

Bank Loan Reliance and Inflation Inattention^{*}

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November 14, 2024

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Abstract

Utilizing merged Italian firm-level administrative and survey data, we provide causal evidence that firms heavily reliant on bank loans are better informed about inflation and make smaller forecast errors. We also show that financing composition affects how firms learn from new information with randomized control trials. To explain these findings, we develop a partial equilibrium model featuring rational inattention where firms become more attentive when their financing costs are sensitive to aggregate inflation. Inflation impacts the relative cost of external versus internal funding, leading firms to adjust their investment, capital structure, and attention allocation. This mechanism, in combination with the heterogeneous financing structure across firms, generates dispersion in inflation expectations, replicating the empirical evidence. A novel policy implication of our model is that the aggressiveness of monetary policy toward inflation stabilization affects the degree of monetary non-neutrality through the firms' attention allocation channel and resulting inflation expectations.

Keywords: Inflation expectation, Financing structure, Bank loan reliance, Rational inattention, Monetary non-neutrality

JEL Codes: D8, D25, E31, E32, E52

^{*}We thank Marc Dordal i Carreras, Yuriy Gorodnichenko, Byoungchan Lee, Yang Lu, and seminar participants in Cafe Macro in UCB and Macro Student Mini Symposium in HKUST for their comments and suggestions. Zhenghua Qi thanks UC Berkeley, where the first draft of this paper was written, for the hospitality. The views expressed here should not be interpreted as representing the views of the Bank of Italy or any other institution with which the authors are affiliated.

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1 Introduction

As central banks increasingly focus on steering inflation expectations to stabilize the economy, extensive literature has documented the heterogeneous inflation expectations of firms and the varying degrees of attention to aggregate conditions. However, few studies have causally identified the factors that shape firms' inflation expectations and explore the reasons behind systematic differences in firms' information quality. This paper aims to fill the gap by proposing that the financing structure is an important determinant of firms' attention to inflation. We further illustrate novel insights into the interplay between financing decisions and the effectiveness of monetary policy.

We empirically and theoretically document how firms' financing composition, specifically the degree of bank loan reliance, causally affects the attentiveness to inflation. First, we show that firms more dependent on bank loans are better informed about inflation and make smaller forecast errors. Second, we develop a partial equilibrium rational inattention model where firms can choose their capital structure between external and internal fundings and attention allocation. We find that heterogeneous financing costs and rationally inattentive firms lead to dispersed inflation inattention, as observed in the data. Finally, we document a novel monetary policy implication that the aggressiveness of monetary policy toward inflation stabilization affects the effectiveness of monetary policy through the firm-side attention allocation channel and resulting inflation expectations.

The empirical analysis utilizes a comprehensive dataset compiled from multiple sources, including a firm-level panel survey and administrative records with detailed information on balance sheets and credit positions for Italian firms. The representative survey of the Italian firms asks about firms' economic expectations and incorporates a repeated randomized information treatment that provides publicly available information about inflation to firms. The attentiveness to inflation of each firm is captured by one-year ahead inflation forecast errors, computed using their inflation expectation data in the survey. The financing conditions of each firm are represented by its bank loan reliance, defined as the proportion of bank loans in the firm's total assets.

The endogeneity of expectation formation and financing decisions is a major challenge to identifying the causal relationship between the two. We conduct causal inferences by employing a Bartik instrument ([Bartik, 1991](#); [Blanchard and Katz, 1992](#); [Goldsmith-Pinkham et al., 2020](#)) and leveraging randomized information treatment embedded in the survey design. First, we construct a Bartik

instrument for bank loan reliance using exogenous credit supply shocks following [Khawaja and Mian \(2008\)](#). Our empirical analysis shows that firms relying more on bank loan financing have smaller inflation forecast errors after controlling for size, profitability, liquid asset share, and firm fixed effect, suggesting that the financing structure of a firm affects the incentive to process information regarding inflation dynamics. Second, we investigate the role of financing composition in affecting how firms learn from new information on inflation. Using the randomized control trial (RCT) design of inflation information provision in the survey, we compare firms' inflation forecasts before and after receiving the information on current inflation. The extent of the post-treatment forecast adjustments shed light on the importance of different factors underlying firms' learning process from the new information. We find that firms relying heavily on bank loans do not significantly adjust their inflation expectations post-treatment. In contrast, firms utilizing other funding sources more than bank credit significantly update their forecasts. This result suggests that firms borrowing substantially from banks have actively acquired inflation information and hold well-informed prior beliefs based on publicly available inflation news, resulting in minor revisions because the treatment merely confirms what they already know. Our firm-side explanations for the inflation inattention fluctuations emphasize firm-level financing composition, complementing the findings in, e.g., [Weber et al. \(2023\)](#) and [Pfäuti \(2024\)](#) focusing on the inflation inattention depending on the aggregate economic conditions.

To investigate the underlying mechanisms for this financing composition-driven inflation inattention, we develop a partial equilibrium model of firms and banks. Firms choose a mix of internal funds and bank loans to finance their capital expenditure. Banks obtain deposits at the policy rate and issue loans to firms, where the loan rates are subject to infrequent adjustment a la [Calvo \(1983\)](#). We assume that banks operate in a monopolistic competitive market following [Ulate \(2021\)](#). In this economy, an increase in inflation triggers contractionary monetary policy responses, yielding simultaneous increases in the deposit rates, the bank's funding costs, and, consequently, loan interest rates¹. In this manner, firms' optimal decisions are linked to inflation dynamics, which determine the relative cost of internal funds and bank credit.

To capture the salient inattentiveness to inflation by firms in the data, we assume acquiring and processing information is costly as in the rational inattention framework of, e.g., [Sims \(2003\)](#); [Wood-](#)

¹See [Cuciniello and Signoretti \(2015\)](#) and [Head et al. \(2022\)](#)

ford (2009); Matějka (2016); Maćkowiak et al. (2018). In an imperfect information environment, firms select the quantity and structure of information to inform their investment and capital accumulation decisions. Because firms more dependent on external credit are more exposed to inflation fluctuations via variable bank loan rates, they choose to process more information and form more precise expectations. Consequently, the extent of a firm's bank loan reliance influences its information set choices and, subsequently, shapes its inflation forecasts' accuracy. In this manner, our model rationalizes the empirical finding of the state-dependent learning process of new information by firms. Firms' financing structures affect their attention to inflation and the accuracy of inflation expectations.

We obtain analytical solutions to the model using the methods developed by Maćkowiak et al. (2018). Those tractable model equations allow us to clearly illustrate how structural parameters and the monetary policy stance affect the level of attention to inflation. First, our theory predicts that economies where bank loans are the predominant financing sources, such as Italy and Japan, tend to exhibit more attentive firms to inflation than those with low debt-to-equity ratios, like the United States and Germany, other things being equal. Second, the model implies that firms pay more attention to inflation when inflation is more volatile. Third, when the central bank responds more aggressively to deviations of inflation from its target, inflation news becomes more relevant to firms. These theoretical implications echo the insights from the recent empirical findings in, e.g., Weber et al. (2023) that attention to inflation is endogenous and depends on aggregate economic conditions.

Related literature. Firms' beliefs and actions are interconnected with each other. Their expectations affect decisions and characteristics, which, in turn, influence future expectation formation. Most of the existing empirical literature examines the direction from expectations to decisions by employing experimental designs in surveys to evaluate the impact of information treatments on firms' subsequent decisions. For example, Coibion et al. 2018 investigates firms' expectation formation in New Zealand. Ropele et al. 2022 and Ropele et al. 2024 use the same RCT design in Italian firm surveys focusing on inflation expectations' influence on borrowing decisions and resource misallocation. The knowledge of the determinants of expectation formation is rather limited in the literature and is mostly based on illustrating correlations rather than causality. For example, Coibion et al. (2018), Kumar (2020), Afrouzi (2023), and Yang (2022) study the correlation between expectation and several factors, such as holdings of liquidity assets, competition, and product scope. By exploiting the richness of our combined

dataset, we provide causal evidence on the channel from heterogeneous financing composition to differential information acquisition incentives and inflation forecasts across firms.

This paper is related to studies investigating (1) the role of inflation expectations, (2) rational inattention, and (3) corporate finance. Inflation expectation is at the central position of dynamic macroeconomic frameworks regarding the determinants of inflation and Phillips curves (Coibion and Gorodnichenko, 2015; Hazell et al., 2022; Meeks and Monti, 2023), consumption decisions through intertemporal substitutions (Woodford, 2003), investment and capital prices, asset prices, forward-looking monetary policy rules (Clarida et al., 2000; Orphanides, 2001; Coibion and Gorodnichenko, 2012), and unconventional monetary policy via communication (Coibion et al., 2020). Given the importance of inflation expectations, we complement these studies by causally identifying rather understudied determinants of the inflation expectation, i.e., firms' bank loan reliance.

Our empirical findings support predictions of the rational inattention theory (see Maćkowiak et al., 2023, for a review). Agents have limited information processing capacity; therefore, they will strategically allocate their attention.² Our theoretical framework is based on Maćkowiak et al. (2018), which offers analytical solutions to linear-quadratic-gaussian tracking problem under rational inattention (see also Afrouzi and Yang, 2021; Miao et al., 2022, for related formulation and solutions). Wang et al. (2023) address the role of financial constraints in affecting firms' information acquisition through the channel of strategic complementarity in pricing. In our framework, we focus specifically on the role of reliance on bank credit. Mackowiak and Wiederholt (2024) shows that the rational inattention theory could predict the treatment effects in RCTs, consistent with our results.

Outline. The remainder of this paper is organized as follows. Section 2 presents the data and identification methods. Section 3 discusses the empirical results. Section 4 describes our theoretical framework and model implications based on the analytical solutions to the partial equilibrium rational inattention model. Section 5 concludes the paper.

²See Sims (1998) and Sims (2003) for classic papers on rational inattention, and Mackowiak and Wiederholt (2009) and Maćkowiak and Wiederholt (2015) for applications to pricing and business cycles.

2 Data and Measurement

To analyze how firms' reliance on bank credit influences their attention to inflation, we integrate data from four sources: 1) the Italian Central Credit Register (CCR), which provides detailed information on the volume of loans granted by banks to their customers; 2) the Analytical Survey of Interest Rates (TAXIA), which collects data on lending rates; 3) the Company Accounts Data System (CADS), which contains balance sheet information for Italian limited liability companies; and 4) the Survey of Inflation and Growth Expectations (SIGE), which includes firms' inflation expectations and several relevant variables that help us further investigate firms' (in)attention to inflation.

2.1 CCR

The CCR is an information system managed by the Bank of Italy that collects granular information on loans granted by banks and other financial intermediaries to firms and households. By maintaining this database, the Bank of Italy provides the participating banks and other financial intermediaries with a flow of information that can enhance their ability to assess and monitor their customers' creditworthiness and better manage credit risk. Once a month, banks and other financial intermediaries are required to report the debtor position of each customer at the end of the reference month if it equals or exceeds the relevant threshold value for three different categories of loans, e.g., term loans, revocable loans, and matched loans.³

Term loans are credit transactions with a contractually fixed term and no form of predetermined repayment. For example, term loans include leases, mortgages, personal loans, and subordinated loans. These loans can have different original maturities and types of interest rates (fixed or adjustable). Firms typically use term loans to finance capital expenditures. Revocable loans are current account overdrafts granted for short-term cash needs. In this case, the bank has the right to withdraw from the contract regardless of the existence of just cause. Matched loans include transactions with a form of predetermined redemption. This form of financing allows customers to obtain immediate access to credit that is not yet past due and for which the reporting bank has control over the cash flows (this occurs when the bank acquires the credit, has an irrevocable collection order, and the credit is paid by standing order to the same bank). For example, matched loans include advances on receivables

³For further information on the Italian Central Credit Register, see [Ropele et al. \(2022\)](#).

related to factoring, advances on invoices, and other advances on bills and documents representing trade receivables.

In what follows, we use term loans to construct a benchmark measure of firms' loan reliance for two reasons. First, term loans include leasing, mortgages, personal loans, and subordinated loans, which are primarily used for capital investment. Second, since term loans are mostly long-term loans with more than one-year durations⁴, firms facing intertemporal decisions need to acquire information to assess future credit conditions. We focus on term loans to explore the information acquisition incentives that arise from the financial needs associated with capital investment. As a robustness exercise, we will also consider the total loan amount, i.e., the sum of term, revocable, and matched loans.

2.2 TAXIA

TAXIA has been conducted quarterly by the Bank of Italy since March 2004. The survey collects information on lending rates for each customer, while it collects information on deposit rates on an aggregate basis. With regard to lending rates, the survey considers the cost of loans granted by the Italian branches of reporting banks separately for term loans, revocable loans, and matched loans. Banks are required to provide information for each customer for which the total debtor exposure reported in the CCR at the end of the reference quarter is at least 75,000 euros. For all outstanding loans at the end of each quarter, banks report the "products" and the related "amounts" received by the banks during the reference quarter. The products are computed as the daily balance of credit times the number of days. The amounts received by the banks comprise interest payments, fees, and other expenses. Based on this information, the annualized nominal interest rate effectively charged to a customer j during a quarter t ($rate_{j,t}$) is calculated using the following formula: $rate_{j,t} = \frac{Amounts_{j,t} \times 365}{Products_{j,t}}$.

2.3 CADS

The CADS is a proprietary database owned by Cerved Group S.p.A., a leading information provider in Italy and a major credit rating agency in Europe. CADS includes detailed information on balance

⁴See Table 1 in the Appendix of [Coibion et al. \(2023\)](#)

sheets and income statements for almost all Italian limited liability non-financial companies since 1993. Information is drawn from official data recorded at the Italian Registry of Companies and financial statements filed at the Italian Chambers of Commerce. Companies provide data on a compulsory basis. Each company's financial statement is updated annually. This dataset includes yearly balance sheet information on various assets and liability items, as well as yearly income statement information.

2.4 SIGE

The SIGE is a quarterly business survey run by the Bank of Italy since December 1999.⁵ The survey design targets the universe of firms operating in industry excluding construction and non-financial private services⁶ with administrative headquarters in Italy and employing at least 50 workers. Since the first quarter of 2013, construction firms with at least 50 employees have been included. The sample is stratified by sector of economic activity (industry, non-financial private services, and construction), geographical location (North-West, North-East, Centre, South, and Islands), and number of employees (50-199, 200-999, 1000 and over). In recent years, each wave has seen the participation of about 1,200 firms (500 in industry excluding construction, 500 in non-financial private services, and 200 in construction). Over the years, about 2,500 firms have taken part in the survey. The list of firms from which the sample is extracted is drawn from the Bureau Van Dijk's Aida database and is updated on average every five years. Sampling weights ensure that the distribution of firms (in terms of employment) in the sample matches the distribution of firms in the reference population. Data are mostly collected in the first three weeks of March, June, September, and December. The response rate is about 45 percent on average. The purpose of the survey is to obtain information on firms' expectations concerning inflation, the general economic situation, own-product prices, demand, investment, and employment. Most of the data - with the exception of own-product price changes (past and expected), inflation expectations, and current number of employees - are qualitative and relate to firms' assessments of their own business activity as well as about macroeconomic matters in the reference quarter and looking ahead.⁷

⁵Until October 2018, the survey was conducted jointly with the economic newspaper *Il Sole 24 Ore*.

⁶The survey excludes the following: financial intermediaries and insurance companies, the general government, the educational and healthcare sectors, and other community, social, and personal services.

⁷More information about the survey is provided in [Grasso and Ropele \(2018\)](#).

2.5 Measurements of Italian firms' reliance on bank credit and attention to inflation

In this section, we present the two primary variables of interest for our study: a measure of firms' reliance on bank credit and a measure of firms' attention to inflation. We define a quarterly measure of firm-level reliance on bank credit at the end of period t as

$$\text{Loan Reliance}_{j,t} \equiv \frac{\sum_{i \in \text{banks}} \text{Term Loan}_{i,j,t}}{\text{Asset}_{j,t}}, \quad (1)$$

where the numerator represents the total amount of term loans borrowed by firm j from the banking system (with banks indexed by i), and the denominator represents firm j 's total asset. A few remarks are in order. First, as in other countries, Italian firms typically borrow from several banks. In our sample of firms with at least 50 employees, over the period 2006–2019, firms had, on average, credit relationships with 4.8 banks if we consider only term loans and with 6.9 banks if we consider the three categories of loans. This large cross-bank variation within each firm helps to identify credit supply shocks and, hence, exogenous variation in bank credit dependency.⁸ We turn to this in the next section. Second, as the total liabilities of firms are available in CADS at a yearly frequency, we simply repeat the annual figure for each quarter of the year.

Figure 1 shows that our measure of firms' reliance on bank credit stands on average at approximately 23% during our sample period from 2006Q1–2019Q4, exhibiting some time variation and a slight decline during the sovereign debt crisis. Notable differences arise when splitting firms across observable characteristics. Specifically, the degree of loan reliance appears to be higher for firms operating in the industrial sector, of medium size, and headquartered in the southern regions of Italy.⁹ For such firms, reliance on bank loans is a persistent firm-level characteristic, indicating their consistent demand for bank credit to finance the investment. In the theoretical part, we interpret the persistent demand coming from a relatively lower bank financing cost than other financing methods. The heterogeneity in firms' average bank loan reliance contributes to the inflation expectation dispersion observed in the survey. We calibrate the model parameters to match the empirical distribution of loan reliance.

Turning to the second main variable, we measure firms' attention to inflation by the absolute value

⁸The fraction of firms with only one credit relationship is below 5%. Nearly 15% of firms borrow term loans from only one bank.

⁹Further summary statistics on firms' loan reliance are reported in Table 1.

of their inflation forecast errors since they are positively correlated with the level of inattentiveness to inflation (see [Coibion and Gorodnichenko, 2012](#), Equation (5)).¹⁰ We elicit inflation expectations from SIGE, which asks firms to provide forecasts of future growth in consumer prices in Italy over different horizons (see [Coibion et al., 2020](#), for detail). Specifically, before 2012Q3, all firms in the survey received the following question *"In [previous month], consumer price inflation measured by the 12-month change in the Harmonized Index of Consumer Prices was [X.X]% in Italy and [Y.Y]% in the Euro area. What do you think it will be in Italy . . . "* over three different horizons: six-month ahead, one-year ahead, and two-year ahead. In 2012Q3, the Bank of Italy redesigned the survey to randomly assign firms to one of two separate groups that received different questions. About two-thirds of the sample received the original question above, while the remaining one-third received the following question *"What do you think consumer price inflation in Italy, measured by the 12-month change in the Harmonized Index of Consumer Prices, will be. . . "* over the same horizons. This new formulation does not provide any additional information about the recent inflation dynamics.¹¹

In the following, we focus on 1-year ahead inflation expectations, as they have the longest historical coverage. Hence, our first measure of inflation (in)attention is given by the absolute value of the inflation forecast error:

$$\text{Attention}_{j,t}^{(\pi)} \equiv \left| \pi_t^{(12m)} - F_j \pi_t^{(12m)} \right|, \quad (2)$$

where $\pi_t^{(12m)}$ represents the annualized 12-month ahead inflation rate at quarter t , and $F_j \pi_t^{(12m)}$ is the 12-month ahead inflation forecasts reported by firm j at quarter t . As shown in Figure 2, the cross-sectional mean of firms' attention to inflation has fluctuated significantly over the sample period tracking the level of inflation.

It appears that there is no significant difference in the average level of attention to inflation when firms are grouped according to standard firm-level characteristics. However, the level of inflation attention significantly varies across firms with our novel firm-level characteristic of bank loan reliance.

¹⁰Alternative measures in the literature include the backcast error and willingness to pay for professional inflation forecasts ([Yang, 2022](#)), the nowcast errors capturing the knowledge about industry-level and aggregate inflation ([Afrouzi, 2023](#)), and the probability of forecast update ([Andrade and Le Bihan, 2013](#)). Due to data limitations, we do not have these measures. In Section 3.2, we employ an alternative econometric strategy of utilizing an RCT following [Weber et al. \(2023\)](#) to measure attention to inflation and show that our results are robust to this change.

¹¹In 2017Q2, the Bank of Italy introduced a new randomized treatment, with a fraction of firms being informed about the ECB's inflation target. As our sample ends in 2019Q4, we decided not to consider these data given the short sample period.

Before showing more formal causal inferences in the next sections, here we present reduced-form evidence on the relationship between firms' reliance on bank credit and their attention to inflation. In particular, we construct a binned scatter plot (18 bins) of firms' inflation forecast error versus firms' loan reliance pooled across periods. We residualize both variables, i.e., we remove observable characteristics fixed effects (sector, area, and size) and information treatment fixed effects. As shown in Figure 3, there appears to be a statistically significant negative relationship between the two variables. This evidence indicates that firms with a higher dependence on bank loans exhibit smaller forecast errors, suggesting more significant attention to inflation by those firms. However, this estimate does not necessarily reflect the magnitude of the causal effects of loan reliance on inflation attention. We tackle this issue in the next section.

3 Empirical results

In this section, we identify the causal effect of firms' loan reliance on their attention to inflation using two different approaches. First, we capture an exogenous variation in loan reliance by constructing a Bartik instrument. Second, we leverage the RCT design to directly observe the revision in inflation forecasts after the information treatment.

This section serves two purposes. First, it offers insights into the paper's main mechanism: a firm's reliance on bank loans affects its attentiveness to inflation, from which it can infer the expected aggregate credit condition because inflation influences the loan spreads via policy rates. Second, it motivates the model where paying attention to inflation is an endogenous decision that depends on firm characteristics, including its financing structure.

3.1 Identification using a Bartik instrument

OLS regression. We start from the following ordinary least squares (OLS) regression of firms' inflation inattention in Equation (2) on loan reliance in Equation (1):

$$\text{Attention}_{j,t}^{(\pi)} = \alpha + \beta_{OLS} \cdot \text{Loan Reliance}_{j,t} + \tau_j + \epsilon_{j,t}, \quad (3)$$

where τ_j captures unobserved firm fixed effects. The results are shown in Table 2, column (6). We compute standard errors robust to heteroskedasticity and autocorrelation following Driscoll and Kraay (1998). Note that $\hat{\beta}_{OLS}$ is a small, statistically insignificant, negative number. However, the OLS estimator is subject to potential measurement errors and endogeneity. Our measurement of $\text{Loan Reliance}_{j,t}$ might involve errors because the total liability in the denominator in Equation (1) is available only at an annual frequency. Unobserved time-varying firm-level characteristics, such as the manager’s traits, might also influence both financing decisions and attention allocation¹², inducing potential biases in $\hat{\beta}_{OLS}$.

Bartik instrument for loan reliance. To address the concerns above, we construct a Bartik instrumental variable (IV) for bank loan reliance. We proceed in three steps. First, we obtain bank-specific credit supply conditions from loan markups by decomposing them into bank-specific (credit supplier) and firm-specific (credit demander) factors. Second, we quantify a firm’s exposure to different banks’ credit conditions through its loan portfolios borrowed from multiple banks. Finally, we construct a Bartik instrument by taking an average of bank-specific credit supply shocks weighted by the lagged exposure of a firm to each bank in its loan composition. Our identification assumption is that when facing higher external financing costs due to negative credit supply shocks to banks, firms decrease borrowing from banks and shift parts of their external funds to internal sources, reducing the reliance on more costly bank loans.

We construct our credit supply shocks using the loan markup data, denoted by $\text{Loan Markup}_{i,j,t}$. The loan markup is given by the spread between loan and policy rates ($\text{Loan Interest Rate}_{i,j,t} - \text{Policy Rate}_t$), where the loan rate is the rate for outstanding term loans between bank i and firm j at time t , and the policy rate is the ECB deposit facility rate. Note that this extra interest rate margin of bank loans relative to the risk-free rate approximates the relative cost of external funds compared with cash or retained earnings.

Next, we turn to the identification of the bank-specific credit supply shocks. Similar to Khwaja and Mian (2008) and Amiti and Weinstein (2018), we decompose the loan markup into three terms,

¹²For example, see for Malmendier et al. (2011) evidence on how early experiences shape manager’s debt aversion

representing credit supply shocks, credit demand conditions, and residual shocks:¹³

$$\text{Loan Markup}_{i,j,t} = \phi_{i,t} + \phi_{j,t} + \epsilon_{i,j,t}, \quad (4)$$

where $\phi_{i,t}$ and $\phi_{j,t}$ are bank-time and firm-time fixed effects, respectively. $\epsilon_{i,j,t}$ captures residual shocks. The estimated bank-time fixed effects, $\hat{\phi}_{i,t}$, in the decomposition isolate variations in the loan spread driven by the bank(supply)-side factors, i.e., credit supply shocks. In contrast, the influences of the firm(demand)-side elements, such as size, industry, and riskiness, are captured by $\phi_{j,t}$.

Firms collaborate with multiple banks in Italy, as discussed in Section 2.5. When firms encounter higher loan rates from a particular counterpart i , it is captured by a larger $\phi_{i,t}$ than the other bank-time fixed effects. We interpret it as representing the fact that bank i is in a relative shortage of credit and, consequently, charges higher borrowing costs.¹⁴

Credit supply shocks to banks are transmitted to firms borrowing from the affected banks. In light of this channel, we construct a measure of each firm's exposure to each bank in Italy as follows:

$$Exposure_{i,j,t-1} = \frac{\text{Outstanding Term Loan}_{i,j,t-1}}{\sum_{i \in \text{banks}} \text{Outstanding Term Loan}_{i,j,t-1}}.$$

The numerator is the outstanding term loan amount between bank i and firm j at time $t - 1$ obtained from the CCR database. The denominator is the total volume of firm j 's outstanding term loans within the Italian banking sector. Thus, this share measures the reliance and strength of the connection between borrower (firm j) and lender (bank i).

Finally, the Bartik instrument is an average of credit supply conditions weighted by lagged exposures:

$$\bar{\phi}_{j,t} = \sum_{i \in \text{banks}} Exposure_{i,j,t-1} \cdot \hat{\phi}_{i,t}.$$

¹³Khawaja and Mian (2008) and Amiti and Weinstein (2018) utilized the growth rates of loan sizes. Instead, we focus on the loan markup, the spread between loan and policy rates, because the relative financing costs directly affect firms' financing composition decisions.

¹⁴A potential concern for this decomposition method might be its omission of bank-firm-time interaction terms. As a result, our decomposition might overlook the possibilities of bank-lending and firm-lending specializations. That is, to gauge demand factors using firm-time fixed effects, we implicitly assume that a firm's credit demand remains the same across all banks. Similarly, a given bank's credit supply is assumed to be uniform across all firms. However, Amiti and Weinstein (2018) formally proved that including the bank-firm-time fixed effects does not change the estimates of $\phi_{i,t}$ in the decomposition. Thus, integrating the triple interaction terms does not affect the consistency of our estimator.

2SLS regression. Equipped with the Bartik instrument for loan reliance, we estimate the causal effect of loan reliance on inflation (in)attention using the following two-stage least squares (2SLS) regression:

$$\begin{aligned} \text{First-stage:} \quad & \text{Loan Reliance}_{j,t} = \beta_{\text{Bartik}} \cdot \bar{\phi}_{j,t} + \tau_j^f + \nu_{j,t}, \\ \text{Second-stage:} \quad & \text{Attention}_{j,t}^{(\pi)} = \beta_{2SLS} \cdot \widehat{\text{Loan Reliance}}_{j,t} + \tau_j + \epsilon_{j,t}, \end{aligned} \quad (5)$$

where $\widehat{\text{Loan Reliance}}_{j,t}$ represents the fitted value from the first-stage regression, and τ_j^f and τ_j are the firm-fixed effects in the first- and second-stage regressions, respectively. Those fixed effect terms, controlling for unobserved firm-level determinants of the attention level and loan reliance, allow us to sharpen the identification of β_{2SLS} in combination with the use of the Bartik instrument, $\bar{\phi}_{j,t}$. In the second-stage regression, we consider additional firm-level controls, such as the firm size measured by the number of employees,¹⁵ the profitability captured by the return on equity (ROE), and the asset liquidity represented by the ratio of the cash and cash-equivalent assets to the total asset. Finally, we include an RCT dummy, being equal to one if firm j is treated in quarter t and zero otherwise. We use [Driscoll and Kraay \(1998\)](#) standard errors given cross-sectional and time correlation in the error terms.

We report the 2SLS estimates in Table 2, columns (1)-(5). Three results are worth mentioning. First, $\hat{\beta}_{2SLS}$ is statistically significantly negative, which is robust to the inclusion of different sets of control variables. Thus, the accuracy of inflation forecasts improves when loan reliance intensifies. The estimate in column (1) implies that a one percentage point exogenous increase in loan reliance leads to a 0.12 percentage point reduction in the size of inflation forecast errors. Second, our instrumental variable is strong. In all cases, the first-stage F-statistic is greater than 10, the threshold for strong IV suggested by [Staiger and Stock \(1997\)](#). Also, $\hat{\beta}_{\text{Bartik}}$ is negative because negative credit supply shocks to the banking sector raise loan markups and external funding costs for firms (see Equation (4)), making them less rely on bank loans. Finally, the OLS estimate in column (6) is small and statistically insignificant. This difference between the OLS and 2SLS estimates emphasizes the importance of addressing the potential endogeneity of inattention in relation to firms' financing composition.

¹⁵Some papers find little evidence that size is systematically related to firms' inflation expectations and inattention (see, e.g., [Coibion et al. \(2018\)](#) for New Zealand, and [Candia et al. \(2021\)](#) for the US.)

In summary, our causal evidence is consistent with the hypothesis that when firms rely more on bank loans as a funding source, they are more attentive to inflation. Our interpretation of this result is that this differential attentiveness to inflation depending on the financing structure across firms arises because external funding costs (i.e., bank lending rates) are more tightly connected to inflation than internal funding costs. Thus, to comprehensively understand the impact of firms' financing structure on their inflation expectations, it is essential to examine the endogenous nature of the attention allocation decisions made by firms.

3.2 Assessing (in)attention to inflation using randomized information treatments

Our second empirical exercise utilizes random information treatments embedded in the SIGE following [Weber et al. \(2023\)](#). The RCT design enables us to identify how firms update their inflation expectations in response to the treated inflation information depending on the degree of loan reliance and the prior belief on future inflation. In light of the three explanatory variables, including the prior, information treatment, and loan reliance, we employ a regression with the triple interaction term. We examine whether the treatment effects on the weights on the priors when forming inflation expectations vary with firms' reliance on loans.

We use the two specific SIGE waves: the 2013Q1 and 2017Q3 waves. Prior to 2013Q1, all firms in the survey received information on consumer price inflation over the last year, which was also publicly available. Then, in the 2013Q1 wave, the RCT was first introduced into the survey, where a randomly selected sample of firms stopped receiving information about the recent inflation dynamics. In the 2017Q3 wave, the treatment and control groups were reshuffled, moving some firms from the control group to the treated group and vice versa. Given the RCT structure in these two waves, we compare the relationship between firms' prior and posterior inflation expectations to evaluate the causal effect of the information treatment on expectation formation.

In the following analysis, the inflation expectations reported in the previous wave, $\pi_{t-1}^{(12m)}$, is used as a proxy for the prior belief about inflation over the next year. The posterior belief, formed after the information treatment for the firms in the treated group, is measured by the inflation expectations

reported in the current wave, $\pi_t^{(12m)}$. Our regression equation is given by:

$$\begin{aligned} F_j \pi_t^{(12m)} = & \alpha_1 \cdot F_j \pi_{t-1}^{(12m)} + \alpha_2 \cdot F_j \pi_{t-1}^{(12m)} \times \text{Loan Reliance}_{j,t} \\ & + \gamma_1 \cdot F_j \pi_{t-1}^{(12m)} \times RCT_{j,t} + \gamma_2 \cdot F_j \pi_{t-1}^{(12m)} \times RCT_{j,t} \times \text{Loan Reliance}_{j,t} \\ & + \delta_1 \cdot RCT_{j,t} + \delta_2 \cdot \text{Loan Reliance}_{j,t} + \delta_3 \cdot RCT_{j,t} \times \text{Loan Reliance}_{j,t} + \epsilon_{j,t}, \end{aligned} \quad (6)$$

where $RCT_{j,t}$ is a dummy variable equal to one if firm j received information treatment in quarter t . The first line in Equation (6) captures how the prior belief influences the posterior for the control group, depending on the level of loan reliance. The second line represents the contribution of information treatment on the posterior.

Our hypothesis is that γ_1 is negative and γ_2 is positive. When $\gamma_1 < 0$, the treated firms utilize the provided information and put smaller weight on their priors. γ_2 reflects how this change in the weight on priors varies with the reliance on bank credit. For firms relying more on bank loans, inflation dynamics are more relevant to their investment decisions because inflation is tightly connected to fluctuations in lending rates and the corresponding external funding costs. Thus, those firms endogenously choose to acquire more information about inflation and form more informed priors. Then, the information treatment based on publicly available past consumer inflation figures would conform to what they already know, making the post-treatment revision of inflation expectations smaller and the weight on priors larger, i.e., $\gamma_2 > 0$.

We summarize the influence of firms' loan reliance on their learning process of the treated inflation information by the scaled slope, $\frac{\hat{\gamma}_1 + \hat{\gamma}_2 \text{Loan Reliance}}{\hat{\alpha}_1 + \hat{\alpha}_2 \text{Loan Reliance}}$. This moment represents the difference between the marginal effects of priors on posteriors between the treated and control groups (numerator) relative to that of the control group (denominator). In short, it captures how sensitively the treated firms revise their inflation forecasts according to the treated information, conditioning their financing structure.

Figure 4 plots the scaled slope against the degree of loan reliance. Panels (a) and (b) are based on the 2013Q1 and 2017Q3 waves, respectively. In both cases, the scaled slopes are negative and diminish with the loan reliance measure. Because the treated firms allocate less weight on their prior and more weight on the information ($\gamma_1 < 0$), the scaled slope tends to be negative. Furthermore, the fact that highly bank-loan-reliant firms respond less to the information treatment and rely more

on their priors ($\gamma_2 > 0$) leads to the diminishing treatment effect with the loan reliance. Thus, the results based on the randomized information provision are consistent with our hypothesis: firms relying more on bank loans actively acquire inflation information. Then, the inflation information provided in the RCT is already in their information set, leading to minor adjustments in posterior inflation expectations.

Our empirical results above emphasize the endogenous nature of information acquisition and expectation formation. Firms choose their attention level to inflation news in light of the incentive to do so, which in turn is affected by their financing structure, specifically, the reliance on bank loans.

4 A Theoretical Model

This section develops a stylized partial equilibrium model with endogenous financing composition and imperfect information to shed light on the mechanisms underlying our empirical findings. In the model, firms combine internal funds and bank loans to finance investment spending. Banks absorb deposits at the policy rate and set the loan interest rates. The monetary authority determines the policy rate according to an exogenously given level of inflation. Thus, fluctuations in the level of inflation could affect the policy rate and the operational cost for banks, which further passed to firms through the loan rates, influencing their intertemporal investment decision. Then, rationally inattentive firms choose the attention level to inflation, given this channel from inflation to their external funding costs. We obtain an analytical solution to the model using the methods developed by [Maćkowiak et al. \(2018\)](#).

Notation. Below, capital and lower-case letters represent the values in level and logarithm, respectively. Hatted variables mean the log deviations from the steady state values, where the latter are denoted by capital letters without time subscripts.

4.1 Firms

Firms, indexed by j , produce goods using the firm-specific capital stock.¹⁶ They choose the optimal combination of internal financing and bank loans to minimize the unit cost of capital installment.

¹⁶We abstract away from labor because it is less relevant to our main mechanism. The working capital constraints for wage payment are known to be short-term and flexible, usually financed by matched loan and revocable loans. Instead, as in our empirical analysis, we focus on capital investment financed by term loans.

Then, with the minimized unit financing cost, firms determine the investment level to maximize the present discounted value of profits. We do not explicitly model corporate bonds because their workings are similar to bank loans in our context, and they only account for a small portion (approximately 10 %) of the total liability for Italian firms (see [Meucci and Parlapiano, 2021](#)). Furthermore, as shown in Table 1, the median bank credit-to-debt ratio is 95%, implying that the other forms of debt are not popular in Italy.

Financing decision. Firms choose an optimal combination of internal and external funding to finance their capital expenditure in each period. The cost of internal funding (e.g., retained earnings) is normalized to 1. The relative cost of the bank loan equals the loan markup, $\Phi_{j,t} = \frac{R_t^B}{R_t} > 1$, where R_t^B and R_t denote the bank loan and deposit rates (gross), respectively. We assume that the deposit rate is equal to the policy rate set by the monetary authority. The internal ($\Gamma_{j,t}^I$) and external ($\Gamma_{j,t}^B$) financing is summed up to the total available credit using a constant elasticity of substitution (CES) technology with elasticity of substitution $\rho > 1$. This specification flexibly and tractably captures several sources of imperfect substitutability between internal and external funds, such as bankruptcy risks, asymmetric information ([Ross, 1977](#)), and moral hazard ([Jensen and Meckling, 1976](#)) (see also [Myers 1984](#) for the pecking order theory justifying the imperfect substitutability between internal and external funding). Note that the case of perfect substitutability is nested in this formulation with $\rho = 1$.

The cost minimization problem for the unit credit is given by:

$$\begin{aligned} \min_{\Gamma_{j,t}^I, \Gamma_{j,t}^B} & \Gamma_{j,t}^I + \Phi_{j,t} \Gamma_{j,t}^B \\ \text{s.t.} & \left[(\Gamma_{j,t}^I)^{\frac{\rho-1}{\rho}} + (\Gamma_{j,t}^B)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} = 1 \end{aligned}$$

The optimal financing composition follows from the first-order conditions (FOCs) and the constraint:

$$\Gamma_{j,t}^I = \left(\frac{1}{1 + \Phi_{j,t}^{1-\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad \Gamma_{j,t}^B = \left(\frac{\Phi_{j,t}^{1-\rho}}{1 + \Phi_{j,t}^{1-\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad \Gamma_{j,t}^I + \Phi_{j,t} \Gamma_{j,t}^B = (1 + \Phi_{j,t}^{1-\rho})^{\frac{1}{1-\rho}} \equiv M_{j,t},$$

Thus, bank loan reliance and unit financing costs are endogenously pinned down by the loan markups charged by banks. It is straightforward to show that when the loan markup, $\Phi_{j,t}$, decreases (and

$\rho > 1$), the bank loan reliance, $\Gamma_{j,t}^B$, increases and the unit cost of the capital installment, $M_{j,t}$, decreases.

Profit maximization. Firms produce using a decreasing return-to-scale production function ($0 < \phi < 1$) and make an investment decision financed by the optimal bundle of internal and external credit. φ_k and δ represent the magnitude of investment adjustment costs and capital depreciation rates, respectively. Firms maximize the present discounted value of profits:

$$\max_{K_{j,t}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left[K_{j,t}^\phi - M_{j,t} [K_{j,t} - (1 - \delta)K_{j,t-1}] \right].$$

We assume that firms keep enough amount of internal reserve by, e.g., retaining earnings, for $\Gamma_{j,t}^I [K_{j,t} - (1 - \delta)K_{j,t-1}]$.

Taking first-order condition with respect to capital stock yields the following equation:

$$\phi K_{j,t}^{\phi-1} = M_{j,t} - (1 - \delta)\beta \mathbb{E}_t M_{j,t+1}$$

With the decreasing return to the scale production function, the optimal capital level, $K_{j,t}$, negatively depends on the loan markup, $M_{j,t}$, given their expected values in the next period. Intuitively, when the loan markup increases, the financing cost increases, and, therefore, firms decrease investment.

Loan markups. We construct the firm-time level loan markup $\hat{\phi}_{j,t}$ by summing across banks with the exposures as the weights. Assuming no response in firms' credit demand or other time-varying firm characteristics affecting the markup¹⁷, we decompose the firm-time loan markup into the firm-level component, $\hat{\phi}_j$, and the time-level component, $\hat{\phi}_t$.

$$\hat{\phi}_{j,t} = \sum_i \text{Exposure}_{i,j,t} \cdot \hat{\phi}_{i,t} + \sum_i \text{Exposure}_{i,j,t} \cdot \hat{\phi}_{j,t} + \sum_i \text{Exposure}_{i,j,t} \cdot \epsilon_{i,j,t} = \hat{\phi}_t + \hat{\phi}_j + \epsilon_{j,t}$$

Inflation affects loan markup through ϕ_t through the banking market with the channel depicted in the following section.

¹⁷Our empirical design controls the demand-driven variation from the firms. The sticky price is the only source of heterogeneous pass-through rates by banks to firms.

4.2 Banks and the central bank

Banks. There exists a continuum of banks, indexed by $i \in [0, 1]$. Banks obtain deposits at the policy rate (R_t) set by the central bank and provide credit to firms in the form of bank loans with the loan rate ($R_{i,t}^B$). Following Ulate (2021), the loan market is monopolistically competitive. Thus, banks have the market power to set their loan interest rates higher than the policy rate and earn positive profits. To capture realistic frictions in the banking business and the dispersion in bank-specific factors in loan markups in the data in a tractable manner, we introduce sluggish dynamics in individual loan rates; Banks could reset their loan interest rates with probability $1 - \omega_b$ in each period in a similar vein to staggered price setting in Calvo (1983).

We assume firms borrow a loan bundle, B_t , which is an aggregated credit of loans initiated by individual banks, $B_{i,t}$. In light of (unmodeled) realistic features in the banking sector, such as switching costs between banks, asymmetric information, menu costs, and regulatory restrictions, we assume that $B_{i,t}$ is not perfectly substitutable across i s and aggregated into B_t via a CES technology (see also Ulate, 2021, for an alternative micro-foundation for the CES specification). The CES structure implies that the demand for bank i 's loan equals $\left(\frac{R_{i,t}^B}{R_t^B}\right)^{-\theta_b} B_t$, where θ_b is the elasticity of substitution across different banks' loans, and $R_t^B = \left[\int (R_{i,t}^B)^{1-\theta_b} di\right]^{\frac{1}{1-\theta_b}}$.

When it is possible to reset the loan rate, bank i chooses the loan interest rate, $R_{i,t}^B$, to maximize the sum of discounted profits while the reset interest rate remains effective.

$$\max_{R_{i,t}^B} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \omega_b^k \beta^k \left[(R_{i,t}^B - R_t) \left(\frac{R_{i,t}^B}{R_t^B} \right)^{-\theta_b} B_t \right] \right\},$$

Central bank and inflation. The monetary policy rule of the central bank is based only on inflation:

$$R_t = R \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\tau_\pi}$$

where the aggressiveness of the monetary policy toward inflation is captured by τ_π . To focus on our novel mechanism behind the endogenous choice of firms' attention to inflation, we assume that the

log-linearized inflation is exogenously given by the AR(1) process:

$$\hat{\pi}_t = \rho_\pi \hat{\pi}_{t-1} + \epsilon_{\pi,t}.$$

Our simple AR(1) specification for inflation dynamics captures the Italy inflation data reasonably well.

Loan markup. Combining the FOC regarding the loan interest rate, $R_{i,t}^B$, with the monetary policy rule, and transfer the system into log-linearized version, we obtain analytical expressions for the dynamics of the loan markup, $\hat{\phi}_t = \hat{r}_t^B - \hat{r}_t$, and the optimal capital, $\hat{k}_{j,t}$, in terms of the inflation shocks, $\epsilon_{\pi,t}$:

$$\hat{\phi}_t = (\omega_b + \rho_\pi) \hat{\phi}_{t-1} - \omega_b \rho_\pi \hat{\phi}_{t-2} + \Omega_2 \epsilon_{\pi,t} + \omega_b \tau_\pi \epsilon_{\pi,t-1}, \quad (7)$$

$$\hat{k}_{j,t} = \rho_1 \hat{k}_{j,t-1} + \rho_2 \hat{k}_{j,t-2} + q_1^j \epsilon_{\pi,t} + q_2^j \epsilon_{\pi,t-1} + q_3^j \epsilon_{\pi,t-2} \quad (8)$$

The explicit expressions for the parameters, $\{\Omega_2, \rho_1, \rho_2, q_1^j, q_2^j, q_3^j\}$ can be found in Appendix. The coefficients on the auto-regressive terms for capital are the same across firms, while the sensitivity of capital to inflation shocks differs across firms depending on the financing composition. Firms with higher loan reliance have larger q_1^j and q_2^j . Note that an unanticipated increase in inflation leads to increases in the policy rate, which squeezes the bank's profit margin ($\Omega_2 < 0$) by reducing the loan markup. In the Appendix, we provide empirical evidence using local projection for this model setting.

4.3 Rational inattention

Now, we move to the scenario where the firm has limited information processing capacity, and a more precise signal is more costly. Following [Maćkowiak et al. \(2018\)](#), we assume the firm chooses the information structure and amount of attention paid at the beginning of the problem at time $t = -1$ subject to an information flow constraint. We put the rational inattention problem and its solution in the Appendix.

4.4 Calibration

The benchmark parameter values are listed in Table 3.

5 Model implications

Using the analytical solution to the model, we illustrate three major implications of the model. First, by varying the level of loan markup, we show that when firms rely more on bank loans, they pay more attention to inflation. As a result, the inflation forecast errors are smaller in response to inflation shocks for firms using more external funding, replicating the first set of our empirical results based on the Bartik instrument. Second, using the simulated data from the model by mimicking the RCT design, we obtain a similar relationship between the loading on prior beliefs and loan reliance to what we obtain from the SIGE data, confirming our interpretation of the empirical results based on endogenously determined (in)attention to inflation according to the financing structure of a firm. Finally, our model provides a novel policy implication that the aggressiveness of monetary policy toward inflation stabilization, captured by τ_π in the policy rule, increases firms' attention level to inflation and, as a result, the accuracy of inflation expectation. Thus, in the case of a hawkish central bank, communication policies aiming at providing information to influence inflation expectations might be less effective.

5.1 Loan markup, bank loan reliance, and inflation forecast errors

5.1.1 Comparative statistics

In this section, we conduct comparative statics analyses on the steady-state equilibrium under varying parameter values. We examine the relationship between the total amount of processed information, κ , and parameters influencing firms' loan reliance and sensitivity to inflation. As shown in the first panel of Figure 6, attention is strictly decreasing in Φ_j . An increase in Φ_j raises the relative cost of external credit, thereby reducing firms' reliance on bank loans. Since inflation shocks affect investment decisions only through the loan financing cost, a financing composition with fewer bank loans makes a firm's investment less sensitive to aggregate inflation. Consequently, firms have a lower incentive to acquire and process information on inflation, replicating the main empirical findings in Section XX.

Moreover, attention increases with the aggressiveness of the central bank's responses to inflation, denoted by τ_π . A higher τ_π implies that the policy rate will rise more sharply after an increase in inflation, leading to higher costs for banks and more expensive loans for firms. Compared to a low-

τ_π regime, the reliance on bank loans decreases more after inflation shocks of the same magnitude, while the bank loan interest rate increases more significantly. Both effects result in greater variations in firms' investment under a high- τ_π scheme, creating incentives to pay more attention to inflation. This state-dependent attention aligns with recent findings by [Coibion et al. \(2023\)](#). The final panel illustrated a scenario where reduced information costs encourage firms to process more information. When the marginal (utility? profit?) cost doubles from the calibrated value of 0.001 to 0.002, the capacity of the information channel, measured by the mutual entropy between XX and YY (is it the case?), decreases by approximately XX% from XX bit to YY bit.

5.1.2 IRFs and Simulation

Inflation shock Figure 7 plots the impulse response functions of various variables in our model to a one standard deviation shock in the inflation process. After a positive shock in $\epsilon_{\pi,t}$, the rising inflation triggers the contractionary monetary policy responses with higher policy rates, reducing the loan markup when banks are limited by the sticky price constraint. This lowers the firm's relative external financing cost and immediately boosts the investment. Subsequently, as banks gradually have the opportunity to adjust loan interest rates, loan markup increases, and the optimal investment ratio decreases. The black solid line ($v_{j,t}^*$) represents the scenario under perfect information where firms could take full advantage of the lower financing cost and increase capital installment. While the firm's perceived optimal investment, $\mathbb{E}_t v_{j,t}^*$, exhibits smaller responses in magnitude compared to the perfect-information case. This difference is due to firms only partially updating their expectations from the new signals received.

We also examine the responses to inflation shocks under varying levels of the firm-side financing cost determinant, Φ_j . Higher loan financing costs mean a lower reliance on bank credit, smaller exposure to inflation shocks, and less incentive to pay attention to inflation. Consequently, responses to inflation shocks are more subdued in a high-financing cost regime (average loan reliance at 11%) compared to scenarios where firms have stronger banking relationships (average loan reliance at 24%). The deviations from the optimal investment rate are larger and more persistent under a more inattentive economy. The second and third plots in Figure 7 illustrate that firms make larger errors in predicting inflation as their signals are less precise. The model suggests that in an economy with a

smaller proportion of firms connected to banks, the response of investment to aggregate shocks would be more limited.

5.2 RCT in the model and the data

To replicate our empirical findings on the varied adjustments to information treatment in RCT, as illustrated in Figure 4, we run the same regression based on the dataset generated from the model. We create the firm-level panel data through the following steps: first, we generate the firm-specific loan markup Φ_j for 375 firms, drawing from a Beta distribution. The Beta distribution parameters are calibrated to match the empirical distribution of loan reliance in Italy during our sample period. We solve each firm's attention problem and its corresponding signal structure. We simulate the model over a span of $T = 115$ periods, cutting the first 100 periods as burn periods. The firms are randomly assigned into the treatment group (250 firms) and control group (125 firms). At time $T_{RCT} = 101$, we give the firms a more precise signal on current inflation and allow firms to revise their expectations based on this new information, thus replicating the RCT's treatment effect. The control group's inflation expectations remain anchored to their exiting information set until time T_{RCT} , while the treated group receives the signal whose standard deviation of the signal's noise term is reduced to 0.0015. This reduced standard deviation is 5% of the level that the firms with benchmark calibration choose. Following the practice in our empirical study, we measure the prior using one-quarter ahead inflation forecasts (at $T_{RCT} - 1$) and the posterior with the reported expectation after the treatment (T_{RCT}) to construct our simulated dataset replicating the 2013Q1 wave. By conducting the same regression analysis on our simulated data, the results, depicted in Figure 5, align with the decreasing adjustment pattern observed in empirical data, thus confirming the validity of our stylized model.

5.3 Policy implications

Investment inefficiency and inflation forecast errors We now examine the relationship between inefficiency on capital installment and inflation inattention, proposing that a firm's inflation forecast error partly mirrors its divergence from ideal investment levels. We then assess the gap between perceived and actual optimal investment—assuming perfect information—to gauge capital investment inefficiency. Additionally, we derive perceived inflation expectations from the model to measure the

forecasting inaccuracies. To facilitate analysis, we conduct simulations under two distinct scenarios, as previously described: the "high scheme" characterized by a higher firm-specific external financing cost Φ_j , or a higher Taylor rule coefficient in front of inflation τ_π , or elevated information processing costs λ_κ . The distribution of deviations from optimal investment and forecast errors are plotted in Figure 8. This comparative approach allows us to assess how different financial and policy conditions affect firms' investment efficiency and forecasting accuracy in inflation.

In the first scenario, where the external financing cost for the firm increases from 1.025 to 3, firms reduce their efforts to gather information, resulting in inflation forecast errors (illustrated by the blue line) being more dispersed and less centered around zero. The impact on the distribution of optimal investment is less significant. With a higher Taylor rule coefficient τ_π , a firm's financing costs become more sensitive to inflation shocks. This sensitivity has a dual effect: on the one hand, the optimal investment rate itself is more sensitive to fluctuations and harder to predict. On the other hand, firms devote more resources to tracking the optimal investment rate, leading to improved signal precision. The net impacts in terms of accuracy on investment are relatively small and less significant than those on inflation forecast errors. Firms under the high- τ_π scheme have better information on the current inflation since the volatility of the inflation process is invariant with τ_π . The information processing cost only affects κ and does not affect the variance of the state space. Consequently, with increased information processing costs, firms' deviations from the ideal scenario are greater due to reduced attention levels.

The firms' rational inattention problem focuses on minimizing the distance between the predicted and optimal investment rate while the accuracy of other variables remains secondary (instead of being targeted), determined by the outcomes of the rational inattention problem. Within our framework, the perceived inflation error is influenced by discrepancies in current and previous investment rates and shocks. This relationship can be explicitly represented as:

$$\mathbb{E}_t v_{j,t}^* = \omega_b \mathbb{E}_t v_{j,t-1}^* + X_1 \mathbb{E}_t \pi_t + X_2 \mathbb{E}_t \epsilon_{\pi,t} + X_3 \mathbb{E}_t \epsilon_{\pi,t-1}$$

where the $v_{j,t}^*$, $v_{j,t-1}^*$, $\epsilon_{\pi,t}$, and $\epsilon_{\pi,t-1}$ are unknown at time t . Due to the information processing costs, even though inflation data is publicly available, firms remain uninformed until they acquire and process the information. X s are coefficients, the details and derivation of which are relegated to the

Appendix. The errors in inflation expectations tend to be more volatile and partially reflect deviations in investment.

6 Conclusion

Understanding how firms form expectations is crucial for assessing the effectiveness of monetary policy. Our empirical analysis, using detailed firm-level administrative and survey data in Italy, shows that firms' financing composition significantly influences their attention to information on current and future inflation, affecting their inflation expectations. We provide causal evidence that firms relying heavily on bank loans are better informed about inflation and make smaller forecast errors. Utilizing an RCT embedded in the survey, we find that highly bank loan-reliant firms are less responsive to information treatment, supporting the state-dependent attention predicted by the rational inattention theory. To explore the underlying mechanisms, we build a partial equilibrium model featuring rationally inattentive firms within a monopolistically competitive banking market. This model demonstrates how bank credit demand and inflation's impact on credit costs create varied incentives for firms' information acquisition. Our model successfully replicates the empirical findings and provides a novel policy implication. The effectiveness of monetary policy is linked to the level of firms' attention to inflation, which is state-dependent and influenced by individual firms' bank loan reliance and the central bank's aggressiveness toward inflation. More hawkish monetary policy leads to more attention to inflation and higher responsiveness from the firms.

This paper focused on the implications of heterogeneous financing conditions across firms for inflation and monetary policy. A potentially important source of heterogeneity in financing methods might be the aggregate economic conditions at the time of entry and early stages of a firm's life cycle. Investigating how such heterogeneity emerges and how firm-level financing structures are shaped is left for future research.

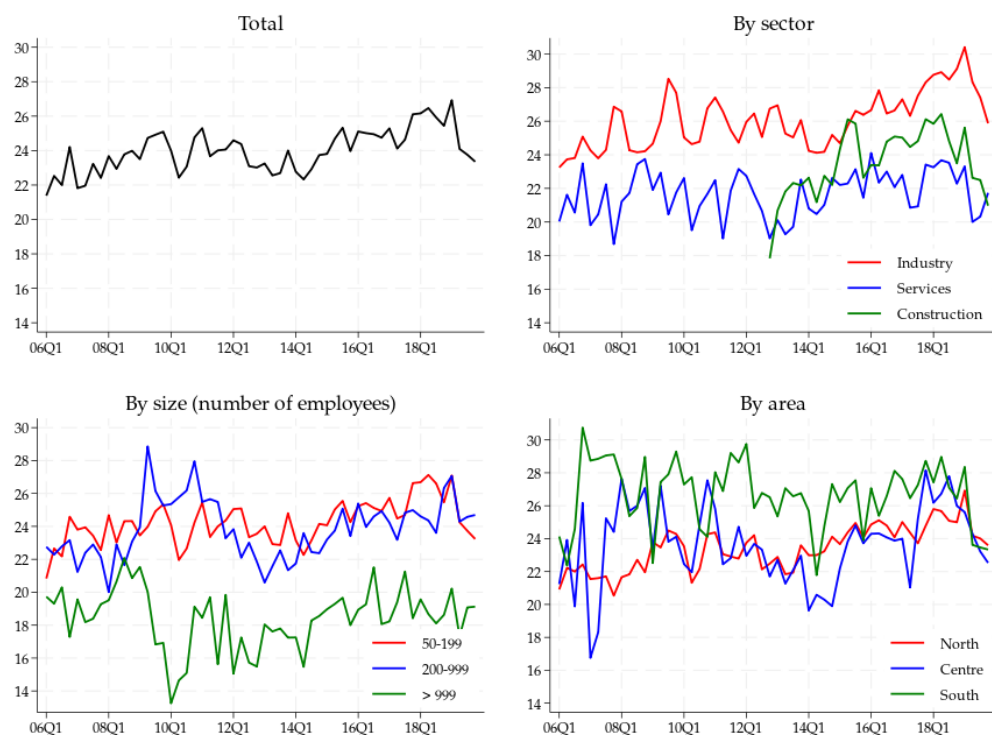
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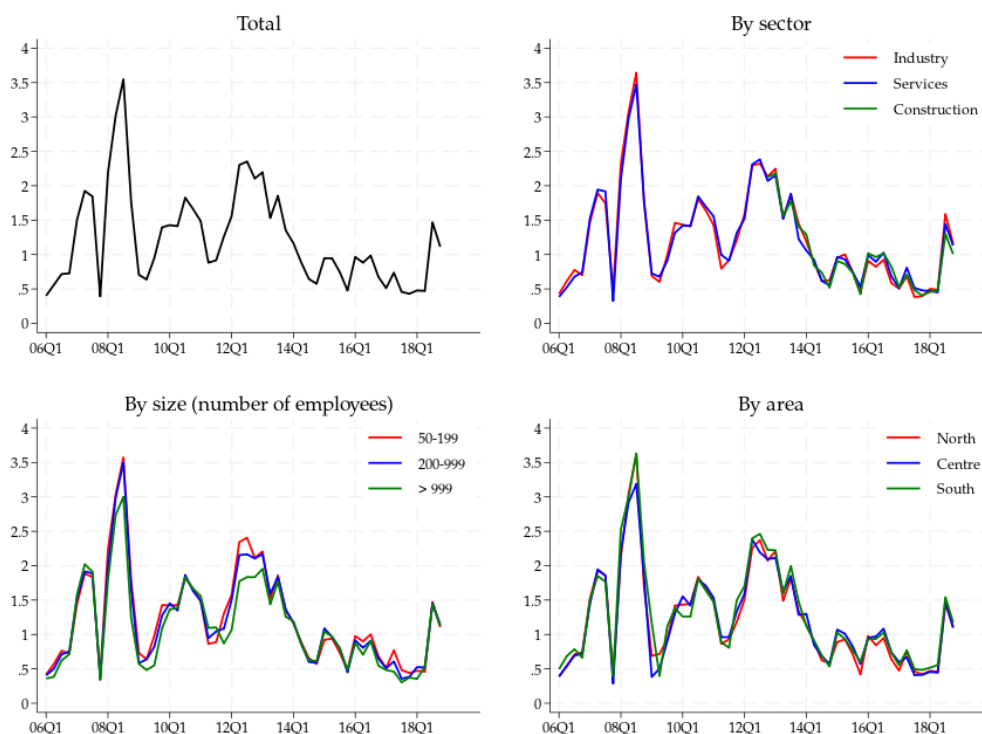
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Figure 1: Developments in Firms' Loan Reliance



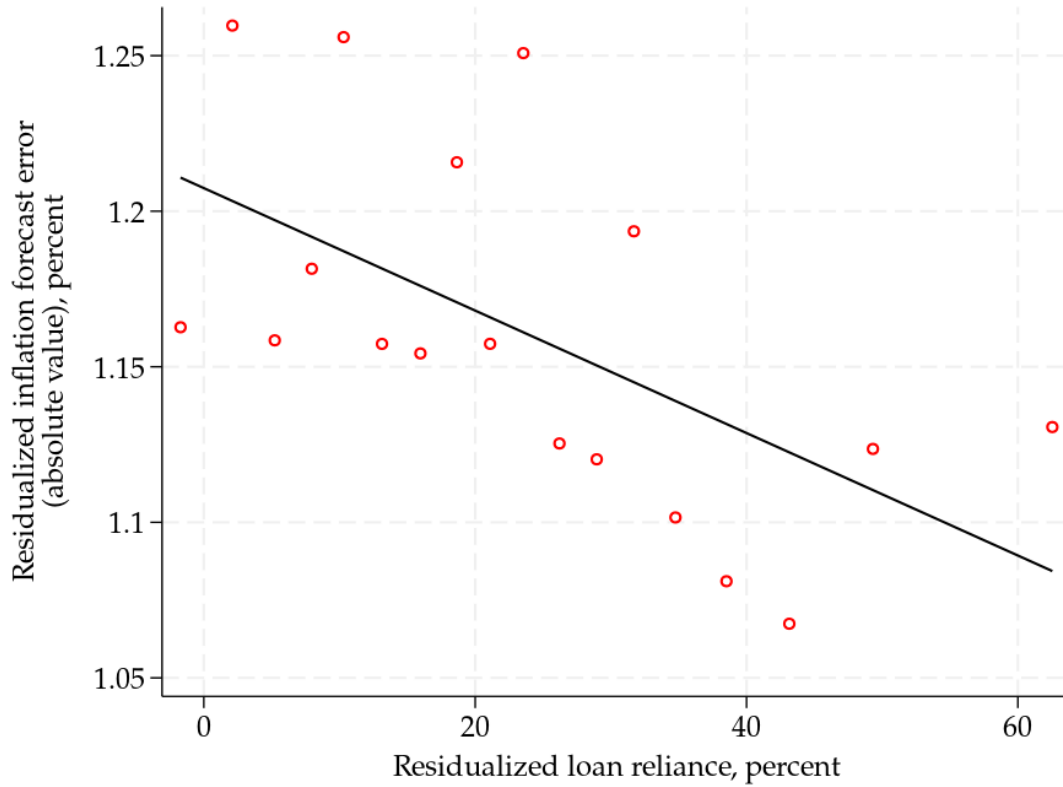
Notes: Each panel reports the evolution over time of firms' reliance on credit, as defined in (1), across all firms (in the top-left panel) and across groups of firms classified by sector, by size, and by geographical area. The sample period is from 2006Q1 to 2019Q4.

Figure 2: Developments in Firms' Attention



Notes: Each panel reports the evolution over time of firms' one-year ahead absolute inflation forecast errors, as defined in (2), across all firms (in the top-left panel) and across groups of firms classified by sector, by size, and by geographical area. The sample period is from 2006Q1 to 2019Q4.

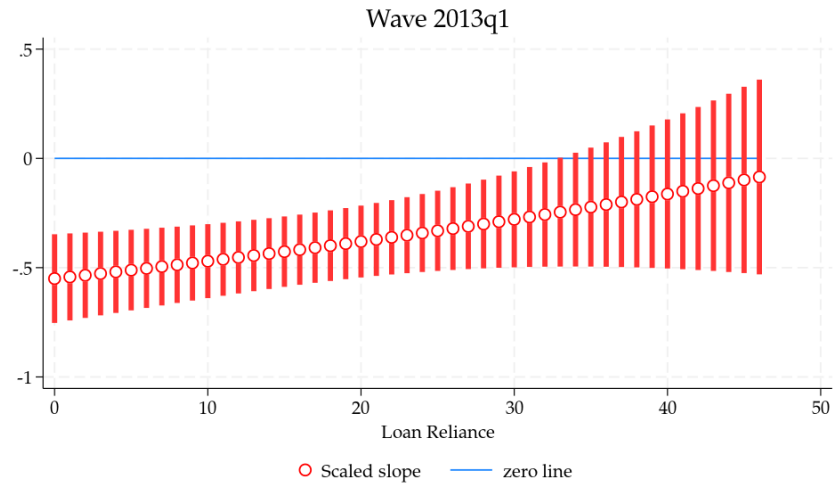
Figure 3: Binned Scatter Plot: Loan Reliance and Inflation Inattention



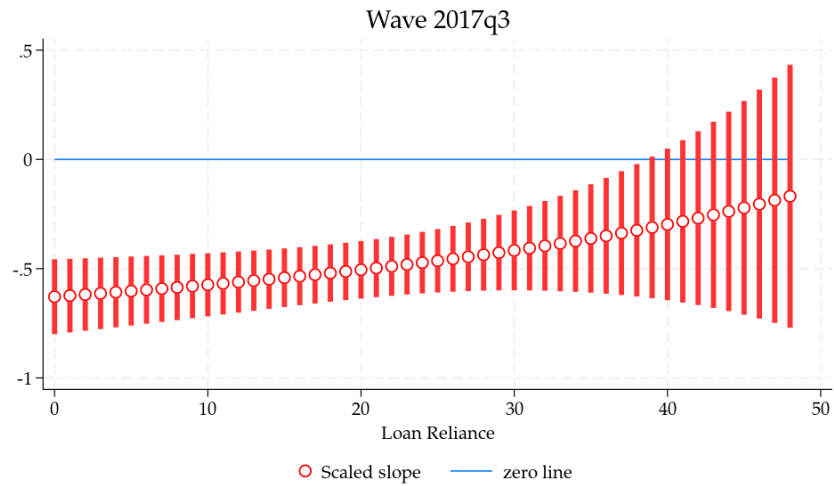
Notes: This figure is a binned scatter plot of loan reliance, as defined in equation (1) and the absolute value of one-year ahead inflation forecast errors. To construct this binned scatter plot, we first regress the absolute forecast errors on the loan reliance controlling fixed effects on sector, area, size, and RCT treatment. The solid line shows the linear estimation results. We then divide the sample into 18 equal-sized groups and plot the mean of the fitted inflation forecast errors against the mean value of the loan reliance within each bin, as denoted by the red circles.

Figure 4: RCT on expectations updates

(a) First RCT



(b) Reshuffling



Notes: This figure plots the adjusted coefficients from the regression specification (6). The red lines are the estimated coefficients with 90% confidence intervals. The x-axis is the loan reliance measure following equation (1). Panel (a) is the 2013Q1 wave when the first RCT is introduced into the survey. Panel (b) is the 2017Q3 wave when the treated group is redrawn from the sample.

Figure 5: RCT on expectations updates: simulated data

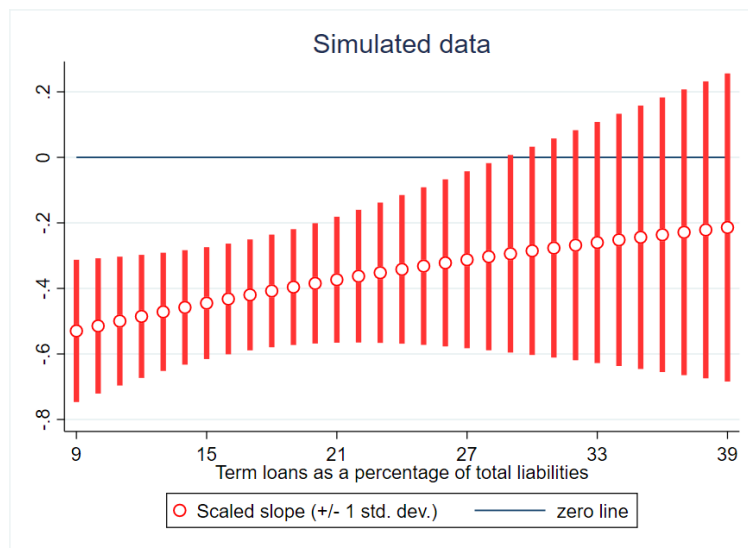
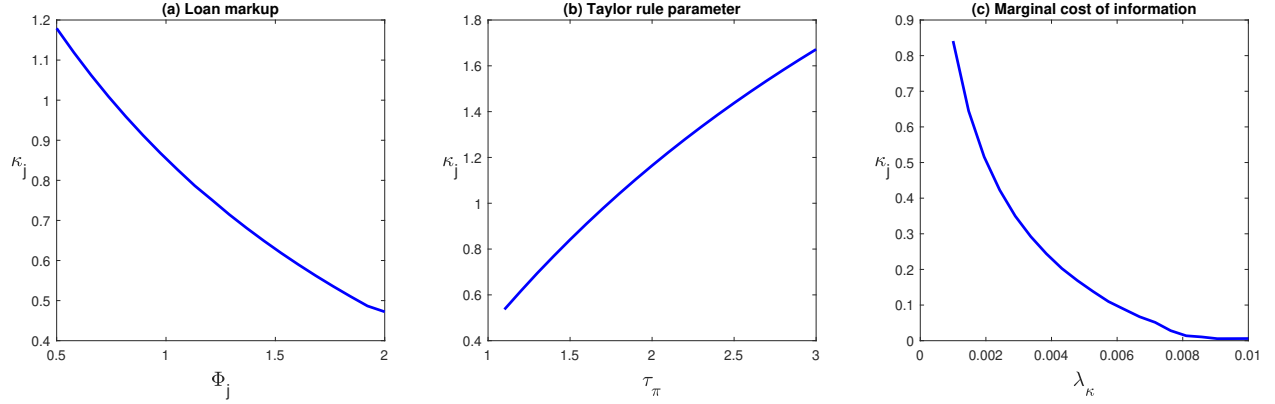
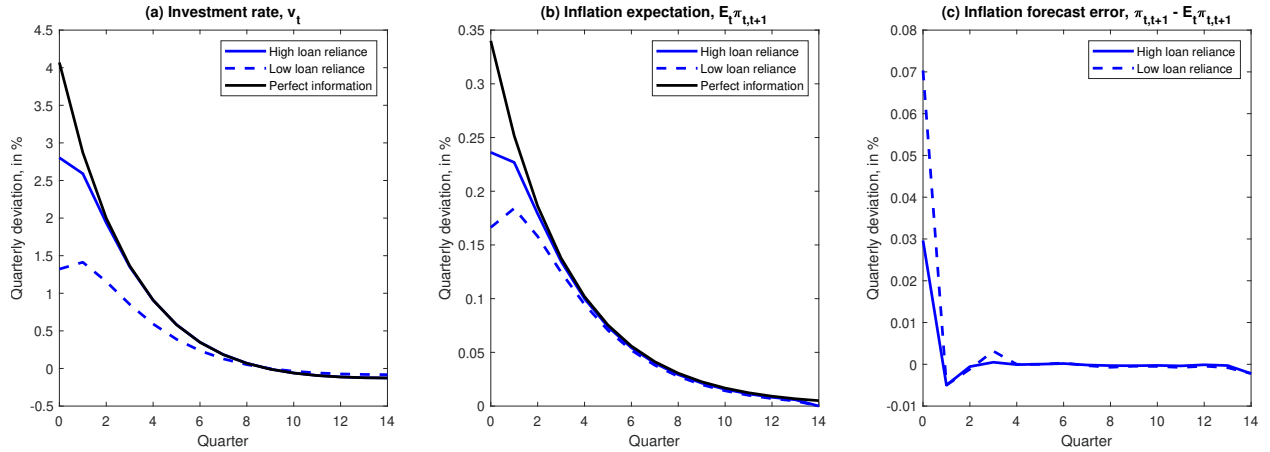


Figure 6: Comparative analysis: attention κ_j



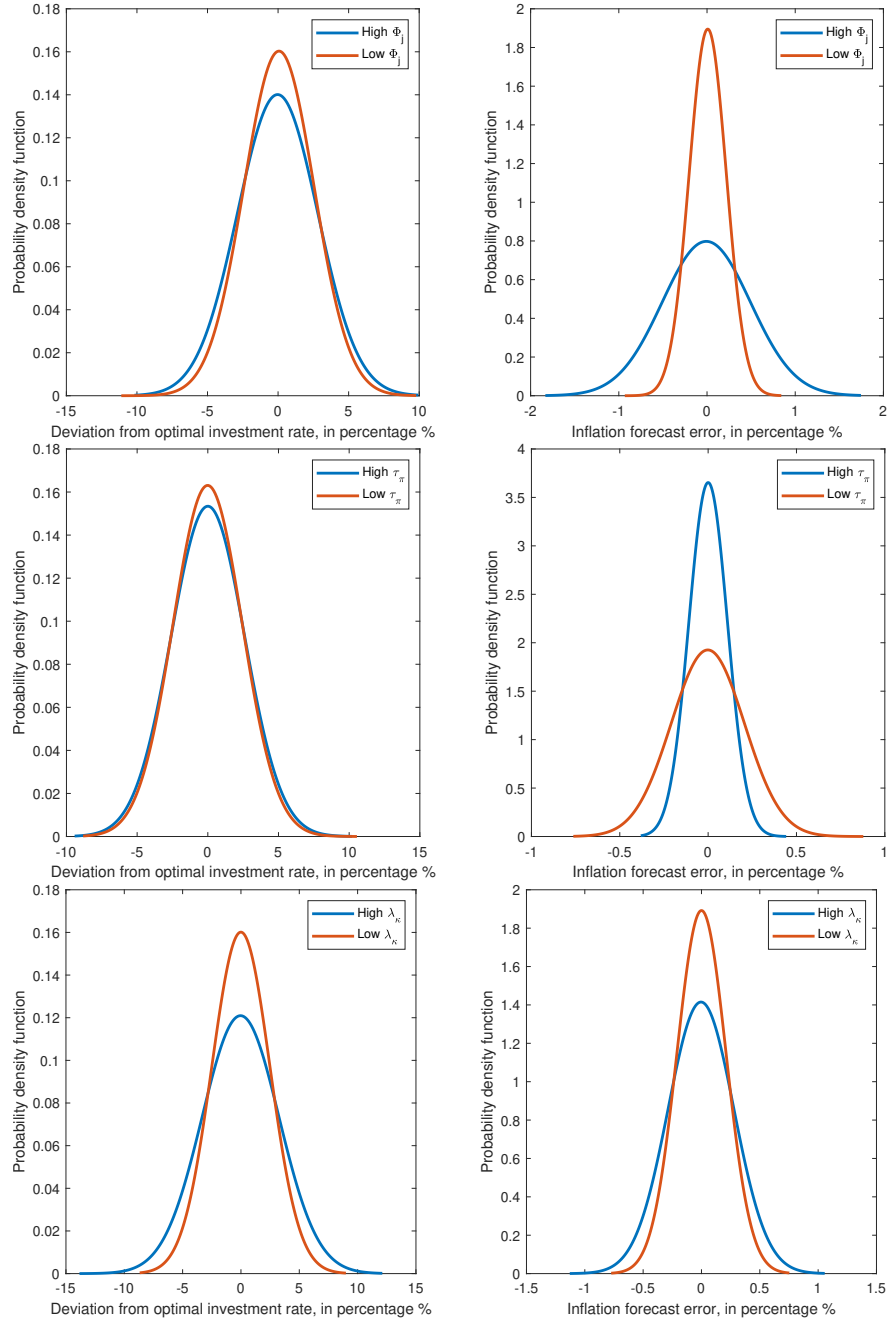
Notes: Benchmark parameters are fixed as listed in Table 3. Ranges for Φ_j , τ_π , and λ_κ are $[0.5, 2]$, $[1, 3]$, and $[0.001, 0.01]$, respectively.

Figure 7: Impulse response function: inflation shock



Notes: The figures display the impulse response functions to 1 positive standard deviation shock in $(0.0034) \epsilon_{\pi,t}$, which increases the annualized inflation by 1.35%. The autoregressive coefficient of the inflation process is 0.74. The black line denotes the responses under the perfect information case without information costs. The solid blue line denotes the responses under the benchmark parameter values with firm-level loan markup $\Phi_j = 1.025$, corresponding to the average loan reliance of 24%. The dashed blue line denotes the responses under $\Phi_j = 2$, leading to the steady state loan reliance of 11%.

Figure 8: Distribution of deviation



Notes: The figure displays the fitted distributions of deviations from optimal investment, $v_{j,t}^* - \mathbb{E}_t v_{j,t}^*$, and the inflation forecast error, $\pi_t - \mathbb{E}_t \pi_t$ under high and low regimes with varying parameter values ($\Phi_j^L = 1.025$, $\tau_\pi^L = 1.5$, $\lambda_\kappa^L = 0.001$; $\Phi_j^H = 3$, $\tau_\pi^H = 3$, $\lambda_\kappa^H = 0.002$). We simulate the time series for 10,000 times and fit the data with normal distribution.

Table 1: Descriptive statistics

	p25	p50	p75	Mean	SD	N
Expected inflation (1-year ahead)	0.600	1.400	2.200	1.531	1.236	29793
Inflation inattention (in %)	0.400	1.000	1.700	1.160	0.997	26376
Term loan reliance (in %)	9.767	22.376	35.470	24.105	17.497	24805
Bank credit to debt ratio (in %)	58.156	94.649	100.000	73.184	36.817	27027
log(employees)	4.060	4.635	5.209	4.840	0.961	35316
ROE	0.102	4.105	11.924	4.119	25.967	28457
Liquid asset ratio (in %)	0.556	2.748	8.948	6.505	8.688	29091

Notes: The loan reliance based on term loans is calculated at the firm level. The summary statistics are computed with the sampling weights. The sample period is from 2006Q1 to 2019Q4.

Table 2: Effects of Loan Reliance on Inflation Attention

	Dependent variable: Inattention $_{j,t}^{(\pi)}$					OLS (6)
	2SLS					
	(1)	(2)	(3)	(4)	(5)	
Loan Reliance	-0.121** (0.0562)	-0.120** (0.0553)	-0.101** (0.0467)	-0.116** (0.0523)	-0.0998** (0.0459)	-0.00206 (0.00128)
log(employees)		0.293* (0.151)			0.231* (0.117)	
ROE			-0.00385*** (0.00131)		-0.00357*** (0.00128)	
Liquid asset ratio				-0.0182*** (0.00568)	-0.0163*** (0.00548)	
Observations	16,886	16,886	15,467	15,885	15,282	16,886
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
RCT FE	Yes	Yes	Yes	Yes	Yes	Yes
1st stage F stat	13.33	13.68	16.07	14.76	16.67	
1st stage coeffi.	-0.0540	-0.0550	-0.0660	-0.0580	-0.0660	

Notes: This table displays the results for regression specification in equation (5). Attention_{j,t}^(π) is the one-year ahead absolute forecast error of inflation for firm j in wave t . The column (1) to (5) are the 2SLS with various controls. The last column (6) is the results for OLS regression without the instrument. The RCT dummy is equal to one for treated firms and zero for control firms. The 1st stage coefficients are estimated from regressing the loan reliance on the Bartik instrument. Standard errors reported in parentheses are as in Driscoll and Kraay (1998). ***, **, * denote statistical significance at 1, 5 and 10 percent level.

Table 3: Calibration

	Parameter	Value	Reference
β	Discount factor	0.99	Average annualized real interest rate of 4%.
ρ	CES aggregator between internal and external funding	0.5	Elasticity of substitution equals 2, match the empirical distribution of loan reliance.
ω_b	Calvo (1983) price stickiness in banking market	0.9	Match the newly issued loan to outstanding loan ratio.
δ	Capital depreciation rate	0.025	Standard.
φ_k	Investment adjustment cost	2.5	Standard.
Φ_j	firm-level loan markup	1.025	Annualized loan interest rate of 10%.
τ_π	Taylor rule parameter (inflation)	1.5	Standard.
λ_κ	Marginal cost of attention	0.001	Match the cross-sectional dispersion of inflation expectations.
ρ_π	Autoregression for inflation	0.74	Estimated from Italian inflation time series.
σ_π	Standard deviation of ϵ_π	0.0034	One standard deviation positive shock leads to an annualized increase in inflation of 1.36%.

Appendix

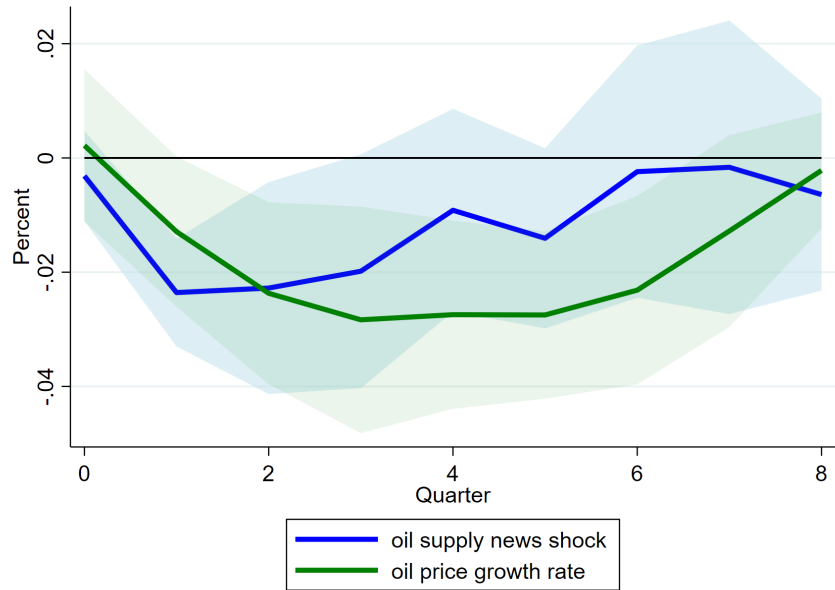
Inflation and bank financing cost

In this section, we provide empirical evidence on our model assumption that the unanticipated increases in inflation will lead to decreases in the loan markup. Our regression specification following the local projection proposed by [Jordà \(2005\)](#):

$$\phi_{t,t+h} = \sum_{q=1}^4 \phi_{t-q} + \sum_{m=0}^4 \beta_{0,m}^{(h)} \epsilon_{t-m}^{\pi} + \sum_{n=1}^4 \text{control}_{t-n} + u_{t+h|t},$$

where $\phi_{t,t+h} = \text{mean}_i(\phi_{i,t})$ is the h quarter ahead loan markup taking the average of bank-time fixed effect constructed from equation (4) across firms. We use oil supply news shocks, ϵ_t^{π} constructed by [Känzig \(2021\)](#) to generate unanticipated movements in inflation. We also present a robustness check using the growth rate of Brent Crude oil prices as exogenous variation in inflation. We add four lags of the industrial production index in Italy and four lags of the ECB deposit facility rate as controls. Figure 9 shows our regression results using both versions of supply shocks. Following one standard deviation increase (1.18, log settlement price in \$) in oil price, the bank-side loan markup decreases by two basis points.

Figure 9: Local projection: oil supply shocks on supply-side financing cost



Notes: The impulse response functions are based on the reduced-form regression of oil shocks on the bank-side financing cost determinant, Φ_t . The Φ_t is constructed from the decomposition stated in equation (4) by taking the average across banks. The shaded areas are 90% confidence intervals.

Derivation for the optimal capital

The log-linearized markup of the loan bundle faced by firms is defined as $\hat{\phi}_t = \hat{r}_t^b - \hat{r}_t$. We could solve the law of motion of optimal loan markup determined by the banks as a function of previous loan markup, previous inflation, and current inflation as below.

$$\begin{aligned}\hat{\phi}_t &= \hat{r}_t^b - \hat{r}_t \\ &= \omega_b \hat{r}_{t-1}^b + (1 - \omega_b) \hat{r}_t^{b,*} - \hat{r}_t \\ &= \omega_b \hat{\phi}_{t-1} + \omega_b \tau_\pi \hat{\pi}_{t-1} + \Omega_2 \hat{\pi}_t \\ (1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) \hat{\phi}_t &= \Omega_2 \epsilon_{\pi,t} + \omega_b \tau_\pi \epsilon_{\pi,t-1}\end{aligned}$$

The loan markup is uniquely pinned down by the ARMA(2,1) process derived above.

Given the process of loan markup, we now derive the law of motion of optimal capital under the perfect information case. We first guess the solution to be in form of $\hat{k}_{j,t} = a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t}$. Plugging the guessed solution into the first order condition.

$$\begin{aligned}(1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) \hat{\phi}_t &= \Omega_2 \epsilon_{\pi,t} + \omega_b \tau_\pi \epsilon_{\pi,t-1} \\ \hat{\phi}_t &= (\omega_b + \rho_\pi) \hat{\phi}_{t-1} - \omega_b \tau_\pi \hat{\phi}_{t-2} + \Omega_2 \epsilon_{\pi,t} + \omega_b \tau_\pi \epsilon_{\pi,t-1} \\ \hat{k}_{j,t} &= a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t} \\ \hat{k}_{j,t} &= \frac{1}{(1 - \beta(1 - \delta))(\phi - 1)} \frac{\Phi_j^{1-\rho}}{1 + \Phi_j^{1-\rho}} \hat{\phi}_t - \frac{\beta(1 - \delta)}{(1 - \beta(1 - \delta))(\phi - 1)} \frac{\Phi_j^{1-\rho}}{1 + \Phi_j^{1-\rho}} \mathbb{E}_t \hat{\phi}_{t+1} \\ a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t} &= A_1^j \hat{\phi}_t + A_2^j \mathbb{E}_t \hat{\phi}_{t+1} \\ a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t} &= A_1^j \hat{\phi}_t + A_2^j [(\omega_b + \rho_\pi) \hat{\phi}_t - \omega_b \tau_\pi \hat{\phi}_{t-1} + \omega_b \tau_\pi \epsilon_{\pi,t}]\end{aligned}$$

The last equation gives us the solution of $\{a_1^j, a_2^j, a_3^j\}$ as functions of $\{\omega_b, \rho_\pi, \tau_\pi, \beta, \delta, \phi, \Phi_j\}$. The solution is unique since if we have the alternative solution with $\hat{k}_{j,t} = a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t} + \hat{w}_{j,t}$, with $\hat{w}_{j,t}$ a disturbance term, the only solution is $w_{j,t} = 0$, which implies the solution from guess-and-verify is unique. The law of motion for capital is:

$$\begin{aligned}\hat{k}_{j,t} &= a_1^j \hat{\phi}_t + a_2^j \hat{\phi}_{t-1} + a_3^j \epsilon_{\pi,t} \\ &= (a_1^j + a_2^j \mathbb{L}) \hat{\phi}_t + a_3^j \epsilon_{\pi,t} \\ (1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) \hat{k}_{j,t} &= (a_1^j + a_2^j \mathbb{L})(1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) \hat{\phi}_t + (1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) a_3^j \epsilon_{\pi,t} \\ &= (a_1^j + a_2^j \mathbb{L}) \Omega_2 \cdot \epsilon_{\pi,t} + \omega_b \tau_\pi \cdot \epsilon_{\pi,t-1} + (1 - \omega_b \mathbb{L})(1 - \rho_\pi \mathbb{L}) a_3^j \cdot \epsilon_{\pi,t} \\ &= (a_1^j \Omega_2 + a_3^j) \cdot \epsilon_{\pi,t} + [a_2^j \Omega_2 + a_1^j \omega_b \tau_\pi - a_3^j (\omega_b + \rho_\pi)] \cdot \epsilon_{\pi,t-1} + (a_2^j \omega_b \tau_\pi + a_3^j \omega_b \rho_\pi) \cdot \epsilon_{\pi,t-2}\end{aligned}$$

The optimal capital follows an ARMA(2,2) process. We rename the coefficients:

$$\hat{k}_{j,t} = p_1^j \hat{k}_{j,t-1} + p_2^j \hat{k}_{j,t-2} + q_1^j \epsilon_{\pi,t} + q_2^j \epsilon_{\pi,t-1} + q_3^j \epsilon_{\pi,t-2} \quad (9)$$

Given the same law of motion of inflation faced by firms, their optimal capital follows different laws of motion, with the coefficients varying across Φ_j .