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Stats C183
HW2

① $$\underset{\sim}{X} = \frac{E - R_F}{(\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1})' \underset{\sim}{\Sigma}^{-1} (\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1})} \underset{\sim}{\Sigma}^{-1} (\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1})$$

$$\underset{\sim}{Z} = \underset{\sim}{\Sigma}^{-1}(\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}) \rightarrow \underset{\sim}{X} = \frac{\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]}{\underset{\sim}{1}'\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]} \rightarrow \frac{\underset{\sim}{\Sigma}^{-1}\bar{\underset{\sim}{R}} - R_F \underset{\sim}{\Sigma}^{-1}\underset{\sim}{1}}{\underset{\sim}{1}'\underset{\sim}{\Sigma}^{-1}\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{1}} \rightarrow$$

$$\left.\begin{array}{l} A = \underset{\sim}{1}'\underset{\sim}{\Sigma}^{-1}\bar{\underset{\sim}{R}} \\ C = \underset{\sim}{1}'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{1} \end{array}\right\} \underset{\sim}{Z} = \underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}] = \underset{\sim}{\Sigma}^{-1}\bar{\underset{\sim}{R}} - R_F \underset{\sim}{\Sigma}^{-1}\underset{\sim}{1} \rightarrow$$

$$\underset{\sim}{X} = \frac{\underset{\sim}{Z}}{\underset{\sim}{1}'\underset{\sim}{Z}} \rightarrow \begin{array}{c} \min \frac{1}{2}\underset{\sim}{X}'\underset{\sim}{\Sigma}\underset{\sim}{X} \\ \text{s.t. } E = R_F + (\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1})'\underset{\sim}{X} \end{array} \rightarrow \begin{array}{c} \min Q = \frac{1}{2}X'\Sigma X - (\lambda)[E] \\ \text{for } \underset{\sim}{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \end{array} \rightarrow$$

$$\left. x_{n+1} = \underset{\sim}{1} - \underset{\sim}{1}'\underset{\sim}{X} \right\} R_F \rightarrow \text{combining all written above to get...} \rightarrow$$

Expected Return of...

Portfolio G		Req. of E
$\bar{R}_G = \underset{\sim}{X}'\bar{\underset{\sim}{R}}$	= (equal)	$E = \underset{\sim}{X}'\bar{\underset{\sim}{R}}$

$\rightarrow$ as for the weights... $\rightarrow$ $$\boxed{\underset{\sim}{X} = \frac{\underset{\sim}{Z}}{\underset{\sim}{1}'\underset{\sim}{Z}} = \frac{\underset{\sim}{Z}}{\lambda} \longleftrightarrow \underbrace{\underset{\sim}{X} = \frac{\underset{\sim}{Z}}{\lambda}}_{\text{weights from Handout \#15}}}$$

② $$\underset{\sim}{X}_1 = \frac{[E_1 - R_F]\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]}{[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]'\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]} \quad \& \quad \underset{\sim}{X}_2 = \frac{[E_2 - R_F]\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]}{[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]'\underset{\sim}{\Sigma}^{-1}[\bar{\underset{\sim}{R}} - R_F \underset{\sim}{1}]} \rightarrow$$

$$\left.\begin{array}{l} R_{P_1} = \underset{\sim}{X}_1'\bar{\underset{\sim}{R}} \\ R_{P_2} = \underset{\sim}{X}_2'\bar{\underset{\sim}{R}} \end{array}\right\} Cov(\underset{\sim}{X}_1'\underset{\sim}{R}, \underset{\sim}{X}_2'\underset{\sim}{R}) = \underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_2 \rightarrow \left.\begin{array}{l} Var(\underset{\sim}{X}_1'\underset{\sim}{R}) = \underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_1 \\ Var(\underset{\sim}{X}_2'\underset{\sim}{R}) = \underset{\sim}{X}_2'\underset{\sim}{\Sigma}\underset{\sim}{X}_2 \end{array}\right\}$$

$$\rho_{P_1, P_2} = \frac{Cov(\underset{\sim}{X}_1'\underset{\sim}{R}, \underset{\sim}{X}_2'\underset{\sim}{R})}{\sqrt{Var(\underset{\sim}{X}_1'\underset{\sim}{R})} \cdot \sqrt{Var(\underset{\sim}{X}_2'\underset{\sim}{R})}} = \frac{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_2}{\sqrt{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_1} \cdot \sqrt{\underset{\sim}{X}_2'\underset{\sim}{\Sigma}\underset{\sim}{X}_2}} \rightarrow \text{substitute either } \underset{\sim}{X}_1 \text{ OR } \underset{\sim}{X}_2 \rightarrow$$

$$\frac{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}(\underset{\sim}{X}_1)}{\sqrt{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_1} \cdot \sqrt{(\underset{\sim}{X}_1')\underset{\sim}{\Sigma}(\underset{\sim}{X}_1)}} = \boxed{\frac{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_1}{\underset{\sim}{X}_1'\underset{\sim}{\Sigma}\underset{\sim}{X}_1} = 1 \longleftrightarrow \text{perfectly correlated}}$$

③ 900K → diversified portf. →
100K → ABC stock

Type	$\bar{R}$ (monthly)	σ (monthly)
Portfolio	0.67%	2.37%
ABC Co.	1.25	2.95

$\rightarrow \rho_{ABC, portf.} = 0.40$

(a1) $\text{Expected Return} = (900{,}000 \times 0.0067) + (100{,}000 \times 0.0125) = (6030) + (1250) = \boxed{\$7280}$ OR $\boxed{0.728\% \text{ monthly}}$

(a2) $\rho = 0.40 = \dfrac{Cov(ABC, Portf.)}{\sqrt{Var(ABC)} \cdot \sqrt{Var(Portf.)}} \Rightarrow Cov(\cdots) = (0.0237)(0.0295)(0.40)$
$= \boxed{0.00027966}$

(a3) $\sigma_{all} = \sqrt{(w^2_{port.})(\sigma^2_{port.}) + (w^2_{ABC})(\sigma^2_{ABC}) + 2(w_{port.})(w_{ABC})(Cov(ABC, Port.))} \longrightarrow$

$w^2_{port} = 0.81$
$w^2_{ABC} = 0.01$
$w_{port.} = 0.9$
$w_{ABC} = 0.1$

$\left.\begin{array}{l} \sigma^2_{port} = 0.00056169 \\ \sigma^2_{ABC} = 0.00087025 \\ Cov(ABC, Port.) = 0.40 \end{array}\right\} \sigma_{all} = \sqrt{(0.81)(0.0237)^2 + (0.01)(0.0295)^2 + (2)(0.0237)(0.0295)(0.40)(0.9)(0.1)} \rightarrow$

$\boxed{\sigma_{all} = 0.022672}$

(b1) $\text{Expected Return} = (0.9 \times 0.0067) + (0.1 \times 0.0042) = \boxed{0.00645 \text{ OR } 0.645\% \text{ monthly}}$

(b2) $Cov(Port., Govt.) = (0)(0.0237)(0) = \boxed{0}$

(b3) $\sigma_{all(new)} = \sqrt{(0.9^2 \times 0.0237^2) + (0.10^2 \times 0^2) + (2 \times 0.9 \times 0.1 \times 0 \times 0)} = \sqrt{(0.9^2 \times 0.0237^2) + 0 + 0}$

$\boxed{\sigma_{all(new)} = 0.02133}$

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④ "Work is done over RStudio"

Answers in Pdf of RStudio

stats_c183_hw2_Charles_Liu

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Setting up for Problem 4:

```
inverse_Q <- matrix(c(166.21139, -22.40241, -22.40241, 220.41076), nrow = 2, ncol = 2)
inverse_Q  # solve(Q) = inverse of Covmat of Q
```

```
##           [,1]      [,2]
## [1,] 166.21139 -22.40241
## [2,] -22.40241 220.41076
```

```
ones <- rep(1, 2)
```

4a)

```
X_numerator <- inverse_Q %*% ones
X_denominator <- t(ones) %*% inverse_Q %*% ones # = 341.8173
X <- X_numerator/341.8173

# Composition of Minimum Risk Portfolio:
X
```

```
##           [,1]
## [1,] 0.4207188
## [2,] 0.5792812
```

4b)

```
sigma_min <- 0.05408825
R_bar_min <- 0.01315856
R_bar_B <- 0.01219724
R_f <- 0.011


R_B <- R_bar_min - (R_f * ones)
X_B_numerator <- inverse_Q %*% R_B
X_B_denominator <- t(ones) %*% inverse_Q %*% R_B # = 0.7378332
X_B <- X_B_numerator/0.7378332

# Composition of Portfolio B in terms of Portfolio A and Risk Free Asset:
X_B
```

```
##           [,1]
## [1,] 0.4207188
```

```
## [2,] 0.5792812
```

4c)

Given this level of risk (0.03), you can do better for Expected Returns for Portfolio B becasuse any portfolio on the Capital Allocation Line (CAL) will yield a higher return for the same amount of risk. The CAL is just above the Minimum Risk Portfolio and attached to the Point of Tangency (G) for the Risk Free Asset. Essentially, yes you can do better Expected Returns for a lower amount of risk (i.e. #4b has risk of 0.01219724).