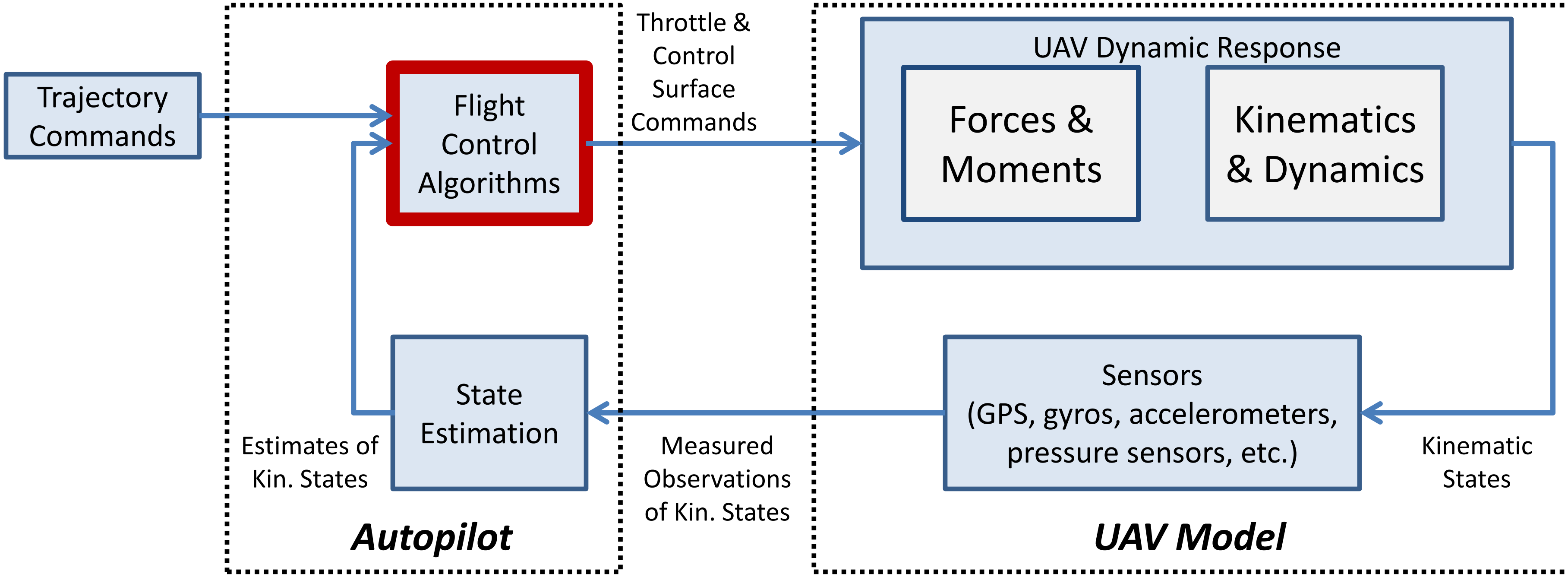
UAV Systems & Control
Lecture 7

Linear Response Models

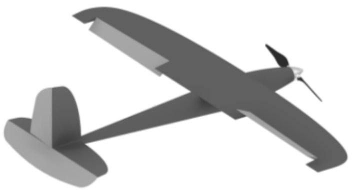
Roll Autopilot

UAV System



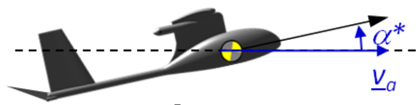
- In the previous lectures we:
  - Developed the equations of motion: a complicated set of 12 nonlinear, coupled, differential equations
  - Discussed autopilot control methods, and introduced manual tuning
- In this section we will use the EoMs to develop simple linear transfer functions for both the longitudinal and lateral flight control channels
  - These simplified models will provide a "cookie-cutter" method of developing autopilot control suitable for many fixed-wind UAVs

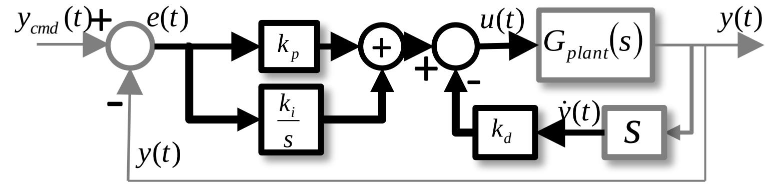
Path to Autopilot Development



Develop the non-linear model of motion

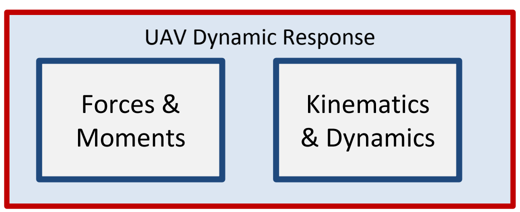
Find "trim" point that balances forces and moments at a nominal speed

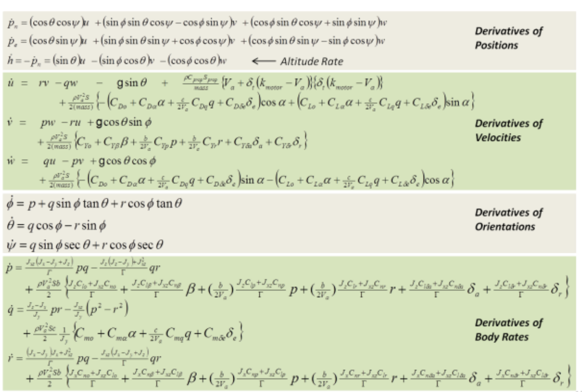




Determine controller structure, e.g.:
- Classical: PID or PI w/ rate feedback, etc.
- Modern: LQR, Full-State FB, H-∞, etc.

Equations of Motion





12 EoMs

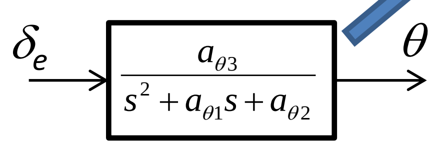
"Linearize" EoMs about trim point to make de-coupled linear response models

Two options:

a) Numerically compute linearized state-space model (Representative, but high order)

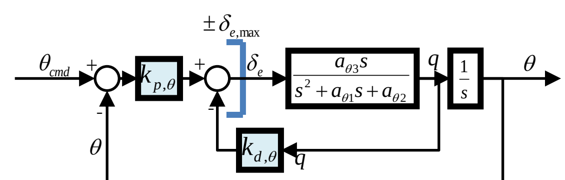
$$\dot{\bar{\mathbf{x}}} = A\bar{\mathbf{x}} + B\bar{\mathbf{u}}$$

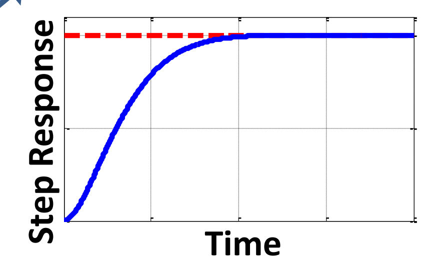
b) Analytically derive simpler 1st & 2nd order linear transfer function response models



Our method

Select controller-specific gains to meet desired performance around "linearized" operating point, e.g.





Fixed Wing EoMs as 12 Scalar Functions

(We'll use these to develop an autopilot)

$$\dot{p}_n = (\cos\theta\cos\psi)u + (\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi)v + (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)w$$

$$\dot{p}_e = (\cos\theta\sin\psi)u + (\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi)v + (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)w$$

$$\dot{h} = -\dot{p}_d = (\sin\theta)u - (\sin\phi\cos\theta)v - (\cos\phi\cos\theta)w \longleftarrow \text{Altitude Rate}$$

Derivatives of Positions

$$\dot{u} = rv - qw - g\sin\theta + \frac{\rho C_{prop} S_{prop}}{mass}\{V_a + \delta_t(k_{motor} - V_a)\}\{\delta_t(k_{motor} - V_a)\}$$
$$+ \frac{\rho V_a^2 S}{2(mass)}\left\{-\left(C_{Do} + C_{D\alpha}\alpha + \frac{c}{2V_a}C_{Dq}q + C_{D\delta e}\delta_e\right)\cos\alpha + \left(C_{Lo} + C_{L\alpha}\alpha + \frac{c}{2V_a}C_{Lq}q + C_{L\delta e}\delta_e\right)\sin\alpha\right\}$$

$$\dot{v} = pw - ru + g\cos\theta\sin\phi$$
$$+ \frac{\rho V_a^2 S}{2(mass)}\left\{C_{Yo} + C_{Y\beta}\beta + \frac{b}{2V_a}C_{Yp}p + \frac{b}{2V_a}C_{Yr}r + C_{Y\delta a}\delta_a + C_{Y\delta r}\delta_r\right\}$$

$$\dot{w} = qu - pv + g\cos\theta\cos\phi$$
$$+ \frac{\rho V_a^2 S}{2(mass)}\left\{-\left(C_{Do} + C_{D\alpha}\alpha + \frac{c}{2V_a}C_{Dq}q + C_{D\delta e}\delta_e\right)\sin\alpha - \left(C_{Lo} + C_{L\alpha}\alpha + \frac{c}{2V_a}C_{Lq}q + C_{L\delta e}\delta_e\right)\cos\alpha\right\}$$

Derivatives of Velocities

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta$$

Derivatives of Orientations

$$\dot{p} = \frac{J_{xz}(J_x - J_y + J_z)}{\Gamma}pq - \frac{J_z(J_z - J_y) + J_{xz}^2}{\Gamma}qr - \frac{J_z}{\Gamma}k_{Tp}k_\Omega^2\delta_t^2$$
$$+ \frac{\rho V_a^2 Sb}{2}\left\{\frac{J_z C_{lo} + J_{xz}C_{no}}{\Gamma} + \frac{J_z C_{l\beta} + J_{xz}C_{n\beta}}{\Gamma}\beta + \left(\frac{b}{2V_a}\right)\frac{J_z C_{lp} + J_{xz}C_{np}}{\Gamma}p + \left(\frac{b}{2V_a}\right)\frac{J_z C_{lr} + J_{xz}C_{nr}}{\Gamma}r + \frac{J_z C_{l\delta a} + J_{xz}C_{n\delta a}}{\Gamma}\delta_a + \frac{J_z C_{l\delta r} + J_{xz}C_{n\delta r}}{\Gamma}\delta_r\right\}$$

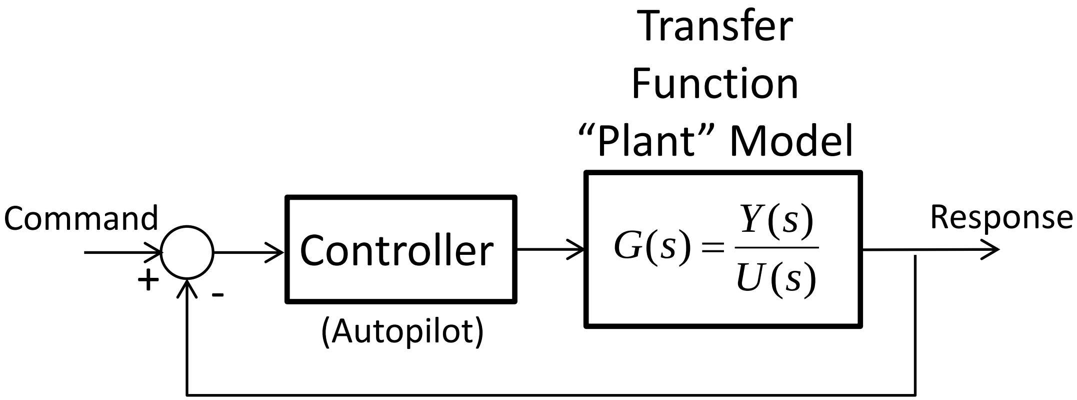
$$\dot{q} = \frac{J_z - J_x}{J_y}pr - \frac{J_{xz}}{J_y}(p^2 - r^2)$$
$$+ \frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}\left\{C_{mo} + C_{m\alpha}\alpha + \frac{c}{2V_a}C_{mq}q + C_{m\delta e}\delta_e\right\}$$

$$\dot{r} = \frac{(J_x - J_y)J_x + J_{xz}^2}{\Gamma}pq - \frac{J_{xz}(J_x - J_y + J_z)}{\Gamma}qr - \frac{J_{xz}}{\Gamma}k_{Tp}k_\Omega^2\delta_t^2$$
$$+ \frac{\rho V_a^2 Sb}{2}\left\{\frac{J_x C_{no} + J_{xz}C_{lo}}{\Gamma} + \frac{J_x C_{n\beta} + J_{xz}C_{l\beta}}{\Gamma}\beta + \left(\frac{b}{2V_a}\right)\frac{J_x C_{np} + J_{xz}C_{lp}}{\Gamma}p + \left(\frac{b}{2V_a}\right)\frac{J_x C_{nr} + J_{xz}C_{lr}}{\Gamma}r + \frac{J_x C_{n\delta a} + J_{xz}C_{l\delta a}}{\Gamma}\delta_a + \frac{J_x C_{n\delta r} + J_{xz}C_{l\delta r}}{\Gamma}\delta_r\right\}$$

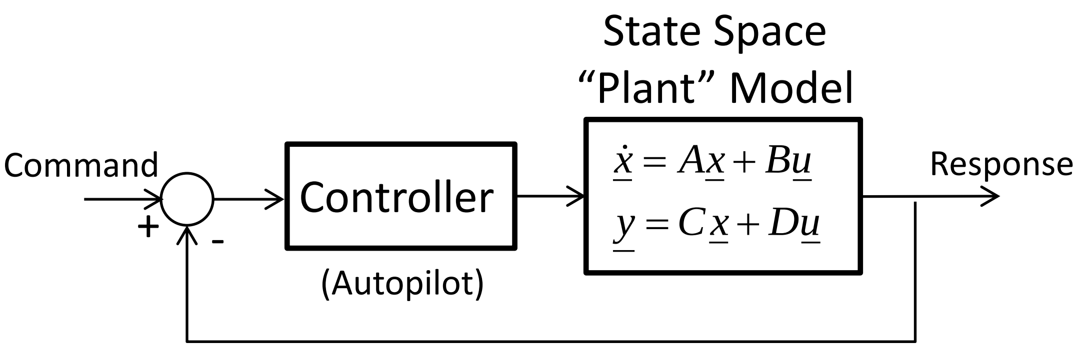
Derivatives of Body Rates

Linear Models in a Control System

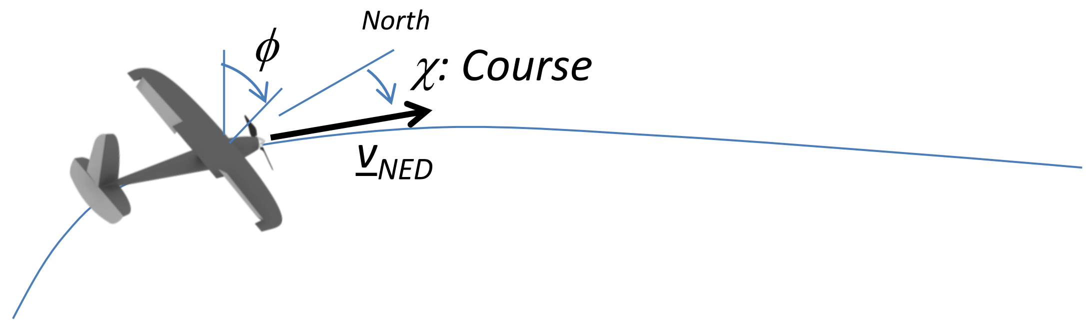
- In control systems, we use linear models to represent input/output relationships of the "plant" model
  - Laplace Transfer Functions (e.g. H(s))
  - State Space Models (e.g. A, B, C, D)

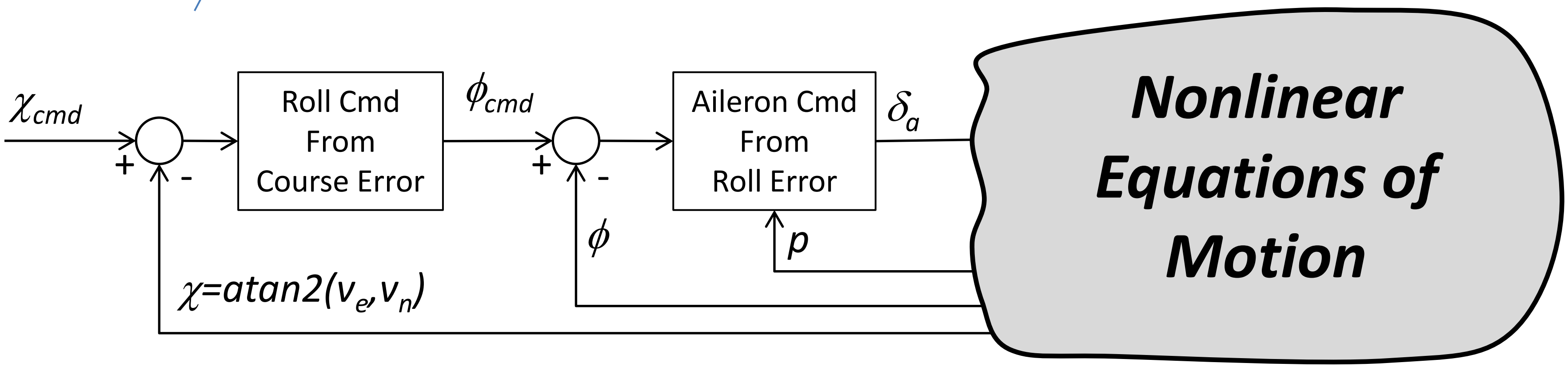


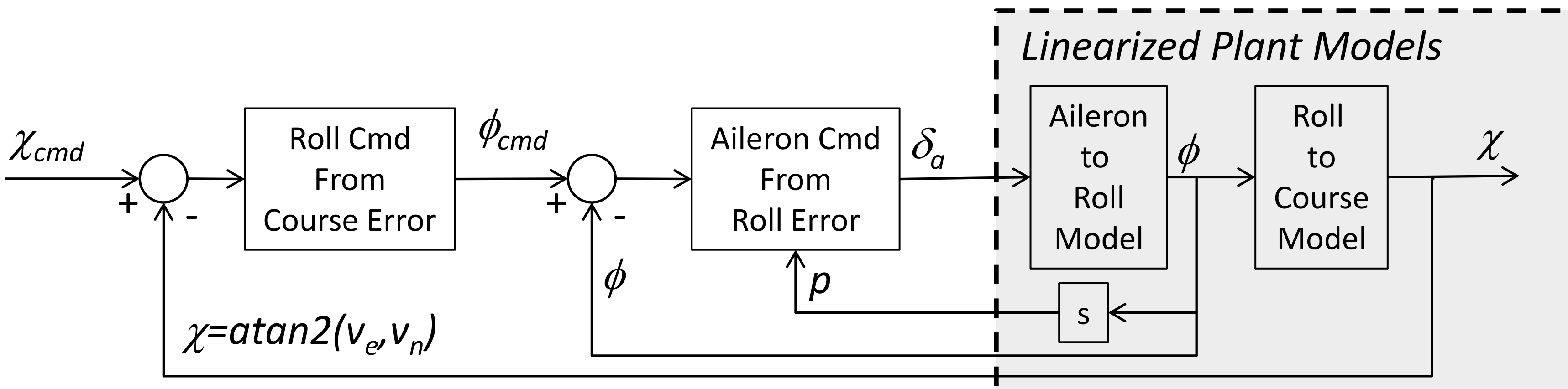
- Control systems engineers (or autopilot designers) develop feedback controllers to regulate overall system performance
  - In unmanned vehicles, the feedback controller is called the "autopilot"



Example: Course Control Autopilot Design



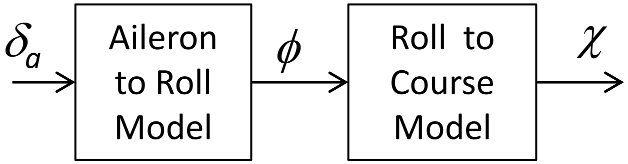


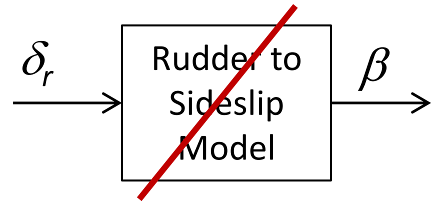


Linear Models for Fixed Wing Flight

- Linear models we'll develop to aid in autopilot design:

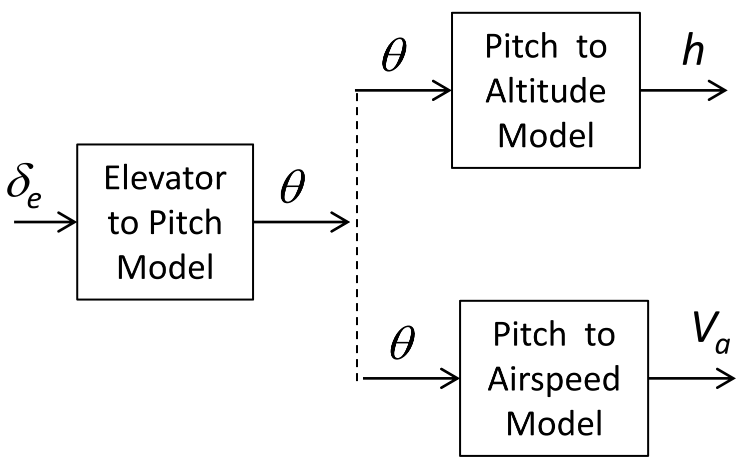
Lateral Channel

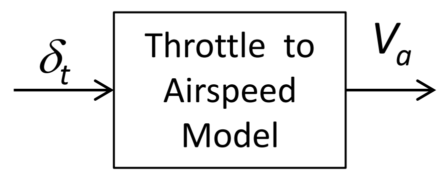




Book develops a sideslip controller using rudder. We won't because:
1) Most UAVs are naturally yaw-stable, and many only use aileron and elevator. (We'll do this.)
2) Sideslip is *very* difficult to measure/estimate, so most UAVs do not try to actively control it. Instead, rudder would control yaw rate for improved turns.

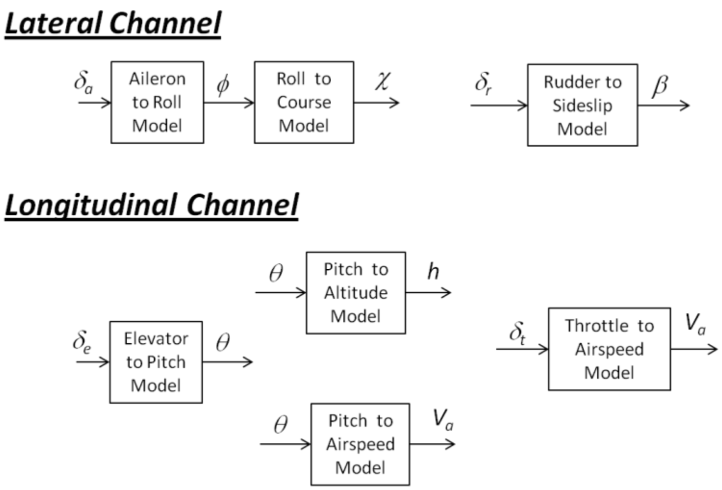
Longitudinal Channel



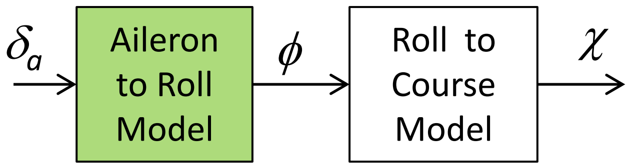


Methods to Acquire Linear Models

- How do we acquire these linear models?
  - Numerical Method: Numerically derive response models using simulation models
    - We did this in a previous lecture
    - We *could* use the numerically derived models to develop an autopilot, but we won't in this course
  - Analytical Method: Analytically derive response models from Equations of Motion
    - We will do this, and use these models to develop an Autopilot
    - Resulting models are "simpler" and hence easier to develop an autopilot controller around



Linear Model for Fixed Wing Roll Response



- To derive the dynamical response of roll, first start with the equation of motion describing the derivative of roll:

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

- Vehicle pitch (θ) is generally fairly small, so the second two terms are generally negligible. Consider them a *disturbance*.

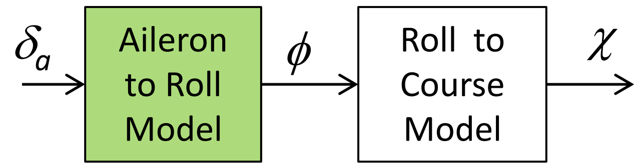
$$\dot{\phi} = p + \underbrace{q\sin\phi\tan\theta + r\cos\phi\tan\theta}_{d_{\phi 1}\ \text{(small)}} \qquad \Rightarrow \dot{\phi} = p + d_{\phi 1}$$

- Differentiate again (because *p* is a state with a governing EoM).

$$\ddot{\phi} = \dot{p} + \dot{d}_{\phi 1}$$

$$\ddot{\phi} = \overbrace{\frac{J_{xz}\left(J_x - J_y + J_z\right)}{\Gamma} pq - \frac{J_z\left(J_z - J_y\right) + J_{xz}^2}{\Gamma} qr - \frac{J_z}{\Gamma} k_{Tp} k_\Omega^2 \delta_t^2 + \frac{\rho V_a^2 S b}{2} \left\{ \begin{array}{c} \frac{J_z C_{lo} + J_{xz} C_{no}}{\Gamma} + \frac{J_z C_{l\beta} + J_{xz} C_{n\beta}}{\Gamma} \beta + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} p \\ + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lr} + J_{xz} C_{nr}}{\Gamma} r + \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \delta_a + \frac{J_z C_{l\delta r} + J_{xz} C_{n\delta r}}{\Gamma} \delta_r \end{array} \right\}}^{\dot{p}} + \dot{d}_{\phi 1}$$

Linear Model for Fixed Wing Roll Response



- From previous, the second derivative of roll is:

$$\ddot{\phi} = \frac{J_{xz}(J_x - J_y + J_z)}{\Gamma} pq - \frac{J_z(J_z - J_y) + J_{xz}^2}{\Gamma} qr - \frac{J_z}{\Gamma} k_{Tp} k_\Omega^2 \delta_t^2 + \frac{\rho V_a^2 S b}{2} \left\{ \begin{array}{c} \frac{J_z C_{lo} + J_{xz} C_{no}}{\Gamma} + \frac{J_z C_{l\beta} + J_{xz} C_{n\beta}}{\Gamma} \beta + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} p \\ + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lr} + J_{xz} C_{nr}}{\Gamma} r + \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \delta_a + \frac{J_z C_{l\delta r} + J_{xz} C_{n\delta r}}{\Gamma} \delta_r \end{array} \right\} + \dot{d}_{\phi 1}$$

- Noting that: $p = \dot{\phi} - d_{\phi 1}$

$$\ddot{\phi} = \frac{J_{xz}(J_x - J_y + J_z)}{\Gamma} pq - \frac{J_z(J_z - J_y) + J_{xz}^2}{\Gamma} qr - \frac{J_z}{\Gamma} k_{Tp} k_\Omega^2 \delta_t^2 + \frac{\rho V_a^2 S b}{2} \left\{ \begin{array}{c} \frac{J_z C_{lo} + J_{xz} C_{no}}{\Gamma} + \frac{J_z C_{l\beta} + J_{xz} C_{n\beta}}{\Gamma} \beta + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \left(\dot{\phi} - d_{\phi 1}\right) \\ + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lr} + J_{xz} C_{nr}}{\Gamma} r + \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \delta_a + \frac{J_z C_{l\delta r} + J_{xz} C_{n\delta r}}{\Gamma} \delta_r \end{array} \right\} + \dot{d}_{\phi 1}$$

- Rearranging to relate roll dynamics with aileron:

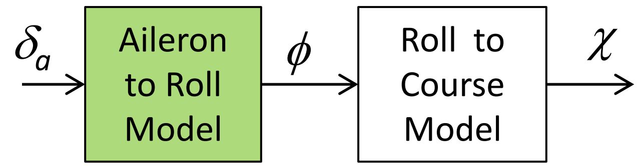
$$\ddot{\phi} = \left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \frac{b}{2V_a} \right) \dot{\phi} + \left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \right) \delta_a$$

$$- \frac{J_z}{\Gamma} k_{Tp} k_\Omega^2 \delta_t^2 + \frac{J_{xz}(J_x - J_y + J_z)}{\Gamma} pq - \frac{J_z(J_z - J_y) + J_{xz}^2}{\Gamma} qr + \frac{\rho V_a^2 S b}{2} \left\{ \begin{array}{c} \frac{J_z C_{lo} + J_{xz} C_{no}}{\Gamma} + \frac{J_z C_{l\beta} + J_{xz} C_{n\beta}}{\Gamma} \beta - \left(\frac{b}{2V_a}\right) \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} d_{\phi 1} \\ + \left(\frac{b}{2V_a}\right) \frac{J_z C_{lr} + J_{xz} C_{nr}}{\Gamma} r + \frac{J_z C_{l\delta r} + J_{xz} C_{n\delta r}}{\Gamma} \delta_r \end{array} \right\} + \dot{d}_{\phi 1}$$

- Let's call everything other than roll and aileron terms a disturbance ☺

$$\ddot{\phi} = \left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \frac{b}{2V_a} \right) \dot{\phi} + \left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \right) \delta_a + d_{\phi 2}$$

Linear Model for Fixed Wing Roll Response



- From previous, roll dynamics is governed by:

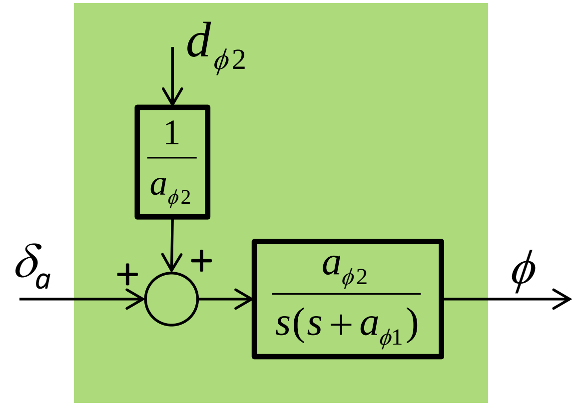
$$\ddot{\phi} = \underbrace{\left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \frac{b}{2V_a} \right)}_{-a_{\phi 1}} \dot{\phi} + \underbrace{\left(\frac{\rho V_a^2 S b}{2} \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma} \right)}_{a_{\phi 2}} \delta_a + d_{\phi 2}$$

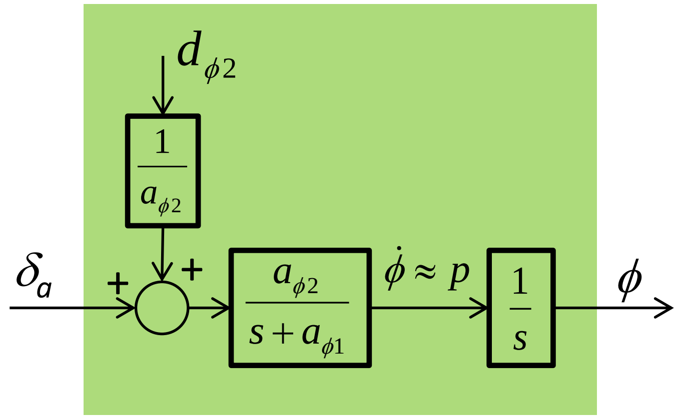
$$\ddot{\phi} = -a_{\phi 1}\dot{\phi} + a_{\phi 2}\delta_a + d_{\phi 2}$$

- Converting to Laplace Domain

$$s^2\phi(s) = -a_{\phi 1} s\phi(s) + a_{\phi 2}\delta_a(s) + d_{\phi 2}(s)$$

$$\Rightarrow \phi(s) = \left(\frac{a_{\phi 2}}{s(s+a_{\phi 1})} \right) \left(\delta_a(s) + \frac{1}{a_{\phi 2}} d_{\phi 2}(s) \right)$$

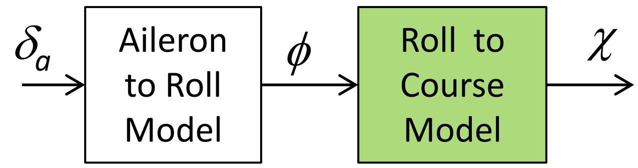




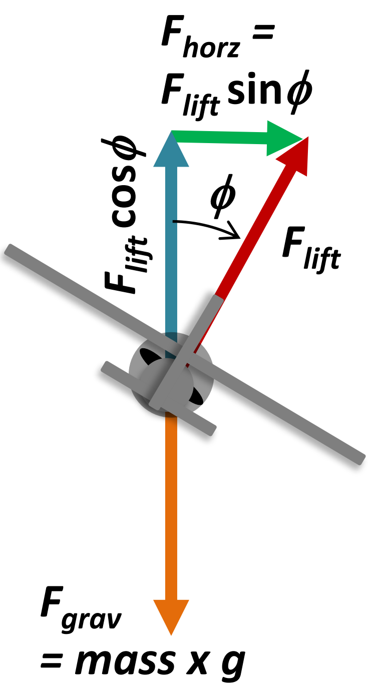
Note 1: This provides a transfer function from aileron to roll for our roll control autopilot. This also shows the affect of a lumped disturbance.

Note 2: Notice that the roll response is a function of airspeed.

Linear Model for Fixed Wing Course Response



- Fixed wing aircraft perform *coordinated turns*
  - Assuming no wind or sideslip and constant altitude, we can relate roll to turn radius and course rate
  - Book gives a more complete (but complicated) derivation



Balancing vertical forces yields:

$$F_{lift} = \frac{mass \cdot g}{\cos\phi}$$

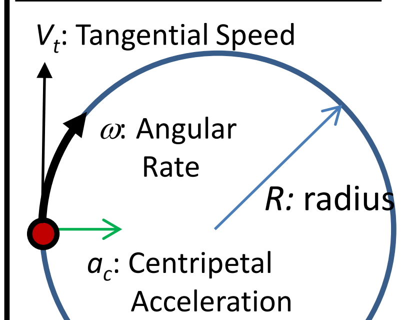
Then, horizontal force is:

$$F_{horz} = mass \cdot g \tan\phi$$

Thus, steady-state horizontal acceleration is:

$$a_{horz} = g \cdot \tan\phi$$

Curvilinear Motion



$$a_c = \frac{V_t^2}{R}\ \frac{\text{m}}{\text{s}^2},\quad \omega = \frac{V_t}{R}\ \frac{\text{rad}}{\text{s}}$$

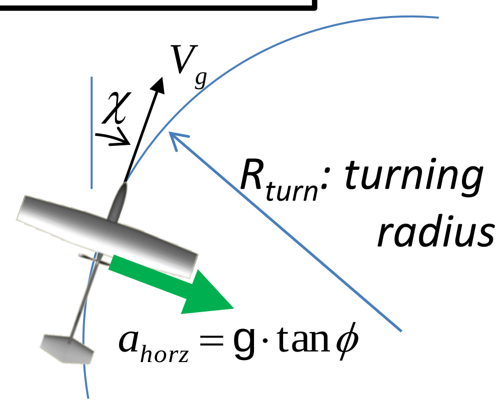
From curvilinear motion:

- Turning Radius, m:

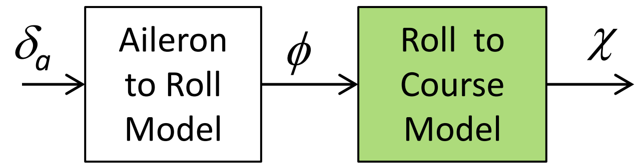
$$R_{turn} = \frac{V_g^2}{a_{horz}} = \frac{V_g^2}{g \cdot \tan\phi}$$

- Course rate, rad/s (aka turning rate):

$$\dot{\chi} = \frac{V_g}{R_{turn}} = \frac{g}{V_g}\tan\phi$$

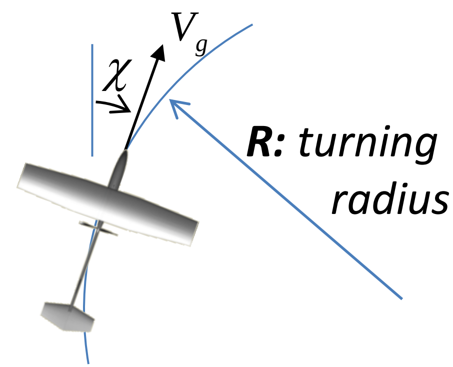


Linear Model for Fixed Wing Course Response



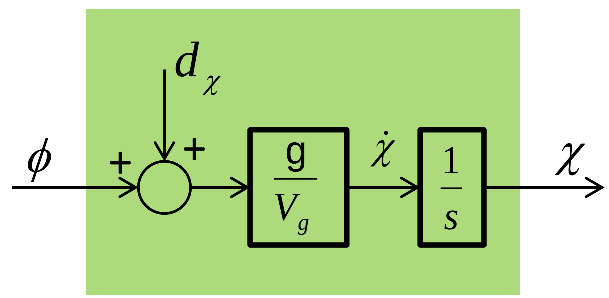
- Given a coordinated turn and constant altitude assumption, a simple model relating roll to course rate is:

$$\dot{\chi} = \frac{g}{V_g} \tan \phi$$



- Re-arranging and calling what we can't deal with linearly a "disturbance":

$$\dot{\chi} = \frac{g}{V_g}\left(\phi + \{\tan(\phi) - \phi\}\right) = \frac{g}{V_g}\left(\phi + d_\chi\right)$$



- In Laplace:

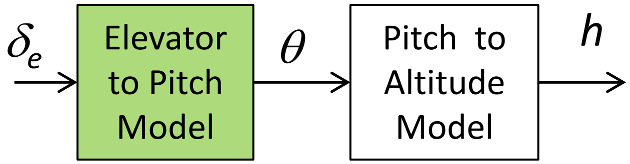
$$s\chi(s) = \frac{g}{V_g}\left(\phi(s) + d_\chi(s)\right)$$

$$\chi(s) = \frac{1}{s}\frac{g}{V_g}\left(\phi(s) + d_\chi(s)\right)$$

Note 1: This provides a transfer function from roll to course. This also shows the affect of a lumped disturbance..

Note 2: Notice that the course response is a function of ground speed.

Linear Model for Fixed Wing Pitch Response



- Similar to deriving "aileron-to-roll", pitch dynamical response is derived from the EoM:

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

- Rearranging and calling nonlinear terms a *disturbance*:

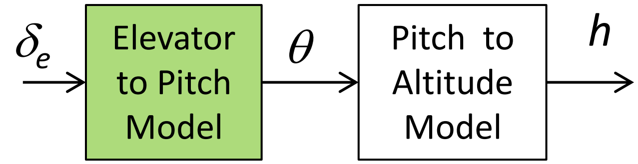
$$\dot{\theta} = q + \underbrace{q(\cos\phi - 1) - r\sin\phi}_{d_{\theta 1}} \qquad \Rightarrow \dot{\theta} = q + d_{\theta 1}$$

- Differentiate again (because *q* is a state with a governing EoM).

$$\ddot{\theta} = \dot{q} + \dot{d}_{\theta 1}$$

$$\ddot{\theta} = \overbrace{\frac{J_z - J_x}{J_y} pr - \frac{J_{xz}}{J_y}\left(p^2 - r^2\right) + \frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\left\{C_{mo} + C_{m\alpha}\alpha + \frac{c}{2V_a}C_{mq}q + C_{m\delta e}\delta_e\right\}}^{\dot{q}} + \dot{d}_{\theta 1}$$

Linear Model for Fixed Wing Pitch Response



- From previous, the second derivative of pitch is:

$$\ddot{\theta} = \frac{J_z - J_x}{J_y} pr - \frac{J_{xz}}{J_y}\left(p^2 - r^2\right) + \frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\left\{C_{mo} + C_{m\alpha}\alpha + \frac{c}{2V_a} C_{mq} q + C_{m\delta e}\delta_e\right\} + \dot{d}_{\theta 1}$$

- Noting that $q = \dot{\theta} - d_{\theta 1}$ and $\alpha \approx \theta - \gamma$ (assuming negligible wind)

$$\ddot{\theta} = \frac{J_z - J_x}{J_y} pr - \frac{J_{xz}}{J_y}\left(p^2 - r^2\right) + \frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\left\{C_{mo} + C_{m\alpha}(\theta - \gamma) + \frac{c}{2V_a} C_{mq}\left(\dot{\theta} - d_{\theta 1}\right) + C_{m\delta e}\delta_e\right\} + \dot{d}_{\theta 1}$$

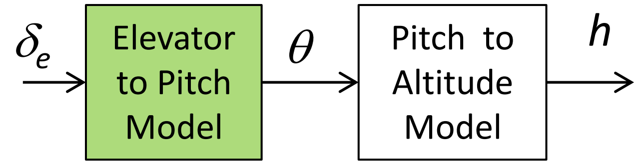
- Rearranging to relate pitch dynamics with elevator:

$$\ddot{\theta} = \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\frac{c}{2V_a} C_{mq}\right)\dot{\theta} + \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y} C_{m\alpha}\right)\theta + \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y} C_{m\delta e}\right)\delta_e$$
$$+ \frac{J_z - J_x}{J_y} pr - \frac{J_{xz}}{J_y}\left(p^2 - r^2\right) + \frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\left\{C_{mo} + C_{m\alpha}\gamma - \frac{c}{2V_a} C_{mq} d_{\theta 1}\right\} + \dot{d}_{\theta 1}$$

- Let's call everything other than pitch and elevator terms a disturbance ☺

$$\ddot{\theta} = \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y}\frac{c}{2V_a} C_{mq}\right)\dot{\theta} + \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y} C_{m\alpha}\right)\theta + \left(\frac{\rho V_a^2 S c}{2}\frac{1}{J_y} C_{m\delta e}\right)\delta_e + d_{\theta 2}$$

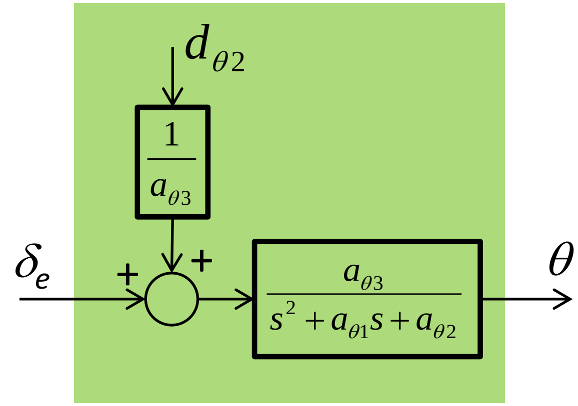
Linear Model for Fixed Wing Pitch Response



- From previous, pitch dynamics is governed by:

$$\ddot{\theta} = \underbrace{\left(\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}\frac{c}{2V_a}C_{mq}\right)}_{-a_{\theta 1}}\dot{\theta} + \underbrace{\left(\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}C_{m\alpha}\right)}_{-a_{\theta 2}}\theta + \underbrace{\left(\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}C_{m\delta e}\right)}_{a_{\theta 3}}\delta_e + d_{\theta 2}$$

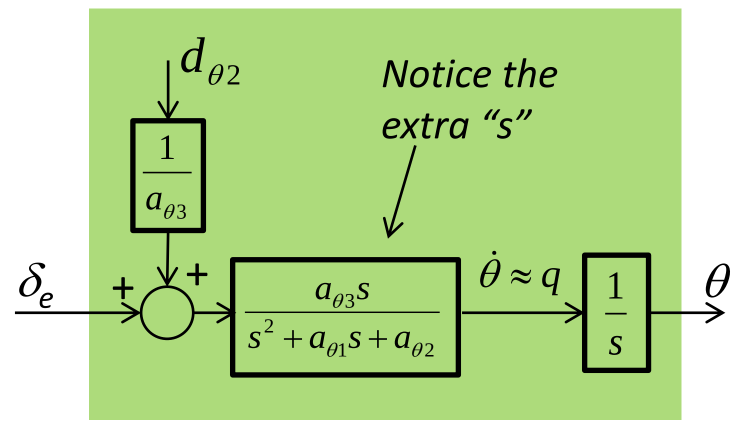
$$\ddot{\theta} = -a_{\theta 1}\dot{\theta} - a_{\theta 2}\theta + a_{\theta 3}\delta_e + d_{\theta 2}$$



- Converting to Laplace Domain

$$s^2\theta(s) = -a_{\theta 1}s\theta(s) - a_{\theta 2}\theta(s) + a_{\theta 3}\delta_e(s) + d_{\theta 2}(s)$$

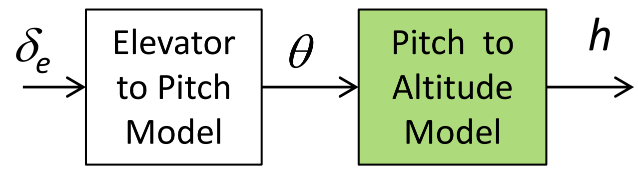
$$\Rightarrow \theta(s) = \left(\frac{a_{\theta 3}}{s^2 + a_{\theta 1}s + a_{\theta 2}}\right)\left(\delta_e(s) + \frac{1}{a_{\theta 3}}d_{\theta 2}(s)\right)$$



Note 1: This provides a transfer function from elevator to pitch for our pitch control autopilot. This also shows the affect of a lumped disturbance.

Note 2: Notice that the pitch response is a function of airspeed.

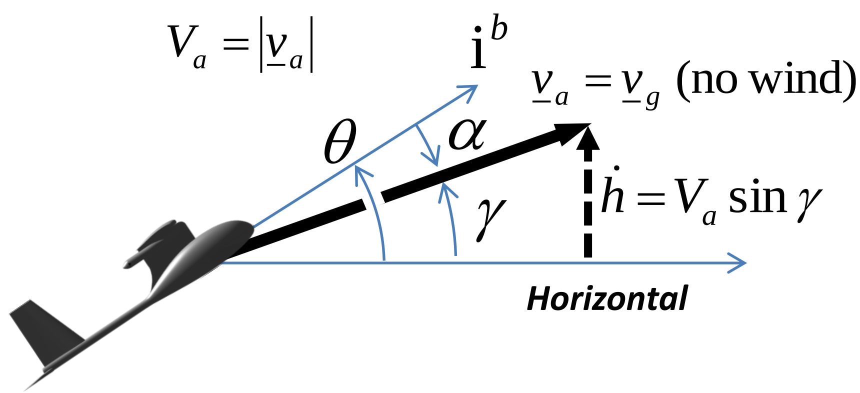
Linear Model for Fixed Wing Altitude Response



- For constant airspeed, pitch directly influences climb rate.
- In the absence of wind and sideslip:

$$\dot{h} \approx V_a \sin\gamma$$

$$\dot{h} \approx V_a \sin(\theta - \alpha)$$



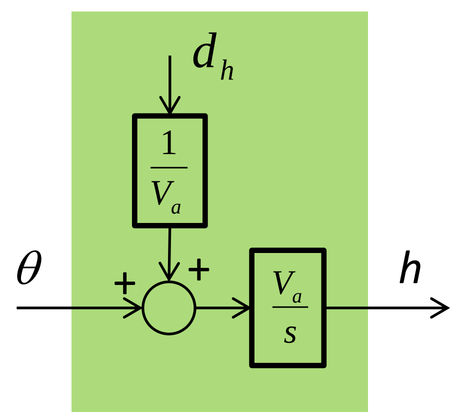
- Using small angle approximation:

$$\dot{h} \approx V_a(\theta - \alpha) = V_a\theta - V_a\alpha$$

- Calling non-pitch stuff "disturbances", and converting to Laplace:

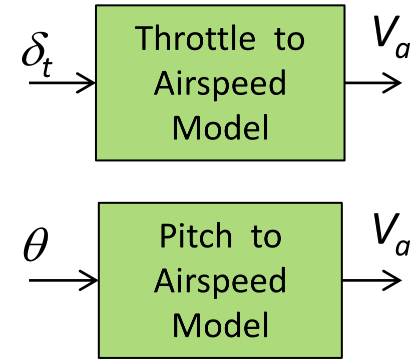
$$\dot{h} = V_a\theta + d_h$$

$$h(s) = \frac{V_a}{s}\left(\theta(s) + d_h(s)\right)$$



This provides a transfer function from pitch to altitude for our altitude control autopilot. This also shows the affect of a lumped disturbance.

Linear Model for Fixed Wing Airspeed Response

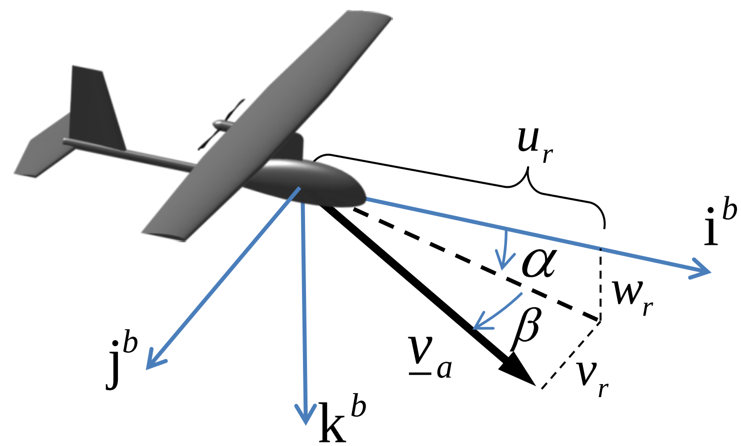


- Airspeed is more complicated because it is a strong function of both throttle and pitch
  - During launch, climbs and descents UAVs often use pitch to control airspeed (with a constant throttle)
  - During level flight, UAVs use throttle to control airspeed

- Recall airspeed body-frame vector relationships:

$$\underline{v}_a^b = \begin{bmatrix} u - u_w \\ v - v_w \\ w - w_w \end{bmatrix} = \begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \begin{bmatrix} V_a \cos\alpha\cos\beta \\ V_a \sin\beta \\ V_a \sin\alpha\cos\beta \end{bmatrix}$$

$$V_a = \left|\underline{v}_a^b\right| = \sqrt{u_r^2 + v_r^2 + w_r^2}$$



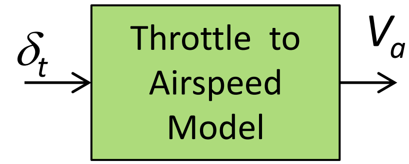
- Differentiating airspeed:

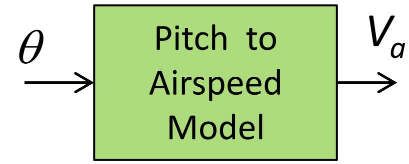
$$\dot{V}_a = \frac{u_r \dot{u}_r + v_r \dot{v}_r + w_r \dot{w}_r}{V_a}$$

- Using aerodynamic angle relationships:

$$\dot{V}_a = \dot{u}_r \cos\alpha\cos\beta + \dot{v}_r \sin\beta + \dot{w}_r \sin\alpha\cos\beta$$

Linear Model for Fixed Wing Airspeed Response





- Derivative of airspeed in body coordinates:

$$\dot{V}_a = \dot{u}_r \cos\alpha \cos\beta + \dot{v}_r \sin\beta + \dot{w}_r \sin\alpha \cos\beta$$

- Re-arrange so sideslip terms are a "disturbance"

$$\dot{V}_a = \dot{u}_r \cos\alpha + \dot{w}_r \sin\alpha + \left\{ \dot{u}_r \cos\alpha(\cos\beta - 1) + \dot{v}_r \sin\beta + \dot{w}_r \sin\alpha(\cos\beta - 1) \right\}$$

$$\dot{V}_a = \dot{u}_r \cos\alpha + \dot{w}_r \sin\alpha + d_{V1}$$

- Assuming no wind, $u_r = u$, and $w_r = w$

$$\dot{V}_a = \dot{u} \cos\alpha + \dot{w} \sin\alpha + d_{V1}$$

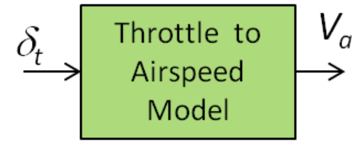
Recall: This expression differs from book due to corrected propeller force equation

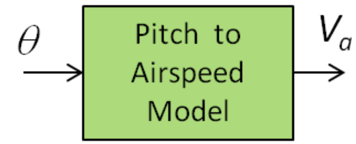
- Plugging in EoMs:

$$\dot{V}_a = \left\{ \begin{array}{c} rv - qw - g\sin\theta + \frac{\rho C_{prop} S_{prop}}{\text{mass}} \{V_a + \delta_t(k_{motor} - V_a)\}\{\delta_t(k_{motor} - V_a)\} \\ + \frac{\rho V_a^2 S}{2(mass)} \left\{ -\left(C_{Do} + C_{D\alpha}\alpha + \frac{c}{2V_a} C_{Dq} q + C_{D\delta e}\delta_e\right)\cos\alpha + \left(C_{Lo} + C_{L\alpha}\alpha + \frac{c}{2V_a} C_{Lq} q + C_{L\delta e}\delta_e\right)\sin\alpha \right\} \end{array} \right\} \cos\alpha$$

$$+ \left\{ \begin{array}{c} qu - pv + g\cos\theta\cos\phi \\ + \frac{\rho V_a^2 S}{2(mass)} \left\{ -\left(C_{Do} + C_{D\alpha}\alpha + \frac{c}{2V_a} C_{Dq} q + C_{D\delta e}\delta_e\right)\sin\alpha - \left(C_{Lo} + C_{L\alpha}\alpha + \frac{c}{2V_a} C_{Lq} q + C_{L\delta e}\delta_e\right)\cos\alpha \right\} \end{array} \right\} \sin\alpha + d_{V1}$$

Linear Model for Fixed Wing Airspeed Response





Simplifications of airspeed rate equation **for the interested reader**:

- Clean up and use: *cos²α + sin²α=1* (C_L terms disappear)

$$\dot{V}_a = rv\cos\alpha - qw\cos\alpha + qu\sin\alpha - pv\sin\alpha - g\sin\theta\cos\alpha + g\cos\theta\cos\phi\sin\alpha$$
$$+\frac{\rho C_{prop}S_{prop}}{mass}\{V_a+\delta_t(k_{motor}-V_a)\}\{\delta_t(k_{motor}-V_a)\}\cos\alpha - \frac{\rho V_a^2 S}{2(mass)}\left(C_{Do}+C_{D\alpha}\alpha+\frac{c}{2V_a}C_{Dq}q+C_{D\delta e}\delta_e\right)+d_{V1}$$

- Simplify assuming no wind:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \begin{bmatrix} V_a\cos\alpha\cos\beta \\ V_a\sin\beta \\ V_a\sin\alpha\cos\beta \end{bmatrix} \Rightarrow \begin{aligned} &rv\cos\alpha - qw\cos\alpha + qu\sin\alpha - pv\sin\alpha \\ &= (rV_a\cos\alpha - pV_a\sin\alpha)\sin\beta \end{aligned}$$

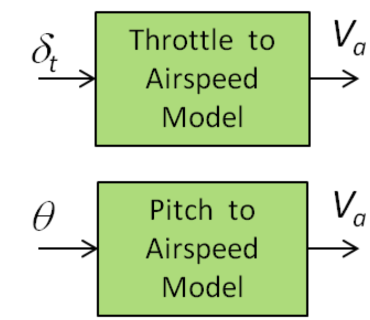
- Add: $zero = g\cos\theta\sin\alpha - g\cos\theta\sin\alpha$

$$\Rightarrow -g\sin\theta\cos\alpha + g\cos\theta\cos\phi\sin\alpha + (g\cos\theta\sin\alpha - g\cos\theta\sin\alpha)$$
$$= (-g\sin\theta\cos\alpha + g\cos\theta\sin\alpha) + g\cos\theta\sin\alpha(\cos\phi - 1)$$
$$= -g\sin(\theta-\alpha) + g\cos\theta\sin\alpha(\cos\phi-1)$$

- Use [cos*α* = *1*+ cos*α* - *1*] and rearrange:

$$\dot{V}_a = -g\sin(\theta-\alpha) - \frac{\rho V_a^2 S}{2(mass)}\left(C_{Do}+C_{D\alpha}\alpha+\frac{c}{2V_a}C_{Dq}q+C_{D\delta e}\delta_e\right)+\frac{\rho C_{prop}S_{prop}}{mass}\{V_a+\delta_t(k_{motor}-V_a)\}\{\delta_t(k_{motor}-V_a)\}$$
$$+\underbrace{\left\{d_{V1}+(rV_a\cos\alpha - pV_a\sin\alpha)\sin\beta - g\cos\theta\sin\alpha(1-\cos\phi)+\frac{\rho C_{prop}S_{prop}}{mass}\{V_a+\delta_t(k_{motor}-V_a)\}\{\delta_t(k_{motor}-V_a)\}(1-\cos\alpha)\right\}}_{d_{V2}}$$

Linear Model for Fixed Wing Airspeed Response



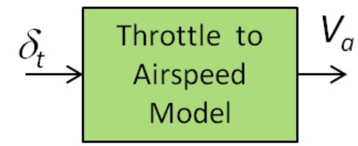
- After simplifying (see previous slide for details):

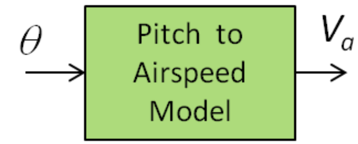
$$\dot{V}_a = \underbrace{\tfrac{\rho C_{prop} S_{prop}}{mass}\{V_a + \delta_t(k_{motor} - V_a)\}\{\delta_t(k_{motor} - V_a)\}}_{\text{Thrust/mass}} \underbrace{- \tfrac{\rho V_a^2 S}{2(mass)}\left(C_{Do} + C_{D\alpha}\alpha + \tfrac{c}{2V_a} C_{Dq} q + C_{D\delta_e}\delta_e\right)}_{\text{minus Drag/mass}} \underbrace{- g\sin(\theta - \alpha)}_{\text{minus gravity along axial}} + d_{V2}$$

- We have a simplified expression for airspeed rate, but it is a non-linear function of airspeed, pitch and throttle:

$$\dot{V}_a = f(V_a, \delta_t, \theta)$$ We'll have to linearize

Linear Model for Fixed Wing Airspeed Response





- Must linearize our expression for airspeed rate:

$$\dot{V}_a = f(V_a, \delta_t, \theta)$$

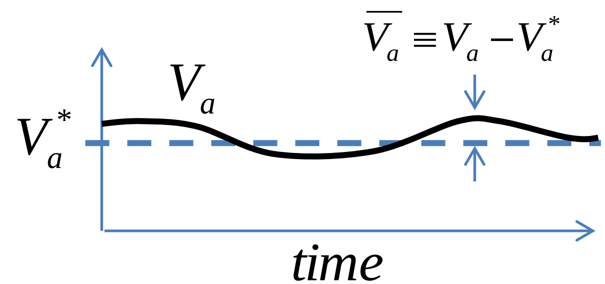
- Define a trim (steady-flight) condition:

$$V_a = V_a^*,\quad \theta = \theta^*,\quad \alpha = \alpha^*,\quad \delta_e = \delta_e^*,\quad \delta_t = \delta_t^*,\quad q = q^* = 0$$

- Define deviations from trim:

$$\overline{V}_a \equiv V_a - V_a^*,\quad \overline{\delta}_t \equiv \delta_t - \delta_t^*,\quad \overline{\theta} \equiv \theta - \theta^*$$

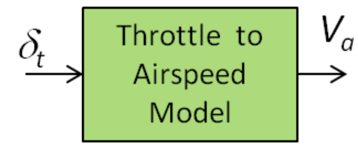
Airspeed Deviation Example

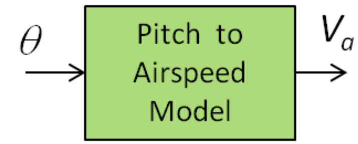


- Linearize deviations about trim:

$$\begin{aligned}
\dot{\overline{V}}_a &= \dot{V}_a - \dot{V}_a^* \\
&= f(V_a, \delta_t, \theta) - f(V_a^*, \delta_t^*, \theta^*) \\
&= f(V_a + V_a^* - V_a^*,\ \delta_t + \delta_t^* - \delta_t^*,\ \theta + \theta^* - \theta^*) - f(V_a^*, \delta_t^*, \theta^*) \\
&= f(V_a^* + \overline{V}_a,\ \delta_t^* + \overline{\delta}_t,\ \theta^* + \overline{\theta}) \quad - \quad f(V_a^*, \delta_t^*, \theta^*) \\
&= \left\{ f(V_a^*, \delta_t^*, \theta^*) + \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial V_a^*}\overline{V}_a + \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial \delta_t^*}\overline{\delta}_t + \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial \theta^*}\overline{\theta} + H.O.T. \right\} - f(V_a^*, \delta_t^*, \theta^*) \\
&= \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial V_a^*}\overline{V}_a + \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial \delta_t^*}\overline{\delta}_t + \frac{\partial f(V_a^*, \delta_t^*, \theta^*)}{\partial \theta^*}\overline{\theta} + H.O.T.
\end{aligned}$$

Linear Model for Fixed Wing Airspeed Response





- Non-linear airspeed function: $\dot{V}_a = f(V_a, \delta_t, \theta)$

$$\dot{V}_a = \frac{\rho C_{prop} S_{prop}}{mass}\{V_a + \delta_t(k_{motor} - V_a)\}\{\delta_t(k_{motor} - V_a)\}$$
$$- \frac{\rho V_a^2 S}{2(mass)}\left(C_{Do} + C_{D\alpha}\alpha + \frac{c}{2V_a}C_{Dq}q + C_{D\delta e}\delta_e\right) - g\sin(\theta - \alpha) + d_{V2}$$

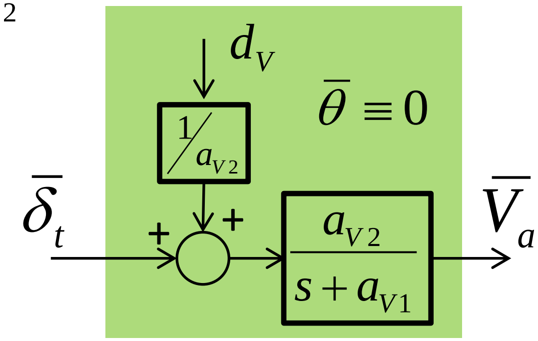
- Linearization about trim condition:

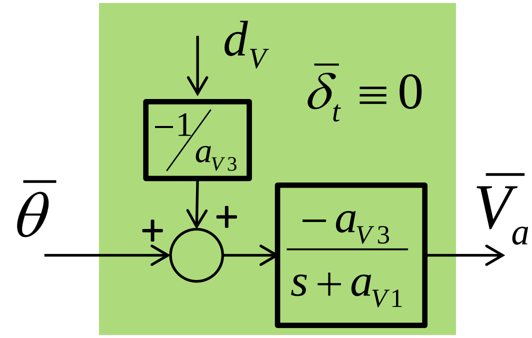
$$\dot{\bar{V}}_a = \underbrace{\frac{\partial f(V_a^*, \theta^*, \delta_t^*)}{\partial V_a^*}}_{-a_{V1}}\bar{V}_a + \underbrace{\frac{\partial f(V_a^*, \theta^*, \delta_t^*)}{\partial \delta_t^*}}_{+a_{V2}}\bar{\delta}_t + \underbrace{\frac{\partial f(V_a^*, \theta^*, \delta_t^*)}{\partial \theta^*}}_{-a_{V3}}\bar{\theta} + d_V$$

$$a_{V1} = -\frac{\rho C_{prop} S_{prop}}{mass}\{\delta_t^*(1 - 2\delta_t^*)(k_{motor} - V_a^*) - \delta_t^* V_a^*\}$$
$$+ \frac{\rho V_a^* S}{mass}\left(C_{Do} + C_{D\alpha}\alpha^* + C_{D\delta e}\delta_e^*\right)$$

$$a_{V2} = \frac{\rho C_{prop} S_{prop}}{mass}(k_{motor} - V_a^*)\left(V_a^* + 2\delta_t^*(k_{motor} - V_a^*)\right)$$

$$a_{V3} = g\cos(\theta^* - \alpha^*)$$





- To Laplace:

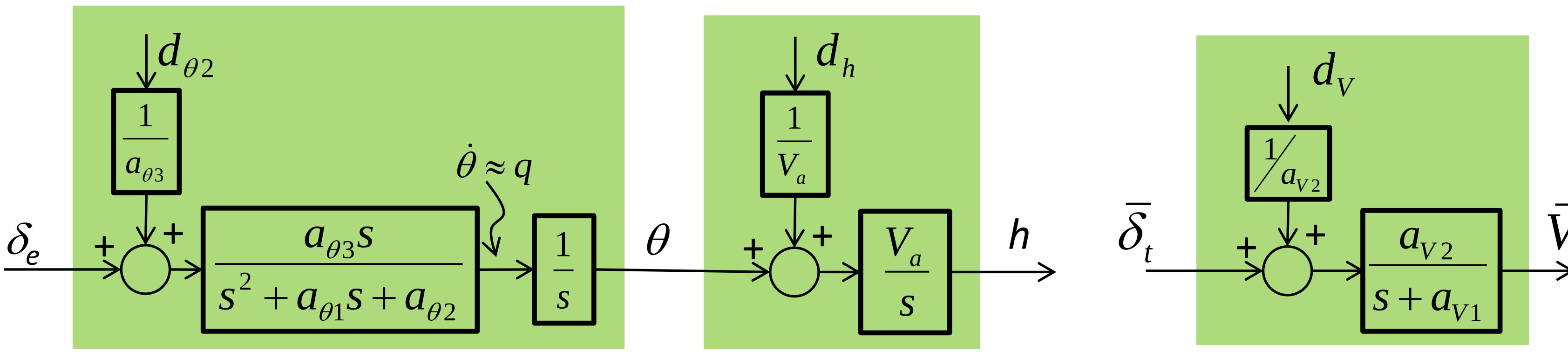
$$s\bar{V}_a(s) = -a_{V1}\bar{V}_a(s) + a_{V2}\bar{\delta}_t(s) - a_{V3}\bar{\theta}(s) + d_V(s)$$

$$\bar{V}_a(s) = \frac{1}{s + a_{V1}}\left(a_{V2}\bar{\delta}_t(s) - a_{V3}\bar{\theta}(s) + d_V(s)\right)$$

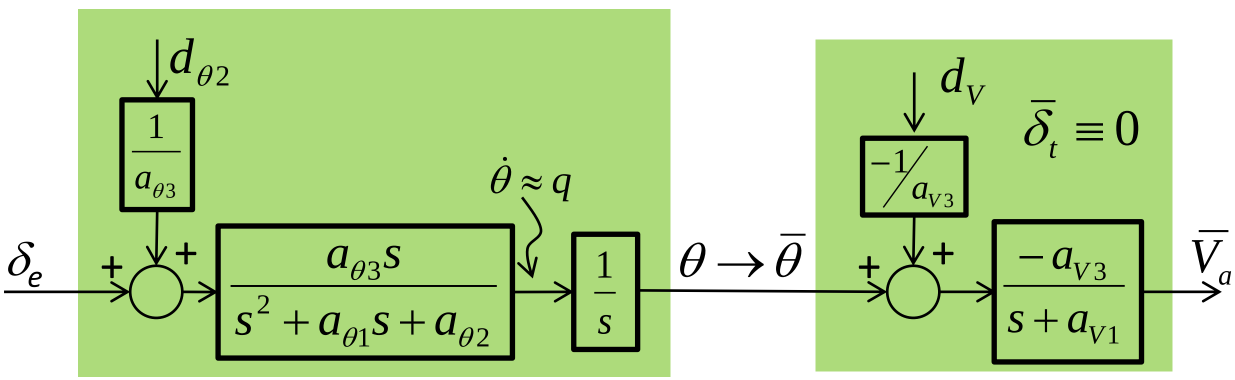
Note: The ins and outs of these models are "variations from trim" values, rather than full values. But, we'll use these models directly in our autopilot development. (An integrator will account for "trim" values.)

Fixed Wing Longitudinal Linear Models

Level Flight Mode: Pitch controls Altitude, Throttle controls Airspeed



Launch/Climb/Descend Modes: Pitch controls Airspeed, Throttle is fixed



$$a_{\theta 1} = -\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} \frac{c}{2V_a} C_{mq}$$

$$a_{\theta 2} = -\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} C_{m\alpha}$$

$$a_{\theta 3} = +\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} C_{m\delta e}$$

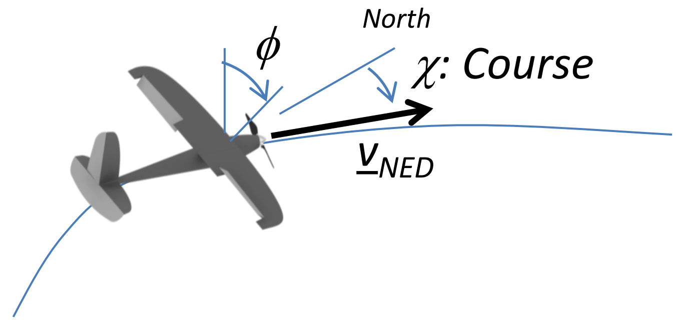
$$a_{V1} = -\frac{\rho C_{prop} S_{prop}}{mass} \left\{ \delta_t^* \left(1 - 2\delta_t^*\right)\left(k_{motor} - V_a^*\right) - \delta_t^* V_a^* \right\} + \frac{\rho V_a^* S}{mass}\left(C_{Do} + C_{D\alpha}\alpha^* + C_{D\delta e}\delta_e^*\right)$$

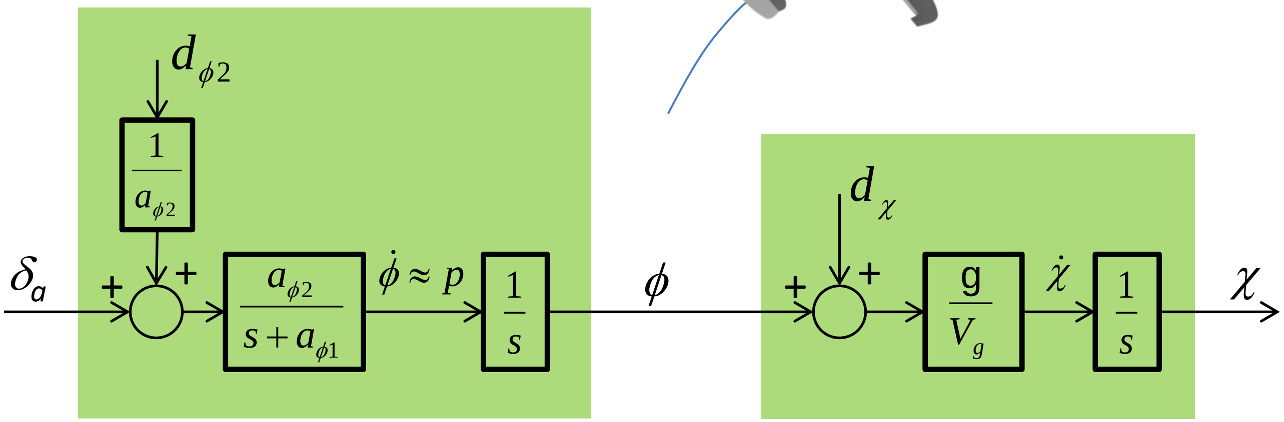
$$a_{V2} = \frac{\rho C_{prop} S_{prop}}{mass}\left(k_{motor} - V_a^*\right)\left(V_a^* + 2\delta_t^*\left(k_{motor} - V_a^*\right)\right)$$

$$a_{V3} = g\cos\left(\theta^* - \alpha^*\right)$$

*x** values are "trim" values. Response models are a function of nominal flight condition.

Fixed Wing Lateral Linear Models





$$a_{\phi 1} = -\frac{\rho V_a^2 S b}{2} \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \frac{b}{2V_a}$$

$$\Gamma = J_x J_z - J_{xz}^2$$

$$a_{\phi 2} = +\frac{\rho V_a^2 S b}{2} \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma}$$

$$V_g \approx V_a$$

Linear Model Coefficients

Longitudinal Coefficients, repeated for readability

$$a_{\theta 1} = -\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}\frac{c}{2V_a}C_{mq}$$

$$a_{\theta 2} = -\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}C_{m\alpha}$$

$$a_{\theta 3} = +\frac{\rho V_a^2 Sc}{2}\frac{1}{J_y}C_{m\delta e}$$

$$a_{V1} = -\frac{\rho C_{prop} S_{prop}}{mass}\left\{\delta_t^*\left(1-2\delta_t^*\right)\left(k_{motor}-V_a^*\right)-\delta_t^* V_a^*\right\} + \frac{\rho V_a^* S}{mass}\left(C_{Do}+C_{D\alpha}\alpha^*+C_{D\delta e}\delta_e^*\right)$$

$$a_{V2} = \frac{\rho C_{prop} S_{prop}}{mass}\left(k_{motor}-V_a^*\right)\left(V_a^*+2\delta_t^*\left(k_{motor}-V_a^*\right)\right)$$

$$a_{V3} = g\cos\left(\theta^*-\alpha^*\right)$$

x^* values are "trim" values. Response models are a function of nominal flight condition.

Lateral Coefficients, repeated for readability

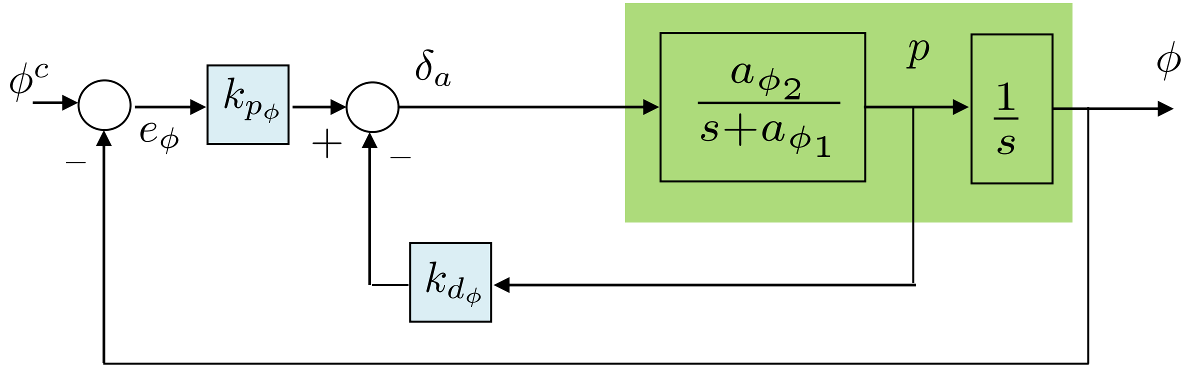
$$a_{\phi 1} = -\frac{\rho V_a^2 Sb}{2}\frac{J_z C_{lp}+J_{xz}C_{np}}{\Gamma}\frac{b}{2V_a}$$

$$a_{\phi 2} = +\frac{\rho V_a^2 Sb}{2}\frac{J_z C_{l\delta a}+J_{xz}C_{n\delta a}}{\Gamma}$$

$$\Gamma = J_x J_z - J_{xz}^2$$

Roll Control Autopilot

- Using the linear design models, we can develop a "cookie-cutter" approach to autopilot design using Proportional with Rate Feedback
- Consider the closed-loop roll control transfer function, compared to a canonical 2nd order TF



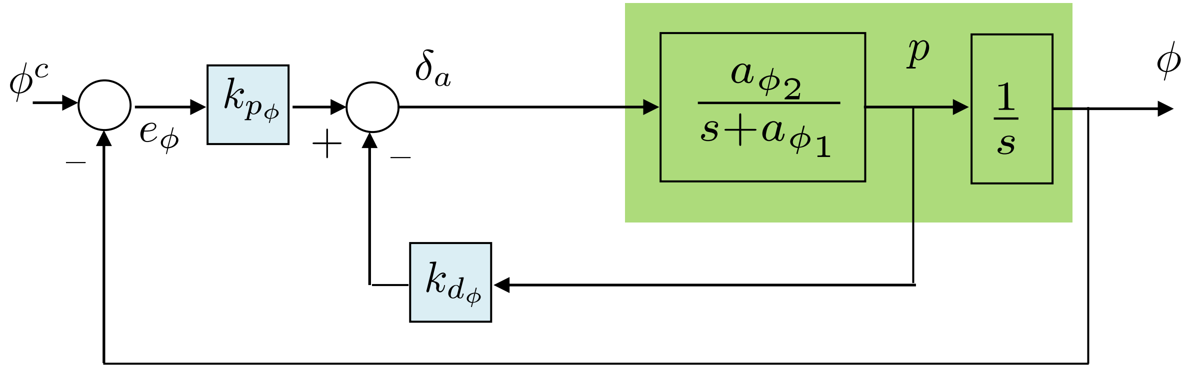
$$H_{\phi/\phi^c}(s) = \underbrace{\frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + a_{\phi_2} k_{d_\phi})s + k_{p_\phi} a_{\phi_2}}}_{\text{Closed Loop TF}} = \underbrace{\frac{\omega_{n_\phi}^2}{s^2 + 2\zeta_\phi \omega_{n_\phi} s + \omega_{n_\phi}^2}}_{\text{Canonical } 2^{nd}\text{-order TF}}$$

Roll Control Autopilot

- Clearly, by satisfying the following two equalities, we can achieve any 2nd order response we want, right? Two equations, two unknowns?

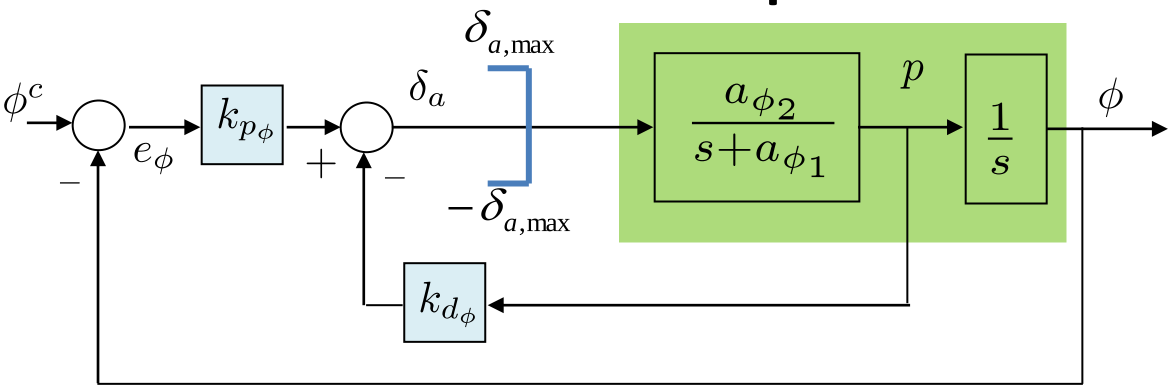
$$k_{p,\phi}\, a_{\phi 2} = \omega_{n,\phi}^2 \qquad\qquad a_{\phi 1} + a_{\phi 2} k_{d,\phi} = 2\zeta_\phi \omega_{n,\phi}$$

- Not quite… because we need to account for aileron limits!



$$H_{\phi/\phi^c}(s) = \underbrace{\frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + a_{\phi_2} k_{d_\phi})s + k_{p_\phi} a_{\phi_2}}}_{\text{Closed Loop TF}} = \underbrace{\frac{\omega_{n_\phi}^2}{s^2 + 2\zeta_\phi \omega_{n_\phi} s + \omega_{n_\phi}^2}}_{\text{Canonical } 2^{nd}\text{-order TF}}$$

Roll Control Autopilot



$$H_{\phi/\phi^c}(s) = \frac{k_{p_\phi} a_{\phi_2}}{s^2 + \underbrace{(a_{\phi_1} + a_{\phi_2} k_{d_\phi})}_{2\zeta_\phi \omega_{n,\phi}} s + \underbrace{k_{p_\phi} a_{\phi_2}}_{\omega_{n,\phi}^2}}$$

- Ideally, we'd like to avoid hitting the aileron limit for any reasonable roll step changes. We can design for this!
  - Select $k_{p\phi}$ such that the ailerons saturate only when the roll error (e_ϕ) exceeds some design threshold, $e_{\phi,max}$. (Also, positive roll error should cause a positive p.)

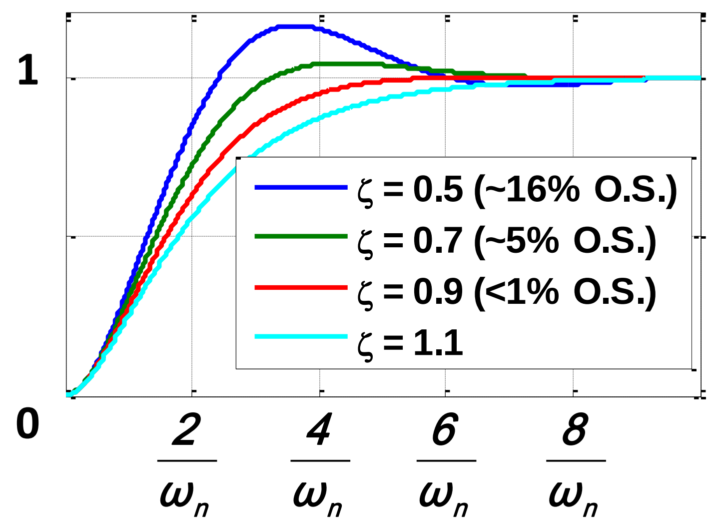
$$k_{p,\phi} = \frac{\delta_{a,\max}}{e_{\phi,\max}} sign\left(a_{\phi 2}\right)$$

  - Then, $k_{p\phi}$ dictates the natural frequency:

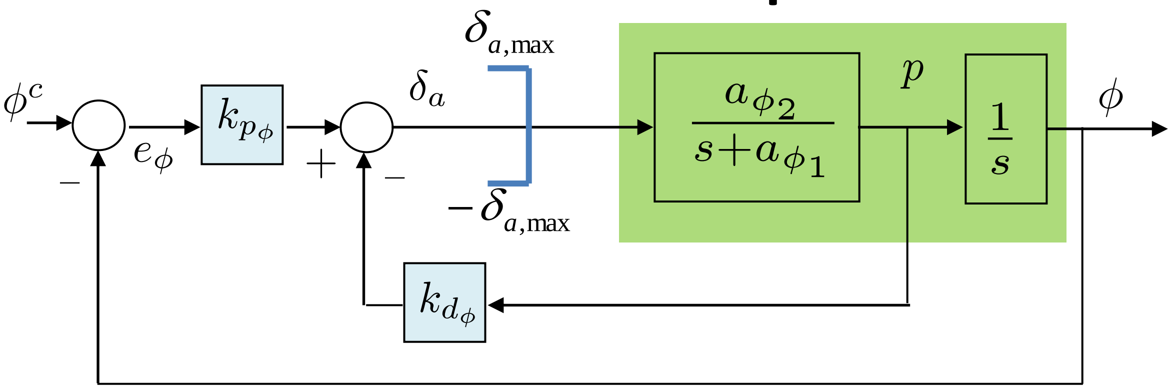
$$\omega_{n,\phi} = \sqrt{k_{p,\phi}\, a_{\phi 2}}$$

  - Finally, we can use ζ_ϕ as a 2[nd] design parameter (i.e. to choose overshoot), resulting in an expression for $k_{d\phi}$:

$$k_{d,\phi} = \frac{2\zeta_\phi \omega_{n,\phi} - a_{\phi 1}}{a_{\phi 2}}$$



Roll Control Autopilot



$$H_{\phi/\phi^c}(s) = \frac{k_{p_\phi} a_{\phi_2}}{s^2 + \underbrace{(a_{\phi_1} + a_{\phi_2} k_{d_\phi})}_{2\zeta_\phi \omega_{n,\phi}} s + \underbrace{k_{p_\phi} a_{\phi_2}}_{\omega_{n,\phi}^2}}$$

- Using a simplified linear design model, we've developed a method to analytically choose autopilot gains for roll control
  - Choose design parameters:
    - $e_{\phi,max}$: Amount of roll error which will cause aileron saturation (smaller value means faster response, but more saturation)
    - ζ_ϕ : Use to choose overshoot
  - Then:

$$k_{p,\phi} = \frac{\delta_{a,\max}}{e_{\phi,\max}} sign\left(a_{\phi 2}\right) \qquad \omega_{n,\phi} = \sqrt{k_{p,\phi}\, a_{\phi 2}} \qquad k_{d,\phi} = \frac{2\zeta_\phi \omega_{n,\phi} - a_{\phi 1}}{a_{\phi 2}}$$

  - Note: Book describes using an integral gain to remove steady-state error due to disturbances. We won't. Integrators add delay and instability, which isn't desired on inner loops. An integrator on the course loop will correct any steady state errors. (Per advice on book website.)

Assumptions/Simplifications we've made:

- NED (attached to ground) is an inertial reference frame
- Presuming a flat earth
- Vehicle has constant mass and inertia
- Vehicle is a rigid body
- Vehicle has symmetry about x-z plane
- Small angle-of-attack for linear and laminar flow
- Aero moments provided about c.g.
- Forces & moments derived for single axial propeller
- Numerous assumptions made to develop simplified linear models (only used in autopilot design, not in simulation)
  - e.g. No wind, small angle approximations, disturbance lumping, etc.

Lecture 7 Homework, 1/4

Start from `load_uavsim` with a trim at Va=13 m/s

Suggested Reading: In Chapter 5: 60-77. In Chapter 6: 95-102

1) Modify `compute_tf_models.m` to generate the linear design models described in this lecture. Print out and turn in each of the resulting transfer functions. (e.g. type: `models.G_de2q`**, etc.)**

Where necessary, use trim values:
`P.Va0, P.theta0, P.alpha0, P.delta_e0, P.delta_t0`

2) Compare the elevator to pitch-rate simplified linear transfer function (models.G_de2q) with the (higher-fidelity) linearized models from the state-space linearization we created a few weeks ago, H(kq,kde).
a) How do the transfer functions compare (poles and zeros)
b) How do the step responses compare? (Hint: Limit the step response to the first 5-10 seconds.) How do they compare during the first portion of a second? How do they compare thereafter?

3) Similarly, compare models.G_da2p with H(kp,kda).

Lecture 7 Homework, 2/4

4) Modify `compute_autopilot_gains.m` to generate the roll control PIR gains described in this lecture. Design for a nominal airspeed of 13 m/s, and use the following design parameters: $e_{\phi,max}$=45deg, and ζ_ϕ=0.9. (Note: integrator gain $k_{\phi i}$ will be 0.)

a) What are your resulting gains?

b) Plot the resulting closed loop roll step response using both the simple linear design model and the (higher fidelity) transfer function from the linearized state space model:

```
Gcl_roll_low =PI_rateFeedback_TF(models.G_da2phi,P.roll_kp,P.roll_ki,P.roll_kd);

Gcl_roll_high =PI_rateFeedback_TF( H(kphi,kda) ,P.roll_kp,P.roll_ki,P.roll_kd);

step(Gcl_roll_low, Gcl_roll_high, 2)   % 2 seconds
```

How do they compare?

See next slide for Problem 5….

Lecture 7 Homework, 3/4

5) Modify `uavsim_control.m` to cause uavsim to perform a roll step. `uavsim_control.m` outputs the control deflection channels [delta_e delta_a delta_r delta_t]. At each iteration, they are initialized to the initial (e.g. trim) values from the P structure. Create a subfunction PIR_roll_hold() in `uavsim_control.m`. Modify uavsim_control.m to implement the roll controller with a 1 deg step, by adding:

```
phi_c = 1*pi/180;
delta_a = PIR_roll_hold(phi_c, phi_hat, p_hat, firstTime, P);
```

In this function call:

phi_c:	Commanded roll angle, rad
phi_hat:	Roll estimate (truth for now, as we haven't learned state estimation), rad
p_hat:	Roll body rate estimate (truth), rad/s
firstTime:	A flag for initializing the PIR integrator
P:	Parameters structure with gains, limits, etc.

In load_uavsim.m, uncomment the lines calling `compute_tf_models` and `compute_autopilot_gains`, call "`load_uavsim`" and then run uavsim.

a) Run uavsim with no wind starting from its trim condition. Compare the resulting step response with the linear step responses Gcl_roll_low and Gcl_roll_high. How do they compare? Was using the simplified model in our autopilot design a valid approach? Why or why not?

b) How well does the roll controller hold the 1 deg roll command in the presence of gusting?

Lecture 7 Homework, 4/4

6) a) Verify that the uavsim roll controller can achieve and maintain a 45 degree roll command in presence of gusting. (Note that the UAV may lose altitude if you haven't already implemented an altitude controller.) How long does it take to perform the 45deg roll step (e.g. 90% rise time)? What is the maximum body roll rate (p) achieved during the roll step? Plot and describe the aileron motion during the 45deg roll step and after it settles. How does the gusting affect the aileron motion?

b) With gusting turned off, explore how changing the roll control design parameters affects a 45 degree roll step. E.g. what if e_phi_max were 15 deg or 90 deg? What if damping were 0.5 or 1.5? Show and describe the effects, specifically looking at the roll response and the aileron commands.

Note: Future homeworks will presume you've used e_phi_max=45*pi/180 and zeta_roll=0.9, so remember to revert back to these suggested values when you are done.