Assignment 3

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Q1 : Discrete Probability Theory

1. (a) $\frac{^5C_4}{^{25}C_4} = 3.9526 \times 10^{-4}$

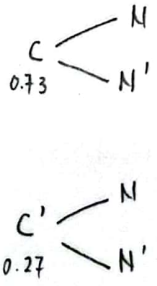
(b) $\frac{^5C_2 \times ^{20}C_2}{^{25}C_4} = 0.1502$

2. $P(C \cap N) = 0.37$

$P(N|C) = \frac{P(N \cap C)}{P(C)}$

$= \frac{0.37}{0.73}$

$= 0.5068$



3. (a) $A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

(b) $B = \{(3,6), (4,5), (5,4), (6,3)\}$

$|S| = 36$

$P(B) = \frac{|B|}{|S|}$

$= \frac{4}{36}$

$= 0.1111$

(c) $C = \{(1,6), (2,5), (2,6), (3,4), (3,5), (4,3), (4,4), (5,2), (5,3), (6,1), (6,2)\}$

$$P(C) = \frac{|C|}{|S|}$$

$$= \frac{11}{36}$$

$$= 0.3056$$

Q2 : Bayes' Theorem

1. A = computer from Acme

D = computer from DotCom

N = computer from Nuclear

B = defective

(a) $P(A) = \frac{55}{100} = 0.55$

$P(D) = \frac{10}{100} = 0.10$

$P(N) = \frac{35}{100} = 0.35$

(b) $P(B)$

$= P(B \cap A) + P(B \cap D) + P(B \cap N)$

$= P(B|A)P(A) + P(B|D)P(D) + P(B|N)P(N)$

$= 0.01(0.55) + 0.03(0.10) + 0.03(0.35)$

$= 0.019$

2. Let C = have covid-19 active infection

Let P = positive on RT-PCR test

$P(C) = 0.15 \quad P(P|C) = 0.95 \quad P(P|C') = 0.02$

(a) $P(C|P) = \frac{P(C \cap P)}{P(P)}$

$= \frac{P(P|C)P(C)}{P(P \cap C) + P(P \cap C')}$

$= \frac{P(P|C)P(C)}{P(P|C)P(C) + P(P|C')P(C')}$

$= \frac{0.95(0.15)}{0.95(0.15) + 0.02(0.85)}$

$= \frac{0.1425}{0.1425 + 0.017}$

$= 0.8934$

$P(C') = 1 - 0.15$

$= 0.85$

(b) $P(C'|P) = \frac{P(C' \cap P)}{P(P)}$

$= \frac{P(P|C')P(C')}{P(P)}$

$= \frac{0.02(0.85)}{0.1595}$

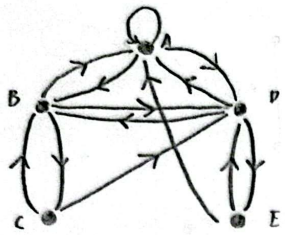
$= 0.1066$

$P(P) = 0.1425 + 0.017$

$= 0.1595$

Q3 : Graph Definition and Notation

1. (a)



(b) deg(A) = 7

deg(B) = 6

deg(C) = 3

deg(D) = 7

deg(E) = 3

2. (a) Vertice is a fundamental unit of graph, representing a point or node where edges meet.

(b) An edge is a line or connection between two vertices in a graph.

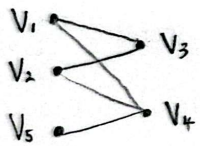
(c) A loop is an edge that connects a vertex to itself

(d) Parallel edges are two or more edges that connect the same pair of vertices.

(e) Degree of vertex is the number of edges incident to that vertex

Q4 : Representation of Graphs

1. (a)

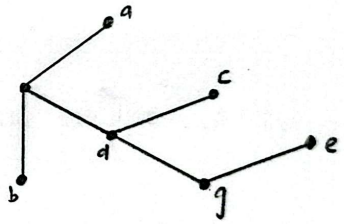


$S_1 = \{V_1, V_2, V_5\}$

$S_2 = \{V_3, V_4\}$

(b) undirected graph, simple graph, connected graph

(b)



total costs = 5+6+5+7+8+20

= 51

Q5 : Isomorphism of Graph

1. no. of vertices :
   G1 = 7
   G2 = 7

   no. of edges :
   G1 = 7
   G2 = 7

   no. of degree for each vertex :
   G1 = all vertices have degree of 2
   G2 = all vertices have degree of 2

Incident function $f: G1 \to G2$, where $G1 = \{a, b, c, d, e, f, g\}$ and $G2 = \{1, 2, 3, 4, 5, 6, 7\}$

$f(a) = 1$, $f(b) = 3$, $f(c) = 5$, $f(d) = 7$, $f(e) = 2$, $f(f) = 4$, $f(g) = 6$

Adjacency matrices for both graph :

$A_{G1} =$

	a	b	c	d	e	f	g
a	0	1	0	0	0	0	1
b	1	0	1	0	0	0	0
c	0	1	0	1	0	0	0
d	0	0	1	0	1	0	0
e	0	0	0	1	0	1	0
f	0	0	0	0	1	0	1
g	1	0	0	0	0	1	0

$A_{G2} =$

	1	3	5	7	2	4	6
1	0	1	0	0	0	0	1
3	1	0	1	0	0	0	0
5	0	1	0	1	0	0	0
7	0	0	1	0	1	0	0
2	0	0	0	1	0	1	0
4	0	0	0	0	1	0	1
6	1	0	0	0	0	1	0

∴ Both A_{G1} and A_{G2} are the same. Therefore, they are isomorphic

2. (a) no. of vertices :
   G1 = 6
   G2 = 6

   no. of edges :
   G1 = 9
   G2 = 9

   no. of degree for each vertex :
   G1 = all vertices has degree of 3
   G2 = all vertices has degree of 3

∴ $f: G1 \to G2$ is undefined. Hence, G1 and G2 is not isomorphic.

(b) no of vertices :
   G1 = 6
   G2 = 6

   no. of edges :
   G1 = 11
   G2 = 11

   no. of degree for each vertex :
   G1 = 4 vertices has degree of 4 and 2 vertices has degree of 3
   G2 = 4 vertices has degree of 4 and 2 vertices has degree of 3

∴ $f: G1 \to G2$ is undefined. Hence, G1 and G2 is not isomorphic

3.