COEN 240 Machine Learning

Homework #4

Guideline: Please complete the following problems and generate a PDF file. Please submit the PDF file and a separate zip file that contains all source code to Camino. Please refer to HomeworkFormat.pdf for the format of the submitted PDF file.

Problem 1 Suppose that X is a discrete random variable with the following probability mass function where $0 \leq \theta \leq 1$ is a parameter:

X	0	1	2	3
$P(X)$	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

We observe the following 10 independent observations taken from the above distribution: (3,0,2,1,3,2,1,0,2,1). What is the maximum likelihood estimate (MLE) of θ?

Problem 2 The Pareto distribution has been used in economics as a model for a probability density function with a slowly decaying tail:

$$f(x|x_0,\theta) = \theta x_0^{\theta} x^{-\theta-1}, \quad x \geq x_0, \quad \theta > 1$$

Assume that $x_0 > 0$ is a given constant and that $X_1, X_2, \ldots, X_n$ are i.i.d. samples from such distribution. Find the MLE of θ using these n observation samples.

Problem 3: Bayes Rule.

I don't have a car. I come to work either by bike or by bus. If I take the bus, there is a 10% chance that I'm late. If I take the bike, there is a 2% chance that I'm late. I take the bike 4 days out of 5. Today I was late.

Calculate the probability that I took the bus today.

Problem 4 Bayesian Decision Theory.

4.1 Write the minimum probability of decision error (min Pe) classifier for the binary classification problem. Prove (derive) that the min Pe decision criterion is equivalent to the maximum a posteriori probability (MAP) decision criterion for the binary classification problem.

4.2 Derive the MAP decision criterion for an M-class classification problem.

4.3 Write the Naïve Bayes classifier for an M-class classification problem. What assumption do we have for the Naïve Bayes classifier?

Problem 5 Skin Detection.

Implement the skin detection classifier given in the lecture slides, using the Naïve Bayes classifier. The training image and its ground truth are family.jpg and family.png. The test image and its ground truth are portrait.jpg and portrait.png.

5.1 Show the test image, its ground truth, and the detected binary mask in the same figure, as in the lecture slides.

5.2 Calculate the true positive rate, true negative rate, false positive rate, and false negative rate for the test result.