Chapter 11
Conservation of Energy and Momentum in a Collision

[Version: October 5, 2018]

11.1 Safety

- Be careful to not let the two carts touch because it can damage the carts and magnets.

11.2 Guiding Question

As already introduced in Chs. 9 and 10, an important principle in physics is the conservation of energy. In addition, also the momentum is conserved in a system with no external forces and cannot be created or destroyed. Can this claim be tested?

11.3 Experimental Setup

One way to test momentum conservation is to conduct an elastic collision and make sure that all the momentum transfers between the colliding objects. This can be done by colliding two carts of varying sizes.

Concept behind the Experiment

The experiment will start out with the condition that one cart is colliding with a second cart at rest (Fig. 11.1). It is furthermore assumed that the collision is elastic, meaning that neither energy nor momentum are lost. In general, both carts will be moving after the collision. The kinematics will be worked out in the

Figure 11.1: The airtrack setup with both carts on it.

following assuming energy and momentum conservation. Furthermore, the carts are moving on a level track, which means that there is no change in potential energy. Therefore, only kinetic energy needs to be studied:

$$T_i = T_f. \tag{11.1}$$

The total kinetic energy T_i (i for initial) before the collision is the total kinetic energy T_f (f for final) after the collision. Using the relationship for kinetic energy results in:

$$T_i = \frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = T_f \tag{11.2}$$

with m_1 and m_2 being the masses of the carts and v_{1i}, v_{1f}, v_{2f} being their corresponding velocities. The total momentum before (p_i) and after (p_f) the collision is the same:

$$p_i = p_f. \tag{11.3}$$

Using the relationship for momentum gives:

$$p_i = m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} = p_f. \tag{11.4}$$

Combining Eqs. (11.2) and (11.4) results in an expression for the final velocity of the second cart as a function of the masses and the initial velocity of the first cart:

$$v_{2f} = \frac{2v_{1i}}{1 + \frac{m_2}{m_1}}. \tag{11.5}$$

The ratio of energy transferred from object 1 to object 2 is called the energy transfer ratio TE:

$$TE = \frac{E_2}{E_1} = \underbrace{\frac{m_2}{m_1}}_{\mu} \frac{\frac{1}{2}v_{2f}^2}{\frac{1}{2}v_{1i}^2} = \mu \frac{v_{2f}^2}{v_{1i}^2} \tag{11.6}$$

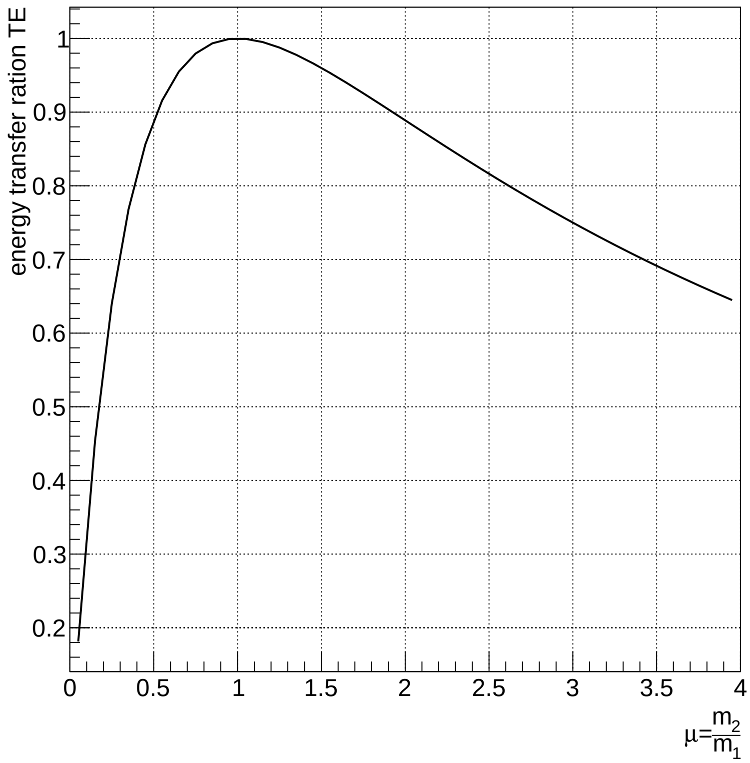

Figure 11.2: Energy transfer ratio.

Combining Eqs. (11.5) and (11.6) results in a relationship for the energy transfer ratio that is independent of the velocities:

$$TE = \frac{4\mu}{(1+\mu)^2}. \tag{11.7}$$

This function is plotted in Fig. 11.2.

11.4 Experimental Procedure

The available carts have different masses. Every group will use at least one mass combination. The results from all groups will be combined in the final step. Time permitting, groups can measure additional cart combinations. As the first step, measure the masses of the two carts. Then, turn on the air flow to eliminate friction and level the track as discussed in Sec. 9.4. Trials can be run by pushing cart 1 into cart 2. Push lightly so the carts do not actually touch, but are repelled by the magnets (Fig. 11.3)[1]. This will ensure that the collision is elastic.

[1] *Pushing cart 1 too strongly slows the acceleration process due to the creation of eddy currents in the track that effectively brake the cart.*

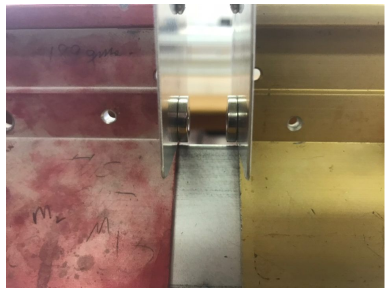

Figure 11.3: The collision with the repelling magnets.

Recording Data

The velocity of the two carts will be measured using the ultrasound sensors (Sec. 9.4). Press down the trigger to start the measurement after pushing the first cart. Release the trigger after the second cart started moving and before it hits the end of the track. The data from the sensors are outputted online at: **http://go.hawaii.edu/2Yf**. Make sure to download data from the correct setup for both sensors and note the conditions of the trial. Also write down, which sensor was closer to cart 1 and which was closer to cart 2. All the times ΔT that the pulses took from the sensor to the carts and back to the sensor need to be converted into distance measurements using Eq. (9.6). For the speed of sound v_s the value that was found during the inclined track experiment can be used.

In the next step the initial velocity of cart 1 and the final velocity of cart 2 need to be calculated. For this purpose plot the distance measurement for both sensors as a function of the time stamp. The goal is to conduct a straight line fit with the slope being the velocity:

$$v = \frac{s}{t} \tag{11.8}$$

For the sensor closer to cart 1 select the data after the initial acceleration where the increase in distance as a function of time is linear. Make sure to not include the data after the collision for cart 1. For cart 2 only use the data after the collision. Also here, only use the data points after the initial acceleration where the increase in distance as a function of time is linear. Make sure to not include any data points after hitting the end of the track.

Run four trials and calculate the energy transfer ratio for every trial from Eq. (11.6). Calculate the average TE and the corresponding sdm(TE). Then compare the measurement with the theoretical value from Eq. (11.6). Time permitting, repeat the measurements for other cart combinations.

Graphing

For the overall class analysis, each group will report their TE with error for their particular mass ratio μ. Then TE can be plotted as a function of μ together with the theoretical curve given in Eq. (11.7).

11.5 Questions

1. Describe how well the data fit the theoretical curve?
2. Which groups agree with the theory? Which groups disagree?
3. What is the maximum value for TE that was measured? What value of μ gives that maximum?