

# ELE075 - Sistemas Nebulosos

## Atividade Prática 2 - Parte 1

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I.

$$Q = \begin{vmatrix} 0 & 0.8 & 0.6 & 0.25 \\ 0.7 & 0.98 & 0.15 & 0.5 \end{vmatrix}$$

$$R = \begin{vmatrix} 1 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.7 \\ 0.4 & 0.15 & 0.05 \\ 0.85 & 0.3 & 0.1 \end{vmatrix}$$

$$L = \begin{vmatrix} 1 & 0.2 & 0.6 & 0.8 \\ 0.85 & 0.3 & 0.8 & 0.88 \end{vmatrix}$$

$$M = Q \wedge \neg L = \begin{vmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \end{vmatrix}$$

em que  $m_{ij} = \min(Q_{ij}, 1 - L_{ij})$

$$\begin{aligned} m_{11} &= \min(0, 0) = 0 \\ m_{12} &= \min(0.8, 0.8) = 0.8 \\ m_{13} &= \min(0.6, 0.4) = 0.4 \\ m_{14} &= \min(0.25, 0.2) = 0.2 \end{aligned}$$

$$\begin{aligned} m_{21} &= \min(0.7, 0.15) = 0.15 \\ m_{22} &= \min(0.98, 0.7) = 0.7 \\ m_{23} &= \min(0.15, 0.2) = 0.15 \\ m_{24} &= \min(0.5, 0.12) = 0.12 \end{aligned}$$

$$M = Q \wedge \neg L = \begin{vmatrix} 0 & 0.8 & 0.4 & 0.2 \\ 0.15 & 0.7 & 0.15 & 0.12 \end{vmatrix}$$

$$P = Q \circ R = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{vmatrix}$$

em que  $p_{ij} = \max(Q_{i,*} * R_{*,j})$  com a operação  $*$  representando multiplicação par a par.

$$\begin{aligned} p_{11} &= \max((1, 0.1, 0.4, 0.85) * (0, 0.8, 0.6, 0.25)) \\ p_{11} &= \max(0, 0.081, 0.24, 0.2125) \\ p_{11} &= 0.24 \end{aligned}$$

$$\begin{aligned} p_{12} &= \max((0.4, 0.4, 0.15, 0.3) * (0, 0.8, 0.6, 0.25)) \\ p_{12} &= \max(0, 0.32, 0.09, 0.075) \\ p_{12} &= 0.32 \end{aligned}$$

$$\begin{aligned} p_{13} &= \max((0.2, 0.7, 0.05, 0.1) * (0, 0.8, 0.6, 0.25)) \\ p_{13} &= \max(0, 0.56, 0.03, 0.025) \\ p_{13} &= 0.56 \end{aligned}$$

$$\begin{aligned} p_{21} &= \max((0.1, 0.1, 0.4, 0.85) * (0.7, 0.98, 0.15, 0.5)) \\ p_{21} &= \max(0.7, 0.098, 0.06, 0.425) \\ p_{21} &= 0.7 \end{aligned}$$

$$\begin{aligned} p_{22} &= \max((0.4, 0.4, 0.15, 0.3) * (0.7, 0.98, 0.15, 0.5)) \\ p_{22} &= \max(0.28, 0.392, 0.0225, 0.15) \\ p_{22} &= 0.392 \end{aligned}$$

$$\begin{aligned} p_{23} &= \max((0.2, 0.7, 0.05, 0.1) * (0.7, 0.98, 0.15, 0.5)) \\ p_{23} &= \max(0.14, 0.686, 0.0075, 0.05) \\ p_{23} &= 0.686 \end{aligned}$$

$$P = Q \circ R = \begin{vmatrix} 0.24 & 0.32 & 0.56 \\ 0.7 & 0.392 & 0.686 \end{vmatrix}$$

II.

$$A = \begin{vmatrix} 1 & 0.5 & 0.4 & 0.2 \end{vmatrix}$$

$$R = \begin{vmatrix} 1 & 0.8 & 0 & 0 \\ 0.8 & 1 & 0.8 & 0 \\ 0 & 0.8 & 1 & 0.8 \\ 0 & 0 & 0.8 & 1 \end{vmatrix}$$

$$B = A \circ R = \begin{vmatrix} b_1 & b_2 & b_3 & b_4 \end{vmatrix}$$

em que  $b_i = \max(\min(A_i, R_{*,i}))$

$$\begin{aligned} b_1 &= \max(\min((1, 0.5, 0.4, 0.2), (1, 0.8, 0, 0))) \\ b_1 &= \max(1, 0.5, 0, 0) = 1 \end{aligned}$$

$$\begin{aligned} b_2 &= \max(\min((1, 0.5, 0.4, 0.2), (0.8, 1, 0.8, 0))) \\ b_2 &= \max(0.8, 0.5, 0.4, 0) = 0.8 \end{aligned}$$

$$b_3 = \max(\min((1, 0.5, 0.4, 0.2), (0, 0.8, 1, 0.8)))$$

$$b_3 = \max(0, 0.5, 0.4, 0.2) = 0.5$$

$$b_4 = \max(\min((1, 0.5, 0.4, 0.2), (0, 0, 0.8, 1)))$$

$$b_4 = \max(0, 0, 0.4, 0.2) = 0.4$$

$$B = A \circ R = \begin{bmatrix} 1 & 0.8 & 0.5 & 0.4 \end{bmatrix}$$

### III.

As curvas de pertinências são como na Figura 1.

$$\mu_{young}(x) = \text{gaussian}(x, 0, 20)$$

$$\mu_{old}(x) = \text{gaussian}(x, 100, 30)$$

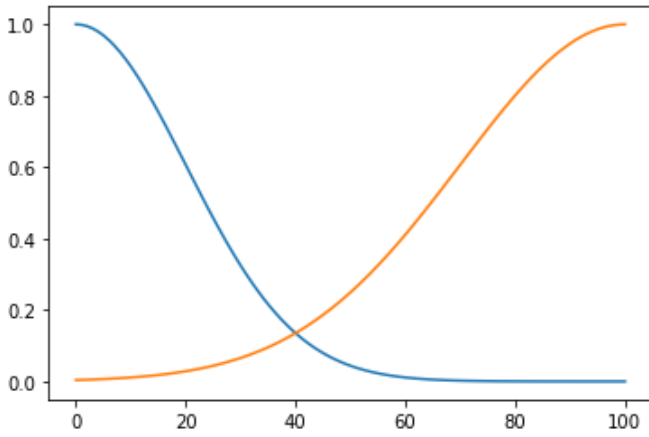


Figura 1. Curvas de pertinência, em azul  $\mu_{young}$  e em laranja  $\mu_{old}$

### IV.

As curvas de pertinências são como na Figura 2.

$$\mu_a(x) = \neg \mu_{young}^2 \wedge \neg \mu_{old}^2$$

$$\mu_b(x) = \mu_{young}^2 \wedge \mu_{old}^2$$

### V.

$$A_1 = \begin{bmatrix} 0.2 & 0.4 & 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 1 & 0.3 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}$$

$$A_1 \rightarrow B_1$$

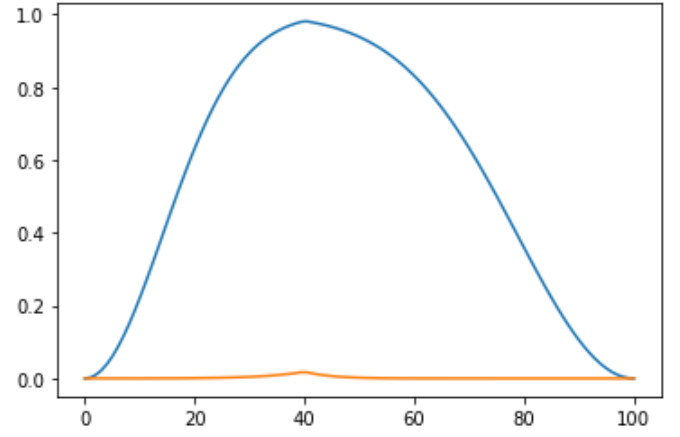


Figura 2. Curvas de pertinência, em azul  $\mu_a$  e em laranja  $\mu_b$

$$A_2 \rightarrow B_2$$

$$A' = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\mu_{B'} = \bigvee_i [\bigvee (\mu_{A'} \wedge \mu_{A_i}) \wedge \mu_{B_i}]$$

$$i = 1$$

$$\mu_{B'_1} = \bigvee (\mu_{A'} \wedge \mu_{A_1}) \wedge \mu_{B_1} = \omega_1 \wedge \mu_{B_1}$$

$$\mu_{B'_1} = \bigvee (\begin{bmatrix} 0 & 0.4 & 0 \end{bmatrix}) \wedge \mu_{B_1} = 0.4 \wedge \mu_{B_1}$$

$$\mu_{B'_1} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}$$

$$i = 2$$

$$\mu_{B'_2} = \bigvee (\mu_{A'} \wedge \mu_{A_2}) \wedge \mu_{B_2} = \omega_2 \wedge \mu_{B_2}$$

$$\mu_{B'_2} = \bigvee (\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}) \wedge \mu_{B_1} = 1 \wedge \mu_{B_2}$$

$$\mu_{B'_1} = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}$$

$$\mu_{B'} = \mu_{B'_1} \vee \mu_{B'_2} = \begin{bmatrix} 0.6 & 0.3 \end{bmatrix} = 0.6/y_1 + 0.3/y_2$$

## VI.

A curva de pertinência é como na Figura 3.

$$\mu_{A_1} = \text{trapmf}(x, [3 \quad 4 \quad 5 \quad 6])$$

$$\mu_{A_2} = \text{trapmf}(x, [6 \quad 6.5 \quad 7 \quad 7.5])$$

$$\mu_{C_1} = \text{trimf}(x, [3 \quad 4 \quad 5])$$

$$\mu_{C_2} = \text{trimf}(x, [4 \quad 5 \quad 6])$$

$$A_1 \rightarrow C_1$$

$$A_2 \rightarrow C_2$$

$$\mu_{A'} = \text{trimf}(x, [5 \quad 6 \quad 7])$$

$$\mu_{C'} = \bigvee_i [\bigvee (\mu_{A'} \wedge \mu_{A_i}) \wedge \mu_{C_i}]$$

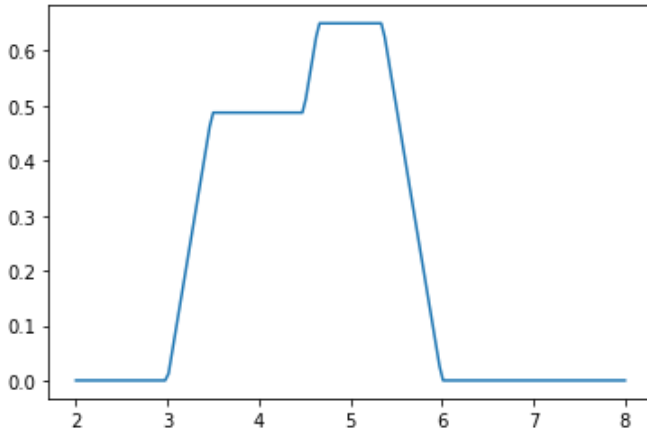


Figura 3. Curvas de pertinência de  $C'$ .