Linear Combination Transcript

In this example we have been asked to compute the expected value and the variance of random variable y here. And random variable y is a linear combination of three random variables x1, x2, and x3. The first random variable comes from a normal distribution which has parameters mu equals 1 and sigma equals 2. The second random variable comes from an exponential distribution with parameter lambda equals 3 and the last random variable comes from the t distribution with a parameter degree of freedom equal to 4 here.

So how do we do this? We just learned the property of linear combination of random variables so obviously we are going to use that. But before we apply that formula, clearly, we need to first find out the expected value and the variance for each one of them. The first one is normal, so it's easy, mu is the expected value and that is 1. The variance would be sigma squared, so that means it should be 2-squared, equals to 4. The second random variable is exponential, so the expected value should be 1 over lambda, and that's 1 over 3 here. And the variance should be 1 over lambda squared, which is 1 over 3 squared that means 1 over 9. The third one is a t-distribution and you can look up in the table, the expected value should be 0 and the variance should be computed as degree of freedom divided by degree of freedom minus two. So that's 4 over 2 and you get 2 here.

Now we are ready to compute the expected value of y, which according to the property is just the linear combination of the expected value for each one of the three random variables, and since we already computed the expected value of each one of the three random variables, we just need to plug them in. That means its 2 times 1 plus 3 times 1/3 minus 5 times 0. That's easy: 2 plus 1 minus 0, so the answer should be 3. The expected value of y should be 3.

What about the variance? To apply the property of the variance, remember that the three random variables have to be independent. In this case we do know they are independent—this is given in the problem. And what that means is now we can go directly apply the formula, which says well notice here you have to square the coefficient of each one and then do the linear combination. So that means I have 2 squared times variance of the first one plus 3 squared times the variance of the second one and then plus minus 5 squared, variance of the third one.

Now we just need to plug in numbers. 2 squared is 4 and the variance of the first one is 4, 3 squared is 9, the variance of the second one that's 1 over 9, and negative 5 squared, the negative sign goes away it's just25, and the variance of the last one is 2, so we do some arithmetic, and the answer is 67. So we know the expected value of y is 3 and the variance of y is 67.