

① a) $\mu = 0.20$ $\sigma = 0.25$ $S = \$50$ $\Delta t = 1 \text{ week}$ $\left\{ \frac{\Delta S}{S} \sim N(\mu \cdot \Delta t, \sigma^2 \cdot \Delta t) \right\} \rightarrow$

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$\Delta t = \frac{1 \text{ week}}{52 \text{ total weeks}} = 0.01923 \rightarrow \Delta S / (50) \sim N(0.20 \cdot (0.01923), (0.25)^2 \cdot (0.01923))$

$\rightarrow \Delta S \sim N[(0.20) \cdot (0.01923) \cdot (50), (0.25)^2 \cdot (0.01923) \cdot (50)^2] \rightarrow \Delta S \sim N(0.1923, 3.0047)$

① done in RStudio code

$S_{\Delta t=1 \text{ week}} \sim N(50.1923, 1.7334)$
(μ) (σ)

② a) $\mu = 0.16$ $\sigma = 0.30$ $S = \$50$ $\Delta t = \frac{1 \text{ day}}{365 \text{ total days}} = 0.00274 \rightarrow \frac{\Delta S}{(50)} \sim N[(0.16) \cdot (0.00274), (0.30)^2 \cdot (0.00274)]$

$\rightarrow \Delta S \sim N[(0.16) \cdot (0.00274) \cdot (50), (0.30)^2 \cdot (0.00274) \cdot (50)^2] \rightarrow \Delta S \sim N(0.02192, 0.6164)$

$\rightarrow S_{\Delta t=1 \text{ day}} \sim N(0.02192, \sqrt{0.6164}) \rightarrow$ Expected stock price at $\Delta t = 1 \text{ day} = \50.02 $\mu = 0.02192$

① Standard deviation at $\Delta t = 1 \text{ day} = \sqrt{0.6164} = 0.7851$

③ a) $\mu = 0.16$ $\sigma = 0.35$ $S = \$38$ $\left\{ \ln(\text{"current price"}) + \left(\mu - \frac{\sigma^2}{2} \right) \cdot \left(\frac{1}{2} \right), \left(\frac{\sigma^2}{2} \right) \right\} \rightarrow$

$\Phi \left\{ \ln(38) + \left(0.16 - \frac{(0.35)^2}{2} \right) \cdot \left(\frac{1}{2} \right), \frac{(0.35)^2}{2} \right\} \rightarrow \Phi \{ 3.6376 + 0.049375, 0.06125 \}$

$\Phi \{ 3.6870, (0.247)^2 \} \rightarrow P(S_T > 40) = (1 - P(S_T \leq 40)) \rightarrow \text{using z-table} \rightarrow$

$1 - P\left(z \leq \frac{\ln(S_T) - \mu_\Phi}{\sigma_\Phi}\right) \rightarrow 1 - P\left(z \leq \frac{\ln(40) - 3.6870}{0.2475}\right) \rightarrow 1 - P(z \leq 0.0076) \rightarrow$

$1 - (0.50303) \rightarrow = 0.49697 \}$ probability

① $1 - (0.49697) = 0.50303 \}$ probability

* $1 - P(\text{"EU Put w/ exercise price of \$40 \& expiration date 6 months"})$

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$$④ P\left(S_0 e^{(\mu - \frac{\sigma^2}{2})(T-t) - 1.96\sigma\sqrt{T-t}} \leq S_T \leq S_0 e^{(\mu - \frac{\sigma^2}{2})(T-t) + 1.96\sigma\sqrt{T-t}}\right) = 0.95 \rightarrow$$

$$S = \$40 \quad \sigma = 0.15 \rightarrow \ln(S) + (\mu - \frac{\sigma^2}{2})(\Delta t) \pm 1.96 \cdot \sigma \cdot \sqrt{\Delta t} \rightarrow \Delta t = \frac{2 \text{ months}}{12 \text{ total months}} \rightarrow$$

$$\left[\ln(40) + (0.10 - \frac{0.15^2}{2}) \left(\frac{2}{12}\right) \right] \pm (1.96)(0.15) \sqrt{\frac{2}{12}} \rightarrow$$

≈ 3.703671121 ≈ 0.120024997

Confidence Interval (95%)

$$P(a < S_T < b) = 0.95 \rightarrow$$

$a = 32.0861$
 $b = 51.3631$

$$⑥ \frac{\Delta S}{S} \sim N(\mu \cdot \Delta t, \sigma^2 \cdot \Delta t) \rightarrow$$

$$\ln(S_T) \sim N\left(\ln(40) + (0.10 - \frac{0.15^2}{2}) \left(\frac{2}{12}\right), (0.15) \cdot \sqrt{\frac{2}{12}}\right) \rightarrow \sim N(3.7037, 0.0612)$$

Expected return in 2 months = 3.7037 = \$40.67 (value price of stock)

Standard deviation in 2 months = 0.0612

$$⑤ \text{ (Black-Scholes Model) on Call } \rightarrow * C = S_0 \cdot P(z \leq d_1) - \frac{E}{e^{rt}} \cdot P(z \leq d_2) \rightarrow$$

for $d_1 = \frac{\ln(\frac{S_0}{E}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$ $d_2 = d_1 - \sigma\sqrt{t}$

$S_0 = \$95$ $E = \$105$ $r = 0.08$
 $\sigma = 0.60$ $t = 8 \text{ months}$

$$d_1 = \frac{\ln(\frac{95}{105}) + (0.08 + \frac{0.6^2}{2}) \left(\frac{8}{12}\right)}{(0.6) \cdot \sqrt{8/12}} \approx 0.1495$$

$$d_2 = (0.1495) - (0.6) \cdot \sqrt{8/12} \approx -0.3404$$

$$\rightarrow C = (95) \cdot (0.55942) - (99.5467) \cdot (0.36678) \rightarrow C = \$16.63$$

⑥ Done in RStudio