University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 2

EXERCISE 1
Consider a portfolio consisting of $n+1$ assets: n risky assets and the $(n+1)$st asset is the risk free asset with guaranteed return R_f. When short sales allowed, the efficient frontier of all feasible portfolios which can be constructed from these $n+1$ assets is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return E is determined by solving the problem

$$\begin{aligned} \min \quad & \frac{1}{2}\mathbf{x}'\mathbf{\Sigma}\mathbf{x} \\ \text{subject to} \quad & E = R_f + (\bar{\mathbf{R}} - R_f\mathbf{1})'\mathbf{x} \end{aligned}$$

Definitions:

$\mathbf{x}$	Vector of the weight of the n risky assets.
$\mathbf{\Sigma}$	Variance covariance matrix of the n risky assets.
$\bar{\mathbf{R}}$	Vector of the expected returns of the n risky assets.
$\mathbf{1} = (1, 1, \ldots, 1)'$	$n \times 1$ vector of ones.
R_f	Risk free rate.
E	Required expected return (combination of the n risky assets and R_f).

We showed in class that the weights of the n risky assets are given by

$$\mathbf{x} = \frac{E - R_f}{(\bar{\mathbf{R}} - R_f\mathbf{1})'\mathbf{\Sigma}^{-1}(\bar{\mathbf{R}} - R_f\mathbf{1})}\mathbf{\Sigma}^{-1}(\bar{\mathbf{R}} - R_f\mathbf{1}). \quad (1)$$

Consider now the point of tangency G. The composition of portfolio G is computed by finding first $\mathbf{Z} = \mathbf{\Sigma}^{-1}(\bar{\mathbf{R}} - Rf\mathbf{1})$. Show that when the required expected return E is equal to the expected return of portfolio G then using (1) the weights are exactly the same as the ones obtained using handout #15.

EXERCISE 2
Show that two portfolios on the capital allocation line are perfectly correlated.

EXERCISE 3
Answer the following questions:

a. An investor has \$900000 invested in a diversified portfolio. Subsequently the investor inherits `ABC` company stock worth \$100000. His financial adviser provided him with the following forecast information:

	$\bar{R}$ (monthly)	σ (monthly)
Portfolio	0.67%	2.37%
`ABC` Compnay	1.25	2.95

The correlation coefficient between `ABC` company stock returns and the portfolio is 0.40.
Assume that the investor keeps the `ABC` company stock. Answer the following questions:

1. Calculate the expected return of the new portfolio which includes the `ABC` company stock.
2. Calculate the covariance between `ABC` company stock and the portfolio.
3. Calculate the standard deviation of his new portfolio which includes the `ABC` company stock.

b. Refer to question (a). If the investor sells the `ABC` company stock, he will invest the proceeds in risk-free government securities yielding 0.42% per month. Calculate the:

1. Expected return of the new portfolio which includes the government securities.
2. The standard deviation of his new portfolio which includes the government securities.

EXERCISE 4

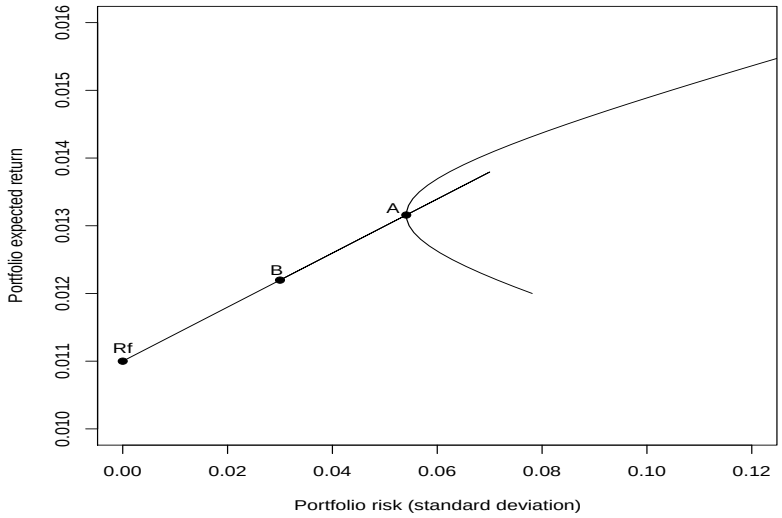
The covariance matrix **Q** of the returns of two stocks has the following inverse:

```
> solve(Q)
          [,1]      [,2]
[1,] 166.21139 -22.40241
[2,] -22.40241 220.41076
```

Answer the following questions:

a. Find the composition of the minimum risk portfolio.

b. It is given that the minimum risk portfolio (point A on the graph below) has standard deviation equal to 0.05408825 and expected return equal to 0.01315856. Portfolio B (see graph below) has expected return equal to 0.01219724. What is the composition of portfolio B in terms of portfolio A and the risk free asset? Assume $R_f = 0.011$.



c. The standard deviation of portfolio B is equal to 0.03. Given this level of risk, can you do better than the expected return of portfolio B? Please explain.