

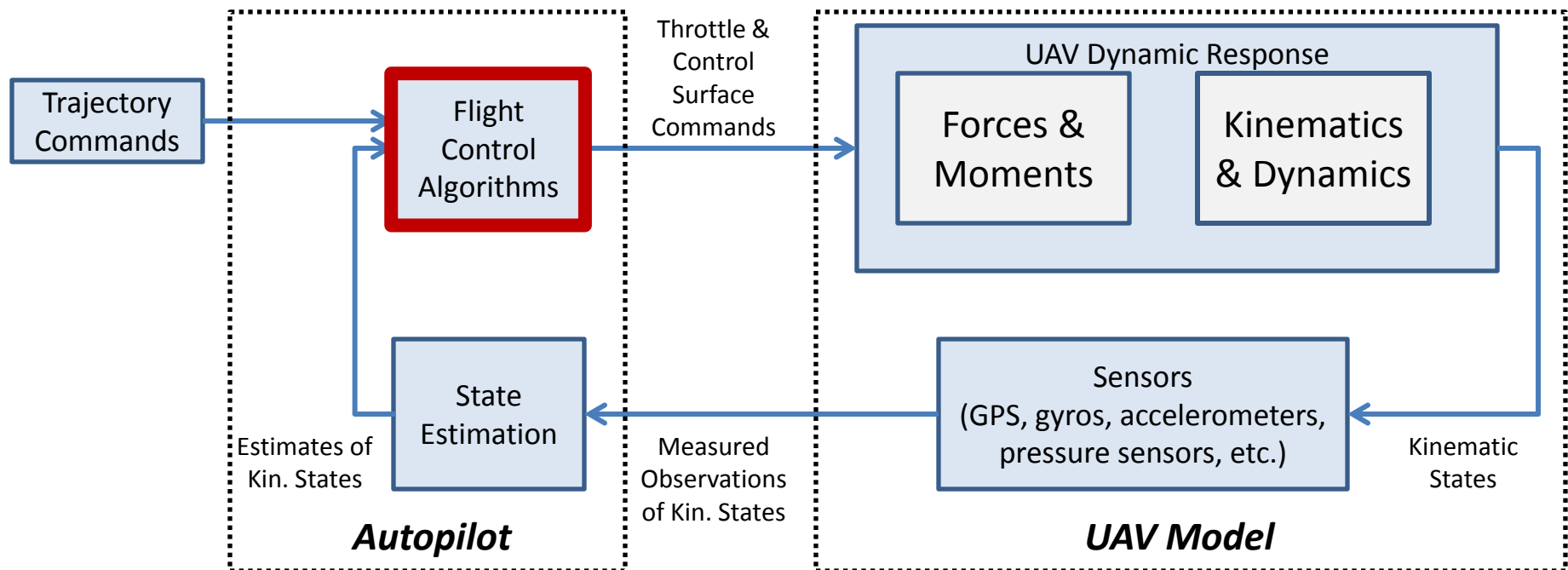
# UAV Systems & Control

## Lecture 8

Analytically-derived PID gains for:

- Lateral channel
- Longitudinal channel

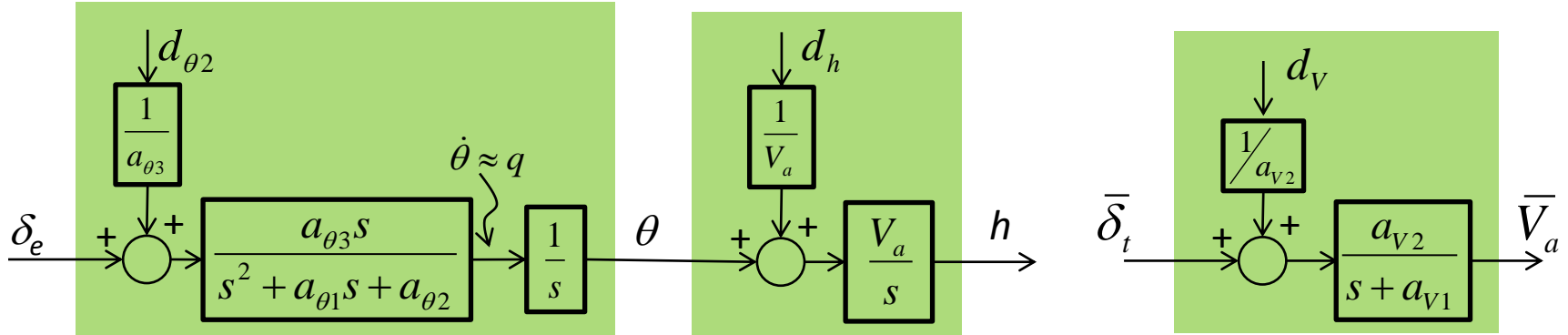
# UAV System



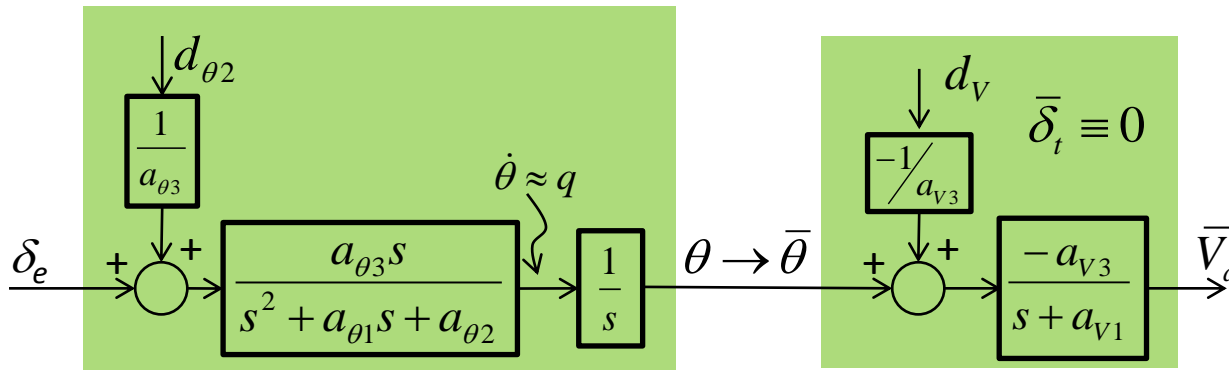
- In the previous lectures we:
  - Developed the simplified models we'll use for a “cookie-cutter” method of autopilot gain selection
  - Used method to develop roll hold autopilot
- In this section we will further develop the “cookie-cutter” autopilot gain selection method develop lateral and longitudinal autopilot gains
  - Autopilot logic will be implemented in “Flight Control Algorithms”

# Fixed Wing Longitudinal Linear Models

**Level Flight Mode:** Pitch controls Altitude, Throttle controls Airspeed



**Launch/Climb/Descend Modes:** Pitch controls Airspeed, Throttle is fixed



$$a_{\theta 1} = -\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} \frac{c}{2 V_a} C_{mq}$$

$$a_{\theta 2} = -\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} C_{m\alpha}$$

$$a_{\theta 3} = +\frac{\rho V_a^2 S c}{2} \frac{1}{J_y} C_{m\delta e}$$

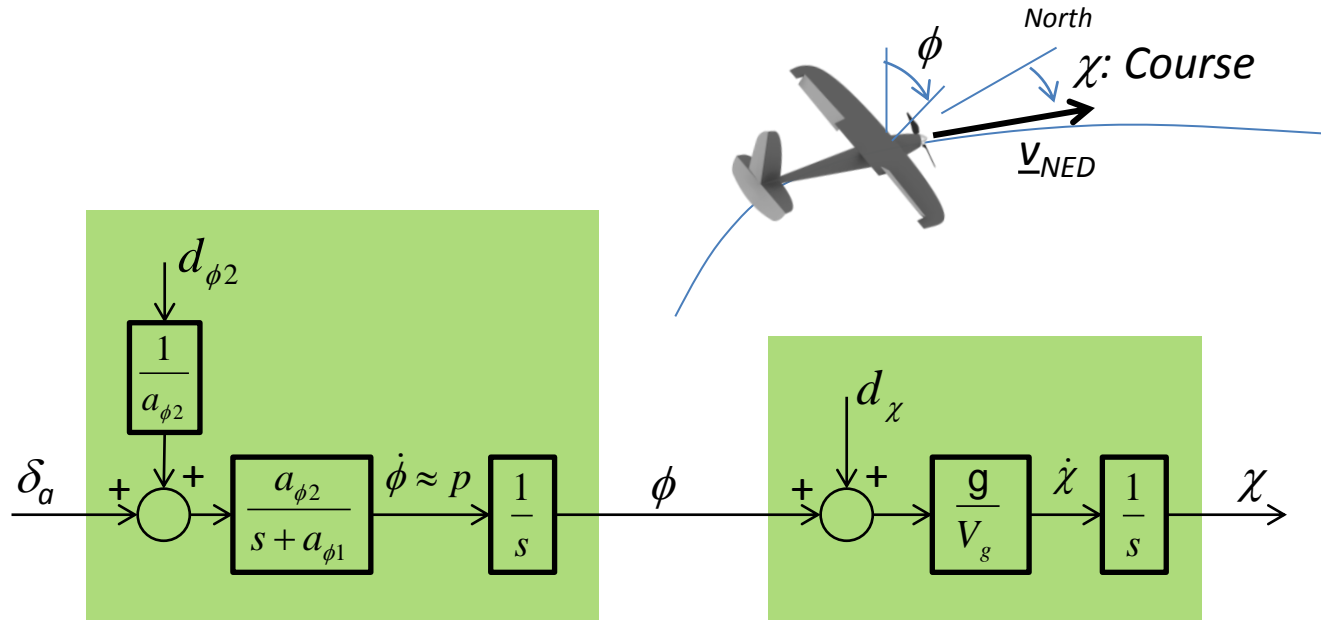
$$a_{V1} = -\frac{\rho C_{prop} S_{prop}}{mass} \left\{ \delta_t^* (1 - 2\delta_t^*) (k_{motor} - V_a^*) - \delta_t^* V_a^* \right\} + \frac{\rho V_a^* S}{mass} (C_{Do} + C_{D\alpha} \alpha^* + C_{D\delta e} \delta_e^*)$$

$$a_{V2} = \frac{\rho C_{prop} S_{prop}}{mass} (k_{motor} - V_a^*) (V_a^* + 2\delta_t^* (k_{motor} - V_a^*))$$

$$a_{V3} = g \cos(\theta^* - \alpha^*)$$

$x^*$  values are “trim” values. Response models are a function of nominal flight condition.

# Fixed Wing Lateral Linear Models



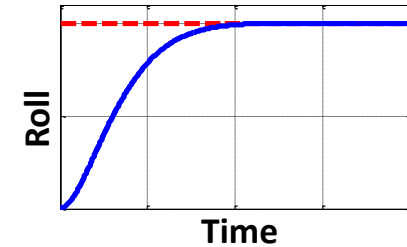
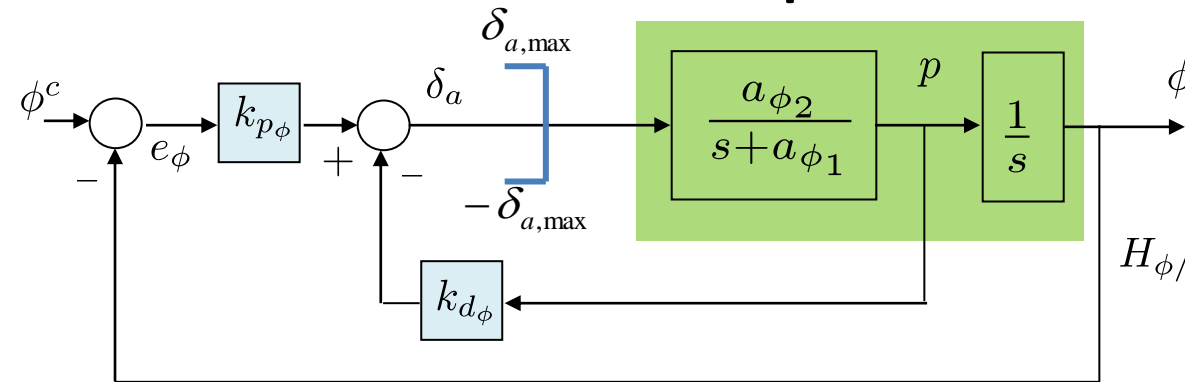
$$a_{\phi 1} = -\frac{\rho V_a^2 S b}{2} \frac{J_z C_{lp} + J_{xz} C_{np}}{\Gamma} \frac{b}{2 V_a}$$

$$a_{\phi 2} = +\frac{\rho V_a^2 S b}{2} \frac{J_z C_{l\delta a} + J_{xz} C_{n\delta a}}{\Gamma}$$

$$\Gamma = J_x J_z - J_{xz}^2$$

$$V_g \approx V_a$$

# Roll Control Autopilot



$$H_{\phi/\phi^c}(s) = \frac{k_{p\phi} a_{\phi 2}}{s^2 + \underbrace{(a_{\phi 1} + a_{\phi 2} k_{d\phi})}_{2\zeta_{\phi} \omega_{n,\phi}} s + \underbrace{k_{p\phi} a_{\phi 2}}_{\omega_{n,\phi}^2}}$$

- Using a simplified linear design model, we've developed a method to analytically choose autopilot gains for roll control

- Choose design parameters:

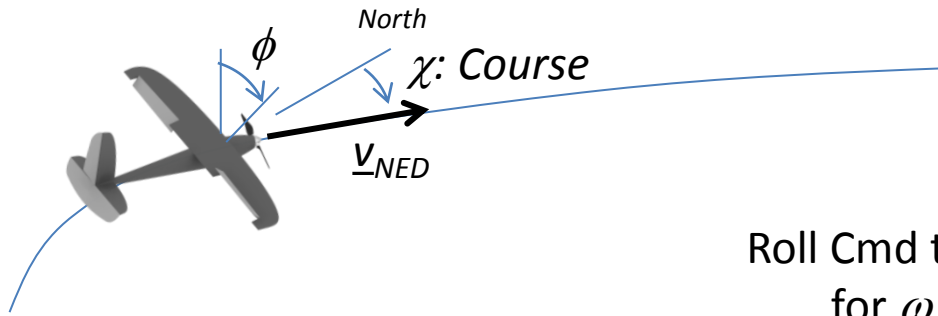
- $e_{\phi, \max}$  : Amount of roll error which will cause aileron saturation (smaller value means faster response, but more saturation)
- $\zeta_{\phi}$  : Use to choose overshoot

- Then:

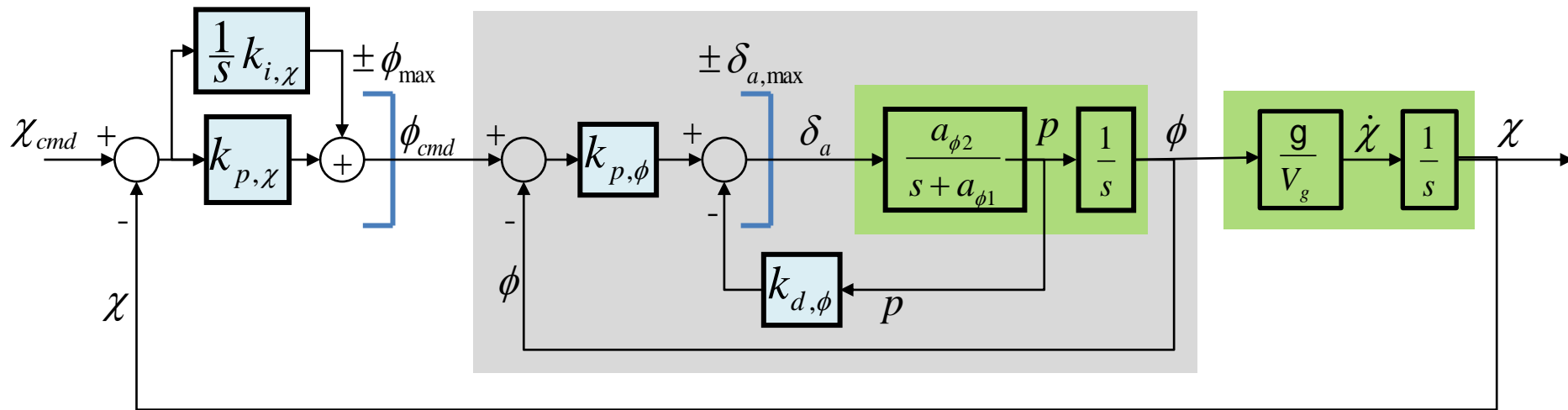
$$k_{p,\phi} = \frac{\delta_{a,\max}}{e_{\phi,\max}} \text{sign}(a_{\phi 2}) \quad \omega_{n,\phi} = \sqrt{k_{p,\phi} a_{\phi 2}} \quad k_{d,\phi} = \frac{2\zeta_{\phi} \omega_{n,\phi} - a_{\phi 1}}{a_{\phi 2}}$$

- Note: Book describes using an integral gain to remove steady-state error due to disturbances. We won't. Integrators add delay and instability, which isn't desired on inner loops. An integrator on the course loop will correct any steady state errors. (Per advice on book website.)

# Course Control Autopilot

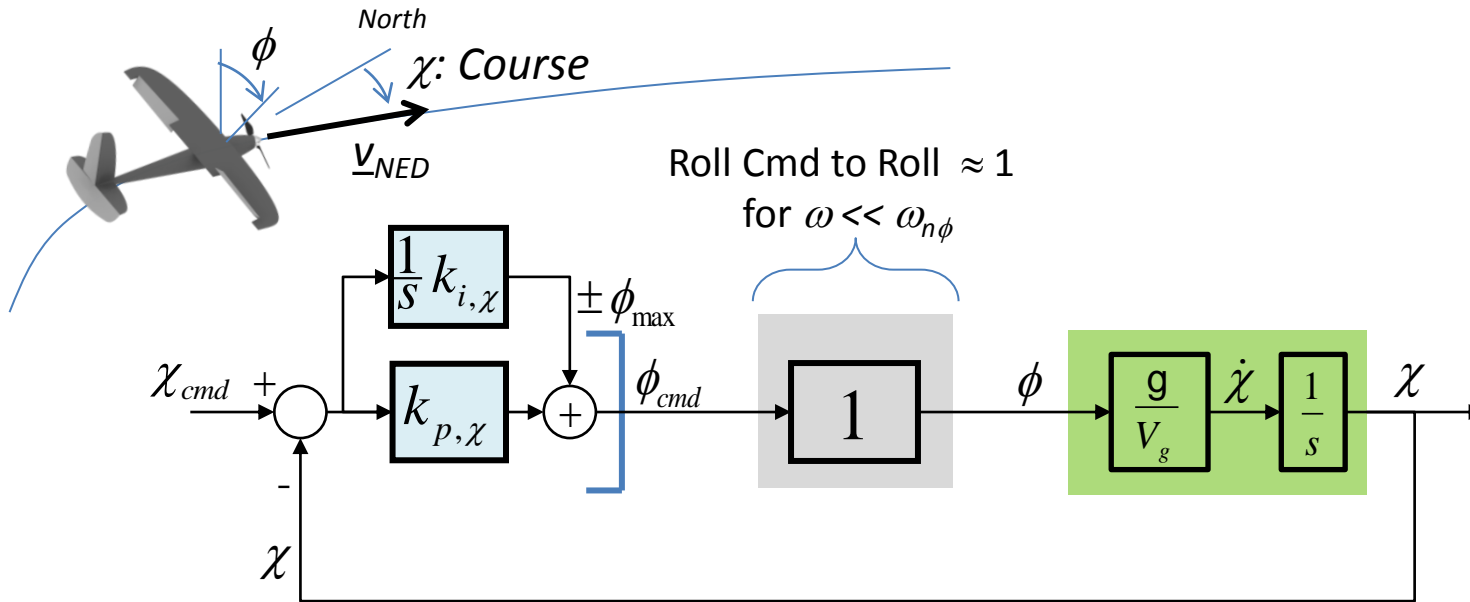


Roll Cmd to Roll  $\approx 1$   
for  $\omega \ll \omega_{n\phi}$



- A bank-to-turn aircraft changes course by rolling into a turn
- We will use a PI controller to control our course angle (Roll Cmd is a function of course error)
- We can simplify our course controller design by assuming that the roll response is *quick*
  - i.e. We will *slow down* the course control enough to assume [Roll Cmd to Roll]  $\approx 1$

# Course Control Autopilot

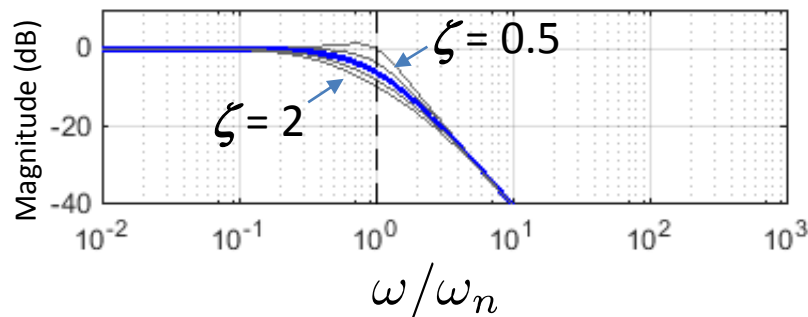
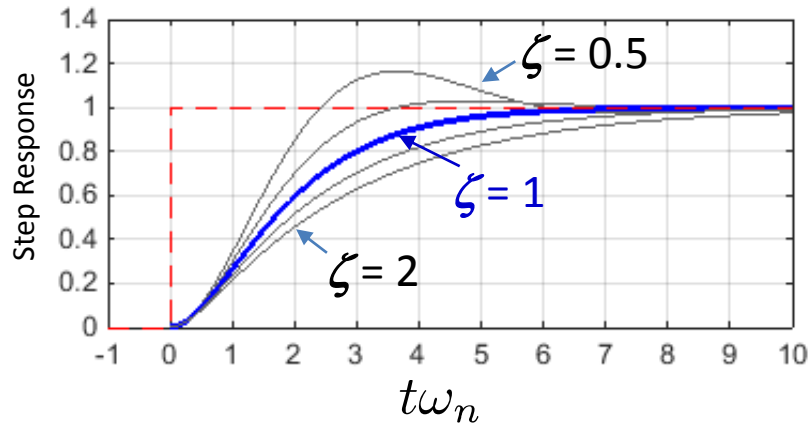


$$H_{\chi/\chi_c} = \frac{k_{p,\chi} g/V_g s + k_{i,\chi} g/V_g}{s^2 + k_{p,\chi} g/V_g s + k_{i,\chi} g/V_g} = \frac{2\zeta_{\chi} \omega_{n,\chi} s + \omega_{n,\chi}^2}{s^2 + 2\zeta_{\chi} \omega_{n,\chi} s + \omega_{n,\chi}^2}$$

- Treating the “Roll Cmd to Roll” response as unity simplifies the course control loop
  - Valid assumption if course control is notably slower than roll control
- Resulting course response has a 2<sup>nd</sup>-order denominator, but with a numerator zero
  - We’ll want:  $\omega_{n\chi} \ll \omega_{n\phi}$

# Canonical 2<sup>nd</sup> Order vs. 2<sup>nd</sup> Order with Zero

$$H = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

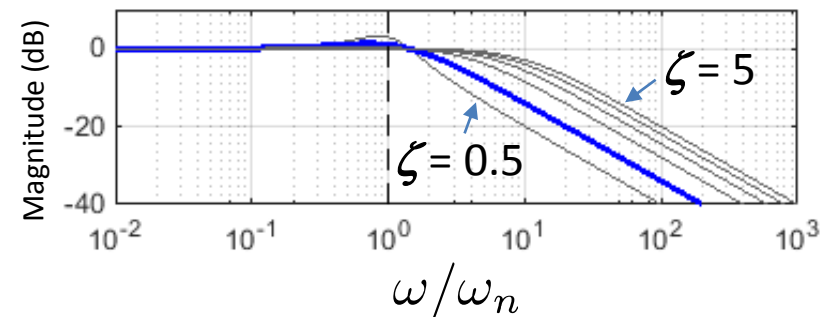
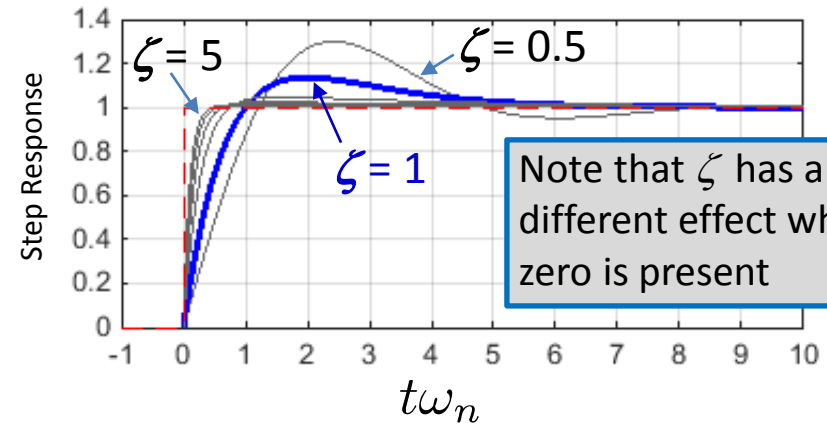


Canonical  
2<sup>nd</sup> Order:

As  $\zeta$  increases:

- Overshoot decreases
- Rise time increases
- Bandwidth decreases slightly

$$H = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



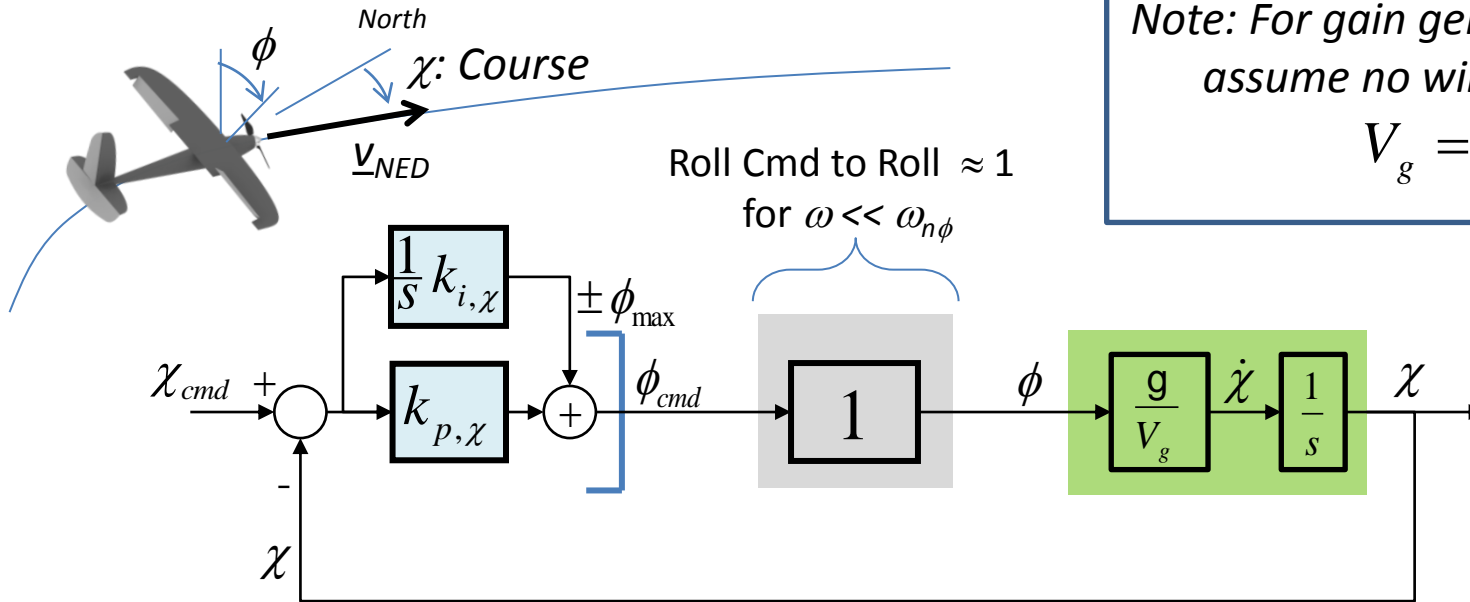
2<sup>nd</sup> Order  
with a zero:

As  $\zeta$  increases:

- Overshoot decreases
- Rise time decreases
- Bandwidth increases



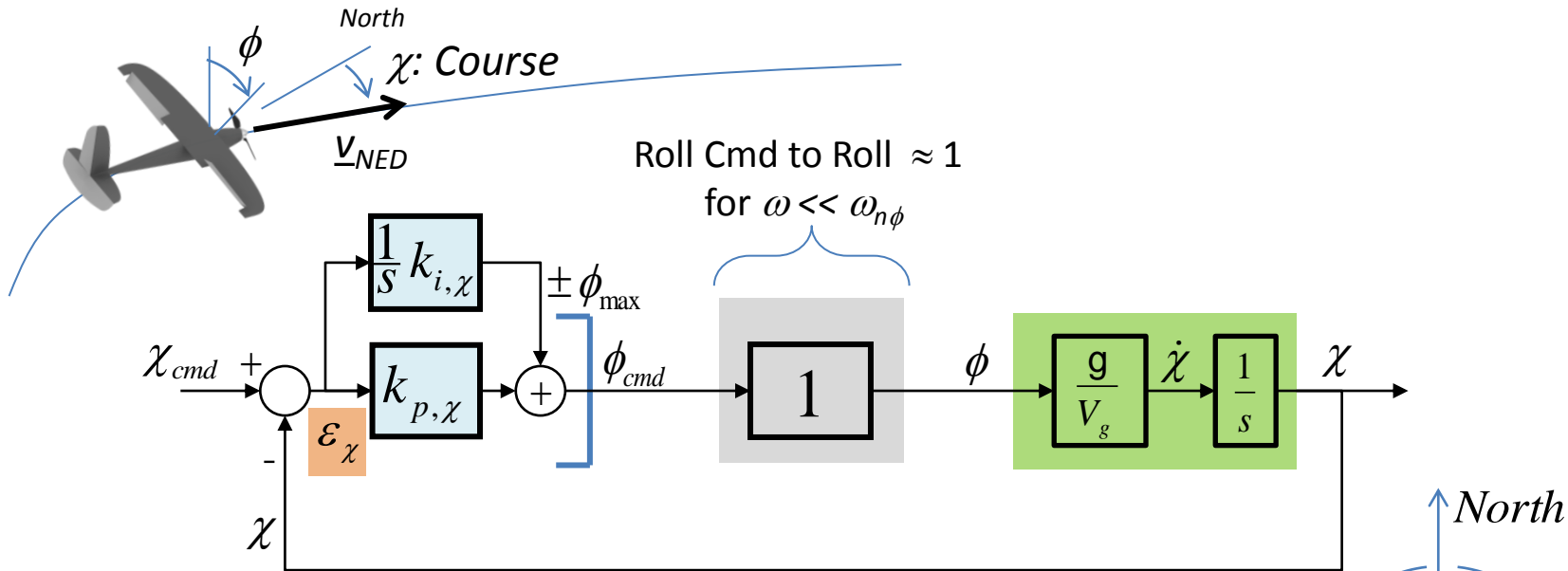
# Course Control Autopilot



$$H_{\chi/\chi_c} = \frac{k_{p,\chi} g/V_g s + k_{i,\chi} g/V_g}{s^2 + k_{p,\chi} g/V_g s + k_{i,\chi} g/V_g} = \frac{2\zeta_\chi \omega_{n,\chi} s + \omega_{n,\chi}^2}{s^2 + 2\zeta_\chi \omega_{n,\chi} s + \omega_{n,\chi}^2}$$

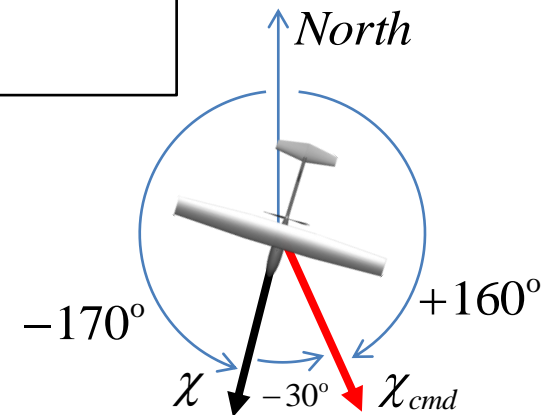
- Equating coefficients:  $k_{p,\chi} = 2\zeta_\chi \omega_{n,\chi} V_g / g$      $k_{i,\chi} = \omega_{n,\chi}^2 V_g / g$
- Let:  $\omega_{n,\chi} = \frac{1}{W_\chi} \omega_{n,\phi}$ ,  $W_\chi \gg 1$ , e.g. 5-50
- Course Loop  
Design Parameters:  $\zeta_\chi$  : Damping Ratio (higher due to numerator zero)

# Course Control Autopilot



## Implementation Note:

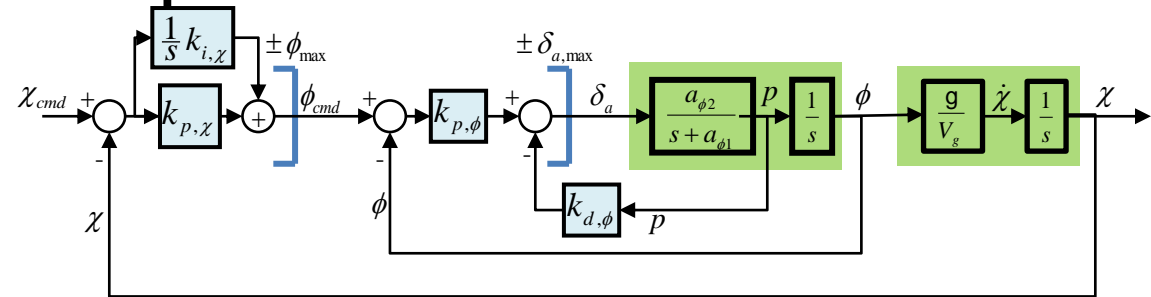
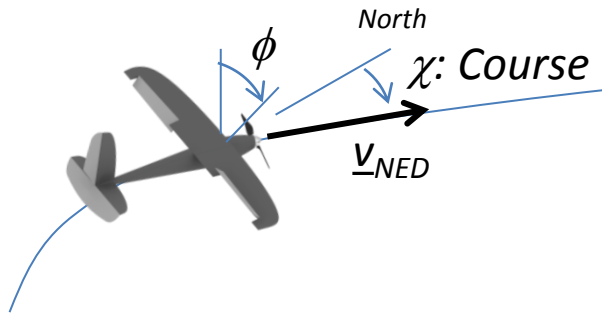
- $\epsilon_{\chi}$  is the error between the course command and the achieved course:  $\epsilon_{\chi} = \chi_{cmd} - \chi$
- Care must be taken because  $\chi_{cmd}$  and  $\chi$  “wrap” every  $2\pi$  radians (or  $360^\circ$ )
- Want:  $-\pi \leq \epsilon_{\chi} \leq \pi$  ( $\pi \text{ rad} = 180^\circ$ )
- Use:  $\epsilon_{\chi} = \text{mod}(\chi_{cmd} - \chi + \pi, 2\pi) - \pi$



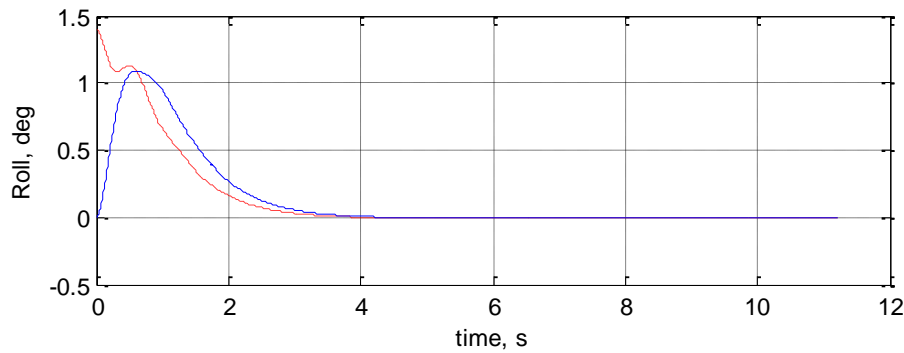
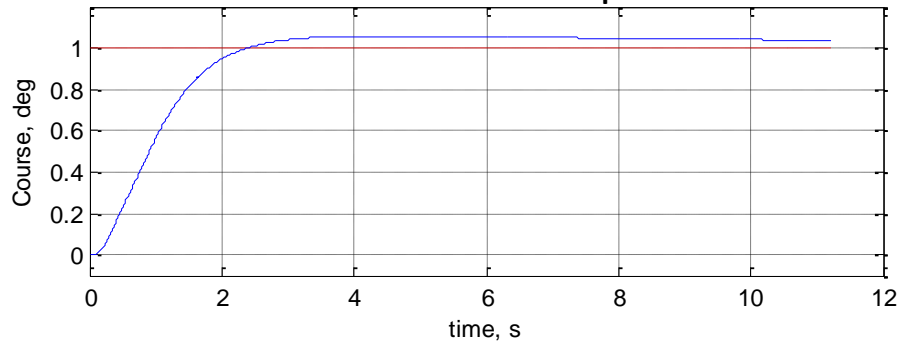
$$\chi_{cmd} - \chi = 160^\circ - (-170^\circ) = +330^\circ$$

Would cause UAV to turn  $330^\circ$  CW instead of  $30^\circ$  CCW!!

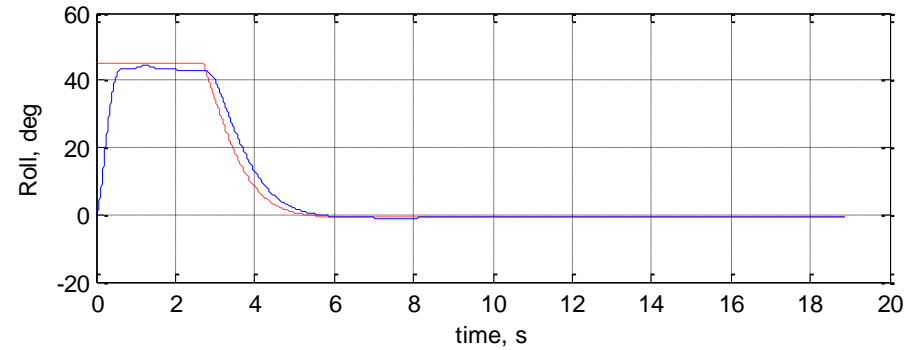
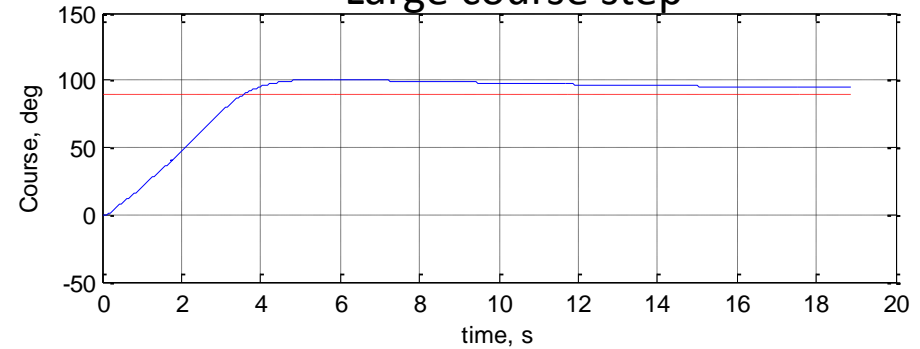
# Course Control Example



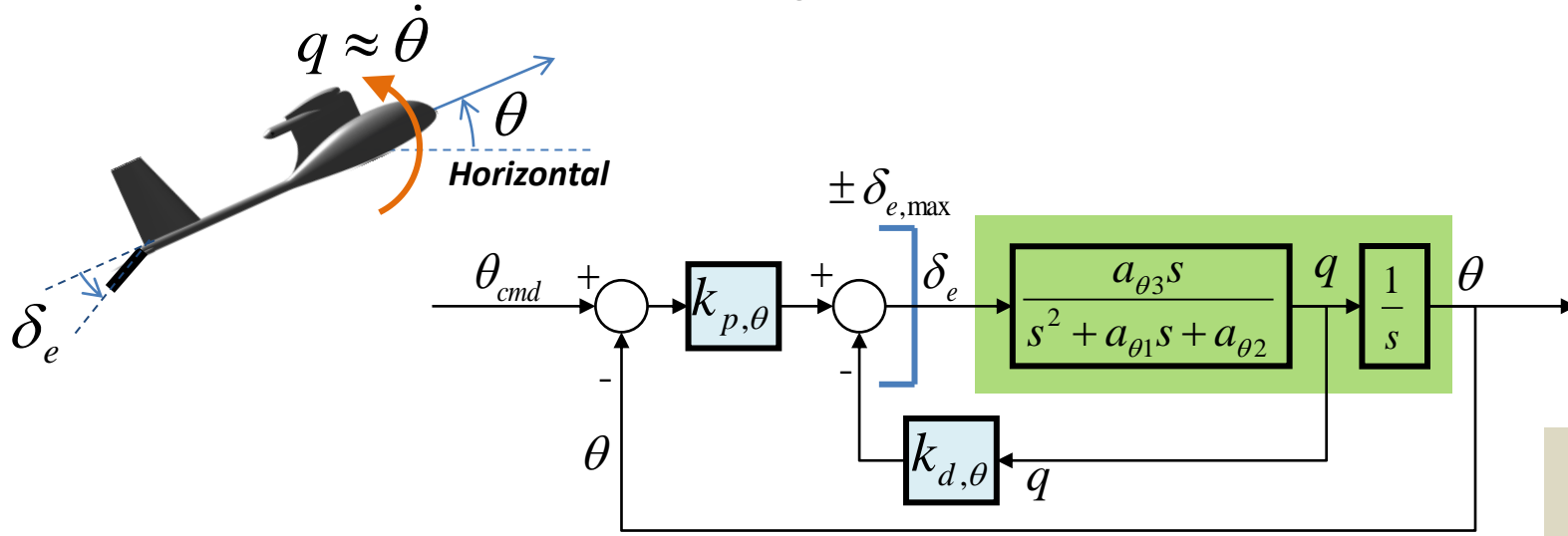
## Small course step



## Large course step



# Pitch Control Autopilot

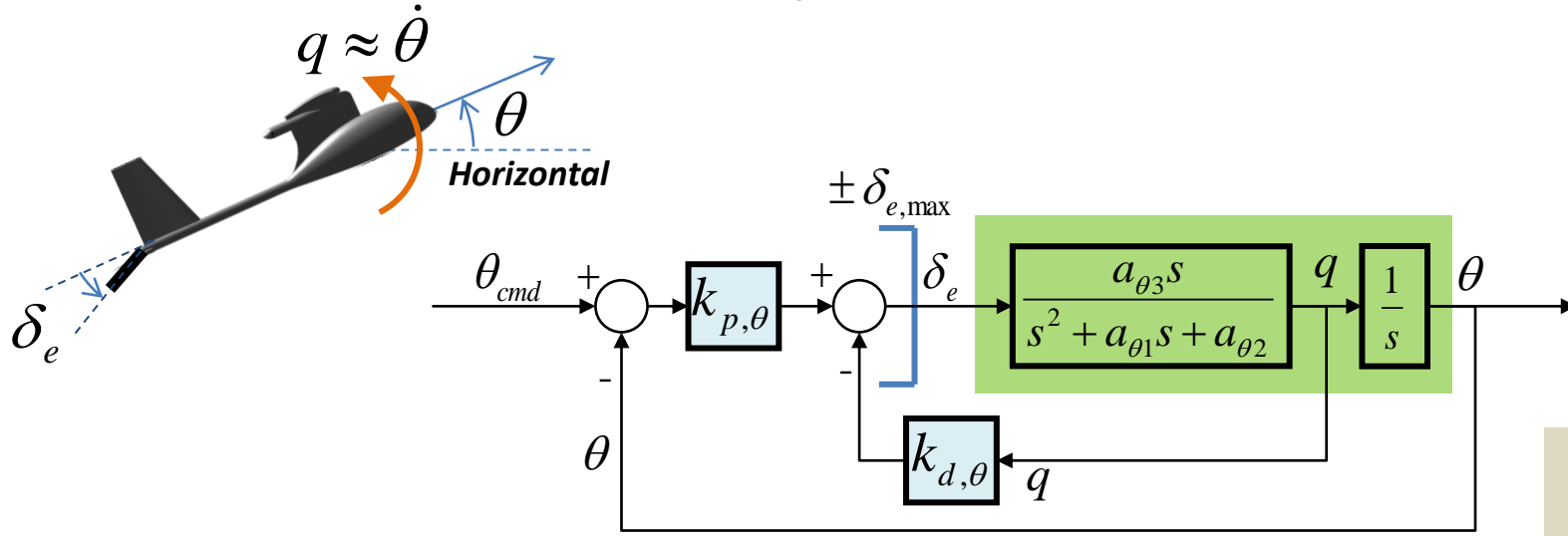


Note:  
NOT unity gain

$$H_{\theta/\theta_c} = \frac{k_{p,\theta} a_{\theta 3}}{s^2 + (a_{\theta 1} + k_{d,\theta} a_{\theta 3})s + (a_{\theta 2} + k_{p,\theta} a_{\theta 3})} = \frac{K_{\theta,DC} \omega_{n,\theta}^2}{s^2 + 2\zeta_{\theta} \omega_{n,\theta} s + \omega_{n,\theta}^2}$$

- Equating coefficients:  $2\zeta_{\theta} \omega_{n,\theta} = a_{\theta 1} + k_{d,\theta} a_{\theta 3}$        $\omega_{n,\theta}^2 = a_{\theta 2} + k_{p,\theta} a_{\theta 3}$ 
  - Provides 2 equations and 2 unknowns, but we need to account for  $\delta_{e,max}$
- Select  $k_{p,\theta}$  such that the elevator saturates only when the pitch error exceeds some design threshold,  $e_{\theta,max}$ .
 
$$k_{p,\theta} = \frac{\delta_{e,max}}{e_{\theta,max}} \text{sign}(a_{\theta 3})$$
  - Also, positive pitch error should cause a positive  $q$ .

# Pitch Control Autopilot



Note:  
NOT unity gain

$$H_{\theta/\theta_c} = \frac{k_{p,\theta} a_{\theta 3}}{s^2 + (a_{\theta 1} + k_{d,\theta} a_{\theta 3})s + (a_{\theta 2} + k_{p,\theta} a_{\theta 3})} = \frac{K_{\theta,DC} \omega_{n,\theta}^2}{s^2 + 2\zeta_{\theta} \omega_{n,\theta} s + \omega_{n,\theta}^2}$$

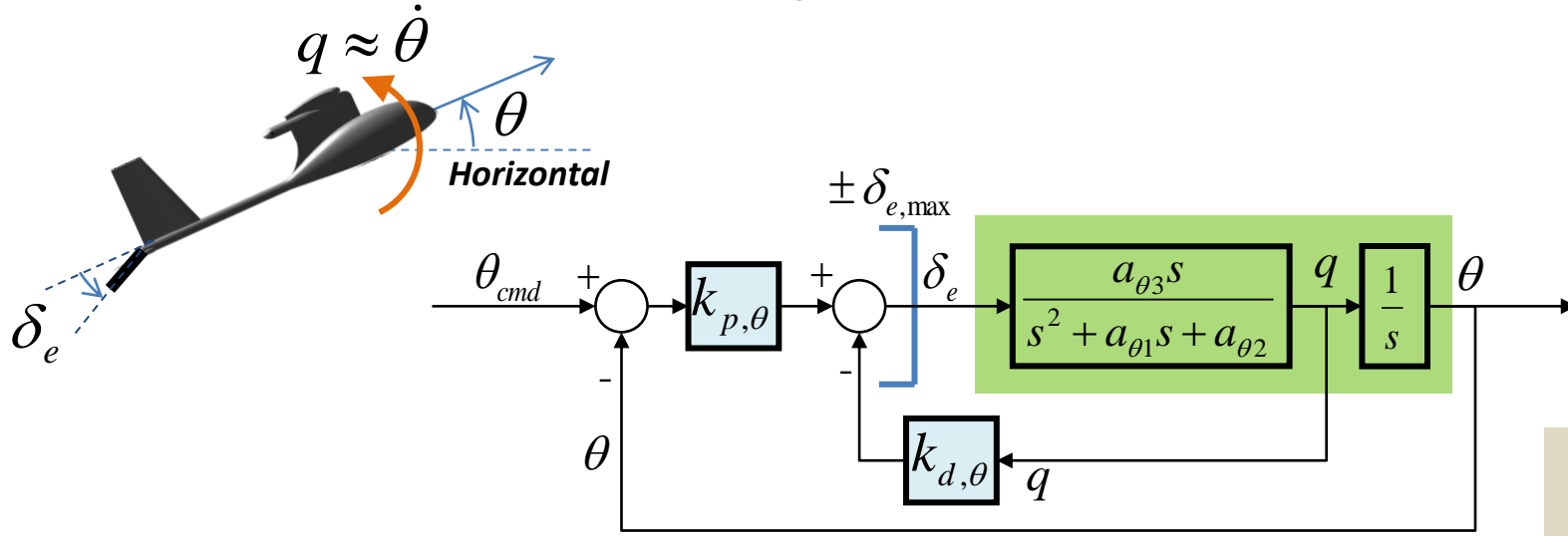
– Choose design parameters:

- $e_{\theta,max}$  : Amount of pitch error which will cause elevator saturation (smaller value means faster response, but more saturation)
- $\zeta_{\theta}$  : Use to choose overshoot

– Then:

$$k_{p,\theta} = \frac{\delta_{e,max}}{e_{\theta,max}} \text{sign}(a_{\theta 3}) \quad \omega_{n,\theta} = \sqrt{a_{\theta 2} + k_{p,\theta} a_{\theta 3}} \quad k_{d,\theta} = \frac{2\zeta_{\theta} \omega_{n,\theta} - a_{\theta 1}}{a_{\theta 3}}$$

# Pitch Control Autopilot

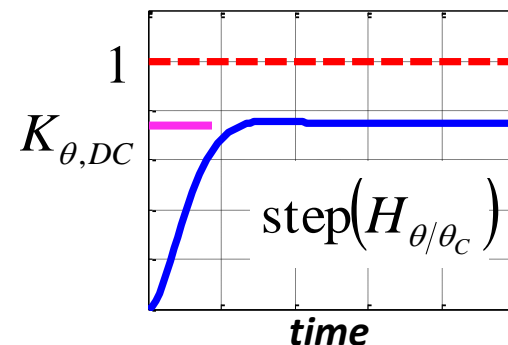


Note:  
NOT unity gain

$$H_{\theta/\theta_c} = \frac{k_{p,\theta} a_{\theta 3}}{s^2 + (a_{\theta 1} + k_{d,\theta} a_{\theta 3})s + (a_{\theta 2} + k_{p,\theta} a_{\theta 3})} = \frac{K_{\theta,DC} \omega_{n,\theta}^2}{s^2 + 2\zeta_{\theta} \omega_{n,\theta} s + \omega_{n,\theta}^2}$$

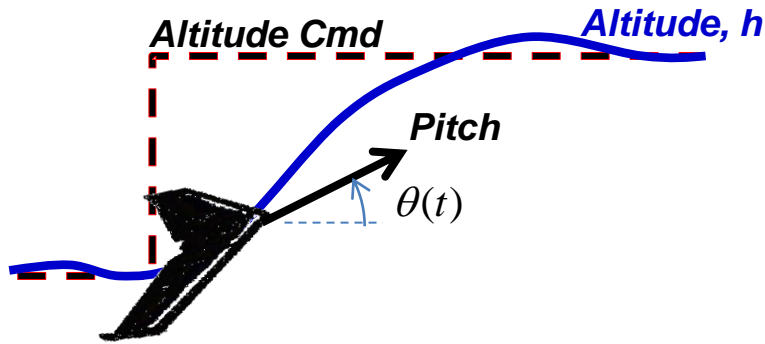
- Pitch loop has 2<sup>nd</sup> order response, but it does not have unity gain.
- Pitch loop response gain:

$$K_{\theta,DC} = \frac{k_{p,\theta} a_{\theta 3}}{a_{\theta 2} + k_{p,\theta} a_{\theta 3}}$$

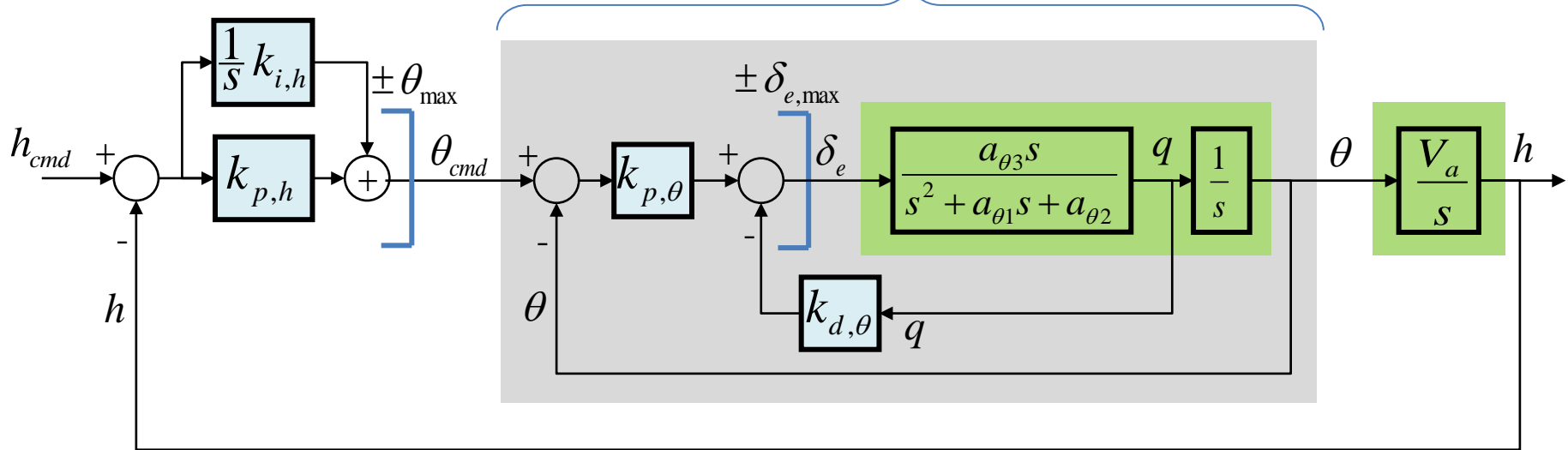


- We could eliminate this steady-state error with an integrator (e.g.  $k_{i,\theta}$ ), but that would slow down the response. Outer loop will compensate instead.

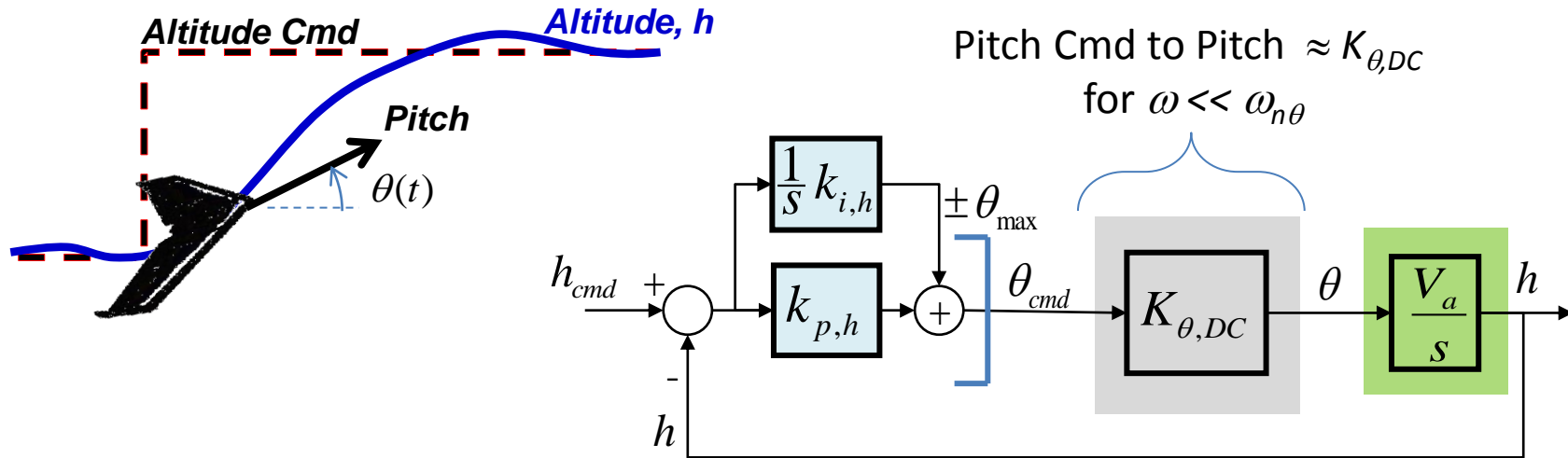
# Altitude Control using Pitch



Pitch Cmd to Pitch  $\approx K_{\theta,DC}$   
for  $\omega \ll \omega_{n\theta}$



# Altitude Control using Pitch



$$H_{h/h_c} = \frac{K_{\theta,DC} V_a k_{p,h} s + K_{\theta,DC} V_a k_{i,h}}{s^2 + K_{\theta,DC} V_a k_{p,h} s + K_{\theta,DC} V_a k_{i,h}} = \frac{2\zeta_h \omega_{n,h} s + \omega_{n,h}^2}{s^2 + 2\zeta_h \omega_{n,h} s + \omega_{n,h}^2}$$

- Equating coefficients:

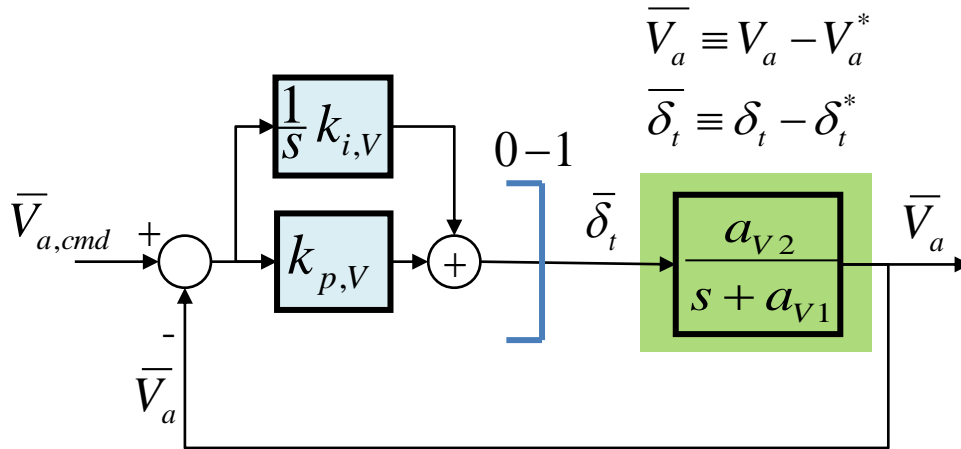
$$k_{p,h} = \frac{2\zeta_h \omega_{n,h}}{K_{\theta,DC} V_a} \quad k_{i,h} = \frac{\omega_{n,h}^2}{K_{\theta,DC} V_a}$$

- Let:  $\omega_{n,h} = \frac{1}{W_h} \omega_{n,\theta}$ ,  $W_h \gg 1$ , e.g. 5-100

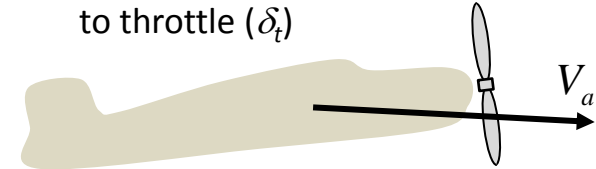
- Altitude Loop  
Design Parameters:  $\zeta_h$ : Damping Ratio (higher due to numerator zero)



# Airspeed Control using Throttle



Propeller speed is proportional to throttle ( $\delta_t$ )



Recall: The “throttle-to-airspeed” model was derived as a “variations from trim” model. The autopilot integrator will account for the “trim” values.

$$H_{V_a/V_{a_c}} = \frac{a_{V2}k_{p,V}s + a_{V2}k_{i,V}}{s^2 + (a_{V1} + a_{V2}k_{p,V})s + a_{V2}k_{i,V}} \approx \frac{2\zeta_V\omega_{n,V}s + \omega_{n,V}^2}{s^2 + 2\zeta_V\omega_{n,V}s + \omega_{n,V}^2}$$

Approximate, because  $a_{V1}$  is small

- Equating coefficients:

$$k_{p,V} = \frac{2\zeta_V\omega_{n,V} - a_{V1}}{a_{V2}} \quad k_{i,V} = \frac{\omega_{n,V}^2}{a_{V2}}$$

- Airspeed using Throttle Loop Design Parameters:

$\omega_{n,V}$  : Airspeed using throttle natural frequency

Advice: Use bandwidth separation from pitch loop

e.g.:  $\omega_{n,V} = \frac{1}{W_V} \omega_{n,\theta}$ ,  $W_V \gg 1$ , e.g. 10-50

$\zeta_V$  : Damping Ratio (higher due to numerator zero)

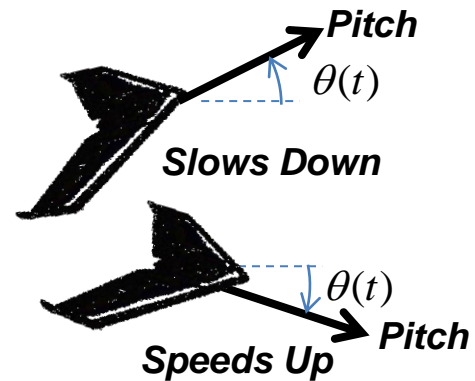
# Longitudinal Control Modes

- Altitude Hold Mode:  
*Pitch controls altitude, Throttle controls airspeed*
  - Tight altitude and speed control for small deviations
  - Undesirable for large altitude steps
    - For climbs, results in large pitch up and potential slow-down
    - For descents, results in large pitch downs and massive speed-ups!!!
- Climb Mode:
  - *Go full throttle, and use pitch to control airspeed*
- Descent Mode:
  - *Go zero throttle, and use pitch to control airspeed*



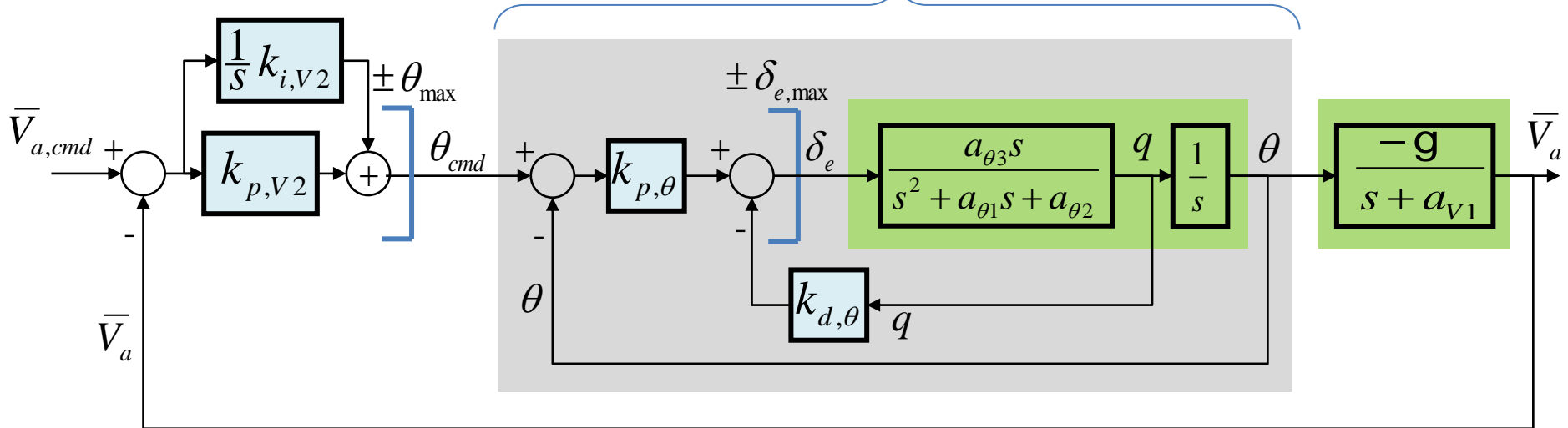
Need another controller:  
*Airspeed control using Pitch*

# Airspeed Control using Pitch

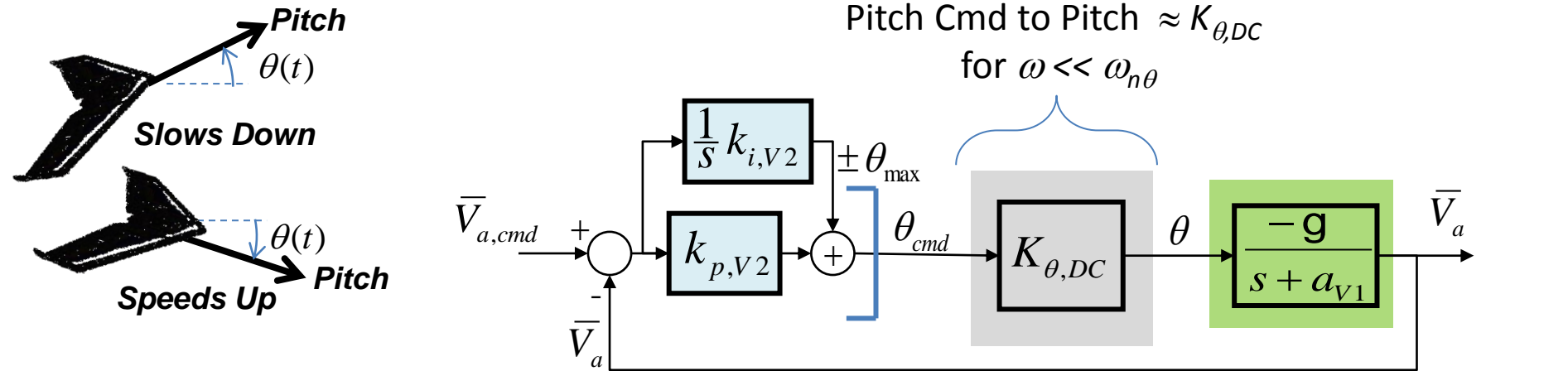


Pitch Cmd to Pitch  $\approx K_{\theta,DC}$   
for  $\omega \ll \omega_{n\theta}$

$$\bar{V}_a \equiv V_a - V_a^*$$



# Airspeed Control using Pitch



$$H_{V_a/V_{aC}} = \frac{-K_{\theta,DC} g k_{p,V2} s - K_{\theta,DC} g k_{i,V2}}{s^2 + (a_{V1} - K_{\theta,DC} g k_{p,V2}) s - K_{\theta,DC} g k_{i,V2}} \approx \frac{2\zeta_{V2} \omega_{n,V2} s + \omega_{n,V2}^2}{s^2 + 2\zeta_{V2} \omega_{n,V2} s + \omega_{n,V2}^2} \quad \text{Approximate, because } a_{V1} \text{ is small}$$

- Equating coefficients:

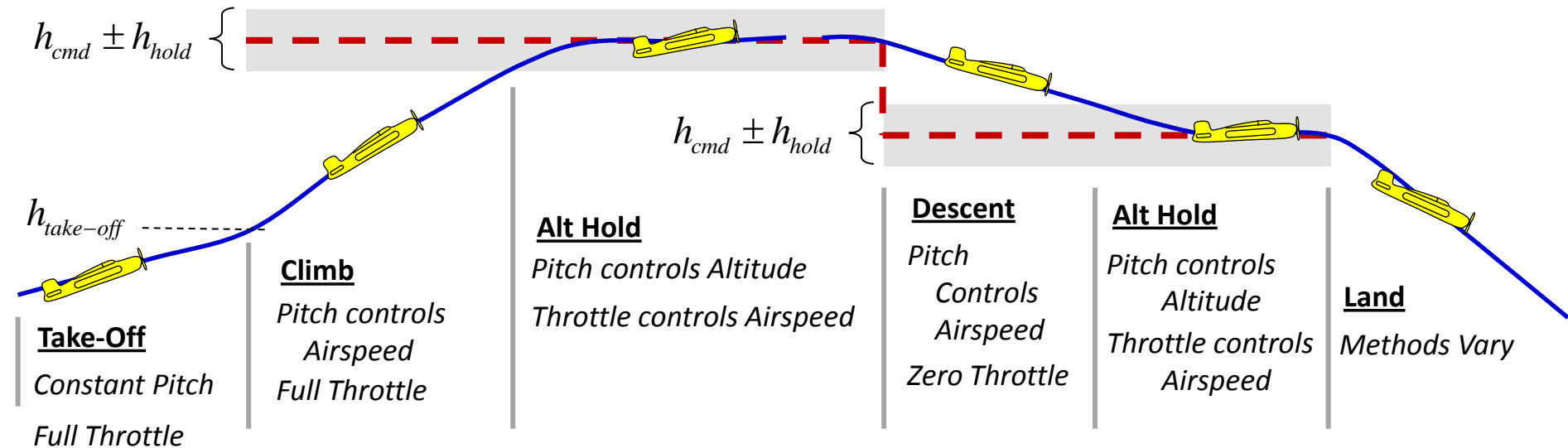
$$k_{p,V2} = \frac{a_{V1} - 2\zeta_{V2} \omega_{n,V2}}{K_{\theta,DC} g} \quad k_{i,V2} = \frac{-\omega_{n,V2}^2}{K_{\theta,DC} g}$$

- Let:  $\omega_{n,V2} = \frac{1}{W_{V2}} \omega_{n,\theta}$ ,  $W_{V2} \gg 1$ , e.g. 5-50

- Airspeed with Pitch  
Loop Design Parameters:  $W_{V2}$ : Bandwidth separation factor  
 $\zeta_{V2}$ : Damping Ratio (higher due to numerator zero)

# Altitude & Airspeed Control Modes

A UAV will generally switch between different Longitudinal Control Modes during flight



Example  
Mode  
Logic

**Take-Off:** Full throttle, and regulate pitch to  $\theta_{take-off}$

- Start at Launch, end upon initially reaching an altitude of  $h_{take-off}$

**Climb:** Full throttle, and use pitch to regulate airspeed to  $V_{a,cmd}$

- Whenever  $h < h_{cmd} - h_{hold}$  (and not in *Take-Off* or *Land*)

**Alt Hold:** Pitch controls altitude to  $(h_{cmd})$ , and throttle controls airspeed ( $V_{a,cmd}$ )

- Whenever  $|h - h_{cmd}| < h_{hold}$  (and not in *Take-Off* or *Land*)

**Descend:** Zero throttle, and use pitch to regulate airspeed to  $V_{a,cmd}$

- Whenever  $h > h_{cmd} + h_{hold}$  (and not in *Take-Off* or *Land*)

Alternative Descent:

- Zero Throttle
- $\theta_{cmd} = \theta_{descent}$

**Land:** Methods vary (e.g. fly down a prescribed glideslope, etc.). Initiated by command.

# Longitudinal Autopilot - Summary

If model is known, the the design parameters are

## Inner Loop (pitch attitude hold)

- $e_{\theta}^{\max}$  - Error in pitch when elevator just saturates.
- $\zeta_{\theta}$  - Damping ratio for pitch attitude loop.

## Altitude Hold Outer Loop

- $W_h > 1$  - Bandwidth separation between pitch and altitude loops.
- $\zeta_h$  - Damping ratio for altitude hold loop.

## Airspeed Hold Outer Loop

- $W_{V_2} > 1$  - Bandwidth separation between pitch and airspeed loops.
- $\zeta_{V_2}$  - Damping ratio for airspeed hold loop.

## Throttle hold (inner loop)

- $\omega_{n_V}$  - Natural frequency for throttle loop.
- $\zeta_V$  - Damping ratio for throttle loop.

# Lateral Autopilot - Summary

If model is known, the the design parameters are  
**Inner Loop (roll attitude hold)**

- $e_{\phi}^{\max}$  - Error in roll when aileron just saturates.
- $\zeta_{\phi}$  - Damping ratio for roll attitude loop.

**Outer Loop (course hold)**

- $W_{\chi} > 1$  - Bandwidth separation between roll and course loops.
- $\zeta_{\chi}$  - Damping ratio for course hold loop.

# General Gain Tuning Procedures

*Decide which knobs  
will you turn*

*Decide goodness  
criteria*

*Decide how you will test  
response*

## Gains

- e.g.  $k_p$ ,  $k_v$ ,  $k_d$
- Pros:
  - Easy to do
- Cons:
  - Not always intuitive
  - May be unstable

## Design Parameters

- e.g.  $e^{max}$ ,  $\zeta$ ,  $W$
- Pros:
  - More intuitive result
  - Stable for reasonable parameter choices
- Cons:
  - Additional math & effort involved

## Examples

- Rise time
- Settling time
- Overshoot
- Avoid oscillations
- Stability margins
- Avoiding limits
- etc.

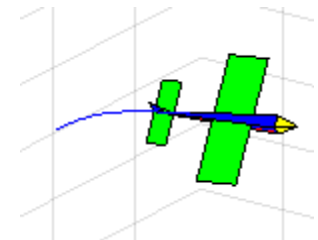
## In-flight tuning

- Pros:
  - Less math
- Cons:
  - Requires expertise
  - May be disastrous



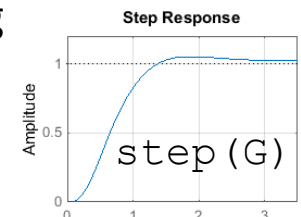
## In-Simulation tuning

- Pros:
  - Not disastrous
- Cons:
  - Requires a sim
  - Responses may be coupled



## Transfer Function tuning

- Pros:
  - Quick
- Cons:
  - Still need to simulate/fly





# Useful Transfer Functions for Tuning

Specifically using methods described in this class:

```
%% Lateral Channel

% Open Loop response from aileron to roll
H(kphi,kda) % Higher fidelity than models.G_da2phi
% Closed Loop response from roll command to roll
G_phic2phi = PI_rateFeedback_TF(H(kphi,kda),P.roll_kp,P.roll_ki,P.roll_kd)
% Closed Loop response from course command to course
G_chic2chi = PI_rateFeedback_TF(G_phic2phi*models.G_phi2chi, ...
                                P.course_kp,P.course_ki,P.course_kd)

%% Longitudinal Channel

% Open Loop response from elevator to pitch
H(ktheta,kde) % Higher fidelity than models.G_de2theta
% Closed Loop response from pitch command to pitch
G_thetac2theta = PI_rateFeedback_TF(H(ktheta,kde),P.pitch_kp,P.pitch_ki,P.pitch_kd)
% Closed Loop response from alt command to alt
G_altc2alt = PI_rateFeedback_TF(G_thetac2theta*models.G_theta2h, ...
                                P.altitude_kp,P.altitude_ki,P.altitude_kd)
% Closed Loop response from airspeed command to airspeed using pitch
G_vac2va_pitch = PI_rateFeedback_TF(G_thetac2theta*models.G_theta2Va, ...
                                    P.airspeed_pitch_kp,P.airspeed_pitch_ki,P.airspeed_pitch_kd)

% Open Loop response from throttle to airspeed
models.G_dt2Va % Note: Can't use H(ku,kdt) because it is dominated by phugoid
% Closed Loop response from airspeed command to airspeed using throttle
G_vac2va_throttle = PI_rateFeedback_TF(models.G_dt2Va, ...
                                        P.airspeed_throttle_kp,P.airspeed_throttle_ki,P.airspeed_throttle_kd)
```

# Lecture 8 Homework, 1/5

*Note: re-run `compute_autopilot_gains` (or `load_uavsim`) each time you modify the gains!*

- 1) Use design parameters to tune and simulate course control.
  - Modify `compute_autopilot_gains.m` to generate the course control gains.
  - Use design parameters and transfer functions (`G_chic2chi`) to achieve design goals:
    - <5% overshoot and a 95% rise time of under 3 seconds
    - Minimal oscillations
  - In `uavsim_control.m`, generate a `chi_hat` (in radians) using `Vn_hat` and `Ve_hat`.
  - Augment `uavsim_control.m` to have a `PIR_course_hold()` routine, and use it to generate the `phi_c` used in the existing roll controller. Use appropriate limits on the routine output (i.e. `P.phi_max`). NOTE: Make sure to “wrap” the course error to +/- 180deg!)
  - Run simulation to achieve a course command of 140 degrees with no winds.
- a. Show derivation of closed-loop course-control transfer function and gains.
- b. Turn in code that generates gains, highlighting chosen design parameters.
- c. Turn in step response of `G_chic2chi`
- d. Turn in the portion of `PIR_course_hold()` sufficient to show PIR “set up” and course error “wrapping”.
- e. Run sim for 10 seconds and turn in plots:
  - Comparing course with course command
  - Comparing roll with roll command
  - Showing aileron
- f. Test error wrapping by verifying that a course command of 270deg results in a UAV turning left to fly due west. (i.e. same result as a command of -90deg)

Enable plotting of commands by modifying  
`uavsim_display: plot_commands = 1;`  
 Enable logging of commands by modifying  
`uavsim_logging: log_commands = 1;`

- 2) Use design parameters to tune and simulate pitch control. (Remove manual tuning gains!)
  - Modify `compute_autopilot_gains.m` and `uavsim_control.m` as necessary.
  - For design parameters, use  $e_{\theta, \max} = 30^\circ$  and  $\zeta_\theta = 0.9$ .
  - Run simulation to achieve a pitch command of 20 degrees with no winds and  $\chi_{cmd} = 0$ .
  - Note: Without an integrator, achieved pitch will have steady-state error.
  - a. Show derivation of closed loop pitch control transfer function and gains.
  - b. Turn in code that generates gains.
  - c. Turn in resulting gains, including `P.K_theta_DC`.
  - d. Run for 10 seconds and turn in plots:
    - 1) Comparing  $\theta$  &  $\theta_{cmd}$ , and 2) showing elevator
  - e. Verify that the steady-state pitch value is *approximately*  $\theta_{cmd} * K_{\theta, DC}$ .

Note:  $K_{\theta, DC}$  was derived from the linear model, so the resulting steady-state pitch may still differ from  $\theta_{cmd} * K_{\theta, DC}$  by a degree or so.

- 3) Use design parameters to tune altitude control using pitch. (Remove manual gains!)
  - Modify `compute_autopilot_gains.m` and `uavsim_control.m` as necessary.
  - Modify `PIR_alt_hold_using_pitch()` to account for the known steady-state pitch error:
 

```
u_lower_limit = -P.theta_max/P.K_theta_DC;
u_upper_limit = +P.theta_max/P.K_theta_DC;
```
  - Simulate 50 seconds of no-wind flight with  $\chi_{cmd} = 0$  and altitude command from:
 

```
if mod(time, 20) < 10, h_c = 50; else, h_c = 51; end
```
  - Adjust “altitude via pitch” control design parameters to achieve “good” response.
  - a. Show derivation of closed loop altitude control transfer function and gains
  - b. Turn in code that generates gains, highlighting chosen design parameters.
  - c. Turn in resulting gains.
  - d. Run for 50 seconds and turn in plots:
    - 1) Comparing altitude with altitude command
    - 2) Comparing  $\theta$  &  $\theta_{cmd}$ , and 3) showing elevator

Student's choice what constitutes a “good” response

# Lecture 8 Homework, 3/5

- 4) Use design parameters to tune airspeed control using throttle.
- Modify `compute_autopilot_gains.m` and `uavsim_control.m` as necessary.
  - Create `PIR_airspeed_hold_using_throttle()` and limit throttle output ( $u$ ) to  $[0 \ 1]$ .
  - Simulate 50 seconds of no-wind flight with  $\chi_{cmd}=0$ ,  $h_{cmd}=50$ , and airspeed cmd from:  
`if mod(time,20)<10, Va_c=13; else, Va_c=16; end`
  - Use the following design parameters for “airspeed via throttle”:
    - $\omega_{n,V} = \omega_{n,\theta} / 40$ ; and  $\zeta_V = 1$ .
  - a. Show derivation of closed loop speed control using throttle transfer function and gains
  - b. Turn in code that generates gains.
  - c. Turn in resulting gains.
  - d. Run for 50 seconds and turn in plots:
    - Comparing airspeed with airspeed command
    - Showing throttle
  - e. Re-run simulation for 100 seconds with following commands (large altitude steps):  
`Va_c = 13;`  
`chi_c = 0;`  
`if time<30, h_c=100; else, h_c=50; end; % 100m for time<30, then 50m`  
 Is the result desirable? If not, why not?

# Lecture 8 Homework, 4/5

## 5) Develop airspeed-using-pitch controller, and implement altitude mode control logic.

- Create `PIR_airspeed_hold_using_pitch`. Limit pitch output to:  $\pm P.\theta_{\max}/P.K_{\theta DC}$ .
- Modify `uavsim_control.m` to achieve the provided over-simplified altitude mode logic (We won't worry about launch and land modes):
- Use the following design parameters for “airspeed via pitch”:  $\omega_{n,V2} = \omega_{n,\theta} / 40$ ; and  $\zeta_{V2} = 1$ .
- a. Show derivations (transfer function and gains)
- b. Turn in code that generates gains, and gain values.
- c. Turn in altitude mode code.
- d. Run simulation for 100

seconds with following commands (large alt. steps):

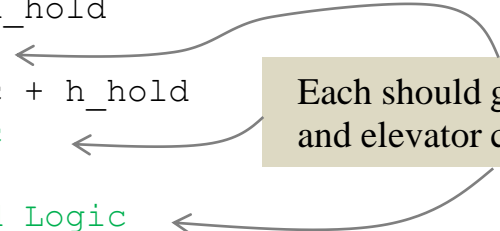
```
Va_c = 13;
chi_c = 0;
if time<30, h_c=100;
else, h_c=50; end;
```

Plot commands (where applicable) and responses for altitude, airspeed, pitch, and throttle. Highlight modes and transitions.

- e. (See question at right...)

- e. Did the longitudinal performance improve with the altitude moding logic? What would you do to improve it further? (You don't need to implement anything more, just discuss.)

```
if(firstTime)
    % Initialize integrators
    PIR_pitch_hold(0,0,0,firstTime,P);
    PIR_alt_hold_using_pitch(0,0,0,firstTime,P);
    PIR_airspeed_hold_using_throttle(0,0,0,firstTime,P);
    PIR_airspeed_hold_using_pitch(0,0,0,firstTime,P);
end
h_hold = 5; % m, alt threshold
if h_hat < h_c - h_hold
    % Climb Logic
elseif h_hat > h_c + h_hold
    % Descend Logic
else
    % Altitude Hold Logic
end
```

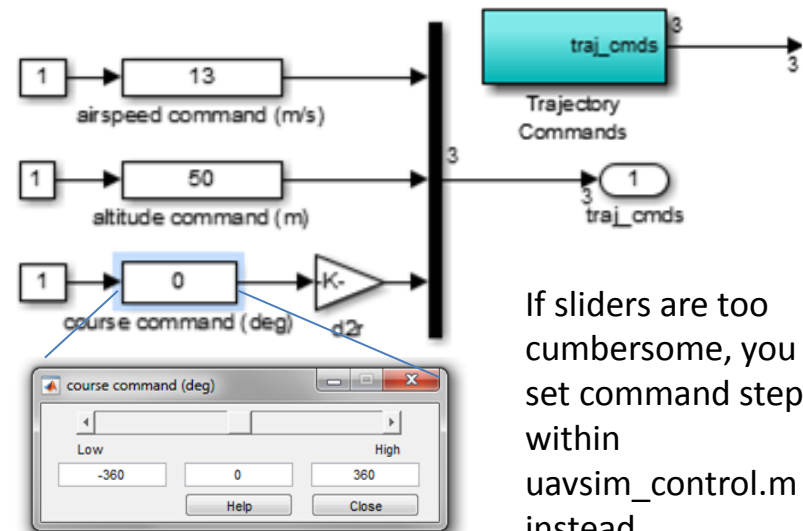


Each should generate throttle and elevator commands

# Lecture 8 Homework, 5/5

## 6) Demonstrate complete flight control in a gusting environment.

- Turn on gusting winds (steady winds can remain zero)
- Remove any `h_c`, `Va_c`, and `chi_c` overrides in `uavsim_control.m`.
- Run simulation for at least 100 seconds and manually perform multiple large step commands in airspeed, altitude and course using the sliders in the “Trajectory Commands” block.
- Plot the following longitudinal variables:
  - Altitude command and response
  - Airspeed command and response
  - Pitch command and response
  - Elevator
  - Throttle
- Plot the following lateral variables:
  - Course command and response
  - Roll command and response
  - Aileron
- Hopefully, your existing controllers will be robust enough for these commands and disturbances. If not, re-tune using the design parameters until they are.



If sliders are too cumbersome, you may set command steps within `uavsim_control.m` instead.

**Recommended Reading:**  
Beard & McLain Chapter 6

**Errata:** - Extra “-” signs in Figures 6.6, 6.7, 6.8, 6.9  
 - Equation on p101: Numerator should be  $a_{\phi 2} k_{p,\phi}$   
 - P116 & 117, line number references incorrect  
 (but we’re not using the book’s exact algorithm anyway)