

**Movement Strategies During Haptic Search:
Implications for the Learning of Blind Populations**

by

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Abstract

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This research investigates the finger-movement strategies used by blindfolded sighted participants when completing a search task on a tactile map. These participants can be considered similar to individuals with recent vision loss, for example due to age-related causes that are responsible for the majority visual impairments in the developed world. The results could also be extended to children with acquired visual impairments, who are likely to encounter a large number of tactile maps and graphics in their continuing education. Tactile maps, and tactile graphics in general, provide opportunities to individuals with low or no vision to access educational, professional, and other graphical materials. However, performance with tactile graphics is highly dependent on the finger movement strategies used. Results of this dissertation can provide insight into the type of movement strategies that are likely to be used by individuals with a recently acquired visual impairment, and can inform tactile training and the design of tactile displays for these individuals.

Finger-position data were collected from nine right-handed blindfolded sighted participants as they searched for a landmark on a tactile map using either one or five fingers. Three separate sets of analyses were conducted. In the first, the spatial and temporal distributions of finger locations during the haptic search task were analyzed to investigate why five-finger search is significantly faster than one-finger search. Results supported the possibility that five-finger search was benefited by greater throughput of information, with information flowing through multiple fingers, and the larger detection radius of the hand during five-finger search. The results did not support faster index-finger movement in five-finger search, nor the use of fingers as non-moving anchors to promote better allocentric spatial representations.

The second set of analyses investigated the use of systematic strategies: spirals, zigzags, and parallel sweeps, during haptic search. Monte Carlo simulations were used to demonstrate that a systematic strategy is more efficient than a random walk (Brownian, Lévy, or ballistic) When searching for a non-revisitable target in an enclosed space. However, this efficiency benefit decreases with increased detection radius. Systematic strategies were observed and coded in the finger-movement trajectories, and were found to be more prevalent in one-

finger search than five-finger search, agreeing with predictions based on the Monte Carlo simulations. This effect was not explained by longer one-finger search times.

Finally, an initial model of finger-movement trajectories was constructed based on the Langevin equation, a stochastic differential equation describing correlated random motion. This model allowed for stimulus features to attract or repel finger movements through instantiation of a potential function. Ornstein-Uhlenbeck schemes of attraction and repulsion were activated through spatial thresholds, determined using Maximum Likelihood Estimation. These models revealed benefits of five-finger search over one-finger search that were not apparent in previous analyses. Namely, attraction to distractors was reduced in five-finger compared to one-finger search.

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Chapter 1

General Introduction

1.1 Motivation

An important application of finger-movement research is the development of tactile interfaces for individuals with and without visual impairments. The current dissertation uses data from blindfolded sighted participants, who would be equivalent to sighted people working in low-vision environments and individuals who have recently lost their vision. Low-vision environments could include situations where light is low or nonexistent, for example due to fog or smoke. Results may also be applicable to individuals with long-term blindness, although blindfolded sighted participants have more recent visual experience and lack similar tactile experiences as those with long-term blindness.

The majority of visual impairments in the developed world can be due to age-related diseases such as macular degeneration, cataracts, diabetic retinopathy, and glaucoma. Therefore, most individuals with visual impairments are not congenitally affected, but acquired their visual impairments later in life. Visual impairment is an umbrella term that includes total blindness, acuity worse than 20/400 or a visual field of 10° or less, legal blindness, acuity worse than 20/200 or a visual field of 20°, and low vision, some loss of acuity that requires adaptations.

This dissertation uses tactile maps as stimuli, mostly because tactile maps, in contrast to raised-line drawings of objects, do not necessarily contain visual cues that may be present in raised-line drawings, such as object self-occlusion. Furthermore, results from the current dissertation are immediately relevant to the design and use of tactile maps for people with visual impairments. As with sighted people using visual maps, blind/low-vision individuals can effectively use tactile maps for gaining spatial knowledge (Perkins & Gardiner, 2003), and the spatial image gained from a tactile map may be functionally equivalent to that from a conventional visual map (Giudice, Betty, & Loomis, 2011). Exploring the tactile map of an unknown environment can provide equivalent or better information on the spatial layout of an environment to a blind user than directly exploring the environment (Bentzen, 1972; Blades, Ungar, & Spencer, 1999; Espinosa, Ungar, Ocharta, Blades, & Spencer, 1998; Ungar,

2000).

Efficacy of tactile-map use is affected by the hand-movement strategies that the explorer employs (Berla & Butterfield, 1977; Blades et al., 1999). My previous research has shown that individuals who are blind are better than blindfolded sighted individuals at using their hands to quickly and accurately gather information from a tactile map (Morash, Connell Pensky, & Miele, 2013; Morash, Connell Pensky, Tseng, & Miele, 2013). It is likely that this can be explained by the specific hand and finger movement strategies that blind individuals use with tactile materials, which would need to be learned by individuals with recent vision loss. If the hand movement patterns used by blindfolded sighted and blind individuals can be understood, training can be designed to help both congenitally and adventitiously blind individuals more effectively interface with tactile maps or tactile graphics in general.

Using poor strategies for reading tactile graphics may present larger problems than making tactile maps inaccessible. As tactile graphics are ubiquitous in educational materials, inability to access these materials may ultimately result in poor educational and employment outcomes. In a previous study, I found that children's abilities in matching objects to tactile graphics predicted their achievement in mathematics containing tactile graphics, but not in mathematics without tactile graphics (Morash & McKerracher, 2014). Furthermore, the blind students showed differential item functioning for tactile graphics problems, but not those that were tactile-graphics free, compared to sighted norms. Therefore, when math problems contain graphics, the ordering of difficulty for sighted students is not the same for blind students. This underscores a need to understand how tactile graphics are best explored, so that translations from visual to tactile educational materials do not complicate the content through inaccessible design.

The current dissertation addresses the issue of how finger movements are used to gather information from a tactile map by investigating finger movements associated with haptic search for a landmark on the map. Future work may investigate how finger-movement strategies differ for different degrees of visual impairment, totally blind versus sighted, and onset of visual impairment, adventitious versus congenital. Future work could also investigate other types of graphics, such as raised-line drawings, math charts, or infographics, and different types of tasks, such as distance measurement and object identification. This dissertation lays the groundwork for such studies.

This work examines finger movement trajectories during haptic search. Three specific sets of results are presented. First, chapter 2 describes the characteristics of finger movements, and explores how these contribute to faster search times when the participant uses five fingers instead of one finger. Chapter 3 addresses the use of random and systematic movement strategies, and how these are modulated by the hand's detection radius. Chapter 4 investigates a possible model of finger-movement trajectories using stochastic differential equations. The remainder of the current chapter provides a background on each of these topics as they relate to previous studies on visual and animal search. The focus on visual and animal search is due to the tremendous amount of relevant research in these areas, which far exceed in quantity the amount of research on haptic search. Previous research on haptic search will be left to the introductions of the specific empirical chapters.

1.2 Visual Search

Eye movements during visual search alternate between fixations on various image features and high-speed saccades between fixations. During fixation, the eye is held in position so that a feature's visual information projects onto the highest acuity section of the retina, the fovea. The fovea is the only portion of the retina with high enough acuity to allow for high-detail tasks like reading (Henderson, 2003; Howard, 1982). During a fixation, the eye is almost completely stationary, except for the presence of small amounts of drift, tremor, and microsaccades, the purpose of which are not completely understood, but without which visual perception stops (Credidio, Teixeira, Reis, Moreira, & Andrade, 2012).

Visual saccades are ballistic in the sense that they are pre-planned, and cannot be altered once underway. For example, if the target were to move in the course of an eye movement towards the target, the saccade would miss the target (Howard, 1982). Furthermore, during an eye movement the visual scene is blurred across the retina, and this information is suppressed (Bridgeman, Hendry, & Stark, 1975; Dodge, 1900; Matin, 1974).

There are predominantly two types of visual search that occur when the searcher is looking for a certain item presented in an array of distractor items. Feature search occurs when the target differs from the distractors in a single dimension, such as color, shape, or orientation. In this scenario, the time to find the target or determine its absence is nearly independent of the number of distractors. In contrast, during conjunctive search, where the target differs from distractors by a combination of features, the search times are typically proportional to the number of distractors (Treisman & Gelade, 1980). Feature search is often referred to as pop-out or parallel search, while conjunctive search is often referred to as serial search.

A popular model for the programming of saccades is that a bottom-up saliency mechanism drives eye movements, originally conceived to explain attentional capture in pop-out search (Tatler, Hayhoe, Land, & Ballard, 2011; Treisman & Gelade, 1980). Covert attention (without moving the eyes) or overt attention (gaze locations) are attracted to items that are more salient, i.e., have features in common with the search target or are different from their surround on one or more image dimension, such as color, intensity, or contrast, over multiple spatial scales (Henderson, 2003). Saliency may be encoded in a scalar two-dimensional saliency map, that topographically represents feature saliency, without information on what specific feature drives saliency at each location (Itti & Koch, 2001). A single saliency map explicitly assigns regions saliency values, divorced from the specific features that drive saliency. This is possibly created from multiple saliency maps that represent saliency due to a particular feature, e.g., color, through weighted linear combination (Gilchrist & Harvey, 2006; Henderson, 2003; Itti & Koch, 2000). The saliency of stimulus features outside of the fovea are available through the visual periphery. Both pop-out (feature) and serial (conjunctive) search behaviors can be derived from these saliency-based models of overt attention (Itti & Koch, 2000). Some researchers have found fixated locations to be correlated with saliency during visual search (Foulsham & Underwood, 2008; Parkhurst, Law, & Niebur, 2002). However, other researchers found that fixation selection is poorly accounted for by local salience,

and alternative models, such as those that account for high-level object information, can better predict fixations (Einhäuser, Spain, & Perona, 2008; Henderson, Brockmole, Castelhano, & Mack, 2007; Nyström & Holmqvist, 2008; Tatler, Baddeley, & Gilchrist, 2005).

Salience-based mechanisms of visual overt attention predict that the next attended location will be that which is the most salient. Based on this winner-take-all scheme, participants are predicted to never move their gaze from the most salient feature. Therefore, inhibiting previously visited locations is necessary to allow attention to shift between different areas (Itti & Koch, 2000). An essential computational strategy is to, at least briefly, inhibit locations that have been recently visited, similar to the well-known visual phenomenon referred to as “inhibition of return” (IOR) (Posner & Cohen, 1984). If IOR’s duration is reduced, e.g., from 900 to 50 ms, a saliency-based algorithm will fail to find less salient visual targets as it endlessly cycles through a small set of salient features (Itti & Koch, 2000).

Given that saccades only direct fixations to locations with high saliency or interesting task-relevant features, it is unlikely for fixations to occur on areas that are empty, uniform, and uninformative. This agrees with even early studies on gaze control, that indicated that fixations are not placed randomly in a scene. Instead, fixations tend to be clustered on informative visual regions, presumably indicated by their low-level salience (Henderson, 2003). Similarly, fixation models do not predict that eye movements follow a systematic pattern, such as a parallel sweep, spiral, or zigzag, unless the visual scene is arranged such that these patterns could be elicited due to the specific arrangement of salient visual features (Foulsham & Underwood, 2008). Historically, eye movement patterns, referred to as scanpaths, were proposed to function for moving gaze and facilitating scene recognition (Noton & Stark, 1971a, 1971b). The same scan path might be used upon two separate viewings of the same scene, and thereby the motor components of these viewings could be matched, facilitating scene/object recognition. However, the support for scanpaths was reduced by findings that eye movements were not necessary for scene recognition, which can be accomplished with a single fixation (no saccades), and that patterns of fixations on a visual scene were not consistent over multiple viewings, even within the same participant (Henderson, 2003).

With diminished interest in scanpaths came a paucity of studies on fixation patterns in general, and little attention on systematic eye movements. However, it is known that when the elements (target and distractors) in a visual scene are arranged in a regular array of rows and columns, eye movements can follow a systematic pattern that follows the rows and columns of the array. Systematic movements are more likely when there are few distractors in the array, and less likely when the display becomes more dense with distractors (Credidio et al., 2012). These systematic patterns are stimulus-driven, and when elements are not placed in a structured array, as in a complex image like a “where’s Waldo?” page, eye movements do not follow a systematic pattern and appear random (Credidio et al., 2012).

1.3 Animal Search

Random Strategies: Brownian, Lévy, and Ballistic

Animals execute search in the context of foraging for food and water resources, looking for a mate, and locating their nest. The canonical searching problems that animals face, such as foraging for food (one of many targets) and locating a nest (a single target) are analogous in the case that the animal has no information to guide its search (Bell, 1991). A connection may be drawn between these animal search processes and human perceptual search processes, for locating candidate targets (animal foraging) and local search to confirm/deny that a candidate is the target (animal single-target search). Generally, animal search trajectories are considered to follow a random walk or systematic strategy.

The simplest type of random walk is Brownian motion, an unbiased and uncorrelated random walk. In this scheme, movement is equally probable in any direction (unbiased) and the step directions and lengths are independent from those taken earlier (uncorrelated) (Codling, Plank, & Benhamou, 2008; Wu, Li, Springer, & Neill, 2000). A downside of a Brownian strategy is that the walker is likely to revisit locations already searched, which can be a disadvantage if the goal is to minimize search time or distance (Bell, 1991).

One of the most popular models of animal movement is the Lévy walk, whose step lengths are referred to as Lévy flights (Benhamou, 2007). This type of walk has been used to describe movements of albatrosses (Viswanathan & Afanasyev, 1996), deer (Viswanathan et al., 1999), and bumblebees (Heinrich, 1979), among others (Edwards et al., 2007). Lévy walks are characterized by a step-length distribution with heavy tails, which results in clusters of short steps that are connected by long steps (Benhamou, 2007; Edwards et al., 2007; Viswanathan & Afanasyev, 1996). This may be an appropriate strategy when food is concentrated in a few places separated by large fruitless areas, such as schools of fish in the sea or clusters of flowers in a meadow (Travis, 2007; Viswanathan et al., 1999).

However, surrounding the popularity of using Lévy walks to describe animal search is a considerable amount of controversy. Typically, animal movements have been compared to Lévy walks using a graphical method, in which the log-log histogram of movement step lengths or times was fit with a line, and if the line had a slope between -1 and -3, the data were considered consistent with a Lévy walk (Edwards et al., 2007; Viswanathan & Afanasyev, 1996). However, more rigorous methods, such as the likelihood-ratio test (G-test) and/or comparison to alternative distributions, have revealed that this graphical approach was not satisfactory, and has led to many challenges against previous Lévy walk results (Edwards et al., 2007; Travis, 2007). Also, re-analysis of albatross flight times, with removal of spurious entries where birds were actually in their nests and not in flight, proved inconsistent with a Lévy walk (Edwards et al., 2007; Travis, 2007). Similarly, deer foraging times that included times spent handling food at a site rather than just times between sites were also inconsistent with a Lévy walk (Edwards et al., 2007).

Lévy walks (in unbounded search areas) are characterized by step lengths ℓ_j , with

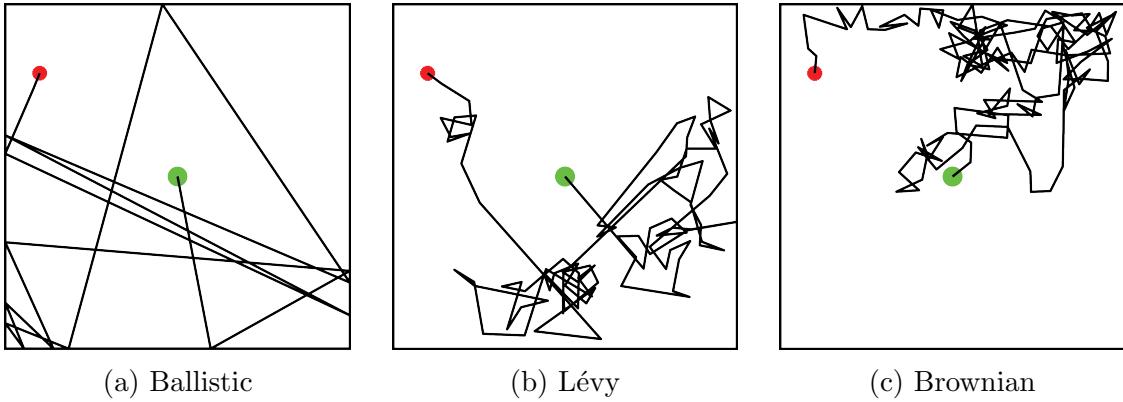


Figure 1.1: Example random search strategies, starting in the center of the search space (green circle), ending when the target is found in the upper left (red circle).

$j = 1, 2, 3, \dots$, drawn from a probability density function with a tail that follows,

$$f(\ell_j) \sim \ell_j^{-\mu} \quad (1.1)$$

where $1 < \mu < 3$, omitting the normalizing constant (Benhamou, 2007; Edwards et al., 2007; Viswanathan et al., 1999). This same distribution can be used to describe Brownian motion, with $\mu > 3$, and a ballistic strategy when $\mu = 1$. A ballistic movement strategy involves moving in a straight line, in a random direction, until movement is no longer possible due to encountering a barrier or resource. Simulated searches using ballistic, Lévy, and Brownian strategies are shown in figure 1.1.

It is important to note that in all random-walk descriptions of animal movements, it is assumed that the animal continues moving until a target is within some detection radius, often the detection radius is how far the searcher can see, smell, or hear. Therefore, the animal may encounter a target within its detection radius between the start and end of a movement step. This contrasts with a ballistic eye movement, during which the observer cannot process information between the start and end points of a step.

Modifications to a random strategy may include adding a bias in movement direction, attraction, repulsion, and boundaries. A directional bias is perhaps the easiest to incorporate into a random walk, where each step includes not only a random component, but a persistent value, e.g., $\Delta x_i = C + \epsilon_i$ will force each step in x towards the right if the constant $C > 0$ and towards the left if $C < 0$, with random perturbances added through $\epsilon_i \sim N(0, \sigma^2)$, (Preisler, Ager, Johnson, & Kie, 2004). In such a scheme, C serves as a constant drift term. Attraction, repulsion, and boundaries can be installed through use of a potential function, whose force field exerts pressure on the random walker to move towards and away from certain areas (Brillinger, Preisler, Ager, & Kie, 2001). This will be discussed in depth in chapter 4.

Correlated random walks describe situations when the walker has a tendency, due to inertia or momentum, to keep walking in the current direction (Holmes, 1993). In this

situation, the walker's position is no longer a Markov process, but its velocity is (Codling et al., 2008). For example, humans will follow correlated random walk patterns when trying to navigate a straight course through featureless terrain (Souman, Frissen, Sreenivasa, & Ernst, 2009). Correlated walks can be described as having a local bias in trajectory, in contrast with a global bias associated with drift (Codling et al., 2008). Correlation can be added to a random walk model by including one or more autoregressive terms in the movement step model (Preisler, Ager, & Wisdom, 2013), also to be discussed in chapter 4.

Systematic Strategies

In contrast to a random search strategy, animals may use systematic approaches to search, such as an expanding square, a spiral, a straight line, or parallel sweeps (Bell, 1991). Such strategies are most attractive when the target is not moving, or is moving slower than the searcher, and the searcher has no information on the target's location (Banks, Vincent, & Phalp, 2009; Bell, 1991). The benefit of a systematic search, rather than a random one, is that the searcher can avoid recovering ground (Bartumeus, Catalan, Fulco, Lyra, & Viswanathan, 2002). Although multiple systematic strategies have been proposed, there are few real-world examples of non-human animals using these strategies (Banks et al., 2009). One of a handful of examples is the desert ant, who, upon finding that its nest is not where it thought it was, will spiral outwards from the believed location (Müller & Wehner, 1994). Mexican bean beetles will also spiral during a search for food, which cannot be explained by environmental features nor a correlated random walk (Turchin, 1998, pp. 147-150). Another example is Eukaryotic cells, which move in a zig-zag pattern when searching for nutrients when no chemical gradients are present (Li, Nørrelykke, & Cox, 2008). Although systematic strategies are rarely used by animals, they are often studied and used by humans to plan optimal movements during war time, search and rescue, and other large-scale activities, which I will discuss in chapter 3.

Multi-State Mixtures

Animals may switch between two or more states during search, including random walks and/or systematic strategies (Bell, 1991; Bénichou, Loverdo, Moreau, & Voituriez, 2011). There are various subtypes of this switching behavior, including trapline foraging, composite random walks, saltatory search, and intermittent strategies, which differ only in the number and types of movements that are involved. Transitioning between states can be initiated by a variety of triggers. For example, hummingbirds and bumblebees may transition from a systematic search to a random search when information on the target's location becomes degraded or proves unreliable (Bartumeus & Catalan, 2009).

Random-walk mixtures can intersperse periods of intensive local search with faster relocation movements. These types of movements may be very common if, during fast ballistic-type movement between resource patches, the searcher has no or limited ability to detect targets. Therefore, slower intensive (diffusive) searches may be necessary to detect

targets between fast relocation phases (Bartumeus & Catalan, 2009; Bénichou, Coppey, Moreau, Suet, & Voituriez, 2005; Bénichou, Loverdo, Moreau, & Voituriez, 2006; Li et al., 2008). This type of behavior, which has the appearance of a Lévy walk, can be related to environmental and/or behavioral states when the movement process is conceptualized as a multi-state mixture (Benhamou, 2007).

There are two general methods by which mixtures of states can be examined. The first is to break the trajectory into segments belonging to each state, and examine the states in isolation (Jonsen, Myers, & Flemming, 2003; Turchin, 1998). This is appropriate when transitions between states can be easily determined based on some time-based or environmental trigger (Jonsen et al., 2003). In the latter case, the process could be Markov. This is the approach taken in this dissertation.

Alternatively, a state-space approach can be used to model movements as a mixture of random walk and systematic processes. In this scheme, movements are not separated into states *a priori*. Instead, movements are assigned to states using maximum likelihood. This approach is carried out in the modeling, and not by the researcher. State selection and transition could be contingent on observable environmental features or internal, unobservable behavioral states. In the latter case, the model would be a hidden Markov model (Jonsen et al., 2003; McClintock et al., 2012; Morales, Haydon, Frair, Holsinger, & Fryxell, 2004). This could be undertaken in future work, specifically using hidden Markov model switching, with transition probabilities dependent on observable external features such as locations and distances to stimulus features. Such future models could be informed by the analyses contained within the current work.

1.4 Overview

In conclusion, the motivation for the current dissertation is to study finger movements used by blindfolded sighted participants when searching for a landmark on a tactile map. The methods used are inspired by those for studying human visual search and animal search, which have been broadly reviewed above.

This dissertation analyzes finger trajectory data in three sections, each structured as a self-contained exposition. The first, chapter 2, investigates the general statistics of finger movements during haptic search. Specifically, what role multiple fingers may play in speeding task completion, whether it be through faster finger movements, the ability to process local information through multiple fingers, a larger detection radius to find important features, or through independent movements of the fingers. Then, in chapter 3, systematic strategies used during haptic search are analyzed. Finally, in chapter 4, an initial model of finger-movement trajectories is developed, using discretized stochastic differential equations.

Chapter 2

Why is Two-Dimensional Haptic Search Slower with One Finger and Faster with Five Fingers?

2.1 Introduction

Previous research indicates that haptic search over a two-dimensional stimulus with raised and textured features is slowest with one finger and fastest with ten fingers, or five fingers if using only one hand. In two separate studies using tactile maps as stimuli, search times were reduced with each additional finger by 14% for sighted individuals (Morash, Connell Pensky, & Miele, 2013) and 12% for blind individuals (Morash, Connell Pensky, et al., 2013). Therefore, a five-finger search was 40-45% faster than a one-finger search.

This chapter investigates why using more fingers during haptic search reduces the time it takes to find a target on an unstructured tactile display. One possibility is that the hand moves faster during five-finger search, enabling the index finger to cover more area on the haptic display more quickly. This proposal assumes that the index finger is the most important haptic sensor, and the other fingers contribute only in somehow allowing the index finger to move more quickly. This possibility is supported by prior reports that suggest that the index finger is the only finger processing information on a tactile display. These include non-significant performance differences between one and two fingers (Loomis, Klatzky, & Lederman, 1991), and one and three fingers (Jansson & Monaci, 2003), in identification of raised-line pictures and shapes; and one and three fingers in searching for a target in a raised-line grid of symbols (Overvliet, Smeets, & Brenner, 2007). The importance of the index finger as the sole haptic sensor is also supported by the finding that sighted individuals tend to use only a single index finger when exploring raised-line drawings (Symmons & Richardson, 2000). Braille reading typically emphasizes the index fingers and middle fingers of both hands.

Alternatively, using more fingers may increase the rate of information through the

hand, assuming that information can be processed in parallel through the different fingers. Although this seems like a likely benefit, several reports have found that information presented to multiple fingers on the same hand produces interference (Lappin & Foulke, 1973), or is processed serially (Overvliet, Smeets, & Brenner, 2010). If fingers on the same hand interfere with one another, it is difficult to explain the search-time reduction provided by multiple fingers, unless, as mentioned above, the multiple fingers somehow speed the movements of the index finger. Alternatively, if information is processed serially through multiple fingers, throughput could still be improved by reduced need to move the hand - the hand could sit still, processing information under each finger serially, instead of moving the index finger to each location. This assumes that it is faster to switch attention between fingers than move a single finger. Increases in throughput of information would be implied by reduced time in contact with stimulus features.

In addition to reducing the time spent on stimulus features through increased throughput, using multiple fingers could reduce the time spent in “empty space,” the space between stimulus features. This is because the detection radius of a single finger is small, the size of a fingertip, and the detection of five fingers is much larger. With a larger detection radius, candidate search targets can be detected during exploration more quickly. Therefore, although the absolute amount of time spent investigating stimulus features may be reduced by multiple fingers (via increased throughput), the *relative* amount of time on map features may stay the same or increase, as the amount of time in empty space decreases.

Lastly, multiple fingers could benefit haptic search through independent positioning of the fingers. In contrast to the hand scanning a stimulus in a static pose, the fingers may react to the stimulus - expanding to increase the hand’s detection radius, contracting to focus the fingers over an important stimulus features, spanning distances with a scissoring movement, or using one or more fingers as a stationary anchor. Anchors have been noted in several previous studies as being beneficial for marking a location for reference. For example, completely traversing a looped path is made easier by marking the start/stop location with an anchor finger (Berla, Butterfield, & Murr, 1976). Using an anchor as a reference may also improve the spatial processing of the stimulus by promoting allocentric versus egocentric spatial representations. For example, Millar and Al-Attar (2004) found that spatial relationships on a tactile map were better reconstructed despite a 90° rotation of the map when the participant learned the map while applying an anchor hand to the map border.

The current chapter specifically examines the possible benefits of multiple fingers in haptic search: higher index-finger speeds, increased throughput, larger detection radius, and positional independence of the fingers. Previous results indicate that there is definite benefit from using multiple fingers during tactile tasks (Morash, Connell Pensky, & Miele, 2013; Morash, Connell Pensky, & Miele, 2012), but through which of these possible mechanisms this benefit manifests is unclear. This is a fundamental question, analogous to asking what benefits multiple eyes or ears provide visual and auditory perception. In vision and audition, multiple sensors provide a larger field of “view” and also spatial information through stereopsis and interaural time differences. The approach taken here is to track the positions

of fingers captured in video during one-finger and five-finger search to directly address how fingers are placed and for how long.

2.2 Methods

Participants

Nine right-handed participants volunteered to participate in this research. Seven of the participants were female, and the participants' average age was 27.2 years ($SD = 6.1$ years). The protocol was approved by University of California, Berkeley's Committee for Protection of Human Subjects, and informed consent was obtained from all participants prior to their participation.

Stimuli

The stimuli were taken from the Tactile Map Open Stimulus Set (TMOSS), which consists of 7 groups of 8 maps in total (Morash et al., 2012), and were manufactured from laser cutting/etching clear acrylic. One group of TMOSS maps (group 3) was used for one-finger trials, and a separate TMOSS group (group 4) was used for five-finger trials. The value provided by using one group of stimuli for all one-finger trials, and a separate group for all five-finger trials, not randomizing group assignment between participants, was in providing the opportunity to visualize spatial distributions of explorations. This necessitated averaging finger locations across participants to provide enough data for two-dimensional density estimation. However, the TMOSS maps were developed such that map parameters, including distances, angles, and symbol types, numbers, and configurations were controlled to be the same across map groups. An example map layout is shown in figure 2.1.

Each tactile map represented a fictitious park, and contained 13 point symbols: circles, squares, ovals, Ts, or triangles, to represent the locations of features, such as trash cans and picnic benches. Solid lines on the maps represented walking paths, and a large textured area on each map represented a lake. All maps were surrounded by a dotted line, 0.635 cm (0.25 in) from the map edge, to indicate the map border. On every map, there were three clusters of symbols, each configured as a square, diamond, vertical line, horizontal line, or triangle shaped arrangement of a single symbol type, e.g., a square-shaped cluster of triangles. These clusters served as landmarks that could be unambiguously referenced.

Most importantly, the target symbol clusters on the one-finger and five-finger maps were not distributed differently. The average location of symbol-cluster centers, relative to the bottom left of the map, towards the right on one-finger maps was 10.29 cm ($SD = 6.50$ cm) and on five-finger maps was 15.42 cm ($SD = 6.59$ cm), $t(14) = -1.57$, $p = 0.139$; and upwards on one-finger maps was 13.06 cm ($SD = 7.89$ cm) and on five-finger maps was 11.68 cm ($SD = 7.12$ cm), $t(14) = 0.37$, $p = 0.719$. The average distance of symbol-cluster centers from the center of the map on one-finger maps was 10.76 cm ($SD = 2.36$ cm) and on

five-finger maps was 9.44 cm ($SD = 2.60$ cm), $t(14) = 1.06$, $p = 0.306$. The two groups of maps also had the same distribution of target symbols (3 Ts, 1 circle, 2 triangles, 2 squares, and 0 ovals) and target cluster shapes (2 vertical lines, 1 diamond, 2 triangles, 2 squares, and 1 horizontal line).

Procedure

Participants were blindfolded and sat at a clear table on which stimuli were placed. The participants were trained how to recognize map symbols and clusters before beginning the main study. During the training, four of the participants were randomly selected to use their right index finger, and their other fingers were bent over and taped to their palm. The other five participants used all fingers on their right hand during training. The training consisted of presenting a participant with each type of map symbol and each shape of landmark (symbol cluster). Then, the participant was tested on naming the five symbols, e.g., “oval,” and the five cluster shapes, e.g., “a square cluster (of ovals).” These were presented in random order, and each participant successfully named the five symbols and five cluster shapes with no mistakes on their first try. The participants were allowed to freely explore three practice maps, and also execute mock trials on these maps to ensure that they understood the task.

During the main experiment, the participants completed 16 trials. In each trial, a map was placed on the table in front of the participant. The maps were in random order, with the constraint that one-finger and five-finger maps alternated. The same maps were used for one-finger trials (TMOSS group 3) and five-finger trials (TMOSS group 4) for all participants. Before starting one-finger trials, the participant’s other fingers were taped to the palm, and before starting five-finger trials, the tape was removed. Then, the participant’s right index finger was placed at approximately the middle of the map, avoiding contact with any path or point symbols. The participant was asked to hold his/her hand in that position, without moving, until he/she was asked a question to search for a specific symbol cluster on the map and a beep indicated it was time to start moving, e.g., “Please locate the cluster of squares and say ‘here,’ {beep}.” The participant was instructed to complete trials as quickly as possible, and once he/she had found the landmark to stop moving his/her hand, say “here,” and remove his/her hand from the map.

Below the clear table and tactile map a video camera was positioned (Canon Vixia HF R21, with a Canon 3.28-megapixel full HD CMOS image sensor). It pointed upwards and was focused to the depth plane of the tactile map. The camera’s field of view was filled by the tactile map (30.5 cm x 30.5 cm) plus some additional space on each side (roughly 8 cm). The video camera recorded at a 1920 x 1080 resolution at 29.97 frames per second (commonly referred to as 30 Hz). Above the table was a white shelf that provided the video a white backdrop. Video clips of each trial were cropped so that they began when the participant initiated movement, and ended when the movement completely stopped, so that there was no footage of a stationary hand.

Finger Tracking

Each trial's video was run through a finger-tracking algorithm (Morash & van der Velden, 2014; van der Velden & Morash, 2011). This algorithm applied a ridge-detection transform to each video frame, after which fingertip locations could be picked out using value thresholding. Example finger paths are shown in figure 2.1. The algorithm's estimates of fingertip location were measured to have standard deviations $\sigma_x = 0.16$ cm and $\sigma_y = 0.21$ cm, and the correlation $\rho_{x,y} = -0.02$ not significantly different from zero (Morash & van der Velden, 2014). In this research, the x-axis refers to the left-right direction on a stimulus, and the y-axis to the up-down direction. Each finger position was a single point in (x,y) . The algorithm did not disambiguate contact and non-contact, due to the participant lifting his/her hand from the stimulus. Anecdotally, participants did not appear to lift their fingers from the stimulus. However, future research may investigate this phenomenon, and address possible approaches to handling "missing" non-contact location data.

Missing Data

The data contained some missing positions for thumb and little fingers that occurred when the fingers were out of camera view. The missing positions were at a considerable distance from the stimulus, and were therefore not in contact with any map features. In total, 7.27% of thumb and 0.06% of little finger positions were missing in five-finger trials.

Analyses

Distance calculations between finger positions and stimulus features were done in Matlab (Mathworks Inc., Natick, MA). All statistical tests were run in R (The R Foundation for Statistical Computing, Vienna, Austria). Repeated-measure ANOVAs were estimated using the `car` package (Fox & Weisberg, 2011), and are reported with the generalized eta-squared measure (η_G^2) of effect size (Bakeman, 2005). Assumptions of sphericity were tested using Mauchly's test (reported statistic W), and violations of sphericity were dealt with using the Greenhouse-Geisser correction (ϵ) to degrees of freedom. Note that Mauchly's test is only reported for factors with 3 or more levels, because sphericity can be automatically assumed for factors with 2 degrees of freedom or less.

Post-hoc tests, in cases with and without violations of sphericity, using t-tests or contrasts, are reported with p-values adjusted by the Bonferroni method, which is the adjustment method most resistant to violations of sphericity (Field, 1998). Post-hoc comparisons following an ANOVA, where paired t-tests would not suffice due to the presence of more than one independent factor, were made using type II/III contrasts (equivalent in all tests in this research due to equal numbers of observations at different factor levels). Contrast effects were estimated and tested using the `lmer4` (Bates, Maechler, Bolker, & Walker, 2013) and `contrast` (Kuhn, Weston, Wing, & Forester, 2013) packages in R.

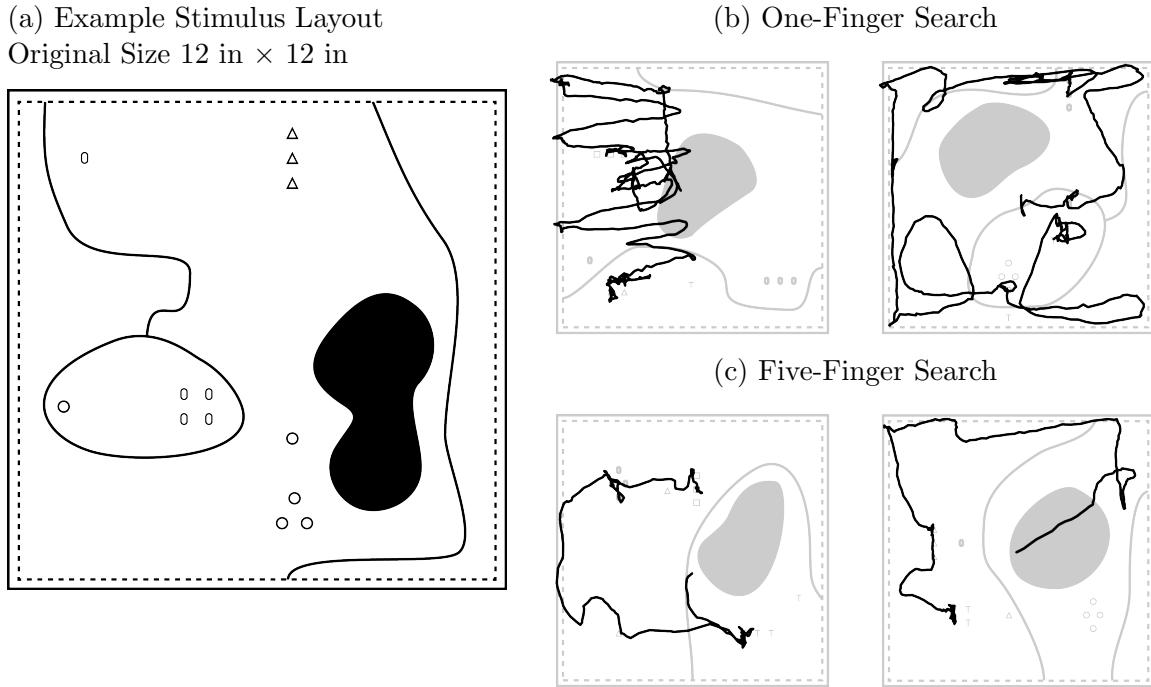


Figure 2.1: Example stimulus layout and index-finger trackings.

Search Times

Participants' mean one-finger and five-finger trial lengths, from first frame of movement to last frame of movement, were compared using paired t-tests. Note that the maps in one-finger and five-finger trials were different, but had similar types and locations of targets.

Index-Finger Speeds in One-Finger and Five-Finger Search

If the index finger is the primary haptic sensor, and the other fingers provide no sensory benefit, then the finding that five-finger search is faster than one-finger search could be attributable to faster movement of the hand in five-finger trials. To examine this possibility, plots were created of the empirical distributions of speeds in one-finger and five-finger search. Speeds were calculated by finding the Euclidean distance between index-finger positions in consecutive frames, divided by the time between frames (≈ 0.03 s). Empirical distributions were created for each trial using kernel density estimation (kernel with $\sigma = 0.5$ cm/s). Trial distributions were averaged within a participant for one-finger and five-finger trials separately to create participants' mean empirical distributions. Participants' mean empirical distributions were then averaged for one-finger and five-finger trials separately to form overall averages. The same procedure was used to create overall averages with cumulative densities.

Wald confidence intervals (CIs) were constructed for overall averages based on the standard error (SE) across participants. In addition to plotting the empirical distributions of speeds in one-finger and five-finger search, participants' mean speeds were compared using a paired t-tests.

Spatial Distributions

To investigate the overall concentration of participants' finger locations on different types of map features, finger locations were averaged within each map, across participants, to construct two-dimensional spatial densities using kernel density estimation, with a Normal kernel, bandwidth chosen using the Normal reference distribution, about 3cm (p. 130 Venables & Ripley, 2002). This was possible because each participant used the same maps for one-finger search, and a different but consistent group of maps for five-finger search. Analyses of trajectory data from Morash, Connell Pensky, and Miele (2013) revealed that a single participant's data and data averaged across as many as 5 participants did not contain a sufficient number of finger locations to produce high-resolution estimates of spatial densities.

Properties of Five-Finger Search

Throughput of Information To investigate how time was spent in relation to map features in one-finger and five-finger search, the amount of time and percentage of time that the participants' fingers were within 0.5 cm of each map-feature type (symbol, path, lake, and border) were calculated. The value 0.5 cm was chosen because it is approximately half a finger width, but changing this threshold to be larger or smaller within reasonable biological limits (0.25 cm - 1 cm) did not alter results. The same analyses were conducted using these data, separately, on absolute time and percentage of time spent in contact with map features. The first set of analyses examined whether the amount or percentage of time spent on map features was different in five-finger and one-finger search using repeated-measures ANOVAs, with within-participant factors of map feature (symbol, path, lake, border) and condition (one-finger, five-finger). The second set of analyses investigated whether participants' index fingers, alone, touched different map features in one-finger and five-finger search using repeated-measures ANOVAs on absolute and percentage of time, with within-participant factors of map feature (symbol, path, lake, border) and condition (one-finger, five-finger). The last set of analyses investigated differences in map-feature contact between fingers in five-finger search using repeated-measures ANOVAs on absolute and percentage of time with within-participant factors of map feature (symbol, path, lake, border) and finger (thumb, index, middle, ring, little).

Detection Radius Five-finger search could be benefited by the larger detection radius of five fingers than one finger. To characterize this size difference, participants' average separations between adjacent fingers were calculated. These were compared using a repeated-

measures ANOVA, with the within-participant factor of finger pair (thumb-index, index-middle, middle-ring, and ring-little).

Using the larger detection radius of five fingers would lead to first encountering the target, leading to detection, with any of the fingers, not just the index finger. First contact with the target was examined by comparing the percentage of trials in which first contact was made by each finger using a repeated-measures ANOVA, with the within-participant factor of finger (thumb, index, middle, ring, little). First contact occurred when one of the fingers was within 0.5 cm of the target (any part of the target, not just the center), and the fingers remained within 0.5 cm of the target, with no more than a 20 sample (0.67 s) departure, until the end of the trial. Changing these time and distance thresholds, within reasonable limits, did not change results. The departure allowed the participant to briefly move his/her hand away from the target to look for nearby symbols that, together with the found symbol, constituted a symbol cluster.

For further analysis, the detection radius, here defined as the length from thumb to little finger positions, was calculated and compared when fingers were touching (within 0.5 cm) and not touching different map features. The logic was that the detection radius would shrink when the hand was concentrated over features that needed careful examination, and would expand when searching for the next feature to investigate. The participants' average thumb-to-little-finger distances were compared for each symbol type separately using a repeated-measures ANOVA, with within-participant factors of contact (contact, no contact) and finger making contact (thumb, index, middle, ring, little).

Finger Independence Participants' average r^2 values between every pair of fingers were compared to examine independence in finger positioning during five-finger search. An r^2 equal to zero would imply that the positioning of the fingers was completely unrelated, a highly unlikely scenario given that fingers are attached to the same hand. An r^2 equal to one would indicate that the position of one finger (f_1) was completely explained by the position of the other finger (f_2) through a linear relationship, $f_2 = \alpha + \beta f_1$ (presumably β would have an estimate close to 1). In general, r^2 values reflect the proportion of a finger's positional variance that was explained by a linear relationship with another finger, as specified in the previous equation.

The r^2 values were compared with a repeated-measures ANOVA, with within-participant factors of finger pair (thumb-index, thumb-middle, etc., for 10 pairs) and measurement axis (x or y). The participants' mean speeds for all five fingers were calculated to disambiguate whether lower r^2 values could be due to anchoring (fingers not moving) or independent finger movements. Finger speeds were compared using a repeated-measures ANOVA, with within-participant factor of finger (thumb, index, middle, ring, little).

2.3 Results

Search Times - Trial Lengths

The current study's participants were significantly faster at five-finger search than one-finger search, $t(8) = 2.59$, $p = 0.032$. Average time to complete a one-finger search was 24.39 s ($SE = 3.70$ s), and to complete a five-finger search was 14.98 s ($SE = 1.61$ s). On average, five-finger search was almost 10 s, or 40 % faster than one-finger search. This finding is consistent with that from a previous study, using the same stimuli with a different group of sighted participants, which did not involve finger tracking (Morash, Connell Pensky, & Miele, 2013).

Index-Finger Speeds in One- and Five-Finger Search

Average empirical distributions of index-finger speeds for one-finger and five-finger searches are shown in figure 2.2. Participants' mean one-finger index speeds, 12.82 cm/s ($SE = 1.03$ cm/s), were significantly faster than their mean five-finger index speeds, 10.31 cm/s ($SE = 0.67$), $t(8) = 2.49$, $p = 0.037$. Therefore, faster index-finger speeds cannot explain why five-finger search takes less time than one-finger search.

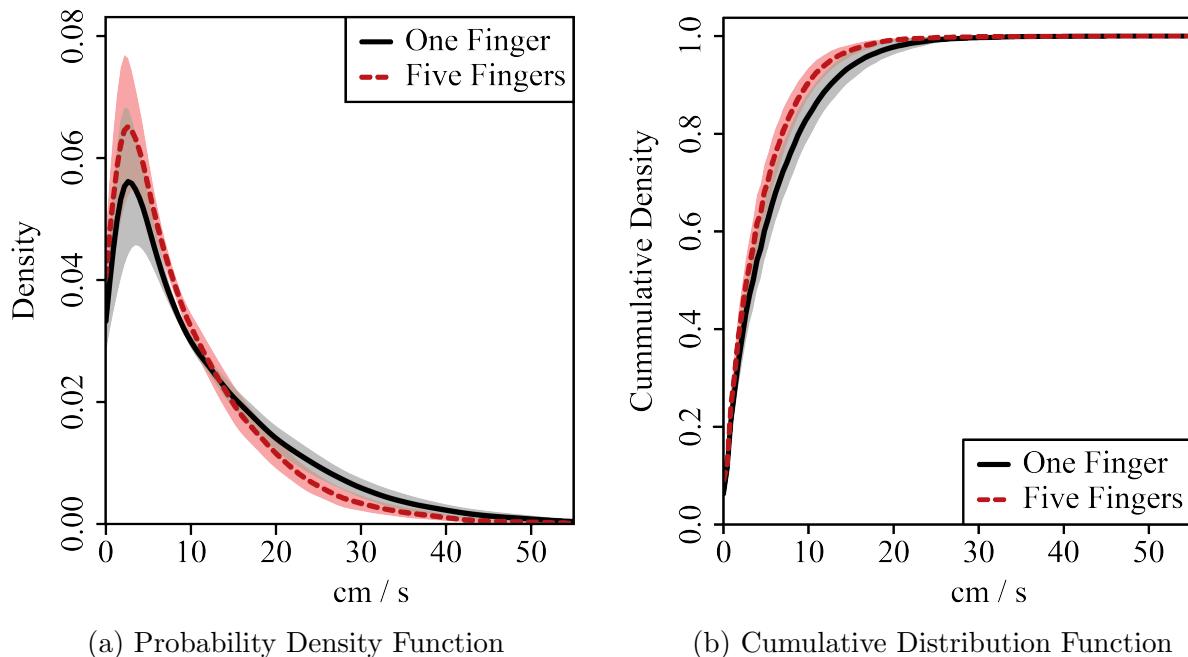


Figure 2.2: Average empirical distributions of index-finger speeds, with 95% CIs.

Spatial Distributions

The spatial distributions associated with a random selection of four one-finger and four five-finger search maps are shown in figure 2.3. The highest densities of finger positions were located on target clusters. A lower, but apparent increase in density was also associated with distractor clusters and loose symbols. There were no apparent affects of map borders, paths, or lakes on finger location distributions.

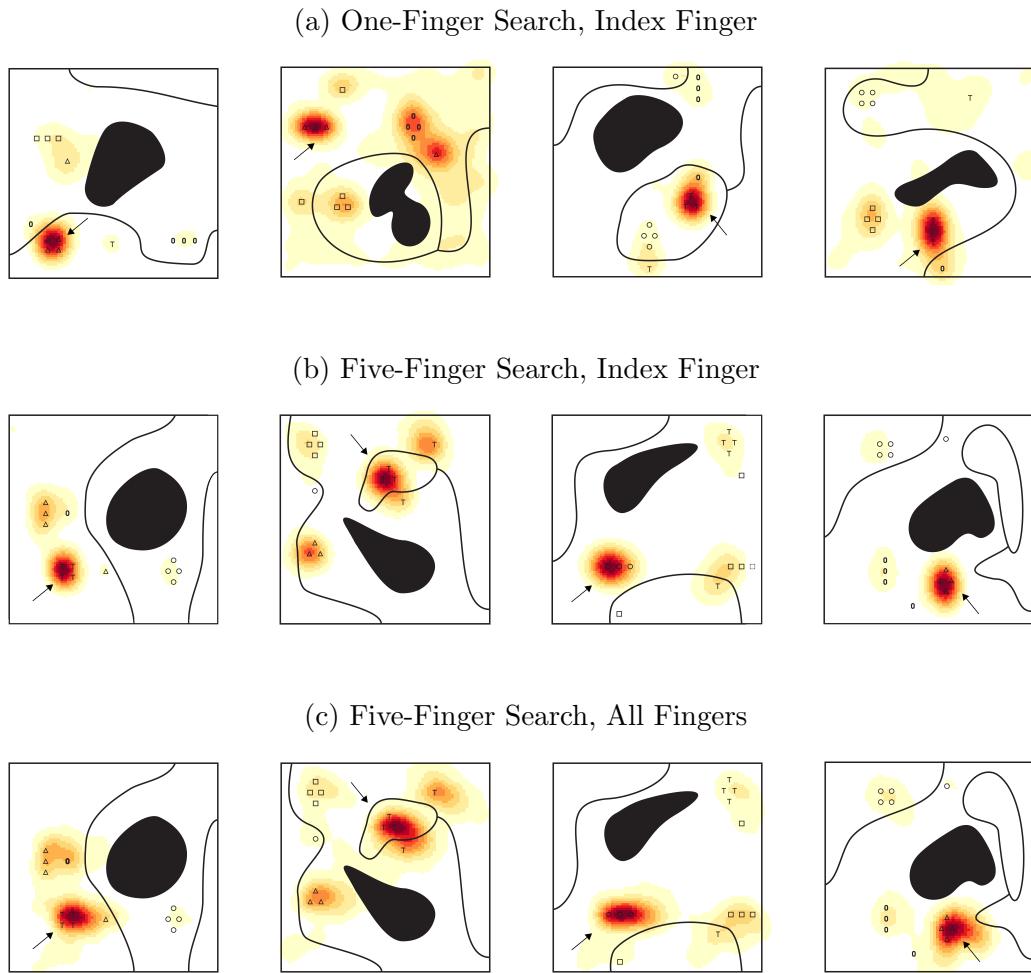


Figure 2.3: Two-dimensional kernel density estimates of finger locations, averaged across participants. Target clusters are marked with arrows, and black lines/areas are map features.

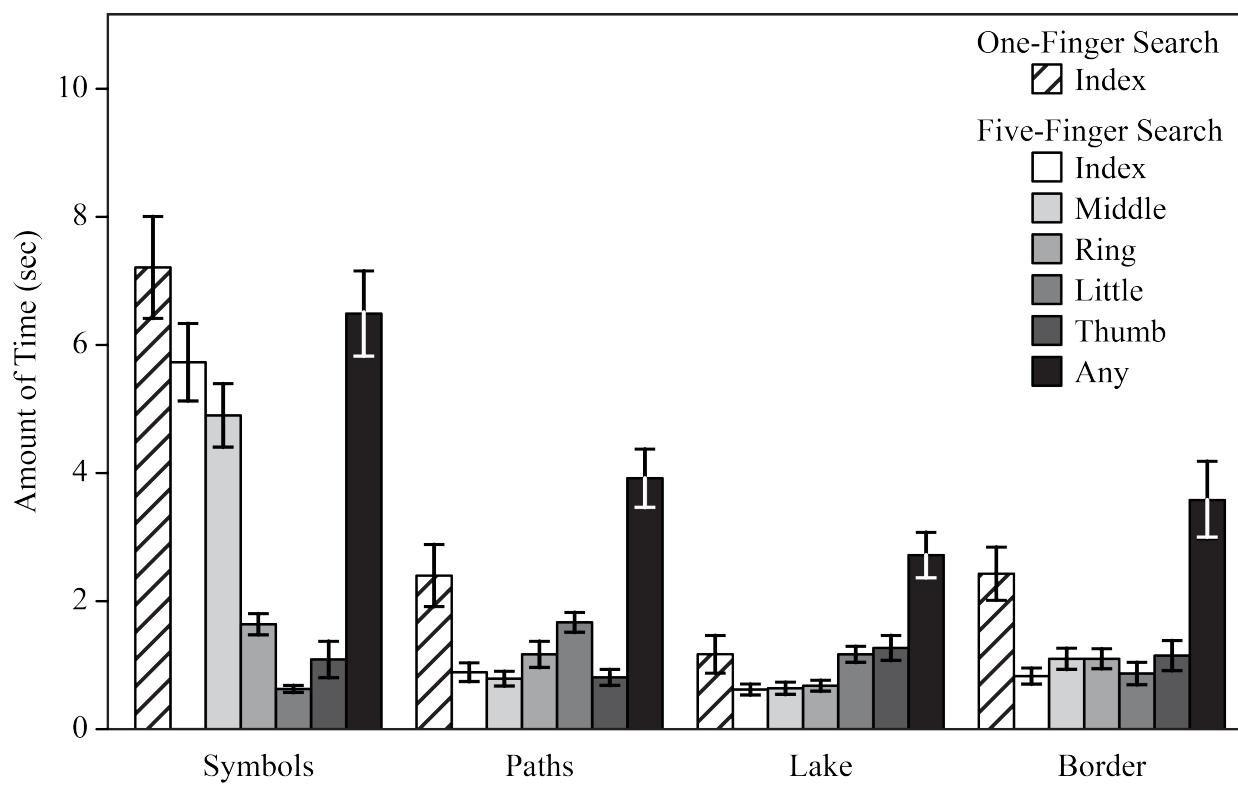


Figure 2.4: Average time (mean \pm SEM) spent within 0.5 cm of map feature.

Properties of Five-Finger Search

Throughput of Information

Absolute Time in Contact with Map Features The average amounts of time that fingers were within 0.5 cm of map features are shown in figure 2.4.

Comparing overall absolute contact time (in seconds) with features in one-finger and five-finger search, the main effect of feature was significant ($W(5) = 0.48, p = 0.43$), $F(3, 24) = 82.35, p < 0.001, \eta_G^2 = 0.58$; the main effect of condition (one versus five fingers) was not significant, $F(1, 8) = 2.22, p = 0.174, \eta_G^2 = 0.07$; and the interaction was significant ($W(5) = 0.48, p = 0.427$), $F(3, 24) = 9.51, p < 0.001, \eta_G^2 = 0.08$. Post hoc contrasts were used to compare one-finger and five-finger search for each map feature, shown in table 2.1. Overall, more time was spent touching symbols than any other map feature, and this effect was more pronounced in one-finger search than five-finger search. More time was spent touching paths than lakes overall, and more time was spent on the border relative to the lake in one-finger search than five-finger search.

Comparing index-finger contact with map features in one-finger and five-finger search, the main effect of feature was significant ($W(5) = 0.06, p = 0.002, \epsilon = 0.042$), $F(1.25, 9.99) =$

103.82, $p < 0.001$, $\eta_G^2 = 0.75$; the main effect of condition was significant, $F(1, 8) = 7.45$, $p = 0.026$, $\eta_G^2 = 0.20$; and the interaction was not significant ($W(5) = 0.55$, $p = 0.544$), $F(3, 24) = 1.96$, $p = 0.147$, $\eta_G^2 = 0.03$. Post hoc contrasts were used to investigate the main effect of map feature, by comparing time spent on pairs of map features, averaged across conditions, shown in table 2.1. The estimated contrast between the time spent on map features in one-finger and five-finger (significant main effect) was: 1.28 ($SE = 0.27$). The index finger was in contact with symbols more than any other map feature, and in contact with map features for more time in one-finger search than five-finger search.

Comparing the five fingers' contact with map features in five-finger search, the main effect of feature was significant ($W(5) = 0.07$, $p = 0.004$, $\epsilon = 0.5143$), $F(1.54, 12.34) = 58.96$, $p < 0.001$, $\eta_G^2 = 0.68$; the main effect of finger was significant ($W(9) = 0.18$, $p = 0.29$), $F(4, 32) = 43.26$, $p < 0.001$, $\eta_G^2 = 0.37$; and the interaction was significant ($W(77) = 0.00$, $p < 0.001$, $\epsilon = 0.22$), $F(2.58, 20.66) = 31.490$, $p < 0.001$. Post hoc contrasts were used to compare pairs of fingers for every map feature, and pairs of features for every finger, shown in table 2.1. During five-finger search, index and middle fingers spent more time in contact with symbols than any other map feature. The ring finger spent more time in contact with symbols than lakes, and the little finger spent more time in contact with paths than symbols. Symbols were touched more by the index and middle fingers than by any other finger, and more by the ring finger than the little finger.

Table 2.1: Contrast results for analyses on absolute amount of time within 0.5 cm of map feature (s). SEs in parentheses.

Contrast	Estimate (SE)	<i>z</i>	<i>p</i> ^a
<i>Overall</i>			
symbol-path	3.69 (0.41)	8.88	< 0.001***
symbol-lake	4.90 (0.41)	11.82	< 0.001***
symbol-border	3.84 (0.41)	9.26	< 0.001***
path-lake	1.22 (0.41)	2.93	0.040*
path-border	0.15 (0.41)	0.37	1.000
lake-border	-1.06 (0.41)	-2.56	0.126
one-five symbol-path	3.28 (0.59)	5.6	< 0.001***
one-five symbol-lake	4.49 (0.59)	7.65	< 0.001***
one-five symbol-border	3.63 (0.59)	6.18	< 0.001***
one-five path-lake	-0.32 (0.59)	-0.54	1.000
one-five path-border	-1.18 (0.59)	-2.01	0.528
one-five lake-border	-2.41 (0.59)	-4.11	0.001**
<i>Index Fingers</i>			
symbol-path	4.83 (0.38)	12.72	< 0.001***
symbol-lake	5.57 (0.38)	14.68	< 0.001***
symbol-border	4.84 (0.38)	12.75	< 0.001***
path-lake	0.74 (0.38)	1.96	0.300
path-border	0.01 (0.38)	0.03	1.000
lake-border	-0.73 (0.38)	-1.93	0.321
<i>Five Fingers</i> ^b			
index symbol-path	4.85 (0.28)	17.53	< 0.001***
index symbol-lake	5.11 (0.28)	18.48	< 0.001***
index symbol-border	4.9 (0.28)	17.73	< 0.001***
middle symbol-path	4.11 (0.28)	14.85	< 0.001***
middle symbol-lake	4.26 (0.28)	15.39	< 0.001***
middle symbol-border	3.8 (0.28)	13.73	< 0.001***
ring symbol-lake	0.96 (0.28)	3.47	0.036*
little symbol-path	-1.05 (0.28)	-3.78	0.011*
symbol index-thumb	4.64 (0.28)	16.78	< 0.001***
symbol index-ring	4.09 (0.28)	14.79	< 0.001***
symbol index-little	5.1 (0.28)	18.46	< 0.001***
symbol middle-thumb	3.8 (0.28)	13.75	< 0.001***
symbol middle-ring	3.25 (0.28)	11.76	< 0.001***
symbol middle-little	4.27 (0.28)	15.43	< 0.001***
symbol ring-little	1.02 (0.28)	3.67	0.017*

^a Bonferroni adjusted

^b Only significant effects shown

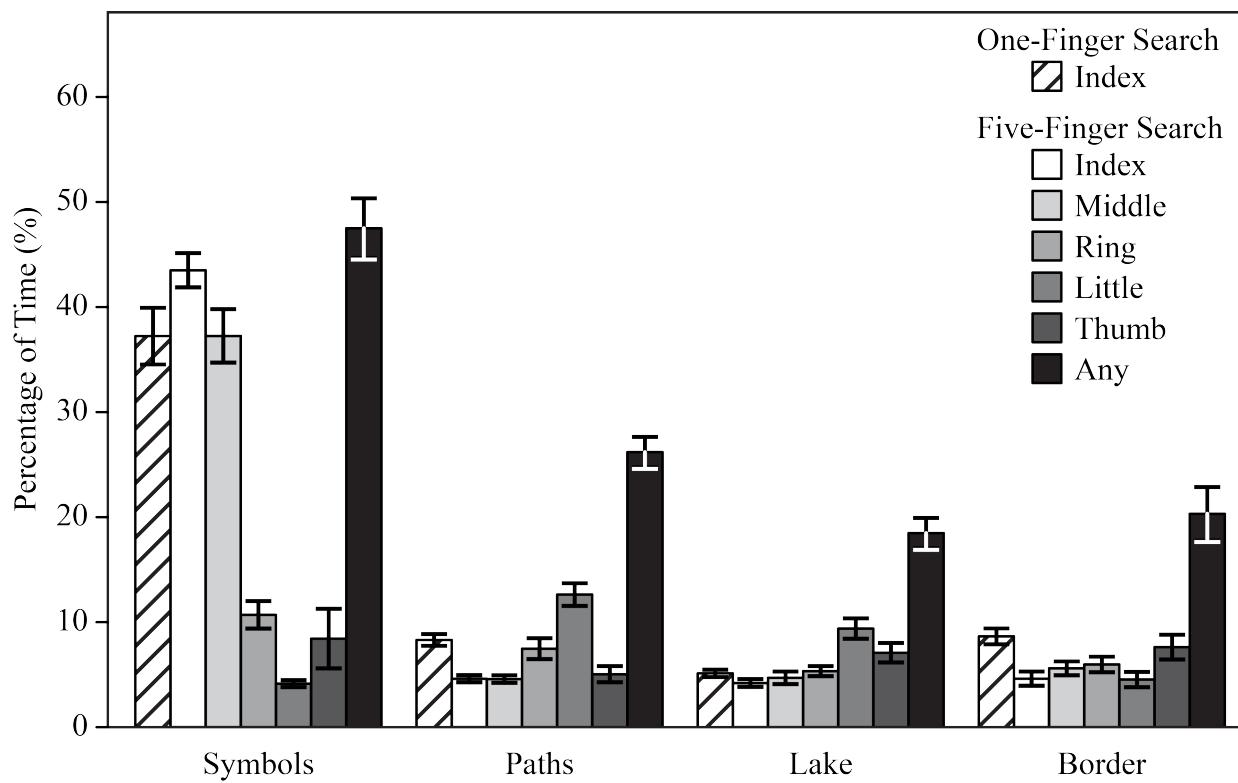


Figure 2.5: Average percentage time (mean \pm SEM) spent within 0.5 cm of map feature.

Relative Time in Contact with Map Features The average percentages of time that fingers were within 0.5 cm of map features are shown in figure 2.5. The average percentage of time in which no fingers were in contact with any map features was 18.45% ($SE = 1.99\%$).

Comparing overall percentages of contact time (in percentages) with map features in one-finger and five-finger search, the main effect of map feature was significant ($W(5) = 0.33, p = 0.191$), $F(3, 24) = 117.86, p < 0.001, \eta^2_G = 0.82$; the main effect of condition was significant $F(1, 8) = 134.62, p < 0.001, \eta^2_G = 0.58$; and the interaction was not significant ($W(5) = 0.10, p = 0.011, \epsilon = 0.495$), $F(1.48, 11.88) = 1.03, p = 0.364, \eta^2_G = 0.06$.

Post hoc contrasts were used to investigate the main effect of map feature, by comparing percentages of time spent on pairs of map features, averaged across conditions, shown in table 2.2. The estimated contrast between the percentage of time spent on map features in one-finger and five-finger search (significant main effect) was: -13.29 ($SE = 1.42$). A greater percentage of time was spent in contact with symbols than any other map feature, and greater percentage of time was spent in contact with paths than lakes. The fingers were in contact with map features for a smaller percentage of time in one-finger than five-finger search.

Comparing index-finger contact with map features in one-finger and five-finger search,

the main effect of feature was significant ($W(5) = 0.08, p = 0.005, \epsilon = 0.51$), $F(1.54, 12.31) = 388.45, p < 0.001, \eta_G^2 = 0.94$; the main effect of condition was not significant $F(1, 8) = 1.30, p = 0.287, \eta_G^2 = 0.01$; and the interaction was not significant ($W(5) = 0.01, p < 0.001, \epsilon = 0.37$), $F(1.10, 8.81) = 4.15, p = 0.07, \eta_G^2 = 0.23$. Post hoc contrasts were used to investigate the main effect of map feature, by comparing percentages of time spent on pairs of map features, averaged across conditions, shown in table 2.2. The index finger was in contact with symbols for a larger percentage of time than any other map feature, for both one-finger and five-finger search.

Comparing the five fingers' contact with map features in five-finger search, the main effect of feature was significant ($W(5) = 0.03, p < 0.001, \epsilon = 0.46$), $F(1.39, 11.09) = 59.28, p < 0.001, \eta_G^2 = 0.75$; the main effect of finger was significant ($W(9) = 0.16, p = 0.24$), $F(4, 32) = 51.00, p < 0.001, \eta_G^2 = 0.42$; and the interaction was significant ($W(77) = 0.00, p < 0.001, \epsilon = 0.33$), $F(3.96, 31.70) = 74.68, p < 0.001, \eta_G^2 = 0.82$. Post hoc contrasts were used to compare pairs of fingers for every map feature, and pairs of features for every finger, shown in table 2.2. Symbols were touched more by index and middle fingers for a larger percentage of time than any other fingers (index and middle were not different from each other). Symbols were also touched by the ring finger more than the little finger. Paths were touched by the little finger more than the thumb, index finger, and middle finger. For index and middle fingers, contact was made with symbols for a greater percentage of time than with any other map features. For the little finger, contact was made with paths for a larger percentage of time than with symbols or the border.

Table 2.2: Contrast results for analyses on percentage of time within 0.5 cm of map feature (s). SEs in parentheses.

Contrast	Estimate (SE)	<i>z</i>	<i>p</i> ^a
<i>Overall</i>			
symbol-path	25.11 (2.00)	12.52	< 0.001 ***
symbol-lake	30.56 (2.00)	15.24	< 0.001 ***
symbol-border	27.88 (2.00)	13.9	< 0.001 ***
path-lake	5.45 (2.00)	2.72	0.040 *
path-border	2.76 (2.00)	1.38	1.000
lake-border	-2.68 (2.00)	-1.34	1.000
<i>Index Fingers</i>			
symbol-path	33.9 (1.36)	24.97	< 0.001 ***
symbol-lake	35.7 (1.36)	26.29	< 0.001 ***
symbol-border	33.73 (1.36)	24.84	< 0.001 ***
path-lake	1.79 (1.36)	1.32	1.000
path-border	-0.17 (1.36)	-0.13	1.000
lake-border	-1.97 (1.36)	-1.45	0.882
<i>Five Fingers</i> ^b			
symbol index-thumb	35.07 (1.87)	18.73	< 0.001 ***
symbol index-ring	32.81 (1.87)	17.52	< 0.001 ***
symbol index-little	39.35 (1.87)	21.02	< 0.001 ***
symbol middle-thumb	28.81 (1.87)	15.39	< 0.001 ***
symbol middle-ring	26.55 (1.87)	14.18	< 0.001 ***
symbol middle-little	33.1 (1.87)	17.68	< 0.001 ***
symbol ring-little	6.55 (1.87)	3.5	0.033 **
path little-thumb	7.58 (1.87)	4.05	0.004 **
path little-index	8.01 (1.87)	4.28	0.001 **
path little-middle	8.04 (1.87)	4.30	0.001 **
index symbol-path	38.88 (1.87)	20.77	< 0.001 ***
index symbol-lake	39.29 (1.87)	20.99	< 0.001 ***
index symbol-border	38.88 (1.87)	20.77	< 0.001 ***
middle symbol-path	32.66 (1.87)	17.45	< 0.001 ***
middle symbol-lake	32.54 (1.87)	17.38	< 0.001 ***
middle symbol-border	31.64 (1.87)	16.9	< 0.001 ***
little path-symbol	8.48 (1.87)	4.53	< 0.001 ***
little path-border	8.09 (1.87)	4.32	0.001 **

^a Bonferroni adjusted

^b Only significant effects shown

Detection Radius

Separation between Fingers The average separations between adjacent fingers are shown in table 2.3.

Table 2.3: Average separation between adjacent fingers (cm). SEs in parentheses.

Thumb - Index	Index - Middle	Middle - Ring	Ring - Little
5.91 (0.48)	2.17 (0.09)	1.95 (0.12)	2.97 (0.14)

These were compared using a repeated-measures ANOVA, with the within-participant factor of finger pair. The main effect of finger pair was significant ($W(7) = 0.011, p < 0.001, \epsilon = 0.36$), $F(1.08, 8.64) = 58.49, p < 0.001, \eta_G^2 = 0.82$. Post-hoc paired t-tests indicated that the average separation between thumb-index was significantly larger than for index-middle $t(8) = 7.67, p < 0.001$; middle-ring $t(8) = 8.70, p < 0.001$; and ring-little $t(8) = 6.32, p = 0.001$. Also, the average separation between ring-little was significantly larger than that for index-middle $t(8) = 9.05, p < 0.001$; and middle-ring $t(8) = 10.68, p < 0.001$. There was not a significant difference in the average separation between index-middle and middle-ring $t(8) = 2.71, p = 0.16$. Therefore, the average distribution of space between the fingers was not even. The space between the thumb and index finger was largest, that between the ring and little fingers was second largest, and the middle three fingers were evenly spaced.

Finger to Make First Contact The average percentages of trials in which first contact was made by each finger with the target, leading to target identification, are shown in table 2.4. There was not a significant difference in the fingers' occurrences of making first contact ($W(9) = 0.62, p = 0.965$), $F(4, 32) = 1.72, p = 0.169$.

Table 2.4: Average percentage each finger was the first to contact the target (%). SEs in parentheses.

Thumb	Index	Middle	Ring	Little
18.06 (6.94)	33.33 (6.25)	12.50 (4.17)	15.28 (5.01)	20.83 (4.66)

Spread of the Hand in Relation to Map Features Table 2.5 shows the average thumb-to-little-finger distance when contact was or was not made by different fingers with different map features. The overall average across participants, including all parts of each trial, was 9.03 cm ($SE = 0.60$ cm).

Table 2.5: Average thumb-to-little-finger distance when fingers were within 0.5 cm of map features (cm), “contact,” or not within 0.5 cm of map features, “no contact.” SEs in parentheses.

	Thumb	Index	Middle	Ring	Little
<i>Symbols</i>					
contact	8.27 (0.69)	8.16 (0.66)	8.17 (0.61)	8.43 (0.53)	9.34 (0.44)
no contact	9.29 (0.64)	9.78 (0.64)	9.65 (0.64)	9.15 (0.6)	9.04 (0.6)
<i>Paths</i>					
contact	9.67 (0.62)	10.29 (0.75)	10.49 (0.87)	9.99 (0.66)	9.25 (0.48)
no contact	9.03 (0.60)	8.99 (0.6)	8.99 (0.6)	9.02 (0.61)	9.03 (0.61)
<i>Lake</i>					
contact	10.05 (0.75)	10.28 (0.82)	9.87 (0.88)	9.87 (0.88)	9.97 (0.82)
no contact	8.99 (0.61)	8.96 (0.6)	8.94 (0.6)	8.94 (0.59)	8.93 (0.60)
<i>Borders</i>					
contact	9.91 (0.7)	10.42 (0.78)	10.61 (0.70)	10.2 (0.57)	10.13 (0.76)
no contact	9 (0.59)	8.98 (0.60)	8.98 (0.60)	9.02 (0.6)	9 (0.60)

Comparing the span of the hand when different fingers were touching or not touching map symbols, the main effect of contact/no-contact was significant, $F(1, 8) = 7.58$, $p = 0.25$, $\eta_G^2 = 0.07$; the main effect of finger was not significant ($W(9) = 0.01$, $p < 0.001$, $\epsilon = 0.40$), $F(1.60, 12.77) = 0.145$, $\eta_G^2 = 0.01$; and the interaction was significant ($W(9) = 0.00$, $p < 0.001$, $\epsilon = 0.36$), $F(1.43, 11.50) = 5.52$, $p = 0.028$, $\eta_G^2 = 0.04$. Post hoc contrasts were used to compare contact/no-contact for each finger. The estimates were: thumb 1.02 ($SE = 0.36$), $z = 2.84$, $p = 0.023$; index finger 1.63 ($SE = 0.36$), $z = 4.51$, $p < 0.001$; middle finger 1.49 ($SE = 0.36$), $z = 4.12$, $p < 0.001$; ring finger 0.72 ($SE = 0.36$), $z = 1.99$, $p = 0.235$; and little finger -0.3 ($SE = 0.36$), $z = -0.83$, $p = 1.000$. Hand span was significantly smaller when symbols were touched by the thumb, index finger, or middle finger.

For paths, the main effect of contact/no-contact was significant, $F(1, 8) = 6.38$, $p = 0.036$, $\eta_G^2 = 0.06$; the main effect of finger was not significant ($W(9) = 0.03$, $p = 0.006$, $\epsilon = 0.41$), $F(4, 32) = 3.93$, $p = 0.54$, $\eta_G^2 = 0.01$; and the interaction was significant ($W(9) = 0.03$, $p = 0.007$, $\epsilon = 0.41$), $F(4, 32) = 4.08$, $p = 0.048$, $\eta_G^2 = 0.02$. Post hoc contrasts were used to compare contact/no-contact for each finger. The estimates were: thumb -0.64 ($SE = 0.36$), $z = -1.79$, $p = 0.371$; index finger 1.3 ($SE = 0.36$), $z = -3.61$, $p = 0.001$; middle finger -1.51 ($SE = 0.36$), $z = -4.18$, $p < 0.001$; ring finger -0.97 ($SE = 0.36$), $z = -2.69$, $p = 0.035$; and little finger -0.22 ($SE = 0.36$), $z = -0.6$, $p = 1.000$. Hand span was significantly larger when paths were touched by the index finger, middle finger, or ring finger.

Table 2.6: The r^2 between pairs of fingers' positions. Values for x positions are in the upper right, those for y positions in the lower left. SEs in parentheses.

	Thumb	Index	Middle	Ring	Little	x Average
Thumb	-	0.85 (0.03)	0.80 (0.04)	0.79 (0.04)	0.76 (0.06)	0.80 (0.02)
Index	0.86 (0.04)	-	0.93 (0.02)	0.89 (0.03)	0.84 (0.05)	0.88 (0.02)
Middle	0.79 (0.06)	0.95 (0.01)	-	0.96 (0.01)	0.89 (0.03)	0.90 (0.02)
Ring	0.79 (0.06)	0.94 (0.01)	0.98 (0.01)	-	0.94 (0.02)	0.90 (0.02)
Little	0.82 (0.05)	0.92 (0.02)	0.94 (0.02)	0.97 (0.01)	-	0.86 (0.02)
y Average	0.82 (0.03)	0.92 (0.01)	0.92 (0.02)	0.92 (0.02)	0.91 (0.02)	-

For lakes, the main effect of contact/no-contact was not significant, $F(1, 8) = 4.20$, $p = 0.074$, $\eta_G^2 = 0.06$; the main effect of finger was not significant ($W(9) = 0.08$, $p = 0.07$), $F(4, 32) = 1.78$, $p = 0.159$, $\eta_G^2 = 0.00$; and the interaction was not significant ($W(9) = 0.07$, $p = 0.048$, $\epsilon = 0.53$), $F(2.13, 17.04) = 1.24$, $p = 0.317$, $\eta_G^2 = 0.00$. Hand span was unaffected by fingers' contact with the lake.

For map borders, the main effect of contact/no-contact was significant, $F(1, 8) = 13.55$, $p = 0.006$, $\eta_G^2 = 0.10$; the main effect of finger was not significant ($W(9) = 0.02$, $p = 0.002$, $\epsilon = 0.49$), $F(1.99, 15.67) = 1.012$, $p = 0.413$, $\eta_G^2 = 0.00$; and the interaction was not significant ($W(9) = 0.02$, $p = 0.002$, $\epsilon = 0.49$), $F(1.97, 15.76) = 0.99$, $p = 0.394$, $\eta_G^2 = 0.00$. The estimated contrast between contact/no-contact (main effect) was: -1.25 ($SE = 0.16$). Hand span was smaller when contact was made with the map border.

Finger Independence

Positional r^2 Positional r^2 values, in x and y coordinates, between pairs of fingers are shown in table 2.6.

Comparing fingers' r^2 values across finger pair and measurement axis, the main effect of finger-pair was significant ($W(44) < 0.001$, $p < 0.001$, $\epsilon = 0.18$), $F(1.65, 13.18) = 17.40$, $p < 0.001$, $\eta_G^2 = 0.31$; the main effect of measurement axis was not significant $F(1, 8) = 2.25$, $p = 0.172$, $\eta_G^2 = 0.02$; and the interaction was not significant ($W(44) < 0.001$, $p < 0.001$, $\epsilon = 0.21$), $F(1.87, 14.92) = 0.73$, $p = 0.490$, $\eta_G^2 = 0.01$. Post hoc contrasts were used to compare pairs of fingers, e.g., the r^2 values for the thumb to those for the little finger. The only significant ($p < 0.05$) estimates were: thumb-index -0.09 ($SE = 0.01$), $z = -8.78$, $p < 0.001$; thumb-middle -0.1 ($SE = 0.01$), $z = -9.41$, $p < 0.001$; thumb-ring -0.1 ($SE = 0.01$), $z = -9.53$, $p < 0.001$; thumb-little -0.08 ($SE = 0.01$), $z = -7.37$, $p < 0.001$. The thumb had lower r^2 values than any other finger.

The amount of variance in finger position attributable to the finger-tracking algorithm was calculated as the ratio of finger-tracking variance to total position variance. In the x-

Table 2.7: Mean finger speeds (cm/s). SEs in parentheses.

	One-Finger		Five-Finger Search			
	Index	Thumb	Index	Middle	Ring	Little
Mean	12.82 (1.03)	10.32 (0.34)	10.31 (0.67)	10.67 (0.62)	10.55 (0.55)	10.17 (0.50)

dimension, this ratio ranged from 0.0050 to 0.0097 across the five fingers. In the y-dimension, this ratio ranged from 0.0037 to 0.0046.

Average Finger Speeds The average speeds of the fingers during five-finger search are shown in table 2.7. For completeness, that for the index finger in one-finger search is included in the table, but not analyzed in this section. Comparing speeds across the fingers, there was not a significant difference in speeds between fingers, ($W(9) < 0.001$, $p < 0.001$, $\epsilon = 0.34$), $F(1.34, 10.73) = 1.27$, $p = 0.301$, $\eta_G^2 = 0.01$. Therefore, the lower r^2 value of the thumb cannot be attributed to anchoring.

2.4 Discussion

The participants in the current study were 40% faster at completing five-finger search than one-finger search, which agrees with previous findings (Morash, Connell Pensky, & Miele, 2013). The decrease in search time cannot be explained by an increase in index-finger speed because participants moved their index fingers more slowly in five-finger than in one-finger search. Therefore, the contribution of the non-index fingers during haptic search is not merely allowing the index finger to cover more of the stimulus in shorter time.

Three possible contributions of multiple fingers to haptic search were examined: increased information throughput, larger detection radius, and positional independence of the five fingers. These were examined by tracking the positions of the fingers during one-finger and five-finger search in relation to stimulus (tactile map) features. First, the general findings regarding fingers' interaction with stimulus features will be discussed, and then the evidence for and against the three possible benefits of multiple fingers during haptic search.

Finger Interaction with Map Features

Symbols appeared to be the most important map feature to participants, and were processed mainly by the index and middle fingers during five-finger search. This is reasonable, considering that the search target consisted of map symbols, and non-symbol features were less relevant to the search task. More absolute and percentage of time were spent touching symbols than any other map feature, in both one-finger and five-finger search. In five-finger

search, the index and middle fingers spent more absolute and percentage of time touching symbols than any other map feature, and symbols were touched for more absolute and percentage of time by the index and middle fingers than by any other finger. Based on the time spent in contact with map features, the third most used finger for processing symbols appeared to be the ring finger. The ring finger spent more time touching symbols than touching lakes, and symbols were touched for more absolute and percentage of time by the ring finger than by the little finger. These results support previous findings that endorsed the index finger as the most used finger for tactile processing (Overvliet et al., 2007; Symmons & Richardson, 2000). However, these results further suggest that the middle finger, and possibly to a lesser extent the ring finger, are similarly important.

When the hand was in contact with symbols, it contracted, suggesting that all fingers were focused over the symbols for processing. In particular, hand span was significantly smaller when symbols were touched by the thumb, index finger, or middle finger. The hand did not contract when fingers touched any other map feature type. Focusing of the hand when the middle and index fingers were in contact with symbols provides additional support for the importance of these fingers in tactile feature processing. The results also suggest that the thumb may provide an important contribution to the perception of certain stimulus features, such as raised symbols. One possibility is that hand contraction associated with the thumb reflects the use of the thumb and index fingers to pinch map symbols, which was anecdotally observed, but the benefit of which is unclear.

Unexpectedly, the results imply that the little finger was often allocated to paths. Specifically, the little finger spent more time touching paths than symbols, and a larger percentage of time touching paths than symbols or borders. Furthermore, paths were touched by the little finger for a larger percentage of time than by the thumb, index finger, and middle finger. This may reflect a strategy reported by one participant, of using the paths to separate the map into spaces that were searched in sequence. Perhaps staying within these areas was accomplished by keeping the little finger in contact with the paths.

In contrast, some participants followed the paths as a strategy for finding symbols, acknowledging that this was not a successful strategy and was typically abandoned. This may be reflected by the result that hand span was significantly larger when paths were contacted by the index, middle, or ring fingers. Given that searching for symbols would benefit from a larger detection radius (hand span), it may be the case that while tracing the paths the participants expanded their hand to look for symbols.

The lakes should have been largely ignored by the participants, given that no search targets ever appeared in the lakes. Supporting this expectation, hand span was unaffected by contact with the lake. For one-finger and five-finger search, more absolute and percentage of time was spent touching paths and symbols than lakes. Perhaps the only map feature less useful to the search task than the lakes was the map border. Hand span was smaller when in contact with the map border, likely because there was no purpose in extending the hand off of the stimulus. More time was spent in contact with the border than the lake for both one-finger and five-finger search (not significant), with this disparity being larger in one-finger than five-finger search (significant).

Throughput

Greater throughput of information would be implied by less absolute time spent on map features in five-finger than one-finger search, because with greater throughput it would take the hand less time to process these features. Although this was not the case over all fingers, the index finger spent less time in contact with map features in five-finger than one-finger search. Therefore, some processing of map features must have been offloaded to the non-index fingers in five-finger search. This benefit could be due to either serial or parallel processing. A previous study indicated that fingers on the same hand are more likely to process tactal information in serial than in parallel (Overvliet et al., 2010). However, serial processing could still lead to faster processing times, if switching of attention between fingers is faster than moving the index finger between stimulus features.

The current results suggest that the task of processing tactile information is not restricted to the index finger. At least in the case of processing point symbols, the index and middle fingers were the primary sensors, with the thumb and ring fingers possibly providing a lesser benefit. However, other tasks, besides identification of point symbols, may be offloaded to fingers differently, as the results imply that the index, middle, and ring fingers may be responsible for centering the hand over raised lines during line following, while the little finger may be used to track lines to ensure the hand stays within an area limited by the line.

Detection Radius

Although the fingers were in contact with important stimulus features (symbols) for less time in five-finger than one-finger search, they were in contact with features for a larger percentage of time. This implies that not only did using five fingers reduce the time to process tactile features, it also reduced the time spent looking for new features to examine. The larger detection radius associated with five fingers could explain the shorter search time in five-finger than one-finger search.

This possibility was supported by two results. First, the hand span was smaller when touching symbols than when not. This could reflect bringing fingers together to process symbol features (providing increased throughput) or spreading the hand out when searching for new symbols features to examine (providing increased detection radius), or both. Stronger evidence is the result that any of the five fingers could be the first to contact the target symbol-cluster leading to identification. Therefore, participants were most certainly using the larger detection radius of five fingers to find candidate targets. Interestingly, the fingers were equally likely to make first contact with the target symbol-cluster, even though they were not evenly spaced.

The fact that the stimulus was unstructured, so that participants did not know where distractors and targets were located a priori, made finding candidate targets particularly important in the current task. A previous study, where the target and distractors were arranged in a grid, did not observe a reduction in time between one-finger and three-finger

search (Overvliet et al., 2007). This contrasting result can be explained by the reduced benefit of a larger detection radius for finding candidate targets compared to the current research.

Independent Movements

The positioning of each finger was highly correlated with that of the other fingers ($r^2 = 0.79 - 0.98$). This is unsurprising given the physical constraint all of the fingers are on the same hand. Non-perfect prediction, r^2 not exactly 1, could be caused by rotations of the hand, instead of translations, even when the hand was held in a static pose. However, the thumb had lower correlation values than any other finger. This could be caused by more expansion and contraction movements occurring between the thumb and other fingers, e.g., scissoring or pinching movements between the thumb and index fingers, or the thumb could move more/less than the other fingers. The thumb is more capable of being moved independently of the other fingers. In particular, the thumb may move less than the other fingers if it is used as a pivot or spatial anchor. This possibility was not supported by comparing the fingers' speeds, which found no significant differences. Therefore, it's likely that the thumb's lower r^2 values were not due to anchoring, but instead due to expansion and contraction movements between the thumb and the rest of the hand. This is not to say that anchoring does not occur in haptic perception, but that there is no evidence of it in the current search task. Anchoring may be more prominent in line-tracing (Berla et al., 1976) or spatial (Millar & Al-Attar, 2004) tasks.

The thumb clearly takes on a special role during haptic exploration, given its lower r^2 values and greater separation from the hand. Further research may investigate what function pinching raised-line figures has in their identification. Pinching, using the index and thumb, was anecdotally observed in the current study, and consistent with the lower r^2 values for the thumb. If this behavior is common, it would be worth investigating whether it is effective for identifying raised symbols. It's possible that this common strategy, like using only one finger (Symmons & Richardson, 2000), provides no perceptual benefit.

Conclusions

The current chapter investigates why one-handed haptic search is faster with five fingers than one finger. The results provide no evidence of anchoring or faster index-finger movement in five-finger search. Instead, the results support the possibility that greater throughput of information, with information flowing through multiple fingers either in parallel or in serial, and the larger detection radius of the hand benefits five-finger search. These specific benefits imply that using a single finger with tactile displays is not optimal. In fact, the current study's participants examined important stimulus features using foremost their index and middle fingers to an equal extent. As such, training should encourage individuals to use more than a single index finger, which they may prefer when encountering raised-line displays (Symmons & Richardson, 2000). More generally, this research provides strong evidence that

the hand is able to integrate information through multiple fingers during a tactile search task, which contrasts with previous findings (e.g., Overvliet et al., 2007). In particular, haptic search on a two dimensional display is able to utilize the larger sensory area and detection radius associated with multiple fingers.

Chapter 3

Detection Radius Modulates Systematic Strategies in Unstructured Haptic Search

3.1 Introduction

When searching for a target in an enclosed space, the searcher can adopt either a systematic or a random movement strategy. Systematic strategies include movement patterns such as spirals, parallel sweeps, and expanding squares (Bell, 1991), which prevent the searcher from re-covering ground (Bartumeus & Catalan, 2009; Baum, 1987). However, the efficiency benefits of a systematic strategy compared to a random strategy diminish with increased detection radius, which can be illustrated with Monte Carlo simulations. In the case of a large detection radius, the memory and movement-planning costs associated with a systematic search may outweigh the efficiency benefits, and the searcher may opt to use a random movement strategy. Applications in operations research, such as search and rescue, may make use of systematic strategies as a result of planning and training (Champagne, Carl, & Hill, 2003). However, this chapter provides evidence that humans will spontaneously use systematic strategies in an unfamiliar and unstructured haptic search task. Furthermore, the use of systematic strategies is modulated by the haptic detection radius as would be predicted by search efficiency.

Although a systematic search strategy, such as those shown in figure 3.1, would be the most efficient approach during foraging when targets are non-revisitable, i.e., destructive foraging, and scattered, the locations of which are not known *a priori*, there are few examples of animals using systematic search (Banks et al., 2009). Instead, animal movements typically follow a random walk pattern, with random directions and step lengths ℓ_j drawn from a distribution $P(\ell_j) \sim \ell_j^{-\mu}$, $\ell_j > \ell_{\min}$, continuing until the target is within the searcher's detection radius, often defined as within sight, sound, or smell. When $\mu \rightarrow 1$, the animal's movement pattern is ballistic, the animal turns a random direction and continues straight

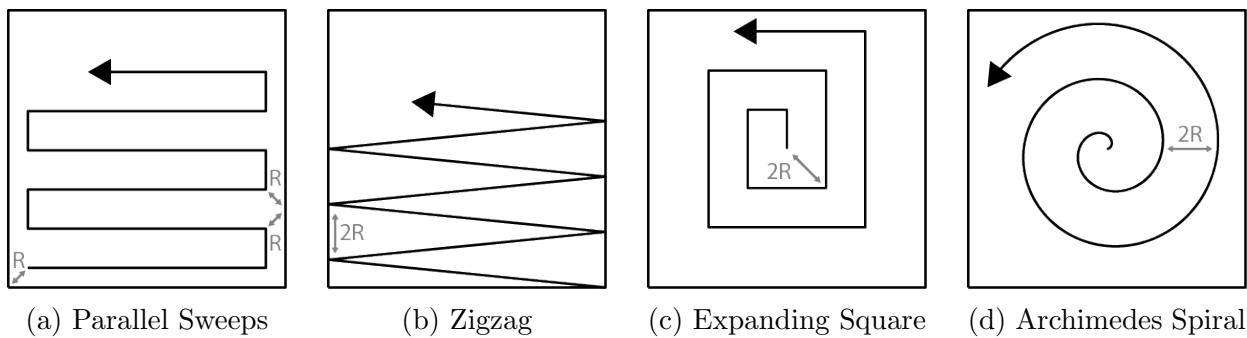


Figure 3.1: Example systematic strategies for a square search area with detection radius R .

until hitting a resource or barrier. When $1 < \mu \leq 3$ the step length distribution is heavy tailed and the movement process is called a Lévy walk, where clusters of short steps are connected by long steps. When $\mu > 3$, the movement process is Brownian (James, Plank, & Brown, 2008; Viswanathan et al., 2000).

Several animals have been reported to forage for resources following a Lévy walk with $\mu \approx 2$, which is optimal for sparse revisitable targets (Viswanathan et al., 2000, 1999); but the most efficient strategy in the case when the targets are non-revisitable is a systematic strategy, followed by ballistic movements, then a Lévy walk, and finally Brownian motion (Banks et al., 2009; Baum, 1987; James et al., 2008; Viswanathan et al., 1999). Higher efficiency can be attributed to better avoidance of previously searched ground. This efficiency benefit decreases with increasing detection radius, as shown by Monte Carlo simulations in figure 3.2. When the detection radius is large, the benefits associated with a systematic strategy may be small enough that the searcher elects a random strategy that requires less memory and motor planning.

Given that systematic search strategies are uncommonly observed in (non-human) animals, the current research examines their use by humans. Systematic search strategies have been used in operations research beginning in WWII, with the allies' search for German U-boats in the Bay of Biscay (Benkoski, Monticino, & Weisinger, 1991; Koopman, 1946), and are currently used for the planning of large-scale human movements in military maneuvers, search and rescue operations, and aerial monitoring (Champagne et al., 2003). However, the use of systematic strategies in these applications was not automatic, only initially emerging under the pressures of war, and are orchestrated through substantial research and planning. Of interest is whether humans will spontaneously use systematic search strategies without extensive experience or training, and if this behavior is more common with a smaller detection radius, as would be predicted based on search efficiency.

The current chapter examines the prevalence of systematic strategies in search for a landmark on an unstructured tactile map, the data being the same throughout this dissertation, described in chapter 2. This search task is analogous to destructive foraging, where distractor and target landmarks are the patches of non-revisitable resources, because there is

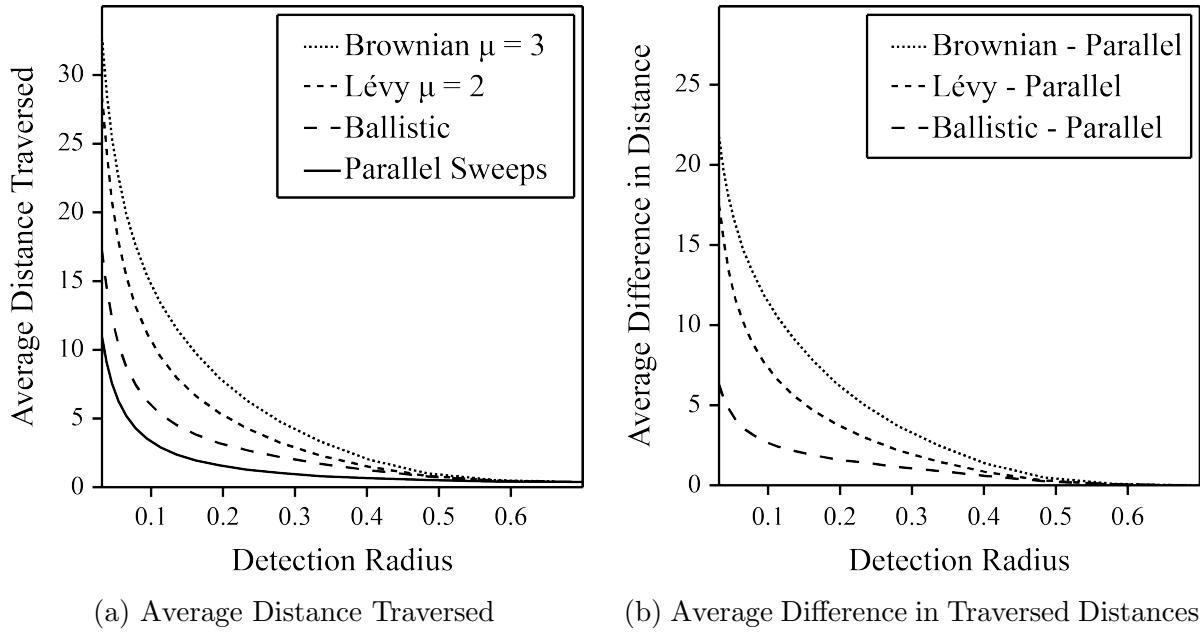


Figure 3.2: Simulations of search for a randomly located target for different sizes of detection radii (0.03 - 0.7 times square search area width and height). The only systematic strategy shown is parallel sweeps, but other systematic strategies produce similar results. The relative ratios of fingertip radius (1 cm) to stimulus width/height is approximately 0.03, and hand radius (fingers spread, 9 cm) to stimulus width/height approximately 0.15.

no value in participants revisiting previous distractors. Because the participants were blindfolded sighted, they did not have focused training on exploring tactile materials, which often includes instructions on using systematic strategies, as these are not automatically adopted by blind/low-vision individuals (Berlá & Murr, 1974).

3.2 Methods

Monte Carlo Simulations

To illustrate the benefits of different search strategies, search movement simulations were created for detection radii 0.03 - 0.70 in a square (side length = 1) search area, for a randomly located target. Simulations were executed for ballistic, Lévy ($\mu = 2$), Brownian ($\mu = 3$), and a parallel sweep search (figure 3.1a), until the searcher was within detection radius of the target. Traversed distances, from start position to target, were averaged across 100,000 simulations to create figure 3.2. Random strategy simulations began in the search area's center, and were simulated $\ell_j = \ell_{\min} u_j^{1/(1-\mu)}$, where $u \sim \text{unif}(0, 1)$, and $\ell_{\min} = 0.05$ was equivalent to a 1.5 cm finger movement (Bartumeus et al., 2002; Bartumeus, da Luz, Viswanathan, &

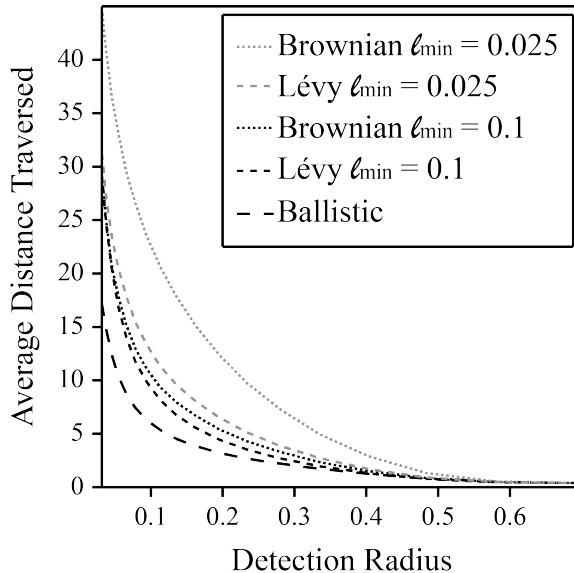


Figure 3.3: Simulations of search for a randomly located target for different sizes of detection radii (0.03 - 0.7 times search area width/height) with different values of ℓ_{\min} .

Catalan, 2005; James et al., 2008). Smaller ℓ_{\min} would further reduce the efficiency of Lévy and Brownian strategies. The effect of ℓ_{\min} is shown in figure 3.3.

Experiment

The data were the same as in chapter 2. Index finger trajectories were analyzed for systematic movements through visual inspection. Trajectories were printed on a blank background, and identified only by a random code that could not be associated with a condition or participant. Two coders inspected each trajectory for zigzags, parallel sweeps, and spirals, the author and a coder with no knowledge of the study procedures or hypotheses. Coder percent agreement and Cohen's Kappa were used to establish that the systematic strategies were clearly apparent and reliably coded. Typical disagreements between coders were over whether a trajectory contained a parallel sweep, zigzag, or combination, as these patterns could be very similar. Therefore, the codes for zigzags and parallel sweeps were combined into a single indication of zigzag or parallel sweep.

3.3 Results

Participants' index-finger scan paths included zigzags, parallel sweeps, and spirals (figure 3.4). The inter-reliability of the coders indicated that the presence/absence of systematic strategies was readily apparent, based on percent agreement and Cohen's Kappa (Landis &

Koch, 1977). Agreement for spirals was 99.3%, $\kappa = 0.93$; and for zigzags or parallel sweeps, which were combined into a single code, 98.6%, $\kappa = 0.97$.

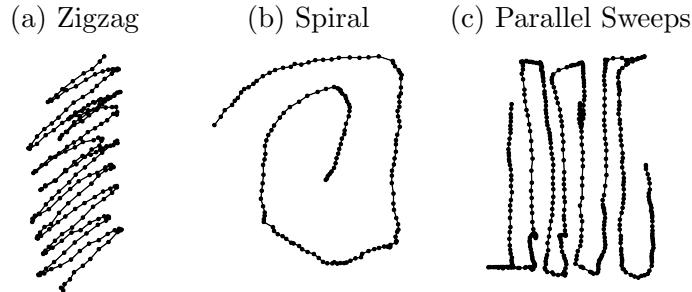


Figure 3.4: Example systematic strategies observed in the current study.

The number of trials containing systematic strategies (figure 3.5) was significantly greater in one-finger than five-finger trials based on nonparametric Mann-Whitney U tests (spiral $z = 1.99$, $p = 0.047$; zigzags or parallel sweeps $z = 2.67$, $p = 0.008$). Recoding truncated one-finger searches ($M = 24.39$ s), which were no longer than average five-finger search times ($M = 14.98$ s), blind to the original codes eliminated five zigzags or parallel sweeps and no spirals, and did not eliminate the significant effect for zigzags or parallel sweeps ($z = 2.62$, $p = 0.009$).

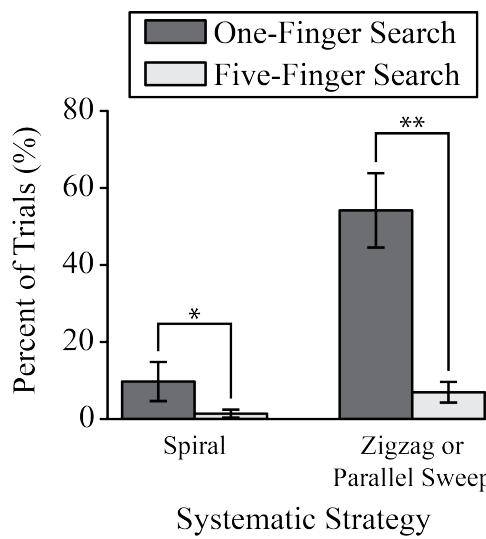


Figure 3.5: Data are represented as mean \pm SEM.

3.4 Discussion

The results of this chapter indicate that untrained humans spontaneously use systematic search strategies: spirals, zigzags, and parallel sweeps, during one-handed haptic search on an unstructured display. Differences in five-finger and one-finger search patterns align with Monte Carlo simulations that indicate that when the detection radius is large, the efficiency benefits from systematic search are reduced, and the searcher may opt for a more random strategy that has lower costs in memory or motor planning. One explanation for the presence of systematic finger/hand movements could be that these movements arise naturally from the mechanics of the arm. For example, opening and closing the elbow could create a horizontal zigzag pattern in the finger trajectory. However, such an explanation cannot account for zigzags and parallel sweeps oriented vertically, parallel sweeps that contain right angles, nor spirals, which are unlikely to be a symptom of the arm's kinematics. Furthermore, the mechanics of the arm cannot account for the differential use of systematic strategies in one-finger and five-finger search.

To my knowledge, systematic strategies are rarely observed in human perceptual processes with an unstructured stimulus, and this is the first study to show that this behavior is modulated by detection radius. Human visual search does not typically produce a systematic scan pattern, and is better described by movements driven by bottom-up feature salience (Henderson, 2003; Itti & Koch, 2000). The exception is when the stimulus is structured in a grid or concentric circles so that serially searching features induces reading-like patterns (Credidio et al., 2012) or spirals (Noton & Stark, 1972), respectively. Haptic search can also follow reading-like movements when the stimulus is structured as a grid (Overvliet, Smeets, & Brenner, 2008). Research on unstructured haptic search, which has been conducted without the specific focus of examining systematic strategies, has sometimes observed participants using systematic parallel sweeps (Plaisier, Kappers, Tiest, & Ernst, 2010; Smith, Gosselin, & Houde, 2002), but other studies have only observed movements that follow straight lines or complex patterns (Plaisier, Bergmann Tiest, & Kappers, 2008; van Polanen, Tiest, & Kappers, 2012).

The current result provides a possible explanation for such disagreement in previous haptic search studies, where some reports observed participants using systematic strategies (Plaisier et al., 2010; Smith et al., 2002), while others did not (Plaisier et al., 2008; van Polanen et al., 2012). In hindsight, systematic strategies were not observed when the hand's detection radius was large (e.g., three or five fingers) relative to the search area, and were observed when the detection radius was small (e.g., one finger). Interestingly, a previous study observed that one-finger search without vision was anecdotally associated with systematic scanning paths, while the same search task with visual information on target/distractor locations (without revealing which was the target) eliminated systematic movements (Plaisier et al., 2010). This can be explained by considering that vision effectively extended the participants' haptic detection radius.

The participants in the current study chose to use systematic strategies when the efficiency benefits were substantial, and used random movement strategies when the efficiency

benefits were presumably not worth the additional movement planning and memory resources. This result underscores the importance of not only information passed through the fingers and proprioceptive information, but also patterns of exploratory movements in haptic perception. Due to the training phase in the current study, it is unclear how this behavior was affected by learning. Presumably the participants could have used their experiences with the stimuli to parametrize their systematic patterns, such as the spacing between parallel sweeps, so as to minimize their search distances/times. This could be the topic of future research.

Chapter 4

Stochastic Differential Equation Models of Haptic Search

4.1 Motivation

The approach taken here is to model fingertip position as a Brownian particle. Besides a random component, the finger's movements are affected by the stimulus being explored. This is achieved by formulating a force field associated with the stimulus. This force field pushes the finger towards points of attraction, e.g., useful stimulus features, and away from points of repulsion, e.g., the stimulus border or features not pertinent to the task. The force field is generated through a potential function, with areas of high potential that are repulsive, like a hill, and areas of low potential that are attractive, like a valley. The potential function and force field are formally related through partial derivatives, and their parameters are estimated using linear regression with the observed finger positions.

There are several scientific questions that motivate estimating these models. First, if stochastic models do not fit observations well, this may imply non-random, possibly systematic finger movements. One interesting outcome would be if one-finger searches are more systematic, and less random, than five-finger searches. This would be indicated by stochastic models fitting the five-finger data better than the one-finger data (“better” fit could be determined using a combination of likelihood-based tests, such as the Bayesian information criterion, BIC, or by comparing real and synthetic data). Such an outcome would provide further evidence that participants alternate between random and systematic search depending on their detection radius (number of fingers). Such switching may be optimal, given that random search can result in the searcher recovering ground (Bartumeus & Catalan, 2009). Therefore, with a small detection radius, it may be prudent to avoid this problem by adopting a systematic strategy. The optimality of such behavior could be proven if one-finger searches are consistently (proportion < 0.05) faster than random-movement simulations based on stochastic models with a fingertip-sized detection radius.

A second motivation for these models is that estimates of the trajectory parameters

may be of interest. For example, do the random components have larger variance or greater momentum for one-finger or five-finger search? This would imply different movement characteristics depending on the number of fingers used. Lastly, differences in the parameters of the stimulus's potential function would reveal differences in stimulus effects. For example, irrelevant features may be more "attractive" in one-finger than five-finger search. This would imply that it takes longer to identify and disregard features with one finger than five fingers, and would be one explanation for why one-finger search takes longer than five-finger search (Morash, Connell Pensky, & Miele, 2013) despite one-finger movements being faster.

4.2 Modeling Background

Stochastic Models of Brownian Motion

Historical Background on Brownian Motion

In 1827 the botanist Robert Brown was studying plant reproduction when he noticed that pollen particles, suspended in water under a microscope, appeared to be in constant random motion. He went on to observe this type of motion, now called Brownian motion, in many types of particles. This provided important support for the idea that matter is composed of individual molecules and atoms, whose constant collisions result in the observed random perturbances. However, it was not until 1905 that Einstein provided the first mathematical formulation of Brownian motion (Coffey, Kalmykov, & Waldron, 2004; Lemons & Gythiel, 1997).

Einstein started with the assumption that a suspended particle would be agitated by constant collisions between it and other suspended particles. These collisions were the natural result of the molecular-kinetic theory of heat, and their cumulative result, assuming the fluid was viscous, was random jumps in a particle's position (Einstein, 1905/1956). Taking the limit of the jump sizes to zero resulted in a partial differential equation, the simplest case of a class of equations now known as the Fokker-Planck equations, which specified the time evolution of the probability density of a Brownian particle. Thereby, Einstein motivated and described the motion of a Brownian particle based on the molecular-kinetic theory of heat, while having never observed nor been familiar with previous work on Brownian motion (Coffey et al., 2004; Einstein, 1905/1956).

Although Einstein's description of Brownian motion agreed with experimental results, it was far removed from the Newtonian dynamics of particles. In contrast, shortly after Einstein's formulation, in 1908, Langevin applied Newton's second law to the motions of a representative Brownian particle, resulting in a simpler formulation called the Langevin equation. Although both Einstein's and Langevin's approaches have been used to derive similar results, the latter is viewed as slightly more general and correct, while also being vastly simpler and with immediate connection to Newtonian dynamics (Coffey et al., 2004; Lemons & Gythiel, 1997).

Langevin's equation is also notable as the first example of a stochastic differential equation (Coffey et al., 2004). The "stochastic" aspect comes from the inclusion of a random (probabilistic) force on the particle. Langevin introduced the random component, which he called the "complementary force," to maintain the particle's agitation. Without the complementary force, the viscous resistance (frictional force) would stop (Lemons & Gythiel, 1997). The random force takes the form of a Wiener process, i.e., standard Brownian motion, defined below.

Brownian Motion - Wiener Process

Note that a Wiener process, also referred to as Brownian motion or standard Brownian motion, is a specific stochastic process, i.e., a collection of random variables indexed by time. This is not to be confused with "Brownian motion" used to describe the movements of a Brownian particle, e.g., through the Langevin equation.

A Wiener process, $W(t)$, can easily be understood by considering its simulation for fixed times t_i separated by $\Delta t > 0$, where $i = 1, \dots, N$. The simulation algorithm, starting with $W(t_1) = 0$ and $i = 1$ is as follows,

1. Generate a random number $z \sim N(0, \sigma^2)$
2. Set $i = i + 1$
3. Set $W(t_i) = W(t_{i-1}) + z\sqrt{\Delta t}$
4. If $i < N$, go to step 1

Note that this algorithm implies, from step 3, that $W(t + \Delta t) - W(t) \sim N(0, \sigma^2 \Delta t)$. More generally, the Wiener process $W = \{W(t), t \geq 0\}$ with variance parameter σ^2 is the integral of Gaussian white noise, with $W(t_1) = 0$, $\mathbb{E}(W(t)) = 0$, and $\mathbb{V}(W(t) - W(s)) = \sigma^2(t-s)$ for $s \leq t$. Any disjoint intervals, e.g., (t_1, t_2) and (t_3, t_4) with $t_1 \leq t_2 \leq t_3 \leq t_4$, have independent increments, $W(t_2) - W(t_1)$ and $W(t_4) - W(t_3)$, (Iacus, 2008; Nelson, 1967).

The Langevin Equation

Starting with Newton's second law, Langevin wrote the position $\mathbf{r}(t) = (x(t), y(t))^T$ of a Brownian particle with mass m at time t , assuming $d\mathbf{r}(t)/dt = \mathbf{v}(t)$ and $d^2\mathbf{r}(t)/dt^2 = d\mathbf{v}(t)/dt$ exist, as

$$\underbrace{m \frac{d^2\mathbf{r}(t)}{dt^2}}_{\text{force}} = \underbrace{-b \frac{d\mathbf{r}(t)}{dt}}_{\text{friction}} + \underbrace{\boldsymbol{\eta}(t)}_{\text{random}} \quad (4.1)$$

where b is a friction constant, governed by Stokes's law, and $\boldsymbol{\eta}(t)$ is a bivariate Gaussian random variable. This expresses the force on the particle in two parts, a frictional force, $-b\mathbf{v}(t)$, and a fluctuating random force due to molecular collisions, $\boldsymbol{\eta}(t)$, (Coffey et al., 2004; Nelson, 1967).

External Force Field In the case that the particle is in an external force field, where the acceleration due to this force field is $K(\mathbf{r}(t), t)$, then the Langevin equation becomes (Chandrasekhar, 1943, eqs. 317-318; Nelson, 1967, eq. 10.1 & ch. 12),

$$m \frac{d^2\mathbf{r}(t)}{dt^2} = \underbrace{K(\mathbf{r}(t), t)}_{\text{force field}} - \underbrace{b \frac{d\mathbf{r}(t)}{dt}}_{\text{friction}} + \underbrace{\frac{d\mathbf{B}(t)}{dt}}_{\text{random}} \quad (4.2)$$

where $\mathbf{B}(t)$ is a Wiener process with variance parameter $2b^2D$.¹ Einstein was the first to derive $D = (kT)/b$, where k is the Boltzmann constant and T is absolute temperature.

Smoluchowski Approximation Under the assumption that the coefficient of friction, $\beta = b/m$, is large, or equivalently mass, m , is small, the so-called Smoluchowski approximation to this equation is (Nelson, 1967, pp. 58 & ch. 10),

$$b \frac{d\mathbf{r}(t)}{dt} = K(\mathbf{r}(t), t) + \frac{d\mathbf{B}(t)}{dt} \quad (4.3)$$

where \mathbf{B} is a Wiener process with variance parameter $2b^2D$.

The Potential Function

In many applications, the force field, $K(\mathbf{r}(t), t)$, is taken to be a conservative force field, derived from a smooth real-valued potential function $H(\mathbf{r}(t), t)$,

$$K(\mathbf{r}(t), t) = -\nabla H(\mathbf{r}(t), t) = \boldsymbol{\mu}(\mathbf{r}(t), t) \quad (4.4)$$

where $\nabla = (\partial/\partial x, \partial/\partial y)^T$ is the gradient operator (Hirsch, Smale, & Devaney, 2004, pp. 280), and the negative sign results from conservation of energy. The force-related acceleration $\boldsymbol{\mu}(\mathbf{r}(t), t) = (\mu_x(\mathbf{r}(t), t), \mu_y(\mathbf{r}(t), t))^T$ is often thought of as a drift parameter (Brillinger et al., 2001; Brillinger, Stewart, & Littnan, 2006; Preisler et al., 2004).

In the current application, the potential function is constructed to examine how finger movements interact with a stimulus. The potential function, $H(\mathbf{r}(t), t)$, and associated force field, $\boldsymbol{\mu}(\mathbf{r}(t), t)$, will be functions of the finger position, $\mathbf{r}(t)$, and important stimulus features, which may be points, lines, or regions (Brillinger et al., 2001). Frequently, the potential is a function of the shortest distance between the finger and a map feature, i.e., $H(\mathbf{r}(t), t) = h(d(\mathbf{r}, t))$ for some function $h(\cdot)$ and $d(\mathbf{r}(t))$ being the shortest distance between the finger and feature (Preisler et al., 2004).

Several potential functions of this type, labeled $k = 1, \dots, n$, can be incorporated linearly,

$$H(\mathbf{r}(t), t) = h_1(d_1(\mathbf{r}(t)), t) + h_2(d_2(\mathbf{r}(t), t)) + \dots + h_n(d_n(\mathbf{r}, t)) \quad (4.5)$$

¹This formula is alternatively often expressed with $\beta = b/m$ and $K = F/m$ for force F , here I use K as the force field directly.

Then their combined force fields are also combined linearly,

$$\boldsymbol{\mu}(\mathbf{r}(t), t) = -2(\mathbf{r}(t) - \mathbf{a}_1)h'_1 - 2(\mathbf{r}(t) - \mathbf{a}_2)h'_2 - \cdots - 2(\mathbf{r}(t) - \mathbf{a}_n)h'_n \quad (4.6)$$

where $\mathbf{a}_k = (x_k, y_k)^T$ is the location of the k th attraction or repulsion region, and h'_k is the partial derivative of h_k with respect to $d_k^2 = (x(t) - x_k)^2 + (y(t) - y_k)^2$ (Preisler et al., 2004). In general, multiple potential functions can be added linearly to create a composite potential function whose force fields also add linearly.

A common scheme is one of attraction/repulsion, where $h(d(\mathbf{r}(t), t)) = \alpha (\mathbf{r}(t) - \mathbf{a})^2$, which is the Ornstein-Uhlenbeck (O-U) process, and describes attraction/repulsion to a point \mathbf{a} . This potential surface either takes the form of a repulsive hill ($\alpha < 0$) or an attractive valley ($\alpha > 0$), centered on point \mathbf{a} with height proportional to the constant α . Example O-U potential surfaces are shown in figure 4.1. The steepness of the hill/valley can vary depending on the axis if the amplitude is a two-valued constant vector $\boldsymbol{\alpha} = (\alpha_x, \alpha_y)^T$.

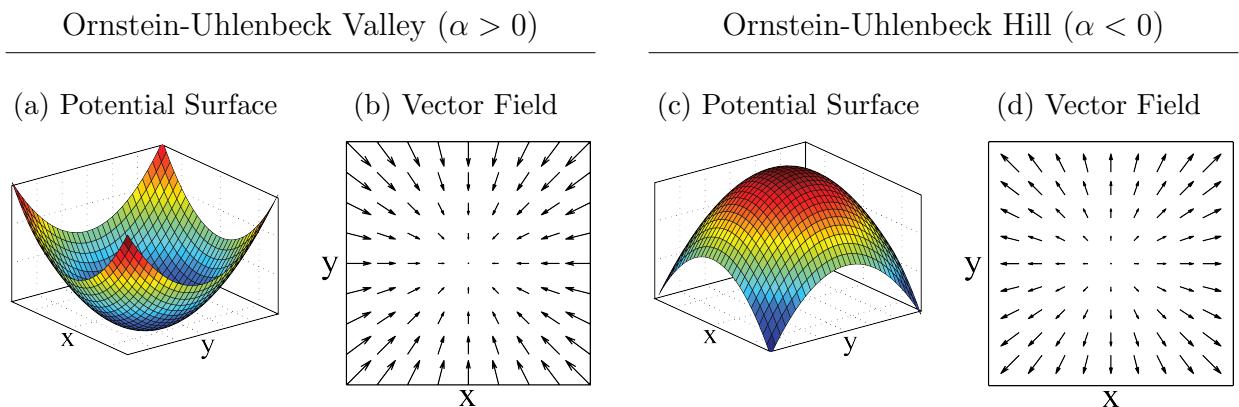


Figure 4.1: Ornstein-Uhlenbeck potential surfaces and vector fields. The vector fields are the gradients of the potential surfaces, and provide an estimate of the average velocity experienced at each point.

Alternatively, the potential function may be a smooth non-parametric function or an arbitrary high-degree polynomial (Brillinger et al., 2001; Brillinger, Preisler, Ager, & Kie, 2004; Preisler et al., 2004). It is important to note that when the potential function is a polynomial, the degree of the polynomial is arbitrary, and may be chosen only to provide a nonlinear shape. As such, interpretation comes from the overall shape of the potential function, and not from individual parameters (Preisler et al., 2013).

Discretization, Estimation, & Simulation

There are several numerical methods for discretizing differential equations. I will use the Taylor series approximations for the time derivatives of position (velocity and acceleration).

I will write $\Delta t_i = t_{i+1} - t_i = \Delta t$, $i = 1, \dots, N$, specifying that the data were sampled at a constant rate.

Discretization: Taylor Series Expansion The Taylor series that approximates a function $f(x)$ evaluated at $x = t + c\Delta t$ for some constant c is equal to,

$$f(t + c\Delta t) = f(t) + c\Delta t f'(t) + \frac{1}{2!}c^2 \Delta t^2 f''(t) + \frac{1}{3!}c^3 \Delta t^3 f'''(t) + \dots$$

This expansion can be used to approximate velocities and accelerations from recorded positions. A well known example is the Euler method, where the first-order approximation to $f(t - \Delta t) \approx f(t) - \Delta t f'(t)$ can be arranged to yield $f'(t) \approx \Delta t^{-1}(f(t) - f(t - \Delta t))$. This will be used to discretize the Smoluchowski approximation, but to discretize the Langevin equation, a second-order expansion is necessary to approximate acceleration.

It is straightforward to show that, using neighboring positions and second-order Taylor expansions, velocity and acceleration at time t can be expressed as,

$$\begin{aligned} \text{Velocity: } \mathbf{r}'(t_i) &\approx \frac{\mathbf{r}(t_{i+1}) - \mathbf{r}(t_{i-1})}{2\Delta t} \\ \text{Acceleration: } \mathbf{r}''(t_i) &\approx \frac{\mathbf{r}(t_{i-1}) - 2\mathbf{r}(t_i) + \mathbf{r}(t_{i+1})}{\Delta t^2} \end{aligned} \tag{4.7}$$

For simulation purposes, it is useful to derive a third-order approximation to acceleration and second order approximation to velocity for the initial and final time points at $i = 1$ and $i = N$, respectively. The approximations for t_1 and t_N are,

$$\begin{aligned} \mathbf{r}'(t_1) &\approx \frac{-\frac{3}{2}\mathbf{r}(t_1) + 2\mathbf{r}(t_2) - \frac{1}{2}\mathbf{r}(t_3)}{\Delta t} \\ \mathbf{r}'(t_N) &\approx \frac{\frac{1}{2}\mathbf{r}(t_{N-2}) - 2\mathbf{r}(t_{N-1}) + \frac{3}{2}\mathbf{r}(t_N)}{\Delta t} \\ \mathbf{r}''(t_1) &\approx \frac{2\mathbf{r}(t_1) - 5\mathbf{r}(t_2) + 4\mathbf{r}(t_3) - \mathbf{r}(t_4)}{\Delta t^2} \\ \mathbf{r}''(t_N) &\approx \frac{-\mathbf{r}(t_{N-3}) + 4\mathbf{r}(t_{N-2}) - 5\mathbf{r}(t_{N-1}) + 2\mathbf{r}(t_N)}{\Delta t^2} \end{aligned} \tag{4.8}$$

The order of these approximations (equation 4.8), and those for non-endpoint velocity and acceleration (equation 4.7), all have second-order error.

Estimating the Langevin Parameters Momentum can only exist for a particle that has mass. Therefore, by taking the limit as $m \rightarrow 0$, the Smoluchowski approximation removes

momentum from the Langevin equation. A discretized version of the original Langevin equation is,

$$m \frac{\mathbf{r}(t_{i+1}) - 2\mathbf{r}(t_i) + \mathbf{r}(t_{i-1})}{\Delta t^2} = \boldsymbol{\mu}(\mathbf{r}(t_i), t_i) - b \frac{\mathbf{r}(t_{i+1}) - \mathbf{r}(t_{i-1})}{2\Delta t} + \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i) \mathbf{Z}(t_i) \Delta t^{-1/2} \quad (4.9)$$

and of the Smoluchowski approximation is,

$$b \frac{\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i)}{\Delta t} = \boldsymbol{\mu}(\mathbf{r}(t_i), t_i) + \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i) \mathbf{Z}(t_i) \Delta t^{-1/2} \quad (4.10)$$

where entries of $\mathbf{Z}(t_i)$ are independent standard normals. The random term is setup such that $\text{Cov}[\boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i) \mathbf{Z}(t_i) \Delta t^{-1/2}] = \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i)$.

The important thing to note about equations 4.9 and 4.10 is that Δt is a known constant, and that the positions, $\mathbf{r}(t_i) = (x_i, y_i)$, are observed. The goal is to estimate the parameters of μ_x , μ_y , b , and m , and possibly $\boldsymbol{\Sigma}$. These unknown parameters can be estimated using conventional schemes, such as ordinary least squares (OLS) or maximum likelihood estimation (MLE).

For such purposes, it is useful to re-write equations 4.9 and 4.10 as autoregressive models,

Langevin, with momentum:

$$\begin{aligned} \mathbf{r}(t_{i+1}) &= \left(\frac{2m}{\frac{1}{2}b\Delta t + m} \right) \mathbf{r}(t_i) + \left(\frac{\frac{1}{2}b\Delta t - m}{\frac{1}{2}b\Delta t + m} \right) \mathbf{r}(t_{i-1}) + \\ &\quad \left(\frac{\Delta t^2}{\frac{1}{2}b\Delta t + m} \right) \boldsymbol{\mu}(\mathbf{r}(t_i), t_i) + \left(\frac{\Delta t^2}{\frac{1}{2}b\Delta t + m} \right) \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i) \mathbf{Z}(t_i) \Delta t^{-1/2} \end{aligned} \quad (4.11)$$

Smoluchowski, without momentum:

$$\mathbf{r}(t_{i+1}) = \mathbf{r}(t_i) + \frac{\Delta t}{b} \boldsymbol{\mu}(\mathbf{r}(t_i), t_i) + \frac{\Delta t}{b} \boldsymbol{\Sigma}(\mathbf{r}(t_i), t_i) \mathbf{Z}(t_i) \Delta t^{-1/2}$$

Letting γ_1 and γ_2 be the coefficients in front of $\mathbf{r}(t_i)$ and $\mathbf{r}(t_{i-1})$ in the momentum case, respectively, then there is evidence for momentum when $\gamma_2 \neq 0$, and no evidence for momentum when $\gamma_2 = 0$. Solving for m and b in terms of γ_1 and γ_2 is under constrained, because $\gamma_1 + \gamma_2 = 1$. Choosing not to fix the variance of the stochastic term, I'll instead fix $b = 1$, permitting the comparison of momentum in stochastic models based on mass, m , alone. Then,

$$\begin{aligned}\gamma_1 &= \frac{2m}{\frac{1}{2}b\Delta t + m} & b &\equiv 1 \\ \gamma_2 &= \frac{\frac{1}{2}b\Delta t - m}{\frac{1}{2}b\Delta t + m} & m &= \frac{\gamma_1 \Delta t}{2(\gamma_2 + 1)}\end{aligned}\quad (4.12)$$

Simultaneous Simulation Using the equations in the section on discretization using the Taylor series expansion, a matrix system can be established to solve for all positions simultaneously. I will illustrate this for the scenario with no force field and non-zero mass. In this case, the following equation describes the position of the particle,

$$(mA + bV)\bar{r} = \bar{\eta} \quad (4.13)$$

where

$$\begin{aligned}A &= \frac{1}{\Delta t^2} \begin{bmatrix} 2 & -5 & 4 & -1 & & & & 0 \\ 1 & -2 & 1 & & & & & \\ 1 & & -2 & 1 & & & & \\ & & & & 1 & -2 & 1 & \\ 0 & & & & 1 & -2 & 1 & \\ & & & & -1 & 4 & -5 & 2 \end{bmatrix} \quad \bar{r} = \begin{bmatrix} r(t_1) \\ r(t_2) \\ \vdots \\ r(t_N) \end{bmatrix} \\ V &= \frac{1}{\Delta t} \begin{bmatrix} -\frac{3}{2} & 2 & -\frac{1}{2} & & & & & 0 \\ -1 & 0 & 1 & & & & & \\ -1 & 0 & 1 & & & & & \\ & & & -1 & 0 & 1 & & \\ 0 & & & -1 & 0 & 1 & & \\ & & & & -1 & 0 & 1 & \\ & & & & \frac{1}{2} & -2 & \frac{3}{2} & \end{bmatrix} \quad \bar{\eta} = \begin{bmatrix} \eta(t_1) \\ \eta(t_2) \\ \vdots \\ \eta(t_N) \end{bmatrix}\end{aligned}$$

This system can be solved for \bar{r} by setting two initial conditions. For example, the first initial condition would set the first row of $(mA + bV)$ to $(1, 0, 0, \dots, 0)$ and the first element of \bar{r} to an initial position, e.g., $r(t_1) = (0, 0)^T$ to set an initial position at zero. The second initial condition would set the last row of $(mA + bV)$ to $(-\frac{3}{2}, 2, -\frac{1}{2}, 0, 0, \dots, 0)$ and the last row of \bar{r} to produce an initial velocity, e.g., $r(t_n) = (0, 0)^T$ to set an initial velocity of zero.

Solving such a system may be difficult in the case of a force field, depending on the dependence of the force field on position, $r(t)$. Furthermore, this system of equations cannot be used in time marching simulations, as it is non-causal and must be solved simultaneously.

Non-causality is apparent in the fact that non-zero elements of \mathbf{A} and \mathbf{V} are not strictly above or below the diagonal.

Time Marching Simulation The time marching simulation of the Smoluchowski approximation is straightforward using the Euler approximation, and can be used directly from equation 4.11. However, in the case of the Langevin equation, the second-order differential equation,

$$\mathbf{r}''(t) = -\frac{b}{m}\mathbf{r}'(t) - \frac{1}{m}\nabla H + \frac{1}{m}\boldsymbol{\eta}(t) \quad (4.14)$$

where $\mathbf{B}(t)$ is a two-dimensional Wiener process, for positions $\mathbf{r}(t) = (x(t), y(t))^T$, and $\mathbf{W}'(t) = \boldsymbol{\eta}(t)$, must be rewritten as a first-order system for time marching. It is straightforward to rewrite the second-order differential equation for the Langevin equation as the following first-order differential system,

$$\mathbf{X}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{r}(t) \\ m\mathbf{r}'(t) - w(t) \end{bmatrix} \quad \mathbf{X}'(t) = \begin{bmatrix} x'_1(t) \\ x'_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(x_2(t) + w(t)) \\ -\frac{b}{m}(x_2(t) + w(t)) - \nabla H(\mathbf{r}(t), t) \end{bmatrix} \quad (4.15)$$

Then a time-marching simulation can then be performed through the Euler approximation,

$$\mathbf{X}(t_{i+1}) = \mathbf{X}(t_i) + \Delta t \mathbf{X}'(t_i) \quad (4.16)$$

Example Simulations

Momentum Three simulations are shown in figure 4.2 of a particle with momentum on a flat potential surface. With increased mass, the particle has more momentum, and the simulations become more compact and more smooth, due to greater resistance to movement perturbations. With low mass, the simulation resembles Brownian motion. True Brownian motion occurs in the limit $m \rightarrow 0$ (the Smoluchowski approximation).

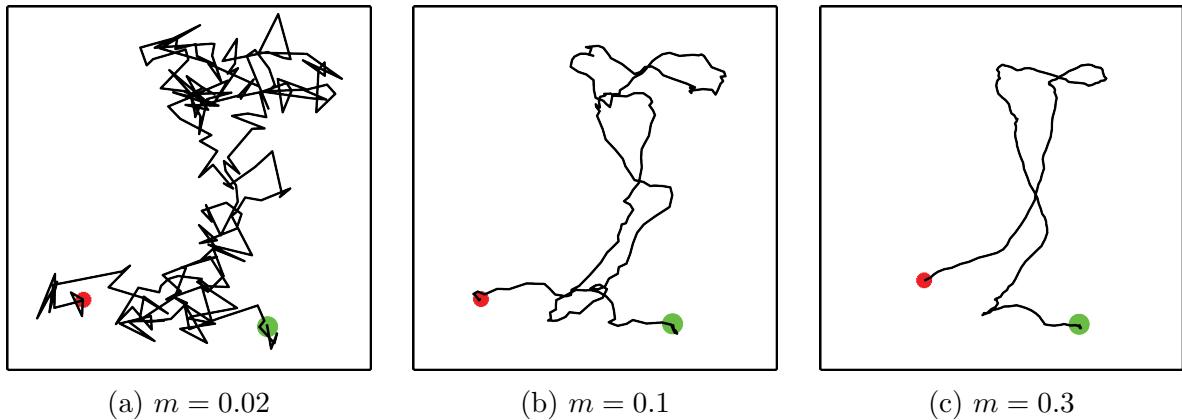
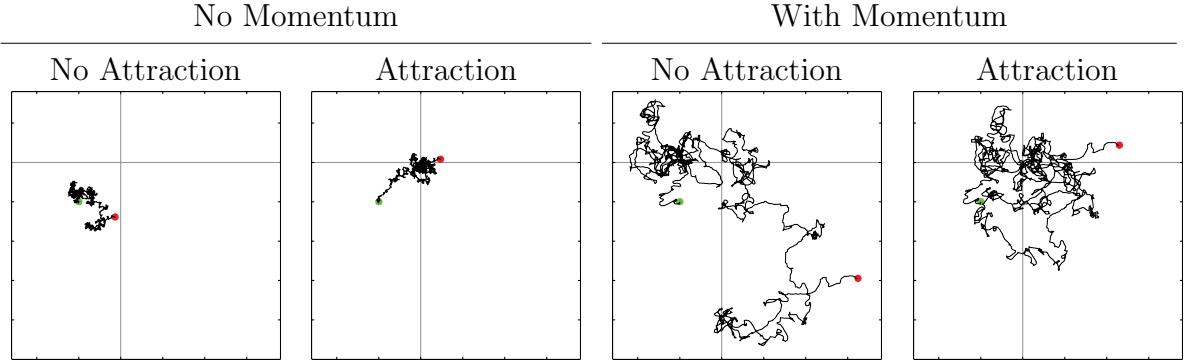


Figure 4.2: Simulations of random particle movement with momentum (friction $b = 1$, $\beta = b/m$, time step $\Delta t = 0.025$, number of samples $N = 200$). Walks start at the green dots and end on the red dots.

Simulation and Estimation: Momentum and Attraction Simulated trajectories with fitted models are shown in figure 4.3.



Model: $\mathbf{r}(t_{i+1}) = \beta_1 \mathbf{r}(t_i) + \beta_2 \mathbf{r}(t_{i-1}) - 2\alpha(\mathbf{r}(t_i) - \mathbf{a}) + \epsilon(t_i), \quad \epsilon(t_i) \sim N(0, \sigma^2)$

$\hat{\beta}_1$	1.01 ***	10.00×10^{-1} ***	1.61 ***	1.61 ***
$\hat{\beta}_2$	-1.37×10^{-3}	2.03×10^{-4}	-6.07×10^{-1} ***	-6.08×10^{-1} ***
$\hat{\alpha}$	7.12×10^{-5}	2.17×10^{-3} ***	1.42×10^{-4}	8.26×10^{-4} *
$\hat{\sigma}^2$	6.32×10^{-3}	6.32×10^{-3}	2.53×10^{-2}	2.53×10^{-2}

Figure 4.3: Simulated trajectories starting at $(0, 0)$, at the green dot, and ending at $N = 2000$, at the red dot. A point of attraction at $\mathbf{a} = (5, 5)$ with $\alpha = 0.05$, located by gray lines, was present as indicated. For all simulations, $\Delta t = 0.004$ and $\sigma^2 = 2$. Models were fit by least squares, the estimated coefficients and variance of the residuals are shown.

The simulations that did not have momentum coincided with non-significant second-order autoregressive coefficients (β_2), and those that did not include a point of attraction coincided with non-significant attraction coefficients (α). Momentum also affected the residuals, as expected from equations 4.9 and 4.10. Had an intercept been included in the linear model, this would have discovered any consistent movement bias (however no bias was included in the simulations).

4.3 Methods: An Initial Model

The model I fit is a threshold model, in which attraction to a feature (through an Ornstein-Uhlenbeck scheme) is only present when the finger is within some distance of that feature. The index finger positions, in one- and five-finger were analyzed, using the following basic model,

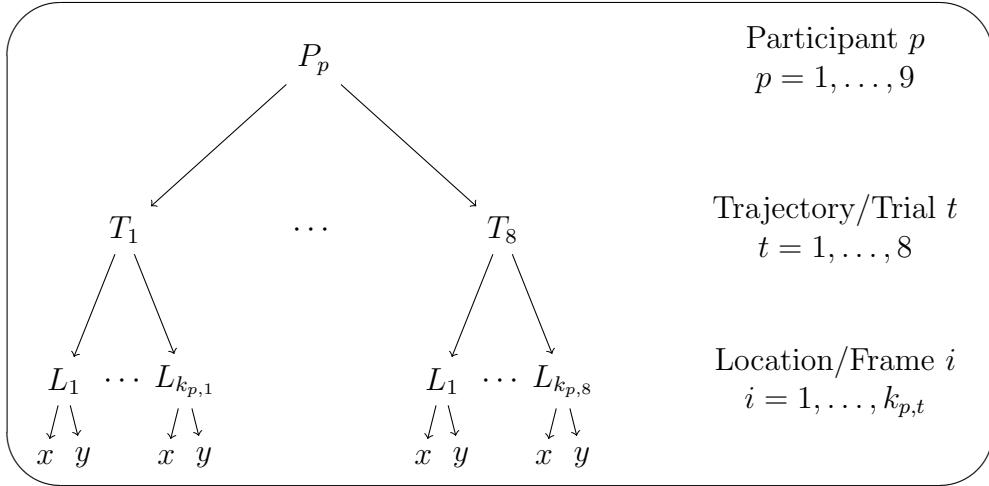
$$\begin{aligned}
\mathbf{r}(t_{i+1}) = & \alpha_1 \mathbf{r}(t_i) + \alpha_2 \mathbf{r}(t_{i-1}) \dots \\
& -2 \times \beta_1 (\mathbf{r}(t_i) - \mathbf{T}) \times \mathbb{1}_{\text{target}} \dots \\
& -2 \times \beta_2 (\mathbf{r}(t_i) - \mathbf{D}) \times \mathbb{1}_{\text{distractor}} \dots \\
& -2 \times \beta_3 (\mathbf{r}(t_i) - \mathbf{L}) \times \mathbb{1}_{\text{loose symbol, target}} \dots \\
& -2 \times \beta_4 (\mathbf{r}(t_i) - \mathbf{M}) \times \mathbb{1}_{\text{loose symbol, other}} \dots \\
& (-\beta_5 \dots \\
& -2 \times \beta_6 (\mathbf{r}(t_i) - \mathbf{B}) \dots \\
& -3 \times \beta_7 (\mathbf{r}(t_i) - \mathbf{B})^2 \dots \\
& -4 \times \beta_8 (\mathbf{r}(t_i) - \mathbf{B})^3) \times \mathbb{1}_{\text{outside boundary}} \\
& -2 \times \beta_9 (\mathbf{r}(t_i) - \mathbf{B}) \times \mathbb{1}_{\text{inside boundary}} \\
& -2 \times \beta_{10} (\mathbf{r}(t_i) - \mathbf{P}) \times \mathbb{1}_{\text{path}}
\end{aligned} \tag{4.17}$$

Where bold variables indicate locations, \mathbf{T} target cluster center, \mathbf{D} distractor cluster center, \mathbf{L} loose symbol not of target type center, \mathbf{M} loose symbol of target type center, \mathbf{B} closest section of the border, and \mathbf{P} closest section of the path. The indicator variables indicate the distance from these locations. In the case of the symbol clusters and loose symbols, the value of the indicator variable is 1 when the finger position is within an oval that scales with the width and height of the feature by some multiple, e.g., an oval with vertical axis two times the height and horizontal axis two times the width of the feature. Based on BIC values, I found this parametrization to describe the data better than being within some absolute range of the feature contours. In the case of the boundary and paths, the indicator variable is 1 when the finger position is within some absolute range, e.g., 1 cm, of the contour. The indicator variables take a value of 0 when the finger position is outside of the specified range. Values of the coefficients and thresholds were determined using maximum likelihood, confidence-interval based significance tests were determined using likelihood profiles.

In the current study, the observations are structured within clusters. There is both a repeated-measures structure to the data, with observations nested within participants, and each observation is contained within a time series within each trial. The observations within a cluster are expected to be more similar than between clusters.

In clustered data, it is important to allow for and model the dependence within clusters. Variance-components models are designed to model and estimate such clustered relationships (Rabe-Hesketh & Skrondal, 2008, ch. 2). Ignoring the clustering of observations will produce underestimation of variance, and lead to spurious significance.

The following nesting assumes separate models are run for the index-finger positions in one- and five-finger search. In a particular model, nested within each participant is their 8 trajectories (one per map). Within these trajectories are the finger locations, indexed from 1 to the number of frames for that particular trajectory. Each location contains an x and y coordinate. This is illustrated in figure 4.4. To incorporate multiple fingers, I allow a finger indicator (thumb, index, middle, ring, little) to interact with each model parameter. Therefore, this nesting was accounted for using linear mixed effects models.

Figure 4.4: Nesting of finger-location observations (x, y).

The units of the data, in x and y are in “pixels.” The conversion follows 1,000 pixels to 11.5 inches, or 29.21 cm. This was the length within the border of each tactile map.

4.4 Results

The thresholds found for the models are shown in table 4.1. The only significant differences (based on CIs found using likelihood profiles) between one-finger and five-finger trajectories were for thresholds controlling (turning on and off) attraction to the target clusters and loose symbols of the same type as the target. These thresholds were significantly larger for five-finger search than one-finger search in both instances.

Table 4.1: Threshold values for one-finger and five-finger search index-finger models. Significant differences between one-finger and five-finger thresholds based on likelihood profile CIs marked with *.

Condition	Target Cluster	Distractor Cluster	Symbol Target Type	Symbol Other Type	Boundary	Path
One-Finger Search	1.54*	4.41	3.94*	4.31	26.68	32.03
Five-Finger Search	2.18*	4.57	5.79*	4.21	25.51	29.92

The model coefficients for the models of one-finger and five-finger search index finger trajectories, respectively, are shown in tables 4.2 and 4.3. All coefficients except for the outside boundary coefficients can be interpreted in as traditional Ornstein-Uhlenbeck attractors/repellers. The potential function outside of the boundary is shown in figure 4.5.

Table 4.2: Coefficients for one-finger index-finger model.

Coefficient	Value	Std. Error	Test, t(52466)	p-value
AR(1), α_1	1.77	1.96×10^{-3}	9.05×10^2	< 0.001 ***
AR(2), α_2	-0.77	1.96×10^{-3}	-3.95×10^2	< 0.001 ***
Target, β_1	8.27×10^{-3}	6.92×10^{-4}	1.20×10^1	< 0.001 ***
Distractor, β_2	3.89×10^{-5}	1.04×10^{-4}	3.76×10^{-1}	0.707
Symbol, Target, β_3	8.08×10^{-3}	1.16×10^{-3}	6.96	< 0.001 ***
Symbol, Other, β_4	4.86×10^{-3}	1.01×10^{-3}	4.83	< 0.001 ***
Boundary, Outside, β_5	6.92×10^{-1}	2.13×10^{-1}	3.25	0.001 **
Boundary, Outside, β_6	1.03×10^{-2}	3.19×10^{-3}	3.24	0.001 **
Boundary, Outside, β_7	-8.24×10^{-5}	1.46×10^{-5}	-5.65	< 0.001 ***
Boundary, Outside, β_8	-4×10^{-7}	7×10^{-8}	-4.80	< 0.001 ***
Boundary, Inside, β_9	-3.11×10^{-2}	3.87×10^{-3}	-8.03	< 0.001 ***
Path, β_{10}	8.45×10^{-3}	2.37×10^{-3}	3.57	< 0.001 ***

Table 4.3: Coefficients for five-finger index-finger model.

Coefficient	Value	Std.Error	Test, t(32163)	p-value
AR(1), α_1	1.78	2.44×10^{-3}	7.30×10^2	< 0.001 ***
AR(2), α_2	-0.78	2.44×10^{-3}	-3.20×10^2	< 0.001 ***
Target, β_1	6.83×10^{-3}	5.33×10^{-4}	1.28×10^1	< 0.001 ***
Distractor, β_2	3.22×10^{-4}	1.14×10^{-4}	2.82	0.005 **
Symbol, Target, β_3	4.95×10^{-3}	6.22×10^{-4}	7.95	< 0.001 ***
Symbol, Other, β_4	1.08×10^{-3}	7.97×10^{-4}	1.35	0.177
Boundary, Outside β_5	8.94×10^{-1}	2.89×10^{-1}	3.10	0.002 **
Boundary, Outside β_6	-4.70×10^{-3}	3.79×10^{-3}	-1.24	0.216
Boundary, Outside β_7	-6.35×10^{-5}	2.46×10^{-5}	-2.58	0.010 **
Boundary, Outside β_8	-1.00×10^{-7}	6×10^{-8}	-2.41	0.016 *
Boundary, Inside β_9	-1.99×10^{-2}	5.78×10^{-3}	-3.44	< 0.001 ***
Path, β_{10}	7.90×10^{-4}	3.15×10^{-3}	0.25	0.802

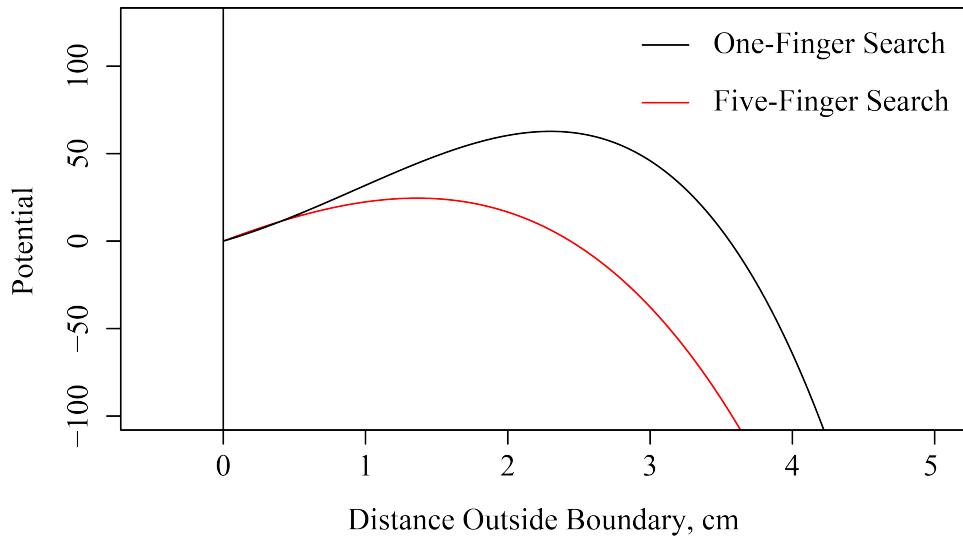


Figure 4.5: Potential function outside the map border as a function of distance.

4.5 Discussion

Both one-finger and five-finger search index-finger models had significant second-order autoregressive terms. This is consistent with finger-movement trajectories following a correlated, rather than an uncorrelated random walk pattern. This also agrees with visualizations of trajectories, in chapter 2 figure 2.1. The coefficients for the one-finger and five-finger autoregressive terms were nearly identical, suggesting similar “momentum” of the index finger in one-finger and five-finger search.

The threshold parameters associated with map features were largely the same for one-finger and five-finger search, but significantly smaller in one-finger search for target clusters and loose symbols of the target type. Specifically, attraction to target clusters seemed to “turn on” once the index finger was within a distance of 1.54 times the cluster size in one-finger search (i.e., an oval defined by 1.54 times the width of the cluster and 1.54 times the height of the cluster) and 2.18 times the cluster size in five-finger search. Similarly, attraction to loose symbols of the target type turned on when the index finger was within 3.94 times the symbol size in one-finger search, and 5.79 times the symbol size in five-finger search. Arguably, this reflects the participants’ use of non-index fingers in five-finger search to seek out and process stimulus features. As discussed in chapter 2, participants used their non-index fingers to not only find the target and distractors, but also to share in the exploration of these features with the index finger. The fact that threshold parameters were only significantly different in cases involving the target symbols suggests that the functioning of multiple fingers were mainly exploited for map features involving target symbols.

Attraction to the target clusters were apparent in both one-finger and five-finger search models, with significant and positive coefficients associated with the Ornstein-Uhlenbeck processes. The magnitudes of these attractions were similar, as apparent by the fact that 95% CIs based on their standard errors overlap. In contrast, the effect of distractor clusters was non-significant in the one-finger search model, but significant in the five-finger search model. However, the attraction of distractor clusters in five-finger search had a very small magnitude compared to the magnitude of attraction to target clusters. This is consistent with the spatial densities of finger locations shown in chapter 2, figure 2.3, which indicated lower densities associated with distractor clusters than target clusters in both one-finger and five-finger searches. Participants spent less time, and were therefore less attracted in the scheme of the models, to distractor clusters than target clusters.

Loose symbols on the tactile map, also acting as distractors, attracted index-finger movements in both one-finger and five-finger search. Interestingly, in one-finger search, attraction to loose symbols that shared the same type with the target cluster was of larger magnitude than to loose symbols of another type. In five-finger search, although there was a significant attraction to target-typed loose symbols, there was not a significant attraction to non-target typed loose symbols. Therefore, when using multiple fingers, the participants were able to more successfully avoid exploring non-target typed loose-symbol distractors with their index finger than in one-finger search. One possibility is that the participants did not actually spend less time exploring these distractors in five-finger search than one-finger search, but processed the symbols using a non-index finger. However, the significant attraction to target-typed loose symbols implies that when a distractor is similar to the target, the index finger is recruited even in five-finger trials for processing. Avoiding this recruitment may enable the participant to save time, and contribute to the finding that five-finger search is faster than one-finger search.

The boundary's potential functions effectively reflected the fact that participants kept their fingers within the tactile map stimuli. When the finger was outside the boundary, the potential functions "pushed" the finger towards it. This was true for both one-finger and five-finger search as shown in figure 4.5. These potential functions appeared to repel the finger from the map once the finger was roughly 2.5cm from the border. However, this should not be taken too seriously given that very few observations, fewer than 0.01%, were located this far outside the stimulus borders. When the index fingers were within the map borders, the border had a repelling effect that was significant in both one-finger and five-finger search models. This was unexpected, given that I had observed some participants to trace the borders. However, this finding is consistent with the spatial distributions of finger positions, shown in chapter 2 figure 2.3. Therefore, although border tracing may have been used sporadically by the participants, there is no evidence that this behavior was a common strategy.

Lastly, the path had an attractive effect on index finger positions in one-finger search but not five-finger search. Although attraction to paths, associated with path tracing or following, was not apparent in the spatial distributions of finger positions, shown in chapter 2 figure 2.3, these model results suggest that participants did path follow in one-finger

search, but not five-finger search. Path following would not be a particularly useful strategy in the current search task, given that target clusters were never located along a path. However several participants reported using path following as a strategy. Using five fingers appears to have mitigated the use of this inefficient strategy. Possibly, participants used path following in one-finger search in an effort to organize a systematic strategy. Whatever the cause, adopting the poor strategy of path following more commonly in one-finger search may contribute to why one-finger search is slower than five-finger search.

Using discretized stochastic differential equations provided significant insight into the movement strategies used by blindfolded sighted participants in their search for a landmark on a tactile map. The current models make use of thresholding to keep the effect of stimulus features on the movement trajectories local. An alternative approach could make use of hidden Markov state space models, in which the participants are assumed to have hidden behavioral states, such as examination of a distractor or general exploration. Being in a behavioral state would be probabilistic, and could depend on the proximity of the participant's finger(s) to stimulus features. These types of models are becoming more common in descriptions of animal movements (e.g., Jonsen et al., 2013; Silva et al., 2014), where they do not currently incorporate potential functions, to my knowledge, but are instead used to describe switching between high- and low-variance random walks. Extending the potential function approach to a hidden Markov state space model could provide an alternative to the scheme used here, and may provide different insights into finger-movement strategies. Such a model could include and test inhibition of return in an exploratory behavioral state, and could also explicitly model noise associated with the finger-tracking method.

Chapter 5

General Discussion

This dissertation investigates the finger-movement strategies used by blindfolded sighted participants when completing a search task on a tactile map. These participants can be considered similar to individuals who have recently experienced vision loss, for example due to age-related ailments, which represent the majority of individuals with vision loss in the developed world. The results could also be extended to children with acquired visual impairments, possibly due to genetic causes, injury, or disease. Tactile maps, and tactile graphics in general, provide an opportunity for blind and low vision individuals to access information for educational or professional purposes and to function independently. However, success using tactile graphics depends heavily on the finger-movement strategies used.

My investigations into the distributions of finger locations in space and time suggest that sighted blindfolded participants use all of their fingers, not just their index fingers, to search for landmarks on tactile maps. They use their multiple fingers to speed finding and recognizing map features. This implies that, although training may be necessary to prompt newly blind individuals to use multiple fingers during haptic tasks (Symmons & Richardson, 2000), newly blind individuals will immediately be able to use multiple fingers in these ways.

However, sighted blindfolded participants demonstrated no evidence of spontaneous anchoring, where one finger is stationary during exploration to either mark a location or facilitate allocentric spatial representations (Berla et al., 1976; Millar & Al-Attar, 2004). Future work is needed to investigate whether long-term blind individuals use anchoring during haptic search, which may enable them to complete search tasks faster than blindfolded sighted individuals (Morash, Connell Pensky, et al., 2013). If blind individuals do use anchoring during haptic search, newly blind individuals may need to be taught this strategy.

Previous research has indicated that systematic strategies are not spontaneously used by blind individuals in exploring a tactile map or graphic, and must therefore be explicitly taught (Berla & Murr, 1974). In contrast, blindfolded sighted participants enrolled in the current research spontaneously used systematic strategies. Furthermore, these strategies were modulated by their detection radius as would be expected based on the relative efficiency of systematic and random strategies depending on the search detection radius. With a small detection radius of one finger, when a random strategy would be particularly onerous to

cover the tactile map, the participants used more systematic strategies, such as spirals, zigzags, and parallel sweeps. With a large detection radius of five fingers, participants used less systematic strategies, using instead movement patterns that appeared similar to random walks. Therefore, not only did sighted blindfolded participants use systematic strategies, but they used them in intelligent ways depending on the circumstances of the search task. This implies that newly blind individuals may not need specific instruction in using systematic strategies. Alternatively, they may spontaneously use such strategies, which could be further encouraged with training. Long-term blind individuals may be more likely to use systematic strategies than blindfolded sighted or newly-blind individuals, due to training. However, they may also experience less pressure to use systematic strategies due to overall faster hand movements.

The random walk and systematic strategies seen in haptic search trajectories are wholly unlike the patterns observed for eye movements during visual search, and are more similar to the movements used by animals when foraging. This is likely because eye movements are driven by stimulus features observable in the visual periphery. In contrast, the fingers do not have access to such a periphery to drive movements. As such, the fingers spend a considerable amount of time during haptic search in empty space between possible targets. This can be mitigated by using multiple fingers to increase the hand's detection radius, using multiple fingers to increase the hand's sensory processing area, and by using systematic strategies to avoid recovering ground. All of these strategies were used by the sighted blindfolded participants in the current study.

Lastly, I fit an initial model to finger-movement trajectories based on stochastic differential equations. Although future modeling development is needed, for example to incorporate hidden behavioral states, the current model provides additional information about the use of fingers during haptic search that was not revealed in analysis of finger location distributions, in space and time. Namely, participants' index fingers were attracted to distractor loose symbols, which did not have the same symbol type as the target cluster, in one-finger search. Index fingers were not attracted to these non-target type distractor loose symbols in five-finger search. Similarly, index fingers were attracted to paths in one-finger search, but not in five-finger search. These effects suggest that participants were better able to avoid distractors on the tactile map during five-finger search than one-finger search. This underscores the importance to encourage newly blind individuals to use multiple fingers when exploring a tactile map or other tactile graphic.

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