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Stats C183
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HW 3

① (2) $MSE = (\bar{A}-\bar{P})^2 + (1-\beta_1)^2 S_P^2 + (1-r_{AP}^2) S_A^2 = \frac{1}{m}\Sigma \underbrace{(A_j - P_j)^2}_{\pm \bar{A} \pm \bar{P}}$

$\rightarrow \frac{1}{m}\Sigma\left[A_j - \bar{A} - (P_j - \bar{P}) + \bar{A} - \bar{P}\right]^2 \rightarrow \frac{1}{m}\Sigma(A_j-\bar{A})^2 + \frac{1}{m}\Sigma(P_j-\bar{P})^2 + (\bar{A}-\bar{P})^2$

$+ \cancel{\frac{2}{m}(\bar{A}-\bar{P})\Sigma(A_j-\bar{A})} - \cancel{\frac{2}{m}(\bar{A}-\bar{P})\Sigma(P_j-\bar{P})}$ $=0$

$- \frac{2}{m}\Sigma(A_j-\bar{A})(P_j-\bar{P})$

$\rightarrow \underbrace{\frac{1}{m}\Sigma(A_j-\bar{A})^2}_{=S_A^2} + \underbrace{\frac{1}{m}\Sigma(P_j-\bar{P})^2}_{=S_P^2} + (\bar{A}-\bar{P})^2 - \underbrace{\frac{2}{m}\Sigma(A_j-\bar{A})(P_j-\bar{P})}_{=(S_P^2)(\hat{\beta}_1)}$ $\xrightarrow{\text{by definition...}}$

$\circledast\ R^2 = \frac{(\hat{\beta}_1)\Sigma(P_j-\bar{P})^2}{\Sigma(A_j-\bar{A})^2} = \frac{(\hat{\beta}_1)(S_P^2)}{S_A^2}$ & $\circledast\ \hat{\beta}_1 = \frac{\Sigma(A_j-\bar{A})(P_j-\bar{P})}{\Sigma(P_j-\bar{P})^2} \Rightarrow (S_P^2)(\hat{\beta}_1) = \Sigma(A_j-\bar{A})(P_j-\bar{P})$

back to problem $\rightarrow MSE = S_A^2 + S_P^2 + (\bar{A}-\bar{P})^2 - 2(S_P^2)(\hat{\beta}_1) \xrightarrow{\pm S_P^2 \hat{\beta}_1} S_A^2 + S_P^2(1 - 2\hat{\beta}_1 + \hat{\beta}_1^2) + (\bar{A}-\bar{P})^2 - \hat{\beta}_1^2 S_P^2 \rightarrow$

$(\bar{A}-\bar{P})^2 + S_P^2(1-\hat{\beta}_1)^2 + S_A^2 - S_A^2 \underbrace{R^2}_{=r_{AP}} \xrightarrow{\text{rearrange}} \boxed{MSE = (\bar{A}-\bar{P})^2 + (1-\beta_1)^2 S_P^2 + (1-r_{AP}^2) S_A^2}$ ✓

$\boxed{\text{RStudio for components of PRESS}}$

(2a) $\bar{R}_m = 0.10 \quad \sigma_m^2 = 0.20$; $\beta_p = 0.90 \quad \sigma_{\varepsilon i}^2 = 0.05$ $\Big\}\ \sigma_j^2 = \beta_i^2 \sigma_m^2 + \Sigma X_i^2 \sigma_{\varepsilon i}^2 \rightarrow \boxed{\sigma_P^2 = (0.90)^2 \cdot (0.20) + \frac{0.05}{20} = 0.1645}$

(2b) RStudio for 2b

(2c) (SIM)

stock	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}_\varepsilon^2$
A	0.0082	0.7900	0.0270
B	0.0099	1.1200	0.0060

$\frac{1}{T}\sum_{i=1}^{60}(R_{mt}-\bar{R}_m)^2 = 0.13$

$\sigma_m^2 = 0.0022$

$\underbrace{\Sigma(R_{it}-R_i)^2}_{SST} = \underbrace{\Sigma e_{it}^2}_{SSE} + \underbrace{\Sigma(\hat{R}_{it}-\bar{R})^2}_{SSR}$ $\longrightarrow$

(*) $Cov(R_A, R_B) = \beta_A \cdot \beta_B \cdot Var(R_m) \longrightarrow Cov(R_A, R_B) = (0.79)(1.12)(0.0022) = 0.00195 \longrightarrow$

regress R_A on $R_m \longrightarrow$ (*) $Var(R_i) = \beta_i^2 \cdot Var(R_m) + Var(e_i) \longrightarrow \sigma_{R_A}^2 = (0.79)^2(0.0022) + (0.027) = 0.02837$

$$R^2 = \rho^2 = \frac{\beta_i^2 \sigma_m^2}{Var(R_i)} = \frac{(0.79)^2(0.0022)}{(0.02837)} = \boxed{0.048397 = R^2}$$

(3) $\Sigma_i^{30} P_i = 32.44349$ (sum of β's in P1) $\longrightarrow$ $PRESS = (\bar{A}-\bar{P})^2 + (1-\beta_1)^2 S_P^2 + (1-R^2)S_A^2$

$\Sigma_i^{30} A_i = 32.26206$ (sum of β's in P2)

$\Sigma_i^{30} P_i^2 = 51.70104$ (sum of β²'s in P1)

$\Sigma_i^{30} A_i^2 = 48.43207$ (sum of β²'s in P2)

$\underbrace{\Sigma(A_i-\bar{A})(A_j-\bar{A}) = 4.299189}_{(SSR)}$ (fitted of S.R. of A on P)

$\bar{A} = \frac{32.44349}{30} = 1.08145$ & $\bar{P} = \frac{32.26206}{30} = 1.07540$

$S_A^2 = \frac{1}{30}\left[48.43207 - \frac{(32.26206)^2}{30}\right] = 0.45791$ $\longrightarrow$

$S_P^2 = \frac{1}{30}\left[51.70104 - \frac{(32.44349)^2}{30}\right] = 0.55383$

$SSR = \hat{\beta}_1^2 \Sigma(P_j-\bar{P})^2 \longrightarrow \hat{\beta}_1^2 = \frac{4.299189}{51.70104 - \frac{(32.44349)^2}{30}} = 0.25875 \longrightarrow \hat{\beta}_1 = \sqrt{0.25875} = 0.50867$

$\longrightarrow R^2 = \frac{SSR}{SST} = \frac{4.299189}{48.43207 - \frac{(32.26206)^2}{30}} = 0.31295$

$\longrightarrow PRESS = (0.000036603) + (0.13370) + (0.31460) = \boxed{0.4483 = PRESS}$

(4) $E(X)=3$ $\quad E(X^2)=10$
$E(Y)=2$ $\quad E(Y^2)=29$
$E(XY)=0$

$Var(X) = E(X^2) - E(X)^2 = (10)-(3)^2 = 1 \longrightarrow \sigma_X = 1$

$Var(Y) = E(Y^2) - E(Y)^2 = (29)-(2)^2 = 25 \longrightarrow \sigma_Y = 5$

$Cov(X,Y) = E(XY) - E(X)\cdot E(Y) = (0)-(3)(2) = -6$ $\longrightarrow$

$$\rho_{xy} = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{-6}{(1)\cdot(5)} = \boxed{-1.2} \xrightarrow{\text{However}} -1 \le \rho \le 1 \longrightarrow \boxed{\text{Not true}}$$