Decidability Problem on Planar Protien Folding

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Abstract

We look into the decidability of whether a hinged configuration locks.

1 Introduction

We look into the decidability of continuity on planar configuration space using regular, unitary hexagonal polygons. These polygons can also represent unit disk configurations [1]

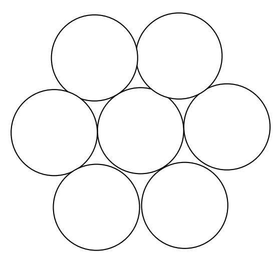

Figure 1: A locked 7 ball configuration

2 Background

Here we review some of the necessary mathematics behind the problem.

2.1 SAT Problems

Problem 2.1 (Satisfiability Problem). Let $\{x_i\}_{i=1}^n$ be boolean variables, and $t_i \in \{x_i\}_{i=1}^n \cup \{\bar{x}_i\}_{i=1}^n$. A *clause* is is said to be a disjuction of distinct terms:

$$t_1 \vee \cdots \vee t_{j_k} = C_k$$

Then the *satisfiability problem* is the decidability of a conjuction of a set of clauses, i.e.:

$$\wedge_{i=1}^m C_i$$

2.1.1 3-SAT Problems

A 3-SAT problem is a SAT problem with all clauses having only three boolean variables.

3 Linkages

Definition 3.1 (Linkage)**.** A collection of fixed-length 1D segments joined at their endpoints to form a graph.

Definition 3.2 (Graph)**.** An ordered pair $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines

Definition 3.3 (Cycle)**.** A closed walk with no repetitions of vertices or edges allowed, other than the repetition of the starting and ending vertex

Definition 3.4 (Configuration)**.** A specification of the location of all the link endpoints, link orientations and joint angles.

4 Circle Packing

Definition 4.1 (Intersection Graph)**.** Given a family of sets $\{S_i\}_{i=1}^{n}$, the intersection graph $G = (V, E)$ such that:

$$\begin{aligned} V &= \left\{ v_i \in \mathbb{R}^2 \,|v_i \text{ corresponds to } S_i \right\} \\ E &= \left\{ l_{i,j} \subset \mathbb{R}^2 \,|\text{if } S_i \cap S_j \neq \emptyset, \text{ then } l_{i,j} \text{ is an edge from } v_i \text{ to } v_j \right\} \end{aligned} \tag{1}$$

Theorem 4.1 (Circle Packing Theorem)**.** *For every connected simple planar graph G there is a circle packing in the plane whose intersection graph is (isomorphic to) G.*

4.1 Circle Packings and Polygonal Linkages

4.2 Hinged Polygons

4.2.1 Hinged Hexagons

Central Scaling

The Shapes

Junctions

Junctions in Conjunctive Normal Form Explain the configurations we're interested in.

4.2.2 Configurations and Locked Configurations

5 Problem

5.1 Problem Statement

text

5.2 Decidability of Problem

test

5.3 Hexagonal Locked Configuration

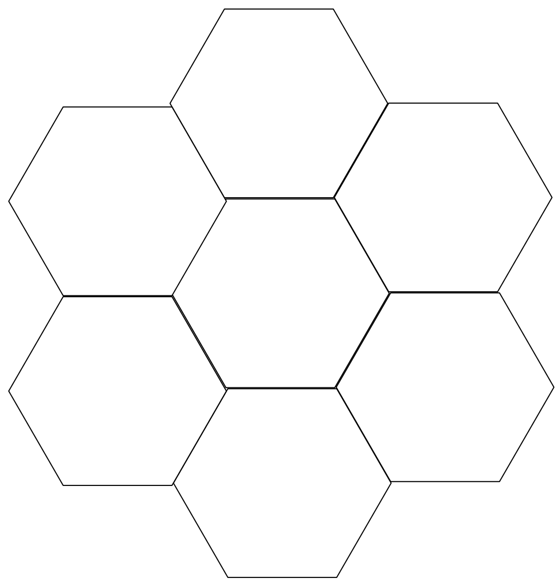

Figure 2: A locked 7 hexagonal configuration

6 Conclusion

We conclude..

References

[1] Heinz Breu and David G. Kirkpatrick. Unit disk graph recognition is np-hard. *Computational Geometry*, 9(12):3 – 24, 1998. Special Issue on Geometric Representations of Graphs.

[2] E.D. Demaine and J. O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*. Cambridge University Press, 2008.

[3] G.N. Frederickson. *Dissections: Plane and Fancy*. Cambridge University Press, 1997.

[4] K. Stephenson. *Introduction to Circle Packing: The Theory of Discrete Analytic Functions*. Cambridge University Press, 2005.