# Biostatistics
# Assignment 1

## Instructions

1. Download the `R markdown` assignment template from Blackboard or the web
2. Open the file `dynamics-assnTemplate.Rmd` in RStudio
3. Insert the relevant information (i.e., assignment number, student name)
4. Save the file as `assnNUMBER-yourLastName.Rmd` and replace `NUMBER` with the assignment number
5. For each question, your answer should consist of (1) the `R` code used to generate the results and (2) the interpretation of the results
6. Generate a PDF of the `R Markdown` document by clicking on the **"`Knit PDF`"** icon in RStudio
7. Email a PDF and `R Markdown` version of your completed assignment to the instructor
8. The assignment is due on Monday February 23

## Problem 1: Designing optimal harvesting regimes (30 points)

You are tasked by NOAA to determine the optimal harvesting regime for the Cod population in the southern Gulf of Maine. Specifically, you are asked to determine the **maximum sustainable yield** (MSY; i.e., the maximum number of individuals that can be harvested from the population without reducing its growth rate). NOAA asks you to determine the constant (i) *per capita* and (ii) *per* population harvesting rates corresponding to the MSY. Assume that the population reproduces continuously and largely experiences logistic growth.

1. Write down the model equations for the constant (i) *per capita* vs. (ii) *per* population harvesting rate and conduct a dimensional analysis to compare and contrast these mathematical depictions (**4 points**)
2. Find the maximum sustainable yield $h$ for the constant *per* population harvesting model. To do so, you will need to (i) determine the population size $N$ at which the *per* population growth rate $\frac{\mathrm{d}N}{\mathrm{d}t}$ reaches its maximum and (ii) find the corresponding value of $h$ (**4 points**)
3. Verify that your solution is correct by plotting the growth rate $\frac{\mathrm{d}N}{\mathrm{d}t}$ as a function of abundance $N$ in red and determining computationally whether the the population size $N$ at which the population growth rate is maximal corresponds to the value that you found analytically. To do so, you will need to use function `which.max` to find the index of the maximum value in the vector of population sizes. Plot the population size at which the population growth rate is maximal using a blue dashed line. Assume that $r = 0.5$, $K = 100$, $h = 0.1$ (**4 points**)
4. Verify your results by simulating the dynamics of the constant *per* population harvesting model for 100 time steps with $r = 0.5$, $K = 100$, $N_0 = K$, and set $h$ to the maximum sustainable yield you calculated previously. Plot population size as a function of time in red and add a black horizontal dashed line to represent the population size at which the population growth rate is maximal. Does the behavior of the model confirm the validity of your harvesting strategy? Justify your answer (**5 points**)

5. Find the maximum sustainable yield $h$ for the constant *per capita* harvesting model. To do so, you will need to (i) determine the population size $N$ at which the *per* population growth rate $\frac{dN}{dt}$ reaches its maximum and (ii) find the corresponding value of $h$ (**4 points**)
6. Verify that your solution is correct by plotting the growth rate $\frac{dN}{dt}$ as a function of abundance $N$ in red and determining computationally whether the the population size $N$ at which the population growth rate is maximal corresponds to the value that you found analytically. To do so, you will need to use function `which.max` to find the index of the maximum value in the vector of population sizes. Plot the population size at which the population growth rate is maximal using a blue dashed line. Assume that $r = 0.5$, $K = 100$, $h = 0.1$ (**4 points**)
7. Verify your results by simulating the dynamics of the constant *per capita* harvesting model for 100 time steps with $r = 0.5$, $K = 100$, $N_0 = K$, and set $h$ to the maximum sustainable yield you calculated previously. Plot population size as a function of time in red and add a black horizontal dashed line to represent the population size at which the population growth rate is maximal. Does the behavior of the model confirm the validity of your harvesting strategy? Justify your answer (**5 points**)

## Problem 2: Dealing with environmental stochasticity (20 points)

The models developed in the previous section assumed that there was no environmental stochasticity affecting the reproductive rate and population size of Cod. To determine the sensitivity of your maximum sustainability yield calculations, you decide to develop a discrete-time logistic growth model with constant *per* population harvesting and time-variable carrying capacity.

1. Find the maximum sustainable yield $h$ for the discrete-time constant *per* population harvesting model with constant carrying capacity. To do so, you will need to (i) determine the population size $N$ at which the *per* population growth rate $\Delta N = N_{t+1} - N_t$ reaches its maximum and (ii) find the corresponding value of $h$ (**5 points**)
2. To simulate the effects of environmental stochasticity on harvesting regimes, assume that the population's carrying capacity $K$ varies in time according to a normal distribution with a mean of $K = 10$ and 100 linearly-spaced standard deviation values ranging from 0.1 to 2. Assume that $N0 = K$, $R = 0.5$, and 100 total time steps. You will test three separate harvesting regimes: MSY, 95% of MSY and 90% of MSY. For each of these MSY values, run 1,000 replicate simulations of the discrete-time logistic growth model for each standard deviation value and record whether the population size at the end was greater than 0. Use function `aggregate` to average these persistence values across the distinct standard deviation values to determine how the mean persistence probability under each MSY varies with the standard deviation of $K$. Finally, plot three curves on the same panel showing the mean probability of persistence (y-axis) as a function of the variability of the carrying capacity $K$ (x-axis). Note that these simulations will take 5-15 minutes to complete depending on your computer's CPU (**10 points**).
3. What do these results suggest regarding the sensitivity of your MSY calculations to environmental stochasticity affecting the carrying capacity? Based on the simulations, what harvesting regime will you recommend and why? (**5 points**)