#2.)

$$3x + ay = 10 \qquad\qquad a = 2.100 \pm 5 \times 10^{-4}$$
$$5x + by = 20 \qquad\qquad b = 3.300 \pm 5 \times 10^{-4}$$

solve for x in one, sub in other

$$\frac{a}{3}y - \frac{b}{5}y = -\frac{2}{3} \qquad\qquad y = \frac{-2/3}{a/3 - b/5} = \frac{-10}{5a - 3b} \longrightarrow \boxed{y = -16.67}$$

$$\sigma_y^2 = \left(\frac{\partial y}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial y}{\partial b}\right)^2 \sigma_b^2 \qquad\qquad \sigma_a = \sigma_b = 5 \times 10^{-4}$$
$$\frac{\partial y}{\partial a} = \frac{50}{(5a - 3b)^2} = 138.9 \qquad\qquad \frac{\partial y}{\partial a} = \frac{-30}{(5a - 3b)^2} = -83.33$$
$$\left(\frac{\partial y}{\partial a}\right)^2 = 1.929 \times 10^4 \qquad\qquad \left(\frac{\partial y}{\partial b}\right)^2 = 6.944 \times 10^3$$
$$\left(\frac{\partial y}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial y}{\partial b}\right)^2 \sigma_b^2 = 6.559 \times 10^{-3} \qquad\qquad \longrightarrow \boxed{\sigma_y = 8.098 \times 10^{-2}}$$

$$\boxed{y = -16.67 \pm 0.081}$$

#5.) For the 3rd formula each successive term is smaller. The n-th term in sum, a_n, is $\sim 1/4n^2$. 4-byte reals can only hold ~ 7 digits of precision. For each decadal decrement in a_n, new terms contribute 1 fewer digit of precision. Once $a_n < 10^{-8}$ ($n \sim 4000$) new terms no longer contribute to sum, so the 3rd formula with single precision can not converge to true answer.

b.) many ways to solve this, here's one

$$\sum_{n=1}^{\infty} \frac{1}{2n(2n+1)} = \ln\frac{e}{2} = 0.30685281944005466...$$
$$\sum_{n=1}^{N} \frac{1}{2n(2n+1)} + \sum_{n=N+1}^{\infty} \frac{1}{2n(2n+1)} = \ln\frac{e}{2} \qquad\qquad \text{want} \sum_{n=N+1}^{\infty} \frac{1}{2n(2n+1)} \leq 10^{-7}$$
$$\sum_{n=N+1}^{\infty} \frac{1}{2n(2n+1)} = \frac{1}{2}\left(\psi^{(0)}(N + 3/2) - \psi^{(0)}(N+1)\right) \qquad \psi^{(0)}(z)\text{– “digamma function”}$$
$$\psi^{(0)}(N + 3/2) - \psi^{(0)}(N+1) \leq 2 \times 10^{-7} \qquad\qquad \longrightarrow \boxed{N = 2.5 \times 10^6}$$