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Stats C183
HW 5
6-5-20

① ⓐ $\mu = 0.20$ $S = \$50$; $\sigma = 0.25$ $\Delta t = 1$ week $\Big\}$ $\frac{\Delta S}{S} = \sim N(\mu \cdot \Delta t, \sigma^2 \cdot \Delta t) \longrightarrow$

$\Delta t = \frac{1 \text{ week}}{52 \text{ total weeks}} = 0.01923 \longrightarrow \Delta S/(50) \sim N((0.20)\cdot(0.01923), (0.25)^2 \cdot (0.01923))$

$\longrightarrow \Delta S \sim N[(0.20)\cdot(0.01923)\cdot(50), (0.25)^2 \cdot (0.01923)\cdot(50)^2] \longrightarrow \boxed{\Delta S \sim N(0.1923, \sqrt{3.0047})}$

$\boxed{S_{\Delta t = 1 \text{ week}} \sim N(\underset{(\mu)}{50.1923}, \underset{(\sigma)}{1.7334})}$

ⓑ $\boxed{\text{done in RStudio code}}$

② ⓐ $\mu = 0.16$ $S = \$50$; $\sigma = 0.30$ $\Delta t = \frac{1 \text{ day}}{365 \text{ total days}} = 0.00274 \longrightarrow \frac{\Delta S}{(50)} \sim N[(0.16)\cdot(0.00274), (0.30)^2 \cdot (0.00274)]$

$\longrightarrow \Delta S \sim N[(0.16)\cdot(0.00274)\cdot(50), (0.30)^2 \cdot (0.00274)\cdot(50)^2] \longrightarrow \Delta S \sim N(\underbrace{0.02192}_{(+50)}, 0.6164)$

$\longrightarrow S_{\Delta t = 1 \text{ day}} \sim N(0.02192, \sqrt{0.6164}) \longrightarrow$ $\boxed{\text{Expected stock price at } \Delta t = 1 \text{ day} = \$50.02}$ $\boxed{\mu = 0.02192}$

ⓑ $\boxed{\text{standard deviation at } \Delta t = 1 \text{ day} = \sqrt{0.6164} = 0.7851}$

③ ⓐ $\mu = 0.16$ $S = \$38$; $\sigma = 0.35$ $\Big\}$ ⊛ $\Phi\left\{\ln(\text{"current price"}) + \left(\mu - \frac{\sigma^2}{2}\right)\cdot\left(\frac{1}{2}\right), \left(\frac{\sigma^2}{2}\right)\right\} \longrightarrow$

$\Phi\left\{\ln(38) + \left(0.16 - \frac{(0.32)^2}{2}\right)\left(\frac{1}{2}\right), \frac{(0.35)^2}{2}\right\} \longrightarrow \Phi\{3.6376 + 0.049375, 0.06125\}$

$\Phi\{3.6870, (0.247)^2\} \longrightarrow P(S_T > 40) = (1 - P(S_T \leq 40)) \longrightarrow$ using z-table $\longrightarrow$

$1 - P\left(z \leq \frac{\ln(S_T) - \mu_\Phi}{\sigma_\Phi}\right) \longrightarrow 1 - P\left(z < \frac{\ln(40) - 3.6870}{0.2475}\right) \longrightarrow 1 - P(z \leq 0.0076) \longrightarrow$

$1 - (0.50303) \longrightarrow = \boxed{0.49697}$ } probability

ⓑ $1 - (0.49697) = \boxed{0.50303}$ } probability

⊛ 1 − P("EU Put w/ exercise price of \$40 & expiration date 6 months")

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④ ⓐ $P\left(S e^{(\mu-\frac{\sigma^2}{2})(T-t)-1.96\sigma\sqrt{T-t}} \leq S_T \leq S e^{(\mu-\frac{\sigma^2}{2})(T-t)+1.96\sigma\sqrt{T-t}}\right)=0.95 \longrightarrow$

$S=\$40 \quad \sigma=0.15$
$\mu=0.10$ $\longrightarrow \ln(S)+\left(\mu-\frac{\sigma^2}{2}\right)(\Delta t) \pm 1.96\cdot\sigma\cdot\sqrt{\Delta t} \longrightarrow \Delta t=\frac{2 \text{ months}}{12 \text{ total months}} \longrightarrow$

$\underbrace{\left[\ln(40)+\left(0.10-\frac{(0.15)^2}{2}\right)\left(\frac{2}{12}\right)\right]}_{\approx 3.703671121} \pm \underbrace{(1.96)\cdot(0.15)\sqrt{\frac{2}{12}}}_{\approx 0.120024997} \longrightarrow$

Confidence Interval (95%)
$P(a < S_T < b)=0.95 \longrightarrow$
$a=32.0861$
$b=51.3631$

ⓑ ⊛ $\frac{\Delta S}{S} \sim N(\mu\cdot\Delta t, \sigma^2\cdot\Delta t) \longrightarrow$

$\ln(S_T) \sim N\left(\ln(40)+\left(0.10-\frac{(0.15)^2}{2}\right)\left(\frac{2}{12}\right), (0.15)\cdot\sqrt{\left(\frac{2}{12}\right)}\right) \longrightarrow \sim N(3.7037, 0.0612)$

$\boxed{\text{Expected return in 2months} = 3.7037}$

ⓒ $\boxed{\text{Standard deviation in 2 months} = 0.0612}$

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⑤ (Black-Scholes Model) on Call $\longrightarrow$ ⊛ $C=S_0\cdot P(z\leq d_1)-\frac{E}{e^{rt}}\cdot P(z\leq d_2) \longrightarrow$

$S_0=\$95 \quad E=\$105 \quad r=0.08$
$\sigma=0.60 \quad t=8 \text{ months}$

for $d_1=\frac{\ln\left(\frac{S_0}{E}\right)+\left(r+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$ & $d_2=d_1-\sigma\sqrt{t}$

$d_1=\frac{\ln\left(\frac{95}{105}\right)+\left(0.08+\frac{(0.6)^2}{2}\right)\left(\frac{8}{12}\right)}{(0.6)\cdot\sqrt{8/12}} \cong 0.1495$

$d_2=(0.1495)-(0.6)\sqrt{8/12} \cong -0.3404$

$C=(95)\cdot P(z\leq 0.1495)-\frac{105}{e^{(0.08)(\frac{8}{12})}}\cdot P(z\leq -0.3404)$

$\longrightarrow C=(95)\cdot(0.55942)-(99.5467)\cdot(0.36678) \longrightarrow \boxed{C=\$16.63}$

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⑥ $\boxed{\text{Done in RStudio}}$

Stats_c183_HW5_Charles_Liu

Charles Liu (304804942)

6/3/2020

Load Necessary Packages:

```
library(readr)
```

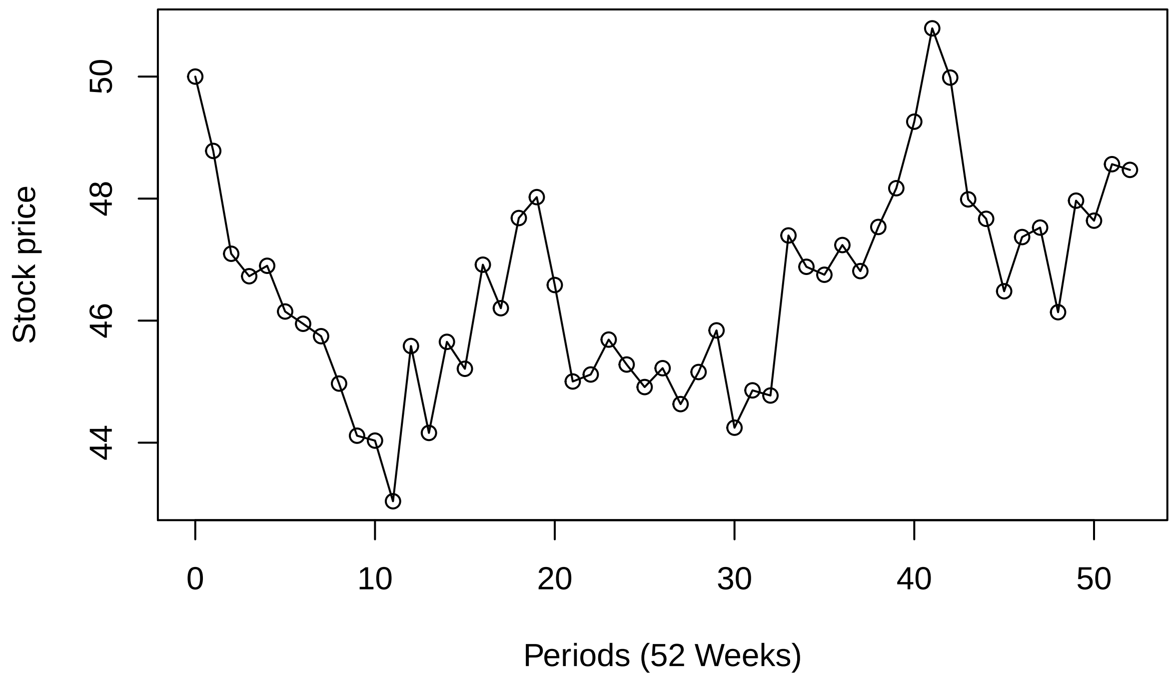
1b)

```
epsilon <- c(0,rnorm(52))
S <- c(50,rep(0,52)) # S_0 = $50 & 52 weeks
DS <- rep(0,53)

for(i in(1:52)) {
  DS[i+1] <- 0.0020*S[i] + 0.025*S[i]*epsilon[i+1]
  S[i+1] = S[i] + DS[i+1]
}

x <- seq(0,52)
xx <- as.data.frame(cbind(x, epsilon, DS, S))

# Plot using 52 weeks
plot(x, S, type="l", xlab="Periods (52 Weeks)", ylab="Stock price")
points(x,S)
```



6)

```r
a <- read.csv("C:/Users/cliuk/Documents/UCLA Works/UCLA Spring 2020/Stats C183/Homeworks/HW 5/AAPL.csv"

# Calculate it by hand
n <- nrow(a)
p <- a[,3]
temp <- p/p[-1]
u <- log(temp)
b <- 1/(n-1)
c <- sum(u^2)
d <- sum(u)
s <-(b*(c - (d^2/n)))
trade_days <- 365 - n
sigma_hat <- sqrt(trade_days) * s

# Value of annual volatility estimation:
sigma_hat
```

```
## [1] 0.03173322
```

```r
# Therefore the annual volatility is sigma = 3/rate(%)
```