Stats_101B_HW_5_Charles_Liu (Dis. 3A)

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Loading Necessary Packages:

```
library(DoE.base)
library(unrepx)
```

Problem 1: (Exercise 7.6)

```
# Set up the problem
response1 <- c(7,9,34,55,16,20,40,60,
               8,10,32,50,18,21,44,61,
               8,12,35,52,15,22,45,65,
               6,10,30,53,15,20,41,63)

k=5
contrast1 <- contr.FrF2(2^(k))
colnames(contrast1) <- c("A", "B","AB", "C", "AC","BC","ABC","D","AD",
"BD","ABD","CD", "ACD","BCD","ABCD","E","AE","BE","ABE","CE","ACE","BCE",
"ABCE","DE","ADE","BDE","ABDE","CDE","ACDE","BCDE","ABCDE")
contrast1
```

```
##     A  B AB  C AC BC ABC  D AD BD ABD CD ACD BCD ABCD  E AE BE ABE CE ACE BCE
## 1  -1 -1  1 -1  1  1  -1 -1  1  1  -1  1  -1  -1    1 -1  1  1  -1  1  -1  -1
## 2   1 -1 -1 -1 -1  1   1 -1 -1  1   1  1   1  -1   -1 -1 -1  1   1  1   1  -1
## 3  -1  1 -1 -1  1 -1   1 -1  1 -1   1  1  -1   1   -1 -1  1 -1   1  1  -1   1
## 4   1  1  1 -1 -1 -1  -1 -1 -1 -1  -1  1   1   1    1 -1 -1 -1  -1  1   1   1
## 5  -1 -1  1  1 -1 -1   1 -1  1  1  -1 -1   1   1   -1 -1  1  1  -1 -1   1   1
## 6   1 -1 -1  1  1 -1  -1 -1 -1  1   1 -1  -1   1    1 -1 -1  1   1 -1  -1   1
## 7  -1  1 -1  1 -1  1  -1 -1  1 -1   1 -1   1  -1    1 -1  1 -1   1 -1   1  -1
## 8   1  1  1  1  1  1   1 -1 -1 -1  -1 -1  -1  -1   -1 -1 -1 -1  -1 -1  -1  -1
## 9  -1 -1  1 -1  1  1  -1  1 -1 -1   1 -1   1   1   -1 -1  1  1  -1  1  -1  -1
## 10  1 -1 -1 -1 -1  1   1  1  1 -1  -1 -1  -1   1    1 -1 -1  1   1  1   1  -1
## 11 -1  1 -1 -1  1 -1   1  1 -1  1  -1 -1   1  -1    1 -1  1 -1   1  1  -1   1
## 12  1  1  1 -1 -1 -1  -1  1  1  1   1 -1  -1  -1   -1 -1 -1 -1  -1  1   1   1
## 13 -1 -1  1  1 -1 -1   1  1 -1 -1   1  1  -1  -1    1 -1  1  1  -1 -1   1   1
## 14  1 -1 -1  1  1 -1  -1  1  1 -1  -1  1   1  -1   -1 -1 -1  1   1 -1  -1   1
## 15 -1  1 -1  1 -1  1  -1  1 -1  1  -1  1  -1   1   -1 -1  1 -1   1 -1   1  -1
## 16  1  1  1  1  1  1   1  1  1  1   1  1   1   1    1 -1 -1 -1  -1 -1  -1  -1
## 17 -1 -1  1 -1  1  1  -1 -1  1  1  -1  1  -1  -1    1  1 -1 -1   1 -1   1   1
## 18  1 -1 -1 -1 -1  1   1 -1 -1  1   1  1   1  -1   -1  1  1 -1  -1 -1  -1   1
## 19 -1  1 -1 -1  1 -1   1 -1  1 -1   1  1  -1   1   -1  1 -1  1  -1 -1   1  -1
```

```
## 20  1  1  1 -1 -1 -1  -1 -1 -1 -1  -1  1   1   1    1  1  1  1   1 -1  -1  -1
## 21 -1 -1  1  1 -1 -1   1 -1  1  1  -1 -1   1   1   -1  1 -1 -1   1  1  -1  -1
## 22  1 -1 -1  1  1 -1  -1 -1 -1  1   1 -1  -1   1    1  1  1 -1  -1  1   1  -1
## 23 -1  1 -1  1 -1  1  -1 -1  1 -1   1 -1   1  -1    1  1 -1  1  -1  1  -1   1
## 24  1  1  1  1  1  1   1 -1 -1 -1  -1 -1  -1  -1   -1  1  1  1   1  1   1   1
## 25 -1 -1  1 -1  1  1  -1  1 -1 -1   1 -1   1   1   -1  1 -1 -1   1 -1   1   1
## 26  1 -1 -1 -1 -1  1   1  1  1 -1  -1 -1  -1   1    1  1  1 -1  -1 -1  -1   1
## 27 -1  1 -1 -1  1 -1   1  1 -1  1  -1 -1   1  -1    1  1 -1  1  -1 -1   1  -1
## 28  1  1  1 -1 -1 -1  -1  1  1  1   1 -1  -1  -1   -1  1  1  1   1 -1  -1  -1
## 29 -1 -1  1  1 -1 -1   1  1 -1 -1   1  1  -1  -1    1  1 -1 -1   1  1  -1  -1
## 30  1 -1 -1  1  1 -1  -1  1  1 -1  -1  1   1  -1   -1  1  1 -1  -1  1   1  -1
## 31 -1  1 -1  1 -1  1  -1  1 -1  1  -1  1  -1   1   -1  1 -1  1  -1  1  -1   1
## 32  1  1  1  1  1  1   1  1  1  1   1  1   1   1    1  1  1  1   1  1   1   1
##    ABCE DE ADE BDE ABDE CDE ACDE BCDE ABCDE
## 1     1  1  -1  -1    1  -1    1    1    -1
## 2    -1  1   1  -1   -1  -1   -1    1     1
## 3    -1  1  -1   1   -1  -1    1   -1     1
## 4     1  1   1   1    1  -1   -1   -1    -1
## 5    -1  1  -1  -1    1   1   -1   -1     1
## 6     1  1   1  -1   -1   1    1   -1    -1
## 7     1  1  -1   1   -1   1   -1    1    -1
## 8    -1  1   1   1    1   1    1    1     1
## 9     1 -1   1   1   -1   1   -1   -1     1
## 10   -1 -1  -1   1    1   1    1   -1    -1
## 11   -1 -1   1  -1    1   1   -1    1    -1
## 12    1 -1  -1  -1   -1   1    1    1     1
## 13   -1 -1   1   1   -1  -1    1    1    -1
## 14    1 -1  -1   1    1  -1   -1    1     1
## 15    1 -1   1  -1    1  -1    1   -1     1
## 16   -1 -1  -1  -1   -1  -1   -1   -1    -1
## 17   -1 -1   1   1   -1   1   -1   -1     1
## 18    1 -1  -1   1    1   1    1   -1    -1
## 19    1 -1   1  -1    1   1   -1    1    -1
## 20   -1 -1  -1  -1   -1   1    1    1     1
## 21    1 -1   1   1   -1  -1    1    1    -1
## 22   -1 -1  -1   1    1  -1   -1    1     1
## 23   -1 -1   1  -1    1  -1    1   -1     1
## 24    1 -1  -1  -1   -1  -1   -1   -1    -1
## 25   -1  1  -1  -1    1  -1    1    1    -1
## 26    1  1   1  -1   -1  -1   -1    1     1
## 27    1  1  -1   1   -1  -1    1   -1     1
## 28   -1  1   1   1    1  -1   -1   -1    -1
## 29    1  1  -1  -1    1   1   -1   -1     1
## 30   -1  1   1  -1   -1   1    1   -1    -1
## 31   -1  1  -1   1   -1   1   -1    1    -1
## 32    1  1   1   1    1   1    1    1     1
```

```
# Estimated Effects w/out Blocks
effect1 <- t(response1)%*%contrast1/(16*1)
names(effect1) <- c("A", "B","AB", "C", "AC","BC","ABC","D","AD",
"BD","ABD","CD", "ACD","BCD","ABCD","E","AE","BE","ABE","CE","ACE","BCE",
"ABCE","DE","ADE","BDE","ABDE","CDE","ACDE","BCDE","ABCDE")
effect1
```

```
##           A       B      AB       C      AC      BC     ABC       D      AD
```

```
## [1,] 11.8125 33.9375 7.9375 9.6875 0.4375 0.0625 -0.4375 -0.8125 -0.0625
##           BD    ABD     CD     ACD    BCD    ABCD      E     AE     BE     ABE
## [1,] -0.6875 0.3125 0.8125 -0.4375 0.4375 -0.0625 0.4375 0.9375 0.5625 -0.1875
##          CE    ACE    BCE   ABCE      DE    ADE    BDE   ABDE     CDE    ACDE
## [1,] 0.3125 0.3125 0.9375 0.1875 -1.1875 0.8125 0.1875 0.9375 -0.8125 -0.3125
##         BCDE   ABCDE
## [1,] -0.9375 -0.1875
## attr(,"names")
##  [1] "A"     "B"     "AB"    "C"     "AC"    "BC"    "ABC"   "D"     "AD"
## [10] "BD"    "ABD"   "CD"    "ACD"   "BCD"   "ABCD"  "E"     "AE"    "BE"
## [19] "ABE"   "CE"    "ACE"   "BCE"   "ABCE"  "DE"    "ADE"   "BDE"   "ABDE"
## [28] "CDE"   "ACDE"  "BCDE"  "ABCDE"
```

```
# Adding in Blocks
block1 <- contrast1[,c("ABCDE")]
da1 <- data.frame(contrast1,block1,response1)
model1 <- aov(response1~factor(A)*factor(B)*factor(C)*factor(D)*factor(E)+factor(block1),data=da1)
summary(model1)
```

```
##                                          Df Sum Sq Mean Sq
## factor(A)                                 1   1116    1116
## factor(B)                                 1   9214    9214
## factor(C)                                 1    751     751
## factor(D)                                 1      5       5
## factor(E)                                 1      2       2
## factor(block1)                            1      0       0
## factor(A):factor(B)                       1    504     504
## factor(A):factor(C)                       1      2       2
## factor(B):factor(C)                       1      0       0
## factor(A):factor(D)                       1      0       0
## factor(B):factor(D)                       1      4       4
## factor(C):factor(D)                       1      5       5
## factor(A):factor(E)                       1      7       7
## factor(B):factor(E)                       1      3       3
## factor(C):factor(E)                       1      1       1
## factor(D):factor(E)                       1     11      11
## factor(A):factor(B):factor(C)             1      2       2
## factor(A):factor(B):factor(D)             1      1       1
## factor(A):factor(C):factor(D)             1      2       2
## factor(B):factor(C):factor(D)             1      2       2
## factor(A):factor(B):factor(E)             1      0       0
## factor(A):factor(C):factor(E)             1      1       1
## factor(B):factor(C):factor(E)             1      7       7
## factor(A):factor(D):factor(E)             1      5       5
## factor(B):factor(D):factor(E)             1      0       0
## factor(C):factor(D):factor(E)             1      5       5
## factor(A):factor(B):factor(C):factor(D)   1      0       0
## factor(A):factor(B):factor(C):factor(E)   1      0       0
## factor(A):factor(B):factor(D):factor(E)   1      7       7
## factor(A):factor(C):factor(D):factor(E)   1      1       1
## factor(B):factor(C):factor(D):factor(E)   1      7       7
```

```
# Estimated effects w/ Blocks
effect1_a <- t(response1)%*%cbind(contrast1)/(16*1)
colnames(effect1_a)[colnames(effect1_a) == "ABCDE"] <- "ABCDE+block1"
```

```
effect1_a # -0.1875 is the estimate of (block effect + effect of interaction ABCDE)
```

```
##            A       B     AB      C     AC     BC     ABC       D      AD
## [1,] 11.8125 33.9375 7.9375 9.6875 0.4375 0.0625 -0.4375 -0.8125 -0.0625
##           BD    ABD     CD     ACD    BCD    ABCD      E     AE     BE     ABE
## [1,] -0.6875 0.3125 0.8125 -0.4375 0.4375 -0.0625 0.4375 0.9375 0.5625 -0.1875
##          CE    ACE    BCE   ABCE      DE    ADE    BDE   ABDE     CDE    ACDE
## [1,] 0.3125 0.3125 0.9375 0.1875 -1.1875 0.8125 0.1875 0.9375 -0.8125 -0.3125
##         BCDE ABCDE+block1
## [1,] -0.9375      -0.1875
```

```
effect1_b <- 2*coef(lm(response1~A*B*C*D*E+block1,data=da1))[-1]
effect1_b
```

```
##         A         B         C         D         E    block1       A:B       A:C
##   11.8125   33.9375    9.6875   -0.8125    0.4375   -0.1875    7.9375    0.4375
##       B:C       A:D       B:D       C:D       A:E       B:E       C:E       D:E
##    0.0625   -0.0625   -0.6875    0.8125    0.9375    0.5625    0.3125   -1.1875
##     A:B:C     A:B:D     A:C:D     B:C:D     A:B:E     A:C:E     B:C:E     A:D:E
##   -0.4375    0.3125   -0.4375    0.4375   -0.1875    0.3125    0.9375    0.8125
##     B:D:E     C:D:E   A:B:C:D   A:B:C:E   A:B:D:E   A:C:D:E   B:C:D:E A:B:C:D:E
##    0.1875   -0.8125   -0.0625    0.1875    0.9375   -0.3125   -0.9375        NA
```

```
# Estimates of effects for treatment factors.
# In other words, remove the highest-order interaction and the blocking factor, as they are confounded.
effect1_c <- effect1_a[setdiff(names(effect1_a),c("block","A:B:C:D:E"))]
effect1_c
```

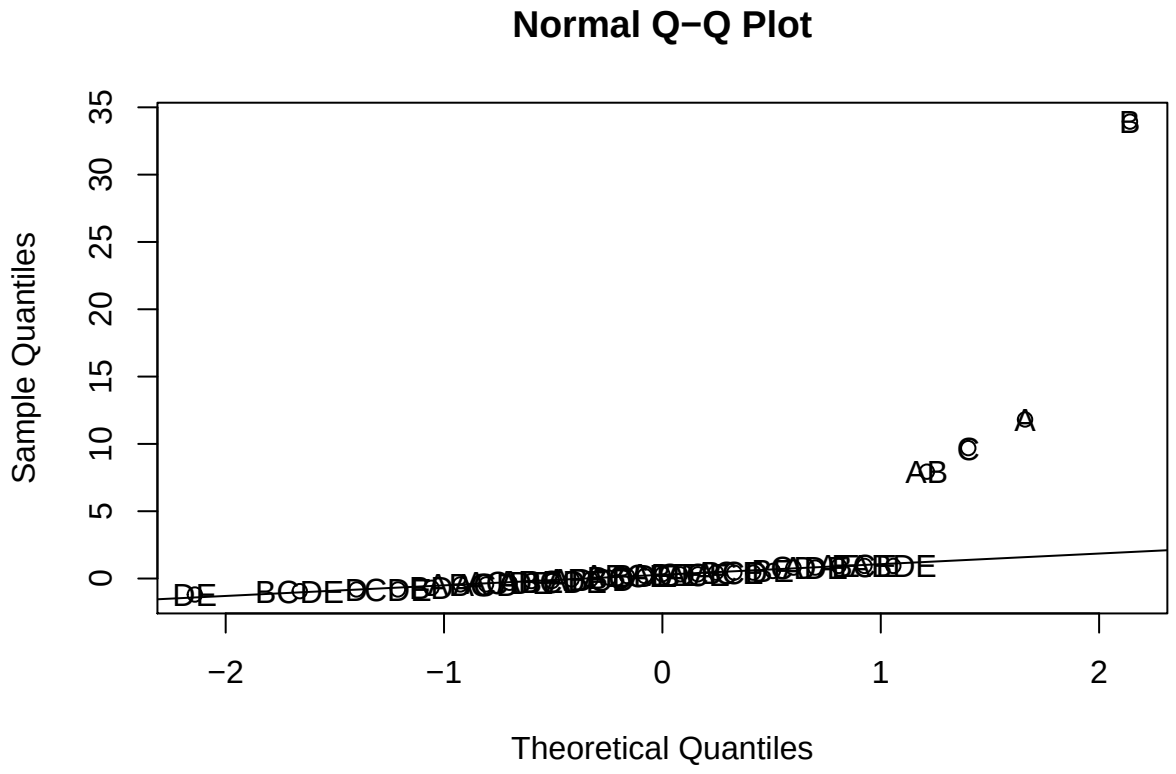
```
## numeric(0)
```

```
# using Lenth's method
t.tests1 <- eff.test(effect1, method = "Lenth")
t.tests1[,-5]
```

```
##        effect Lenth_PSE t.ratio p.value
## B     33.9375   0.65625  51.714  0.0000
## A     11.8125   0.65625  18.000  0.0000
## C      9.6875   0.65625  14.762  0.0000
## AB     7.9375   0.65625  12.095  0.0000
## DE    -1.1875   0.65625  -1.810  0.0808
## BCDE  -0.9375   0.65625  -1.429  0.1571
## ABDE   0.9375   0.65625   1.429  0.1571
## BCE    0.9375   0.65625   1.429  0.1571
## AE     0.9375   0.65625   1.429  0.1571
## CDE   -0.8125   0.65625  -1.238  0.2124
## ADE    0.8125   0.65625   1.238  0.2124
## CD     0.8125   0.65625   1.238  0.2124
## D     -0.8125   0.65625  -1.238  0.2124
## BD    -0.6875   0.65625  -1.048  0.2864
## BE     0.5625   0.65625   0.857  0.3792
## E      0.4375   0.65625   0.667  0.5212
## BCD    0.4375   0.65625   0.667  0.5212
## ACD   -0.4375   0.65625  -0.667  0.5212
## ABC   -0.4375   0.65625  -0.667  0.5212
## AC     0.4375   0.65625   0.667  0.5212
## ACDE  -0.3125   0.65625  -0.476  0.6492
```

```
## ACE    0.3125   0.65625   0.476  0.6492
## CE     0.3125   0.65625   0.476  0.6492
## ABD    0.3125   0.65625   0.476  0.6492
## ABCDE -0.1875   0.65625  -0.286  0.7865
## BDE    0.1875   0.65625   0.286  0.7865
## ABCE   0.1875   0.65625   0.286  0.7865
## ABE   -0.1875   0.65625  -0.286  0.7865
## ABCD  -0.0625   0.65625  -0.095  0.9259
## AD    -0.0625   0.65625  -0.095  0.9259
## BC     0.0625   0.65625   0.095  0.9259
```

```
# Using normal probability plot
qqnorm(effect1)
text(qqnorm(effect1)$x,qqnorm(effect1)$y, names(effect1))
qqline(effect1)
```



ANOVA does not print the ABCDE interaction, since it is confounded with blocking. Also, the estimated block effect includes ABCDE interaction effect. (-0.1875) is the estimate of (block effect + effect of interaction ABCDE). We see that "A, B, C, AB" are significant factors from our T-test using Lenth's Method. We create our Normal Q-Q Plot and see that "A, B, C, AB" are significant.

Problem 2: (Exercise 7.24)

```
mx1_b <- matrix(names(effect1_b), nrow=8, ncol=4, byrow=FALSE)
mx1_b
```

```
##      [,1]     [,2]  [,3]    [,4]
## [1,] "A"      "B:C" "A:B:C" "B:D:E"
## [2,] "B"      "A:D" "A:B:D" "C:D:E"
## [3,] "C"      "B:D" "A:C:D" "A:B:C:D"
## [4,] "D"      "C:D" "B:C:D" "A:B:C:E"
## [5,] "E"      "A:E" "A:B:E" "A:B:D:E"
## [6,] "block1" "B:E" "A:C:E" "A:C:D:E"
## [7,] "A:B"    "C:E" "B:C:E" "B:C:D:E"
## [8,] "A:C"    "D:E" "A:D:E" "A:B:C:D:E"
```

The answer is (d) "BE" is in the same block as "ACDE". We can see that from the 6th row that "B:E" is along the same row as "A:C:D:E".

Problem 3:

```
# Choice 1: B_1 = {1,2,3}, B_2 = {4,5,6,7}, B_3 = {3,4,5}
# Choice 2: B_1 = {1,2,3,4}, B_2 = {3,4}, B_3 = {5,6,7}
# Choice 3: B_1 = {1,2,5,6}, B_2 = {1,2,3,4}, B_3 = {1,3,5,7}

# C1 --> (B_1)(B_2)(B_3) --> (1,2,3)(4,5,6,7)(3,4,5) = (1,2,6,7)
# C2 --> (B_1)(B_2)(B_3) --> (1,2,3,4)(3,4)(5,6,7) = (1,2,5,6,7)
# C3 --> (B_1)(B_2)(B_3) --> (1,2,5,6)(1,2,3,4)(1,3,5,7) = (4,6,7)

# 3 Block Effects!
# In Conclusion,...

# C3 < C1 < C2
```

Since we are using 3 Block Effects, we can see that Choice 3 (C3) is the worst one of the choices. After that, we see that Choice 1 (C1) is the next worst with a confounding effect of (1,2,6,7). Lastly, we see that Choice 2 (C2) has the best confounding effect of (1,2,5,6,7) out of all the choices given.