## Homework 1

Exercise 1
Answer the following questions.

a. Use 5 stocks from the project data. Compute the quantities A, B, C, D as given in the paper "An Analytic Derivation of the Efficient Frontier", by Robert C. Merton, *The Journal of Financial and Quantitative Analysis,* Vol. 7, No. 4. Also compute the values of λ_1 and λ_2 (the two Lagrange multipliers).

b. Use the same data as in part (a). Suppose an investor has a prescribed expected return E. Find the composition of the efficient portfolio given the return E. Note: You need to choose a value of E.

c. Use the same data as in part (a). Plot the frontier in the mean-variance space. Note: See equation (12) in the paper.

Exercise 2
Please answer the following questions.

1. An investor wants to hold portfolio `A` as shown on the plot below. Suppose that short sales are allowed and $R_f = 0.001$. Suggest a better investing strategy. Note: Point `G` on the plot is the point of tangency.
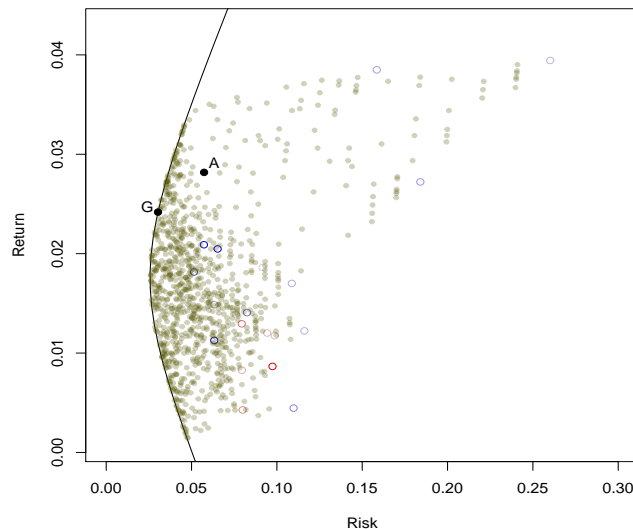
2. Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz? Please explain your answer.

Portfolio	Expected return (%)	Standard deviation (%)
W	15	36
X	12	15
Y	5	7
Z	9	21

3. Suppose all stocks have $E(R) = 15\%$, $\sigma = 60\%$, and common correlation coefficient $\rho = 0.5$. What are the expected return and standard deviation of an equally weighted portfolio of $n = 25$ stocks?

4. Refer to question (3). What is the smallest number of stocks necessary to generate a portfolio with standard deviation of at most 43%?

5. Refer to question (3). As n gets larger, is it true that $\sigma_p = \sigma\sqrt{\rho}$? Please explain your answer.

Exercise 3

The mean returns and variance covariance matrix of the returns of three stocks (`C, XOM, AAPL` and the market `SP500`) are given below:

```
Mean returns:
         C         XOM        AAPL       ^GSPC
   0.005174    0.010617    0.016947    0.010846


Variance-covariance matrix:
          C       XOM      AAPL    ^GSPC
C      0.010025 0.000000 0.000000 0.000000
XOM    0.000000 0.002123 0.000000 0.000000
AAPL   0.000000 0.000000 0.005775 0.000000
^GSPC  0.000000 0.000000 0.000000 0.001217
```

Assume short sales are allowed. Compute the composition of the minimum risk portfolio using only the three stocks (do not use the `SP500`).

Exercise 4

Assume that the average variance of the return for an individual security is 50 and that the average covariance is 10. What is the variance of an equally weighted portfolio of 5, 10, 20, 50, and 100 securities?

Exercise 5

In the paper "An Analytic Derivation of the Efficient Frontier," *The Journal of Financial and Quantitative Analysis*, Vol. 7, No. 4, Robert Merton gives on page 1854 the proportion of the k_{th} risky asset held in the frontier portfolio with expected return E by

$$x_k = \frac{E\sum_{j=1}^m v_{kj}(CE_j - A) + \sum_{j=1}^m v_{kj}(B - AE_j)}{D}, \quad k = 1, \dots, m. \tag{1}$$

Prove equation (1).

On the same page, it is shown that the expected return of the minimum risk portfolio is $\bar{E} = \frac{A}{C}$. Using equation (1) above show that the proportion of the k_{th} risky asset of the minimum risk portfolio is $x_k = \frac{\sum_{j=1}^m v_{kj}}{C}$, $k = 1, \dots, m$.