#2.)
$$H_{ji} = \int_0^1 \sin(j\pi x)\left(i^2\pi^2 + \frac{x^2}{2}\right)\sin(i\pi x)dx$$

$$\text{if} \quad i \neq j: \quad H_{ji} = \frac{2ij(-1)^{i+j}}{(i-j)^2(i+j)^2\pi^2}$$

$$\text{if} \quad i = j: \quad H_{ji} = \frac{1}{24}\left(2 - \frac{3}{i^2\pi^2} + 12i^2\pi^2\right)$$

$$S_{ji} = \int_0^1 \sin(j\pi x)\sin(i\pi x)dx$$

$$\text{if} \quad i \neq j: \quad S_{ji} = 0$$

$$\text{if} \quad i = j: \quad S_{ji} = 1/2$$

$$S_{ji} = \frac{1}{2} \times \mathbb{I}$$

$$H\alpha_n = E_n S\alpha_n \quad \Rightarrow \quad 2H\alpha_n = E_n\alpha_n$$