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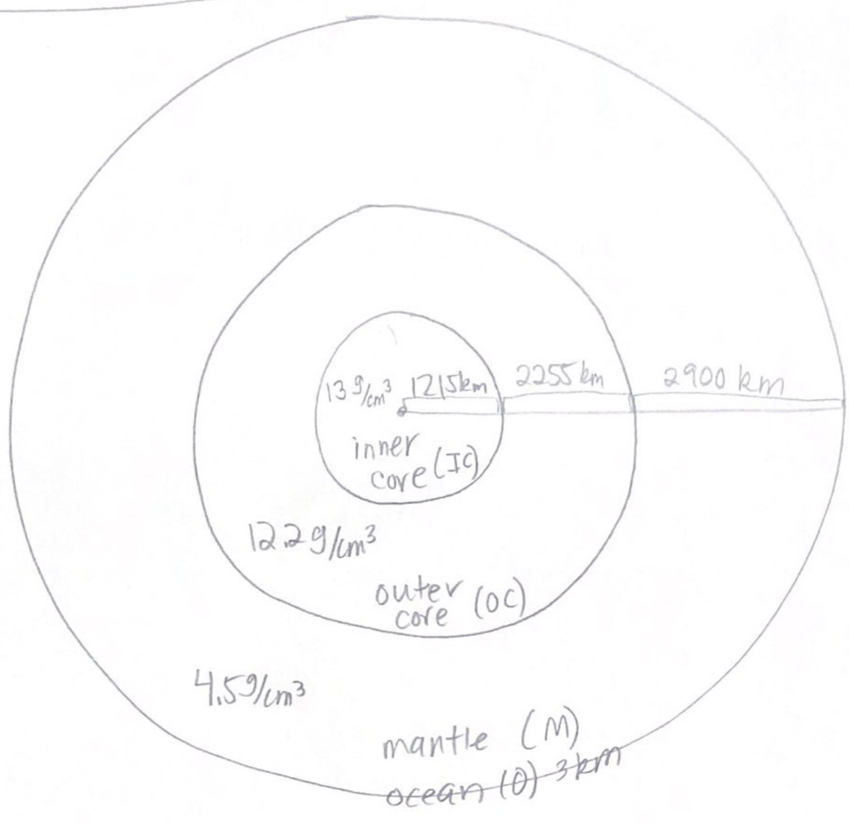
Problem 1.1 a)

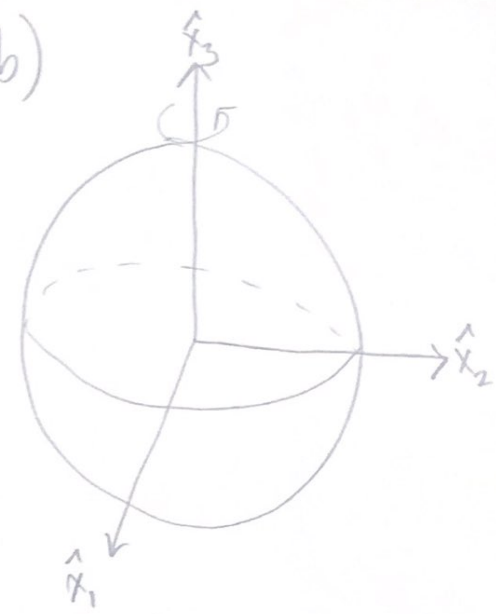


$$I_{Earth} = \frac{2}{5} M_{IC} R_{IC}^2 + \frac{2}{5} M_{OC} R_{OC}^2 + \frac{2}{5} M_M R_M^2 + \frac{2}{5} M_O R_O^2$$

$$M = \frac{4}{3} \pi R^3 \rho \qquad I_{Earth} \approx 6.47 \times 10^{31} \text{ kg km}^2$$

I assumed the Earth is spherical with four layers to include inner core, outer core, mantle, and 3 km thick ocean. I individual calculated the inertia for each layer and subtracted out the inner layers except for the inner core.

1.1b)



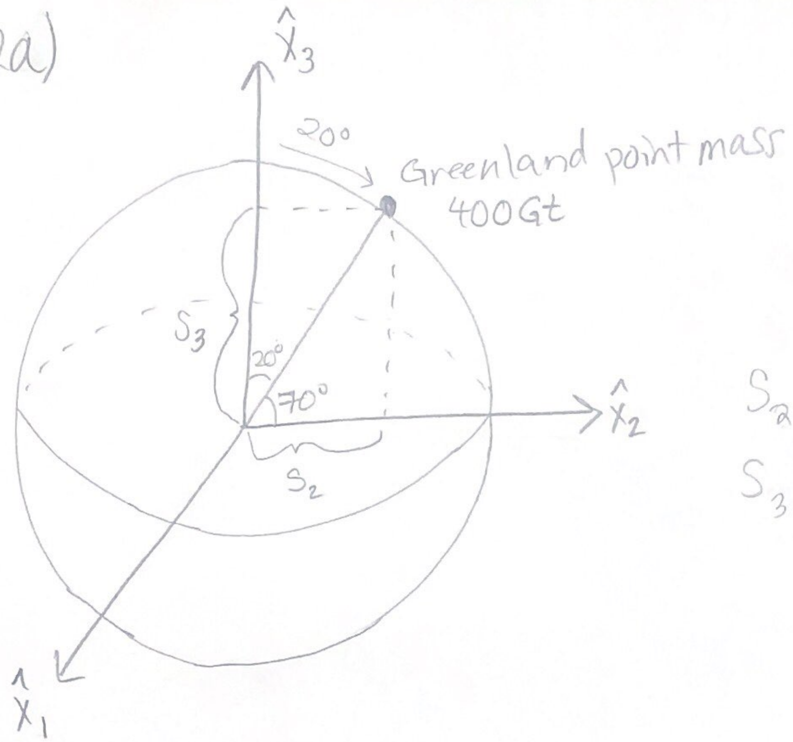
$$M_{Earth} = I_{IC} + I_{OC} + I_M + I_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6.47\times10^{31} & 0 & 0 \\ 0 & 6.47\times10^{31} & 0 \\ 0 & 0 & 6.47\times10^{31} \end{bmatrix}$$

the inertia tensor will not change with a change rotation in the coordinate system. The rotation axis is not part of the tensor.

1.1c) $L_i = M_{ij}\,\Omega_j$ $\qquad \Omega = \begin{bmatrix} 0 \\ 0 \\ 7.292\times10^{-5} \end{bmatrix} \frac{rad}{sec}$

$$L = \begin{bmatrix} 6.47\times10^{31} & 0 & 0 \\ 0 & 6.47\times10^{31} & 0 \\ 0 & 0 & 6.47\times10^{31} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 7.292\times10^{-5} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4.7\times10^{27} \end{bmatrix} \frac{rad\cdot kg\cdot km^2}{s}$$

1.2a)



$S_2 = R_E \sin(70°) = 2.17 \times 10^3$ km
$S_3 = R_E \sin(20°) = 5.98 \times 10^3$ km

The inertia tensor for the updated Greenland point mass distribution is the old point mass inertia tensor subtract a thin spherical inertia for the loss 400Gt water plus the point mass inertia of Greenland.

$$M_{loss\ 400\,Gt} = \left(\tfrac{2}{3}\right) Mass_{400Gt} R^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3.21\times10^{21} & 0 & 0 \\ 0 & 3.21\times10^{21} & 0 \\ 0 & 0 & 3.21\times10^{21} \end{bmatrix} \text{kg km}^2$$

$$M_{point\ mass} = mass_{400Gt} \begin{bmatrix} s_2^2+s_3^2 & 0 & 0 \\ 0 & s_1^2+s_2^2 & -s_2 s_3 \\ 0 & -s_3 s_2 & s_1^2+s_3^2 \end{bmatrix} = \begin{bmatrix} 1.62\times10^{22} & 0 & 0 \\ 0 & 1.43\times10^{22} & -0.5\times10^{22} \\ 0 & -0.5\times10^{22} & 0.19\times10^{22} \end{bmatrix} \text{kg km}^2$$

$$M_{new} = M_{Earth} - M_{loss\,400Gt} + M_{point\ mass} = \begin{bmatrix} 6.47\times10^{31} & 0 & 0 \\ 0 & 6.47\times10^{31} & -5.2\times10^{21} \\ 0 & -5.2\times10^{21} & 6.47\times10^{31} \end{bmatrix} \text{kg km}^2$$

1.2b)

The angular momentum is conserved. The new calculated angular velocity is

$$M_{new} \Omega_{new} = L$$

$$M_{new}^{-1} M_{new} \Omega_{new} = M_{new}^{-1} L$$

$$\Omega_{new} = M_{new}^{-1} L$$

$$\Omega_{new} = \begin{bmatrix} 0 \\ 5.8 \times 10^{-11} \\ 0.729 \times 10^{4} \end{bmatrix} \frac{rad}{sec}$$

1.2c) The new period for Earth is $\approx 8.61 \times 10^4$ s which is approximately 1.7 microseconds shorter than Earth spin. So the Earth would spin faster and compared to the Japan Earthquake it's nearly the same, 1.8 microsec.

Table of Contents


```
% SIO 229 Homework 1
clear all; close all; clc
```

Problem 1.1a

Calculate the interia of Earth using I = mr^2 for four spherical layers

```
% Calculate the mass for the layers if the sphere is filled entire of
 that
% material
% Inner core mass
radiusIC = 1215;            % radius of inner core km
densityIC = 13e12;          % density of inner core kg/km^3
massIC = (4/3)*pi*(radiusIC)^3*densityIC;
% Outer core mass
radiusOC = 2255+radiusIC;            % radius of outer core km
densityOC = 12.2e12;                   % density of outer core kg/km^3
massOC = (4/3)*pi*(radiusOC)^3*densityOC;
% Mantle mass
radiusMan = 2900 +radiusOC;            % radius of mantle km
densityMan = 4.5e12;                    % density of mantle kg/km^3
massMan = (4/3)*pi*(radiusMan)^3*densityMan;
% Ocean mass
radiusOcean = 3 +radiusMan;            % radius of ocean km
densityOcean = 1.03e12;                    % density of mantle kg/km^3
massOcean = (4/3)*pi*(radiusOcean)^3*densityOcean;

% Calculate the inertia for the layers
% Inertia inner core
inertiaIC = (2/5)*massIC*radiusIC^2;
% Inertia outer core
inertiaOC = (2/5)*massOC*radiusOC^2 - (2/5)*massOC*radiusIC^2;
% Inertia mantle
inertiaMan = (2/5)*massMan*radiusMan^2 - (2/5)*massMan*radiusOC^2;
% Inertia ocean
inertiaOcean = (2/5)*massOcean*radiusOcean^2 -
 (2/5)*massOcean*radiusMan^2;

% Sum of all layers (kg)
inertiaEarth = inertiaIC + inertiaOC + inertiaMan + inertiaOcean
```

```
inertiaEarth =

     6.471030383777104e+31
```

Problem 1.1b

```
%  Earth's inerta tensor assuming spherical layers of inner core,
 outer
%  core, mantle, and a layer of ocean

% spherical inertia tensor
Mearth = inertiaEarth*eye(3)


Mearth =

   1.0e+31 *

   6.471030383777104                   0                   0
                   0   6.471030383777104                   0
                   0                   0   6.471030383777104
```

Problem 1.1c

angular momentum of Earth's spin axis assuming spherical and rotation rate of 7.2921150e-5 rad/s

```
earthRot= 7.2921150e-5;         % Earth's rotation rate radian/sec
omega = [0; 0; earthRot];       % Angular velocity radians/ sec
tperiod = 2*pi/omega;           % Earth's period, length of a day in
 sec
L = Mearth*omega                % Angular momentum kg*km^2/sec


L =

   1.0e+27 *

                   0
                   0
   4.718749772699677
```

Problem 2.1a

```
% Thin spherical shell of 400 Gt of ocean inertia
ocean400Gt = 4e14;              % 400 Gt of water to kg
M400Gt = (2/3)*ocean400Gt*radiusOC^2*eye(3) % inertia of water 400Gt

% Point mass location of Greenland approximate 70 degrees N of Equator
% x3 is the axial spin
```

```
s2 = radiusOcean*sind(20)          % km
s3 = radiusOcean*sind(70)          % km
s1 = 0;                             % km

% Inertia of Greenland as a pointmass
greenInertia = [s2^2+s3^2 -s1*s2 -s1*s3; -s2*s1 s1^2+s3^2 -s2*s3; -
s3*s1 -s3*s2 s1^2+s2^2];
Mgreen = ocean400Gt*greenInertia

% Inertia tensor of Earth with Greenland as a pointmass
Mearth2 = Mearth-M400Gt+Mgreen


M400Gt =

   1.0e+21 *

   3.210906666666666                   0                   0
                   0   3.210906666666666                   0
                   0                   0   3.210906666666666


s2 =

     2.179694373414487e+03


s3 =

     5.988661072268595e+03


Mgreen =

   1.0e+22 *

   1.624605160000000                   0                   0
                   0   1.434562457540209  -0.522138033740409
                   0  -0.522138033740409   0.190042702459791


Mearth2 =

   1.0e+31 *

   6.471030385080619                   0                   0
                   0   6.471030384890576  -0.000000000522138
                   0  -0.000000000522138   6.471030383646057
```

Problem 2.1b

```
% Calculate new angular velocity assuming angular momentum vector is
```

```
% conserved.
omega2 = Mearth2\L

% Earth's period, length of a day in seconds
tperiod2 = 2*pi/omega2

tperiod-tperiod2
```

```
omega2 =

   1.0e-04 *

                   0
   0.000000000058839
   0.729211500014767


tperiod2 =

   1.0e+04 *

                   0                   0   8.616410063544450


ans =

   1.0e-05 *

                   0                   0   0.174493470694870
```

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