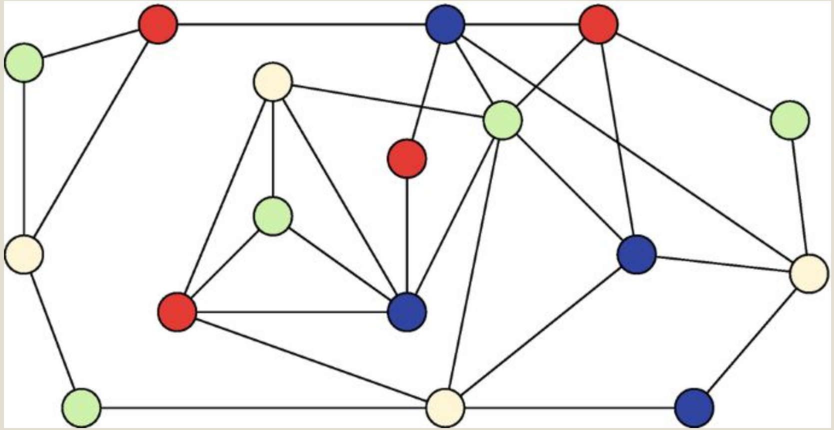
MIN GRAPH COLORING



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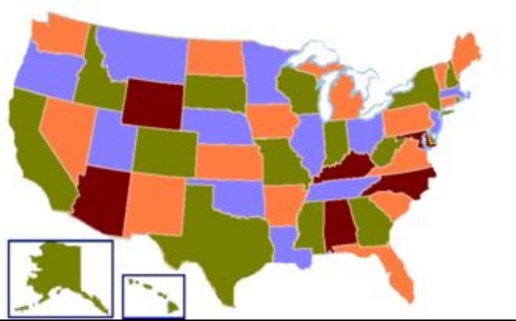
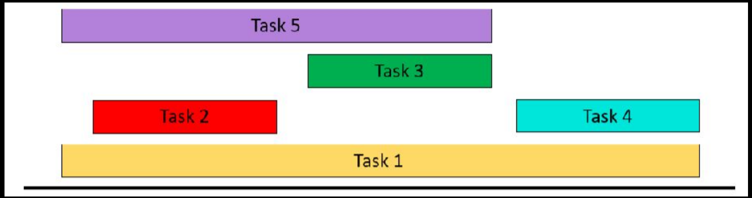
Min Graph Coloring Intro

- Problem
  - How can we color elements in a undirected graph so that no two adjacent vertices have the same color?
- Applications
  - Map Coloring
  - Register Allocation
  - Scheduling
    - Traffic
    - Event Planning





Decision vs. Optimization

- Decision
  - Is it possible to color a graph with at most (K) colors?
  - “Yes” can be verified in polynomial time
  - NP-Complete
- Optimization
  - What is the minimum number of colors (K) needed to color a given graph
  - Must solve the problem in order to verify
  - NP-Hard

```
Is it possible to
color this graph
using two colors?

#Input:
2
a b
b c

#Output:
True
```

```
Minimum number
of colors?

#Input:
2
a b
b c

#Output:
2
a 1
b 0
c 1
```

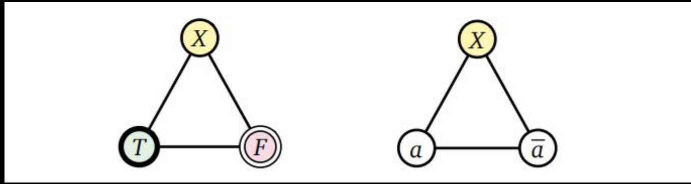
Certifier process

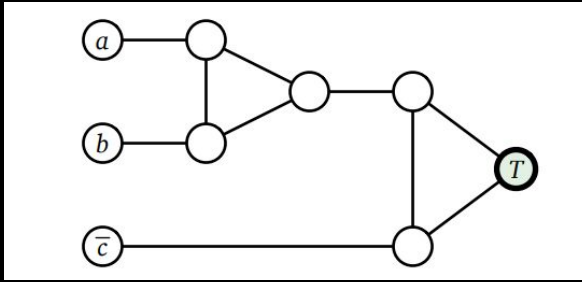
- Checks each vertex’s neighbor once
- Resulting in a time complexity of O(V + E)
- Polynomial time complexity
  - Scales with the number of vertices and edges

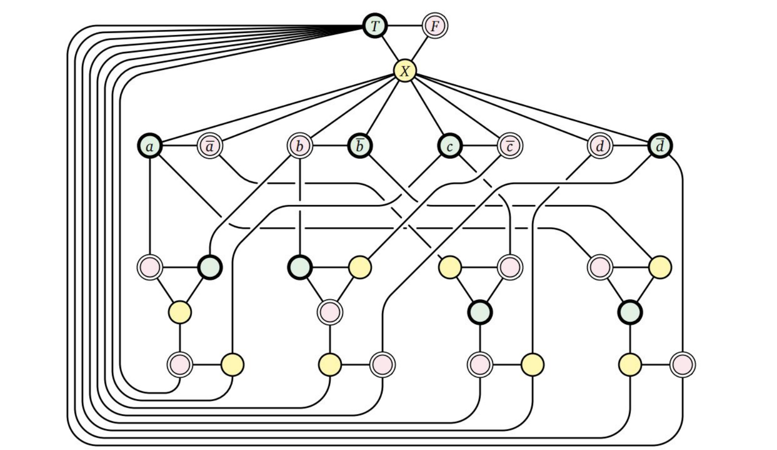
```python
graph = [...]
coloring = {...}
def is_valid_coloring(graph, coloring):
# Loop over each vertex and it's list of neighbors
    for vertex, neighbors in graph: # <- O(V)
        for neighbor in neighbors:  # <- O(E)
            if coloring[vertex] == coloring[neighbor]:
                return False
    return True
```

3-Sat -> 3-Color

- Create Truth gadget - O(1)
  - T, F, and Other
  - Must have different colors
- Create variable gadget - O(n)
  - a, a′, and Other
- Create Clause gadget - O(m)
  - Joins 3 literals to node T
  - Uses 5 unlabeled nodes and 10 edges to ensure one literal is True (Colored T)
- Final graph - O(n + m)
  - Connected Truth, Variable and Clause gadgets
  - If graph is 3-colored then original 3-Sat formula is satisfiable







$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

https://jeffe.cs.illinois.edu/teaching/algorithms/book/12-nphard.pdf

Exact Solution

Code

- Tries all possible assignments for the vertices
- Backtracking occurs when it finds a conflict
- With *m* available colors and *V* vertices
    - Runtime: $O(m^V)$

Time & Input

- More edges = more runtime
- 60 % of max edges
    - 10 vertices= 45 max edges
    - Test with 27 edges

```python
def graph_color_util(self, color_assignment, colors, index) -> bool:
        # Base case
        if index == len(self.vertices):
            return True

        vertex = self.vertices[index]
        for color in colors: # Loop over the colors
            if self.is_safe(vertex, color_assignment, color): # Check if it is safe
                color_assignment[vertex] = color # If it is, then assign the color to the vertex
                if self.graph_color_util(color_assignment, colors, index + 1): # Recursive call
                    return True
                # Backtrack
                del color_assignment[vertex] # Delete color
        return False
```