

COEN 266 Artificial Intelligence

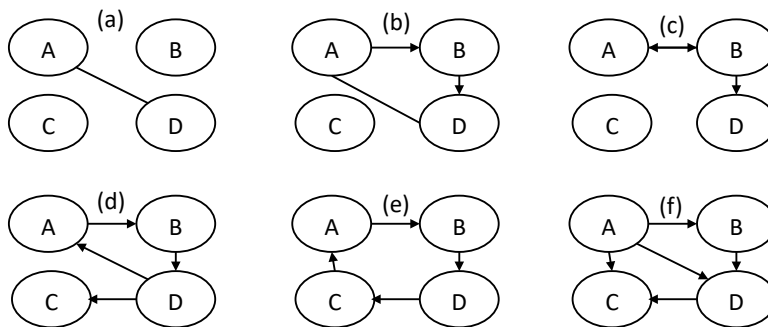
Homework #5

Guideline: Please complete the following problems and submit the answers as a single PDF file to Camino.

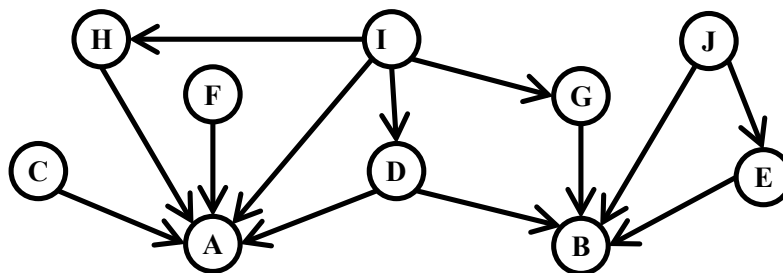
Problem 1

1.a Circle the letter/s that correspond/s to all valid Bayesian networks in the following figure. If there is not any valid Bayesian network, circle None.

Bayesian Networks: None (a) (b) (c) (d) (e) (f)



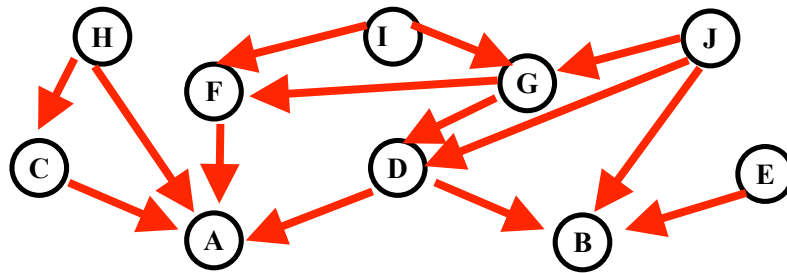
1.b For the following Bayesian Network, write down the joint distribution expression of all the variables/nodes in terms of the product of conditional probabilities of all variables (also called: factored conditional probability expression).



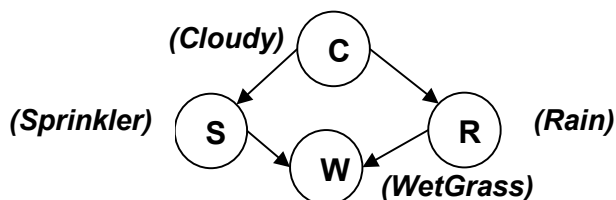
$$P(C) \times P(A|C,H,F,I,D) \times P(H|I) \times P(F) \times P(I) \times P(D|I) \times P(G|I) \times P(B|D,G,J,E) \times P(J) \times P(E|J)$$

1.c Draw the Bayesian Network that corresponds to the following factored conditional probabilities:

$$P(A | C,D,F,H) P(B | D,E,J) P(C | H) P(D | G,J) P(E) P(F | G,I) P(G | I, J) P(H) P(I) P(J)$$



Problem 2. Below is the Bayesian network for the WetGrass problem. Some prior probabilities and conditional probability tables are given. All variables are Boolean variables that can take values true (t) or false (f).



P(C=t)	C	P(S=t C)	C	P(R=t C)	S	R	P(W=t S, R)
0.5	t	0.1	t	0.8	t	t	0.99
	f	0.5	f	0.2	t	f	0.90
					f	t	0.90
					f	f	0.00

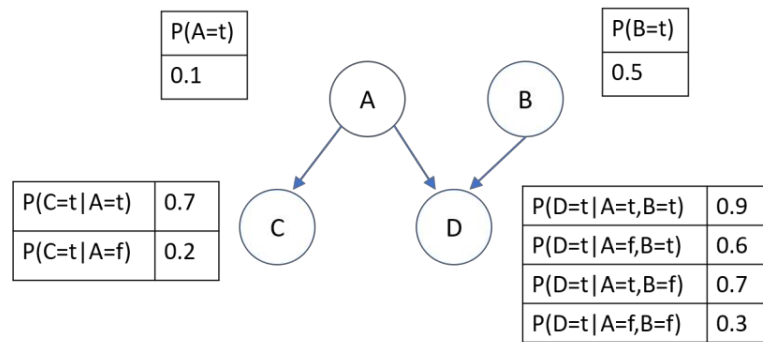
2.a Calculate the value for the joint probability (show your work):

$$\begin{aligned}
 P(C=f, R=f, S=t, W=t) &= P(C) \times P(R|C) \times P(S|C) \times P(W|S,R) \\
 &= 0.5 \times 0.8 \times 0.5 \times 0.9 \\
 &= 0.18
 \end{aligned}$$

2.b You observe that $W=t$ and $S=f$. Perform inference to obtain the posteriori probability that the weather is cloudy, that is: $P(C = t | W = t, S = f)$. Show your work.

$$\begin{aligned}
 &P(C=t | W=t, S=f) \\
 &= P(C=t, W=t, S=f) / P(W=t, S=f) \\
 &= P(C=t, W=t, S=f, R) / (P(C=t, W=t, S=f, R) + P(C=f, W=t, S=f, R)) \\
 &= 0.5 \times 0.9 \times [0.8 \times 0.9 + 0.2 \times 0] / (0.5 \times 0.9 \times [0.8 \times 0.9 + 0.2 \times 0] + 0.5 \times 0.5 \times [0.2 \times 0.9 + 0.8 \times 0]) \\
 &= 0.324 / (0.324 + 0.045) \\
 &= 0.878
 \end{aligned}$$

Problem 3 Consider the following Bayesian Network containing four Boolean random variables.



3.a Compute the joint probability $P(A=f, B=t, C=f, D=t)$.

$$\begin{aligned}
 &P(A) \times P(B) \times P(C|A) \times P(D|A,B) \\
 &= 0.9 \times 0.5 \times 0.8 \times 0.6 \\
 &= 0.216
 \end{aligned}$$

3.b Compute the posteriori probability $P(A = t | B = t, C = t, D = t)$.

$$\begin{aligned}
 &P(A=t | B=t, C=t, D=t) \\
 &= P(A=t, B=t, C=t, D=t) / P(B=t, C=t, D=t) \\
 &= P(A=t, B=t, C=t, D=t) / (P(A=t, B=t, C=t, D=t) + P(A=f, B=t, C=t, D=t)) \\
 &= 0.1 \times 0.5 \times 0.7 \times 0.9 / (0.1 \times 0.5 \times 0.7 \times 0.9 + 0.9 \times 0.5 \times 0.2 \times 0.6) \\
 &= 0.368
 \end{aligned}$$