#2.a) When x is small, sx term dominates. When x is large, x^3 term dominates.

$$\int_0^\infty (x^3+sx)^{-1/2}dx \approx \int_0^{x_0}(sx)^{-1/2}dx + \int_{x_0}^{x_1}(x^3+sx)^{-1/2}dx + \int_{x_1}^\infty x^{-3/2}dx$$
$$\approx 2\sqrt{x_0/s} + \int_{x_0}^{x_1}(x^3+sx)^{-1/2}dx + 2/\sqrt{x_1}$$

Let's set x_0 when ratio between terms is 10^{10}:

$$\frac{sx_0}{x_0^3} = 10^{10} \Rightarrow x_0 = \sqrt{s}\times 10^{-5}$$

Likewise for x_1:

$$\frac{x_1^3}{sx_1} = 10^{10} \Rightarrow x_1 = \sqrt{s}\times 10^{5}$$

We would still need to sum quadrature of x over ~ 10 orders of magnitude, which would take forever for any kind of decent accuracy. Transform $y = \log x$ to remedy this.

$$y = \log x \to dy = dx/x \Rightarrow dx = e^y dy$$
$$\int_{x_0}^{x_1}(x^3+sx)^{-1/2}dx = \int_{y(x_0)}^{y(x_1)}(e^{3y}+se^y)^{-1/2}e^y dy$$

#2.b) e^x goes to zero fast, integrate to where it basically disappears.

$$\int_0^\infty (x^2+1)^{-1/2}e^{-sx}dx \approx \int_0^{x_0}(x^2+1)^{-1/2}e^{-sx}dx$$
$$e^{-sx_0} = 10^{-10} \Rightarrow x_0 = 10\log(10)/s$$