SIO 229 HW#2

Problem 2.1

$$\vec{g} = -\nabla V$$

$$= -\nabla G\left(\frac{M_E}{r} - \frac{a^2}{r^3} J_2 \frac{1}{2}(3\cos^2\theta - 1) + \dots\right)$$

$$\approx -G\left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\phi}\hat{\phi}\right)\left(\frac{M_E}{r} - \frac{a^2}{r^3} J_2 \frac{1}{2}\left(3\cos^2\theta - 1\right)\right)$$

$$= -\frac{\partial}{\partial r}G\left(\frac{M_E}{r} - \frac{a^2}{r^3} J_2 \frac{1}{2}(3\cos^2\theta - 1)\right)\hat{r}$$

$$- \frac{\partial}{\partial\theta}\frac{G}{r}\left(\frac{M_E}{r} - \frac{a^2}{r^3} J_2 \frac{1}{2}\left(3\cos^2\theta - 1\right)\right)\hat{\theta}$$

$$- \frac{G}{r\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{M_E}{r} - \frac{a^2}{r^3} J_2 \frac{1}{2}(3\cos^2\theta - 1)\right)\hat{\phi}$$

$$\vec{g} \approx G\left[\left(\frac{M_E}{r^2} - \frac{3a^2 J_2 (3\cos^2(\theta) - 1)}{2r^4}\right)\hat{r} - \left(\frac{3a^2 J_2 \sin(\theta)\cos(\theta)}{r^4}\right)\hat{\theta}\right]$$

$r_{pole} = 6357$ km

$\theta_{pole} = 0°$

$\vec{g}_{pole} = (9.8633, 0, 0)$ m/s²

$r_{equator} = 6378$ km

$\theta_{equator} = 90°$

$\vec{g}_{equator} = (9.7984, 0, 0)$ m/s²

$$\boxed{|\vec{g}|_{pole} = 9.863 \text{ m/s}^2 \;..> \; |\vec{g}|_{equator} = 9.798 \text{ m/s}^2}$$

Problem 2.2

$$U_0 = -\frac{G m_E}{r_0(\theta)} + \frac{G m_E a^2}{r_0(\theta)^3} J_2 P_2(\cos\theta) - \frac{1}{2}\omega^2 r_0(\theta)^2 \sin^2\theta$$

where $r_0(\theta) = a(1 - f\cos^2(\theta))$

$$f = \frac{3}{2} J_2 + \frac{1}{2}\frac{a^3\omega^2}{G m_E}$$

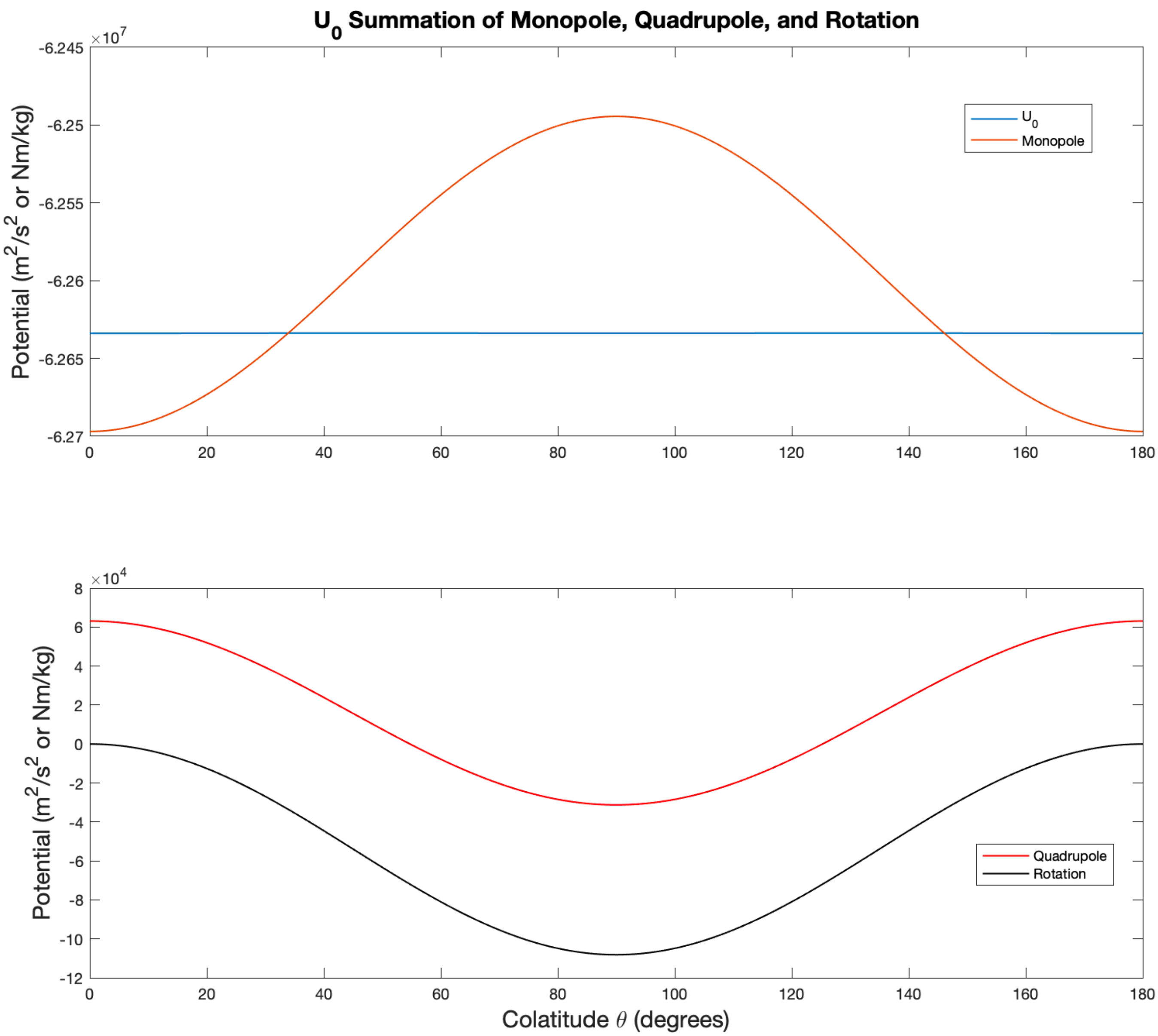
$$J_2 \approx 0.001$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

$$\omega = 7.291150 \times 10^{-5} \ \frac{rad}{s}$$

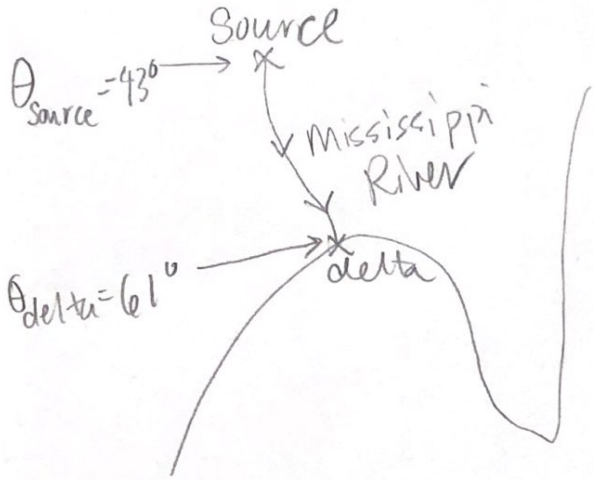
The geopotential U_0 on the geoid is approximately $-6.263 \times 10^7 \ \frac{N \cdot m}{kg}$ or $\frac{m^2}{s^2}$.

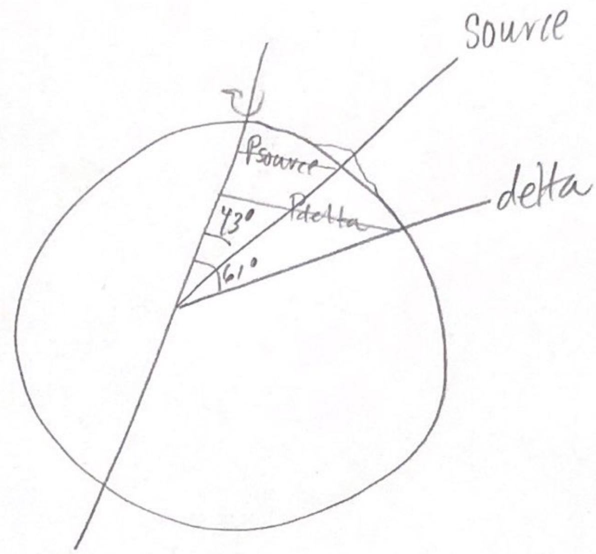
The calculation was done on matlab under problem 2.2. The graph shows U_0 being constant around the Earth. The graph also seperates out the monopole, quadrupole, and spin rotation.

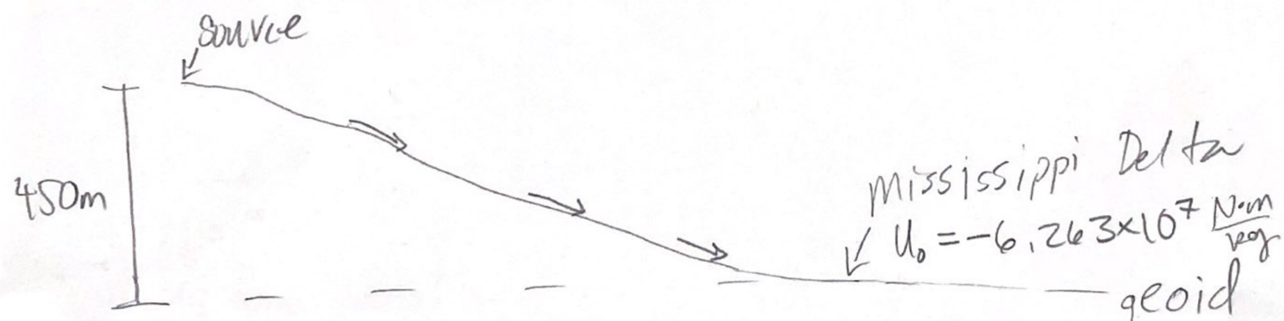


U0 Summation of Monopole, Quadrupole, and Rotation
×10^7
-6.245
-6.25
-6.255
-6.26
-6.265
-6.27
Potential (m^2/s^2 or Nm/kg)
U0
Monopole
0
20
40
60
80
100
120
140
160
180
×10^4
8
6
4
2
0
-2
-4
-6
-8
-10
-12
Potential (m^2/s^2 or Nm/kg)
Quadrupole
Rotation
Colatitude θ (degrees)

Problem 2.3







The Mississippi delta is located on the geoid so the potential difference is the 450 m elevation and the 18° change in colatitude. The Mississippi river will stop flowing when $U_{source} = U_{delta}$.

$$U_{source} = \underbrace{\frac{-Gm_E}{r_0(43°)+450m} - \frac{Gm_E a^2 J_2 P_2(\cos(43°))}{(r_0(43°)+450m)^3}}_{V_{source} \text{ gravitational potential}} - \underbrace{\frac{1}{2}\omega^2 (r_0(43°)+450m)^2 \sin^2(43°)}_{\text{spin potential } S_{source}}$$

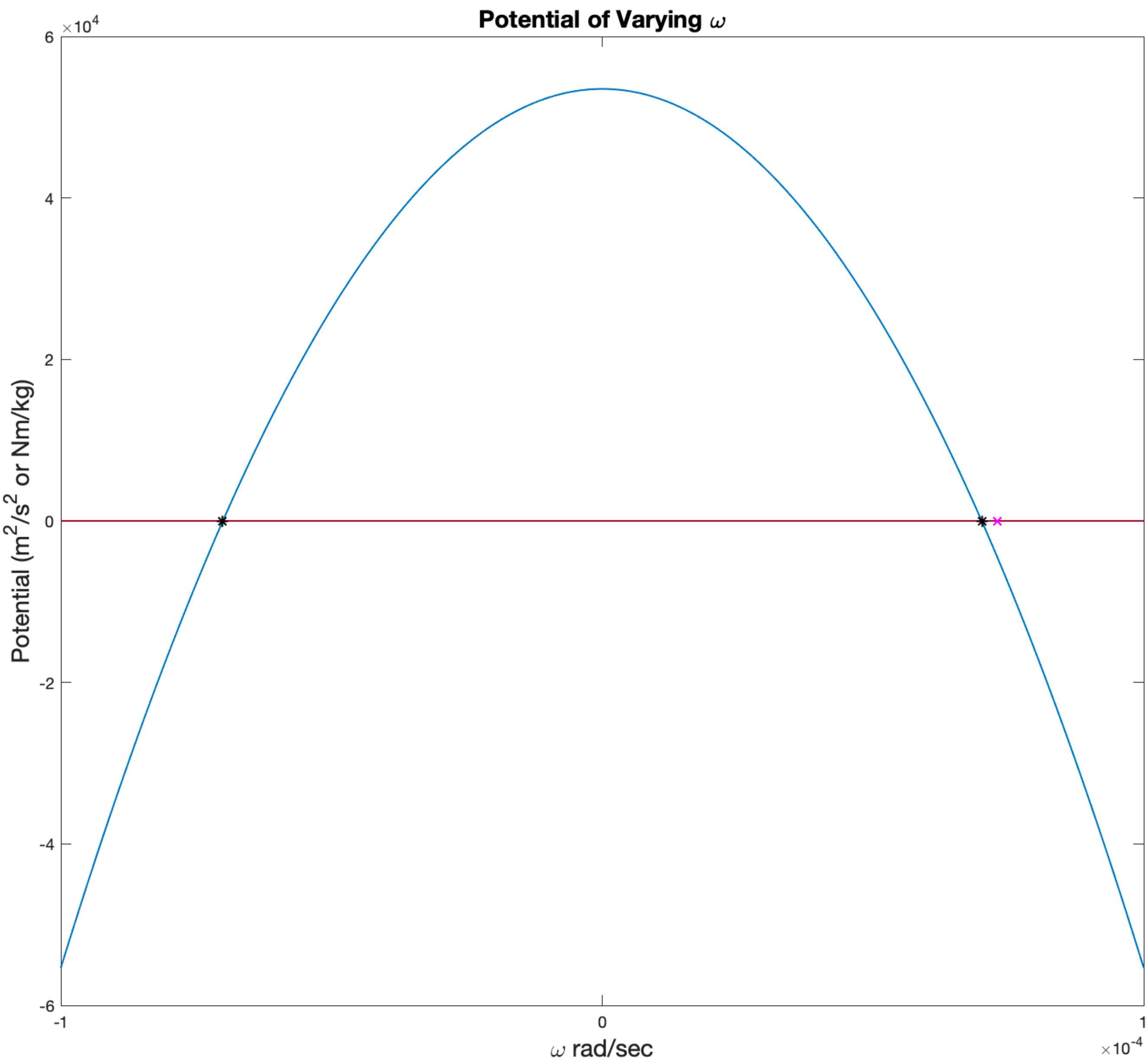
$$U_{delta} = U_0 = \underbrace{\frac{-Gm_E}{r_0(61°)} + \frac{Gm_E a^2 J_2 P_2(\cos(61°))}{(r_0(61°))^3}}_{V_{delta}} \underbrace{- \frac{1}{2}\omega^2 (r_0(61°))^2 \sin^2(61°)}_{S_{delta}}$$

$$V_{soure} + S_{source} = V_{delta} + S_{delta}$$

J_2 and $P_2(\cos\theta)$ does not depend on ω, however, r_0 does. I varied ω from $-1\times10^{-4}\ \frac{rad}{sec}$ to $1\times10^{-4}\ \frac{rad}{sec}$ and recalculateed r_0 for the changing spin. The graph shows where the potential is zero which is the point where there wouldn't be flow.

$$0 = -V_{source} - S_{soure} + V_{delta} + S_{delta}$$

The red x is the current spin rate and the black * are the spin rate when flow stops. The current spin rate would need to slow down to ~24.89 hour days.



Potential of Varying ω
×10^4
6
4
2
0
-2
-4
-6
Potential (m^2/s^2 or Nm/kg)
-1
0
1
×10^-4
ω rad/sec

Table of Contents


```
% SIO 229 Homework 2
clear all; close all; clc
```

Problem 2.1

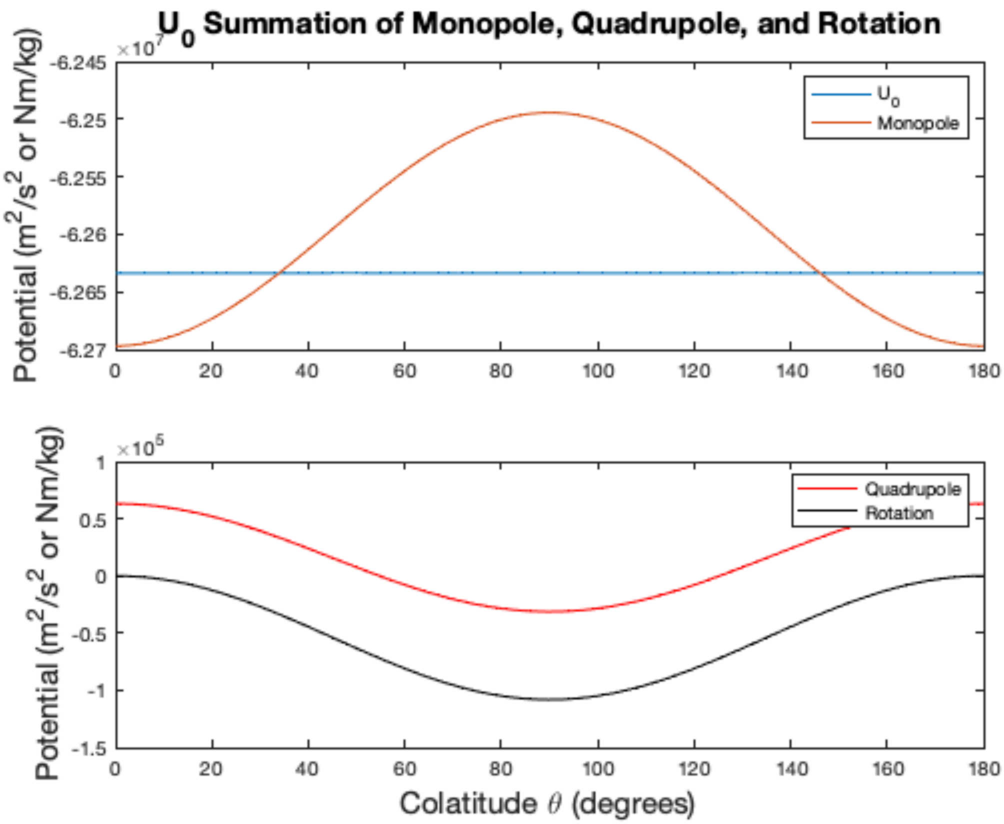
Use MacCullagh's Formula to compare the difference in gravity at the pole and at the equator without Earth's spin and assume present shape and interior structure.

```
% g = -gradient of V
% V is MacCullagh's Formula
mEarth = 5.972e24;           % Mass of Earth (kg)
J = 0.001;                   % J_2 value for flattening roughly,
 unitless
a = 6378000;                 % Radius of Earth at equator m
G = 6.6743e-11;              % Gravitational constant N*m^2/kg^2

% radius and theta
rPole = 6357000;            % radius at the pole meters
thetaPole = 0;              % theta at the pole degrees

rEquator = a;               % radius at the equator meters
thetaEquator = 90;          % theta at the equator degrees

% component for g at the poles without phi
g_rPole = G*(mEarth/rPole^2-(3*a^2*J*(3*cosd(thetaPole)^2-1))/
(2*rPole^4));
g_thetaPole = G*(-3*a^2*J*sind(thetaPole)*cosd(thetaPole))/rPole^4;

% component for g at the equator without phi
g_rEquator = G*(mEarth/rEquator^2-
(3*a^2*J*(3*cosd(thetaEquator)^2-1))/(2*rEquator^4));
g_thetaEquator = G*(-3*a^2*J*sind(thetaEquator)*cosd(thetaEquator))/
rEquator^4;
```

Problem 2.2

```
% Calculate the value of the geopotential U on the geoid

w = 7.2921150e-5;        % Earth's spin rate (rad/s)
theta = 0:180;           % degrees of latitude
```

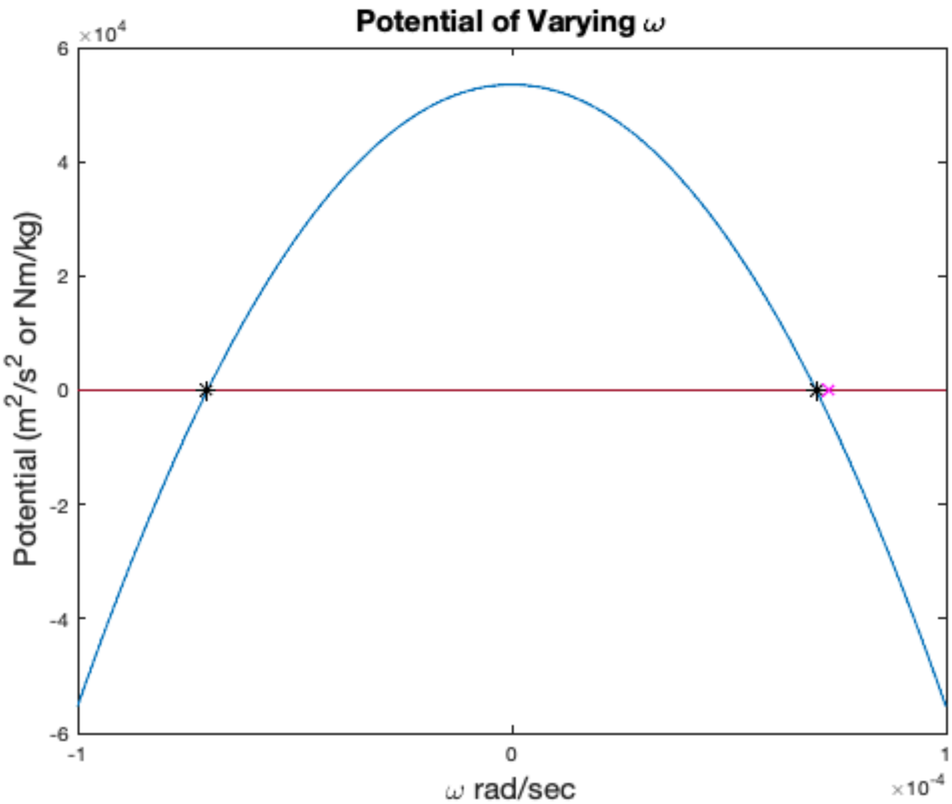
```
f = (3/2)*J+(a^3*w^2)/(2*G*mEarth);      % flattening, unitless
r0 = a*(1-f*cosd(theta).^2);             % changing radius value in
 meters
P = (1/2)*(3*cosd(theta).^2-1);          % Legrende Polynomial,
 unitless

mono = -G*mEarth./r0;                    % Monopole  N*m/kg
quad = (G*mEarth*a^2)*(J*P)./(r0.^3);    % Quadrupole  N*m/kg
spin = -(1/2)*w^2*r0.^2.*sind(theta).^2;  % Rotation  m^2/s^2

U0= mono+quad+spin;                      % Geopoential U on the geoid

U0mean = mean(U0)

figure(1)
subplot(2,1,1)
plot(theta, U0,'LineWidth',1)
hold on
plot(theta, mono,'LineWidth',1)
ylabel('Potential (m^2/s^2 or Nm/kg)','FontSize',15)
legend('U_0','Monopole')
hold off
title('U_0 Summation of Monopole, Quadrupole, and
 Rotation','FontSize',15)
subplot(2,1,2)
plot(theta,quad,'r','LineWidth',1)
hold on
plot(theta, spin,'k','LineWidth',1)
ylabel('Potential (m^2/s^2 or Nm/kg)','FontSize',15)
legend('Quadrupole','Rotation')
xlabel('Colatitude \theta (degrees)','FontSize',15)
set(gcf,'color','w');
hold off


U0mean =

  -6.2634e+07
```



Problem 2.3

```
% Find the rotation rate, omega, when the Mississippi starts to flow
% backward from South to North. Assuming the Earth does not change
 shape.
% The source of the Mississippi is 47 degrees North and the delta is
 29
% degrees North of the Equator.

MissLatS = 43;           % Mississippi source colatitude from the pole
 degrees
MissLatD = 61;           % Mississippi delta colatitude from the pole
 degrees

radiusSource = r0(44)+450;      % radius of source in meters
Psource = (1/2)*(3*cosd(MissLatS).^2-1);         % Legrende Polynomial
 for Source

% Vary omega for when potential for the source and delta are equal
omega = (-1e-4:1e-7:1e-4);    % Spin radian/sec

% Calculate the source gravitational potential, V source in N*m/kg or
% m^2/s^2
Vsource = -G*mEarth/radiusSource + G*mEarth*a^2*J*Psource/
(radiusSource^3);
```

```
% Calculate the source apparent centrifugal force, depends on omega
 m^2/s^2
SpinSource = -(1/2)*omega.^2*radiusSource^2*sind(MissLatS)^2;

% Calculate the delta gravitational potential, geoid potential, V in
 N*m/kg
% depends on omega
f = (3/2)*J+(a^3*omega.^2)/(2*G*mEarth);    % flattening depends on
 rot. rate
r0Delta = a*(1-f.*cosd(MissLatD).^2);            % radius meters
 depends on rotation rate
Pdelta = (1/2)*(3*cosd(MissLatD)^2-1);         % Legrende Polynomial,
 unitless
Vdelta = -G*mEarth./r0Delta + G*mEarth*a^2*J*Pdelta./(r0Delta.^3);

% Calculate the delta apparent centrifugal force m^2/s^2, depends on
 omega
SpinDelta = -(1/2)*omega.^2.*r0Delta.^2*sind(MissLatD)^2;

% Source and Delta potential equalling zero
SandDequal0 = -Vsource - SpinSource + Vdelta + SpinDelta;

T = (2*pi./omega)/(60*60);      % Period of rotation of Earth in hours
Topp = T(300)                   % Period in the opposite direction
Tsame = T(1702)                 % Period for the same direction today

figure(2)
plot(omega, SandDequal0,'LineWidth',1)
hold on
plot(omega, zeros(length(omega)),'LineWidth',1)
plot([omega(1702) omega(300)],[SandDequal0(1702)
 SandDequal0(300)],'k*','LineWidth',1)
% plot(omega(300), SandDequal0(300),'k*','LineWidth',1)
plot(w, 0,'mx','LineWidth',1)
ylabel('Potential (m^2/s^2 or Nm/kg)','FontSize',15)
xlabel('\omega rad/sec','FontSize',15)
title('Potential of Varying \omega','FontSize',15)
set(gcf,'color','w')
hold off


Topp =

  -24.8977


Tsame =

   24.8977
```



Published with MATLAB® R2020a