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Stats C183
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HW 3

① (2) $MSE = (\bar{A}-\bar{P})^2 + (1-P_1)^2 S_P^2 + (1-r_{AP}^2) S_A^2 = \frac{1}{m}\sum (A_j - P_j)^2$

$\pm \bar{A} \pm \bar{P}$

$\rightarrow \frac{1}{m}\sum \left[A_j - \bar{A} - (P_j - \bar{P}) + \bar{A} - \bar{P}\right]^2 \rightarrow \frac{1}{m}\sum (A_j-\bar{A})^2 + \frac{1}{m}\sum (P_j-\bar{P})^2 + (\bar{A}-\bar{P})^2$

$+ \cancel{\frac{2}{m}(\bar{A}-\bar{P})\sum (A_j-\bar{A})} - \cancel{\frac{2}{m}(\bar{A}-\bar{P})\sum (P_j-\bar{P})}$ ($\sum \sim 2R$, $\sum = 0$)

$- \frac{2}{m}\sum (A_j-\bar{A})(P_j-\bar{P})$

$\rightarrow \underbrace{\frac{1}{m}\sum (A_j-\bar{A})^2}_{=S_A^2} + \underbrace{\frac{1}{m}\sum (P_j-\bar{P})^2}_{=S_P^2} + (\bar{A}-\bar{P})^2 - \underbrace{\frac{2}{m}\sum (A_j-\bar{A})(P_j-\bar{P})}_{=(S_P^2)(\hat{B}_1)}$ by definition...

$\circledast R^2 = \frac{(\hat{\beta}_1)\sum (P_j-\bar{P})^2}{\sum (A_j-\bar{A})^2} = \frac{(\hat{B}_1)(S_P^2)}{S_A^2}$ $\overset{\text{sub}}{=}$ $\circledast \hat{B}_1 = \frac{\sum (A_j-\bar{A})(P_j-\bar{P})}{\sum (P_j-\bar{P})^2} \Rightarrow (S_P^2)(\hat{B}_1) = \sum (A_j-\bar{A})(P_j-\bar{P})$

back to problem $\rightarrow MSE = S_A^2 + S_P^2 + (\bar{A}-\bar{P})^2 - 2(S_P^2)(\hat{B}_1) \xrightarrow{\pm S_P^2 \hat{B}_1} S_A^2 + S_P^2(1 - 2\hat{B}_1 + \hat{B}_1^2) + (\bar{A}-\bar{P})^2 -$

$(\bar{A}-\bar{P})^2 + S_P^2(1-\hat{B}_1)^2 + S_A^2 - S_A^2 \underbrace{R^2}_{=r_{AP}} \xrightarrow{\text{rearrange}} \boxed{MSE = (\bar{A}-\bar{P})^2 + (1-B_1)^2 S_P^2 + (1-r_{AP}^2) S_A^2}$ $\hat{B}_1^2 S_P^2$ ✓

RStudio for components of PRESS

(2a) $\bar{R}_m = 0.10$ $\sigma_m^2 = 0.20$
$B_P = 0.90$ $\sigma_{\varepsilon i}^2 = 0.05$
$\Big\} \sigma_j^2 = B_i^2 \sigma_m^2 + \sum X_i^2 \sigma_{\varepsilon i}^2 \rightarrow \boxed{\sigma_P^2 = (0.90)^2 \cdot (0.20) + \frac{0.05}{20} = 0.1645}$

(2b) RStudio for 2b

(2c) (SIM)

stock	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}_\varepsilon^2$
A	0.0082	0.7900	0.0270
B	0.0099	1.1200	0.0060

$\frac{1}{t}\sum_{i=1}^{60}(R_{mt}-\bar{R}_m)^2 = 0.13$

$\sigma_m^2 = 0.0022$

$\sum(R_t-R_i)^2 = \sum e_{it}^2 + \sum(\hat{R}_t-\bar{R})^2$

$SST = SSE + SSR$ →

* $Cov(R_A,R_B) = \beta_A \cdot \beta_B \cdot Var(R_M) \rightarrow Cov(R_A,R_B) = (0.79)(1.12)(0.0022) = 0.00195$ →

regress R_A on R_M → * $Var(R_i) = \beta_i^2 \cdot Var(R_m) + Var(e_i) \rightarrow \sigma_{R_A}^2 = (0.79)^2(0.0022) + (0.027) = 0.02837$

$$R^2 = \rho^2 = \frac{\beta_i^2 \sigma_m^2}{Var(R_i)} = \frac{(0.79)^2(0.0022)}{(0.02837)} = 0.048397 = R^2$$

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(3) $\sum_1^{30} P_i = 32.44349$ (sum of β's in P1)

$\sum_1^{30} A_i = 32.26206$ (sum of β's in P2)

$\sum_1^{30} P_i^2 = 51.70104$ (sum of β^2's in P1)

$\sum_1^{30} A_i^2 = 48.43207$ (sum of β^2's in P2)

$\sum(A_i-\bar{A})(A_j-\bar{A}) = 4.299189$ (fitted of S.R. of A on P) (SSR)

$PRESS = (\bar{A}-\bar{P})^2 + (1-\beta_1)^2 S_p^2 + (1-R^2) S_A^2$

$\bar{A} = \frac{32.44349}{30} = 1.08145$ $\quad \frac{1}{t}\bar{P} = \frac{32.26206}{30} = 1.07540$

$S_A^2 = \frac{1}{30}\left[48.43207 - \frac{(32.26206)^2}{30}\right] = 0.45791$ →

$S_p^2 = \frac{1}{30}\left[51.70104 - \frac{(32.44349)^2}{30}\right] = 0.55383$

$SSR = \hat{\beta}_1^2 \sum(P_j-\bar{P})^2 \rightarrow \hat{\beta}_1^2 = \frac{4.299189}{51.70104 - \frac{(32.44349)^2}{30}} = 0.25875 \rightarrow \hat{\beta}_1 = \sqrt{0.25875} = 0.50867$

$\rightarrow R^2 = \frac{SSR}{SST} = \frac{4.299189}{48.43207 - \frac{(32.26206)^2}{30}} = 0.31295$

$\rightarrow PRESS = (0.000036603) + (0.13370) + (0.31460) = 0.4483 = PRESS$

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(4) $E(X)=3$ $\quad E(X^2)=10$

$E(Y)=2$ $\quad E(Y^2)=29$

$E(XY)=0$

$Var(X) = E(X^2) - E(X)^2 = (10) - (3)^2 = 1 \rightarrow \sigma_X = 1$

$Var(Y) = E(Y^2) - E(Y)^2 = (29) - (2)^2 = 25 \rightarrow \sigma_Y = 5$

$Cov(X,Y) = E(XY) - E(X)\cdot E(Y) = (0) - (3)(2) = -6$ →

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{-6}{(1)\cdot(5)} = -1.2$$

However → $-1 \le \rho \le 1$ → Not true ✓

Stats_C183_HW_3_Charles_Liu

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Loading Necessary Packages:

```
library(readr)
```

1)

```
# Loading the data for (2012 - 2017) & (2017 - 2020):
a1 <- read.csv("C:/Users/cliuk/Documents/UCLA Works/UCLA Spring 2020/Stats C183/Project/stockData_2012-1
a2 <- read.csv("C:/Users/cliuk/Documents/UCLA Works/UCLA Spring 2020/Stats C183/Project/stockData_2017-1

# Convert adjusted close prices into returns:
r1 <- (a1[-1,3:ncol(a1)]-a1[-nrow(a1),3:ncol(a1)])/a1[-nrow(a1),3:ncol(a1)]
r2 <- (a2[-1,3:ncol(a2)]-a2[-nrow(a2),3:ncol(a2)])/a2[-nrow(a2),3:ncol(a2)]

# Compute the variance covariance matrix of the returns for each period:
covmat1 <- var(r1)
covmat2 <- var(r2)

# Compute the betas in each period:
beta1 <- covmat1[1,-1] / covmat1[1,1]
beta2 <- covmat2[1,-1] / covmat2[1,1]

# Vasicek's Technique for PRESS:
beta <- rep(0,30)
alpha <- rep(0,30)
sigma_e2 <- rep(0,30)
var_beta <- rep(0,30)

for(i in 1:30){
q2 <- lm(data = r1, formula = r1[,i+1] ~ r1[,1])
beta[i] <- q2$coefficients[2]
alpha[i] <- q2$coefficients[1]
sigma_e2[i] <- summary(q2)$sigma^2
var_beta[i] <- vcov(q2)[2,2]
}

beta2adj_vasicek <- var_beta*mean(beta)/(var(beta) + var_beta) + var(beta)*beta/(var(beta) + var_beta)
names(beta2adj_vasicek) <- names(r1[-1])

PRESS_Vasicek <- sum((beta2adj_vasicek - beta2)^2) / 30
```

```r
# PRESS Bias:
q <- lm(beta2 ~ beta2adj_vasicek)
SP2 <- sum((beta2adj_vasicek - mean(beta2adj_vasicek))^2) / 30
SA2 <- sum((beta2 - mean(beta2))^2) / 30
PRESS_Bias <- (mean(beta2) - mean(beta2adj_vasicek))^2

# PRESS Inefficiency:
PRESS_Inefficiency <- (1 - q$coefficients[2])^2 * (29/30) * var(beta2adj_vasicek)
attributes(PRESS_Inefficiency) <- NULL


# Components of PRESS from Project 4:
PRESS_Vasicek
```

```
## [1] 0.09436878
```

```r
PRESS_Bias
```

```
## [1] 0.000249397
```

```r
PRESS_Inefficiency
```

```
## [1] 0.0003388024
```

2b)

```r
# r = [(B_hat1) * sd(B1)]/[sd(B2)]
# B_hat0 = B_bar2 - B_bar1 * B_hat1
# B_hatC = B_hat0 + B_hat1 * B_i1
Beta_hat1 <- ((0.2744995)*(0.5225564))/(1.154281)
Beta_hat0 <- (1.042003) - (1.068617 * Beta_hat1)
Beta_hatC <- Beta_hat0 + Beta_hat1 * (2.5513904)

# Beta Adjusted for Stock C (Blume's Technique):
Beta_hatC
```

```
## [1] 1.226266
```