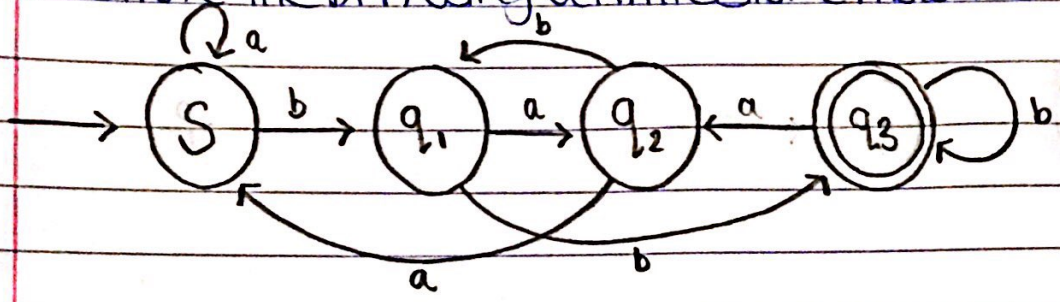
INDIVIDUAL COURSEWORK 2

1.

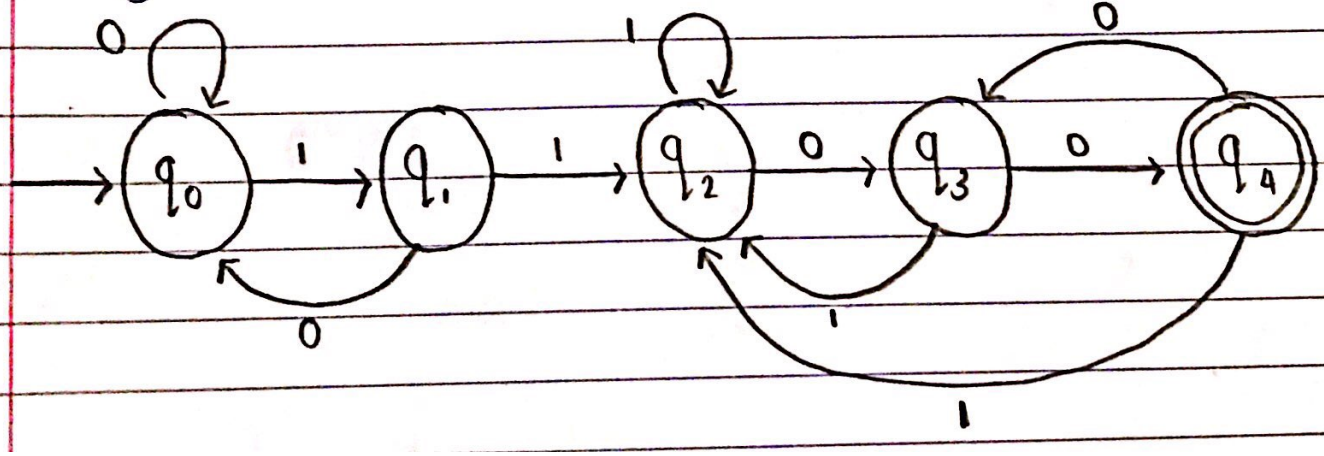
(a.) Describe in english the language recognised by the DFA illustrated in the following finite state machine

The DFA displayed in the finite state machine accepts all binary strings which contain the sequence 1101

(b.) Illustrate the DFA using a finite state machine



(c.) Design a DFA to recognise the language consisting of all binary strings which start with the pattern 11 and end with 00.



2.

(a) Describe in english the language of the NFA

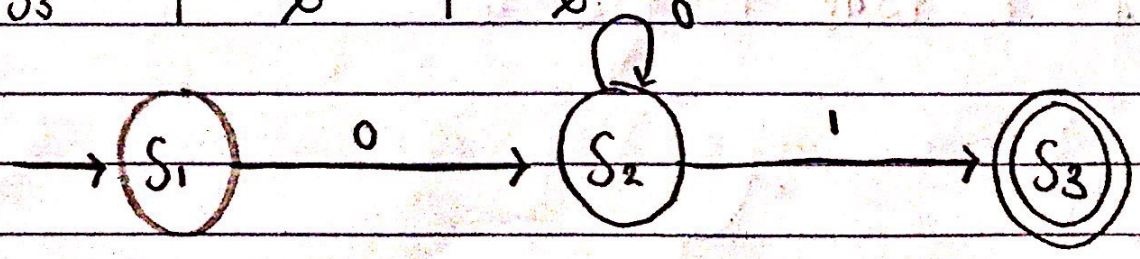
This NFA accepts all binary strings which start with any number number of 0's and ends with 1

(b) Convert the NFA in part A into a DFA which recognises the same language.

Q \ Σ	0	1	ε
q_0	$\{q_1, q_0\}$	$\emptyset$	$\emptyset$
q_1	$[q_0, q_1\}$	$\{q_2\}$	$\{q_0\}$
q_2	$\emptyset$	$\emptyset$	$\emptyset$

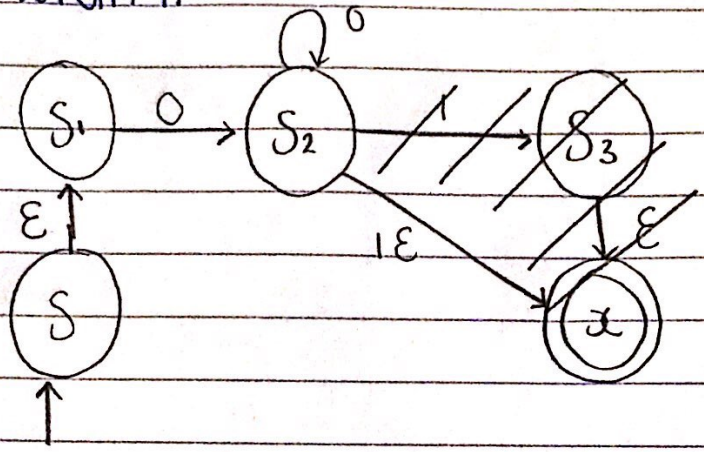
Q \ Σ	0	1
$[q_0]$	$[q_1, q_0]$	$\emptyset$
$[q_1, q_0]$	$[q_1 q_0]$	$[q_2]$
$[q_2]$	$\emptyset$	$\emptyset$

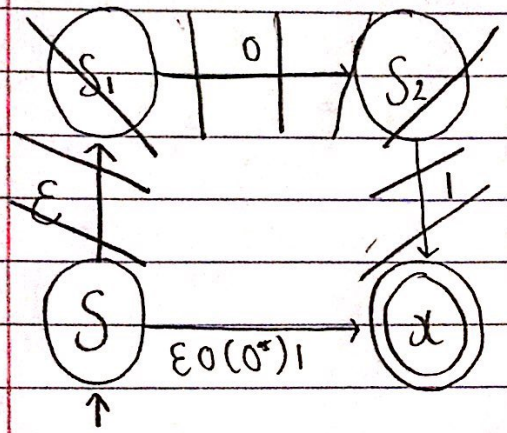
Q \ Σ	0	1
S_1	S_2	$\emptyset$
S_2	S_2	S_3
S_3	$\emptyset$	$\emptyset$



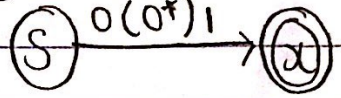
2.

c. Give a regular expression to describe the language of the NFA in 2 Part A.



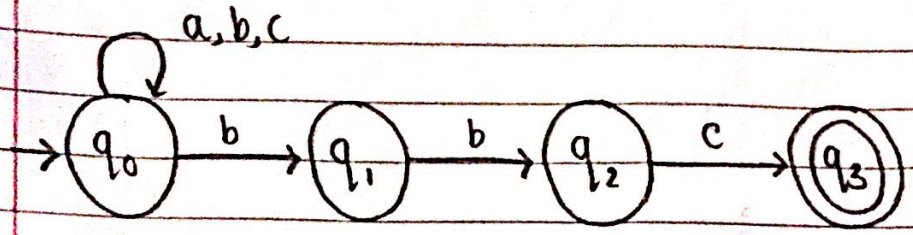


Regular Expression =



$= 0^+ 1$

(a.) Design a NFA to recognise the language consisting of all strings, over the alphabet {a, b, c} that do not end with the pattern bbc.



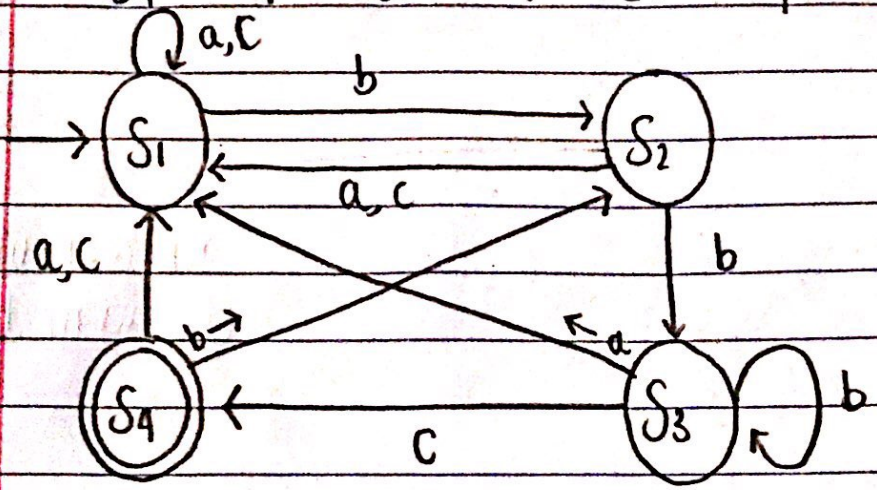
* Accept strings ending in b b c

Q \ Σ	a	b	c
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_2\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\emptyset$	$\emptyset$

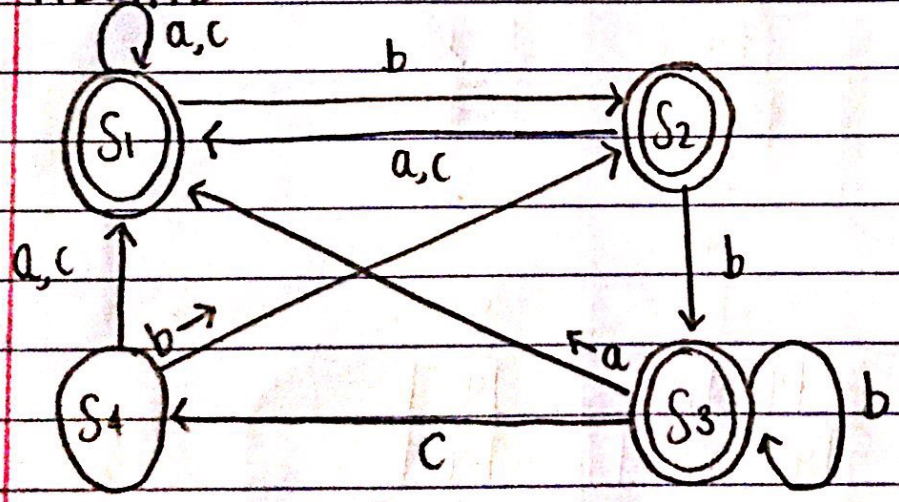
Q \ Σ	a	b	c
$[q_0]$	$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0]$	$[q_0, q_1, q_2]$	$[q_0]$
$[q_0, q_1, q_2]$	$[q_0]$	$[q_0, q_1, q_2]$	$[q_0, q_3]$
$[q_0, q_3]$	$[q_0]$	$[q_0, q_1]$	$[q_0]$

2d continued...

Q \ Σ	a	b	c
S_1	S_1	S_2	S_1
S_2	S_1	S_3	S_1
S_3	S_1	S_3	S_4
S_4	S_1	S_2	S_1



NEGATE :



3.

a. For each of the following languages over alphabet $\Sigma = \{x, y, z\}$ give two strings that are members of the language.

(i) $(xy)^* = (x \circ y)^*$
- xy
- $xyxy$

$\{xy, xyxy\}$

(ii) $y^* \vee zx^+$
- yyy
- zxx

$\{yyy, zxx\}$

(iii) $\Sigma x \Sigma y \Sigma z$
- $xxyyzz$
- $zxzyzz$

$\{xxyyzz, zxzyzz\}$

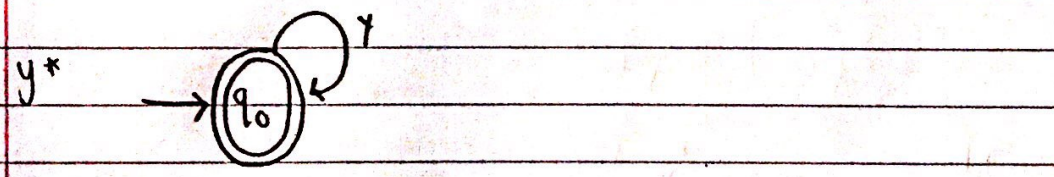
(iv) $y^* z^* x^2$
- $yyyzzzx^2$
- yzx^2

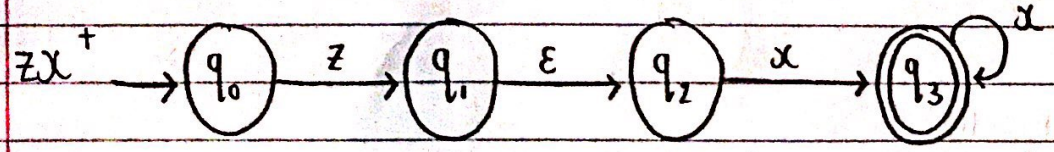
$\{yyyzzzx^2, yzx^2\}$

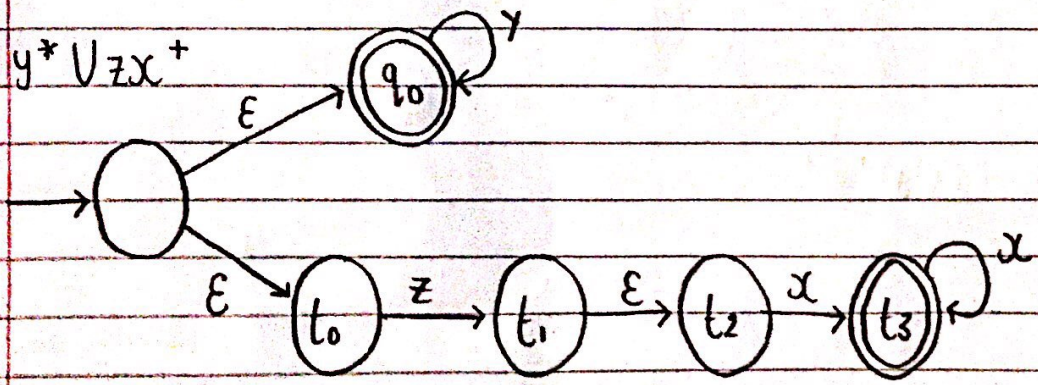
(v) $(x \vee y) z^*$
- xzz
- yz

$\{xzz, yz\}$

(b) $y^* \vee zx^+$

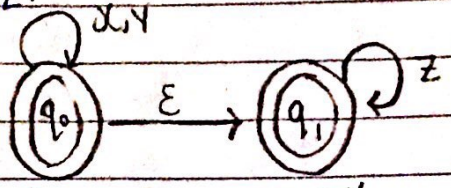




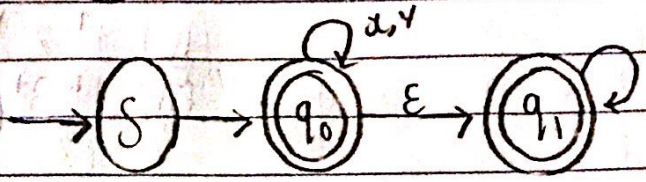


c. Write a regular expression for the following languages over the alphabet $\{x, y, z\}$

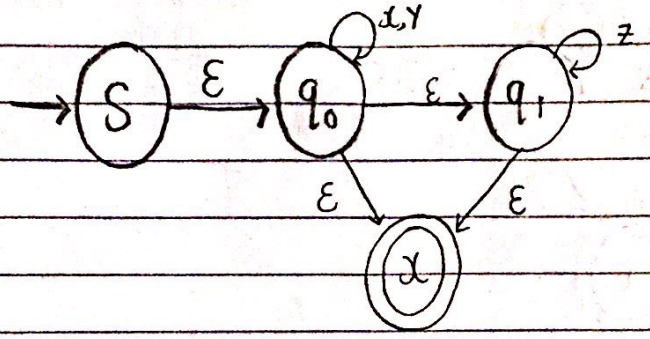
(i) The set of all strings where any and all instances of x and y appear before any and all instances of x and y appear before any and all instances of z.



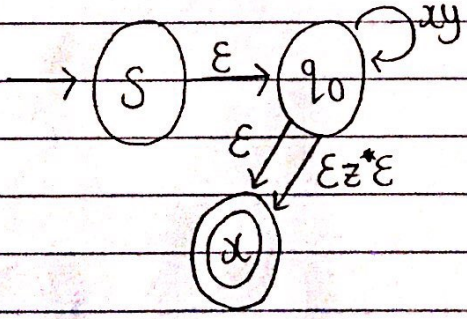
STEP 1:



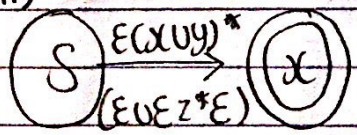
STEP 2:



STEP 3 (i):



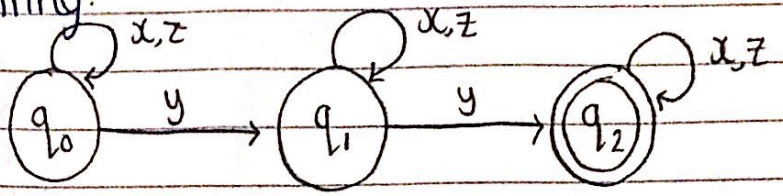
STEP 3 (ii):



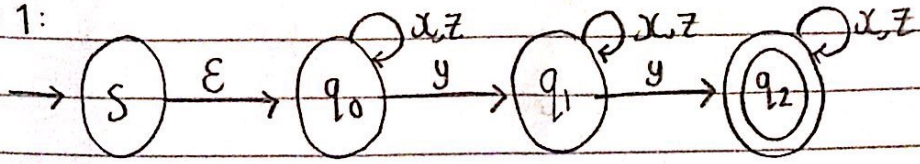
$= (x \cup y)^* z^*$

c.

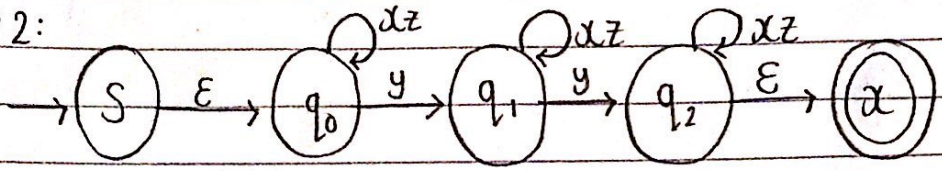
(ii) The set of all strings which exactly 2 y's appearing anywhere in the string.



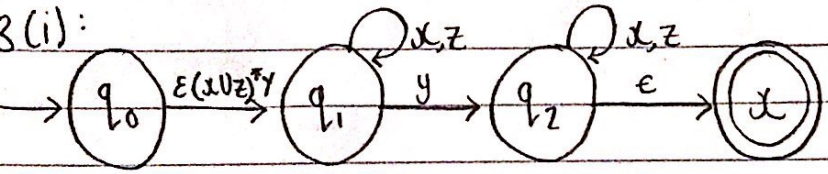
STEP 1:



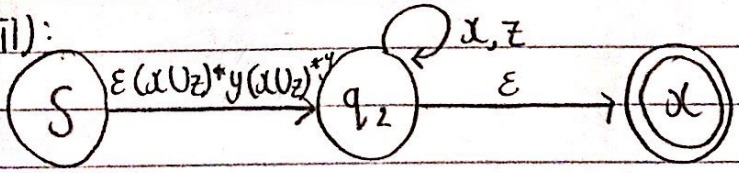
STEP 2:



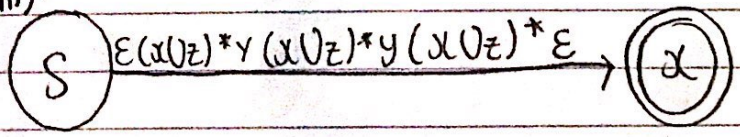
STEP 3 (i):



STEP 3(ii):



STEP 3 (iii):

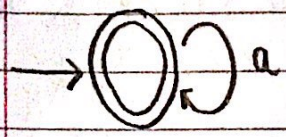


$= (x \cup z)^* y (x \cup z)^* y (x \cup z)^*$

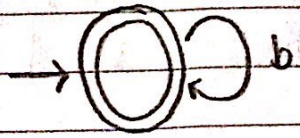
4. Decide whether or not the following language over the alphabet {a, b, c} is regular. If so, design a DFA/NFA to recognise it, and if not give a formal Proof based on Pumping Lemma.

(a.) $\{w \mid w \text{ in } (a^k \cup b^k) \text{ for } k \geq 0\}$

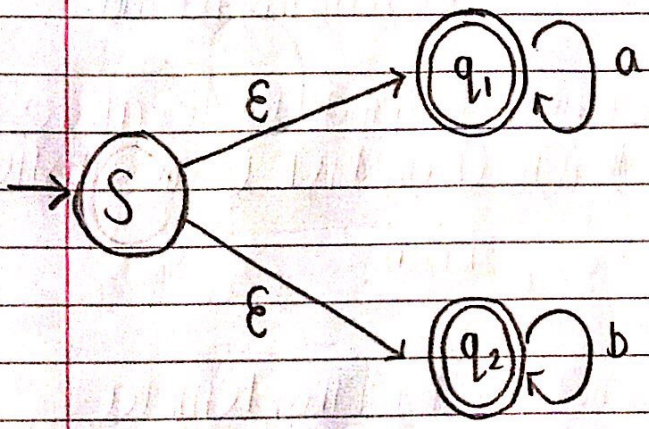
a^k



b^k



Therefore, $(a^k \cup b^k)$



(b.) $\{a^i b^j c^{2i+2j} \mid i,j,k \geq 0\}$

$w = aa\ b\ cccccc$

1. $xy^*z \in L$
2. $|y| > 0$
3. $|xy| \leq k$

- Assume the language is regular.
- By the pumping lemma, there exists a pumping length k which is greater than 0 [$\exists\ k>0$] such that any word in the language which is longer than k characters can be described as xyz. [$|w| > k$ then $w = xyz$]

Consider the word $a^k b^1 c^{2k+2}$

Note that $|a^k b^1 c^{2k+2}| = 3k+3 > k$ where k = pump factor from the pumping lemma.

Let $|y| > 0$ and $xy^*z \in L$ belong to the language for all possible iterations of Y. Note, therefore rule ① and rule ② are satisfied.

$x = a^k \qquad y = b \qquad z = c^{2k+2(1)}$

$|xy| = k+1 > k$ ∴ this condradicts the pumping lemma.
Therefore our assumption is incorrect and the language is non-regular.