① "Problems a-c are done in RStudio code"

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②
ⓘ The investor will move up from Point A until the tangent, or move to the left of Point A until the tangent (Point G). These represent the Efficient Frontier.

ⓘⓘ Portfolio Z cannot be on the Efficient Frontier because the point lies below the Efficient Frontier. It has a higher standard deviation than Portfolio X with a lower expected return.

ⓘⓘⓘ $E(R) = 15\% = 0.15$, $\sigma = 60\% = 0.60$, $\rho = 0.5$, $n = 25$

$$\bar{R}_p = \sum_{i=1}^{n} x_i \bar{R}_i = \frac{1}{n}\sum_{i=1}^{n} \bar{R}_i \Rightarrow \frac{1}{25}\sum_{i=1}^{25}\bar{R}_i = \frac{\cancel{(25)}(0.15)}{\cancel{(25)}} \rightarrow \boxed{\bar{R}_p = 0.15} \rightarrow \sigma_p^2 = \sum_{i=1}^{n} x_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq n}^{n} x_i x_j \sigma_{ij} \Longrightarrow$$

$$\frac{1}{(25)^{\cancel{2}}}\cdot\cancel{(25)}\cdot(0.60)^2 + \frac{1}{(25)^{\cancel{2}}}\cdot\cancel{(25)}(25-1)(0.5)(0.60)^2 = (0.0144) + (0.1728) \rightarrow$$

$$\boxed{\sigma_p^2 = 0.1872} \rightarrow \boxed{\sigma_p = \sqrt{0.1872} = 0.4327}$$

ⓘⓥ We have the formula... $\rightarrow \sigma_p^2 = \frac{1}{n}(\sigma^2) + \frac{1}{n}(n-1)(\rho)(\sigma^2) \rightarrow$ we need $\sigma_p^2 < (0.43)^2$ for an unknown (n) $\rightarrow$

$$\frac{1}{n}(0.36) + \frac{n-1}{n}(0.18) < (0.43)^2 \rightarrow (0.36) + (n-1)(0.18) < 0.1849(n) \rightarrow$$

$$n = 36.7347 \rightarrow \boxed{n = 37}$$

ⓥ $\sigma_p = \sigma\sqrt{\rho}$ (?) $\rightarrow \sigma_p = (0.6)\sqrt{0.5} = 0.4243 \rightarrow$ if $n = 25 \rightarrow \sigma_p = 0.4327$; $n = 37 \rightarrow \sigma_p = 0.4300$; $n = 100 \rightarrow \sigma_p = 0.4264$

$\rightarrow$ Yes, it is <u>true</u> that if (n) increases, then $\sigma_p = \sigma\sqrt{\rho}$. The reason is because the more you increase your sample size ("diversify"), the closer it'll be till it reaches the true minimum risk.

③ "Exercise 3 done in RStudio"

④ Equal Weight Portfolio $\rightarrow \sigma_p^2 = \frac{1}{n}(\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$ for $\bar{\sigma}_i^2 = 50$ & $\bar{\sigma}_{ij} = 10 \rightarrow$

$$* \; (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) = ((50)^{\cancel{2}} - (10)^{\cancel{2}}) = 40\cancel{00}$$

n	$\sigma_p^2 = \frac{1}{n}(\bar{\sigma}_i^2 - \bar{\sigma}_{ij}^2) + \bar{\sigma}_{ij}$
5	18.0
10	14.0
20	12.0
50	10.8
100	10.4

$\leftarrow$ "Answers"

(5) (i) $X_k = \dfrac{E\sum_{j=1}^{m} V_{kj}(CE_j - A) + \sum_{j=1}^{m} V_{kj}(B - AE_j)}{D}$ for $k=1,\dots,m$ $\longrightarrow$ we need equations... $\longrightarrow$

(a) $0 = \sum_1^m x_j \sigma_{ij} - \lambda_1 E_i - \lambda_2$ for $i=1,\dots,m$

(b) $0 = E - \sum_1^m x_i E_i$

(c) $0 = 1 - \sum_{i=1}^m x_i$

$\lambda_1 = \dfrac{(CE-A)}{D}$ $\qquad \lambda_2 = \dfrac{(B-AE)}{D}$

$A = \sum_1^m \sum_1^m V_{kj} E_j$

$B = \sum_1^m \sum_1^m V_{kj} E_j E_k$

$C = \sum_1^m \sum_1^m V_{kj}$

$D = BC - A^2$

$E = B\lambda_1 + A\lambda_2$

$1 = A\lambda_1 + C\lambda_2$

Now to solve... $\longrightarrow$

$X_k = \lambda_1 \sum_1^M V_{kj} E_j + \lambda_2 \sum_1^m V_{kj}$ for $k=1,\dots,m$

$\longrightarrow$ we would simply plug the the (λ_1) & (λ_2) to our (X_k) $\longrightarrow = \left[\dfrac{CE-A}{D}\right] \cdot \sum_1^m V_{kj} E_j + \left[\dfrac{B-AE}{D}\right] \cdot \sum_1^m V_{kj} \longrightarrow$

$$\boxed{X_k = \dfrac{E \cdot \sum_1^m V_{kj}(CE_j - A) + \sum_1^m V_{kj}(B - AE_j)}{D} \text{ for } k=1,\dots,m}$$

✓ frontier portfolio

(ii) say $\overline{E} = \dfrac{A}{C} \longrightarrow$ multiply eq. (a) by (x_i) $\longrightarrow \sum_1^m \sum_1^m x_i x_j \sigma_{ij} = \lambda_1 \sum_1^m x_i E_i + \lambda_2 \sum_1^m x_i$

$\longrightarrow \sigma^2 = \lambda_1 E + \lambda_2$ (from definition of σ^2, (b), & (c)) $\longrightarrow \sigma^2 = \lambda_1 E + \lambda_2 \longrightarrow \sigma^2 = \dfrac{CE^2 - 2AE + B}{D} \longrightarrow$

$\dfrac{\partial \sigma^2}{\partial E} = \dfrac{2[CE-A]}{D} = 0 \longrightarrow \overline{E} = \dfrac{A}{C} \longrightarrow \sigma^2 = \dfrac{1}{C} \longrightarrow$ we define $(\overline{X}_k)$ as the proportion of min.-variance portfolio... $\longrightarrow$

$\overline{X}_k = \dfrac{\sum_1^m V_{kj}}{\sum_1^m \sum_1^m V_{kj}}$ for $k=1,\dots,m \longrightarrow$ we know that $C = \sum_1^M \sum_1^m V_{kj} \longrightarrow$ $\boxed{\overline{X}_k = \dfrac{\sum_1^m V_{kj}}{C} \text{ for } k=1,\dots,m}$

✓ min. risk portfolio

Stats C183 HW 1

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Loading Necessary Packages/Data:

```
library(readr)
a <- read.csv("C:/Users/cliuk/Documents/UCLA Works/UCLA Spring 2020/Stats C183/Project/stockData.csv", s
```

Create Returns and Matrices:

```
# Convert adjusted close prices into returns:
r <- (a[-1,3:ncol(a)]-a[-nrow(a),3:ncol(a)])/a[-nrow(a),3:ncol(a)]

# Compute mean vector:
means <- colMeans(r[-1]) # Without ^GSPC

# Compute variance covariance matrix:
covmat <- cov(r[-1]) # Without ^GSPC

# Compute correlation matrix:
cormat <- cor(r[-1]) # Without ^GSPC

# Compute the vector of variances:
variances <- diag(covmat)

# Compute the vector of standard deviations:
stdev <- diag(covmat)^.5
```

Exercise 1:

1a)

```
# Set up column of Ones, A - E formulas, & both Lagranges:
ones <- rep(1, 30)
A <- t(ones) %*% solve(covmat) %*% means
B <- t(means) %*% solve(covmat) %*% means
C <- t(ones) %*% solve(covmat) %*% ones
D <- B * C - A^2
E <- seq(-5,5,.1)
Lagrange_1 <- ((C * E) - A)/D
Lagrange_2 <- (B - (A * E))/D
```

```
# Values for our formulas:
Exc_1a <- matrix(c(A, B, C, D), byrow = TRUE)
rownames(Exc_1a) <- c("A:", "B:", "C:", "D:")
Exc_1a
```

```
##           [,1]
## A:   20.627969
## B:    1.155529
## C: 2841.013797
## D: 2857.361175
```

```
Lagrange_1
```

```
##   [1] -4.978613512 -4.879185627 -4.779757741 -4.680329856 -4.580901970
##   [6] -4.481474085 -4.382046199 -4.282618314 -4.183190428 -4.083762543
##  [11] -3.984334657 -3.884906772 -3.785478886 -3.686051001 -3.586623115
##  [16] -3.487195230 -3.387767344 -3.288339459 -3.188911573 -3.089483688
##  [21] -2.990055802 -2.890627917 -2.791200031 -2.691772146 -2.592344260
##  [26] -2.492916375 -2.393488489 -2.294060604 -2.194632718 -2.095204833
##  [31] -1.995776947 -1.896349062 -1.796921176 -1.697493291 -1.598065405
##  [36] -1.498637520 -1.399209634 -1.299781749 -1.200353863 -1.100925978
##  [41] -1.001498092 -0.902070207 -0.802642321 -0.703214436 -0.603786550
##  [46] -0.504358665 -0.404930779 -0.305502894 -0.206075008 -0.106647123
##  [51] -0.007219237  0.092208648  0.191636534  0.291064419  0.390492305
##  [56]  0.489920190  0.589348076  0.688775961  0.788203847  0.887631732
##  [61]  0.987059618  1.086487503  1.185915389  1.285343274  1.384771159
##  [66]  1.484199045  1.583626930  1.683054816  1.782482701  1.881910587
##  [71]  1.981338472  2.080766358  2.180194243  2.279622129  2.379050014
##  [76]  2.478477900  2.577905785  2.677333671  2.776761556  2.876189442
##  [81]  2.975617327  3.075045213  3.174473098  3.273900984  3.373328869
##  [86]  3.472756755  3.572184640  3.671612526  3.771040411  3.870468297
##  [91]  3.969896182  4.069324068  4.168751953  4.268179839  4.367607724
##  [96]  4.467035610  4.566463495  4.665891381  4.765319266  4.864747152
## [101]  4.964175037
```

```
Lagrange_2
```

```
##   [1]  0.0365005914  0.0357786677  0.0350567439  0.0343348202  0.0336128965
##   [6]  0.0328909727  0.0321690490  0.0314471252  0.0307252015  0.0300032777
##  [11]  0.0292813540  0.0285594303  0.0278375065  0.0271155828  0.0263936590
##  [16]  0.0256717353  0.0249498116  0.0242278878  0.0235059641  0.0227840403
##  [21]  0.0220621166  0.0213401928  0.0206182691  0.0198963454  0.0191744216
##  [26]  0.0184524979  0.0177305741  0.0170086504  0.0162867266  0.0155648029
##  [31]  0.0148428792  0.0141209554  0.0133990317  0.0126771079  0.0119551842
##  [36]  0.0112332604  0.0105113367  0.0097894130  0.0090674892  0.0083455655
##  [41]  0.0076236417  0.0069017180  0.0061797942  0.0054578705  0.0047359468
##  [46]  0.0040140230  0.0032920993  0.0025701755  0.0018482518  0.0011263280
##  [51]  0.0004044043 -0.0003175194 -0.0010394432 -0.0017613669 -0.0024832907
##  [56] -0.0032052144 -0.0039271382 -0.0046490619 -0.0053709856 -0.0060929094
##  [61] -0.0068148331 -0.0075367569 -0.0082586806 -0.0089806044 -0.0097025281
##  [66] -0.0104244518 -0.0111463756 -0.0118682993 -0.0125902231 -0.0133121468
##  [71] -0.0140340705 -0.0147559943 -0.0154779180 -0.0161998418 -0.0169217655
##  [76] -0.0176436893 -0.0183656130 -0.0190875367 -0.0198094605 -0.0205313842
##  [81] -0.0212533080 -0.0219752317 -0.0226971555 -0.0234190792 -0.0241410029
##  [86] -0.0248629267 -0.0255848504 -0.0263067742 -0.0270286979 -0.0277506217
```

```
##  [91] -0.0284725454 -0.0291944691 -0.0299163929 -0.0306383166 -0.0313602404
##  [96] -0.0320821641 -0.0328040879 -0.0335260116 -0.0342479353 -0.0349698591
## [101] -0.0356917828
```

1b)

```r
E <- seq(-5,5,.1) # Choosing arbitrary E values.

# Efficient Frontier Formulas:
minvar <- 1/C
minE <- A/C

# Efficient Frontier Values:
minvar
```

```
##             [,1]
## [1,] 0.000351987
```

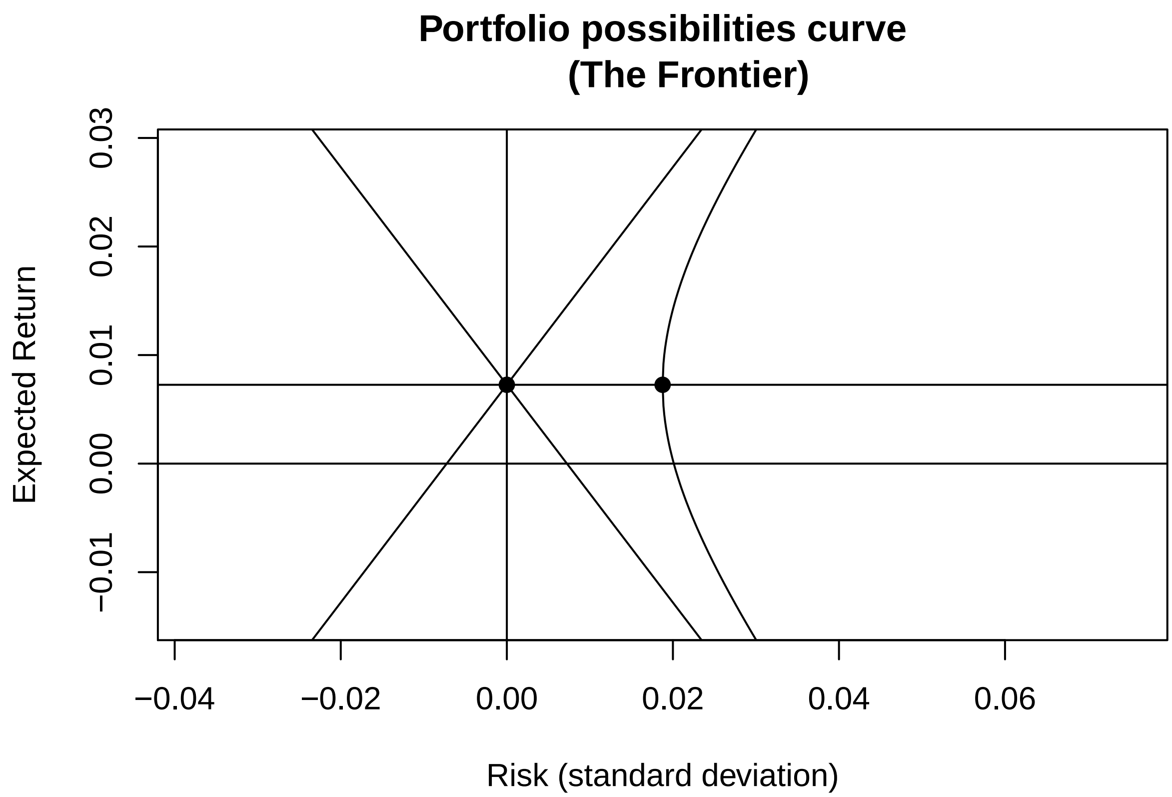
```r
minE
```

```
##             [,1]
## [1,] 0.007260777
```

1c)

```r
plot(0, A/C, main = "Portfolio possibilities curve
     (The Frontier)", xlab = "Risk (standard deviation)",
     ylab = "Expected Return", type = "n",
     xlim = c(-2*sqrt(1/C), 4*sqrt(1/C)),
     ylim = c(-2*A/C, 4*A/C))
points(0, A/C, pch = 19) # Plot center of the hyperbola
abline(v = 0) # Plot transverse and conjugate axes. Also this is the y-axis.
abline(h = A/C)
abline(h = 0) # Plot the x-axis
points(sqrt(1/C), A/C, pch=19) # Plot the minimum risk portfolio
V <- seq(-1, 1, 0.001) # Find the asymptotes
A1 <- A/C + V * sqrt(D/C)
A2 <- A/C - V * sqrt(D/C)
points(V, A1, type = "l")
points(V, A2, type = "l")


sdeff <- seq((minvar)^0.5, 1, by = 0.0001)
options(warn = -1)
y1 <- (A + sqrt(D*(C*sdeff^2 - 1)))*(1/C)
y2 <- (A - sqrt(D*(C*sdeff^2 - 1)))*(1/C)
options(warn = 0)
points(sdeff, y1, type = "l")
points(sdeff, y2, type = "l")
```



Exercise 3:

```
R_bar <- matrix(c(0.005174, 0.010617, 0.016947))
var_cov <- diag(c(0.010025, 0.002123, 0.005775), 3, 3)
ones <- rep(1, 3)

R_bar_min <- (t(ones) %*% solve(var_cov) %*% R_bar)/(t(ones) %*% solve(var_cov) %*% ones)
sigma_min <- (1)/((t(ones) %*% solve(var_cov) %*% ones)^1/2)

X_numer <- solve(var_cov) %*% ones
X_denom <- t(ones) %*% solve(var_cov) %*% ones
# X_denom = 743.9424
X_vec <- X_numer/743.9424
rownames(X_vec) <- c("C", "XOM", "AAPL")

# Composition of the Minimum Risk Portfolio:
X_vec
```

```
##           [,1]
## C    0.1340838
## XOM  0.6331560
## AAPL 0.2327602
```

```
R_bar_min
```

```
##            [,1]
```

```
## [1,] 0.01136055
```

```
sigma_min
```

```
##             [,1]
## [1,] 0.00268838
```