Problem Set 3
BIOE5060: Biomolecular Dynamics and Control
April 7, 2015
Due: 5 p.m. April 22 (by email)

Problem 1. *Bistability and phase portrait for a first-order system.* Consider the situation where a stimulus activates A into A^*, and A^* undergoes first-order deactivation. The dynamics are given by

$$\frac{\mathrm{d}A^*}{\mathrm{d}t} = \left(\text{stimulus} + f\frac{A^{*n}}{K^n + A^{*n}}\right)(A_T - A^*) - k_i A^* \tag{1}$$

The parameter values are $n = 7$, $K = 0.5$, $A_T = 1$, $k_i = 0.2$.

(a) Does this system contain a positive or negative feedback?

(b) Plot the phase portrait for the system under the conditions given below and identify the stable and unstable steady-states:

i. $f = 0.3$ and stimulus $= 0$

ii. $f = 0.3$ and stimulus $= 0.06$

iii. $f = 0.3$ and stimulus $= 0.1$

(c) What is the qualitative change in behavior if $f = 0.4$?

Problem 2. *Phase portrait and stability analysis of a linear second-order system.* Consider the following linear system of differential equations:

$$\dot{x} = -3x + 2y \tag{2}$$
$$\dot{y} = x - 2y \tag{3}$$

(a) Construct the phase portrait for this system.

(b) Identify the nullclines and the fixed points, indicating whether they are stable or unstable.

(c) Show the vector field.

(d) Derive the general solution, including the eigenvalues and eigenvectors.

(e) Which is the slow eigendirection?

(f) Draw the manifolds on the phase portrait and indicate whether they are stable or unstable.

Problem 3. *Complex eigenvalues.* Consider the linear system shown below:

$$\dot{x} = x - y \tag{4}$$
$$\dot{y} = x + y \tag{5}$$

Determine the eigenvalues and eigenvectors. Write the general solution in terms of real-valued functions. *Note*: No graphs are requested for this problem.

Problem 4. *Stability analysis for a non-linear system.* In Box 1 of the Elowitz and Leibler (2000) paper, the authors state that the system is unstable (i.e., exhibits oscillations) when

$$\frac{(\beta+1)^2}{\beta} < \frac{3X^2}{4+2X} \tag{6}$$

Derive this condition for instability. *Note*: No graphs are requested for this problem.