Q1)

B - magnetic field (tesla or $\frac{kg}{s^2 A}$)

H - Magnetic Displacement ($\frac{A}{m}$)

E - Electric Field ($\frac{volt}{m}$)

D - Electric Displacement (sec·A)

J - current density ($\frac{A}{m^2}$)

σ - electrical conducitivity ($\frac{S^3 A^2}{kg\ m^3}$) ($\frac{siemens}{m}$)

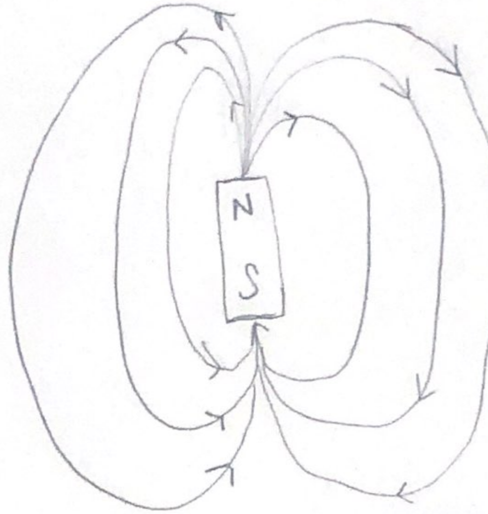
ϵ_0 - permeability of vacuum ($\frac{H}{m}$) ($\frac{Tm}{A}$) ($\frac{kg\ m}{S^2 A}$)

Telsa - $\frac{kg}{S^2 A}$

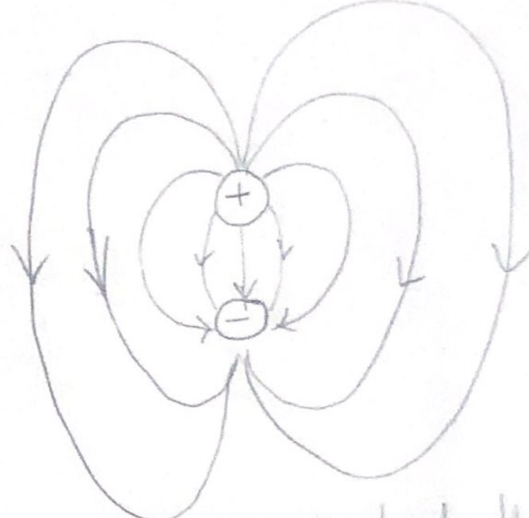
Henrie - $\frac{kg\,m^2}{S^2 A^2}$

$$\frac{Tm}{A} \Rightarrow \frac{kg}{S^2 A} \cdot \frac{m}{A} \Rightarrow \frac{kg}{S^2 A} \cdot \frac{m}{A} \cdot \frac{m}{m} \Rightarrow \frac{kg\,m^2}{S^2 A^2} \cdot \frac{1}{m} \Rightarrow \frac{H}{m}$$

Magnetic Field



Electric Dipole Field



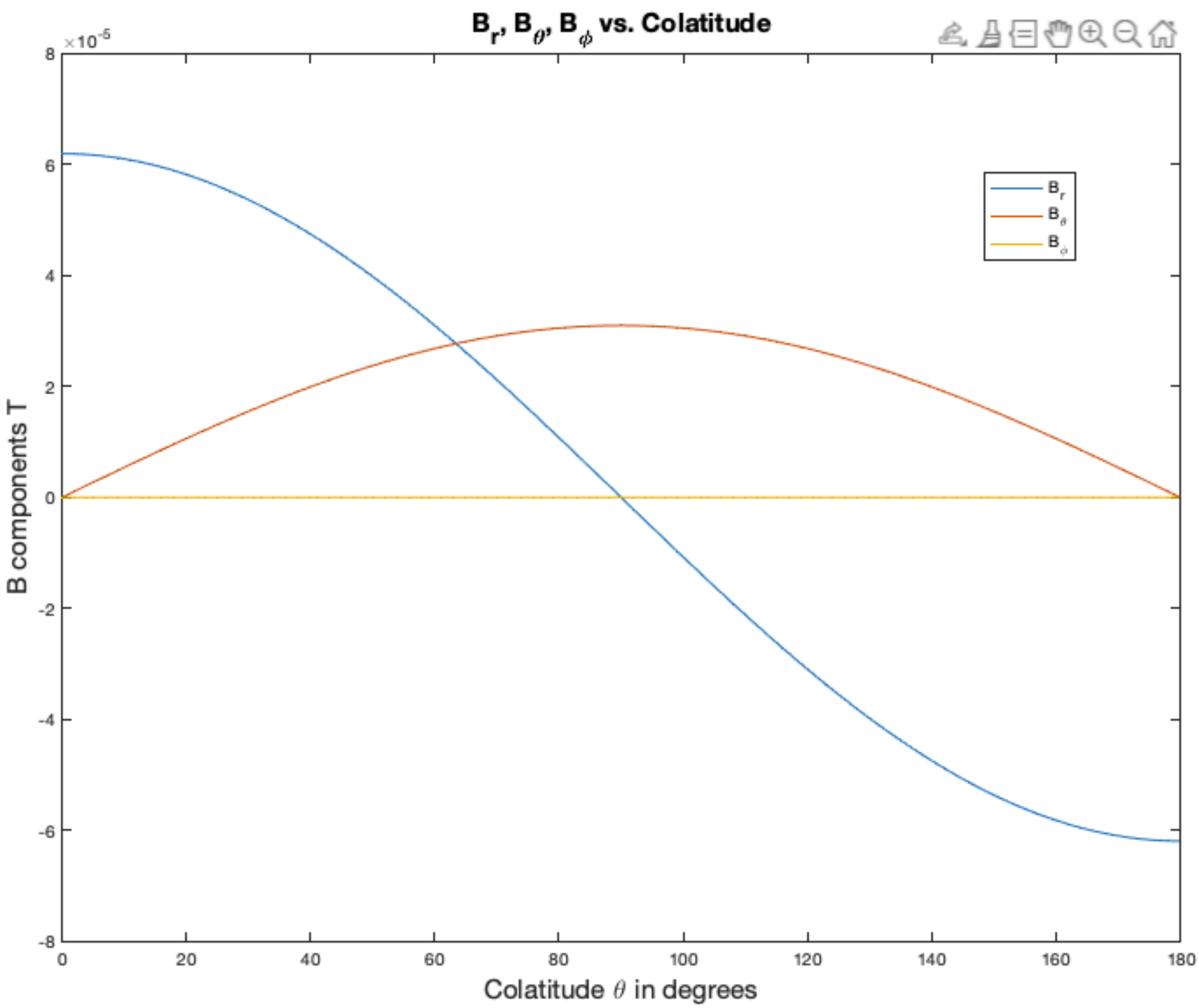
The magnetic and electric field have similar shape but the electric field does not close like the magnetic field.

The magnetic dipole moment, m is measured in $A \cdot m^2$.
the magnetic field elements was plotted with the following equations

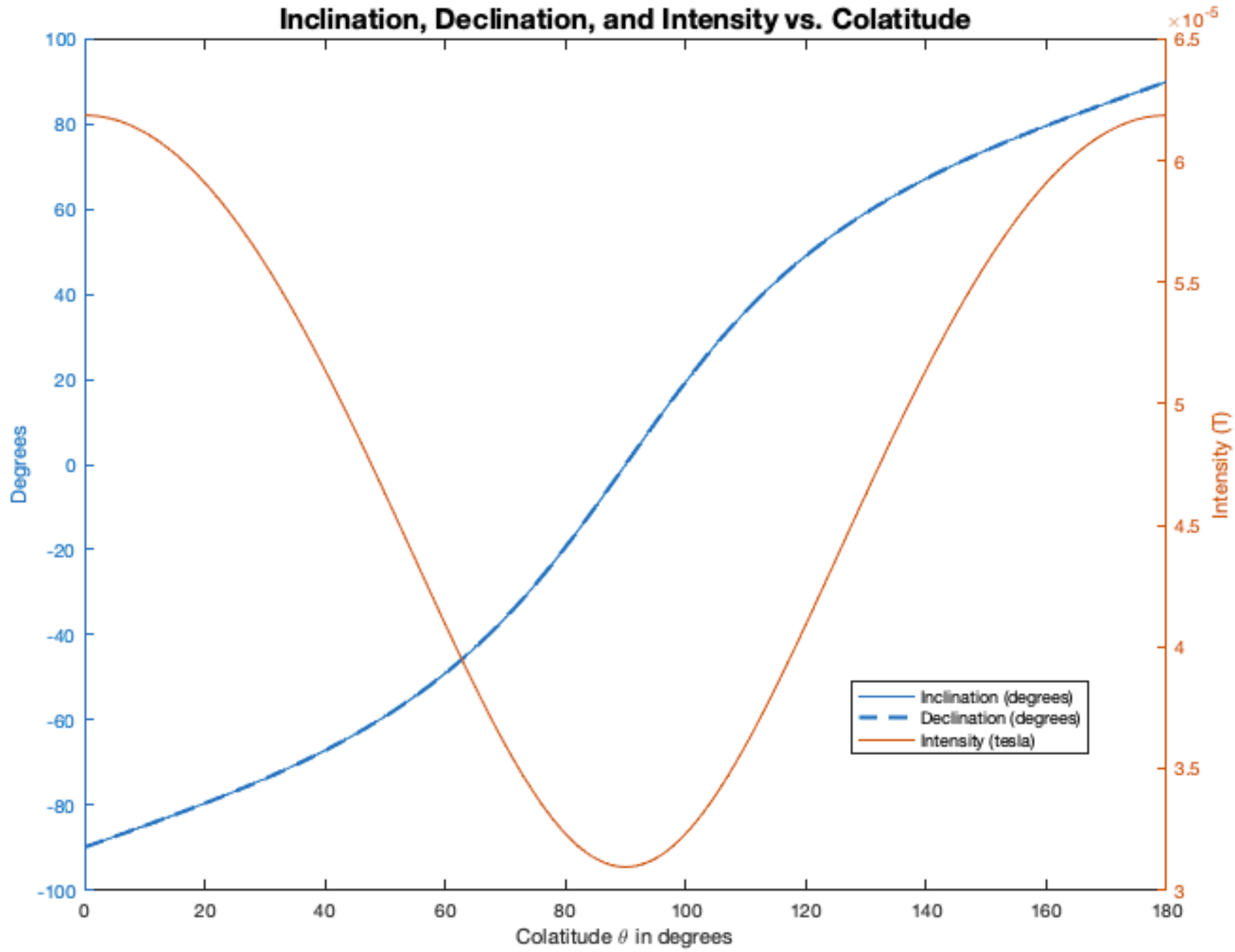
$$B_{dipole}(r) = \frac{\mu_0 \cdot m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$D = \tan^{-1}\left[\frac{B_\phi}{-B_\theta}\right] \qquad I = \tan^{-1}\left[\frac{-B_r}{(B_\theta^2 + B_\phi^2)^{1/2}}\right] \qquad F = (B_\theta^2 + B_\phi^2 + B_r^2)^{1/2}$$

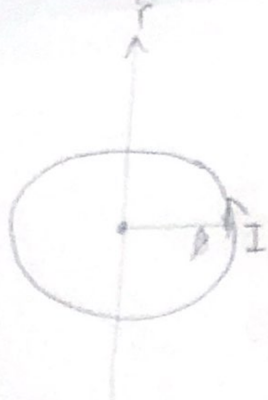
Constants:
$\mu_0 = 4\pi \times 10^{-7}\ \frac{H}{m}$, $m = 8 \times 10^{22}\ mA^2$, $r = 6371\ km$



B_r, B_θ, B_ϕ vs. Colatitude
$\times 10^{-5}$
8
6
4
2
0
-2
-4
-6
-8
B components T
0
20
40
60
80
100
120
140
160
180
Colatitude θ in degrees
B_r
B_θ
B_ϕ


Inclination, Declination, and Intensity vs. Colatitude
Degrees
Intensity (T)
×10^-5
Colatitude θ in degrees
Inclination (degrees)
Declination (degrees)
Intensity (tesla)
100
80
60
40
20
0
-20
-40
-60
-80
-100
6.5
6
5.5
5
4.5
4
3.5
3
0
20
40
60
80
100
120
140
160
180

Q2)



magnetic dipole moment

$m = I(\text{Area})$ $[Am^2]$

Biot Savart Law

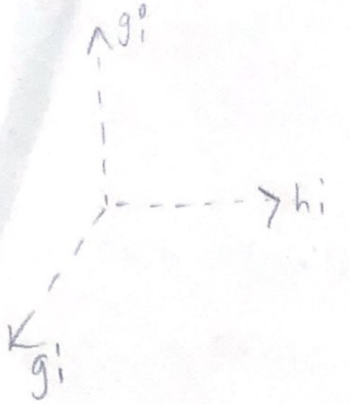
$\nabla \times B = \mu_0 J$ $\qquad J = \frac{m}{(\text{Area})^2}$ from $I = \frac{m}{\text{Area}}$

$\nabla \times B = \mu_0 \frac{m}{(\pi r^2)^2} \hat{\theta} \times \hat{\phi} =$

$$\nabla \times B = \frac{1}{r\sin\theta}\left(\frac{d}{d\theta}(B_\phi \sin\theta) - \frac{\partial B_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial B_r}{\partial \phi} - \frac{\partial}{\partial r}(r B_\phi)\right)\hat{\theta} + \frac{1}{r}\left(\frac{d}{dr}(rA_\theta) - \frac{d B_r}{d\theta}\right)\phi$$

I initially started out with $\nabla \times B = \mu_0 J$ and tried to replace the current density with the dipole moment. To give the current density direction it would need to be the cross product of the components θ and ϕ. I tried to take the curl of B but my set became unbalanced in the end.

(b)



The dipole moment m is related to the Gauss coefficients g_1^0, h_1^1, g_1^1, by being the component of the axial (g_1^0) and the equatorial (h_1^1 and g_1^1) axis. The dipole moment would be the magnitude of the coefficients.

$$m = \frac{4\pi a^3}{\mu_0}\sqrt{(g_1^0)^2+(g_1^1)^2+(h_1^1)^2}$$

I did get this from GPSG ch. 3 but I'm having problem with the units not equalling A·m² for the dipole moment.

(c) 2000 IGRF

$g_1^0 = -29615$ nT

$g_1^1 = -1728$ nT

$h_1^1 = 5186$ nT

2020 IGRF

$g_1^0 = -29404.8$ nT

$g_1^1 = -1450.9$ nT

$h_1^1 = 4652.5$ nT

Using the equations from b I calculated the dipole moment by the magnitude of g_1^0, g_1^1, h_1^1.

The moment for 2000 was 7.78×10^{22} Am^2

and for 2020 was 7.707×10^{22} Am^2

In the past 20 years, the dipole has weakened about a ~1%. If it continues at the same rate it would reverse ~1980 years from now.

I do think it is a possibility for a reversal. The last one was ~780,000 years ago and the average is ~2 million years. We're going due for one in the next million years. This is based off of numbers I remember.

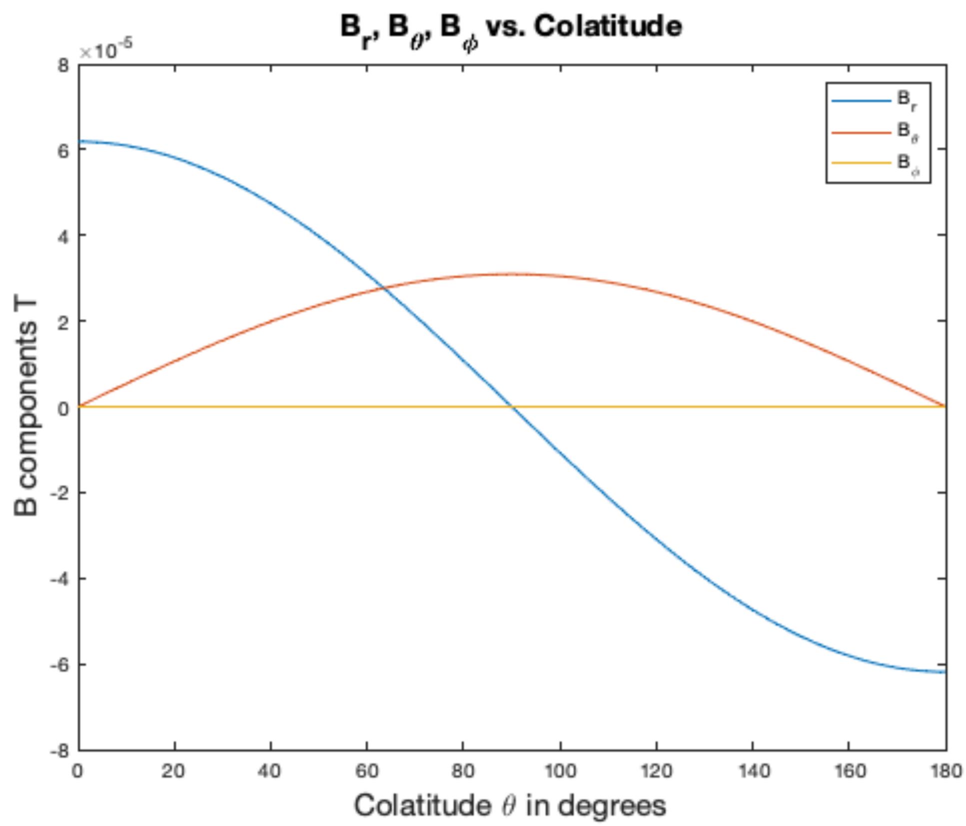
```
% SIO 229 Homework 3, Magnetism HW # 1
clear all; close all; clc
```

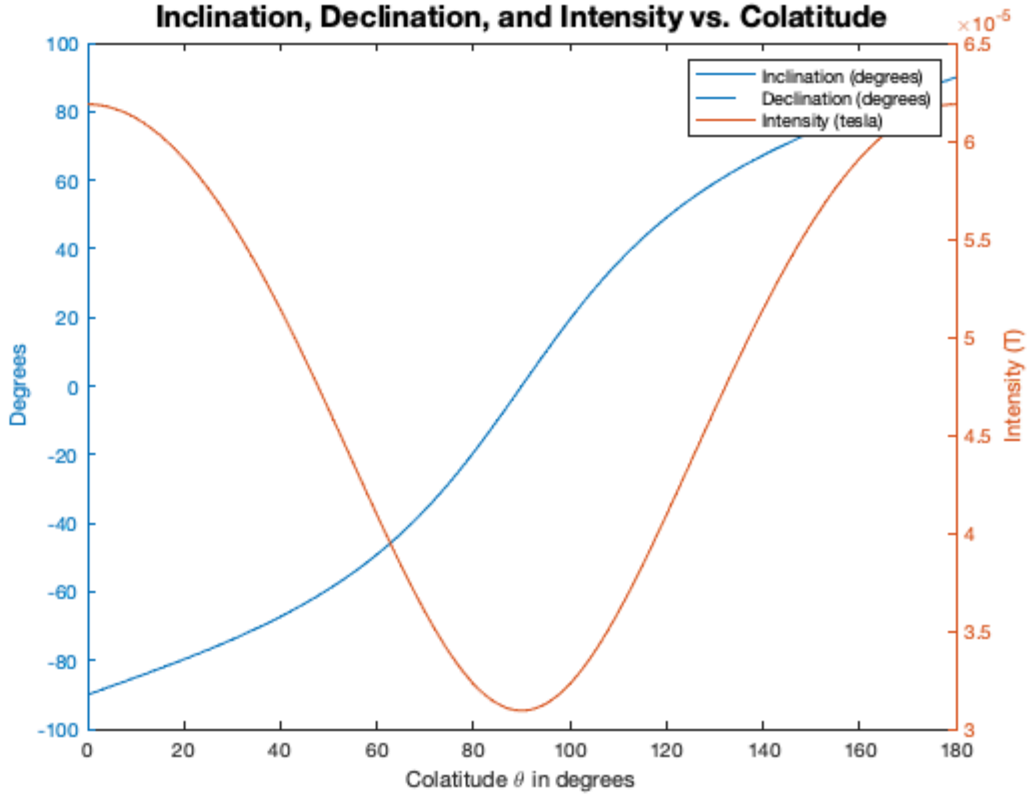
Question 1: Plot magnetic field elements against latitude

Suppose we have a magnetic dipole with dipole moment m at Earth's center and oriented along Earth's axis. Calculate (i) the magnetic field elements B_r, B_{theta}, B_{phi}, and (ii) the declination , inclination, and intensity as a function of latitude.

```
% Calculate the magnetic field as a function of latitude

rEarth = 6371000;                % Earth's radius in m
mu0 = 4*pi*1e-7;                 % Permeability of vacuum in henries/
meters
m = 8e22;                        % Dipole moment in ampere*m^2
theta = 0:180;                   % colatitude in degrees
phi = 0:360;                      % longitude in degrees
B_r = mu0*m*(2*cosd(theta))/(4*pi*rEarth^3); % Magnetic field in tesla
 for unit r
B_theta = mu0*m*(sind(theta))/(4*pi*rEarth^3); % Magnetic field in
 tesla for unit theta
B_phi =zeros(1,length(theta));              % Magnetic field in tesla
 for unit phi

figure(1)
plot(theta, B_r,'LineWidth',1)
hold on
plot(theta, B_theta,'LineWidth',1)
plot(theta, B_phi,'LineWidth',1)
title('B_r, B_{\theta}, B_{\phi} vs. Colatitude','FontSize',15)
xlabel('Colatitude \theta in degrees','FontSize',15)
ylabel('B components T','FontSize',15)
set(gcf,'color','w');
legend('B_r', 'B_{\theta}', 'B_{\phi}')
hold off

% Convert B to inclination in degrees
inclin = atand(-B_r./sqrt(B_theta.^2+B_phi.^2));
% Convert B to declination in degrees
declin = atand(B_r./-B_theta);
% Convert B to intensity in teslas
inten = sqrt(B_r.^2+B_theta.^2+B_phi.^2);

figure(2)
yyaxis left
plot(theta, inclin,'LineWidth',1)
hold on
plot(theta, declin,'LineWidth',1)
ylabel('Degrees')
```

```
yyaxis right
plot(theta, inten,'LineWidth',1)
title('Inclination, Declination, and Intensity vs.
 Colatitude','FontSize',15)
xlabel('Colatitude \theta in degrees')
ylabel('Intensity (T)')
legend('Inclination (degrees)','Declination (degrees)','Intensity
 (tesla)')
set(gcf,'color','w');
hold off
```





Question 2c: Calculate the magnetic dipole from Gauss Coefficients

```
% Gauss Coefficients from IGRF-2000
g10_2000 = -29615e-9;
g11_2000 = -1728e-9;
h11_2000 = 5186e-9;

% Gauss Coefficients from IGRF-2020
g10_2020 = -29404.8e-9;
g11_2020 = -1450.9e-9;
h11_2020 = 4652.5e-9;

% Dipole moment

m_2000=4*pi*rEarth^3*norm([g10_2000 g11_2000 h11_2000])/mu0
m_2020=4*pi*rEarth^3*norm([g10_2020 g11_2020 h11_2020])/mu0
```

```
m_2000 =

   7.7877e+22


m_2020 =
```

```
7.7077e+22
```

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