SIO229 Homework 1

For all problems, include a sketch (if it helps communicate what you are doing), reasonable mathematical detail, succinct explanations of your methods and results, and any code you used. Equations should be developed in symbolic form before being evaluated numerically. A well-presented homework solution strives for the same qualities as a good academic manuscript: clarity, completeness, and just the right amount of reader hand-holding. Good luck!

Problem 1.1

a. Calculate the moment of inertia for a simple, spherically symmetric Earth. You can make whatever reasonable approximations you like about Earth structure, including assuming that any compositional layers are of uniform density. Feel free to use canned formulas for this step (i.e., no need to evaluate any integrals).

b. Write down the inertia tensor for this simple Earth, assuming a coordinate system whose origin is at the center of Earth's mass and whose axes are aligned with Earth's spin axis and equator. How does this inertia tensor change if you rotate the coordinate system by 45 degrees around one of the equatorial axes?

c. What is the angular momentum of this Earth around its spin axis, assuming Earth's current rotation rate of 7.2921150×10^{-5} rad/s? Again, assume that Earth's spin axis is aligned with one axis of an Earth-centered coordinate system.

Problem 1.2

Now imagine that 400 Gt of water (roughly equivalent to the annual mass loss from the Greenland Ice Sheet) is somehow transported from the global oceans to Greenland and frozen into place. To simplify the problem, assume that Earth is entirely covered by ocean and that the 400 Gt deposited onto Greenland is a point mass located at sea level at the approximate center of the ice sheet.

a. Write down the updated inertia tensor for your (otherwise) spherically symmetric Earth.

b. Calculate the change in 1) angular momentum and 2) angular velocity of Earth with this mass redistribution.

c. Finally, what is the associated change in Earth's length of day, and how does it compare with the estimate made by Richard Gross for the 2011 Tohoku Earthquake (you can use the value reported in https://www.nasa.gov/topics/earth/features/japanquake/earth20110314.html)?