recognizing *coin
  graphs* is NP-hard; see also [**?**].

Planar 3SAT
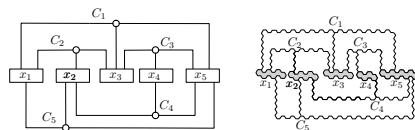
* Define the *associated graph* $A(\Phi)$ as follows: the vertices correspond to the variables and clauses in Φ. We place an edge in the graph if variable x_i appears in clause C_j.

* Given a Boolean formula Φ in 3-CNF such that its associated graph is planar, decide whether it is satisfiable is a *3-SAT problem*.

Figure: Caption Text

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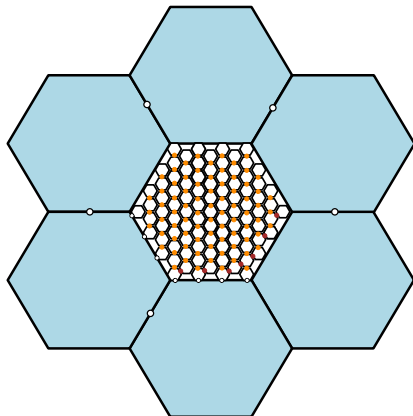

Figure: Caption Text

Transmitter Gadget

* A *transmitter gadget* is constructed for each edge $\{x_i, C_j\}$ of the graph $A(\Phi)$; it consists of a sequence of junctions and corridors from a variable gadget's junction to a clause junction.
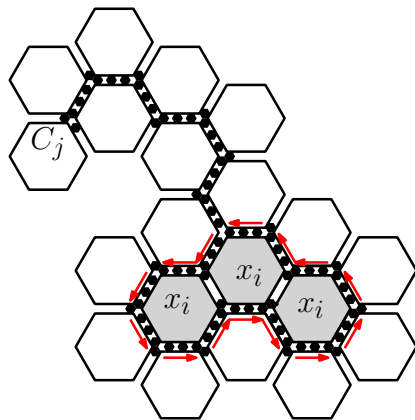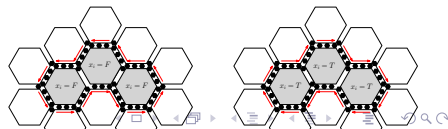

Figure: Caption Text

Variable Gadget

* Variable x_i corresponds to a cycle in the associated graph $\tilde{A}(\Phi)$, which has been embedded as a cycle in the hexagonal tiling, with corridors and junctions. In each junction along this cycle, attach a small hexagon in the common boundary of the two corridors in the cycle.
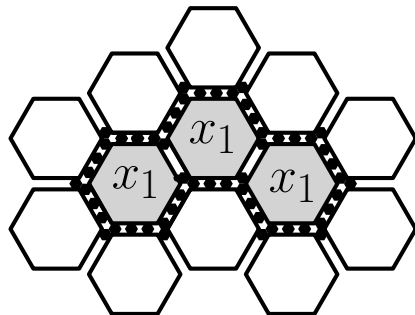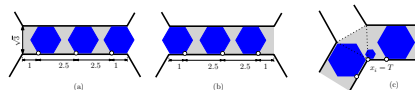

Figure: Caption Text

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Clause Junction Gadget

* The *clause gadget* lies at a
  junction adjacent to three
  transmitter gadgets (see Fig. **??**
  and Section **??**). At such a
  junction, we attach a unit line
  segment to an arbitrary vertex
  of the junction, and a small
  hexagon of side length $\frac{1}{3}$ to the
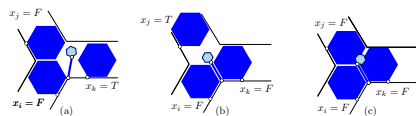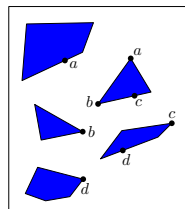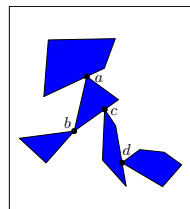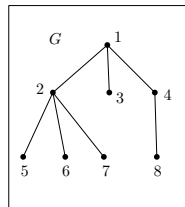 other end of the segment.

Figure: Caption Text

* Is it strongly NP-hard to decide whether a polygonal linkage whose hinge graph is a *tree* can be realized?

* Is it NP-Hard to decide whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with specified radii?
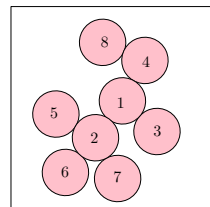

Figure: Caption Text

* The modified auxiliary gadget channels and junctions in a hexagonal grid enclosed by six frame hexagons.

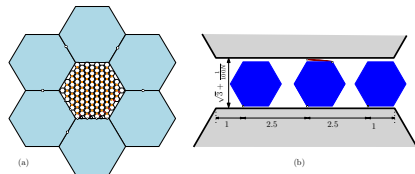* A comment about NAE3SAT....

Figure: Caption Text

Approximation of Hexagon with a Disk Arrangement

* A drawing of a tree T overlayed with a corresponding disk arrangement, each disk with unit radius.
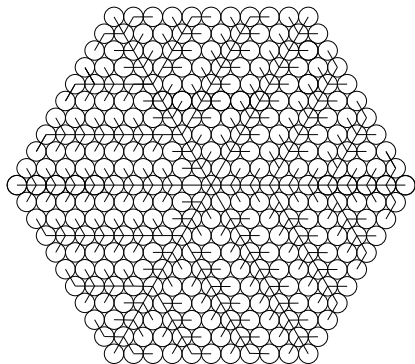
* The nodes of the tree are the centers of the disks.

Figure: A tree overlayed with a disk arrangement