Planar Configuration Spaces

of Disk Arrangements and Hinged Polygons

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Motivation: Hinged Dissection

* *Dudeney Problem*: Can a square and an equilateral triangle of the same area have a common dissection into four pieces?
* Can finite collection of polygons of equal area has a common hinged dissection?

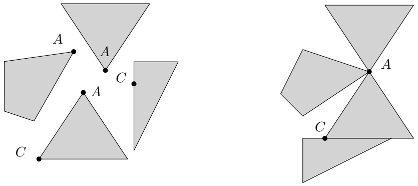

Figure: A collection of polygons with hinges.

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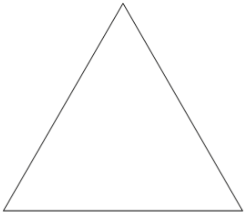

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Protein Folding

Protein folding is the process in which a protein chain acquires its 3-dimensional structure.

- Proteins in an organism fold into a specific geometric pattern (sometimes referred as its *native state*).
- Geometric patterns can determine a protein's function and behavior.

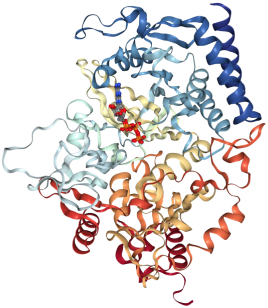
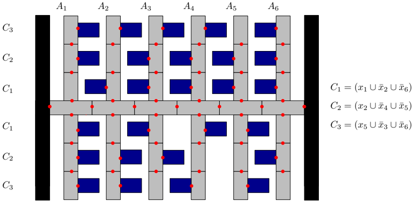


Figure: The structure of rat cytosolic PEPCK variant E89A in complex with oxalic acid and GTP [**?**].

Logic Engine Realized as Hinged Polygons.pdf

* Suppose we are given an Boolean formula with m clauses and n variables in 3-CNF form, Φ, we construct the polygonal linkage similarly to the logic engine.



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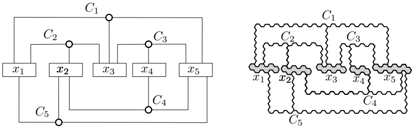
* Breu and Kirkpatrick [**?**] proved that it is NP-hard to decide whether a graph G is the contact graph of unit disks in the plane, i.e., recognizing *coin graphs* is NP-hard; see also [**?**].

* Define the *associated graph* $A(\Phi)$ as follows: the vertices correspond to the variables and clauses in Φ. We place an edge in the graph if variable x_i appears in clause C_j.
* Given a Boolean formula Φ in 3-CNF such that its associated graph is planar, decide whether it is satisfiable is a *3-SAT problem*.



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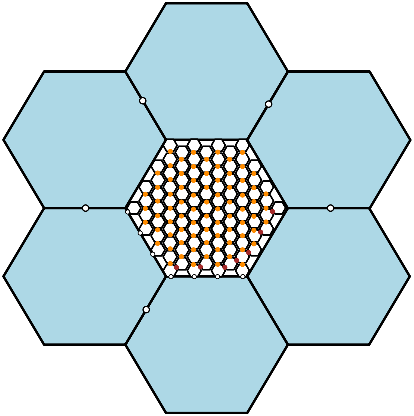

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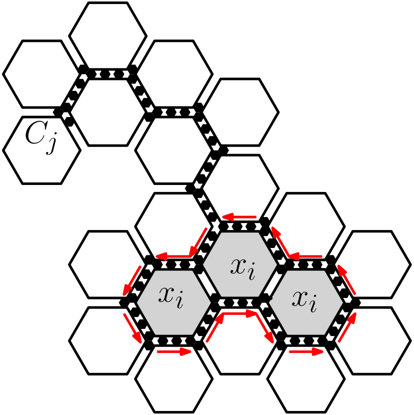
Transmitter Gadget

* A *transmitter gadget* is constructed for each edge $\{x_i, C_j\}$ of the graph $A(\Phi)$; it consists of a sequence of junctions and corridors from a variable gadget's junction to a clause junction.



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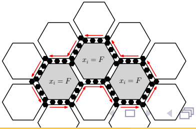
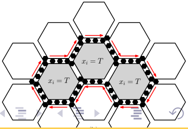
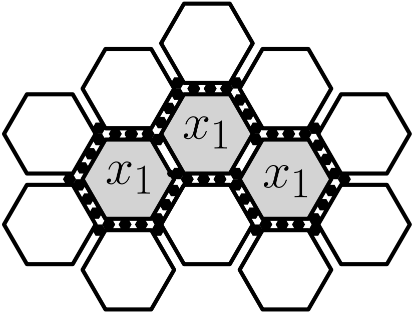




* Variable x_i corresponds to a cycle in the associated graph $\tilde{A}(\Phi)$, which has been embedded as a cycle in the hexagonal tiling, with corridors and junctions. In each junction along this cycle, attach a small hexagon in the common boundary of the two corridors in the cycle.



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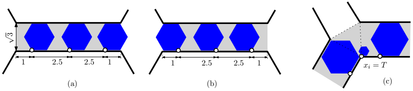

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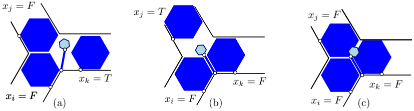
* The *clause gadget* lies at a junction adjacent to three transmitter gadgets (see Fig. **??** and Section **??**). At such a junction, we attach a unit line segment to an arbitrary vertex of the junction, and a small hexagon of side length $\frac{1}{3}$ to the other end of the segment.



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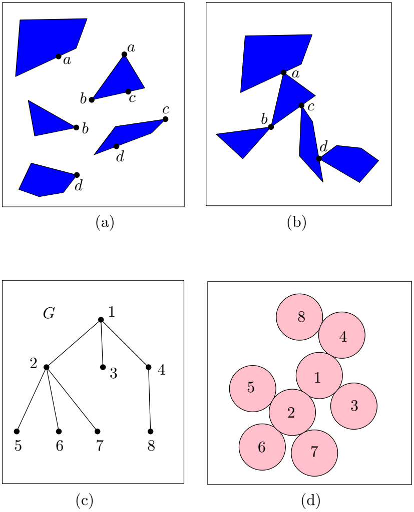
Motivation: Weighted Trees and Disk Arrangements

* Is it strongly NP-hard to decide whether a polygonal linkage whose hinge graph is a *tree* can be realized?
* Is it NP-Hard to decide whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with specified radii?



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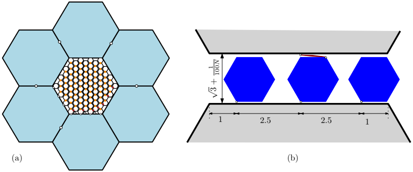
Modified Auxiliary Gadget

* The modified auxiliary gadget channels and junctions in a hexagonal grid enclosed by six frame hexagons.
* A comment about NAE3SAT....



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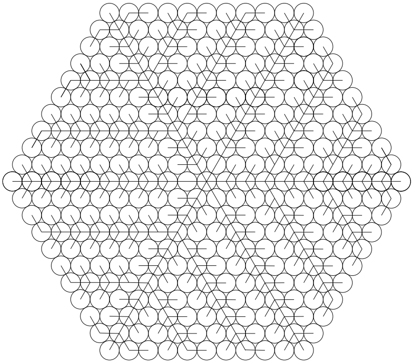
* item 1
* item 2



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honeycomb.pdf

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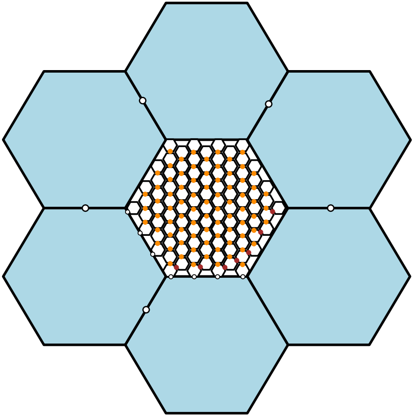

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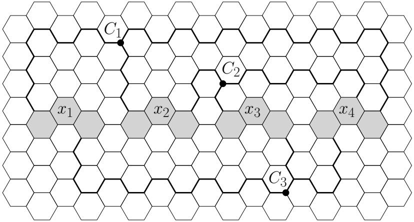

Figure: Caption Text

Problem

It is strongly NP-hard to decide whether a polygonal linkage whose inge graph is a *tree* can be realized.

It is strongly NP-hard to decide whether a polygonal linkage whose inge graph is a *tree* can be realized with fixed orientation.

It is NP-Hard to decide whether a given ordered tree with positive vertex weights is the contact graph of a disk arrangements with pecified radii.