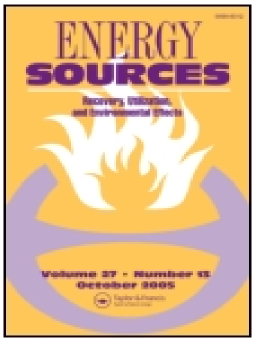

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Optimum Tilt Angle for Solar Collectors

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A solar collector is required to absorb solar radiation and transfer the absorbed energy into a heat transfer fluid with a minimum of heat loss. In assessing the performance of a collector, it is therefore important not only to determine its ability to absorb solar radiation but also to characterize its heat losses. The ability of a collector to absorb solar radiation is largely determined by its optical and geometric properties. One of the important parameters that affect the performance of a solar collector is its tilt angle with the horizontal. This is due to the fact that the variation in tilt angle affects the amount of solar radiation reaching the collector surface. In this study, a mathematical model is used to estimate the total (global) solar radiation on a tilted surface and to determine the optimum tilt angle for a solar collector in Izmir, Turkey. Total solar radiation on the solar collector surface with an optimum tilt angle is computed for specific periods. It is found that the optimum tilt angle changes between 0° (June) and 61° (December) throughout the year. In winter (December, January, and February) the tilt should be 55.7°, in spring (March, April, and May) 18.3°, in summer (June, July, and August) 4.3°, and in autumn (September, October, and November) 43°. The yearly average of this value was found to be 30.3° and this would be the optimum fixed tilt throughout the year.

Keywords solar collector, optimum tilt angle, clearness index, solar radiation

Turkey is geographically well situated with respect to a solar energy potential that is about 2,640 hours per year. Average daily solar energy density is 3.6 kWh/m^2. The total gross solar energy potential of Turkey is calculated to be about 88 mtoe (million tons of oil equivalents), of which 40% can be used economically. Three-fourths of the economically usable potential is efficient for thermal use and the remainder for electricity production. In 1999 and 2000, solar energy use accounted as 112 ktoe (kilo tons of oil equivalent) and 129 ktoe, respectively. It is expected to reach 431 ktoe in 2010 and 828 ktoe in 2020. The yearly average total solar radiation varies from a minimum value of 1,120 kWh/m^2 yr with 1,971 hours of sunshine per year in the Black Sea Region, to a maximum value of 1,460 kWh/m^2 yr with 2,993 hours of sunshine per year in the South East Anatolia (Ulgen and Hepbasli, 2002). Although Turkey has high solar energy potential, residential and industrial utilization of solar energy started in the 1980s. The residential sector accounted for the biggest share at about three times larger than the consumption in the industrial sector between 1986 and 2001 in Turkey. Solar collectors

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mounted in 2001 were 8.2 10^6 m^2, and total energy production related to this amount was 287 ktoe (Hepbasli et al., 2004).

A solar collector is required to absorb solar radiation and transfer the absorbed energy into a heat transfer fluid with a minimum heat loss. In assessing the performance of a collector it is therefore important both to determine its ability to absorb solar radiation and to characterize its heat losses. The ability of a collector to absorb solar radiation is largely determined by the optical and geometric properties of the solar collector. One of the important parameters that affect the performance of a solar collector is its tilt angle with the horizontal. It is generally known that in the northern hemisphere the optimum collector orientation is south facing ($\gamma = 0$) and the optimum tilt depends upon the latitude and the solar declination. Some researchers have given the optimum tilt angle as $\beta_{opt} = \phi \pm 15°$ in the northern hemisphere (Heywood, 1971; Lunde, 1980; Garg, 1982). However, Duffie and Beckman (1991), suggested this value as $(\phi \pm 15°) \pm 15°$. In some articles, different recommendations have been made for the optimum tilt angle depending on the latitude (Kern and Harris, 1975; Iqbal, 1979). Chiou and El-Naggar (1986), have given the optimum tilt angle in the heating season as about $\approx 30°$ ($\beta_{opt} = \phi + 10°$). Elsayed (1989) determined the optimum tilt angle and the surface azimuth angle for absorber plate containing using one and/or two glass covers. He reported that using two glass covers instead of one did not appreciably affect the value of optimum tilt angle, and has shown that changing tilt angle by about $\pm 10°$ around its optimum value reduces the amount of the monthly-absorbed radiation by less than 3%.

For maximum annual energy availability, the slope of the collector should be equal to the angle of latitude for low latitude countries ($\phi \leq 40°$), increasing the latitude by 10° for higher latitude countries ($\phi > 40°$) (Bari, 2000). In another study, the same author installed domestic solar water heaters on the roof with the same slope of the roof, and calculated that common slopes were between 30° and 40° (Bari, 2001).

In most studies different mathematical models were used (isotropic and anisotropic models) in the calculation of total radiation on tilted surface for measurements based on a horizontal surface (Willmott, 1982; Rowe and Willmott, 1984; Abdelrahman and Elhadidy, 1986; Chiou and El-Naggar, 1986; Gopinathan, 1990, 1991; Feuermann and Zemel, 1992; Ibrahim, 1995; Tiris and Tiris, 1998a, 1998b; Hartley et al., 1999; Yakup and Malik, 2001).

The main objective of this study is to determine the optimum tilt angle for a solar collector and the total solar radiation on a tilted surface in Izmir, Turkey. Total solar radiation on the solar collector surface with an optimum tilt angle is computed for specific periods (monthly, seasonal, and yearly).

Solar Data Measurements

The Solar-Wind Meteorological Station located on the roof of the Solar Energy Institute Building in Ege University is used to measure the local potentials of both solar and wind energy data. The station is fully operated with passive energy, and supplementary energy is not needed. Data collection was made at an interval of one second and hourly average values were recorded. The solar radiation measurement system was installed in October 1993, and the measurements have been carried out since then. The monthly-average daily values for extraterrestrial, global, and diffuse solar radiation on a horizontal surface and the monthly-average clearness index values for Izmir, Turkey are given in Table 1, and they are shown graphically in Figure 1 as well.

Table 1
Monthly-average daily values of H_o, H, H_D, K_T for Izmir, Turkey

Months	H_o (MJ/m^2 day)	H (MJ/m^2 day)	H_D (MJ/m^2 day)	K_T (−)
Jan.	16.20	7.35	3.20	0.45
Feb.	21.16	10.42	4.51	0.49
Mar.	28.12	14.48	6.29	0.51
Apr.	35.04	19.27	8.24	0.55
May	39.79	23.26	9.71	0.58
June	41.69	27.07	10.54	0.65
July	40.65	26.75	10.32	0.66
Aug.	36.71	23.38	9.22	0.64
Sep.	30.42	18.64	7.35	0.61
Oct.	23.18	13.08	5.06	0.56
Nov.	17.28	8.50	3.46	0.49
Dec.	14.69	6.07	2.71	0.41

Theoretical Analysis

Solar radiation is the most important parameter in the design of solar energy conversion systems. Solar radiation data are commonly available in two forms, the monthly average daily global solar radiation on a horizontal surface (H) and the hourly total radiation on a horizontal surface (I) for each hour for extended periods such as one or more years. The H data are widely available (Duffie and Beckman, 1991).

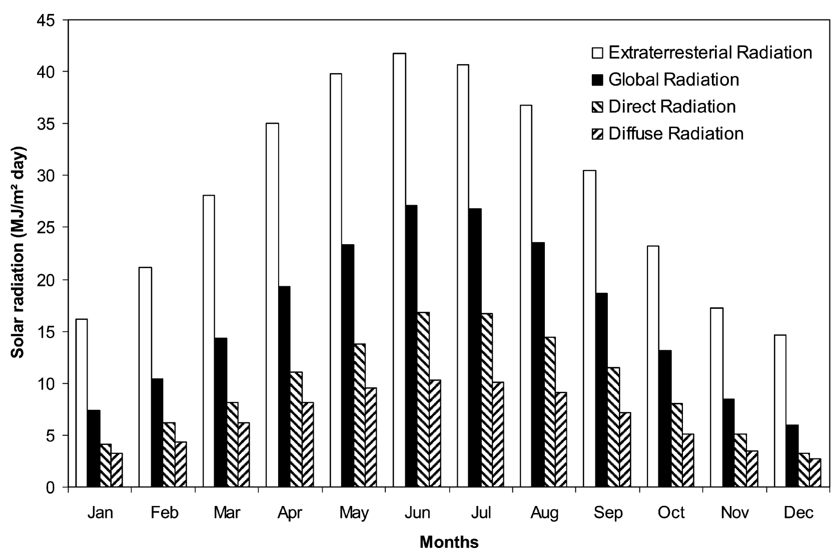


Figure 1. Monthly-average daily extraterrestrial, global, direct and diffuse solar radiation on horizontal surfaces in Izmir, Turkey.

The total solar radiation on a tilted surface (H_T) is made up of the direct or beam solar radiation (H_B), diffuse radiation (H_D), and ground reflected radiation (H_R). Thus, for a surface tilted at slope angle from the horizontal, the incident total solar radiation is

$$H_T = H_B + H_D + H_R. \tag{1}$$

The daily beam radiation received on an inclined surface may be expressed as

$$H_B = (H - H_D)R_b, \tag{2}$$

where H and H_D are the monthly-average daily global and diffuse radiation on a horizontal surface, and R_b is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month. R_b is a function of the transmittance of the atmosphere, which depends upon the atmospheric cloudiness, water vapor, and particulate concentration. However, Liu and Jordan (1960), have suggested that R_b can be estimated to be the ratio of extraterrestrial radiation on the tilted surface to that on a horizontal surface for each month. For a surface facing directly towards the equator,

$$R_b = \frac{\cos(\phi - \beta)\cos\delta\sin\omega_s' + (\pi/180)\omega_s'\sin(\phi - \beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + (\pi/180)\omega_s\sin\phi\sin\delta}, \tag{3}$$

where ω_s' is the sunset hour angle for the tilted surface given by

$$\omega_s' = \min\begin{bmatrix}\omega_s = \cos^{-1}(-\tan\phi\tan\delta\\ \cos^{-1}(-\tan(\phi - \beta)\tan\delta)\end{bmatrix}, \tag{4}$$

where "min" means the smaller of the two items in the bracket. Assuming isotropic reflection, the daily ground-reflected radiation may be written as

$$H_R = H\rho(1 - \cos\beta)/2, \tag{5}$$

where β is the tilt of the surface from horizontal, and ρ is the ground reflectance (≈ 0.2). As a consequence, the monthly-average daily solar radiation on a tilted surface, H_T, may be expressed as follows (Liu and Jordan, 1960):

$$H_T = (H - H_D)R_b + \frac{H_D}{2}(1 + \cos\beta) + \frac{H\rho}{2}(1 - \cos\beta). \tag{6}$$

If we simplify to this equation, the equation

$$H_T = RH = RK_TH_o \tag{7}$$

can be reached, where R is defined to be the ratio of the daily average radiation on a tilted surface to that of on a horizontal surface for each month. R can be estimated by considering individually the beam, diffuse, and ground reflected components of the radiation incidence on the tilted surface. If the diffuse and ground reflected radiations are assumed to be isotropic, then the ratio of the total insolation on a tilted surface to that on a horizontal surface can be expressed as follows (Liu and Jordan, 1960):

$$R = \left(1 - \frac{H_D}{H}\right)R_b + \frac{H_D}{2H}(1 + \cos\beta) + \frac{\rho}{2}(1 - \cos\beta). \tag{8}$$

The monthly-average clearness index (K_T) is the ratio of the monthly average daily radiation on a horizontal surface (H) to the monthly average daily extraterrestrial radiation (H_o) that is (Duffie and Beckman, 1991):

$$K_T = \frac{H}{H_o}. \tag{9}$$

The monthly average daily extraterrestrial radiation on a horizontal surface may be computed from the following equations:

$$H_o = \frac{24}{\pi} G_{on} \left(\cos\phi \cos\delta \sin\omega_s + \frac{\pi}{180} \omega_s \sin\phi \sin\delta \right), \tag{10}$$

and

$$G_{on} = G_{sc} \left(1 + 0.033 \cos \left(\frac{360 n_{day}}{365} \right) \right), \tag{11}$$

where G_{sc} is the solar constant (= 1,367 W/m^2), ϕ is the latitude of the site, δ is the solar declination, ω_s is the main sunshine hour angle for the month, and n_{day} is the number of the day of year starting from the first of January.

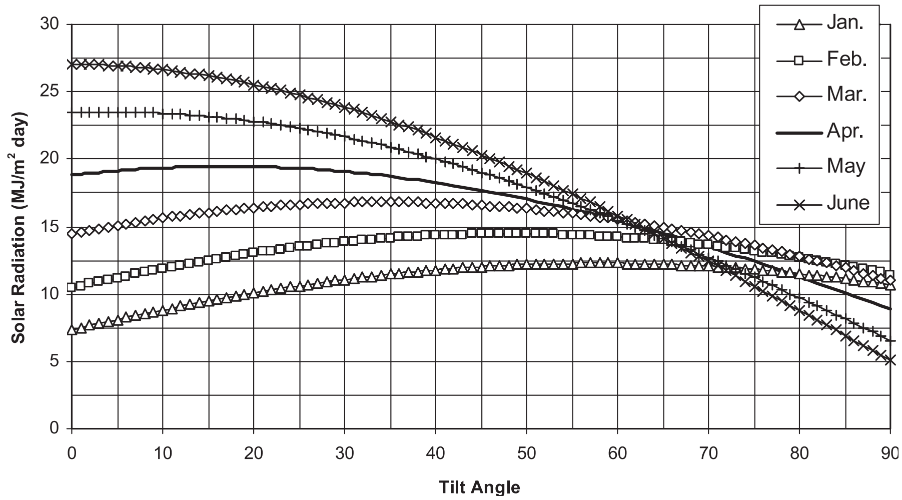
Results and Discussion

An isotropic model was developed using the above formula to calculate the monthly-average daily total radiation on a south-facing surface. Figure 2a and 2b show the average daily total solar radiation on a south-facing surface when the angle of the tilt varied from 0° to 90° in steps of 1°. It is clear from the figures that a unique optimum tilt angle exists for each month of the year that corresponds to the maximum point of each curve. Optimum tilt angle and the monthly-average daily total solar radiation on an optimum tilted surface are given in Table 2. It is found that the optimum tilt angle was changing between 0° (June) and 61° (December) throughout a year.

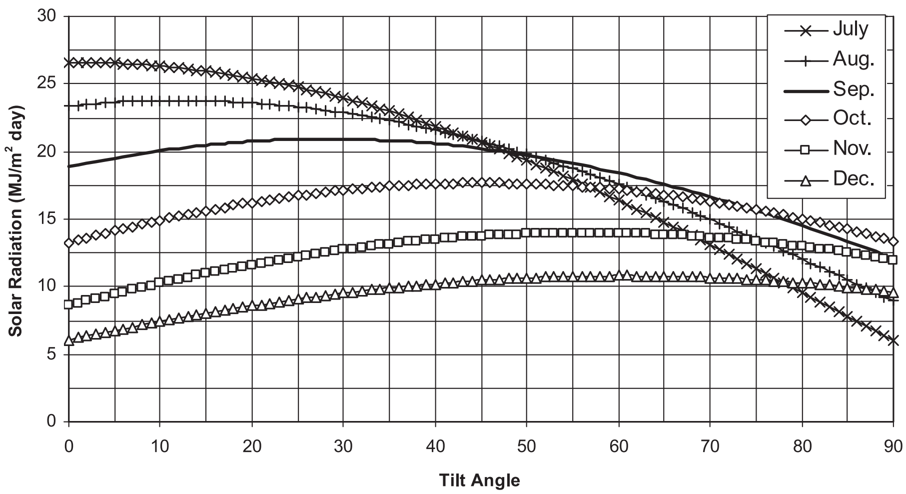
Table 3 and Figure 3 show the tilt angle for each month of the year when the solar collector is tilted at the optimum angle, the seasonal average, and the yearly average.

Table 2
Optimum tilt angle (β_{opt}) and monthly-average daily solar radiation on a tilted surface (H_{opt}) for each month of the year for a south-facing solar collector in Izmir, Turkey

Months	β_{opt} (°)	H_{opt} (MJ/m^2 day)	Months	β_{opt} (°)	H_{opt} (MJ/m^2 day)
Jan.	58	12.29	July	1	26.59
Feb.	48	14.45	Aug.	12	23.78
Mar.	34	16.81	Sep.	28	20.92
Apr.	17	19.46	Oct.	45	17.67
May	4	23.51	Nov.	56	13.98
June	0	27.07	Dec.	61	10.81



(a) January–June



(b) July–December

Figure 2. Monthly-average daily solar radiation availability of tilted surfaces.

The seasonal average was calculated by finding the average value of the tilt angle for each season, and the implementation of this requires that the collector tilt be changed four times a year. In winter (December, January, and February) the tilt should be 55.7°, in spring (March, April, and May) 18.3°, in summer (June, July, and August) 4.3°, and in autumn (September, October, and November) 43°. The yearly average tilt was calculated by finding the average value of the tilt angle for all months of the year. The yearly average of this value found to be 30.3° and this result points out an optimum fixed tilt throughout a year. Figure 4 shows the monthly average solar energy collected when the angle of the tilt is optimum, and when the seasonal and yearly average angles are used.

Table 3
Seasonal and yearly average tilt angle and monthly-average daily solar radiation on a tilted south-facing surface in Izmir, Turkey

		Seasonal average		Yearly average	
Months	Seasons	β (°)	H (MJ/m^2 day)	β (°)	H (MJ/m^2 day)
Dec.			10.77		9.54
Jan.	Winter	55.7	12.28		11.09
Feb.			14.36		13.86
Mar.			16.29		16.78
Apr.	Spring	18.3	19.45		19.05
May			22.94	30.3	21.61
June			26.96		23.80
July	Summer	4.3	26.55		23.88
Aug.			23.62		22.82
Sep.			20.38		20.91
Oct.	Autumn	43.0	17.66		17.16
Nov.			13.65		12.74

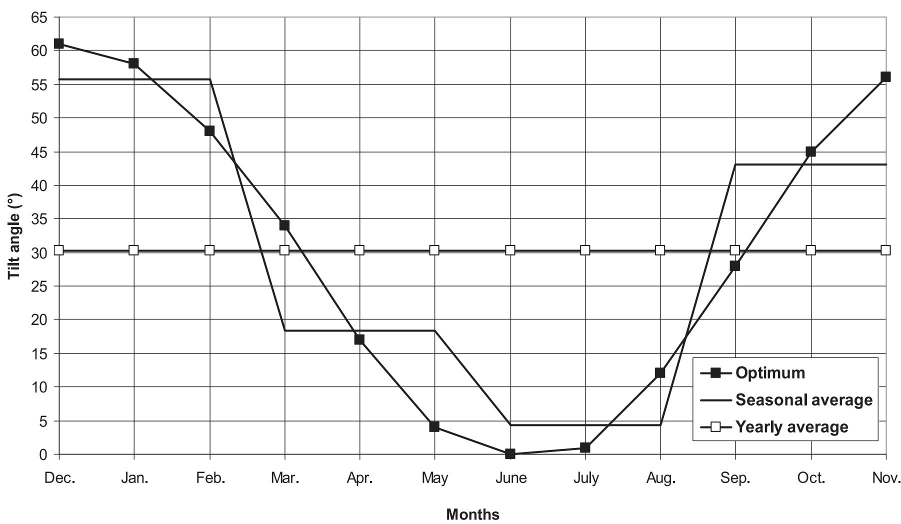


Figure 3. Optimum, seasonal average, and yearly average tilt angles for each month of the year.

Conclusions

In the light of the preceding results, the following conclusions can be drawn:

1. The average daily total solar radiation on a south-facing surface may be determined as the angle of the tilt is varied from 0° to 90° in steps of 1°.
2. The optimum tilt angle in June goes to a minimum of 0° and the monthly-average daily total radiation at this angle is 27.07 MJ/m^2 day. The optimum tilt angle then

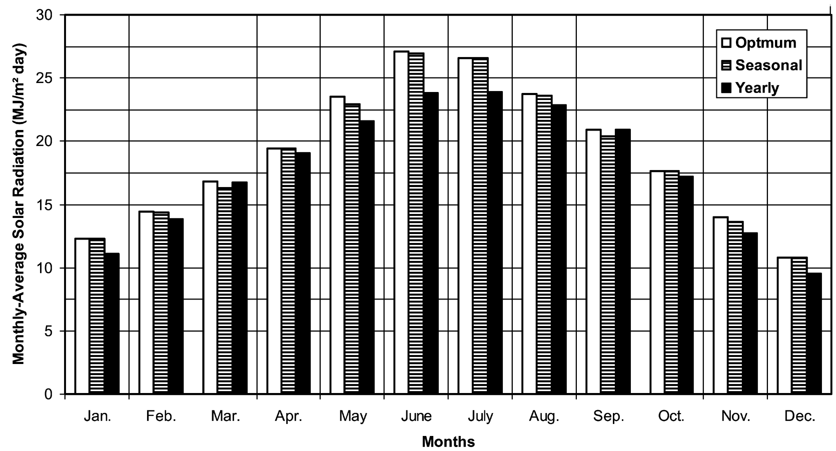


Figure 4. Monthly-average daily solar radiation for optimum, seasonal, and yearly tilt angles in Izmir, Turkey.

increases during the winter months and reaches a maximum of 61° in December, which collects 10.81 MJ/m^2 day of solar energy monthly. When the monthly optimum tilt angle used, the yearly collected solar energy was 6,820.36 MJ/m^2 year.

3. The results show that the average optimum tilt angle for the summer months is 4.3° ($\approx \phi - 34°$) and for the winter months 55.7° ($\approx \phi + 19°$). With the seasonally adjusted tilt angles, the yearly collected solar energy was 6,747.17 MJ/m^2 year.
4. Finally, the yearly-average optimum tilt angle found to be 30.3° for a south-facing solar collector. With the yearly average tilt angle, the yearly collected solar energy was 6,397.78 MJ/m^2 year.

Nomenclature

G_{on}	extraterrestrial radiation (W/m^2)
G_{sc}	solar constant (= 1,367 W/m^2)
H	monthly average daily global radiation on a horizontal surface (MJ/m^2 day)
H_B	monthly average daily direct radiation on a horizontal surface (MJ/m^2 day)
H_D	monthly average daily diffuse radiation on a horizontal surface (MJ/m^2 day)
H_o	monthly average daily extraterrestrial radiation on a horizontal surface (MJ/m^2 day)
H_R	monthly average daily ground reflected radiation on a horizontal surface (MJ/m^2 day)
H_T	monthly average daily total radiation on a tilted surface (MJ/m^2 day)
I	hour total radiation on a horizontal surface (MJ/m^2 day)
K_T	the monthly-average clearness index
n_{day}	number of the day of year starting from the first January

R the ratio of the daily average radiation on a tilted surfaces to that on a horizontal surface for each month
R_b the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month

Greek Letters

β tilt of the surface from horizontal (°)
δ solar declination (°)
ϕ latitude of site (°)
γ azimuth angle (°)
ρ ground reflectance (≈ 0.2)
ω hour angle (°)
ω_s sunshine hour angle for the month (°)
ω_s' sunset hour angle for the tilted surface (°)

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