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Utility of Theoretical Hirschberg Ratio for Gaze Position Calibration

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ABSTRACT

Purpose. Gaze position is calibrated in first Purkinje image–based eye trackers using the population-average Hirschberg ratio (HR) that is prone to inaccuracies or using the individual's HR that is cumbersome to obtain empirically. This study investigated (1) the agreement between HR calculated theoretically from the individual's corneal curvature and anterior chamber (AC) depth and those obtained empirically and (2) the contribution of corneal curvature and AC depth in the intersubject variance of the two HRs.

Methods. Twenty-four subjects (mean ± SD age, 23.6 ± 3.5 years) fixated monocularly on a light-emitting diode array spanning ±24 degrees of horizontal or vertical gaze angle, in 4-degree steps, at 95 cm viewing distance. Empirical HR was determined using a custom-designed infrared eye tracker as the magnitude of separation between Purkinje image position and entrance pupil center per unit change in angular eccentricity. Theoretical HR was calculated from the subject's corneal curvature and AC depth using the model of Brodie (1987).

Results. Empirical and theoretical HRs for horizontal and vertical gaze directions were well correlated ($r \geq 0.83$) and not significantly different from each other ($p \geq 0.23$; mean difference [±95% limits of agreement], −0.35 [0.85 to −1.55] degrees/mm for horizontal HR and −0.16 [1.01 to −1.33] degrees/mm for vertical HR). Corneal curvature and AC depth together accounted for greater than or equal to 80% and greater than or equal to 91% of intersubject variance in empirical and theoretical HR, respectively ($p < 0.001$). Hirschberg ratios changed at −2.3 to −2.8 degrees/mm per millimeter change in corneal curvature and at 2.0 to 2.4 degrees/mm per millimeter change in AC depth.

Conclusions. Theoretical HR calculated from the individual's corneal curvature and AC depth can be used in lieu of the empirical HR for gaze position calibration to within approximately 2 degrees/mm of accuracy. Gaze position accuracy significantly improves by using the theoretical HR, relative to the population-average HR. Corneal curvature and AC depth combined explain the majority of intersubject variability in HR.

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Key Words: anterior chamber depth, calibration, corneal curvature, entrance pupil, gaze position, Hirschberg ratio, Purkinje image

Video-based eye trackers that use specific ocular landmarks (e.g., first Purkinje image [PI], limbus, blood vessel configuration, etc.) to track gaze position are widely used to study the oculomotor behavior of human and nonhuman primates.[1–3] Quantitative estimates of gaze position can be readily obtained using this technology in a rapid and noninvasive fashion. Within these breed of eye trackers, those using the relative location of the first PI and entrance pupil center (PC; i.e., virtual image of the anatomical pupil as seen through the cornea and anterior chamber [AC][4]) as landmarks are of specific interest to the present study. A conversion factor—the Hirschberg ratio (HR)—is used in these trackers to convert the millimeter separation between the PI and PC into angular units of gaze position (degrees or prism diopters [ΔD]).[5–7] A population-average value of HR is incorporated into some eye trackers to report gaze position directly in angular units (e.g., PowerRefractor[6,8]) or a separate calibration routine is used to measure the individual's HR, which is then used to convert raw data into angular units (e.g., EL-MAR eye tracker[2]).

The HR is typically measured by asking participants to fixate on targets at known angular eccentricities and measuring the separation between the PI and PC for each of these positions using a

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high-resolution video camera. The reciprocal of the slope of a linear regression fit between the measured separation between the PI and PC and the corresponding eye position gives the HR in degrees (or ΔD) per millimeter.[9–11] The *y*-intercept of this function gives the angle κ—the angle between the visual fixation axis and the pupillary axis.[9] Using this procedure, previous studies on adults[6,11–18] and infants[19,20] have obtained an average HR value of about 12 degrees/mm (or 21 ΔD/mm) with intersubject variability ranging from 7 to 16 degrees/mm (or 12 to 28 ΔD/mm). This large intersubject variability is a hallmark of HR measurements and it is of concern because of the potential inaccuracies that would be induced in gaze position estimates when using this average value as the calibration factor. Gaze position will be overestimated if the population-average HR is greater than the individual's HR, whereas it will be underestimated if the population-average HR is less than the individual's HR. The inaccuracy will also increase with the magnitude of gaze position owing to the multiplicative nature of this error. Several studies have recognized the importance of this problem, especially when estimating the magnitude of eye deviation in strabismus, and have recommended the use of the individual's own HR to improve the accuracy of results.[15–17,21] However, currently available techniques for measuring HR, such as the one described above, are time consuming and impractical to use in uncooperative participants such as infants and children. The challenges involved in obtaining empirical measurement of each eye's HR in such cases include (1) a strong resistance to monocular occlusion leading to low success rates in data collection,[22] (2) inaccurate and noisy HR measurement attributed to unsteady head position and variability in where the subject chooses to fixate (infants and young children cannot be instructed to fixate on a given target),[23] and (3) trade-off between the number and angular separation of fixation points required to obtain the HR empirically with the time taken to complete this task. Taken together, these calibration difficulties have deemed video-based analyses of gaze positions to provide only rough estimates of gaze angle.[21,24]

The HR is dependent on two ocular biometric parameters—corneal curvature and AC depth.[9,10] The theoretical relation between HR, corneal curvature, and AC depth can be derived from a geometric model described previously by Brodie.[10,12] If this theoretical relationship is verified empirically and the theoretical and empirical measures of HR match, then it may be possible to predict an individual's HR from their own corneal curvature and AC depth measurements. The predicted HR value can be used as the conversion factor, in lieu of HR determined empirically in each individual. Such a theoretical prediction will overcome the aforementioned challenges encountered in calculating the HR empirically. Corneal curvature and AC depth values can be easily and rapidly obtained using noncontact optical imaging/ultrasound techniques (e.g., corneal topographers[25,26] and A-scan devices[27,28]) on a wide range of age groups, including infants and children with good success rates.

Hasebe et al.,[15] Hunter and Guyton,[17] and Wick and London[20] obtained theoretical measures of HR from the corneal curvature and AC depth of their subjects. They found the HR to increase with the steepening of cornea and with deepening of AC. However, they did not determine the agreement of the theoretically calculated HR values with those obtained empirically on the same subject. The issue of agreement between the two HR measures therefore remains unanswered in the literature. The main aim of this study was therefore to determine the agreement between the HRs obtained from the individual's corneal curvature and AC depth and those measured empirically on the same subjects. This information is crucial for determining the accuracy with which gaze angles can be estimated using the theoretical HR values, vis-à-vis the individual's empirical and population-average HR value. A second related aim was to determine if the large intersubject variability in empirical HR can be explained by the underlying intersubject variance of corneal curvature and AC depth.

METHODS

Theoretical Calculation of HR Using the Geometric Optics Model

The relation between HR, corneal curvature, and AC depth can be derived from a geometric optics model described by Brodie.[10,15] The key equations in this model are shown here in equations 1 and 2. The displacement of the first PI from PC (in millimeters; PI_disp) was calculated over a ±25-degree span of gaze position in 5-degree steps and a linear regression equation was fit to the data of PI_disp versus the corresponding gaze position. The HR was determined as the reciprocal of the slope of this linear regression fit. The ±25-degree span was chosen to ensure that the relation between PI_disp and gaze position was linear, beyond which the relation theoretically tends to become nonlinear.[12]

$$\mathrm{PI}_{\mathrm{disp}} = (\mathrm{CC} + \mathrm{EP}_{\mathrm{distance}}) \times \sin(\Theta) \quad (1)$$

where CC is the corneal curvature (in millimeters), $\mathrm{EP}_{\mathrm{distance}}$ is the distance between entrance pupil and corneal apex (in millimeters), and Θ is the gaze position. $\mathrm{EP}_{\mathrm{distance}}$ can be calculated as:

$$\mathrm{EP}_{\mathrm{distance}} = 1000/[K - (1000 * n/\mathrm{ACD})] \quad (2)$$

where n is the refractive index of the AC (1.336 from the Bennett and Rabbetts schematic eye model[4]), K is the corneal power (in diopters) calculated as $K = 1000 \times [(n - 1)/\mathrm{CC}]$, and ACD is the AC depth (in millimeters). The refractive index of the cornea (1.3375 from the Bennett and Rabbetts schematic eye model[4]) is marginally different from that of the aqueous humor (1.336). Using different refractive indices for the two components, however, did not impact the results presented in this study significantly. A uniform refractive index (1.336) was therefore used for both cornea and the AC to generate the theoretical results shown below.

To understand the theoretical relation between HR, corneal curvature, and AC depth, HRs were calculated for corneal curvatures from 6.7 to 8.4 mm and AC depths from 2.5 to 3.5 mm, both varying in 0.1-mm steps. These values of corneal curvature and AC depth were chosen to represent the typical range encountered in the general population[29] and to match the range of empirical values obtained in the current study. The calculations indicated that the HR increased with steepening of the cornea and with the deepening of AC (Fig. 1). The rate of increase in HR with AC depth was greater for steeper than flatter corneas—for example, across the entire range of AC depths tested, the delta change in HR was 1.5 degrees/mm for an 8.4-mm corneal

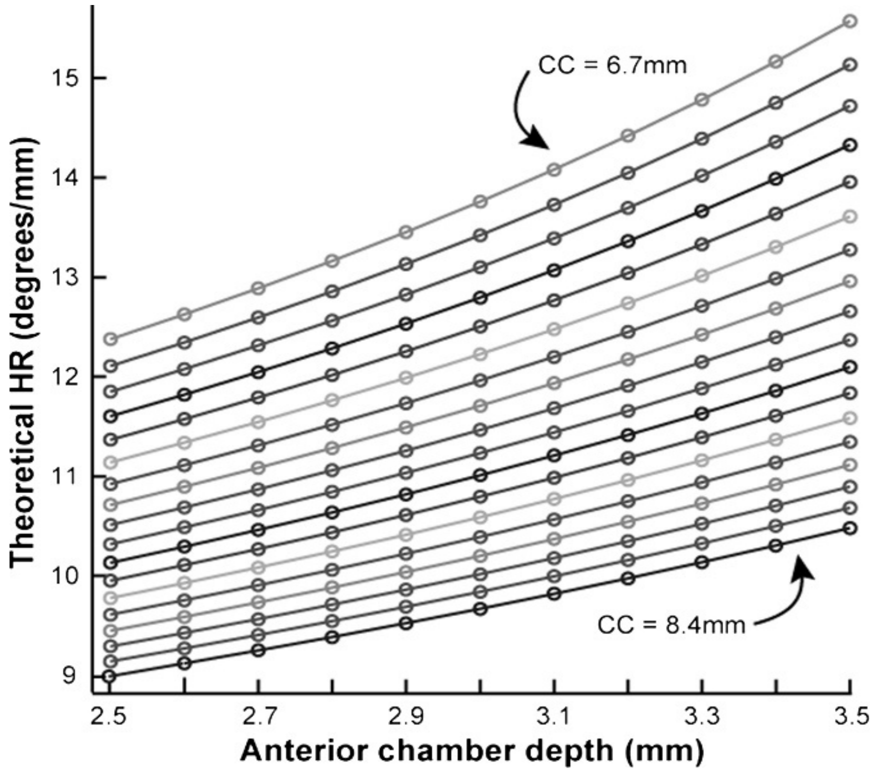

FIGURE 1.

Theoretical HR calculated from the model of Brodie[10,12] plotted as a function of AC depth for a range of corneal curvatures (CC). Hirschberg ratios were calculated for CCs ranging from 6.7 to 8.4 mm and AC depths ranging from 2.5 to 3.5 mm, both varying in 0.1-mm steps.

curvature (i.e., flatter cornea) and 3.2 degrees/mm for a 6.7-mm corneal curvature (i.e., steeper cornea) (Fig. 1). The increase in HR with steepening of corneal curvature and deepening of AC appeared nonlinear, suggesting multiplicative effects of the two biometric parameters on the HR (Fig. 1). As expected from a multiplicative nonlinearity, this effect was most apparent for the steepest cornea (radius of curvature, 6.7 mm) and the deepest AC (AC depth, 3.5 mm) (Fig. 1). Based on these results, the empirical HR was hypothesized to be inversely related to corneal curvature (in millimeters) and directly related to AC depth (in millimeters). Other biometric parameters such as axial length and the location/translation of the eye's center of rotation are not expected to impact the HR greatly and hence they were not considered in these calculations. Eskridge et al. obtained only a −0.15 correlation coefficient between axial length and HR.[14,30] Quick and Boothe[23] obtained an error of less than 0.2 degrees/mm in HR for a 2-mm error in the location of the eye's center of rotation for a camera distance of 60 cm whereas Barry et al.[31] obtained an error of approximately 1% in the displacement of the first PI per degree of eye rotation when the center of rotation was positioned between 9.6 and 17.6 mm behind the cornea.

Empirical Determination of HR, Corneal Curvature, and AC Depth

Twenty-four visually healthy volunteers (mean ± SD age, 23.6 ± 3.5 years) participated in the study. Subjects were purposefully sampled to represent a range of corneal curvatures and AC depths that is normally seen in the population. The subject's refractive error (0.25 to 4.0 diopters [D] of myopia and ≤3 D of astigmatism), if any, remained uncorrected during the study. The visual stimuli remained visible even without their refractive correction. The study adhered to the tenets of the Declaration of Helsinki and it commenced after subjects provided written informed consent, duly approved by the institutional review board of L V Prasad Eye Institute, Hyderabad.

The HR was measured as follows.[9–11] Subjects monocularly fixated on an array of visible light-emitting diodes (LEDs) arranged in a rotatable mount at 95 cm viewing distance in an otherwise dark room (Fig. 2A). Each LED was separated from the adjacent one by 4 degrees, with the LEDs on the left and right extremes subtending 24 degrees of gaze angle from straight-ahead viewing. High-resolution images of the eye were captured at each gaze angle using a custom-designed infrared (IR) eye tracker at 30 fps (Fig. 2A). The eye tracker consisted of a high-resolution digital camera (PGR Dragonfly Express) with a 4 by 6 rectangular array of IR LEDs (peak spectral emission at 850 nm) that acted as the PI generating light source and as the illumination source for pupil edge detection.[32] The top row of LEDs were 4 mm below the camera aperture whereas the other three rows were placed at 7 mm from the previous row.[32] This eye tracker was originally designed as an IR photorefractor for obtaining refractive error estimates from both eyes at 75 cm viewing distance.[32] To improve the spatial resolution of eye position (14 pixels/mm), the camera was moved to 30 cm and the image was brought into focus using a zoom lens. The eye tracker was placed at right angles to the subject's eye at 30 cm and was aligned with the eye using an IR reflecting hot mirror (Tower Optical Corporation, Florida) that was oriented at 45 degrees to the subject (Fig. 2A).

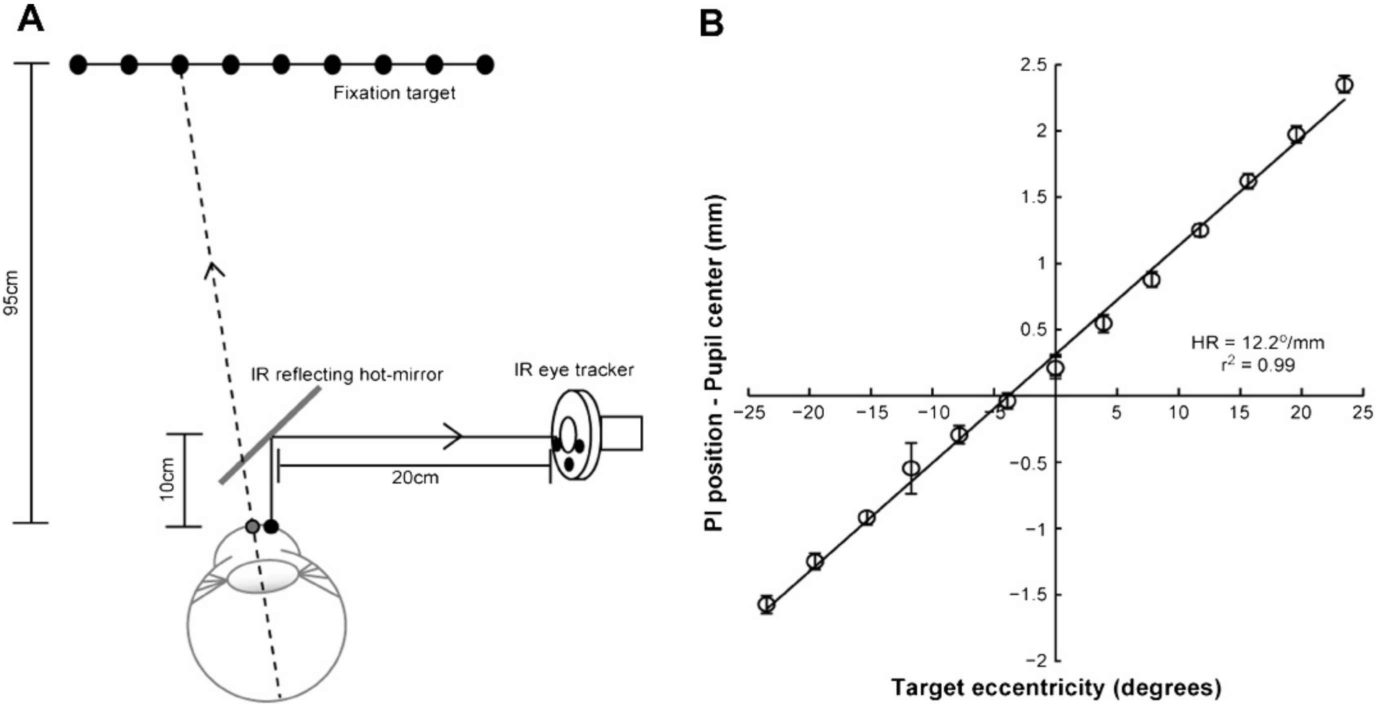

FIGURE 2.

The experimental setup (a). Raw data of separation between PI and PC plotted as a function of angular eccentricity for one representative subject who participated in the study (b). Each data point represents the mean (±95% confidence interval) separation between PI and PC obtained by averaging data from all usable frames for that gaze position. The solid line through the data represents the best-fit linear regression equation.

The pupils were dilated to greater than 6 mm using 10% phenylephrine hydrochloride eye drops to reduce measurement noise from random pupil fluctuations, improve the signal-to-noise ratio of the measurements, and minimize any errors in HR calculation owing to variations in pupil centration with pupil miosis.[33] The subject's head was stabilized using a head and chin rest. Subjects fixated on the central LED of the fixation array at the beginning of the experiment and they changed fixation to the adjacent LED on the left once every 4 seconds (4 seconds × 30 fps = 120 frames) until the last LED was reached. This task was repeated once more in the opposite direction. The entire experiment was repeated for the same range of vertical gaze positions by rotating the LED array by 90 degrees. Data were collected from both eyes of 10 subjects and from only one eye in the remaining 14 subjects.

The subject's horizontal and vertical corneal curvature and AC depth were measured using the Bausch & Lomb Orbscan IIz topographer thrice and averaged. Corneal curvature and AC depth are measured noninvasively by this instrument using a combination of Placido disc imaging and slit-scanning technology.[34,35] Subjects with irregularities in corneal topography of greater than ±1.50 D within 3 mm of the PC (e.g., those with early forms of keratoconus) were excluded from the study. This value represents the upper limit of irregularity observed in otherwise normal and healthy corneas.[36,37] Anterior chamber depth was measured as the distance between the corneal apex and the front surface of the crystalline lens. This is somewhat different from the traditional definition of AC depth, which is the distance from the corneal apex to the anatomical pupil plane used to calculate the location of PC.[4] However, since the edge of the iris forming the pupil boundary is approximately in the same plane as the front vertex of the crystalline lens, any difference in AC depth should be negligible and could be ignored.[4]

Data Analyses

Eye position videos were analyzed using custom-written MATLAB software. Video frames with blinks and eyelid/eyelash artifacts that occluded the pupil were removed from analysis. Pupil center was determined using standard centroid-based edge detection algorithm whereas PI was determined as the brightest grayscale value within the pupil. The separation between PI and PC was calculated for each usable video frame and averaged for each gaze position. This separation was plotted against the corresponding angular eccentricity, and the reciprocal of the slope of a linear regression equation fit to this data gave the eye's HR (Fig. 2B).

The theoretical HR of each subject was calculated from their individual corneal curvature and AC depth measurements using equations 1 and 2. The agreement between the theoretical and empirical HRs were examined using Bland-Altman plots.[38] The relation between HR, corneal curvature, and AC depth was determined using multiple regression analyses. Data from horizontal and vertical gaze positions were analyzed separately. As observed previously,[6,15] the empirical HR data obtained from both eyes of 10 subjects showed a very high degree of intraclass correlation ($r \geq 0.92$). Therefore, data from only one eye of these individuals were randomly selected for all analyses.[39]

RESULTS

Data were successfully collected from all subjects who participated in the study. Subjects' corneal curvatures ranged from 6.97 to 8.20 mm (mean ± SD, 7.68 ± 0.32 mm) for the horizontal meridian and from 6.86 to 8.09 mm (mean ± SD, 7.49 ± 0.32 mm) for the vertical meridian. The AC depth of subjects ranged from 2.6 to 3.3 mm (mean ± SD, 2.97 ± 0.19 mm) for both meridians. There was no correlation between the corneal curvature and AC depth for both meridians ($r = -0.18$ for horizontal and $r = -0.15$ for vertical; $p > 0.40$ for both). The empirically measured HRs ranged from 8.30 to 13.06 degrees/mm (mean ± SD, 10.62 ± 1.1 degrees/mm) and from 9.25 to 13.95 degrees/mm (mean ± SD, 11.32 ± 1.2 degrees/mm) for horizontal and vertical gaze positions, respectively. The linear regression equations from which these HRs were calculated had r^2 values greater than 0.90 in all subjects (Fig. 2B).

Theoretical HRs calculated from each subject's corneal curvature and AC depth ranged from 9.25 to 13.02 degrees/mm (mean ± SD, 10.97 ± 0.93 degrees/mm) and from 10.06 to 13.64 degrees/mm (mean ± SD, 11.48 ± 0.92 degrees/mm) for horizontal and vertical gaze positions, respectively. The theoretical HRs were well correlated with the corresponding empirical HR values (Pearson correlation coefficient; $r \geq 0.83$ for both gaze angles; $p < 0.001$) and they were not significantly different from each other for both gaze angles ($p \geq 0.23$ for both). The mean difference (±95% limits of agreement [LOA]) between the two HR measures was −0.35 (0.85 to −1.55) degrees/mm for horizontal gaze angles and −0.16 (1.01 to −1.33) degrees/mm for vertical gaze angles (Fig. 3). The difference in HR for both gaze directions did not vary significantly across the entire range of mean HR (slope of linear regression equation of difference in HR vs. mean HR was 0.16 for horizontal gaze angles [$df = 22$; $t = 1.26$; $p = 0.89$] and 0.26 for vertical gaze angles [$df = 22$; $t = 2.35$; $p = 0.92$]) (Fig. 3).

The relation between empirical HR, corneal curvature, and AC depth followed the overall expectations of the theoretical analysis described earlier (Fig. 1). Empirical HR was negatively correlated with corneal curvature for horizontal ($r = -0.84$; $p < 0.001$) and vertical ($r = -0.81$; $p < 0.001$) gaze directions. Empirical HR was also positively correlated with AC depth for horizontal ($r = 0.50$; $df = 22$; $p = 0.007$) and vertical ($r = 0.51$; $df = 22$; $p = 0.005$) gaze directions.

Multiple linear regression analysis indicated that the combination of corneal curvature and AC depth accounted for 81.4% ($F_{2,21} = 49.33$; $p < 0.001$) and 79.5% ($F_{2,21} = 42.94$; $p < 0.001$) of variance in the empirical HRs for horizontal and vertical gaze directions, respectively. The coefficients of the regression model indicated that a unit increase in corneal curvature (i.e., flattening of the cornea) resulted in a decrease of 2.62 ± 0.32 degrees/mm and 2.76 ± 0.36 degrees/mm in the empirical HR for horizontal and vertical gaze directions, respectively (equations 3 and 4). Similarly, a unit increase in AC depth resulted in an increase of 2.03±0.53 degrees/mm and 2.47±0.61 degrees/mm in the empirical HR for horizontal and vertical gaze directions, respectively (equations 3 and 4). Various multiple regression models with higher-order polynomial terms were attempted to capture the nonlinear change in HR with corneal curvature and AC depth apparent in Fig. 1. This was, however, not the case for HR in horizontal direction. For HR in the vertical direction, a multiple regression model with third-order polynomial in the AC depth parameter accounted for 82.3% of variance ($F_{4,19} = 27.71$; $p < 0.001$). The coefficients of the squared and cubic terms of the AC depth polynomial did not, however, reach statistical significance ($p > 0.05$), indicating that the contributions of the nonlinear elements were negligible and within the experimental variability.

$$\text{Empirical horizontal HR} = 24.70 - 2.62 * \text{Corneal curvature} + 2.03 * \text{AC depth} \left(r^2 = 0.81\right) \quad (3)$$

$$\text{Empirical vertical HR} = 24.68 - 2.76 * \text{Corneal curvature} + 2.47 * \text{AC depth} \left(r^2 = 0.80\right) \quad (4)$$

The relation between theoretical HR values, corneal curvature, and AC depth was very similar to the empirical data. Multiple linear regression analysis indicated that the combination of corneal curvature and AC depth accounted for 91.0% ($F_{2,21} = 106.02$; $p < 0.001$) and 95.0% ($F_{2,21} = 221.59$; $p < 0.001$) of variance in the theoretical HRs for horizontal and vertical directions, respectively (see equations 5 and 6 for coefficients of the multiple regression equations). Other statistical models were not attempted as the linear regression model accounted for such a high percentage of variance in the theoretical HR data.

$$\text{Theoretical horizontal HR} = 22.61 - 2.29 * \text{Corneal curvature} + 2.02 * \text{AC depth}\left(r^2 = 0.91\right) \quad (5)$$

$$\text{Theoretical vertical HR} = 22.94 - 2.36 * \text{Corneal curvature} + 2.09 * \text{AC depth}\left(r^2 = 0.95\right) \quad (6)$$

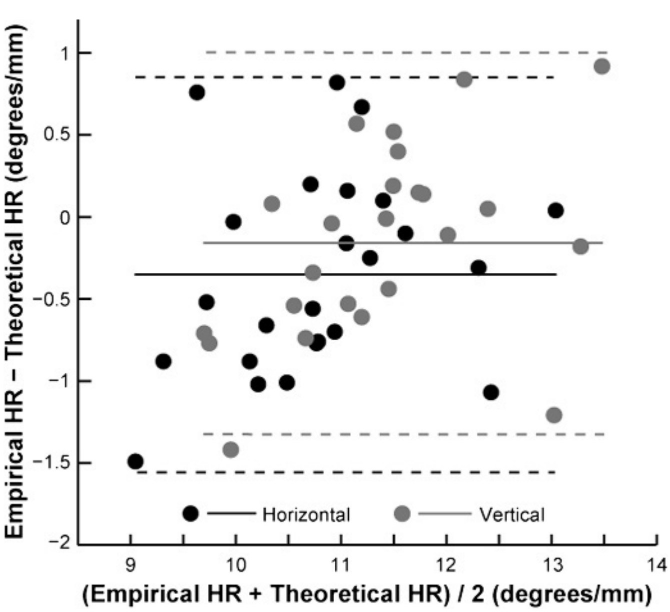

FIGURE 3.
Bland-Altman-type plot showing agreement between theoretical and empirical HR for all subjects who participated in the study. The black and gray symbols indicate data for horizontal and vertical gaze directions, respectively. Solid lines indicate the mean difference and dashed lines indicate the ±95% LOA.

DISCUSSION

The overall purpose of this study was to develop an alternate, easy-to-use technique for obtaining the eye position calibration factor of first PI-based eye trackers (HR). The HR calculated theoretically from the individual's corneal curvature and AC depth using technology available in most clinical or research settings is attractive for this purpose. By doing this, the cumbersome empirical calibration procedure could be entirely circumvented and the dependence on the population-average HR for converting raw data into angular units can also be minimized.

Empirical HR varied from 8 to 14 degrees/mm in this study and from 7 to 16 degrees/mm in previous studies.[6,10–16,19] Using a population-average HR (~11 to 12 degrees/mm) as the conversion factor may therefore induce 33 to 42% of inaccuracy in an individual's gaze angle estimate [e.g., 100 − (7 degrees/mm of individual empirical HR ÷ 12 degrees/mm of population-average HR) × 100 = 41.7% overestimation]. In contrast, the 2.0 to 2.3 degrees/mm of LOA between empirical and theoretical HR results in only 14 to 25% of inaccuracy in gaze angle estimates, with the magnitude of inaccuracy scaling inversely with the baseline HR (Fig. 3). The smallest empirical HR of 8 degrees/mm in this study would result in 25% overestimation or underestimation of gaze angle [e.g., 100 − (6.0 degrees/mm of theoretical HR at the lower LOA ÷ 8 degrees/mm of empirical HR) × 100 = 25% of overestimation], whereas the largest empirical HR of 14 degrees/mm in this study would result in 14% overestimation or underestimation of gaze angle [e.g., 100 − (12 degrees/mm of theoretical HR at the lower LOA ÷ 14 degrees/mm of empirical HR) × 100 = 14.3%]. The accuracy of gaze angles estimated by first PI-based eye trackers therefore nearly doubles by using the individual theoretical HR (14 to 25% of inaccuracy) for calibration, relative to the population-average HR (33 to 42% of inaccuracy). The theoretical calculations, however, slightly overestimated the individual's HR and this needs to be considered during the calibration process (Fig. 3).

The accuracy of gaze angle estimated using this technology would be limited by the intrasubject variability of empirical and theoretical HR. Empirical HR varies by approximately 1.5 to 3.0 degrees/mm over two repeated measurements within an individual,[16,18] presumably attributed to the instability of ocular fixation and to limitations in spatial and temporal resolution of the video recording.[1,3] The repeatability of theoretical HR depends on the repeatability of corneal curvature and AC depth measurements (~0.08 mm for both measures in this study and in previous studies[40–42]). This leads to an intrasubject variability in theoretical HR of approximately 0.5 degrees/mm for an average corneal curvature of 7.68 mm and AC depth of 2.97 mm. The 2.0 to 2.3 degrees/mm LOA between empirical and theoretical HR observed in this study may therefore largely reflect the repeatability of empirical HR, with only a small contribution from the repeatability of corneal curvature and AC depth measurements. Absolute estimates of gaze position require angle κ to also be measured on each individual. Although angle κ was not measured in this study, previous studies indicate that it can vary by up to ±2.5 degrees across individuals[6,19] and up to ±0.6 degrees over repeated measurements within individuals.[5,43] The currently used theoretical way of estimating HR from corneal curvature and AC depth cannot be directly used to estimate angle κ, and this presents a limitation of this technique in obtaining absolute estimates of gaze position. However, HR and angle κ can be estimated simultaneously using the traditional empirical calibration procedure. Other factors contributing to the inaccuracy of gaze position estimates include subject cooperation and random measurement errors from pupil size and shape variations, lid position, and blinks.[1]

Theoretical HRs could be practically applied in first PI-based eye trackers in one of two ways. First, eye trackers currently using the population-average HR (e.g., PowerRefractor[6,8]) could incorporate a measure of the subject's corneal curvature and AC depth instead, and the theoretical HR calculated from these values could be used for converting raw gaze position data into angular units. Second, if the population-average HR used by the eye tracker is known (e.g., 11.8 degrees/mm for PowerRefractor[6]), then a ratio of the individual's theoretical and population-average HR could be derived for scaling data already recorded in angular units. For instance, if the individual's theoretical HR is 16 degrees/mm and the population-average HR is 11.8 degrees/mm, then the recorded eye position would be scaled by a factor of 1.36 (16 degrees/mm ÷ 11.8 degrees/mm = 1.36) for accurate gaze position estimates. Indeed, the same logic is followed in studies that calibrate gaze position using prisms.[22,44] Base-in and base-out prisms of varying powers are introduced before the subject's eye occluded using an IR transmitting filter. The induced gaze deviation recorded by the eye tracker is plotted against the corresponding prism power to obtain the aforementioned scaling factor.[22,44] As with any empirical technique, this prism-calibration procedure is also time consuming and critically dependent on subject cooperation. The prism-calibration procedure could now be replaced by the scaling factor derived from the theoretical HR.

Approximately 80 to 85% of the intersubject variance in empirical HR was explained from the combined intersubject variance of corneal curvature and AC depth. This knowledge has thus far only been an inferred logic in the previous literature. The coefficients of both independent variables were statistically significant, indicating that their combination accounted for a larger proportion of variance in HR than each one considered separately. The standard errors of the corneal curvature coefficient were less than those of the AC depth coefficient, indicating that the former had a more robust influence on HR than the latter. The unaccounted variance may be explained by the aforementioned intrasubject repeatability of empirical HR and other random measurement errors in corneal curvature, thickness, and AC depth that were not included in this statistical model. The aforementioned random measurement errors were however a part of the statistical model that accounted for 90 to 95% of variance in the theoretical HR data. This suggests that these measurement errors account for less than 10% of variance in the empirical HR data set. A larger proportion of unaccounted variance in the empirical data set may therefore arise from the repeatability of the empirical HR values.

The strong inverse relation between HR and corneal curvature implies that individuals with high degrees of astigmatism may show significantly different HRs across their steepest and flattest corneal meridians. In the current data set, the difference in corneal curvature across the two meridians ranged from 0.04 to 0.47 mm across subjects, corresponding to corneal astigmatism of 0.21 to 2.6 D. The difference in corneal curvatures (horizontal − vertical) was positively correlated with the difference in HR obtained across the two gaze directions (r = 0.86; p < 0.001 for theoretical HR and r = 0.40; p = 0.02 for empirical HR). At a mean level, the cornea was flatter horizontally (7.68 ± 0.32 mm) than vertically (7.49 ± 0.32 mm), and the mean horizontal empirical HR (10.62 ± 1.1 degrees/mm) was correspondingly smaller than the vertical empirical HR (11.32 ± 1.2 degrees/mm). Overall, these results are similar to those by Quick and Boothe[18] and suggest that separate HR values may be needed for calibrating horizontal and vertical gaze angles in individuals with toric corneas.

In conclusion, the study shows that HR values calculated theoretically from the individual's corneal curvature and AC depth can be used in lieu of empirical HR measurements for converting raw gaze position into angular units. The theoretical HR is easy to calculate and it nearly doubles the accuracy of gaze positions measured using this technique, relative to the population-average HR value. The hallmark intersubject variability of empirical HR reflects the underlying variance of corneal curvature and AC depth. Furthermore, corneal meridian-specific HR may be needed to obtain accurate estimates of gaze position in eyes with toric corneas.

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