# DIRECTIONAL CONTROLLABILITY OF A HEAVY CARGO CARRIER WITH TWIN-PROPELLER AND TWIN-RUDDER IN WIND AND WAVES

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#### ABSTRACT

Mathematical models describing the manoeuvring motions of a heavy cargo carrier with twin-propeller and twin-rudder are investigated. With the use of hydrodynamic coefficients obtained in captive model tests, numerical simulations of manoeuvring motions are performed using two mathematical models; twin-propeller and twin-rudder model and equivalent single propeller and single rudder model. Comparing the calculated results with those obtained in free-running model tests, applicabilities of the mathematical models are discussed. Making use of the former mathematical model, directional controllability of the heavy cargo carrier in wind and waves are clarified, where the wind resistance coefficients and the wave drifting force coefficients are obtained in model tests.

#### 1. INTRODUCTION

In constructing various plants such as oil plants or power plants, a modularization has been the object of public attention recently, because of the merits in the aspect of quality control, safety control, secure labour powers, and so on. With this trend, it is required of the shipping world to carry large modules safely by sea. For this purpose, so-called module carriers have been built, whose characteristics are as follows:

- (1) Wide Beam, which secure the sufficient deck area for a large module.
- (2) Shallow Draft, which enable the ship to enter into shallow water harbours.
- (3) Strong Wind Resistance, which is caused by the large module on the deck. With respect to these features (1) and (2), twin-propeller and twin-rudder are usually fitted on the carrier. The operational performance of this type of ship is anticipated to differ from the ordinary ships.

The manoeuvrability of ordinary ships can be predicted with sufficient accuracy for practical purpose with the use of mathematical models such as so-called MMG-model[1,2](MMG: Manoeuvring Model Group of the Japan Towing Tank Committee), of which hydrodynamic coefficients are determined by captive model tests. In the

case of such a ship as module carrier, however, few research work has been done to investigate the applicability of mathematical models. In the present paper, the manoeuvrability of a module carrier with twin-propeller/twin-rudder will be investigated: First, a series of captive model tests are performed to determine the various coefficients in the mathematical model for manoeuvring motions. Two kinds of mathematical models are adopted to predict the manoeuvring motions. One is twin-propeller/twin-rudder model and the other is equivalent single propeller/single rudder model. Calculated results are compared with those of free-running model tests. Secondly, the directional controllability of the carrier in wind and waves are investigated with the use of numerical simulations. The course keeping ability of the ship is clarified.

### 2. MATHEMATICAL MODEL OF MANOEUVRING MOTIONS

The manoeuvring motions of a ship in wind and waves can be expressed by the following equations with respect to the body-fixed coordinate system as shown in Fig. 1.

$$\begin{cases} m(\dot{\mathbf{u}} - \mathbf{v} \mathbf{r} - \mathbf{x}_{G} \mathbf{r}^{2}) = \mathbf{X}_{H} + \mathbf{X}_{P} + \mathbf{X}_{R} + \mathbf{X}_{WD} + \mathbf{X}_{WV} \\ m(\dot{\mathbf{v}} + \mathbf{u} \mathbf{r} + \mathbf{x}_{G} \dot{\mathbf{r}}) = \mathbf{Y}_{H} + \mathbf{Y}_{P} + \mathbf{Y}_{R} + \mathbf{Y}_{WD} + \mathbf{Y}_{WV} \end{cases}$$
Fig. 1 Cool
$$I_{\mathbf{x}} \dot{\mathbf{r}} + m \mathbf{x}_{G} (\dot{\mathbf{v}} + \mathbf{u} \mathbf{r}) = \mathbf{N}_{H} + \mathbf{N}_{P} + \mathbf{N}_{R} + \mathbf{N}_{WD} + \mathbf{N}_{WV}$$

Wind Uw Wind Wave

Fig. 1 Coordinate system

•••• (1)

where u, v, r are the surge velocity, sway velocity and yaw angular velocity, and m,  $I_{\underline{w}}$ ,  $x_{\underline{G}}$  are the mass of a ship, mass moment of inertia around the z-axis and x-coordinate of the center of gravity of the ship. X, Y and N in the righthand side of Eq.(1) are surge force, sway force and yaw moment, respectively, in which the subscripts H, P, R, WD and WV mean the hydrodynamic force acting on the ship's hull itself and the forces induced by the propellers, rudders, wind and waves, respectively. The mathematical expressions of individual terms are described in the subsequent sections.

# 2.1 Hydrodynamic Forces and Moment Acting on the Ship's Hull

The forces and moment acting on the ship's hull, namely bare hull, are expressed by the followings:

$$\begin{cases} X_{H} = X_{\dot{\mathbf{u}}} \dot{\dot{\mathbf{u}}} + X_{vv} v^{2} + (X_{vr} - Y_{\dot{\mathbf{v}}}) v r + X_{rr} r^{2} + X_{vvvv} v^{4} - \frac{1}{2} \rho S_{w} c_{T} u^{2} \\ Y_{H} = Y_{\dot{\mathbf{v}}} \dot{\dot{\mathbf{v}}} + Y_{\dot{\mathbf{r}}} \dot{\dot{\mathbf{r}}} + Y_{v} v + (Y_{r} + X_{\dot{\mathbf{u}}} u) r + Y_{vvv} v^{3} + Y_{vvr} v^{2} r + Y_{vrr} v r^{2} \\ + Y_{rrr} r^{3} \\ N_{H} = N_{\dot{\mathbf{v}}} \dot{\dot{\mathbf{v}}} + N_{\dot{\mathbf{r}}} \dot{\dot{\mathbf{r}}} + N_{v} v + N_{r} r + N_{vvv} v^{3} + N_{vvr} v^{2} r + N_{vrr} v r^{2} + N_{rrr} r^{3} \end{cases}$$
(2)

where  $X_{\dot{u}}$ ,  $X_{vv}$ ,  $X_{vr}$ ,  $X_{rr}$ ,  $X_{vvvv}$ ,  $Y_{\dot{v}}$ ,  $Y_{\dot{v}}$ ,  $Y_{\dot{v}}$ ,  $Y_{\dot{v}}$ ,  $Y_{vvv}$ ,  $Y_{vvv}$ ,  $Y_{vvr}$ ,  $Y_{rrr}$ ,  $Y_{\dot{v}}$ 

where  $\rho$  and S are the mass density of water and the wetted surface area of the ship. The total resistance coefficient  $\mathbf{C}_{\overline{\mathbf{T}}}$  is given by

$$C_{T} = (1 + k) C_{F} + C_{W}$$
 (3)

where k is the form factor. The frictional resistance coefficient of flat plate,  $C_{\overline{F}}$ , and the wave-making resistance coefficient,  $C_{\overline{W}}$ , are assumed to be ex-

$$\frac{0.242}{\sqrt{C_F}} = \log(C_F \cdot R_n) \quad : Schönherr's formula$$
 (4)

$$C_{W} = C_{4} F_{n}^{4} + C_{5} F_{n}^{5} + C_{6} F_{n}^{6}$$
(5)

where  $\mathbf{F}_{\mathbf{n}}$  and  $\mathbf{R}_{\mathbf{n}}$  are Froude number and Reynolds number, respectively.

# 2.2 Hydrodynamic Forces and Moment Induced by Propellers

In the present paper, the ship is assumed to have twin-propeller and the interval of the propeller shafts is denoted as b. Disregarding the lateral component of each propeller force, the mathematical expressions of the forces and moment induced by two propellers can be given by

$$\begin{cases} X_{p} = (1 - t) \rho n^{2} D_{p}^{4} (K_{T}^{(p)} + K_{T}^{(s)}) \\ Y_{p} = 0 \\ N_{p} = (1 - t) \rho n^{2} D_{p}^{4} \frac{b}{2} (K_{T}^{(p)} - K_{T}^{(s)}) \end{cases}$$
(6)

where t, n and  $\mathbf{D}_{\mathbf{p}}$  are the thrust deduction fraction, propeller revolution per second and propeller diameter, respectively. It should be noted here that the propeller revolution of two propellers are assumed to be identical. The thrust coefficient,  $K_{\mathrm{T}}$ , is assumed to be expressed by a second-order polynomial of the advance constant, J; that is,

$$K_{T} = a_{0} + a_{1} J + a_{2} J^{2}$$
 (7)

where

$$J = u_p/(n D_p)$$
(8)

The superscripts in Eq.(6), (p) and (s), denote the port-side propeller and the starboard-side propeller, respectively. The longitudinal component of the propeller inflow velocity,  $u_p$ , can be obtained using the wake fraction at propeller,  $w_p$ ;

$$u_{p} = u(1 - w_{p}) \tag{9}$$

The wake fraction depends on the ship motions, i.e. drift angle or yaw rate, and should be given in each propeller. For the practical purpose, however, it is better to use single value of  $\mathbf{w}_{\mathbf{p}}$  for both of propellers. In the present paper, the effect of using the single value of  $\mathbf{w}_{\mathbf{p}}$ , which is called as the equivalent wake fraction at propeller and is denoted as  $\mathbf{w}_{\mathbf{p}}^{(eq)}$  hereafter, will be considered.

#### 2.3 Hydrodynamic Forces and Moment Induced by Steering Rudders

As is the propeller, the rudder is also assumed to be twin-system. The mathematical expressions of the forces and moment induced by steering rudders are as follows:

$$\begin{cases} X_{R} = -(1 - t_{R})(F_{N}^{(p)} + F_{N}^{(s)}) \sin \delta \\ Y_{R} = -(1 + a_{H})(F_{N}^{(p)} + F_{N}^{(s)}) \cos \delta \\ N_{R} = -(x_{R} + a_{H}x_{H})(F_{N}^{(p)} + F_{N}^{(s)}) \cos \delta - (1 - t_{R}) \frac{b}{2}(F_{N}^{(p)} - F_{N}^{(s)}) \sin \delta \end{cases}$$
(10)

where  $t_R$ ,  $x_R$ ,  $a_H$ ,  $x_H$  and  $\delta$  are the fraction of rudder resistance deduction, x-coordinate of rudder, magnitude and the point of application of additional lateral force induced by steering rudders and rudder angle, respectively. The angle of two rudders are assumed to be identical in the present paper. The rudder normal force,  $F_N$ , is expressed by

$$F_{N} = \frac{1}{2} \rho A_{R} (u_{R}^{2} + v_{R}^{2}) f_{\alpha} \sin \alpha_{R}$$
 (11)

where  ${\bf A}_R$  and  ${\bf f}_\alpha$  are the rudder area and gradient of rudder normal force coefficient. The expression of the longitudinal component of rudder inflow velocity,  ${\bf u}_R$ , is given by

$$u_{R} = \varepsilon u_{P} \sqrt{\eta \{1 + \kappa (\sqrt{1 + \frac{8K_{T}}{\pi J^{2}}} - 1)\}^{2} + (1 - \eta)}$$
 (12)

where  $\eta$  and  $\kappa$  are the ratio of propeller diameter to rudder height and intensity of speed increase of the propeller slip stream at rudder position.  $\epsilon$  is defined by  $(1-w_R^{})/(1-w_P^{})$ , where  $w_R^{}$  is the wake fraction at rudder without propeller. The lateral component of rudder inflow velocity,  $v_R^{}$ , and rudder inflow angle,  $\alpha_R^{}$ , are expressed by

$$v_{R} = u_{R} \tan(\delta_{R})$$
 (13)

$$\alpha_{R} = \delta - \delta_{R} \tag{14}$$

where  $\delta_R^{}$  is the rudder angle at which rudder normal force comes to be zero and is the function of the equivalent drift angle at rudder,  $\beta_R^{}$  , defined by

$$\beta_{R} = \beta - \frac{\ell_{R}}{L} \frac{r L}{U} \tag{15}$$

where L is the ship length. From the geometrical consideration, the factor  $\ell_R$  seems to be identical to the horizontal distance between the origin of the coordinates and rudder; however, in some case it is rather close to the ship length. In the present paper,  $\ell_R$  is determined to best fit the experimental results of  $\delta_R$ .

It is also convenient in evaluating rudder force that the equivalent value is used for the rudder angle at which rudder normal force comes to be zero. This equivalent value is denoted as  $\delta_R^{(eq)}$  and the effect of using this value will be also considered in the present paper.

## 2.4 Hydrodynamic Forces and Moment Induced by Wind and Waves

In the present paper, only the steady forces induced by wind and waves are considered; that is, as for waves, wave drifting forces of comparatively short waves are taken into account. The expressions of the forces and moment induced by wind and waves are as follow[3,4]:

$$\begin{cases} X_{WD} = \frac{1}{2} \rho_{a} A_{f} C_{WDX} U_{WR}^{2} \\ Y_{WD} = \frac{1}{2} \rho_{a} A_{s} C_{WDY} U_{WR}^{2} \\ N_{WD} = \frac{1}{2} \rho_{a} A_{s} L C_{WDN} U_{WR}^{2} \end{cases}$$
(16)

$$\begin{cases} X_{WV} = \frac{1}{2} \rho g L C_{WVX} \zeta_o^2 \\ Y_{WV} = \frac{1}{2} \rho g L C_{WVY} \zeta_o^2 \\ N_{WV} = \frac{1}{2} \rho g L^2 C_{WVN} \zeta_o^2 \end{cases}$$
(17)

where  $\rho_a$ ,  $A_f$ ,  $A_s$ , g and  $\zeta_o$  are the mass density of air, effective projected areas of the ship and cargo on deck in front view and in side view, gravitational acceleration and wave amplitude, respectively.  $U_{WR}$  is the relative wind velocity defined by

$$U_{WR} = \sqrt{(u_W + u)^2 + (v_W + v)^2}$$
 (18)

where the longitudinal and lateral component to the ship of wind velocity,  $\mathbf{u}_{\overline{W}}$  and  $\mathbf{v}_{\overline{W}}$ , are given by

$$\begin{cases} u_W = U_W \cos \psi_W \\ v_W = U_W \sin \psi_W \end{cases}$$
 (19)

The coefficients  $C_{WDX}$ ,  $C_{WDY}$ ,  $C_{WDN}$ ,  $C_{WVX}$ ,  $C_{WVY}$  and  $C_{WVN}$ , which are determined by the captive model tests, are the functions of the relative angle of encounter wind or waves,  $\psi_{WR}$ , or  $\chi$ .  $\psi_{WR}$  is given by

$$\psi_{WR} = \tan^{-1} \left( \frac{v_W + v}{u_W + u} \right) \tag{20}$$

Table 1 Principal particulars of model (model scale = 1/50)

Length between perpendiculars (L)	2.34 m
Breadth moulded (B)	0.60 m
Depth moulded (D)	0.14 m
Mean draft (d)	0.0767 m
Trim (t <sub>r</sub> )	0.0162 m
Displacement (A)	80.6 kgf
Center of buoyancy from B (xg)	0.0622 m (fore)
Radius of gyration (kg)	0.2536 L
Propeller diameter (Dp)	0.05856 m
Propeller pitch ratio	0.735
x-coordinate of propeller (xp)	-1.128 m
Rudder area (A <sub>R</sub> )	0.0048655 m <sup>2</sup>
Rudder aspect ratio (A)	1.1255
x-coordinate of rudder (x <sub>R</sub> )	-1.17 m
Interval of two propellers (b)	0.244 m

Table 2 Hydrodynamic coefficients for bare hull and open characteristics of propeller and rudder

m' = 0.3838,	I' = 0.02469	-		
X: -0.02656	Y: -0.1495	N; 0.01149		
X' -0.1361	Y: 0.001709	N: -0.008615		
X'vr -0.02653	Y' -0.2656	N0.04186		
X'r -0.03882	Y: 0.05912	N: -0.03587		
X'vvv 0.3785	Y'vv -0.7712	N' 0.1310		
	Y'vr 0.5397	N'vr -0.2597		
	Y' -0.2374	N' 0.05773		
	Yrr 0.01481	N'rr -0.006686		
$k = 0.2080$ : form factor $C_W = 2.6961 F_n^4 - 18.212 F_n^5 + 38.270 F_n^6$				
$R_{T}^{(p)} = 0.3639 - 0.3533 J - 0.1110 J^{2}$ $R_{T}^{(g)} = 0.3261 - 0.2835 J - 0.1774 J^{2}$				
f <sub>a</sub> = 1.2067				
Non-Dimensionalization				
$\frac{\text{Velocity}}{\text{U}},  \frac{\text{Angular Velocity}}{\text{U/L}},  \frac{\text{Mass}}{\frac{1}{2}\rho \text{ L}^2 \text{ d}},$				
$\frac{\text{Moment of Inertia}}{\frac{1}{2}\rho \text{ L}^4 \text{ d}}, \frac{\text{Force}}{\frac{1}{2}\rho \text{ Ld U}^2}, \frac{\text{Moment}}{\frac{1}{2}\rho \text{ L}^2 \text{ d U}^2}$				

Table 3 Other coefficients

(1 - t)	= 0.7254,	(1 - t <sub>R</sub> )	- 0.8398
a <sub>H</sub>	<b>=</b> 0.2986,	× <sub>H</sub> /L	= -0.4071
(1 - w <sub>R</sub> )	<b>=</b> 0.7655,		
κ <sup>(p)</sup>	= 0.6315,	κ <sup>(s)</sup>	= 0.5855

#### 3. CAPTIVE MODEL TESTS

In order to determine the hydrodynamic coefficients with respect to the mathematical model described in the previous section, a series of captive model tests was performed; that is, towing tests (straight ahead and oblique), planer motion mechanism, circular motion tests, and so on. Principal particulars of the model ship are shown in Table 1.

The hydrodynamic coefficients for bare hull and the open characteristics of the propeller and rudder are shown in Table 2, where ' means a non-dimensional value. Since the beam/draft ratio of this ship is large, the moment of inertia for yaw motion is comparatively small. It should be noted here that the open characteristics of the propellers are different each other in this model. Table 3 shows the interaction coefficients between hull and propeller or hull and rudder obtained in the captive model tests. The coefficients in Eq.(12),  $w_p$  and  $\kappa$ , are also shown in Table 3.

The wake fraction at propeller can be obtained by the thrust identity method. The experimental results are shown in Fig. 2, where the ship is towed straight ahead. In spite of the difference of the open characteristics, the wake fractions at both propellers are identical. In the case where the ship is yawing and/or swaying, however, the wake fractions at each propeller are anticipated to be different by the effects of such motions. Fig. 3 shows

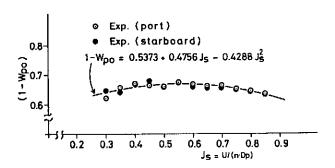


Fig. 2 Wake fraction at propeller (Running straight ahead)

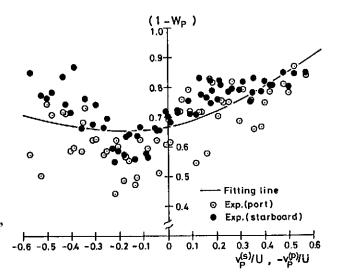


Fig. 3 Effects of ship motions on the wake fraction at propellers

the wake fractions at both propellers obtained by the circular motion tests. The abcissa is the effective lateral velocity at propeller defined by

$$v_p/U = v/U + x_p r/U \ (= v_p^{\dagger})$$
 (21)

where  $\mathbf{x}_{p}$  is the x-coordinate of the propeller; however, it was determined experimentally in this paper as -0.4551L. The solid line in the figure denotes the fitting line represented by

$$(1 - w_p) = (1 - w_{po}) + \tau \{(v_p^{\dagger} + C_p v_p^{\dagger} | v_p^{\dagger} |)^2 + C_{pvr} v_p^{\dagger}\}$$
 (22)

where  $\tau$ ,  $C_p$  and  $C_{pvr}$  are the coefficients determined experimentally. The first term in the righthand side of Eq.(22) is the wake fraction when the ship is running straight ahead. The other terms are the effects of ship motions. It is known that Eq.(22) represents the wake fraction at propeller pretty well for single propeller ships. As for the twin-propeller ship, however, Eq.(22) is not fitted very well in the experimental results. The effects of ship motions on the wake fraction at propeller of twin-propeller ships requires further investigations, besides the propeller wake of running straight ahead conditions.[5]

Fig. 4 shows the longitudinal component of the rudder inflow velocity under the effects of ship motions. The longitudinal component of the rudder inflow velocity is not so much affected by the ship motions. Hence, this inflow velocity component,  $\mathbf{u}_R$ , are assumed to be constant in spite of ship motions in the present paper. This means that the coefficients in Eq.(12),  $\mathbf{w}_P$ ,  $\mathbf{w}_R$  and  $\kappa$ , are the values obtained in the straight ahead towing tests.

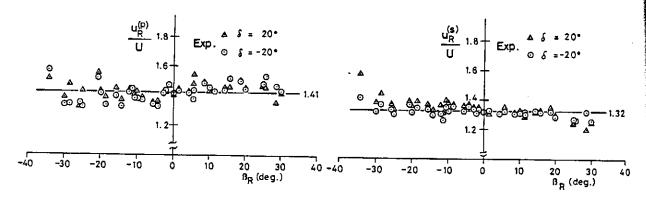


Fig. 4 Effects of ship motions on the longitudinal component of rudder inflow velocity

Fig. 5 shows the rudder angle at which the rudder normal force comes to be zero, where the measured value,  $\ell_R = -0.5\,\mathrm{L}$ , is used in the expression of the effective drift angle,  $\beta_R$ , the abcissa. This value of  $\ell_R$  coincides with the geometrical position of the rudder in this ship. As usual, the experimental data is fitted by three kinds of straight lines. It should be noted that the angle,  $\delta_R$ , is not symmetric between port and starboard sides because of the difference of open characteristics of the propellers.

Figs. 6 and 7 show measured wind resistance coefficients and wave drifting force coefficients, respectively. The measurements were conducted in wind or waves with a model fixed to a force transducer. The cargo on the deck of the model ship is a pair of container cranes, of which height is approximately 1.6m and the effective projected areas for wind resistance, which includes both ship's hull above water and the cargo on deck, are about  $0.3\text{m}^2$  in front view and  $0.7\text{m}^2$  in side view, respectively. Both of the experimental data in Figs. 6 and 7 are fitted by B-Spline and used in numerical simulations.

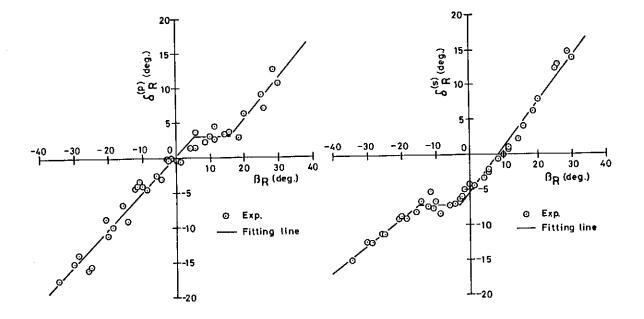
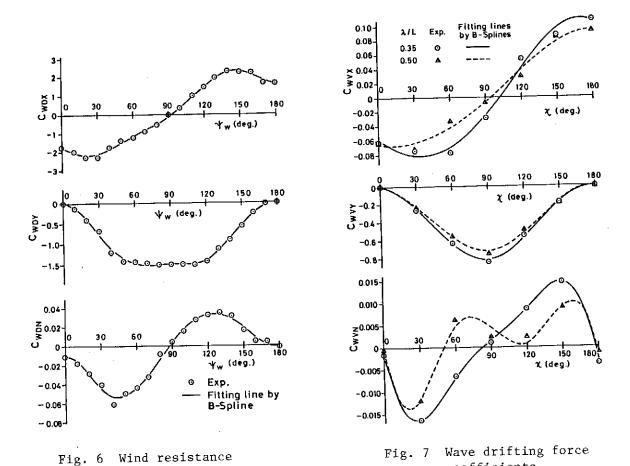


Fig. 5 Rudder angle at which the rudder normal force comes to be zero



4. FREE-RUNNING TESTS IN CALM WATER WITHOUT WIND

coefficients

In order to investigate the applicability of the mathematical model described in the previous section, a series of free-running tests was performed.

coefficients

The results obtained are shown in Figs. 8 and 9. In the numerical simulations, two kinds of mathematical models are used; one is the twin-propeller/twin-rudder model described in the previous section, and the other is the equivalent single propeller/single rudder model. As for the latter model, the equivalent wake fraction at propeller,  $\mathbf{w}_{p}^{(eq)}$  and the equivalent value of the rudder angle at which the rudder normal force comes to be zero,  $\delta_R^{(eq)}$  are assumed to be the mean values of each side's ones. Since the open characteristics of the propellers in each side are different, the calculations were performed using each characteristics, and the longitudinal component of the rudder inflow velocity can be calculated according to these characteristics. In Fig. 8, the manoeuvring motions at larger rudder angle cannot be predicted with accuracy by the twin-propeller/twin-rudder model, although the prediction at smaller rudder angle shows good agreement with the experimental ones. The discrepancies between the calculated results and the experimental ones are greater in the rudder force in Fig. 9. One of the main reasons of these discrepancies might be the poorness in fitting of the experimental wake fraction at propeller under ship motions (see Fig. 3). The single propeller/single rudder model is effective to predict the motions in Fig. 8; however, it is not useful to predict the rudder force in Fig. 9.

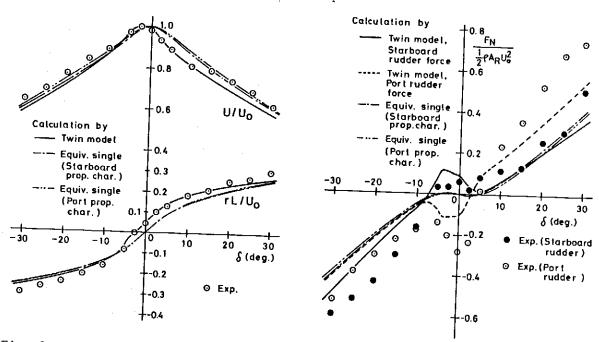


Fig. 8 Steady turning characteristics

Fig. 9 Rudder normal force during steady turning

# 5. DIRECTIONAL CONTROLLABILITY IN WIND AND WAVES

Making use of the twin-propeller/twin-rudder model as the mathematical model for the manoeuvring motions, the course keeping ability of the heavy cargo carrier with twin-propeller and twin-rudder is investigated. Numerical simulations were performed to keep the ship on a straight course in wind or waves with certain check helm. The calculated results are shown in Figs. 10 and 11, where  $\mathbf{U}_0$  is the ship speed without wind and waves. The severest wind direction to keep straight

course is about  $\psi_W=30^\circ-60^\circ$  as is like a tourist vessel.[3] The severest wave direction, on the other hand, is rather beam seas,  $\chi=60^\circ-90^\circ$ . This tendency also hold for any wave length as far as the ship motions in wave encounter frequency are neglected. Since the wave drifting moment is comparatively smaller than that of wind, maximum check helm in waves appears near beam-sea conditions.

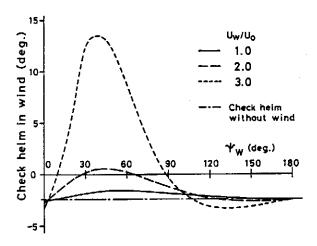


Fig. 10 Check helm in wind (without waves)

Fig. 11 Check helm in waves (without wind)

#### 6. CONCLUSIONS

The main conclusions obtained in the present investigations are as follows:

- (1) The wake fraction at propeller of twin-propeller and twin-rudder ships in motions differs from single propeller and single rudder ships. The effects of manoeuvring motions on the wake requires further investigations.
- (2) With respect to the prediction of manoeuvring motions of twin-propeller and twin-rudder ships, the equivalent single propeller/single rudder model is effective; however, rudder normal forces should be calculated by virtue of the twin-propeller/twin-rudder mathematical model.
- (3) As for the heavy cargo carrier investigated in this paper, the severest wind direction to keep on straight course is about  $\psi_W = 30^\circ-60^\circ$ . On the other hand, the severest wave direction is rather beam seas, i.e.  $\chi = 60^\circ-90^\circ$ .

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