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Stats C183
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① "Problems a–c are done in RStudio code"

② (i) The investor will move up from Point A until the tangent, or move to the left of Point A until the tangent (Point G). These represent the Efficient Frontier.

(ii) Portfolio Z cannot be on the Efficient Frontier because the point lies below the Efficient Frontier. It has a higher standard deviation than Portfolio X with a lower expected return.

(iii) $E(R) = 15\% = 0.15$, $\sigma = 60\% = 0.60$, $\rho = 0.5$, $n = 25$

$$\bar{R}_p = \sum_{i=1}^{n} x_i \bar{R}_i = \frac{1}{n}\sum_{i=1}^{n} \bar{R}_i \Rightarrow \frac{1}{25}\sum_{i=1}^{25} \bar{R}_i = \frac{\cancel{(25)}(0.15)}{\cancel{(25)}} \rightarrow$$

$$\boxed{\bar{R}_p = 0.15} \rightarrow \sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j \neq n} x_i x_j \sigma_{ij} \Longrightarrow$$

$$\frac{1}{(25)^{\cancel{2}}} \cdot \cancel{(25)} \cdot (0.60)^2 + \frac{1}{(25)^{\cancel{2}}} \cdot \cancel{(25)}(25-1)(0.5)(0.60)^2 = (0.0144) + (0.1728) \rightarrow$$

$$\boxed{\sigma_p^2 = 0.1872} \rightarrow \boxed{\sigma_p = \sqrt{0.1872} = 0.4327}$$

(iv) We have the formula... $\rightarrow \sigma_p^2 = \frac{1}{n}(\sigma^2) + \frac{1}{n}(n-1)(\rho)(\sigma^2) \rightarrow$ we need $\sigma_p^2 < (0.43)^2$ for an unknown $(n) \rightarrow$

$$\frac{1}{n}(0.36) + \frac{n-1}{n}(0.18) < (0.43)^2 \rightarrow (0.36) + (n-1)(0.18) < 0.1849(n) \rightarrow$$

$$n = 36.7347 \rightarrow \boxed{n = 37}$$

(v) $\sigma_p = \sigma\sqrt{\rho}$ (?) $\rightarrow \sigma_p = (0.6)\sqrt{0.5} = 0.4243 \rightarrow$ if $n = 25 \rightarrow \sigma_p = 0.4327$, $n = 37 \rightarrow \sigma_p = 0.4300$, $n = 100 \rightarrow \sigma_p = 0.4264$

$\rightarrow$ Yes, it is true that if (n) increases, then $\sigma_p = \sigma\sqrt{\rho}$. The reason is because the more you increase your sample size ("diversify"), the closer it'll be till it reaches the true minimum risk.

③ "Exercise 3 done in RStudio"

④ Equal Weight Portfolio $\rightarrow \sigma_p^2 = \frac{1}{n}(\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$ for $\bar{\sigma}_i^2 = 50$ & $\bar{\sigma}_{ij} = 10 \rightarrow$

$$* \; (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) = ((50)^2 - (10)) = 40$$

n	$\sigma_p^2 = \frac{1}{n}(\bar{\sigma}_i^2 - \bar{\sigma}_{ij}^2) + \bar{\sigma}_{ij}$
5	18.0
10	14.0
20	12.0
50	10.8
100	10.4

$\leftarrow$ "Answers"

5 (i) $$X_k = \frac{E\sum_{j=1}^{m} V_{kj}(CE_j - A) + \sum_{j=1}^{m} V_{kj}(B - AE_j)}{D} \quad \text{for } k=1,\dots,m$$ → we need equations... →

(a) $0 = \sum_1^m x_j \sigma_{ij} - \lambda_1 E_i - \lambda_2$ for $i=1,\dots,m$

(b) $0 = E - \sum_1^m x_i E_i$

(c) $0 = 1 - \sum_{i=1}^m x_i$

$A = \sum_1^m \sum_1^m V_{kj} E_j$

$B = \sum_1^m \sum_1^m V_{kj} E_j E_k$

$C = \sum_1^m \sum_1^m V_{kj}$

$D = BC - A^2$

$E = B\lambda_1 + A\lambda_2$

$1 = A\lambda_1 + C\lambda_2$

$$\lambda_1 = \frac{(CE - A)}{D} \qquad \lambda_2 = \frac{(B - AE)}{D}$$

Now to solve... →

$X_k = \lambda_1 \sum_1^m V_{kj} E_j + \lambda_2 \sum_1^m V_{kj}$ for $k=1,\dots,m$

→ we would simply plug the the (λ_1) & (λ_2) to our (X_k) → $= \left[\frac{CE - A}{D}\right] \cdot \sum_1^m V_{kj} E_j + \left[\frac{B - AE}{D}\right] \cdot \sum_1^m V_{kj}$ →

$$\boxed{X_k = \frac{E \cdot \sum_1^m V_{kj}(CE_j - A) + \sum_1^m V_{kj}(B - AE_j)}{D} \quad \text{for } k=1,\dots,m}$$

✓ frontier portfolio

(ii) say $\overline{E} = \frac{A}{C}$ → multiply eq. (a) by (x_i) → $\sum_1^m \sum_1^m x_i x_j \sigma_{ij} = \lambda_1 \sum_1^m x_i E_i + \lambda_2 \sum_1^m x_i$

→ $\sigma^2 = \lambda_1 E + \lambda_2$ (from definition of σ^2, (b), & (c)) → $\sigma^2 = \lambda_1 E + \lambda_2$ → $\sigma^2 = \frac{CE^2 - 2AE + B}{D}$ →

$\frac{\partial \sigma^2}{\partial E} = \frac{2[CE - A]}{D} = 0$ → $\overline{E} = \frac{A}{C}$ → $\sigma^2 = \frac{1}{C}$ → we define $(\overline{X}_k)$ as the proportion of min.-variance portfolio... →

$\overline{X}_k = \frac{\sum_1^m V_{kj}}{\sum_1^m \sum_1^m V_{kj}}$ for $k=1,\dots,m$ → we know that $C = \sum_1^m \sum_1^m V_{kj}$ → $$\boxed{\overline{X}_k = \frac{\sum_1^m V_{kj}}{C} \quad \text{for } k=1,\dots,m}$$

✓ min. risk portfolio