S T Q Q S S D

José Geraldo [illegible]

(1) $N(a) = (1-a)(1+\lambda a)^{-1} \qquad \lambda \in (-1, \infty)$

* n1: $N(0) = 1, \; N(1) = 0$

$$N(0) = (1-0)(1+0)^{-1} = 1$$

$$N(1) = (1-1)(1+1) = 0$$

* n2: $N(a) \geq N(b), \; a \leq b$

$$a, b \in [0,1] \Rightarrow 1+\lambda a, \; 1+\lambda b > 0 \Rightarrow$$

$$\frac{1-a}{1+\lambda a} \geq \frac{1-b}{1+\lambda b}$$

$$(1-a)(1+\lambda b) \geq (1-b)(1+\lambda a)$$

$$1 - \lambda ab - a + \lambda b \geq 1 - \lambda ab - b + \lambda a$$

$$\lambda b - a \geq \lambda a - b$$

$$\lambda b + b \geq \lambda a + a$$

$$b(\lambda+1) \geq a(\lambda+1) \qquad (\lambda+1) \neq 0$$

$$b \geq a \Leftrightarrow a \leq b$$

n3: $N(N(a)) = a$

$N(N(a)) = (1 - N(a))(1 + \lambda N(a))^{-1}$

$= \left(1 - \frac{1-a}{1+\lambda a}\right)\left(1 + \lambda \frac{1-a}{1+\lambda a}\right)^{-1}$

$= \left(\frac{1+\lambda a - 1 + a}{1+\lambda a}\right)\left(\frac{1+\lambda a + \lambda - \lambda a}{1+\lambda a}\right)^{-1}$

$= (1 + \lambda a - 1 + a)(1 + \lambda a + \lambda - \lambda a)^{-1}$ $\lambda \in (-1, \infty)$

$= (a + \lambda a)(1 + \lambda)^{-1}$

$= a(1+\lambda)(1+\lambda)^{-1}$ $\quad [(1+\lambda) \neq 0]$

$= a$

//

$N(a) = (1 - a^w)^{1/w}$, $w \in (0, \infty)$

n1: $N(0) = 1$, $N(1) = 0$

$N(0) = (1-0)^{1/w} = 1$

$N(1) = (1-1)^{1/w} = 0$

n2: $N(a) \geq N(b)$ ~ $a \leq b$

$(1 - a^w)^{1/w} \geq (1 - b^w)^{1/w}$

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$x \geq y \;, x, y, K \geq 0 \Rightarrow$

$x^K \geq y^K$

$(1-a^{w})^{1/w} \geq (1-b^{w})^{1/w}$

$1-a^{w} \geq 1-b^{w}$

$a^{w} \leq b^{w}$

$a \leq b$

$n^{4}: N(N(a)) = a$

$N(N(a)) = (1 - N(a)^{w})^{1/w}$

$= (1 - ((1-a^{w})^{1/w})^{w})^{1/w}$

$= (1 - (1-a^{w}))^{1/w}$

$= (a^{w})^{1/w} = a$

spiral

(11) $T(a,b) = \min(a,b)$

t1. $T(0,0) = 0, \quad T(a,1) = T(1,a) = a$

$T(0,0) = \min(0,0) = 0$

$T(a,1) = \min(a,1) \qquad a \leq 1 \Rightarrow$
$T(a,1) = a$
$T(1,a) = \min(1,a) = a = T(a,1)$

t2: $T(a,b) \leq T(a,d), \quad b \leq d$

(i) $a \leq b$

$T(a,b) = \min(a,b) = a$
$T(a,d) = \min(a,d) \qquad a \leq b \leq d \Rightarrow a \leq d$
$T(a,d) = a \geq T(a,b)$

(ii) $b \leq a \leq d$

$T(a,b) = \min(a,b) = b$
$T(a,d) = \min(a,d) = a$
$\cancel{\min(a,b)} \quad T(a,b) = b \leq T(a,d) = a$

(iii) $b \leq d \leq a$

$T(a,b) = \min(a,b) = b$
$T(a,d) = \min(a,d) = d$

$b \leq d \Rightarrow T(a,b) \leq T(a,d)$

S T Q Q S S D ___/___/___

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$t3: T(a,b) = T(b,a)$

$T(a,b) = \min(a,b) = \min(b,a) = T(b,a)$

$t4: T(a, T(b,c)) = T(T(a,b),c)$

$T(a, T(b,c)) = \min(a, \min(b,c))$

$T(T(a,b),c) = \min(\min(a,b),c)$

(*)	$T(a, T(b,c))$	$T(T(a,b),c)$
$a \leq b \leq c$	a	a
$a \leq c \leq b$	a	a
$b \leq a \leq c$	b	b
$b \leq c \leq a$	b	b
$c \leq a \leq b$	c	c
$c \leq b \leq a$	c	c

note que (*) cobre todo o domínio da função

(III) $S(a,b) = a+b-ab$

$s1: S(0,0) = 0,\ s(a,0) = S(0,a) = a$

$S(0,0) = 0+0-0 = 0$

$s(a,0) = a+0-0 = a$
$s(0,a) = 0+a-0 = a = s(a,0)$

spiral

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a2: $S(a,b) \leq S(a,d)$, $b \leq d$

$a+b-ab \leq a+d-ad$

$b-ab \leq d-ad$

$b(1-a) \leq d(1-a)$ $\quad 0 \leq a \leq 1 \Rightarrow (1-a) \geq 0$

$b \leq d$

a3: $S(a,b) = S(b,a)$

$a+b-ab = b+a-ba$

a4: $S(a, S(b,c)) = S(S(a,b),c)$

$$
\begin{aligned}
S(a,S(b,c)) &= a + S(b,c) - a\,S(b,c) \\
&= a + (b+c-bc) - a(b+c-bc) \\
&= a+b+c-bc-ab-ac+abc \\
&= a+b+c-ab-c(a+b-ab) \\
&= c+(a+b-ab)-c(a+b-ab) \\
&= c+S(a,b)-c\,S(a,b) \\
&= S(S(a,b),c)
\end{aligned}
$$

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(IV) $N(a) = 1 - a$

$S(a,b) = a + b - ab \qquad T(a,b) = ab$

$$\left|\begin{array}{l} T(a,b) = N(S(N(a), N(b))) \\ S(a,b) = N(T(N(a), N(b))) \end{array}\right.$$

(i) $N(S(N(a), N(b))) = N(S(1-a, 1-b))$

$= N(1 - a + 1 - b - ((1-a)(1-b)))$

$= N(1 - \cancel{a} + \cancel{1} - \cancel{b} - (\cancel{1} - \cancel{b} - \cancel{a} + ab))$

$= N(1 - ab) = 1 - (1 - ab) = ab = T(a,b)$

(ii) $N(T(N(a), N(b))) = N(T(1-a, 1-b))$

$= N((1-a)(1-b)) = N(1 - a - b + ab)$

$= \cancel{1} - (\cancel{1} - a - b + ab) = a + b - ab = S(a,b)$