SIO 229 Gravity and Geomagnetism
Geomag Homework # 1,
Due February 24, 2021

Q1. Magnetic & Electric Fields Refresher.

What are the SI units for **B**, **H**, **E**, **D**, **J**, σ, and ϵ_0? Show that the unit T m A^{-1} is equivalent to H m^{-1} for μ_0, and express the units in terms of another pair of SI quantities.

Draw the magnetic field lines emanating from a magnetic dipole. How does the shape of the field compare to that from an electric charge or an electric dipole?

What are the units for a magnetic dipole moment? Suppose we have a magnetic dipole **m** with dipole moment m at Earth's center and oriented along Earth's rotation axis, calculate (i) the magnetic field elements B_r, B_θ, B_ϕ and (ii) the declination, inclination and intensity as a function of latitude. Feel free to approximate the shape of Earth's surface as a sphere. Make a plot of your results.

Q2. A common first order approximation to Earth's field is to suppose it can be represented by a geocentric dipole that is tilted with respect to the geographic axis. The place where the dipole axis would pierce Earth's surface is called a geomagnetic pole, and the dipole strength is called the geomagnetic dipole moment.

(a) Regardless of whether it is produced by a dipole field a single vector magnetic field observation (let's call it $\mathbf{B}(r, \theta, \phi)$) at any location, (θ, ϕ), on Earth's surface can be transformed into an equivalent virtual geocentric dipole **V**. Show how this can be calculated, and express your result as a linear, position-dependent transformation from **B** to **V**.

(b) How is the dipole moment m related to the Gauss coefficients g_1^0, g_1^1, and h_1^1? What is the difference between m and V?

(c) Calculate Earth's dipole moment using the table of Gauss coefficients provided on p. 33 of the notes. How much has this changed in the past 20 years? See https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html for 2020 IGRF coefficients.

If the dipole part of Earth's field continued to decay at the current rate given by the IGRF when would you expect the next geomagnetic reversal? Do you think this is likely?