Stats_C183_Project_3_Charles_Liu

Charles Liu (304804942)

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Access the following data: http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt. In R you can access the data from the command line as follows: a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt", header=T) These are close monthly prices from January 1986 to December 2003. The first column is the date and P1; P2; P3; P4; P5 represent the close monthly prices for the stocks Exxon-Mobil, General Motors, Hewlett Packard, McDonalds, and Boeing respectively.

a)

Convert the prices into returns for all the 5 stocks.

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt", hea

r1 <- (a$P1[-length(a$P1)]-a$P1[-1])/a$P1[-1]
r2 <- (a$P2[-length(a$P2)]-a$P2[-1])/a$P2[-1]
r3 <- (a$P3[-length(a$P3)]-a$P3[-1])/a$P3[-1]
r4 <- (a$P4[-length(a$P4)]-a$P4[-1])/a$P4[-1]
r5 <- (a$P5[-length(a$P5)]-a$P5[-1])/a$P5[-1]

# And then you can create a data frame with the retuns:
returns <- as.data.frame(cbind(r1,r2,r3,r4,r5))
```

b)

Compute the mean return for each stock and the variance-covariance matrix.

```
means <- colMeans(returns)
covmat <- cov(returns)

# Values for the Means and Covmat for all Stocks:
means
```

```
##           r1           r2           r3           r4           r5
## 0.0027625075 0.0035831363 0.0066229478 0.0004543727 0.0045679106
```

```
covmat
```

```
##             r1          r2          r3          r4          r5
## r1 0.005803160 0.001389264 0.001666854 0.000789581 0.001351044
## r2 0.001389264 0.009458804 0.003944643 0.002281200 0.002578939
## r3 0.001666854 0.003944643 0.016293581 0.002863584 0.001469964
## r4 0.000789581 0.002281200 0.002863584 0.009595202 0.003210827
## r5 0.001351044 0.002578939 0.001469964 0.003210827 0.009242440
```

c)

Use only Exxon-Mobil and Boeing stocks: For these 2 stocks and the composition, expected return, and standard deviation of the minimum risk portfolio.

```
means_b <- colMeans(returns[c(-2,-3,-4)]) # only Exxon-Mobil and Boeing stocks
covmat_b <-cov(returns[c(-2,-3,-4)]) # only Exxon-Mobil and Boeing stocks
ones_b <- rep(1, 2)

R_min_b <- (t(ones_b) %*% solve(covmat_b) %*% means_b)/(t(ones_b) %*% solve(covmat_b) %*% ones_b)
sigma_min_b <- (1)/((t(ones_b) %*% solve(covmat_b) %*% ones_b)^1/2)
X_b <- (solve(covmat_b) %*% ones_b)/as.integer(t(ones_b) %*% solve(covmat_b) %*% ones_b)

# Minumu Risk Portfolio for only Exxon-Mobil and Boeing stocks (composition, expected return, and stand
X_b
```

```
##          [,1]
## r1 0.6399750
## r5 0.3610569
```

```
R_min_b
```

```
##              [,1]
## [1,] 0.003413689
```

```
sigma_min_b
```

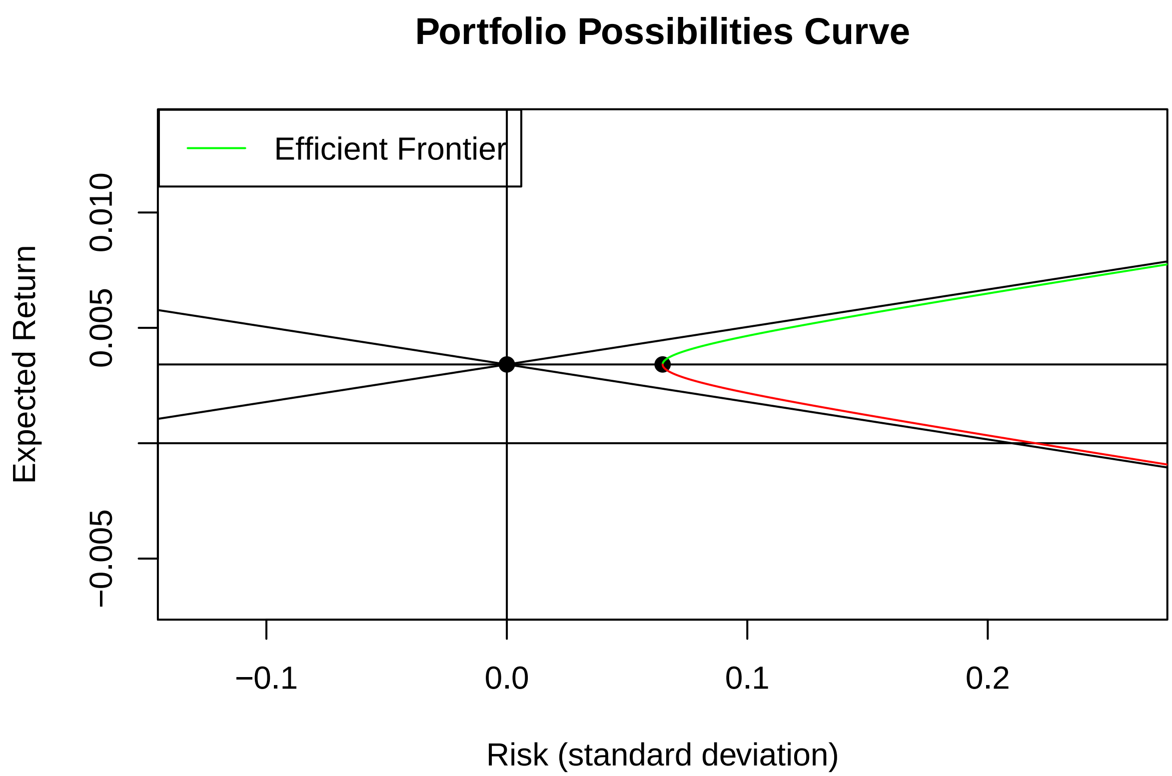
```
##              [,1]
## [1,] 0.008394699
```

d)

Plot the portfolio possibilities curve and identify the efficient frontier on it.

```
A <- t(ones_b) %*% solve(covmat_b) %*% means_b
B <- t(means_b) %*% solve(covmat_b) %*% means_b
C <- t(ones_b) %*% solve(covmat_b) %*% ones_b
D <- B * C - A^2

# Hyperbola:
plot(0, A/C, main = "Portfolio Possibilities Curve", xlab = "Risk (standard deviation)",
     ylab = "Expected Return", type = "n",
     xlim = c(-2*sqrt(1/C), 4*sqrt(1/C)),
     ylim = c(-2*A/C, 4*A/C))

# Plot center of the hyperbola:
points(0, A/C, pch = 19)

# Plot transverse and conjugate axes:
abline(v = 0) # Also this is the y-axis.
abline(h = A/C)

# Plot the x-axis:
abline(h = 0)

# Plot the minimum risk portfolio:
```

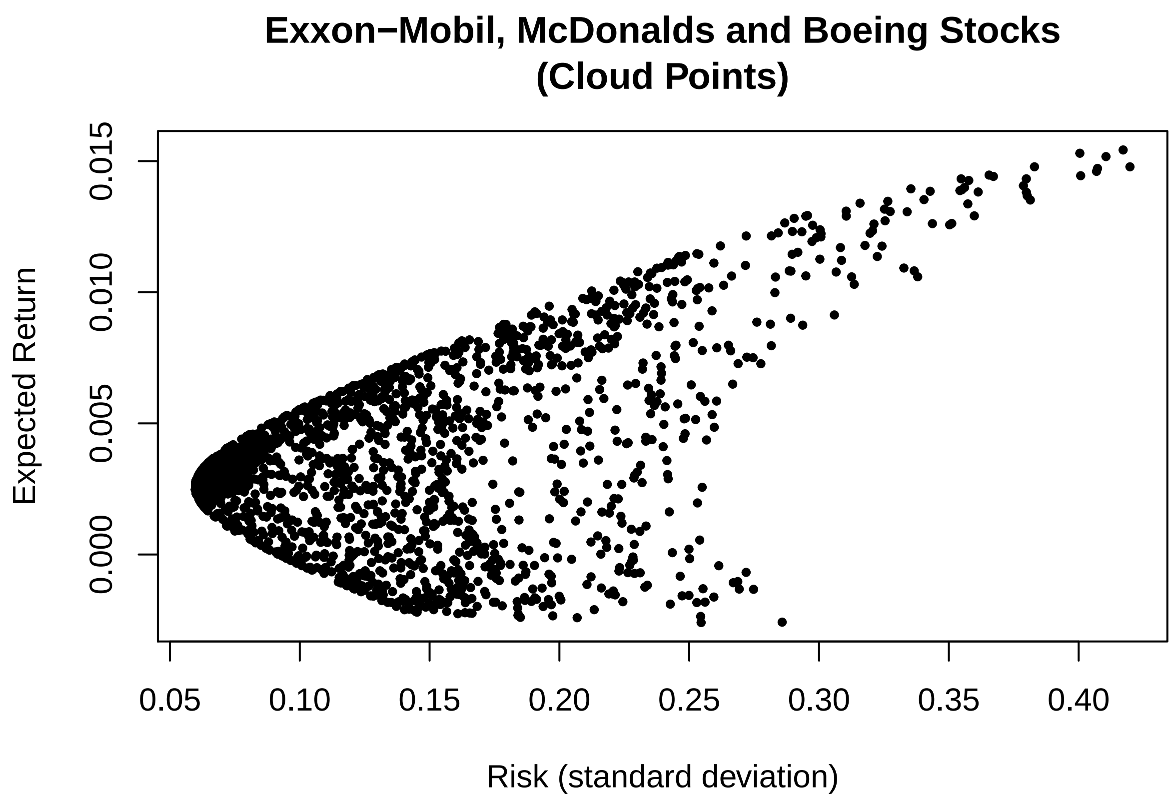
```r
points(sqrt(1/C), A/C, pch=19)

# Find the asymptotes:
V <- seq(-1, 1, 0.001)
A1 <- A/C + V * sqrt(D/C)
A2 <- A/C - V * sqrt(D/C)
points(V, A1, type = "l")
points(V, A2, type = "l")

# Efficient frontier:
minvar_b <- 1/C
minE_b <- A/C
sdeff <- seq((minvar_b)^0.5, 1, by = 0.0001)
options(warn = -1)
y1 <- (A + sqrt(D*(C*sdeff^2 - 1)))*(1/C)
y2 <- (A - sqrt(D*(C*sdeff^2 - 1)))*(1/C)
options(warn = 0)

# The green is the Efficient Frontier
# The red is part of the Frontier
points(sdeff, y1, type = "l", col = "green")
points(sdeff, y2, type = "l", col = "red")
legend(x = "topleft", inset = 0.001, legend = c("Efficient Frontier"), lty = 1, col = c("green"), box.lt
```



e)

Use only Exxon-Mobil, McDonalds and Boeing stocks and assume short sales are allowed to answer the following question: For these 3 stocks compute the expected return and standard deviation for many combinations of xa; xb; xc with xa + xb + xc = 1 and plot the cloud of points. You can use the following combinations of the three stocks: a <- read.table("http://www.stat.ucla.edu/~nchristo/datac183c283/statc183c283_abc.txt", header=T)

```
a_e <- read.table("http://www.stat.ucla.edu/~nchristo/datac183c283/statc183c283_abc.txt",
header=T)

means_e <- means[c(-2, -3)] # Exxon-Mobil, McDonalds, and Boeing stocks
covmat_e <- covmat[c(-2, -3), c(-2, -3)] # Exxon-Mobil, McDonalds and Boeing stocks

expected_returns <- numeric(nrow(a_e))
sdev <- numeric(nrow(a_e))

for(i in 1:nrow(a_e)){
expected_returns[i] <- as.matrix(a_e[i,]) %*% means_e
sdev[i] <- sqrt(as.matrix(a_e[i,]) %*% covmat_e %*% t(as.matrix(a_e[i,])))
}

# Plot the Cloud of Points
plot(sdev, expected_returns, xlab = "Risk (standard deviation)", ylab = "Expected Return",
main = "Exxon-Mobil, McDonalds and Boeing Stocks
(Cloud Points)", pch = 19, cex = 0.5)
```



```r
# Values for the Many Combinations Portfolio (head() only):
head(sdev)
```

```
## [1] 0.4171400 0.4004671 0.4105275 0.4197722 0.4072746 0.3829940
```

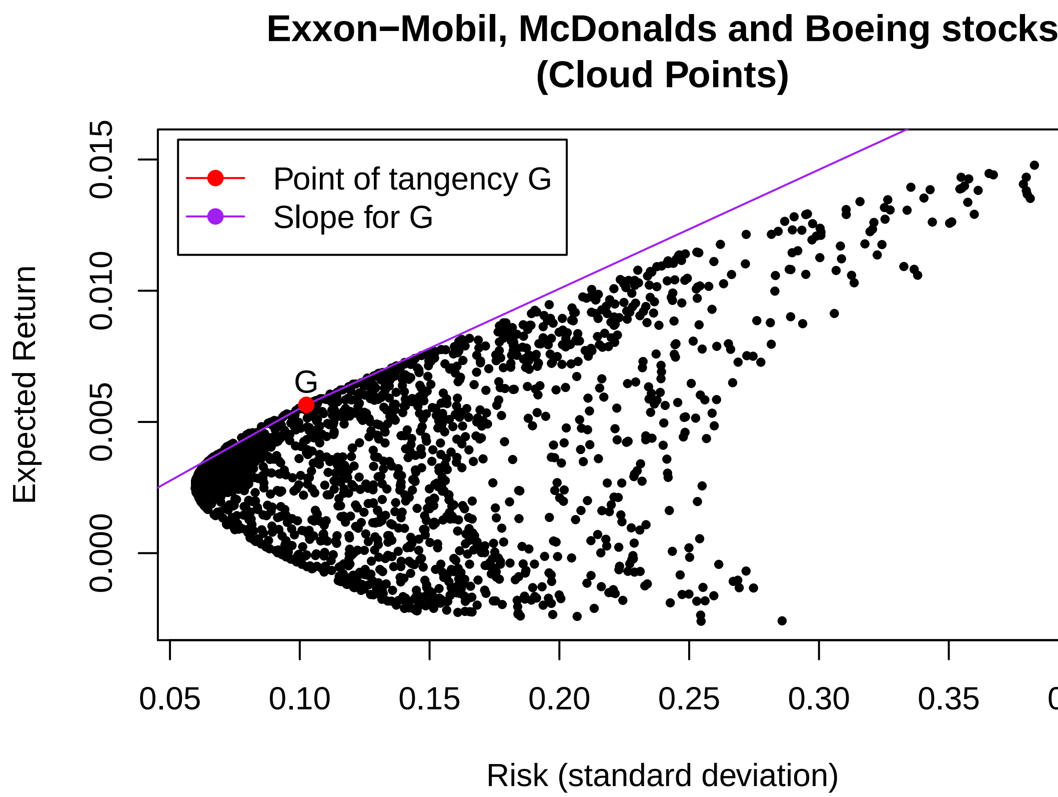
```r
head(expected_returns)
```

```
## [1] 0.01542593 0.01530052 0.01517314 0.01478307 0.01472167 0.01478383
```

f)

Assume Rf = 0.001 and that short sales are allowed. Find the composition, expected return and standard deviation of the portfolio of the point of tangency G and draw the tangent to the efficient frontier of question (e).

```r
Rf <- 0.001
R <- means_e - Rf
z <- solve(covmat_e) %*% R

# Composition of G
x <- z/sum(z)

# Expected Return of G
R_Gbar <- t(x) %*% means_e

# Standard Deviation of G
sd_G <- sqrt(t(x) %*% covmat_e %*% x)

# Slope of Point of Tangency
slope <- (R_Gbar-Rf)/sd_G

# Plot the Point of Tangency G:
plot(sdev, expected_returns, xlab = "Risk (standard deviation)", ylab = "Expected Return",
main = "Exxon-Mobil, McDonalds and Boeing stocks
(Cloud Points)", pch = 19, cex = 0.5)
lines(c(0, sd_G, 4*sd_G), c(0, R_Gbar, Rf + slope*(4*sd_G)), col = "purple")
points(sd_G, R_Gbar, col = "red", pch = 19)
text(sd_G, R_Gbar + 0.0009, "G")
points(0, Rf, pch = 19)
legend(x = "topleft", inset = 0.02, legend = c("Point of tangency G", "Slope for G"), pch = 19, lty = 1
col = c("red", "purple"), box.lty = 1)
```




```
# Values for Composition, Expected Return, and Standard Deviation of G:
x
```

```
##           [,1]
## r1  0.5284782
## r4 -0.4955882
## r5  0.9671100
```

```
R_Gbar
```

```
##             [,1]
## [1,] 0.005652415
```

```
sd_G
```

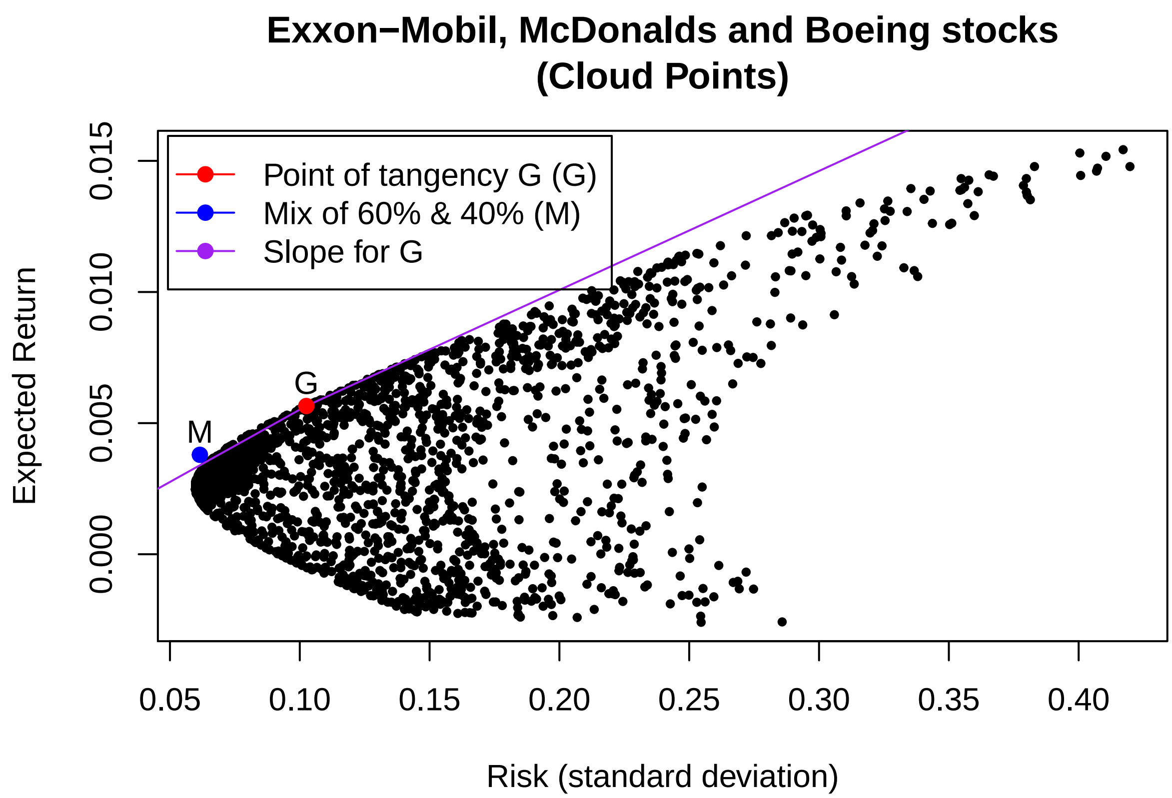
```
##           [,1]
## [1,] 0.1025256
```

g)

Find the expected return and standard deviation of the portfolio that consists of 60% G 40% risk free asset. Show this position on the capital allocation line (CAL).

```
# Mixed Portfolio for Expected Returns and Standard Deviation:
R_mixed <- R_Gbar*0.6 + Rf*0.4
sigma_mixed <- sd_G*0.6

# Plot the Mixed and Point G:
```

```r
plot(sdev, expected_returns, xlab = "Risk (standard deviation)", ylab = "Expected Return",
main = "Exxon-Mobil, McDonalds and Boeing stocks
(Cloud Points)", pch = 19, cex = 0.5)
lines(c(0, sd_G, 4*sd_G), c(0, R_Gbar, Rf + slope*(4*sd_G)), col = "purple")
points(sd_G, R_Gbar, col = "red", pch = 19)
text(sd_G, R_Gbar + 0.0009, "G")
points(0, Rf, pch = 19)
points(sigma_mixed, R_mixed, col = "blue", pch = 19)
text(sigma_mixed, R_mixed + 0.0009, "M")
legend(x = "topleft", inset = 0.01, legend = c("Point of tangency G (G)", "Mix of 60% & 40% (M)", "Slope
```




```r
# Values for our Mixed Portfolio of 60% and 40%:
R_mixed
```

```
##             [,1]
## [1,] 0.003791449
```

```r
sigma_mixed
```

```
##            [,1]
## [1,] 0.06151535
```

h)

Now assume that short sales allowed but risk free asset does not exist.

i)

Using Rf1 = 0.001 and Rf2 = 0.002 and the composition of two portfolios A and B (tangent to the efficient frontier - you found the one with Rf1 = 0.001 in question (f)).

```
# Choose two risk free rates:
Rf1 <- 0.001
Rf2 <- 0.002

# Re-label to make better sense formula-wise:
R_ibar <-  means[c(-2, -3)]
var_covar <- covmat[c(-2, -3), c(-2, -3)]

# Construct the vectors RA and RB:
RA <- R_ibar-Rf1
RB <- R_ibar-Rf2

# Find the composition of the two portfolios A, B:
zA <- solve(var_covar) %*% RA
xA <- zA/sum(zA)
zB <- solve(var_covar) %*% RB
xB <- zB/sum(zB)

# Values for our Composition of A and B:
xA
```

```
##          [,1]
## r1  0.5284782
## r4 -0.4955882
## r5  0.9671100
```

```
xB
```

```
##          [,1]
## r1  0.5312205
## r4 -1.8026632
## r5  2.2714427
```

ii)

Compute the covariance between portfolios A and B?

```
cov_AB <- t(xA) %*% var_covar %*% xB

# Value for our Covariance between A and B:
cov_AB
```

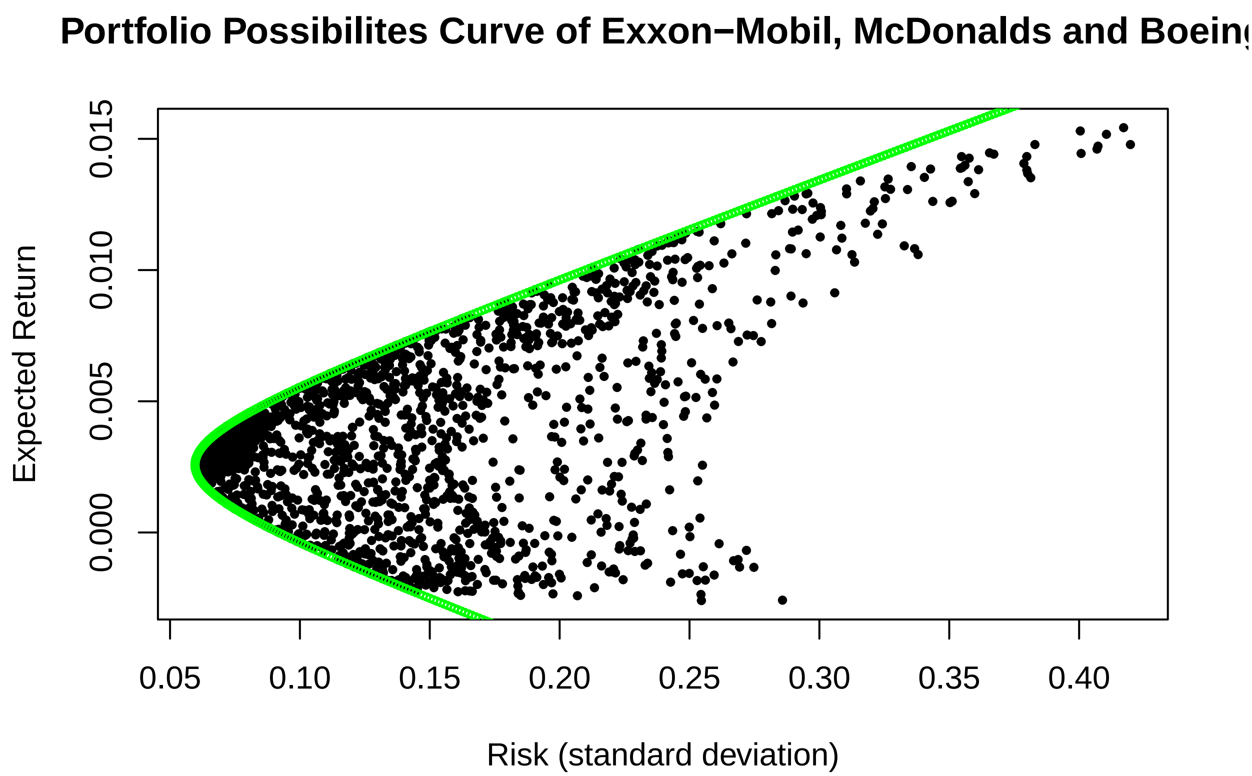
```
##            [,1]
## [1,] 0.02264823
```

iii)

Use your answers to (1) and (2) to trace out the efficient frontier of the stocks Exxon-Mobil, McDonalds, Boeing. Use a different color to show that the frontier is located on top of the cloud of points from question (e). Your graph should look like the one below.

```
# Our composition for Portfolios A and B:
xa <- seq(-3, 5, 0.01)
```

```r
xb <- 1 - xa

# Our Expected Returns from Portfolios A and B:
RA_bar <- t(xA) %*% R_ibar
RB_bar <- t(xB) %*% R_ibar

# Our Variances from Portfolios A and B:
var_A <- t(xA) %*% var_covar %*% xA
var_B <- t(xB) %*% var_covar %*% xB

# Portfolios A and B's sigma_p and rp_bar:
sigma_p <- sqrt(xa^2*var_A + xb^2*var_B+ 2*xa*xb*cov_AB)
rp_bar <- xa*RA_bar + xb*RB_bar


# Plot all below:
plot(sdev, expected_returns, xlab = "Risk (standard deviation)", ylab = "Expected Return",
main = "Portfolio Possibilites Curve of Exxon-Mobil, McDonalds and Boeing", pch = 19, cex = 0.5)
points(sigma_p, rp_bar, col = "green", cex = 0.5)
```



iv)

Find the composition of the minimum risk portfolio (how much of each stock) and its expected return, and standard deviation.

```r
# Create our set of ones vector:
ones <- rep(1,3)

# Composition of Minimum Risk Portfolio:
x_min <- (solve(covmat_e) %*% ones)/as.integer(t(ones) %*% solve(covmat_e) %*% ones)

# Expected Return of Minimum Risk Portfolio:
R_min <- (t(ones) %*% solve(covmat_e) %*% as.matrix(means_e))/as.integer(t(ones) %*% solve(covmat_e) %*

# Standard Deviation of minimum risk portfolio
sigma_min <- (1)/ sqrt(t(ones) %*% solve(covmat_e) %*% ones)

# Values for Our Minimum Risk Portfolio (Compositon, Exepected Returns, Standard Deviation):
x_min
```

```
##         [,1]
## r1 0.5275355
## r4 0.2539561
## r5 0.2197024
```

```r
R_min
```

```
##             [,1]
## [1,] 0.002576292
```

```r
sigma_min
```

```
##            [,1]
## [1,] 0.05961942
```