Computational Earthquake Engineering Report #3 (Delaunay Triangulation & Conjugate Gradient Method)

2020/12/24

Consider a problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 \leq x \leq 1 \ \& \ 0 \leq y \leq 1),$$

with boundary conditions

$$\begin{aligned} u(0, y) &= 0, \quad (0 \leq y \leq 1), \\ u(x, 0) &= 0, \quad (0 \leq x \leq 1), \\ u(1, y) &= y, \quad (0 \leq y \leq 1), \end{aligned}$$

and

$$u(x, 1) = x, \quad (0 \leq x \leq 1).$$

1. Triangulate domain with nodes shown in Fig. 1 using Delaunay triangulation.
2. Triangulate domain with nodes shown in Fig. 2 using Delaunay triangulation.
3. Solve $u(x, y)$ using two-dimensional linear triangle elements with conjugate gradient solver, and compare numerical solution with analytical solution. Here, use mesh generated by Delaunay triangulation using nodes shown in Fig. 1.
4. Solve $u(x, y)$ using two-dimensional linear triangle elements with conjugate gradient solver, and compare numerical solution with analytical solution. Here, use mesh generated by Delaunay triangulation using nodes shown in Fig. 2.
5. Optional (will add points if answered): Compute the solution $u(x, y)$ for different mesh resolution and discuss the accuracy and convergence of the numerical solution.

Note: Implement your own Delaunay triangulation, finite-element program, and conjugate gradient solver for use in your report.

Due date: 2021/1/7 (submit source file [Fortran or C] and report on discussion of results [A4 2-4 pages in pdf] to ITC-LMS).

Questions: Kohei Fujita [fujita@eri.u-tokyo.ac.jp]

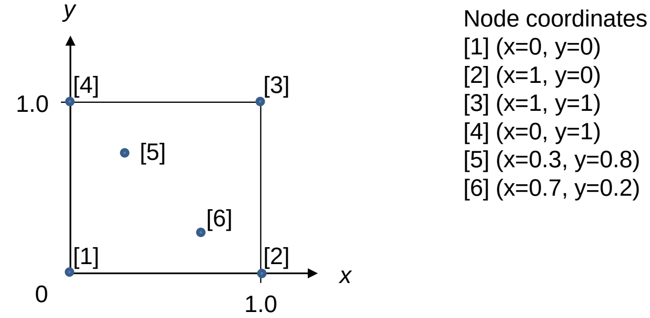

Figure 1

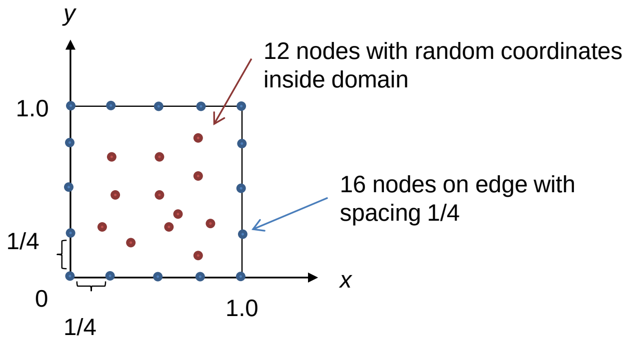

Figure 2