Planar Configuration Spaces

of Disk Arrangements and Hinged Polygons

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* Contribution
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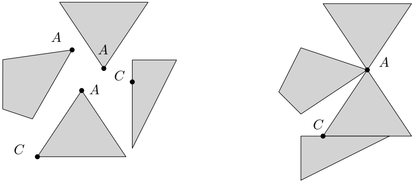
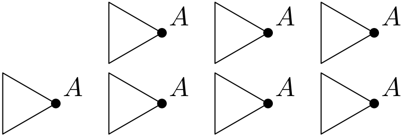
* Conclusion

Motivation: Hinged Dissection

* *Dudeney Problem*: Can a square and an equilateral triangle of the same area have a common dissection into four pieces?
* Can finite collection of polygons of equal area has a common hinged dissection?





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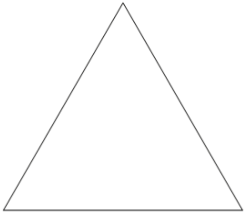
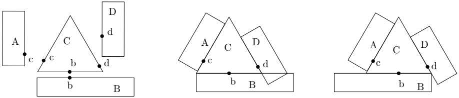


Motivation: Protein Folding

Protein folding is the process in which a protein chain acquires its 3-dimensional structure.

* Proteins in an organism fold into a specific geometric pattern (sometimes referred as its *native state*).
* Geometric patterns can determine a protein's function and behavior.

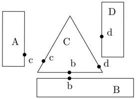


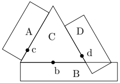
Deciding Realizability of Polygonal Linkages

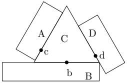
Problem

What is the complexity of deciding whether a polygonal linkage whose hinge graph is a tree can be realized?

Bhatt and Cosmadakis showed realizability of linkages is NP-Complete.







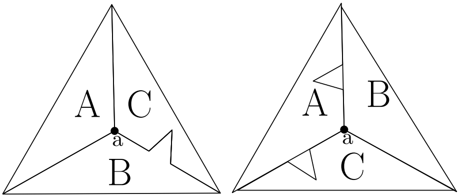
Deciding Realizability of Polygonal Linkages

Problem

What is the complexity of deciding whether a polygonal linkage whose hinge graph is a tree can be realized with fixed orientation?

Bhatt and Cosmadakis showed realizability of linkages is NP-Complete.

Here we have two realizations of a polygonal linkage with two different counter-clockwise order (C,B,A) and (B,C,A) respectively.

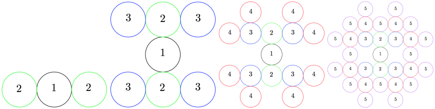


Deciding Realizability of Polygonal Linkages

Problem

What is the complexity of deciding whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with specified radii?

Breu and Kirkpatrick proved that it is NP-Hard to decide whether a graph G is the contact graph of unit disks in the plane, i.e., recognizing coin graphs is NP-Hard



Contributions

Theorem

It is strongly NP-Hard to decide whether a polygonal linkage whose hinge graph is a tree can be realized.

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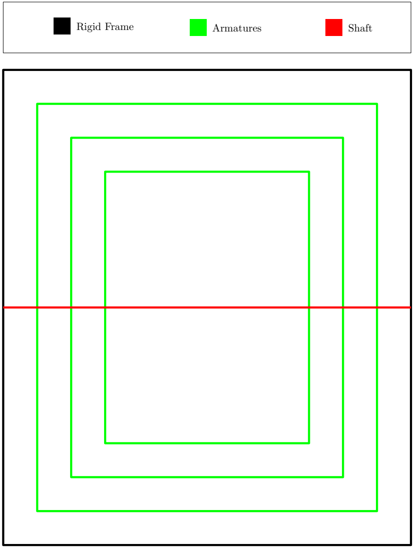
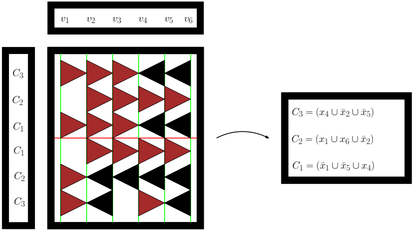
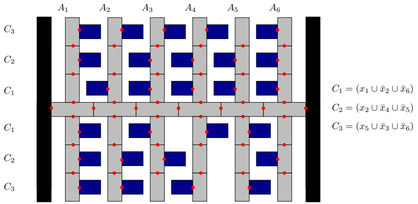
The Logic Engine





* Suppose we are given an Boolean formula with m clauses and n variables in 3-CNF form, Φ, we construct the polygonal linkage similarly to the logic engine.

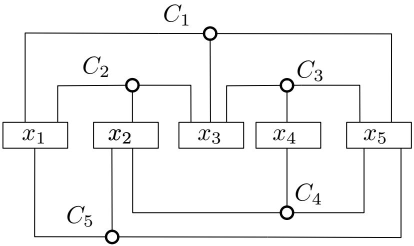


Logic Engine Realized as Hinged Polygons.pdf

* Breu and Kirkpatrick [**?**] proved that it is NP-hard to decide whether a graph G is the contact graph of unit disks in the plane, i.e., recognizing *coin graphs* is NP-hard; see also [**?**].

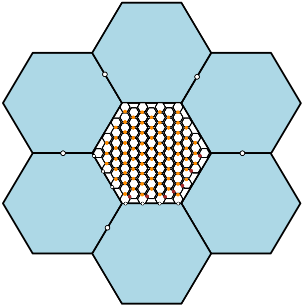
Planar 3SAT

* Given a Boolean formula Φ in 3-CNF such that its associated graph is $A(\Phi)$, decide whether it is satisfiable is a *3-SAT problem*.



Modified Auxiliary Construction

* Define the *associated graph* $A(\Phi)$ as follows: the vertices correspond to the variables and clauses in Φ. We place an edge in the graph if variable x_i appears in clause C_j.
* Given a Boolean formula Φ in 3-CNF such that its associated graph is planar, decide whether it is satisfiable is a *3-SAT problem*.



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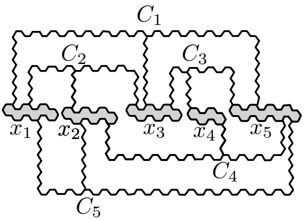
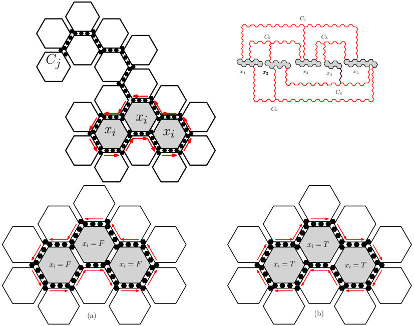


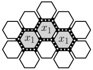
Transmitter Gadget

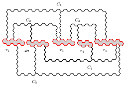
* A *transmitter gadget* is constructed for each edge $\{x_i, C_j\}$ of the graph $A(\Phi)$; it consists of a sequence of junctions and corridors from a variable gadget's junction to a clause junction.



Variable Gadget

* Variable x_i corresponds to a cycle in the associated graph $\tilde{A}(\Phi)$.





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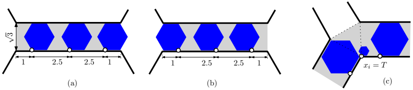
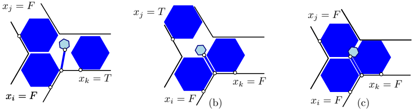
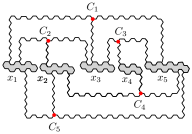


Clause Junction Gadget

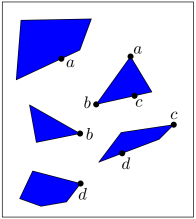
* The *clause gadget* lies at a junction adjacent to three transmitter gadgets.x



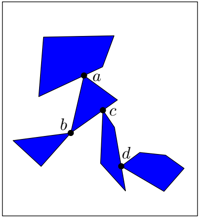


Motivation: Weighted Trees and Disk Arrangements

* Is it strongly NP-hard to decide whether a polygonal linkage whose hinge graph is a *tree* can be realized?
* Is it NP-Hard to decide whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with specified radii?



(a)



(b)

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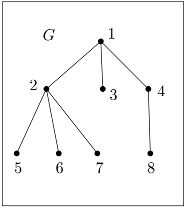

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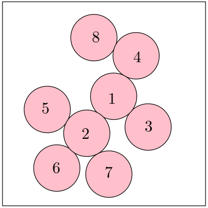
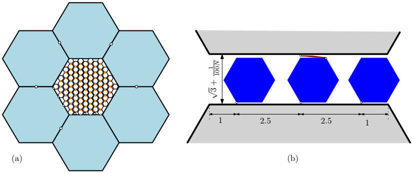


(b)

Modified Auxiliary Gadget

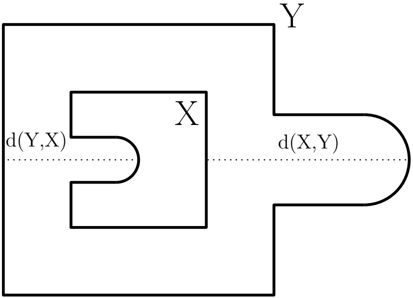
* The modified auxiliary gadget channels and junctions in a hexagonal grid enclosed by six frame hexagons.
* A comment about NAE3SAT....



(a) (b)

Approximation of Hexagon with a Disk Arrangement: Hausdorff Distance

* An illustrative example of $d(X, Y)$ and $d(Y, X)$ where X is the inner curve, and Y is the outer curve.



Approximation of Hexagon with a Disk Arrangement

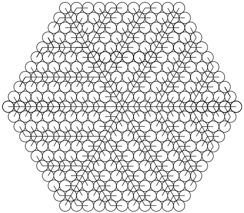
Lemma

For every $\epsilon > 0$ and $x > 0$, there exists an ordered weighted tree T and regular hexagon h of side length x such that:

* *Every realization σ_i of T as an ordered disk contact graph where the radii of the disks equal the vertex weights, approximates the hexagon in the sense that:*

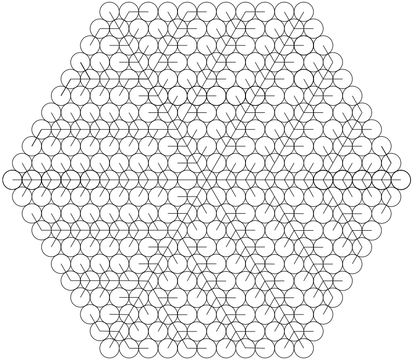
$$H\left(h, \sigma\right) \leq \epsilon$$

* *The number of nodes in T and the weights are polynomial in ϵ and x, the weights $\frac{\epsilon}{10}$ and $\frac{\epsilon}{10} + \zeta$ are polynomial.*



Approximation of Hexagon with a Disk Arrangement

* A drawing of a tree T overlayed with a corresponding disk arrangement, each disk with unit radius.
* The nodes of the tree are the centers of the disks.



Conclusion

Thank You!