Methodology for Optimally Sizing the Combination of a Battery Bank and PV Array in a Wind/PV Hybrid System

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Abstract- **In this paper a methodology for calculation of the optimum size of a battery bank and the PV array for a stand-alone hybrid Wind/PV system is developed. Long term data of wind speed and irradiance recorded for every hour of the day for 30 years were used. These data were used to calculate the average power generated by a wind turbine and a PV module for every hour of a typical day in a month. A load of a typical house in Massachusetts was used as a load demand of the hybrid system. For a given load and a desired Loss of Power Supply Probability, an optimum number of batteries and PV modules was calculated based on the minimum cost of the system.**

I. INTRODUCTION

The rapid depletion of fossil-fuel resources on a worldwide basis has necessitated an urgent search for alternative energy sources. Of the many alternatives, photovoltaic and wind energy have been considered as promising toward meeting the continually increasing demand for energy. The wind and photovoltaic sources of energy are inexhaustible, the conversion processes are pollution-free, and their availability is free. For remote systems such as radio telecommunications, satellite earth stations, or at sites that are far away from a conventional power system, the hybrid systems have been considered as attractive and preferred alternative sources [1-3]. Such systems are usually equipped with diesel generators to meet the peak load demand during short periods when there is a deficit of available energy to cover the load demand.

Diesel generators need their control of operation and supply in fuel. Therefore, the cost of their usage is reasonably expensive.

95 SM 460-6 EC A paper recommended and approved by the IEEE Energy Development and Power Generation Committee of the IEEE Power Engineering Society for presentation at the 1995 IEEE/PES Summer Meeting, July 23-27, 1995, Portland, OR. Manuscript submitted July 22, 1994; made available for printing April 27, 1995.

To eliminate the need of a diesel generator, a battery bank can be used. The configuration of a hybrid Wind/PV system is shown in Fig.1. Battery life is greatest when batteries are kept at near 100% of their capacity, or returned to that state quickly after a partial or deep discharge.

The use of photovoltaic modules only does not protect batteries against deep discharges. During periods of little or no sunshine, the load draws more energy than the photovoltaics can replace. A more dynamic source of energy is a wind turbine. Adding a wind turbine to a system would protect batteries against deep discharges and thus extend their life.

For stand alone applications, storage cost still represents the major economic restraint. Combining both wind power and PV power would lead to minimizing the storage requirements, and hence the overall cost of the system.

For a given load characteristic and a given wind turbine, an algorithm was developed to calculate the optimum number of PV modules and batteries that would achieve a desired Loss of Power Supply Probability.

In this paper 30 years of long term wind speed and solar irradiance data were used. These data were recorded for every hour for 30 years. There are 930 irradiance level and wind speed data for every hour of a typical day in a month, considering a 31-day-long month.

The algorithm is based upon the use of long term data for both wind speed and irradiance for the site under consideration and the energy concept. The variability in the available energy form the Wind/PV system makes it necessary to choose a right size of a battery bank that the system will satisfy the load demand at any hour of a typical day.

Wind turbine and PV power output are matched to a given load demand that is a load of a typical house in Massachusetts. For every hour of a typical day in each month, power output of both a wind turbine and a PV module were calculated. Then for a given Loss of Power Supply Probability, the combinations of a number of PV modules and a number of batteries were calculated. The choice of the optimum number of PV modules and batteries was based on the minimum cost of the system.

The developed methodology is described in steps.

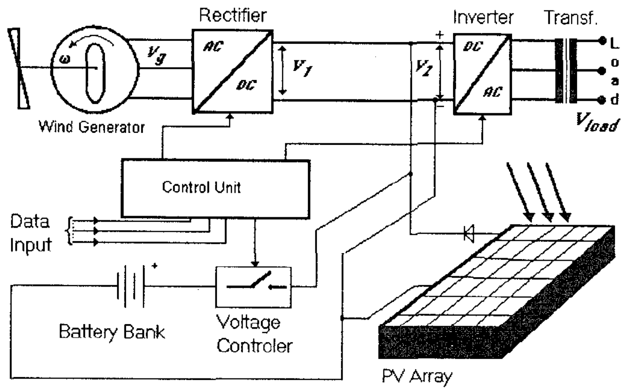

Fig. 1. Wind/PV Hybrid Stand-Alone System

II. METHODOLOGY

A. Step One: Calculation of the Power Output from Wind Turbine

The speed of wind is a random process; therefore it should be described in terms of statistical methods. The wind speed data were recorded near the ground surface. To upgrade wind speed data to a particular hub height, the following equation is commonly used [4]:

$$v = v_i \cdot \left(\frac{H}{H_i}\right)^{\alpha} \tag{1}$$

where: v-wind speed at projected height, H

v_i-wind speed at reference height, H_i

α- power-law exponent (~ 1/7 for open land).

The wind speed distribution is assumed to be a Weibull distribution. Hence the probability density function (pdf) is given by [5]:

$$f(v) = \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{2}$$

where: c - scale factor, unit of speed

k -shape factor, dimensionless

v -wind speed.

The wind speed distribution functions were calculated for each hour of a typical day in every month. An example of the wind speed probability density function calculated at the hub height is shown in Fig.2.

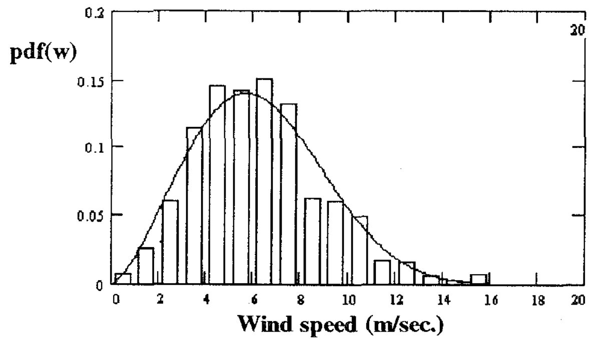

Fig.2. Wind speed histogram and the matched Weibull probability density function plot for a typical day in December at 9:00 a.m.

Once distribution functions for wind speed were calculated for every hour of a typical month, the average power output for every hour of the typical day in each month can be easily calculated using the following equation [6]:

$$P_{w,avg} = \int_0^{\infty} P_w \cdot f(v) \cdot dv \tag{3}$$

where: $f(v)$ is a probability density function given by (2)

P_w is the electrical power output of the turbine.

The available wind generator power output is a function of the wind velocity v [6]:

$$P_w = \begin{cases} P_R \cdot \dfrac{v^k - v_c^k}{v_R^k - v_c^k} & \text{for } v_c \le v \le v_R \\ P_R & \text{for } v_R < v < v_F \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where: P_R is the rated electrical power,

v_c is the cut-in wind speed,

v_R is the rated wind speed,

v_F is the cut-off wind speed,

k is the Weibull shape parameter.

The plot of P_w versus wind velocity v is illustrated in Fig. 3.

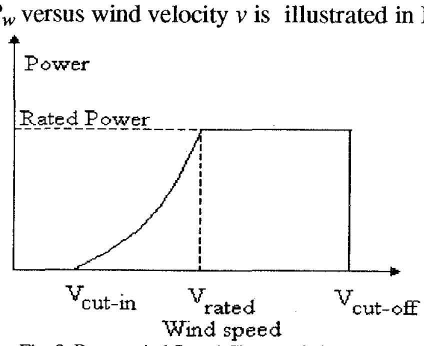

Fig. 3. Power wind Speed Characteristic.

B. Step Two: Calculation of the PV Module Probability Density Functions

For the same hour of a typical day the irradiance data have a bimodal distribution function. Therefore the irradiance data for every hour were divided into two groups, each of them having their own unimodal distribution function. Each of these two groups was matched to one of the three most common probability distribution functions, i.e., Beta, Weibull and Log-Normal. Chi-Square and Kolmogorov-Smirnov tests were used to select the distribution function that best fits the histogram data. An example of the histogram data and the distribution functions is shown in Fig. 4. The irradiance probability density functions for every hour of the typical day in each month were calculated using the methodology described in [6].

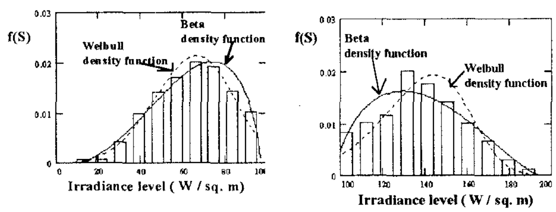

Fig. 4. Histogram and probability density functions plots versus irradiance level for a typical day of December at 9:00 a.m.

C. Step Three: Calculation of the PV Module Average Power Output

The power output of the PV module $P(S)$ is a product of the module output voltage and output current. The equivalent circuit of a PV module used is shown in Fig. 5.

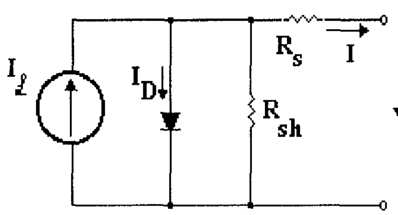

Fig. 5. Equivalent Circuit of a Module.

In the calculation of the power output of a module, we assumed that a maximum power point tracker will be used. Manufacturers of PV modules supply information on the maximum power point voltage and current at reference temperature and reference irradiance. The module equivalent circuit output current I can be expressed as a function of the module output voltage V, as follows [6]:

$$I(V) = I_{sc}\left\{1 - C_1\left[\exp\left(\frac{V+\Delta V}{C_2 V_{oc}}\right) - 1\right]\right\} + \Delta I \tag{6}$$

where:

$$C_2 = \frac{V_{mp}/V_{oc} - 1}{\ln\left(1 - I_{mp}/I_{sc}\right)} \tag{7}$$

$$C_1 = \left(1 - I_{mp}/I_{sc}\right)\cdot \exp\left[-V_{mp}/\left(C_2 \cdot V_{oc}\right)\right]$$

$$\Delta I = \alpha\left(S/S_{ref}\right)\Delta T + \left(S/S_{ref} - 1\right)\cdot I_{sc}$$

$$\Delta V = -\beta \cdot \Delta T - R_s \cdot \Delta I$$

$$\Delta T = T - T_{ref}$$

$$T = T_A + 0.02 \cdot S$$

α Current change temperature coefficient at reference insolation (Amps/°C),
β Voltage change temperature coefficient at reference insolation (Volts/°C),
I Module Current (Amps),
I_{mp} Module Maximum Power Current (Amps),
I_{sc} Module Short Circuit Current (Amps),
S Total Tilt Insolation,
S_{ref} Reference Insolation,
R_s Module Series Resistance (Ohms),
T Cell Temperature (°C),
T_A Ambient Temperature (°C),
T_{ref} Reference Temperature (°C),
ΔT Change in Cell Temperature (°C),
V Module Voltage (Volts),
V_{mp} Module Maximum Power Voltage (Volts),
V_{oc} Module Open Circuit Voltage (Volts),

The I-V characteristics of a module for different insolation levels and different temperatures are shown in Fig. 6.

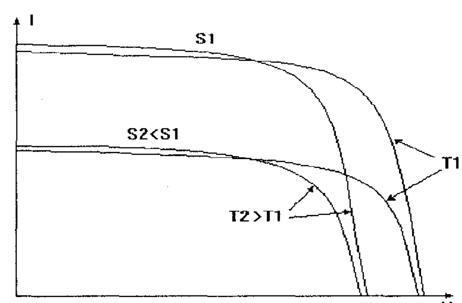

Fig. 6. I-V Characteristics of a Solar Module for different insolation levels S1 and S2 with two different ambient temperatures T1 and T2.

The average power output from a PV module was calculated using the following equation in the integral form [6]:

$$P_{pv,avg} = \int P(S) \cdot f(S) \cdot dS \qquad (5)$$

where: *f(S)* - irradiance probability density function. calculated in Step Two

The PV module power output was calculated for every hour of a typical day in every month.

D. Step Four: Calculation of the Number of Batteries in the Battery Bank

1) Defining Loss of Power Supply Probability (LPSP) in Terms of Battery State of Charge.

The calculation of the optimum number of photovoltaic modules and batteries was based upon a Loss of Power Supply Probability concept [7,8] and the economics of the system.

Loss of Power Supply Probability can be defined as the long-term average fraction of the load that is not supplied by a stand-alone system. A LPSP of 0 means the load will be always satisfied, and the LPSP of 1 means that the load will never be satisfied [8].

The load will not be satisfied when the power generated by both the wind turbine and the PV array is insufficient and the storage is depleted and its voltage has fallen below the allowed value. Energy is stored in batteries when the generated power by the wind turbine and PV array is greater than the load. When the power generated is less than the load, the energy is taken from the batteries. The state of charge of the batteries at any time t_1 depends on the state of charge in the previous moment t_0 and the sequence of generated power and load demand levels in the time interval t_1-t_0.

The state of charge of the batteries was used as a decision variable for the control of the overcharge and discharge. The case of overcharge may occur when high power is generated by the photovoltaics and wind turbine, or when low load demand exists. In such a case when the state of charge of the batteries reaches the maximum value, B_{max}, the control system intervenes and stops the charging process. On the other hand, if the state of charge decreases to a minimum level, B_{min} , the control system disconnects the load. This is important to prevent batteries against shortening their life or even their destruction [7].

In terms of state of charge of batteries, the Loss of Power Supply Probability can be therefore defined as:

$$LPSP = \Pr\{E_B(t) \le E_{B_{\min}};\ \text{for } t \le T\} \qquad (8)$$

where:

$E_B(t)$ - energy stored in batteries at any time t,

E_{Bmin} - battery minimum allowable energy level,

i.e. the probability of the state of charge at any accumulative time t, within the time period T, to be less or equal than the minimum level EB_{min}.

2)Simulation Model

The performance of batteries is complicated and cannot be precisely predicted for uncontrolled charge/discharge cycles in stand-alone systems. It is difficult to measure separate charge and discharge efficiencies. Manufacturers usually specify a round-trip efficiency and a maximum depth-of-discharge. Moreover, the battery capacity is defined in terms of the amount of energy that can be extracted, not the amount that is actually stored [8]. Therefore, the battery charge efficiency was set equal to the round-trip efficiency, and the discharge efficiency was set equal to 1.

The inverter is rated in terms of the peak load demand. The efficiency of the inverter is a function of the ratio of actual load to the inverter's rating. In this paper, we used a constant value of inverter efficiency based on the average load demand. The error introduced with such an approach is negligible in the calculation of the LPSP of the system.

Both the maximum power point tracker (MPPT) and the battery controller draw a small amount of power. We assumed that both have constant efficiencies.

3) Derivation of the Simulation Model.

The energy generated by wind turbine and PV array for hour t , $E_{G(t)}$ can be expressed as follows:

$$E_{G(t)} = E_{w(t)} + N_{PV} \cdot E_{PV(t)} \qquad (9).$$

where: $E_{w(t)}$ - energy generated by wind turbine,

$E_{PV(t)}$ - energy generated by a PV module,

N_{PV} - number of PV modules in a PV Array.

Since we assumed that the battery charge efficiency is set equal to the round-trip efficiency and the discharge efficiency is set equal to 1, we considered two cases in expressing current energy stored in the batteries for hour t .

If the generated energy from the wind turbine and PV array exceeds that of the load demand, the batteries will be charged with the round-trip efficiency:

$$E_{B(t)} = E_{B(t-1)} + \left(E_{G(t)} - E_{L(t)} / \eta_{inv}\right) \cdot \eta_{batt,in} \quad (10).$$

where:

η_{inv} - efficiency of the inverter,

$\eta_{batt,in}$ - round-trip efficiency of the batteries,

$E_{B(t)}$ - energy stored in batteries in hour t,

$E_{B(t-1)}$ - energy stored in batteries in previous hour,

$E_{L(t)}$ - load demand in hour t.

When the load demand is greater than the available energy generated, the batteries will be discharged by the amount that is needed to cover the deficit. It can be expressed as follows:

$$E_{B(t)} = E_{B(t-1)} - \left(E_{L(t)} / \eta_{inv} - E_{G(t)}\right) \qquad (11).$$

The energy stored in batteries at any hour t is subject to the following constraint:

$$E_{B\min} \le E_{B(t)} \le E_{B\max} \qquad (12).$$

That means that the batteries should not be overdischarged or overcharged at any time. That protects batteries from being damaged.

When the available energy generated and stored in batteries is insufficient to satisfy the load demand for hour t, that deficit called Loss of Power Supply for hour t can be expressed as:

$$LPS_{(t)} = E_{L(t)} - (E_{G(t)} + E_{B(t-1)} - E_{B\min}) \cdot \eta_{inv} \qquad (13).$$

The Loss of Power Supply Probability for a considered period of time T is the ratio of all LPS_t values for that period to the sum of the load demand. This can be defined as:

$$LPSP = \frac{\sum_{t=1}^{T} LPS_t}{\sum_{t=1}^{T} E_{L(t)}} \qquad (14).$$

Once the available energy generated from both a wind turbine and a PV module was determined in the previous steps for every hour of a typical day in each month, different combinations of the number of PV modules and the number of batteries could be calculated for a desired LPSP.

The program for calculation of the number of batteries and PV modules was written in PASCAL programming language. A flowchart diagram for this program is shown in Fig. 7.

The input data for this program consists of:

- power generated by the wind turbine for every hour of a typical day in each month calculated in step one,
- power generated by a single PV module for every hour of a typical day in each month calculated in step three,
- depth of discharge of chosen batteries δ,
- round-trip efficiency of the batteries η_{batt},
- capacity of a single battery, C_{batt}.

desired long-term LPSP of the hybrid stand-alone system.

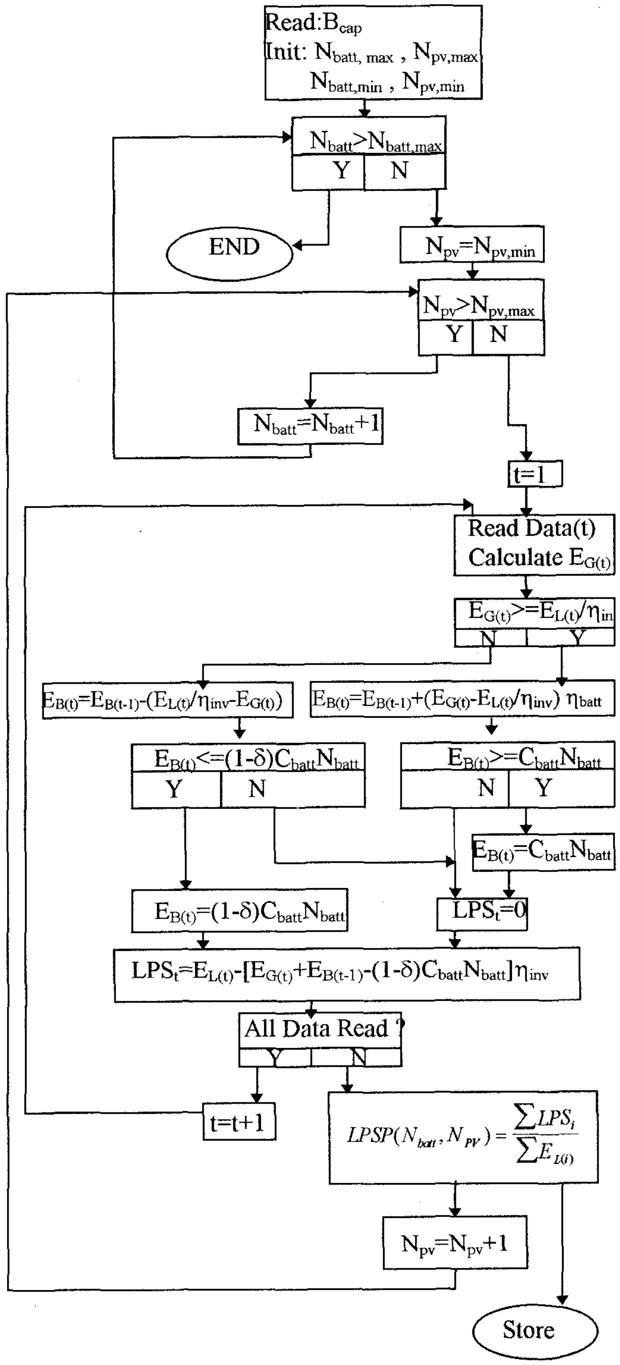

Fig. 7. Flowchart Diagram for the calculation of LPSP.

4) Determination of the Optimum Number of Batteries and PV Modules Based on the Economic Approach

For a given LPSP the number of PV modules as a nonlinear function of the number of batteries was determined in the former point of this step. An example plot of the number of PV modules versus number of batteries is shown in Fig. 8.

It is necessary to determine a PV/battery combination that yields a minimum cost of the system. The cost function of the system can be defined as follows [7]:

$$C = \alpha \cdot N_{PV} + \beta \cdot N_{batt} + C_0 \qquad (15).$$

where:

C - capital cost of the hybrid system,
α - cost of a PV module,
β - cost of a battery,
C_0 - the total constant costs including the cost of design, installation, and a wind turbine.

The condition to obtain the optimum solution of (15) yields:

$$\frac{\partial N_{PV}}{\partial N_{batt}} = -\frac{\beta}{\alpha} \qquad (16).$$

The solution of (16) is graphically illustrated in Fig. 8. The inclination of the line equal to $(-\beta/\alpha)$ is equal to that of the curve in point S.

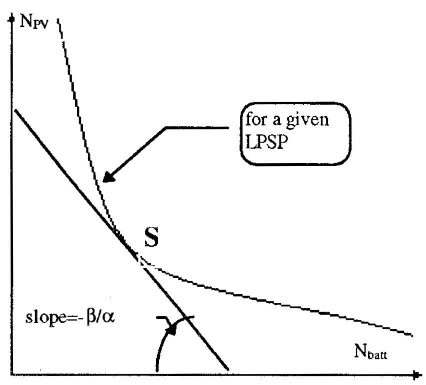

Fig. 8. Plot of Number of PV modules versus Number of Batteries for a given LPSP.

III. ANALYSIS, RESULTS, AND DISCUSSION

The developed method was used to calculate the optimum number of batteries and PV modules for a stand-alone wind/PV hybrid system that has been built on the campus of the University of Massachusetts Lowell. Irradiance and wind speed data for Logan Airport-Boston obtained from the National Solar Radiation Data Base were utilized. The Bergey BWC 1500 type wind turbine of rated power 1.5kW was considered. The rated power of a PV module used was $53W_{peak}$. The capacity of a single battery used was 100Ah. That battery has a round-trip efficiency of 0.85 and 80% of the depth of discharge.

The LPSP was specified at the value of 1 day within 10 years, as recommended by the utility company. The load of a typical house in Massachusetts was also used. The load profile plot is shown in Fig. 9.

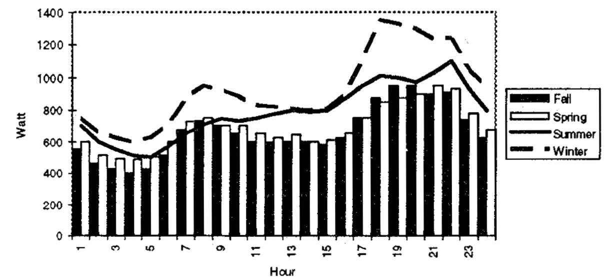

Fig. 9. Plot of load profile for all seasons.

The highest load demand values occur in winter. Therefore, the tilt angle of the PV array was set at 63°27', which is the optimum tilt angle for winter for the considered site.

With the use of the program described in the former section, we calculated a series of possible combinations of the of the number of PV modules and batteries. For a given unit price of batteries and PV modules, an optimum solution that minimizes the cost of the system was found. A plot of the number of PV modules versus the number of batteries for a given LPSP is shown in Fig. 10. On the same plot, the cost function line is shown. The optimum numbers of PV modules and batteries from the point of tangency are 45 and 80 respectively, as indicated in Fig. 10.

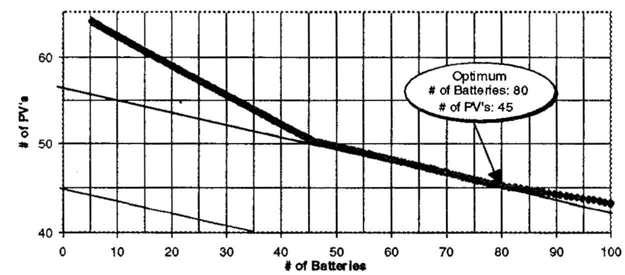

Fig. 10. Plot of the number of PV modules versus the number of batteries for a given LPSP with the optimum solution.

For the found optimum number of batteries and PV modules, we calculated the state of charge of the batteries for a year time period with the use of the data for every hour of a typical day in each month. The state of charge of batteries is plotted in Fig. 11. The plot of the available energy i.e., the generated energy minus load demand, is shown in Fig. 12.

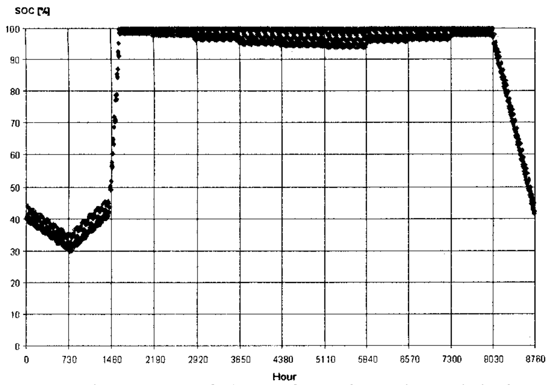

Fig. 11. State of Charge of Batteries on the yearly basis.

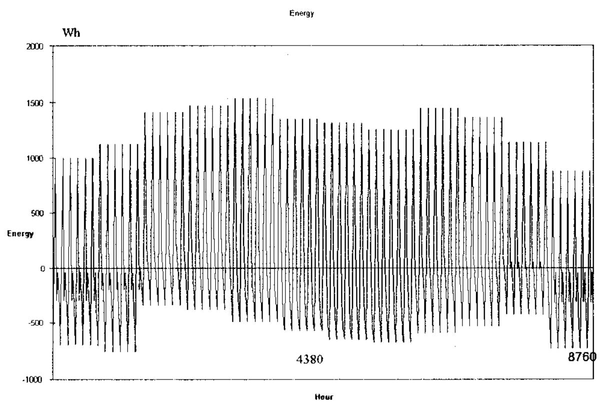

Fig. 12. Measured Available Energy to the Batteries.

IV. CONCLUSION

A methodology for calculation of the optimum size of a battery bank and the optimum size of a PV array in a hybrid wind/PV system for a given load and level of reliability was demonstrated. It is based on the use of long term data for both wind speed and irradiance for the site under consideration.

The average power outputs of both the wind turbine and the PV module were calculated. For a given Loss of Power Supply Probability, a different combination of the number of PV modules and the number of batteries was calculated. An optimum design choice depends on the relative costs of a PV module and a battery. We assumed that total cost of the system is linearly related to both the number of PV modules and the number of batteries. The minimum cost will be at the point of tangency of the line cost and the curve that represents the relationship between the number of PV modules and the number of batteries.

The optimum mix of PV modules and batteries depends on the particular site, load profile, and the desired reliability of the hybrid system.

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DISCUSSION

Lambert Pierrat, Senior Member, IEEE (Electricité de France, General Technical Division, 37 Rue Diderot, 38040 Grenoble, France) and **Yaw-Juen Wang**, Member IEEE (National Yun-Lin Institute of Technology, Taiwan). Authors present an interesting methodology, because it allows to study technico - economic aspects of a hybrid system with two renewable energy sources (wind generator and PV modules) and an electrochemical storage system (battery bank). However, results obtained with the proposed method, depend both on the quality of the simulation model and the representativity of functional data. Next questions have for purpose to obtain precisions and necessary complements.

1. Power output from wind turbine: The parameter (k) intervenes simultaneously in expressions (2) and (4), what seems curious, because they characterise two very different aspects of the problem. On the one hand, the wind speed distribution is a functional statistical data and in the pdf (2), (k) is the Weibull shape parameter. This parameter translates the relative dispersion of observed wind speeds. In general one knows that k=2, this is to said that the Weibull distribution degenerates in a Rayleigh distribution . On the other hand, (4) is the intrinsic power characteristic of the wind generator; in the wind speed range bounded by (v_c) and (v_r), the aerodynamic power varies habitually as: $[(v - v_c)/(v_c - v_r)]^{k'}$, in this expression k'=3 and the real speed (v) is transformed by translation (location parameter v_c) and by standardization (scale parameter $v_c - v_r$).

1.1 Why the Weibull shape parameter (k) does it intervene explicitly in the aerodynamic characteristic of the wind generator?

1.2 By observing the histogram in fig. 2, it is evident that k > 2 (because of pdf behaviour to the vicinity of the origin) and that k < 3.6 (because of the positive skewness of the pdf). Is it possible to justify the numerical equality of two parameters (k=k'=3) in expressions (2) and (4)?

1.3 It is possible that my interpretation of the expression (4) is not correct. Do can authors justify more clearly the genesis of this expression ?

1.4 To note that a typographic error undertakes the equation (1): it is necessary add the power law exponent (α) taking into account the propeller height in open-land.

2. PV Module average power output: Eqs. (6, 7) and figs. (5, 6) show that authors use an accurate parametric model of the PV generator. It results some that the electrical power is a nonlinear function of two functional parameters: the irradiance level (S) and the ambiant temperature (TA).

In the equation (5) the power P (S) vary only with (S) while fig. 6 show that the sensitivity to (TA) is significant.

2.1 In the simulation process, how are taken into account variations of the temperature, correlatively with those of the irradiance?

2.2 Would be it possible to simplify the PV generator model so as to use a constant average ambiant temperature?

2.3 To note that the numeration of equations could have be modified: (5, 6, 7) takes place (6, 7, 5).

3. Wind speed and irradiance level distributions: Statistical data are both precise (temporal sampling: 1 hr) and observed during a long time (30 years); it are therefore compatible with the study of a hybrid system whose characteristics are well defined: hourly cost profile (fig. 9) and important life duration (10 years). In the reality, the wind speed, the irradiance level, the ambiant temperature and the load curve are stochastic processes that are not necessarily mutually independent. Statistical approach adopted by authors consists in time sampling and to use valid hourly distributions for a typical day of each month; This method does not take account explicitly correlation between variables, since distributions are mutually independent.

3.1 Does a correlation exist really between the wind speed and the irradiance level?

3.2 If this correlation exists, what is the consequence of using mutually independent distributions, instead of jointly distributions?

3.3 Is the hourly duration step used to define distributions compatible with the correlation time of each process?

The fig. 9 show that load curves are different for each of 4 seasons; this supposes obviously that energy need of typical house depends on the climate, especially of the ambiant temperature; during the peak load in the morning, the PV generator operates with an ambiant temperature more raised in summer than winter.

3.4 Does a correlation exist between the irradiance level and the ambiant temperature?

3.5 If this correlation exists, is it taken into account indirectly by a hourly ambiant temperature?

4. Analysis of the studied case: The proposed method is illustrated by the study of a particular case. The aim is to define optimal characteristics for the hybrid system, capable to supply a typical house. The annual average power load is order of 0.8 kW and the supply safety correspond to a LISP of 1 day within 10 years. Numerical information contained in the paper, as well as the analysis of figs. (11,12) allow to summarise rated characteristics of 3 main components (optimized values): 1.5kW (wind turbine), 2.4kW (PV module) and 96kWh (battery bank). The rated capacity of the battery is 8000Ah and I suppose that its rated voltage is 12V.

4.1 Are these values exact? What are characteristics of the battery bank (electrochemical principle, rated voltage and capacity)?

By supposing that preceding values are exact, this means that the ratio installed power / average power ≈ 5; moreover the total storage capacity would correspond approximately to 5 days of load supply (corresponding to 1 day with a depth of discharge d=20%). Therefore, the cost and the size of the battery are significant.

4.2 In equation (15), what are relative cost of the battery, the PV generator and the wind generator (respect: βNbatt, αNpv and Co)?

4.3 What is the value of the ratio (β / α) who determined, in equation (16), the optimal solution of the problem?

4.4 Being did the life duration of the system, are the component cost actualized? on the other hand, can one do neglect maintenance and repaired costs of some components?

5. Utilization and extension of the model: To my opinion, the proposed method is interesting, especially because that it allows to study the sensitivity of the solution to the various hypotheses of the problem.

Among the various parameters, two parameters affectstrongly the total cost of the system: the discharge depth of the battery (d), and the LPSP. The effective life duration of a battery depends both on its operating conditions (charge and discharge cycles) and its conditions of supervision and periodic maintenance.

5.1 What is the mathematical expectation the number of cycles to which will be submitted the battery during the life cycle?

5.2 Even if the depth of discharge is strictly limited (d=20%), can we consider that the life duration of the battery will be at least equal to the economic life duration of the system (10 years)?

5.3 Do authors can indicate how varies the optimal solution (Npv, Nbatt) as well as the total cost according to the discharge depth (d)?

5.4 Would be it economically interesting to envisage a battery of weaker capacity, more strongly stressed (d > 50%), therefore having a weaker duration life do including its renewal in the course the life duration of the system?

The relative value specified for the LPSP ($2.74\ 10^{-4}$) can be interpreted practically according to the considered duration: 1day / 10 years, 24hrs / 1 year, 12min / 1 month ...); This specification seems enough severe in the case of a typical house.

5.5 Do authors can indicate how varies the optimal solution (Npv, Nbatt) as well as the total cost according to LPSP (between $2.74\ 10^{-4}$ and $2.74\ 10^{-3}$)?

5.6 To note that the fig. 9 corresponding to the fig. 8 would have to be numbered "fig. 10".

I thank authors for their replies to these questions in order that the readers could surround possibilities and limits of the method.

Manuscript received August 22, 1995.

B. Borowy and Z. Salameh:

The authors wish to thank Dr. L. Pierrat and Dr. Y. Wang for their interests in the paper, and would like to take the opportunity to clarify the issues raised in their discussion. The closure follows the sequence of the discussion.

1.1-1.3) Most good wind regimes have the shape parameter *k* in the range of 1.5 to 3.0, as it appeared in the studied case. It is convenient to define a model for electrical power output P_e that can be used in any wind system. The simplest model would use a straight line to describe the variation in output power between cut-in and the rated wind speeds. However, other monotonic functions will fit the observed data perhaps even better for some machines, and may yield more accurate energy estimates or more convenient analytic results. For a wind turbine with a fixed pitch angle, the output power is proportional to the cubic of wind speed. Since all the calculated shape parameters k of the Weibull distributions fell within the vicinity of 3.0, this type of model was justified to use. Using this type of expression for the power output the average generated power can be easily estimated analytically.

$$P_{avg} = \int_0^\infty P_e f(v)dv = \int_{v_C}^{v_R} (a+bv^k) f(v)dv + P_R \int_{v_R}^{v_F} f(v)dv$$

where: $a = P_R \dfrac{v_c^k}{v_c^k - v_R^k}$, $b = P_R \dfrac{1}{v_R^k - v_c^k}$

By making the change in variable $x = \left(\dfrac{v}{c}\right)^k$, one can easily solve the integral for average output power:

$$P_{avg} = P_R \left\{ \frac{\exp\left[-(v_c/c)^k\right] - \exp\left[-(v_R/c)^k\right]}{(v_R/c)^k - (v_c/c)^k} - \exp\left[-(v_F/c)^k\right] \right\} \text{W}$$

For a given wind regime with known c and k parameters, one could select v_c , v_R , and v_F in order to maximize the average power, and thus maximize the total energy production. The genesis of using this type of approximation of the output power comes from the early stage when we had to choose best turbine for our site. Therefore, the choice of a somewhat complicated model made the later calculations easier, and perhaps more accurate than the simplified straight line model.

1.4) Typographic errors in equation (1) and the Fig. 9 and Fig. 10 have been corrected.

2) The electrical output power P(S) of a PV array in (5) was calculated as a product of array's output voltage V and current I. It is assumed that a maximum power point tracker is used. Therefore, photovoltaics operate at the *maximum power* voltage. This voltage and a corresponding current were calculated using (6) and (7) for actual cell temperature and insolation level. Probably a better notation for the output power in (5) would be P(S,T).

The cell temperature and insolation level relation are expressed in (7) as a sum of the ambient temperature and two percent of irradiance level. The temperature coefficients α and β will vary for different manufacturers [5]. Usually output current only slightly increases if the cell temperature increase. On the other hand, the output voltage is more sensitive to temperature changes in the opposite way. Figure 6 merely only illustrates the effect of a temperature and irradiance changes on the I-V characteristics. It does not represent any realistic example. A fairly good approximation of a PV module could neglect the effects of temperature variations by taking only the average cell temperature.

3) Although the actual wind speed distribution can be described by either a Weibull or a Rayleigh density function, there are other quantities that are better described by a normal distribution. The distribution of yearly or monthly mean speeds is likely to be normally distributed around a long-term mean wind speed. Calculations of a correlation coefficient between wind speed and irradiance level resulted in a lack of any significant level of a correlation between these random variables. Zero value of a correlation coefficient infers that these variables are uncorrelated--not that they are automatically independent. However, wind speed and irradiance distribution functions can be approximated to that with normal distributions (based on a long term data available). Since for two jointly Gaussian random variables that are uncorrelated are also independent, it makes it possible to simplify the problem of calculation power output from these resources. The load demand was entirely treated not as a random variable, since it was explicitly provided by utility company. However, in more general case it should be treated as a random variable. The correlation between the ambient temperature and irradiance level was not considered, as not extremely important to the power generation.

4) In this paper authors presented a general methodology that can be used for optimal sizing of a hybrid system. The results obtained are relevant only for the studied case and for actual price of batteries and photovoltaics on the market at that time. The optimum solution will be different for other values of the price, type of batteries and photovoltaics used.

5) A low value of a given LPSP makes the system to contain a large number of batteries and PVs. It affects strongly the total cost of the system. A slightly higher LPSP results dramatically in the cost reduction.

Because of a page limit to this closure authors cannot give more specific answers with regard to: numerical values for different battery size, their depth of discharge, and different LPSPs. These can be easily found from the flowchart diagram in Fig. 7.

Manuscript received October 11, 1995.