SIO 229 Gravity and Geomagnetism
Geomag Homework # 2,
Due March 17, 2021

1. The radial component of the magnetic field is $B_r = \hat{\mathbf{r}} \cdot \mathbf{B}$. Show that $\nabla^2(rB_r) = 0$, that is rB_r is also harmonic outside of $S(a)$. This is the result we made use of in discussing upward continuation in Section 3.2. You may find this easiest to do using the summation notation.

2. The IGRF for 2020 has been used to make a prediction of expected magnetic field elements at Earth's surface for a small range of sites around the world, and is provided in the file **magdata_2020** linked on the class website. Follow the steps below and use the X, Y, Z components given to make a degree 1, Schmidt normalized spherical harmonic model of the magnetic field based on these data. You may assume that the Earth is a sphere for this purpose.

(a) Write the data vector **d** associated with this data.

(b) Find expressions for the elements of the matrix **G** connecting the $l = 1$ model vector, with magnetic field parameters g_1^0, g_1^1, h_1^1, to the data vector **d**.

(c) Evaluate the elements of this **G** matrix.

(d) Use Matlab or other methods of your choice to calculate $\mathbf{G}^T\mathbf{G}$, $\mathbf{G}^T\mathbf{d}$ and $(\mathbf{G}^T\mathbf{G})^{-1}$.

(e) Find the least squares solution for the $l = 1$ model parameters.

(f) Compare your result with the 2020 IGRF magnetic field predictions and decide whether or not your results make sense in light of what you know about the non-dipole field.

(g) Do your results depend on the available data distribution? Explain your answer.

(h) In Section 3.4 of the notes we noted the difference between the geodetic and geocentric coordinate systems. What is the approximate size of the error you make in neglecting to correct for the ellipsoidal shape of the earth and use geocentric latitude and longitude in calculating the field from a spherical harmonic model?

3. As suggested in the notes if you assume that magnetization is constant throughout a source volume V, Equation (165) can be rewritten in a form that allows you to apply the divergence theorem, and convert it to a surface integral. Show how this can be done and find the surface integral.

4. Find out about Poisson's relationship between gravity and magnetic field anomalies. Explain what it is, how it works, and when its use might be appropriate.