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Parametric investigation of geometric form effects on solar potential of housing units

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Abstract

The paper presents a study of the solar potential of different shapes of two-story single family housing units, located in mid-latitude climate. Seven plan geometries are studied: square, rectangle, trapezoid, L, U, H and T shapes. The study investigates the effect of these shapes on two major response variables – Solar radiation incident on equatorial-facing facades and transmitted by the fenestration of such facades, and electricity production potential of building integrated photovoltaic (BIPV) covering roof surfaces with optimal solar exposure. The parameters, whose effects on the response variables are investigated, include, in addition to the basic shapes and roof design, variations to the geometry of L and U shapes and variations to the roof design. Shape variations include varying values of the relative dimensions of shading and shaded facades and variations to the angle enclosed by the wings of these shapes. Variations of roof design consist of modifications to the tilt and side angles of hip roofs. The results indicate that the number of shading facades in-self shading geometries and their relative dimensions are the major parameters affecting solar incident and transmitted radiation. Manipulation of the orientation of wings in L shape units can result in increased peak electricity generation potential, and in shifting the timing of the peak by up to 2 h either side of solar noon. The shift of peak load may be economically beneficial, facilitating more even distribution of electricity production over an assemblage of buildings. Judicious manipulation of unit shapes and window location can lead to optimization of solar radiation and its utilization for electricity generation and passive solar gain.

Keywords: Active solar system; Energy; Passive solar system; Solar energy; Geometry; Shape

1. Introduction

Building shape plays a key role in influencing energy consumption in buildings. It has a significant effect on thermal performance and can provide advantages in capturing solar energy (Ouarghi and Krarti, 2006; Depecker et al., 2001; Pessenlehner and Mahdavi, 2003; Hachem et al., 2010). Consideration of solar access in the determination of building shape is an essential part of the design of energy efficient buildings. Solar radiation can be exploited in passive design to assist in reducing energy consumption for space heating and daylight (Hastings and Wall, 2007). It can also serve as a source for electricity generation using photovoltaic systems. A well designed passive-solar building can provide 45–100% of daily heating requirements (ASHRAE, 2007). Building shape has significant effect on energy demand and can be manipulated to control it (Knowles, 1981).

Rectangular shape is generally considered as the optimal building shape for passive solar design and the most energy efficient (Chiras, 2002; Hachem et al., 2011). However, under certain design conditions in urban context, this shape may not be optimal (Hachem et al., 2011). The shape

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of a site, the layout of a street, distance of neighboring buildings and their shapes are all factors that interact significantly with the effect of shape on solar access. In addition, it should be born in mind that shape design is governed by many constraints other than energy efficiency, such as functional demands and quality of life of inhabitants. For these reasons it is important to explore the penalties, as well as the benefits associated with plan layouts other than rectangular, and with different roof forms.

The main parameters that are often used to determine a building solar access, assuming that the building is of a given height and is optimally oriented, are aspect ratio, the area of the equatorial-facing or near equatorial-facing fenestration and the roof slope and area. *Aspect ratio* (W/L, in Fig. 1) is defined as the ratio of the equatorial-facing facade width (W) to that of the lateral façade (L). These parameters are illustrated in Fig. 1.

Aspect ratio is a significant parameter in energy efficient design (from energy demand viewpoint), as emphasized in several studies. In cold climate, the ideal aspect ratio for a rectangular shape solar house design ranges from 1.3 to 1.5 (Chiras, 2002; Charron and Athienitis, 2006). Another parameter of significance to energy efficiency encountered in the literature is the surface area (S) to volume (V) ratio (S/V being the three dimensional extrapolation of the perimeter to area ratio) (Behsh, 2002). S/V ratio determines the relation between the outer exposed surface and the contained volume (Knowles, 1981).

Numerous studies have investigated the effect of aspect ratio and surface to volume ratio on solar radiation and energy performance of buildings. For instance, Ling et al. (2007) examined the effect on the total insolation of high-rise buildings of two geometric shapes – rectangular and oval, with different W/L ratios (Ling et al., 2007). The goal of the study was to identify the optimum shape in minimizing total solar insolation in low latitude regions. Ouarghi and Krarti (2006) examined building shape optimization for office buildings, where the building shape was described by its S/V ratio in comparison with a cubical reference building. Various existing built forms have been investigated in an urban context, to determine their energy performance and solar exposure e.g. (Gupta, 1984; Ratti et al., 2003; Compagnon, 2004; Kampf et al., 2010).

The parameters of W/L and S/V are not always sufficient to provide a clear estimation of the solar access of shapes with complex geometry. For this reason, additional parameters should be studied in the optimization of solar potential of shapes. For instance, relative dimensions of shading and shaded facades (see Fig. 1) and the number of mutual shading surfaces in self shading geometries are of primary significance in solar design. The equatorial-facing fenestration area is another crucial parameter in passive solar design. An optimal equatorial-facing glazing area ranges between 7% and 12% of total floor area (Charron and Athienitis, 2006). Under certain design conditions, this fenestration area can be increased up to 20% of the heated floor area (Charron and Athienitis, 2006).

The roof is an important component in the design of building shape for energy efficient solar design. The roof can be designed to perform several functions simultaneously, including integration of photovoltaic systems for electric power generation (Sui and Munem, 2007). Photovoltaic systems are emerging as an important part of the trend to energy source diversification (Wiginton et al., 2010; Neuhoff, 2005; Pearce, 2002). PV technology implementation is still limited however; constituting less than 1% of global energy production (Wiginton et al., 2010). In Canada, building integrated photovoltaic (BIPV) systems are estimated to be capable of providing 46% of total energy demand (Pelland and Poissant, 2006). To maximize the electricity generation of BIPV, a tilt angle equivalent to the latitude of the building location is generally considered optimal (Buresch, 1983; Mondol et al., 2007). During the winter months, insolation can be maximized by using a surface tilt angle that approximately equals the latitude angle plus 10–15°; in summer an inclination of 10–15° less than the site latitude maximizes insolation (Duffie and Beckman, 1991).

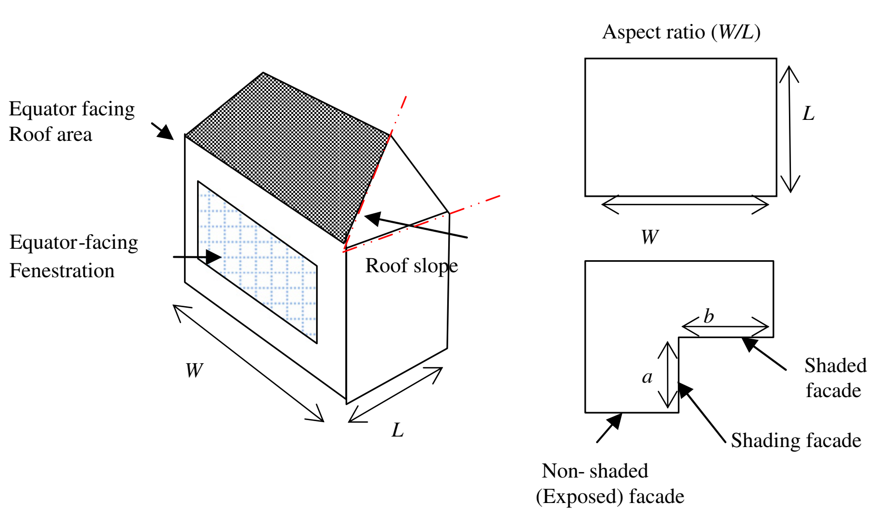


Fig. 1. Design parameters for solar access.

This paper investigates the effects of the geometric shapes of two-storey single family housing units on their solar potential. The solar potential falls under two main categories: (a) Radiation incident on the near equatorial facing façades and radiation transmitted by the fenestration of this façade; (b) Electricity generation potential of PV systems integrated in the near south facing roof surfaces.

Section 2 presents the methodology of the investigation. It starts with details of the basic design of housing units, followed by an outline of the principles of the parametric investigation. The effects of varying shape design parameters on solar potential response as detailed above are assessed, with rectangle serving as reference. The principles and data employed in the *EnergyPlus* simulation program (EnergyPlus, 2010) of the response variables are also outlined. Section 3 presents results of the simulation, highlighting the main effects of the design parameters on the response variables of solar radiation and electricity production potential. The paper concludes with some design recommendations.

2. Methodology

The scope of this study is to explore the solar potential, as defined above, of different housing shapes. Incident radiation on equatorial facing facades, and transmitted by the fenestration of such facades significantly influence the energy performance of a building (Hachem et al., 2011). Detailed investigation of energy performance is, however beyond the scope of this paper.

The First step in the investigation is design of the various plan layouts together with the solar façades and the position of windows on these facades. This is followed by roof design of each shape and the integrated photovoltaic portion of the roof. The selected configurations are then subjected to simulations aimed at estimating solar radiation incident on south facing facades, radiation transmitted by the windows of these facades and the BIPV electricity generation. The simulation employs the *EnergyPlus* building simulation program (EnergyPlus, 2010). The simulations are followed by a comparative analysis to assess the effect of shape parameters on the solar potential, relative to a reference case. A rectangle, with aspect ratio of 1.3 serves as the reference. This shape is selected as it is generally considered optimal for passive solar design for the given climate (Chiras, 2002).

2.1. Basic design

2.1.1. Plan design

Seven commonly built plan layouts of housing units – square, rectangle, trapezoid, L, U, H and T shapes, are considered in this study. These shapes and the main design parameters that characterize them are illustrated in Fig. 2. The units are designed as two-storey single family houses in a northern climate (Montreal, Canada – latitude 45°N). Variations of some parameters governing certain shapes are explored, to identify design possibilities that enhance solar radiation capture potential on near-south facing roofs and facades.

The two-storey option is adopted in this study as it represents one of the most common options of a single family detached solar home in Canada (Charron and Athienitis, 2006). This option requires less land compared with a single floor house (Athienitis, 2007).

The basic design of the plans accounts for passive solar principles (Athienitis and Santamouris, 2002) and rules of thumb (CMHC, 1998). The design ensures that the overall east–west dimension of the house – the solar façade, is

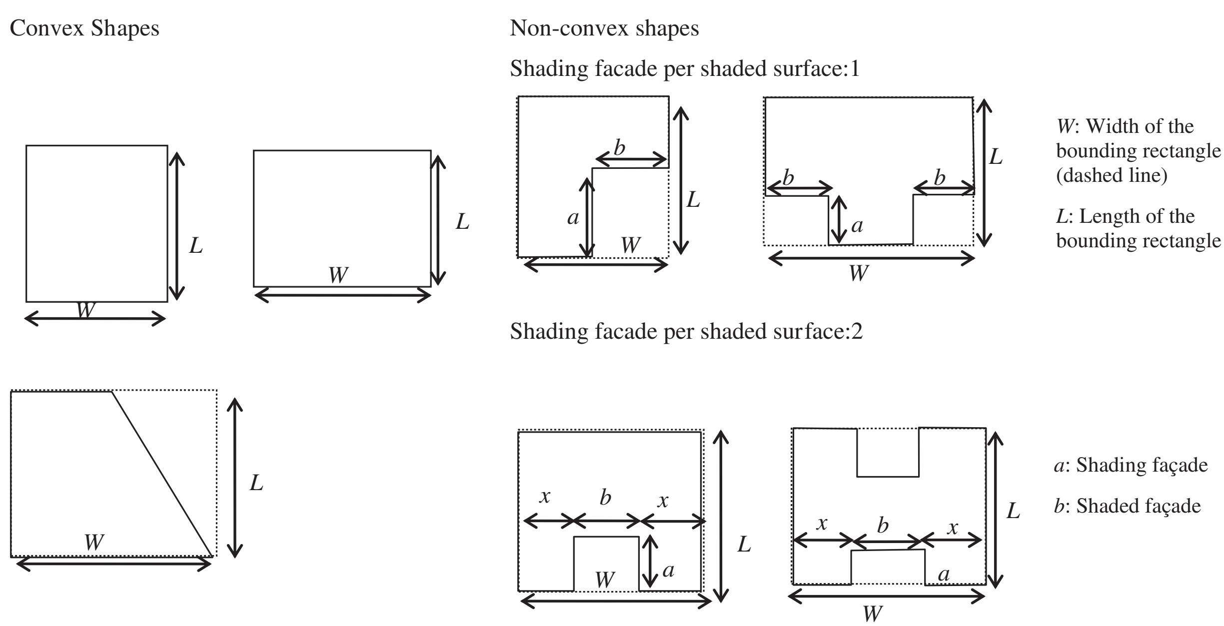


Fig. 2. Ground floor plan layouts of basic shapes.

larger than the perpendicular dimension (north–south), to maximize passive solar gains in winter. The layout of the interior space ensures that the living area and the kitchen, in the ground floor, are adjacent to the south facade (Chiras, 2002; CMHC, 1998). Holding the floor area constant at around 60 m^2, the interior spaces of all units are partitioned to fit a family of four persons. The floor area is based on the need to reduce costs by maintaining a compact design.

An important parameter characterizing non-convex shapes is the relative dimensions of the shading and shaded facades. The ratio of the width of the shading facade to that of shaded façade is termed the *depth ratio* – a/b, in Table 1. The shaded façade's width and the depth ratio are determined so as to maintain a functional interior space. Decisions on the width, length and configurations of non-convex shapes are based on functional partitioning of the interior space, so as to avoid wasted space, or long corridors. The main considerations governing the design of shapes in this study are as follows:

- An aspect ratio of 1.3 should be applied, when possible.
- The basic L shape has an overall aspect ratio of 1. An aspect ratio of 1.3 is used in L shape with depth ratio (a/b) of ½.
- In U, H and T shapes a symmetric design is adopted in order to simplify the analysis.
- Interior dimensions, after cutting out the recess should allow a useful functional distance for interior arrangement. This distance should not be less than 3 m.

The shapes shown in Fig. 2 are characterized by the parameters presented in Table 1.

2.1.2. Facades and windows

The ceiling height of the ground floor is set at 3 m, to enhance daylight penetration (Athienitis, 2007).The first floor ceiling height is about 2.7 m. Double pane, low-emissivity windows (total SHGC = 0.62, Visible transmittance = 0.65) are selected (ASHRAE, 2005). All windows are composed of units having an area of ca. 2.0 m^2. The size of south-facing ground floor windows is 12% of the ground floor area; while first floor south façade windows are 8% of first floor area. The east and west windows are 4% of the total heated floor area. This is based on recommendations to minimize non-south glazing area to 4% or less of the total heated floor area, under the northern climate conditions (Chiras, 2002), while maintaining functional considerations.

Due to the different areas of south facing façades of different shapes, south windows constitute differing percentages of these facades. For instance, the ground floor south window area of the square shape is 26% of ground floor south façade, while it is 20% of the corresponding façade areas of the U shape (Table 1).

For non-convex shapes, the south facades are not coplanar, and therefore the total window area is distributed over the different portions of the facade. This is to accommodate both the predetermined area of window and functional requirements, such as providing daylight for different zones of the plan.

In non-convex shapes, except T, a single window unit is located on the shaded façade, corresponding to a third of the total south window area. The dimensions of the T shape allow a single window unit to be fitted on each of the 3 south facades.

2.1.3. Roofs

The basic roof design in this study is a hip roof with tilt and side angles of 45° (Fig. 3). Variations of tilt and side angles are explored (see parametric investigation below).

The height of the lowest edge of the roof is kept constant at seven meters above ground level. The roofs are designed with their ridge running east–west at the center of the plan area for all shapes, except L and T (Fig. 3a). In the case of L and T shapes the ridge of each wing runs along its center, with a triangular south facing hip at the south end. The north wing roof ends with gables (Fig. 3b).The ridge height varies depending mainly on the width perpendicular to the ridge, and the tilt angle. For a tilt angle of 45°, the ridge can reach a height of 3.5 m from the lower plane of the roof, in the rectangular shape and 4 m in the trapezoid.

U and H shape roofs are designed with a single ridge, in a similar way to the rectangular shape roof, with the central recess cut out (Fig. 3c).

PV modules are assumed to cover the surface of all south-facing and near-south-facing roof surfaces. This includes the triangular portions of hip roofs in L and T shapes.

2.2. Parametric investigation

The study investigates the effects of a number of parameters n the two major response variables – radiation

Table 1
Shape design parameters.

	Square	Rectangle	Trapezoid	L shape	U shape	H shape	T shape
Aspect ratio (W/L)	1	1.3	1.3	1	1.3	1.4	1.6
Number of shading facades	n/a	n/a	n/a	1	2	2	1
Depth ratio a/b –				1	1	1/4	2/3
Ground floor south window as percentage of ground floor south-facing wall area	26%	23%	20%	23%	20%	19%	19%

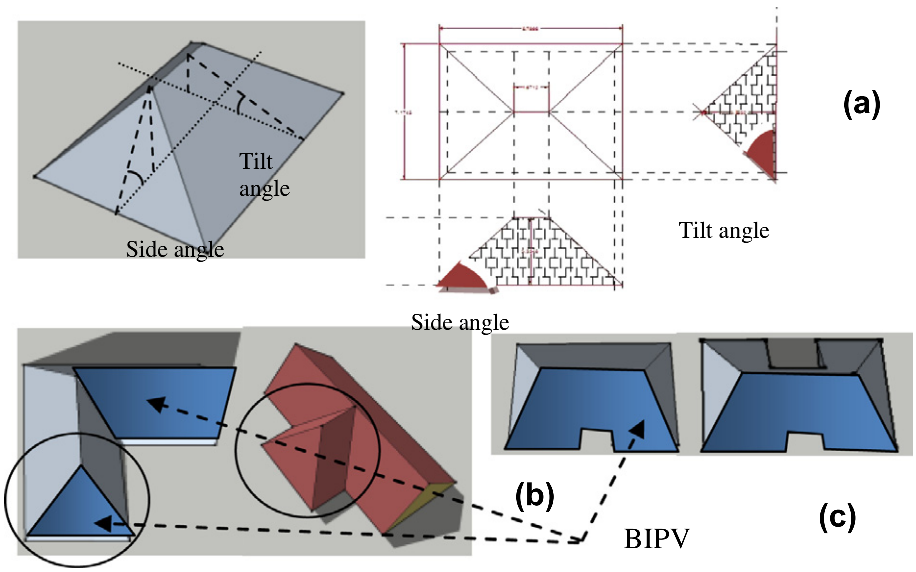


Fig. 3. Roof layouts of basic designs: (a) single ridge designs; (b) double ridge designs in L, T; (c) roofs of U and H shapes.

incident on south facades and transmitted by their windows, and PV electricity production potential. The parameters are summarized in Table 3.

The parameters, whose effects on the response variable are investigated, include, in addition to the basic shapes and roof design, several variations to this design. These additional parameters are variations to the geometry of L and U shapes and variations to the roof design. Shape variations include varying values of the depth ratios – a/b (Fig. 2 and Table 1) and variations to the angle enclosed by the wings of the layout, which in the basic design are at right angles. Variations to the roof design include various combinations of the tilt and side angles. Combinations of parameter values analysed in this study are summarized in Table 3.

2.2.1. Variations of L and U shapes

L and U shapes are selected for studying the effects of modification to the geometry of non convex shapes because they have relatively long wings branching from the main east–west wing, and therefore are likely to highlight effects associated with shading.

2.2.1.1. Depth ratio. The depth of the shadow-receiving facade and the number of shadow projecting facades play an important role in determining the amount of solar radiation incident on the shaded façade, and transmitted by its windows. Two values of the depth ratio, in addition to the value of 1, which is the basic design, are adopted for L and U shapes – 1/2 and 3/2. These depth ratios represent two extreme values, while maintaining functional plan.

In the design of units with varying depth ratios, the floor area is kept constant (ca. 60 m^2). The aspect ratio of U shape is kept constant (1.3). An aspect ratio of 1.3 is also applied in L shape with depth ratio of ½.

In variants of U and L shapes with $a/b = 3/2$, the width of the window on the shaded façade reaches a value of ca. 85% of the façade width.

2.2.1.2. Wing rotation. Six different variations each of L shape and of U shape are studied. The angle β denotes rotation west of the western branch wing of either L or U shape. The angle θ denotes rotation north of the northern wing of L shape and rotation east of the eastern wing of U shape – Table 2. β and θ take three values each, 15°, 30° and 45° (Table 2).

2.2.2. Roof parameters

Variations from the basic roof design of 45° tilt angle and 45° side angle (where applicable), include tilt angle of 30° and side angle of 60°. In general a tilt angle of 45° is considered optimal for northern regions of ca. 45° latitude. Accordingly, an additional roof, termed hereunder the *optimum roof*, is designed to be used as control for comparative evaluation of the electricity generation potential by the south facing BI PV systems of all other roofs. The optimum roof is a gable roof with 45° tilt angle covering the rectangular shape of aspect ratio of 1.3. The ridge of this roof is maintained in the center of the roof.

2.2.3. Simulation modeling

2.2.3.1. Weather data. This study is performed for Montreal, Canada (45°N Latitude). The weather files of the building simulation program EnergyPlus are used for the simulations (EnergyPlus, 2010). The weather data file, which is based on CWEC (Canadian Weather for Energy Calculations), provides hourly weather observations. Two design days are selected from the weather files – a sunny cold winter day (WDD) (in January), and a sunny hot summer day (SDD) (in July). The selection is intended to assess

Table 2
Relative rotation of the wings of L and U shapes.

Variations of L and U shapes Angles of rotations						
$\beta = 45°$	$\beta = 30°$	$\beta = 15°$	Basic L	$\theta = 15°$	$\theta = 30°$	$\theta = 45°$

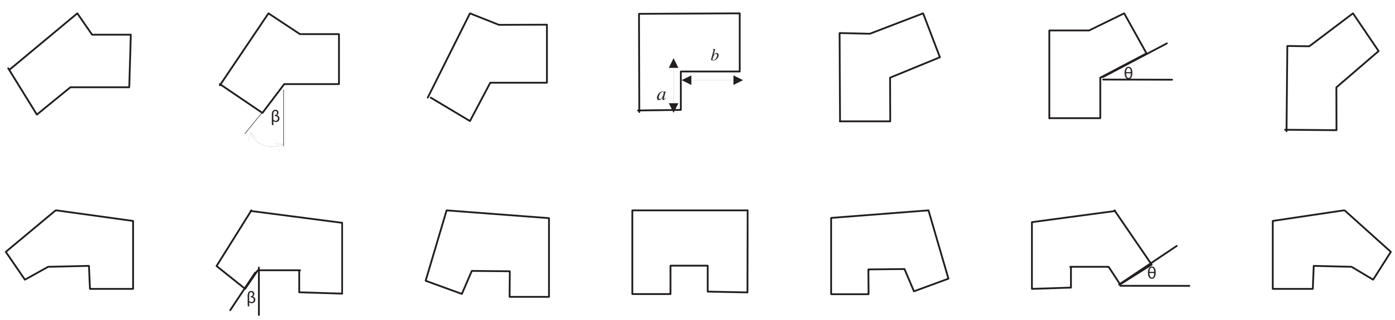


Table 3
Parameter combinations investigated in the study.

Shape	Depth ratio	Rotation of wing (for L and U shapes)	Roof	
			Tilt angle	Side
Basic shapes (Fig. 2)	Basic design	Basic design	45°	45°
			45°	60°
			30°	45°
			30°	60°
L and U shapes variations	1/2–3/2	Basic design (Fig. 2)	45°	45°
	Basic design	$\beta = 15–45°$ (15° step)	45°	45°
		$\theta = 15–45°$ (15° step)	45°	45°

maximum solar potential when the sun is at the two extreme positions.

In addition to the two design days, a whole year weather data set is used to estimate the annual electricity production of the PV system installed on south-facing roof surfaces (details are presented below). These observations simulate a 1-year period, specifically intended for building energy calculations.

2.2.3.2. Energyplus solar radiation computations. The first stage in the model is to compute solar irradiance (solar modeling). The instantaneous solar radiation accounts for direct beam and diffuse radiation, as well as for radiation reflected from the ground and adjacent surfaces. The solar model used in this study employs the ASHRAE Clear Sky model (ASHRAE, 2005). This model is the default model used by EnergyPlus (EnergyPlus, 2010) to estimate the hourly clear-day solar radiation for any month of the year. Sky radiation is calculated using the Perez anisotropic sky model (Perez et al., 1990).

The clear sky model yields values that are representative of conditions on cloudless days for a relatively dry and clear atmosphere. The clearness numbers (Threlkeld and Jordan, 1958) are usually used as correction factors to apply this model to locations with clear, dry skies or locations with hazy and humid conditions (ASHRAE, 2007).

Validation tests show that the simulation codes used in EnergyPlus (in addition to other simulation programs such as ESP-r) are capable of computing total irradiated solar energy on building facades with a high precision for long time periods (such as months) Loutzenhiser et al., 2009. Heat flow through windows was also shown to be predicted by EnergyPlus with a good precision, where the difference with the experimental data was in the order of 5.8% (Loutzenhiser et al., 2007).

2.2.3.3. Shading calculations. To study the solar radiation incident on different shapes it is necessary to allow for mutual shading by surfaces. The shading algorithm accounts for self-shading geometries, such as L shape. This algorithm is based on coordinate transformation methods (Groth and Lokmanhekim, 1969) and the shadow overlap method (Walton, 1983).

2.2.3.4. BIPV model and computation. A simple model roof integrated PV system based on constant electrical conversion efficiency (η) of 12%, is employed in the simulations. This efficiency is based on a nominal PV efficiency of 16% and a PV system performance ratio of 0.75, and is obtained as (Pelland and Poissant, 2006):

$(\eta = 16 * 0.75)$.

The PV performance ratio is the ratio of the actual system yield (kW h/kW) to the reference yield. Reference yield refers to the insolation in the plane of the PV module (kW h/m^2). The performance ratio accounts for all PV system losses, including electrical wiring losses and PV operation under non-optimal conditions (Poissant et al., 2003). The performance ratio of 0.75 is reported by the International Energy Agency as the most commonly used, based on a global monitoring of grid-connected PV systems, built between 1996 and 2002 (Pelland and Poissant, 2006). For Montreal (Quebec province) the annual potential of PV electricity generation of south facing surfaces at latitude tilt angle is about 1200 kW h/kW (Natural Resources Canada, 2007).

The model used by EnergyPlus to compute the electrical power produced by the PV system is expressed as follows:

$$P = A \cdot f_{act} \cdot G_t \cdot \eta \tag{1}$$

where P is the electrical power (W), A is the net area of surface (m^2), f_{act} is the fraction of surface area with active solar cells (m^2), G_t is the total solar radiation incident on PV array (W/m^2), and η is the PV module conversion efficiency.

This model is widely used in quantifying the potential of PV systems e.g. (Pelland and Poissant, 2006; Wiginton et al., 2010). A fixed efficiency is used in this study in order to avoid obscuring the effect of shape, such as shading of the PV array by other surfaces, in self shading geometries, by other, unrelated effects. The conversion efficiency is however a significant design parameter in the optimization of BIPV system. Further investigation is envisaged of this parameter and its correlation with other design parameters such as tilt angle and orientation of the roof.

3. Simulation results

3.1. Solar radiation

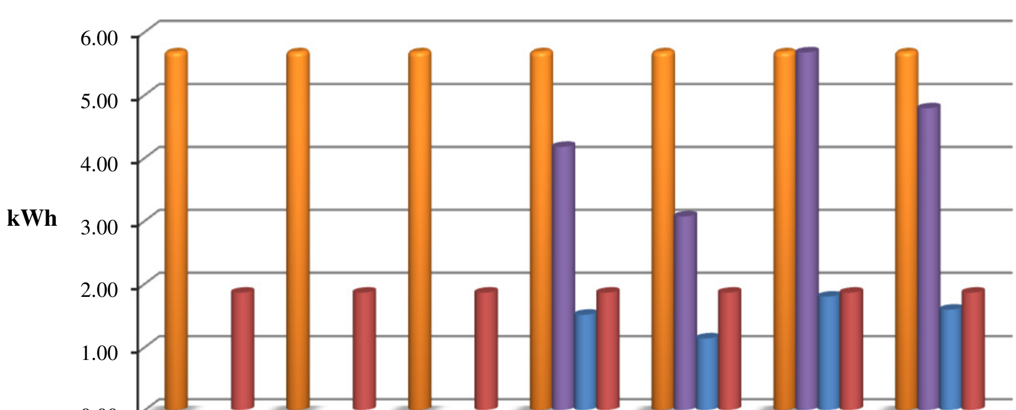
3.1.1. Basic shapes

The mean daily global insolation for the south facing non-shaded facade obtained from EnergyPlus simulations is about 3.23 kW h/m^2. This value falls within the range estimated by the Natural resources Canada (NRCan) (2.5–3.3 kW h/m^2) Natural Resources Canada, 2007.

The results include allowance for shading in non-convex shapes such as L, U, H and T. The Total radiation is compared with the radiation of the rectangular shape, which serves as reference. The total transmitted radiation (in kW h) for a single south facing window unit (2 m^2), for both shaded and non-shaded facades, for winter and summer design days, are displayed in Fig. 4. It should be noted that the total area of south-facing windows is the same for all shapes. Following are some comments on the more significant results.

- The transmitted radiation by the window of the south shaded façade in a winter day is reduced by 26% for the L shape and by 46% for the U shape, as compared with the exposed window. For the summer design day, the transmitted radiation is reduced by ca. 19% and 40% for L and U shapes, respectively.
- In the case of T shape, the transmitted radiation differs slightly between the right and left wings. The table of Fig. 4 presents the mean values (the maximum difference is some 7%). The mean reduction in transmitted radiation for window units of the shaded compared to the exposed façade is 10% in a winter design day and 15% in a summer design day.

3.1.1.1. *Variations of depth ratio in L and U shapes.* Radiation on the shaded façade is significantly dependent on the depth ratio – *a*/*b* (Fig. 2, Table 1). Another important effect is the number of shading facades. The effect of the depth ratio is shown in the study of U and L shapes. Table 4 summarizes these results, for a winter design day. Following are the main observations:



	Cube	Rectangle	Trapezoid	Lshape	Ushape	Hshape	Tshape
WDD-non-shadedfacade (kWh)	5.68	5.68	5.68	5.68	5.68	5.68	5.68
WDD-shaded facade (kWh)				4.17	3.07	5.67	4.79
SDD-shaded facade (kWh)				1.51	1.14	1.79	1.59
SDD-non shaded facade	1.86	1.86	1.86	1.86	1.86	1.86	1.86

Fig. 4. Transmitted radiations of windows in south facades for a WDD and for a SDD.

Table 4
Effect of depth ratio.

Shape	Number of shade projecting facades		$a/b = 1/2$	$a/b = 1$	$a/b = 3/2$
L shape	1	Percentage of reduced radiation on shaded facade	12%	22%	26%
		Percentage of reduced transmitted radiation	7%	27%	34%
U shape	2	Percentage of reduced radiation on shaded facade	23%	43%	53%
		Percentage of reduced transmitted radiation	14%	46%	60%

- The results indicate that the radiation incident, per m^2, on the shaded south facade of L shape is reduced by 12%, 22% and 26%, as compared to the radiation on the exposed south facade, for depth ratio values of 1/2, 1 and 3/2, respectively. The transmitted radiation by the window of the shaded south façade, compared to the transmitted radiation by non shaded window, of the L shape is reduced by 7%, 27% and 34% for *a*/*b* values of ½, 1 and 3/2, respectively.
- The reduction of incident and transmitted solar radiation of the shaded south façade of the U shape is approximately double that of the corresponding L shape (with similar depth ratio) (see Table 4).

3.1.1.2. Variations of wing angles. Following is a summary of the main effects of the variations of wing angles on the incident and transmitted radiation (Fig. 5).

- The solar radiation on the non-shaded facade in L shape is reduced by up to 27% with increasing values of the angle β. This is counterbalanced by a similar increase on the shaded façade. The increase of angle θ (Table 2) produces a decrease of the radiation striking the south façade of this wing by up to 15%.
- In U shape, rotation of the exposed south façade of the east wing towards east and of the west wing towards west, produce practically identical changes in irradiation magnitude. It is also identical with the effect on L shape (up to 27% reduction).
- The radiation incident on the shaded façade in U shape is increased due to wing rotation by almost the same rate as the decrease on the non-shaded facade. It can be inferred that rotation outward of both U branches would result in doubling both effects. The transmitted radiation (Fig. 5a) follows the same trend as the incident radiation.
- The transmitted radiation by the shaded facade window of L shape variations decreases with increasing values of the angle θ. Increasing the angle β reduces the transmitted radiation by the exposed facade window while increasing that of the shaded facade window (Fig. 5b).

3.2. Electricity generation

3.2.1. Basic shapes with varying roof slopes

The mean daily insolation for the roof at 45° tilt angle is about 4.4 (kW h/m^2) (comparable to 4.33 kW h/m^2 for Quebec as obtained from the Natural Resources Canada (Natural Resources Canada, 2007). The rectangular layout with gable roof (The optimum roof) has a total south facing area of ca. 40 m^2, a peak electricity generation for the winter design day of 4.32 kW and annual generation of 8 MW h. These results are comparable with results from Pelland and Poissant (2006) and with published data by the Natural Resources Canada (Natural Resources Canada, 2007) for Montreal location, within 5%, under the conditions employed in this study.

Table 5 presents the comparison of all studied roofs to the optimum roof (rectangular layout with gable roof), expressed as the ratio of annual electricity generation of each roof to that of the optimal roof. Figs. 6 and 7 illustrate the peak electricity generation for the design days and the annual generation, for all variants. The main observations are highlighted as follows.

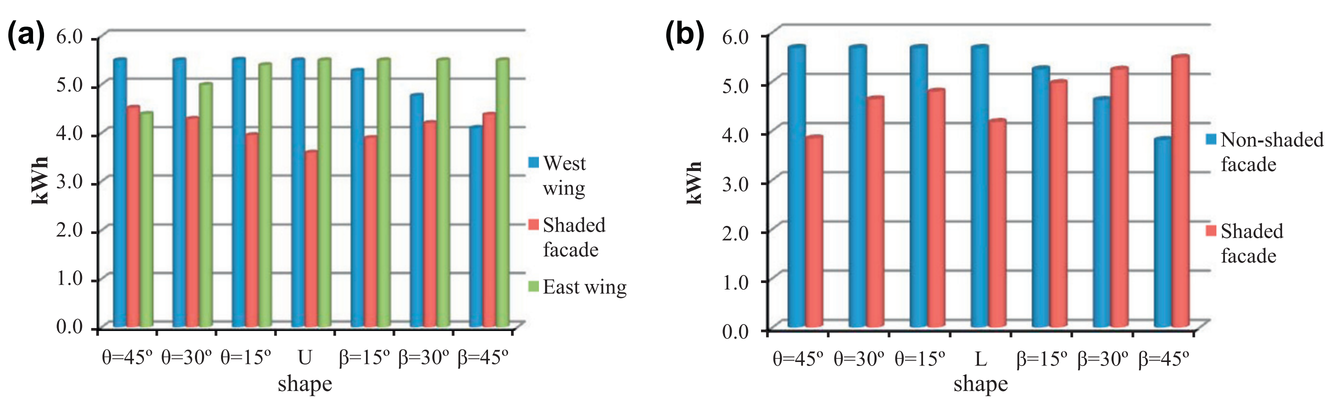
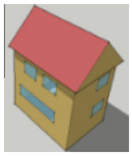


Fig. 5. Effect of wing rotation on transmitted radiation of south-facing windows: (a) U shape; (b) L shape.

Table 5
Ratio of annual electricity generation of different variants of roofs, to the optimum roof (gable roof).

Optimum roof-gable roof		Total area of PV covered surface (m^2) 40	Electricity generated per winter design day (kW h) 26.48	Peak electricity generated-winter design day (kW) 4.32	Electricity generated per summer design day (kW h) 34.81	Peak electricity generated-summer design day (kW) 4.33	Annual electricity 8 (MW h)
							

Tilt-side angles	Ratio of annual electricity generation to gable roof for different shapes						
	Square	Rectangle	Trapezoid	L shape	U shape	H shape	T shape
(1) 45–45°	0.53	0.65	0.85	0.63	0.60	0.77	0.90
(2) 45–60°	0.75	0.81	1.10	0.58	0.76	0.97	0.86
(3) 30–45°	0.61	0.66	0.88	0.47	0.64	0.77	0.70
(4) 30–60°	0.73	0.75	0.95	0.44	0.73	0.86	0.66

- With tilt angle of 30°, the peak generation on the SDD is slightly lower than on the WDD (Fig. 7a). However, with tilt angle of 45°, the peak generation is significantly larger for a WDD (Fig. 7b).
- In general, electricity generation is proportional to roof surface area. Given a fixed floor area, roof surface area can be manipulated through various design options such as:

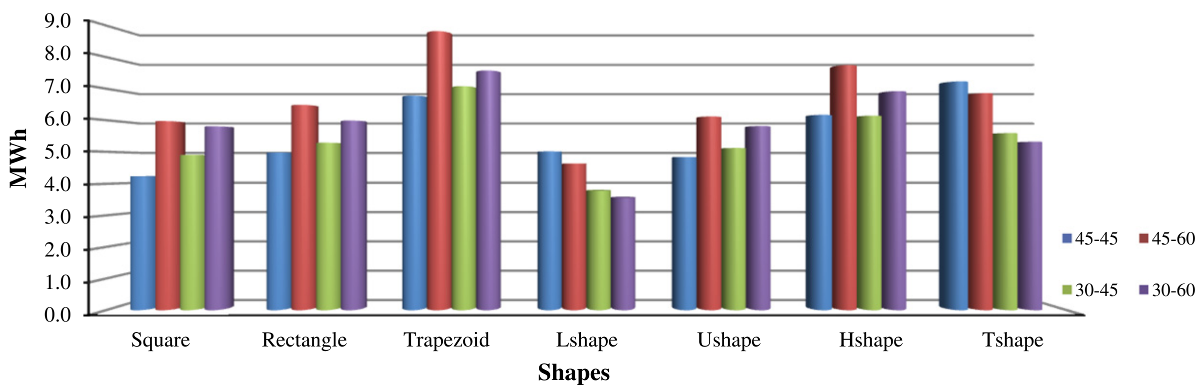


Fig. 6. Annual electricity generation for roofs with differing tilt-side angles and shapes (MW h).

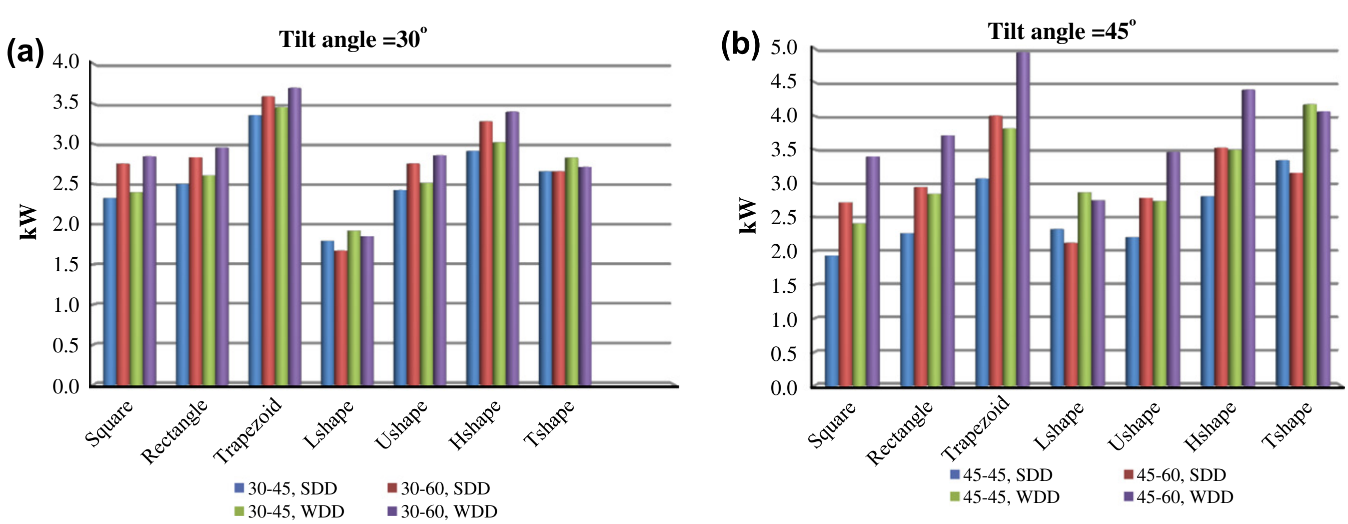


Fig. 7. Peak electricity generations (kW): (a) tilt angle 30°; (b) tilt angle 45°.

– Combination of side angle and tilt angle. For instance, a rectangular layout with side angle of 45° and tilt angle of 30° possesses a larger south-facing surface area than with a tilt angle of 45°. Consequently the yearly electricity generation is larger with the 30° tilt, even though a tilt angle of 45° allows higher radiation per unit area (5.67 kW h/m^2, as against 4.86 kW h/m^2 for 30° tilt angle, for the WDD)
– Different plan shapes offer different south facing roof area. These roof areas, together with tilt and side angle combinations, affect the total electricity generation. Table 5 presents annual electricity generation of roofs of the different shapes with different tilt/side angle combinations relative to the optimum roof (rectangle with gable roof of 45° tilt angle).

- The annual energy generation of shapes with basic roof design (45° tilt and side angles) shows that the trapezoid has the maximum generation (Fig. 6). T and H shapes have larger electricity production than the rectangular shape, while U and L generation approximate the generation of the rectangular shape (difference of 5% or less). Square shape has the lowest production of all shapes.

3.2.2. Effect of depth ratio on the basic roof

Only the basic roof (45° tilt and side angles) is considered for the analysis of the effect of depth ratio on electricity generation. Roofs of L shape variations are compared to the optimum roof. For depth ratio $a/b = 1/2$, the 45–45° L shape roof provides annual electricity generation of ca. 63% of the optimal roof, while for $a/b = 1$ and $a/b = 3/2$ the annual electricity generated is ca. 58% and 50% respectively, of the optimum roof. The 45–45° roof of the U shape with $a/b = 1/2$ generates 63% of the electricity generated by the optimal roof, while the roofs with $a/b = 1$ and $a/b = 3/2$ generate 60% and 54%, respectively. The rectangular 45–45° layout, by comparison, generates 62% of the annual electricity of the optimum gable roof.

The effect of the depth ratio on energy generation of L and U shapes is due primarily to the change in south-facing roof area, rather than to shading effects. The electricity generation of the basic L shape (with depth ratio = 1) is reduced by a maximum of 6% per unit area on a WDD, and by ca. 3% annually, as compared with the electricity generated per unit area of a non-shaded south facing roof. The annual electricity generation per unit area indicates that the maximum difference between different depth ratios is 4% or less for L shape.

3.2.3. Effect of wing angle

The effect of wing rotation on electricity generation is studied for L shape with 45° tilt and side angles. For this shape the angles β and θ affect the south branch and the main wing, respectively (Table 2), while in U shape they affect each of the branch wings and the effects can be expected to be similar. The peak electricity generation for all shapes, both for the south-facing roof of the main wing and the hip of the branch of L shape are displayed in Fig. 8, together with the annual electricity generation. Following are the main observations:

- The largest annual electricity generation is reached by the rotation of 45° of either β or θ. The normalized electricity generation per unit area indicates that this result is due to the large south roof area obtained by such variations (Table 6).
- The normalized data of the total annual electricity generation indicate that the highest electricity generation per m^2 is registered when the angle β or θ are at 15°, reaching some 3% higher value than the electricity generation of the basic shape, where β and $\theta = 0$. The annual electricity generation per unit area of the rectangular shape with 45° tilt angle is ca. 200.7 W/m^2. The rotation west of the south wing (varying angle β) allows in general higher annual electricity generation than rota-

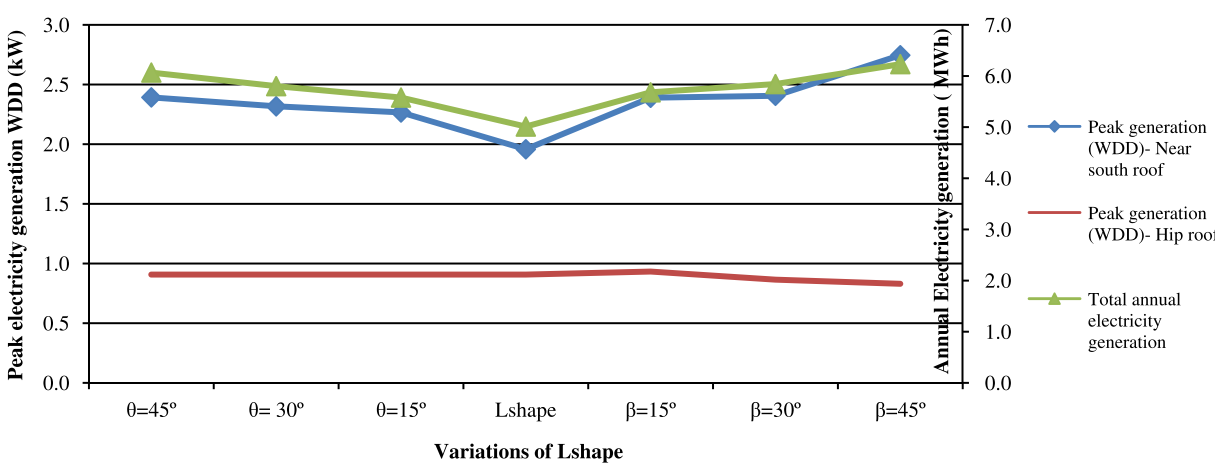
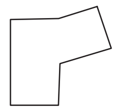
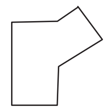
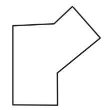
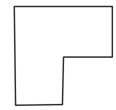
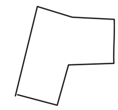
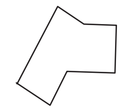
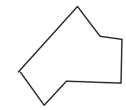


Fig. 8. Electricity generations of L shape variations, peak for WDD (kW) and annual generations (MW h).

Table 6
Summary of results of L shape variations.

Shapes								
								
Angle of variation	θ (°)	15	30	45	0	0	0	0
	β (°)	0	0	0	0	15	30	45
South-facing roof area (m^2)		20.4	21.8	23.9	17.6	20.6	21.6	23.7
Hip area(m^2)		8.1	8.1	8.1	8.1	8.1	8.1	8.1
Normalized total Annual energy(kW h/m^2)		195.5	193.8	189.6	194.7	197.7	196.5	195.8
Ratio of electricity generation to optimum roof		0.70	0.73	0.76	0.63	0.71	0.73	0.78

tion north of the main wing (varying angle θ). This is attributed to the fact that only the small triangular portion of the roof is rotated toward the west, while reducing the shade on the south facing area of the roof.

3.2.3.1. Peak generation. The hourly electricity generation during the WDD indicates that the different variations allow reaching the peak at different times, as shown in Fig. 9. The peak generation is reached in earlier hours than the solar noon, with the variation of the angle θ, while the opposite occurs when the angle β is changed. Fig. 9 shows the normalized peak generation per unit area of different variations of L shape on a WDD. A shift of peak generation of about 2 h can be reached relative to solar noon. A difference of peak of 3 h is observed between L variation with $\theta = 30°$ (peak between 11 and 12 h) and the variation with $\beta = 30°$ (peak between 13 and 14 h). The figure indicates that peak generation itself is not significantly sensitive to wing rotation. The difference in peak generation between the non shaded hip roof of basic L shape ($\beta = \theta = 0$) and that associated with rotation angles as large as 30° is less than 5%.

4. Design recommendations

This study investigates the potential of various geometric shapes of two-storey single family housing units to capture and exploit solar energy from near-south facing facades and roofs. Climatic, environmental and regulatory data employed in the simulations relate to northern regions and particularly to regions of Canada with similar climate to Montreal. While the specific results obtained are applicable to regions of similar climatic conditions, some general observations can be made, and these form the central topic of this section.

4.1. Shape effects on passive solar potential

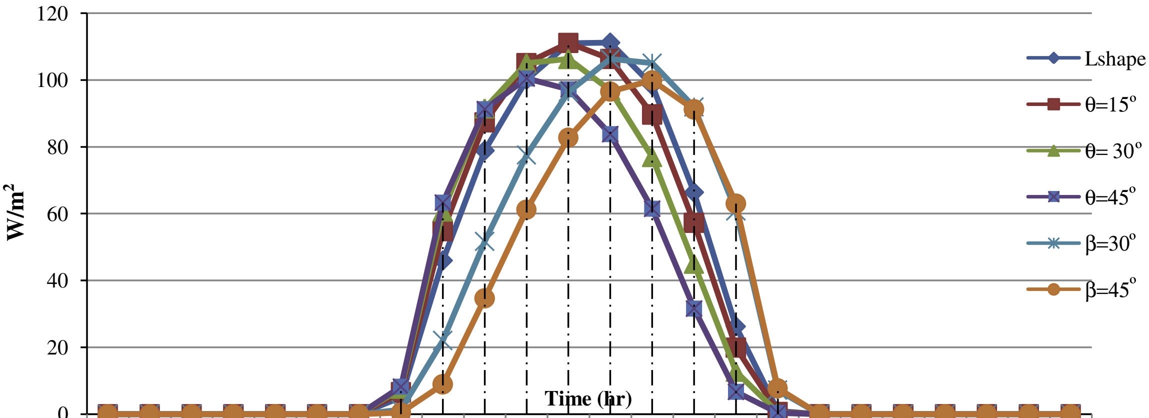
The study shows that several parameters should be considered in the optimization of shapes for passive solar design of housing units. Rectangular layout is generally considered the optimal shape for energy efficiency. Non-rectangular and particularly non-convex shapes offer a wider flexibility in architectural as well as solar design, but due to their complexity their efficient design is influenced by several parameters. Some of the observed effects on such shapes are summarized below.

- Solar radiation on non-convex shapes is significantly affected by the depth ratio – the ratio of shading to shaded façade lengths and the number of shading facades. These two parameters control the extent of shading and consequently reduction in radiation incident on, and transmitted by windows of, the shaded facade. It is therefore desirable to reduce the depth ratio in order to optimize the solar potential of facades. Depth ratio is particularly critical with a larger number of shading facades. For instance, the reduction of solar radiation on U shape, having two shading facades, is approximately double the values for L shape of the same depth ratios.
- Self-shading can be controlled and manipulated by variations of the basic geometry. By increasing the angle enclosed between shading and shaded facades, the shading effect can be mitigated. The present study indicates that while such manipulations may not significantly affect the overall incident or transmitted radiation, it can alter the distribution of radiation between non-shaded and shaded facades.

In general, optimization of energy performance in the climatic conditions studied is associated with maximizing sun exposed surfaces while maintaining a compact shape. Compactness is measured as the ratio of volume to perimeter surface.

4.2. Roof design

The basic design parameters in the presented roofs are the tilt and side angles of hip roofs. The BIPV conversion efficiency, considered constant in this study, is a significant parameter in the optimization of roof design. Further investigations should be conducted to account for this design parameter and its correlation with other roof design parameters such as orientation, shape and tilt angle. The following are important issues that should be considered



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Lshape	0	0	0	0	0	0	0	5	46	79	100	111	111	99	66	26	1	0	0	0	0	0	0	0
θ=15°	0	0	0	0	0	0	0	7	55	87	105	111	106	90	57	20	1	0	0	0	0	0	0	0
θ= 30°	0	0	0	0	0	0	0	8	61	92	105	106	97	77	45	13	0	0	0	0	0	0	0	0
θ=45°	0	0	0	0	0	0	0	8	63	91	101	97	84	62	32	7	0	0	0	0	0	0	0	0
β=30°	0	0	0	0	0	0	0	1	22	52	78	96	106	105	92	61	7	0	0	0	0	0	0	0
β=45°	0	0	0	0	0	0	0	0	9	35	61	83	97	100	91	63	8	0	0	0	0	0	0	0

Fig. 9. WDD electricity generation for L shape with wing rotation (kW).

in the design of hip roofs, to maximize their energy generation potential:

- In general, the energy generation potential of BIPV systems of all building shapes having the same tilt angle and orientation, is affected solely by the near south-facing area of the roof. It is therefore important to maximize the south facing roof area, for a predetermined floor area. This can be achieved through:
  - Optimal combination of side angle and tilt angle of a hip roof, associated with a specific plan design.
  - Manipulation of shape design such as demonstrated in L variations, where rotation of one wing leads to enlargement of the south facing roof area.

  Such considerations enable flexibility of design by providing several possibilities that allows tradeoffs of advantages and disadvantages of each option in a specific situation. For instance, there may be situations where the designer can opt for a larger roof area but a smaller tilt angle, thus saving material and interior space, or alternatively select higher tilt angle and smaller roof area.
- In self shading geometry, the shape produces mutually shading roof surfaces, as in the case of L and T shapes in the present study. In this case the depth ratio and number of shading surfaces should be considered (see above), especially if large tilt angles are used (>45°).
- The integration of PV systems in surfaces with different orientations, as studied in L shape variations, enables shifting the timing of peak generation by up to 2 h relative to solar noon. As a result, a spread of peak generation amounting to 3 h can be achieved among different variants of L shape. Time spread of peak electricity generation among housing units can result in a more even electricity generation profile, thus imposing less demand on the electric grid. This can be economically beneficial, since the cost and price of electricity often vary with time of day.

5. Concluding remarks

The solar potential of various shapes of two-storey houses is evaluated in this paper. Solar potential includes radiation incident on south facing facades and transmitted by the fenestration of these facades, and BIPV electricity generation. The main goal is to assess potential benefits and penalties associated with different plan layouts and roof shapes, as compared with the reference rectangular shape.

In rectangular layout, solar potential is governed by the area of south facing facades and roofs. The solar potential of non-convex building shapes is affected by the depth ratio of the shading to shaded facade (considering that the shaded facade is due south) and the number of the shading facades. The reduction of transmitted solar associated with a depth ratio of 1 on a WDD, can reach 26% for L shape and 46% for U shape, as compared to the rectangular shape. Shape effects, such as shading can be controlled and manipulated by variations of the basic geometry. Increasing the angle enclosed between shading and shaded wings, the shading effect can be mitigated by up to 27%. The radiation on the tilted roof and therefore the electricity generation per unit area by the BIPV system of the basic L shape (depth ratio of 1) is reduced by 6% on a WDD and ca. 3% annually, in comparison with the reference case. The difference of electricity generation between L shapes with depth ratio of 1 and ½ is less than 4%.

The rotation of a part of the roof, as studied in L shape variations, can be advantageous in shifting the timing of peak generation. A difference of about 3 h of peak generation can be achieved by the rotation of roofs between 30° east of south and 30° west of south, without compromising significantly the peak electricity generation (less than 5% difference as compared to south facing roof, for a WDD). Shifting peak generation time towards peak demand time can lower net energy cost and also reduce net peak demand from the grid.

Acknowledgments

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Glossary

Building-integrated PV (BIPV): Photovoltaic system designed as part of the roof system.

Depth ratio: The ratio of the width of a shading facade (orthogonal to south facade) to the width of shaded facade (the recessed south facade) in non convex shapes.

L variations: L shapes with varying angles between the two wings.

Non-convex shape: A shape where there exists at least one line segment drawn between two points located within the unit lies outside the shape. This shape has a self- shading geometry.

Non-shaded facade: South facing facade in a non-convex shape that is not shaded by any other facade, of the same shape.

Shaded facade: Facade in non-convex geometries that are shaded by other facades. In this paper they refer to the recessed south facing facades.

Shading façade: Facade in non convex geometries orthogonal or inclined to a shaded façade and casting shadow on it.

Solar potential: Potential of buildings to capture solar energy. In this paper, it consists of the cumulative solar radiation incident on the near-south facing facade, transmitted by the fenestration of this facade, and the electricity generated by the building integrated photovoltaic system (BIPV) of near south facing roof surfaces.

Summer design day (SDD): In this paper, SSD represents an extreme hot sunny day. SDD is used to check the performance of the building integrated photovoltaic system (BIPV).

Winter design day (WDD): In this paper, represents an extreme cold sunny day. The weather during WDD is ideal for passive solar design, so as to maximize the solar gain. WDD is also used to check the performance of the building integrated photovoltaic system (BIPV).