**Assignment 1**

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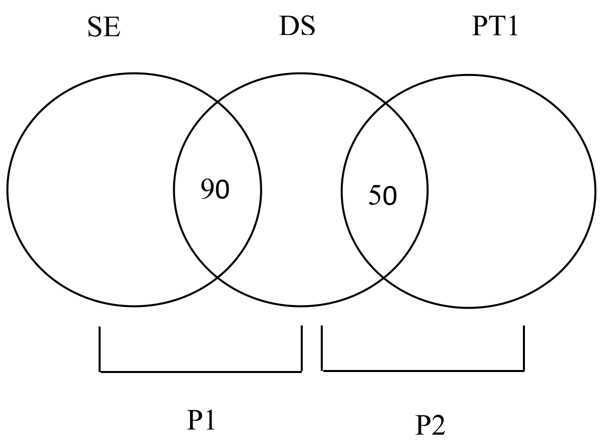
## Question 1

1a. i)




ii) $|DS| = |U| - |SE - (SE \cap DS)| - |PT1 - (PT1 \cap DS)|$

$= 250 - |SE - 90| - |PT1 - 50|$

$= 110 - SE - PT1$

iii) 90 + 50 = 140

iv) 250 − 40 = 110

b. A = {3, 5, 7, 9, 11, 13, 15, 17, 19}

N = {1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

B = {2, 3, 5, 7, 11, 13, 17, 19}

C = {5}

i) |A| = 9,  |B| = 8,  |C| = 1

ii) $|A| = 9$

$|P(A)| = 2^9$

$= 512 - 1$

$= 511$

iii) $C \times B = \{5, 2\}, \{5, 3\}, \{5, 5\}, \{5, 7\}, \{5, 11\}, \{5, 13\}, \{5, 17\}, \{5, 19\}$

**Question 2**

a. i) $m \land n =$ You play table tennis and miss the midterm test.

ii) $\neg(m \lor n) \lor o = \neg m \land \neg n \lor o$ [De Morgan's Law]

= You are not playing table tennis and not miss the midterm test or pass the subject.

b. $(a \to b) \equiv (\neg a \lor b)$

| a | b | $a \to b$ | $\neg a \lor b$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

Based on the truth table above, it is shown that $(a \to b) \equiv (\neg a \lor b)$.

c. i) $x \to y$

ii) $(\neg x \lor \neg z) \to \neg y$

$\neg(x \land z) \to \neg y$

iii) $y \leftrightarrow (x \land z)$

**Question 3**

a. P(n) = n divides 15, Domain of discourse = Z+

i) P(5) = 5 is the value of n, **True**

ii) $\forall nP(n)$ = For all values of n, P(n) is true. **False**

iii) $\exists n\neg P(n)$ = For some values of n, n are not divide 15. **True**

b. P(m, n) = $m \geq n$, Domain of discourse = Z+ × Z+

- $\exists m\exists nP(m,n)$ = For some values of m and n, P(m, n) is true.

The statement above is **true**. Since the domain of discourse is Z+ × Z+, that's means the values of m and n could be 1, 4, 9, 16,…..

Let says,

1. The value of m = 1; the value of n = 1. Hence, m = n. P(m, n) = $m \geq n$
2. The value of m = 9; the value of n = 1. Hence, m > n. P(m, n) = $m \geq n$
3. The value of m = 4; the value of n = 16. Hence, n > m. P(m, n) $\neq m \geq n$

In conclusion, there is more than 1 value of m and n are true for the P(m, n) propositional function. However, it's not all true, some of them are false.

- The negation of $\exists m\exists nP(m,n)$, means that none of the value of m and n is true for P(m, n)

$$\neg(\exists m\exists nP(m,n)) = \neg\forall m\neg\forall nP(m,n)$$

**Question 4**

$\exists$x P(x)

$\exists x\ (x < a < y)$

Domain of discourse is x and y are real numbers and a is a rational number

If $x = 0, y = 1, a = \frac{1}{2}$,

$0 < \frac{1}{2} < 1$ , 0 and 1 are real numbers while $\frac{1}{2}$ is a rational number. Thus, this statement is true.

**Question 5**

a. If 2 divides m - n

$(m, n) \in R, x = \{1, 2, 3, 4, 5\}, \quad x, y \in X$

R = { (1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4 ,4), (5, 1), (5, 3), (5, 5) }

$R^{-1}$ = { (1, 1), (3, 2), (5, 1), (2, 2), (2, 4), (1, 3), (3, 3), (5, 3), (2, 4), (4, 4), (1, 5), (3, 5), (5, 5) }

b. If $m + n \leq 4$,

$x = \{1, 2, 3, 4, 5\}, \quad x, y \in X$

$\mathrm{R} = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

$\mathrm{R}^{-1} = \{(1, 1), (2, 1), (3, 1), (1, 2), (2, 2), (1, 3)\}$

c. Relation of question a and b are both symmetric.

d. i) $(x, y) \in R$

if x and y = 1, R = {(1, 1)}

- symmetric, antisymmetric and transitive

ii) $(x, y) \in R$

if $x = y^2$, R = { (1, 1), (2, 4), (3, 9),…}

- antisymmetric

iii) $(x, y) \in R$

if x = y, R = { (1, 1), (2, 2), (3, 3),…}

- reflexive, symmetric, antisymmetric, transitive and partial order.

## Question 6

a. i) $f(n) = n + 1$

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 + 1$$

$$n_1 = n_2$$

Function is one-to-one

Let f(n) = m,

$$m = n + 1$$

$$n = m - 1$$

m can be any integer. Hence, the function is onto.

∴ function f(n) = n + 1 is both one-to-one and onto.

ii) $f(n) = |n|$

Let $n_1 = 4, n_2 = -4$,

$$f(n_1) = f(n_2)$$

$$|n_1| = |n_2|$$

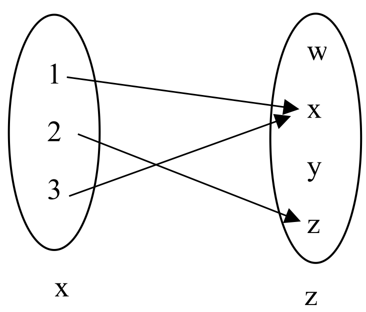
$$|4| = |-4|$$

$$4 = 4$$

Different input, same output.Function is not one-to-one. $|n|$ will only produce positive integers for $f(n)$. No negative integer $f(n)$ will map onto n. Hence, function is not onto.

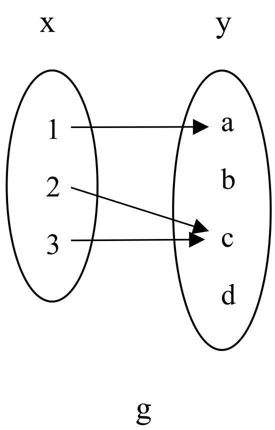
∴ Function $f(n) = |n|$ is neither one-to-one or onto.

b.




$m\ o\ n = \{(1, x), (2, z), (3, x)\}$

c.




$g(s) = \{g(x) \mid x \epsilon S\} = \{g(1)\}$

$g(1) = a$

$g(s) = a$

$g(T) = \{g(x) \mid x \epsilon T\} = \{g(1), g(3)\}$

$g(1) = a,\ \ g(3) = c$

$g(T) = \{a, c\}$

$g^{-1}(U) = \{x \epsilon X \mid g(x) \epsilon U\}$

$g(1) = a \quad \left(1 \epsilon g^{-1}(U)\right)$

$g(2) = c \quad (2 \notin g^{-1}(U))$

$g(3) = c \quad (3 \notin g^{-1}(U))$

$g^{-1}(U) = \{1\}$

$g^{-1}(V) = \{x \in X \mid g(x) \in V\}$

$g(1) = a \quad \left(1 \epsilon g^{-1}(V)\right)$

$g(2) = c \quad (2 \in g^{-1}(V))$

$g(3) = c \quad (3 \in g^{-1}(V))$

$g^{-1}(V) = \{1, 2, 3\}$