EPD894 - Modelos de Regressão Paramétricos e Não-Paramétricos: Teoria e Aplicações
Lista de Exercícios 1

José Geraldo Fernandes
Escola de Engenharia
Universidade Federal de Minas Gerais
Belo Horizonte, Brasil
josegeraldof@ufmg.br

I.

$$F(\beta_0, \sigma) = \prod_i^n \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{-1}{2\sigma^2}(y_i - \beta_0)^2)$$

$$F(\beta_0, \sigma) = (\frac{1}{\sqrt{2\pi\sigma^2}})^n \exp(\frac{-1}{2\sigma^2}\sum_i^n(y_i - \beta_0)^2)$$

$$\frac{\partial}{\partial\sigma^2}F|_{\sigma^2=\hat{\sigma}^2} = 0$$

$$\frac{\partial}{\partial\sigma^2}\log F|_{\sigma^2=\hat{\sigma}^2} = 0$$

$$\frac{\partial}{\partial\sigma^2}[\frac{-n}{2}\log 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_i^n(y_i - \beta_0)^2]|_{\sigma^2=\hat{\sigma}^2} = 0$$

$$[\frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_i^n(y_i - \beta_0)^2]|_{\sigma^2=\hat{\sigma}^2} = 0$$

$$\frac{-n}{2\hat{\sigma}^2} + \frac{1}{2\hat{\sigma}^4}\sum_i^n(y_i - \beta_0)^2 = 0$$

$$\hat{\sigma}^2 \neq 0$$

$$n = \frac{1}{\hat{\sigma}^2}\sum_i^n(y_i - \beta_0)^2$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_i^n(y_i - \beta_0)^2$$

II.

$$F(p) = \prod_i^n p^{y_i}(1-p)^{1-y_i}$$

$$F(p) = p^{\sum_i^n y_i}(1-p)^{n-\sum_i^n y_i}$$

$$\frac{\partial}{\partial p}F|_{p=\hat{p}} = 0$$

$$\frac{\partial}{\partial p}\log F|_{p=\hat{p}} = 0$$

$$\frac{\partial}{\partial p}[\sum_i^n y_i \log p + (n - \sum_i^n y_i)\log(1-p)]|_{p=\hat{p}} = 0$$

$$[\sum_i^n y_i \frac{1}{p} - (n - \sum_i^n y_i)\frac{1}{1-p}]|_{p=\hat{p}} = 0$$

$$\sum_i^n y_i \frac{1}{\hat{p}} = (n - \sum_i^n y_i)\frac{1}{1-\hat{p}}$$

$$(1-\hat{p})\sum_i^n y_i = \hat{p}(n - \sum_i^n y_i)$$

$$\hat{p} = \frac{1}{n}\sum_i^n y_i$$

III.

$$F(\mu) = \prod_i^n \frac{\mu^{y_i} \exp(-\mu)}{y_i!}$$

$$\frac{\partial}{\partial \mu} F|_{\mu=\hat{\mu}} = 0$$

$$\frac{\partial}{\partial \mu} \log F|_{\mu=\hat{\mu}} = 0$$

$$\frac{\partial}{\partial \mu} \sum_i^n [y_i \log \mu - \mu - \log y_i!]|_{\mu=\hat{\mu}} = 0$$

$$\sum_i^n [y_i \frac{1}{\mu} - 1]|_{\mu=\hat{\mu}} = 0$$

$$\frac{1}{\hat{\mu}} \sum_i^n y_i = n$$

$$\hat{\mu} = \frac{1}{n} \sum_i^n y_i$$