

# Photographic Calibration of the Hirschberg Test

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A series of direct photographic measurements of corneal reflex positions was taken to reconcile discrepant interpretations of the Hirschberg test. A value of 21 prism diopters per millimeter was obtained for the conversion factor relating ocular rotations to reflex displacement, valid over a range of 200 prism D. Quantitative interpretation of routine clinical photographs of strabismic patients can thus be easily obtained by including a millimeter scale in each picture. This value for the conversion factor is in agreement with a simple optical model for the corneal reflex. The discrepancy with the traditional value (about 14 prism D/mm) appears to arise from an attempt of most observers to measure reflex displacement from the corneal apex along the surface of the cornea, rather than to project the reflex and pupil onto a true frontal plane, as is done in a photograph. *Invest Ophthalmol Vis Sci* 28:736-742, 1987

The "Hirschberg test" consists of the estimation of strabismic angle by comparison of the positions of the images in the corneas of a light held in front of the patient. While such direct estimation is generally acknowledged to be less satisfactory than the use of prisms, the Hirschberg test is nevertheless frequently employed in practice, both at the bedside, and in cases where poor fixation or lack of cooperation precludes the use of other methods. It is therefore of interest to note a persistent lack of agreement among various authorities concerning the interpretation of Hirschberg estimates. While some authors describe the test in terms of the landmarks furnished by the pupil, iris, and limbus, others suggest that the observer estimate the displacement (in millimeters) of the corneal reflex, and then multiply this number by some suitable conversion factor to obtain the deviation of the eye (in degrees or prism diopters). Surprisingly, there is substantial variation among the values recommended for this coefficient. The current study attempted to explain this inherently contradictory situation from historical, empirical, and theoretical points of view.

The origins of the Hirschberg test as performed in current practice are obscure. Hirschberg's paper of 1885<sup>1</sup> consists of two surgical case reports. Each patient's ocular alignment is described in terms of the strabismic angle in degrees, and graded on a five-point scale. One patient's deviation was also described as "three quarters of the width of the cornea" (possibly

referring to the position of a corneal reflex), but no method for the measurement of the deviation was described.

Hirschberg's paper of 1886<sup>2</sup> described the method in more detail: The image in the deviating eye of a candle flame held 30 cm from the patient was observed with reference to pupillary landmarks. Patients were then placed in one of five groups, each of which corresponded to a range of angular deviations. At the conclusion of the paper, the following statement was included without comment: "It has also been found, with normal eyes, that a rotation of 50 degrees corresponds to the entire radius of the cornea, and a rotation of nearly 8 degrees corresponds approximately to one-sixth of the radius of the cornea, or 1 mm." (This paper also includes a brief description of an adjustable suture recess-resect procedure performed without anesthesia on adults with large-angle deviations!)

In a companion paper, Hirschberg's assistant duBois-Reymond<sup>3</sup> also described the technique of the "reflex test", as employed in Hirschberg's clinic. He made no mention of attempts to numerically convert reflex displacement to strabismic angle; rather, patients were grouped according to the anatomical reference point (pupillary border, corneal limbus etc.) nearest the corneal reflex.

The use of a quantitative estimate of the reflex displacement as an intermediate step in making a Hirschberg estimate of ocular deviation has gradually been adopted into clinical practice. While some authors have continued to describe the test in terms of pupillary landmarks, most authorities now describe the quantitative form. Hirschberg appears to have been aware of a numerical relationship between reflex displacement and ocular deviation angle, though it does not appear that he employed the reflex test routinely in this way; he gives no source for his value of 8 degrees/mm for

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the conversion factor. Other authors have given different values (Table 1).

Krimsky<sup>4</sup> remarked that his value of 7 degrees/mm, (roughly equivalent to 14 prism D/mm) is derived from simple computation, based on the radius of the cornea. Such a calculation can be readily reconstructed: Identify the apparent corneal reflex as the intersection of the line connecting the center of curvature of the cornea to a light source located indefinitely far away, and the surface of the cornea (Fig. 1). As the eye rotates through an angle  $\theta$ , the angle formed at the center of curvature of the cornea by the incoming light and the axis of the eye is likewise  $\theta$ , and the apparent corneal reflex moves a distance  $s = \rho\theta$  along the surface of the cornea. Thus  $\theta$  (radians) =  $s/\rho$ . If we take  $\rho = 7.7$  mm as the radius of curvature of the cornea, then  $\theta$  (degrees) =  $s/7.7 \cdot (180^\circ/\pi)$  or  $\theta = 7.44 \cdot s$ .

Of course, this hypothetical "derivation" seriously misrepresents the Hirschberg measurement. As the eye rotates, the center of the pupil, not the corneal apex, serves as a reference point for localization of the corneal reflex, particularly if the observer is careful to sight monocularly along the incident beam of light, or if observations are made from a formal or informal photograph. Furthermore, as the eye reaches extreme positions of rotation (roughly  $45^\circ$  from primary position), the cornea and iris are seen obliquely by the observer, so that a reflex apparently emanating from near the corneo-scleral limbus cannot possibly appear to be displaced from the pupil by the full frontal radius of the cornea.

This discrepancy in the interpretation of a basic clinical procedure has been all but ignored by ophthalmologists. Current editions of some standard textbooks<sup>9,10</sup> continue to repeat the traditional treatment, while texts<sup>11,12</sup> which include the more recent data have made scant impact. We attempted to assess these concepts, employing simple apparatus available in any office practice, and attempted to extend the range of validation over the entire range of horizontal ocular rotations.

### Materials and Methods

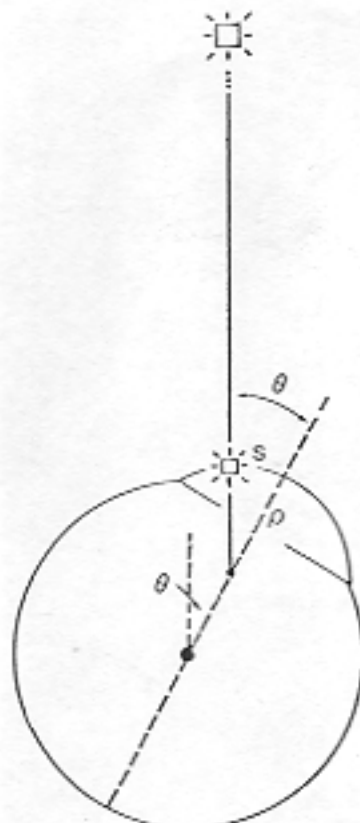
Three adult volunteer human subjects were examined to ensure bifoveal fixation and emmetropia. Informed consent was obtained from each subject prior to participation in the study. Subjects sat 1.5 m from a tangent screen, and occluded one eye with a millimeter ruler. With the other eye, they fixed in turn each of 21 targets equally spaced along the screen. The array of targets subtended a visual angle from 100 prism D nasal to 100 prism D temporal, with a 10 prism D separation between targets. A 35-mm flash camera, aligned with the central target, recorded the position of the corneal reflex (Fig. 2).

**Table 1.** Quantitative interpretations of the Hirschberg Test

| Source                          | "Hirschberg ratio" | Method   |
|---------------------------------|--------------------|--|
| Hirschberg <sup>3</sup>         | nearly 8°/mm       | empirical  |
| Krimsky <sup>4</sup>            | 7°/mm              | geometric model  |
| Wheeler <sup>5</sup>            | 8°/mm              | inference from measurement of direction of gaze when reflex is at limbus |
| Jones and Eskridge <sup>6</sup> | 12°/mm (22Δ/mm)    | photographic measurements and geometric model                            |
| Griffin and Boyer <sup>7</sup>  | 20.75Δ/mm          | photographic measurements  |
| Carter and Roth <sup>8</sup>    | 13.6°/mm           | photographic measurements, correlated with axial length                  |

\* Ratio of angle of ocular deviation to displacement of corneal reflex.

The photographs were projected on a wall, and the apparent width of the cornea, the distance from the corneal reflex to the temporal limbus, and to the nasal limbus were measured with calipers, using the millimeter ruler in each photograph for calibration. The position of the reflex was plotted as a function of the (known) direction of gaze. As the center of the pupil was difficult to locate on the photographs, reflex position was measured instead from the nasal limbus, from the temporal limbus, and from the midpoint of the line connecting the nasal limbus to the temporal limbus.



**Fig. 1.** Plausible derivation of traditional quantitative form of Hirschberg Test. Eye turns through angle  $\theta$  from primary position. Reflex appears to be displaced along arc of cornea of length  $s = \rho\theta$ , where  $\rho$  is corneal radius of curvature.

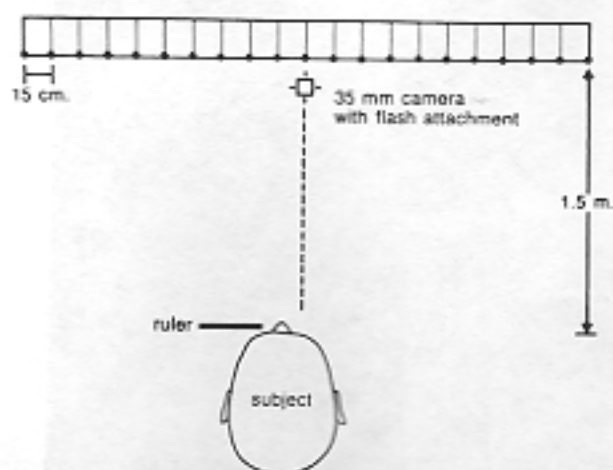


Fig. 2. Arrangement for photographing corneal reflexes. Normal subjects sat 1.5 m from a tangent screen. One eye viewed any of 21 fixation targets placed at intervals of 10 prism D. The other eye was occluded with a millimeter ruler. A 35-mm camera with flash attachment was aligned with the central fixation target.

### Results

Typical results are shown in Figure 3. The plots of reflex position as measured from the limbus showed distinct deviation from linearity. (The geometric basis for this effect is described below). On the other hand, the plots of reflex position as measured from the center of the cornea were nearly linear over the entire 200 prism D range. The displacement data were plotted against fixation angle both in degrees and in prism diopters; the linearity of the dependence of displacement was comparable with the two methods (Fig. 4). (There is therefore little reason to record Hirschberg measurements in degrees, while prismatic measure-

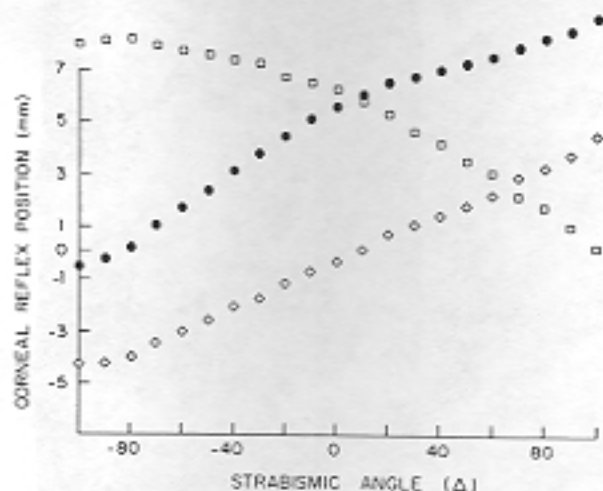


Fig. 3. Corneal reflex position plotted as a function of fixation angle (in prism diopters) for a single subject. (□) distance from nasal limbus to corneal reflex. (●) distance from corneal reflex to temporal limbus. (○) distance from apparent center of cornea to corneal reflex.

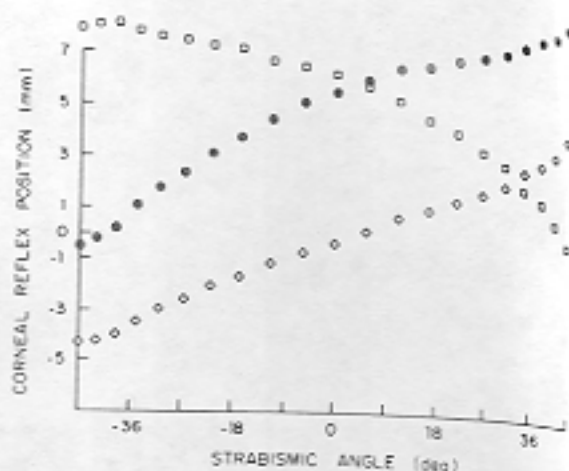


Fig. 4. Corneal reflex position for a single subject plotted as a function of fixation angle in degrees, replotted from Figure 3.

ments are recorded in prism diopters. The consistent use of the same unit for all strabismus measurements would eliminate much needless confusion).

The reflex displacement curves were fit to straight lines according to the method of least squares. The reciprocal of the slope of such lines gives the value of the Hirschberg Ratio (Fig. 5).

Typical photographs are shown in Figure 6. The imprecision of the use of pupillary landmarks is immediately evident. A more subtle question is the role of the angle kappa. The usual definitions of this quantity are vague, and ignore the imperfections in the alignment of the optical elements of the eye. Operationally, one would like to determine that angle of rotation of the globe such that the corneal reflex appears centered,

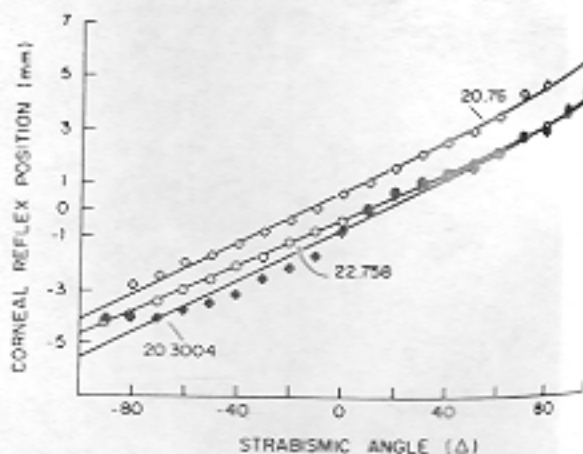


Fig. 5. Corneal reflex displacements, measured from the center of the cornea, plotted for three subjects. Straight lines are fit to the data for each subject according to the method of least squares. The Hirschberg ratio, calculated as the reciprocal of the slope, is shown for each subject.



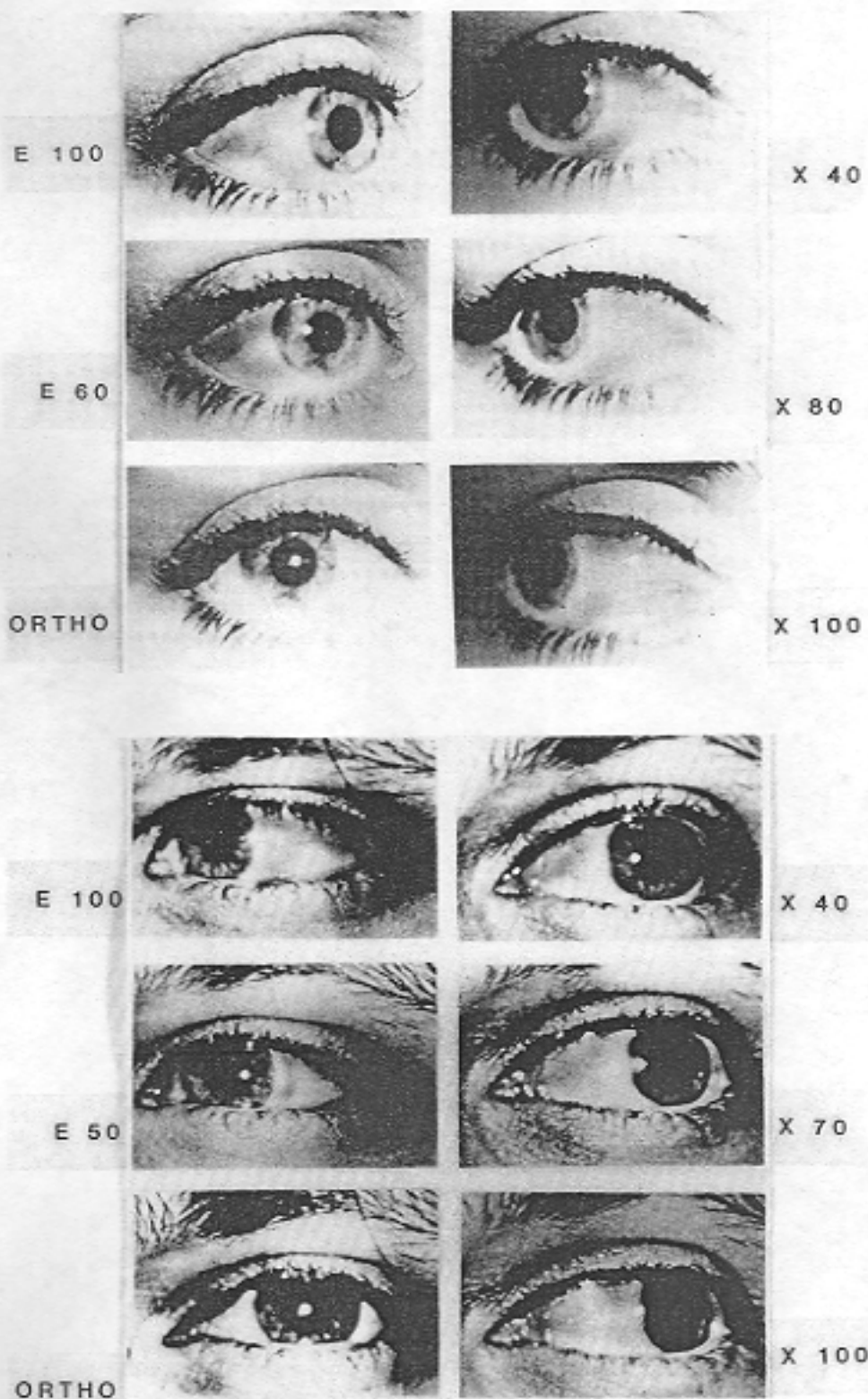


Fig. 6. A, B. Sample corneal reflex photographs for two subjects. (E-fixation nasal to camera, simulating esodeviation; X-fixation temporal to camera, simulating exodeviation; ortho-subject looking directly at camera, simulating orthophoria). Large variations in displacement of reflex are seen for similar reflex displacements. Reflex may even fall entirely outside the cornea.

either on the pupil, or with respect to the cornea itself (see below). If the eye under examination is capable of central fixation, this quantity can be measured directly

(Fig. 7). If not, the fellow eye, if capable of fixation, will provide a useful estimate, unless there are gross anatomic differences between the two eyes.

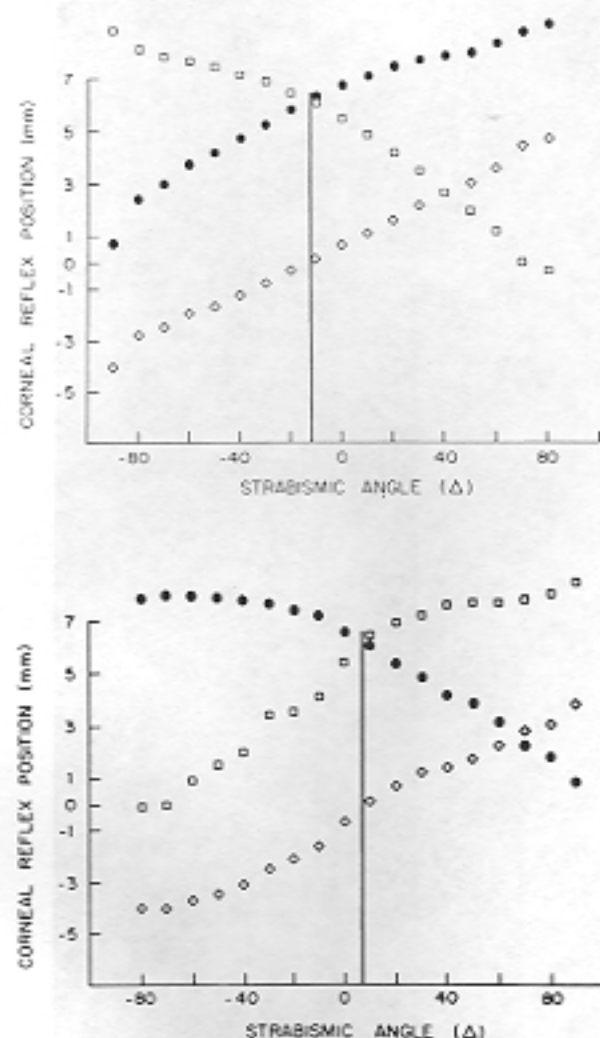


Fig. 7. Measurement of angle  $\kappa$  from reflex displacement curves in eyes capable of central fixation (data from two subjects). Intersection of curves showing distance of reflex from nasal and temporal limbus indicates angle where reflex is centered on cornea.

### Discussion

In Figure 8, the geometry of the Hirschberg test is illustrated for the case of perfect centration of the cornea and pupil on the visual axis (angle kappa equal to zero). If the cornea is assumed to be spherical, the formula for the displacement of the reflex may be calculated by trigonometry:

$$x_2 - x_1 = d \cdot \sin \theta$$

where  $x_1$  is the projection of the reflex onto the frontal plane,  $x_2$  is the projection onto the frontal plane of the center of the circle defined by the corneal limbus,  $d$  is the distance from the center of curvature of the corneal surface to the center of the circle formed by the corneal limbus, and  $\theta$  is the angle of rotation of the eye. For-

mulas for the distance from the corneal reflex to either limbus follow directly:

$$x_4 - x_1 = (x_4 - x_2) + (x_2 - x_1) = c \cdot \cos \theta + d \cdot \sin \theta$$

$$x_1 - x_3 = (x_2 - x_3) - (x_2 - x_1) = c \cdot \cos \theta - d \cdot \sin \theta$$

where  $x_3$  and  $x_4$  are the projections of the nasal and temporal limbus, and  $c$  is the radius of the circle formed by the corneal limbus.

In this context, the parameter  $d$  is generally taken to be the distance from the center of curvature of the corneal surface to the center of the entrance pupil (the image formed by the cornea, as a refracting surface, of the anatomical pupil). I have used instead the center of the circle defined by the corneo-scleral junction, because the limbus landmarks were easier to locate in the experimental photographs, and because the anterior-posterior location of the anatomic pupil with respect to the surface of the cornea (ie, the depth of the anterior chamber) is variable.

It is apparent from Figure 8 that if  $\kappa$  is defined as the angle between the visual axis and the line connecting the center of the corneal limbus with the center of curvature of the cornea, the angle  $\kappa$  effect may be included by replacing  $\theta$  in the formulas above by the quantity  $\theta + \kappa$ ; and the formula of Jones and Eskridge is obtained<sup>6</sup>:

$$\text{reflex displacement} = d \cdot \sin (\theta + \kappa);$$

measuring from either limbus, we obtain

reflex displacement (from limbus)

$$= c \cdot \cos (\theta + \kappa) + d \cdot \sin (\theta + \kappa)$$

or

$$= c \cdot \cos (\theta + \kappa) - d \cdot \sin (\theta + \kappa).$$

If we take as nominal parameter values  $c = 6.0$  mm,  $\rho = 7.7$  mm (after Gullstrand<sup>13</sup>), the Pythagorean theorem yields  $d = \sqrt{(\rho^2 - c^2)} = 4.8259$  mm, a result independent of the geometry of the posterior segment of the eye. Reflex displacement curves based on these nominal values are shown in Figure 9.

These same curves are shown superimposed on reflex position data in Figure 10. The only adjustable parameter for this fit is the angle  $\kappa$ . The data are well-described for rotations within 60 prism D of primary position. For more extreme rotations, the reflex displacements are slightly greater than predicted by the geometric model given above. This discrepancy is consistent with a slight flattening of the peripheral cornea, compared with its central curvature; however, the net effect is to extend even further the range where a linear Hirschberg relation is useful. Using the nominal parameters, the

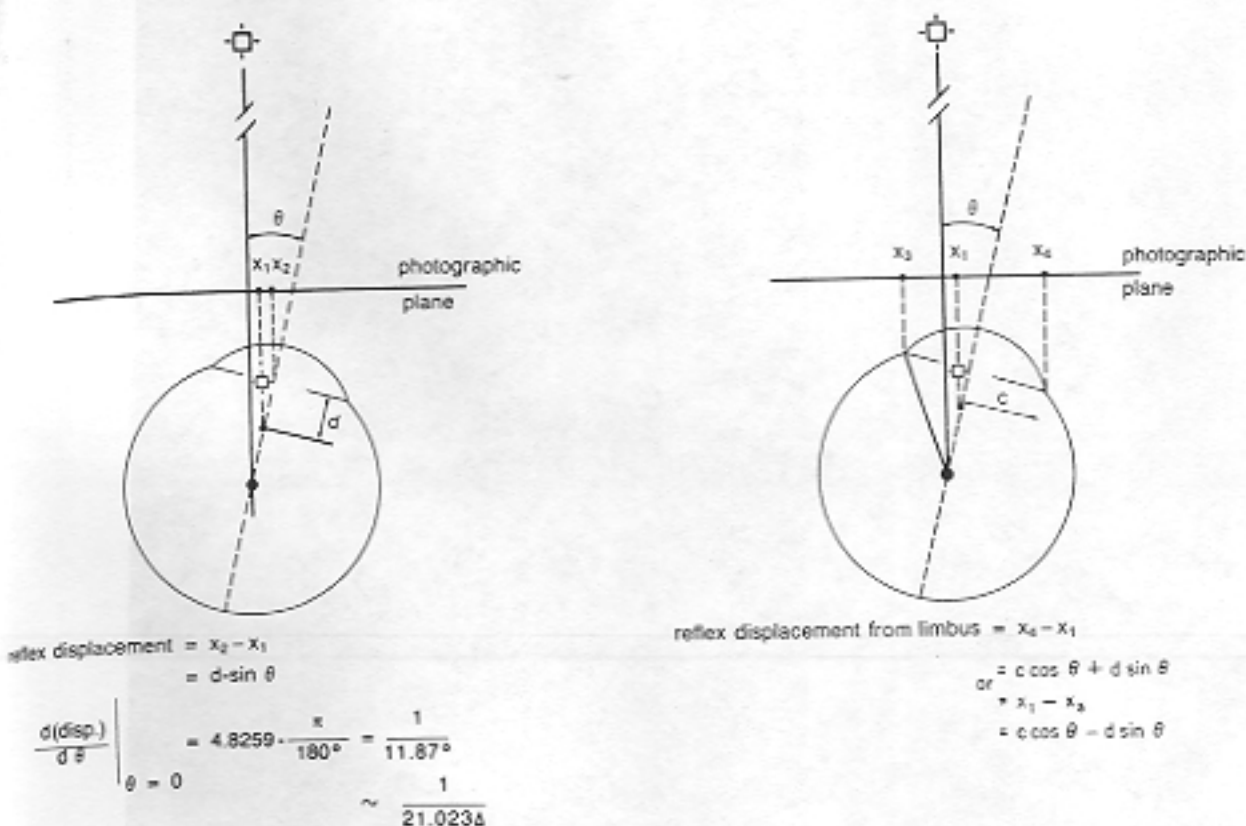


Fig. 8. Geometry of localization of the corneal light reflex. (left) Displacement of reflex from center of pupil.  $x_1$  indicates projection of reflex image (small square) onto frontal (photographic) plane;  $x_2$  indicates projection of center of pupil onto frontal plane. Distance from center of curvature of cornea to plane of pupil is indicated by  $d$ . (right) Displacement of reflex from nasal or temporal limbus.  $x_3$  and  $x_4$  indicate projection of nasal and temporal limbus onto frontal plane.  $c$  indicates the radius of the corneo-scleral limbus.

slope of the curve for displacement from the center of the cornea at  $\theta = 0$  yields a Hirschberg Ratio of 21.023 prism D/mm, in agreement with the photographic measurements.

The simple dependence of the Hirschberg Ratio on the geometric parameter  $d$  has been noticed by other investigators. While discussing the calibration of automated eye-tracker devices, Bronson<sup>14</sup> noted that this parameter appears to remain nearly constant, despite overall growth of the eye from infancy to adulthood. He noted that, while the distance from the corneal apex to the iris plane increases by about 0.90 mm from infancy to adulthood, the radius of curvature of the cornea increases by about 0.75 mm during the same period. Thus, throughout post-natal development, the parameter  $d$  should change by only a small percentage. It follows that the Hirschberg Ratio of about 21 prism D/mm is an appropriate value, regardless of the age of the subject. Bronson was able to verify this directly for a rotation angle of 20 degrees, on subjects ranging in age from 2 months to adulthood. Carter and Roth<sup>8</sup> obtained similar results using eyes of varying axial length as measured by ultrasonography. They found

that the Hirschberg ratio is not correlated with axial length.

It is important to reconcile the present revised value for the Hirschberg Ratio with the widespread experi-

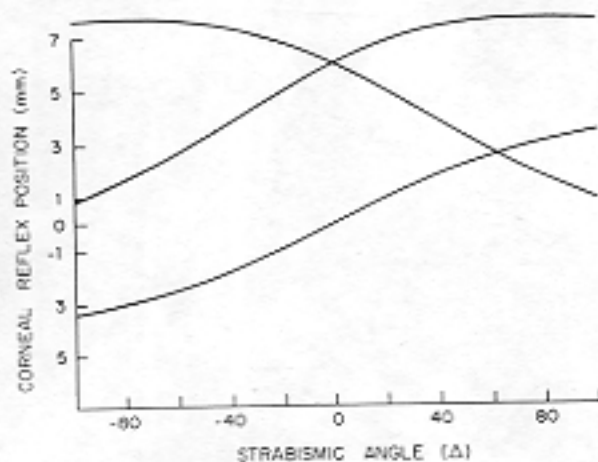


Fig. 9. Reflex displacement curves calculated from the geometric model of Figure 8 based on nominal values for the dimensions of the eye.



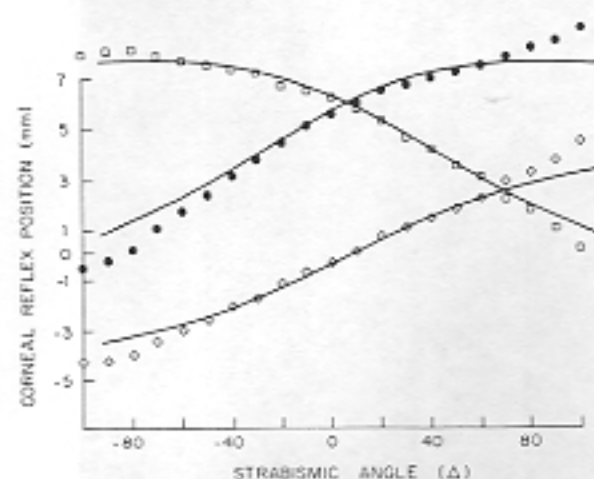


Fig. 10. Comparison of corneal reflex data with theoretical curves from Figure 9. The curves have been slid horizontally to optimize the fit to the data, which corresponds to adjustment for angle  $\kappa$  as defined in text.

ence of clinicians based on the traditional value. Apparently, the frequent feedback available to practitioners, who are able to compare the appearance of corneal reflexes with the results of prism/cover tests on the same patients, trains them in the accurate estimation of ocular deviation. The concurrent estimates of corneal reflex position are then apparently back-calculated according to the nominal Hirschberg ratio. This estimate of reflex displacement, much in the spirit of Wheeler (quoted above), thus describes a presumed fractional displacement relative to the known diameter of the cornea, rather than the actual displacement of the reflex along the frontal plane. Thus, the use of the traditional Hirschberg ratio in this fashion leads to no systematic errors, but is in reality no more accurate than reference to pupillary landmarks.

On the other hand, simply by inclusion of a millimeter scale in routine office photographs of strabismic patients, a quantitative measure of ocular position may be obtained as described above. Such a photograph, however, records the true reflex displacement in the frontal plane, and should be converted to ocular position according to a Hirschberg Ratio of approximately 21 prism D/1 mm. This value appears to be valid over the entire range of ocular rotation, and is essentially independent of age.

Strabismic patients who can fixate with either eye can be photographed with each eye in both the primary and the deviated positions. The difference in the mea-

sured positions of the corneal reflexes will then give a quantitative estimate of the ocular deviation, which automatically corrects for the presence of an angle  $\kappa$ . For patients who are unable to fixate with a deviating eye, an estimate of the origin to be used for Hirschberg calculations can be made by assuming symmetry with the fellow eye. This procedure should be generally valid except in instances where there are gross anatomical differences between the two eyes, such as cases of marked anisometropia or unilateral retinopathy of prematurity.

It is to be emphasized that these measurements require only ordinary photographic equipment. The method may even be used to compare deviation in near and far gaze, or in different fields of gaze, with the use of a telephoto lens and suitable fixation targets.

**Key words:** strabismus, strabismometry, ocular alignment, Hirschberg test, corneal light reflex.

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