NETWORKED CONTROL PROJECT

The Quadruple-Tank Process - 20

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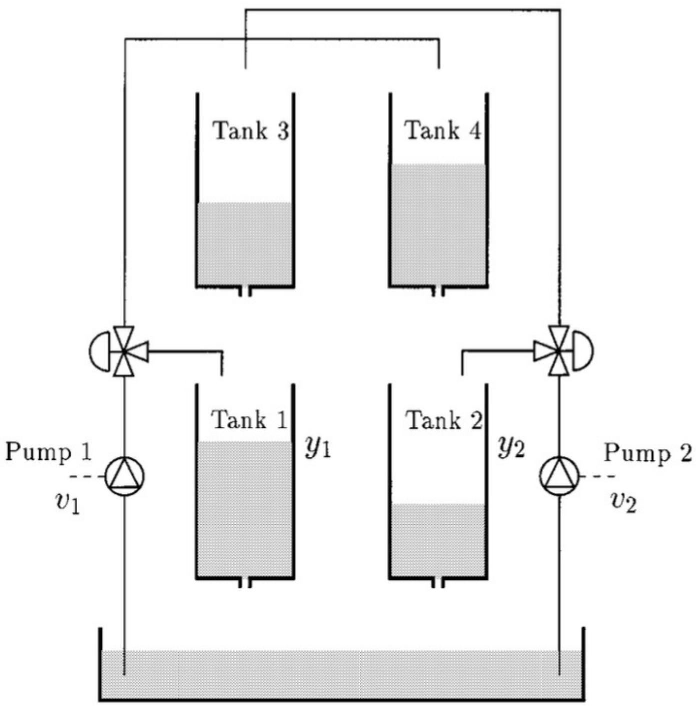
System Description

The system under analysis is a laboratory process that consists of four interconnected water tanks.

The target is to control the level of the lower two tanks with two pumps.

- Process inputs: v1 and v2 (input voltages to the pumps)
- Process outputs: y1 and y2 (voltages from level-measurament devices)



System Equations

$$
\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1
\end{aligned}
$$

A_1, A_3	[cm^2]	28
A_2, A_4	[cm^2]	32
a_1, a_3	[cm^2]	0.071
a_2, a_4	[cm^2]	0.057
k_c	[V/cm]	0.50
g	[cm/s^2]	981.

where

- A_i cross-section of Tank i;
- a_i cross-section of the outlet hole;
- h_i water level.

Continuous Time:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

System Operating Conditions

Two possible operating conditions that depend on the configuration of the system parameters are possible. We will analyse the minimum-phase characteristic.

		P_-	P_+
(h_1^0, h_2^0)	[cm]	(12.4, 12.7)	(12.6, 13.0)
(h_3^0, h_4^0)	[cm]	(1.8, 1.4)	(4.8, 4.9)
(v_1^0, v_2^0)	[V]	(3.00, 3.00)	(3.15, 3.15)
(k_1, k_2)	[cm^3/Vs]	(3.33, 3.35)	(3.14, 3.29)
(γ_1, γ_2)		(0.70, 0.60)	(0.43, 0.34)

Subsystem Decomposition

The system can be decomposed in two subsystems, each one corresponding to one of the two tanks attached to the pump plus a connected thank.

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2$$
$$y_i = C_i x \quad i = 1, 2$$

$$\Downarrow$$

$$\dot{x} = Ax + [B_1 \ \ B_2]\, u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x$$

Input-decoupled system:

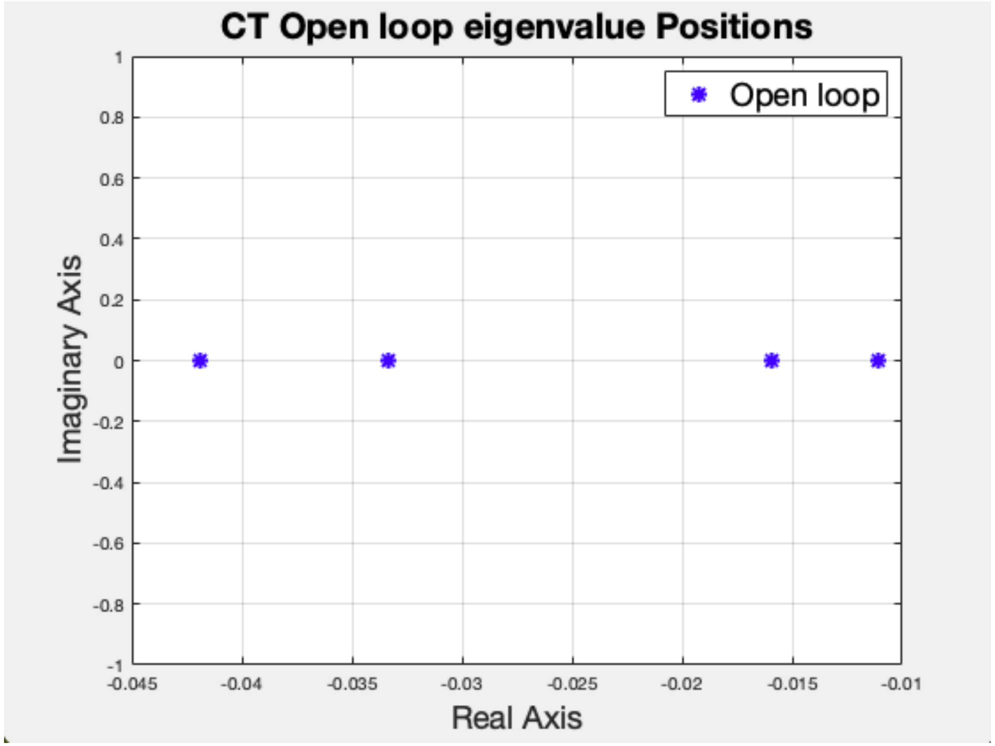
```
Atot =

   -0.0161         0         0    0.0435
         0   -0.0333         0         0
         0    0.0333   -0.0111         0
         0         0         0   -0.0435


B =

    0.0833         0
    0.0312         0
         0    0.0628
         0    0.0479


C =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

Open Loop System Analysis

Spectral abscissa = -0.0111

⇩

All the eigenvalues of the matrix A are strictly negative, so the open-loop system is **asymptotically stable**.



Open Loop System Analysis

Analysing the step response of the Continuous Time System we can have an idea of the dynamics and have an idea of a suitable sampling time.

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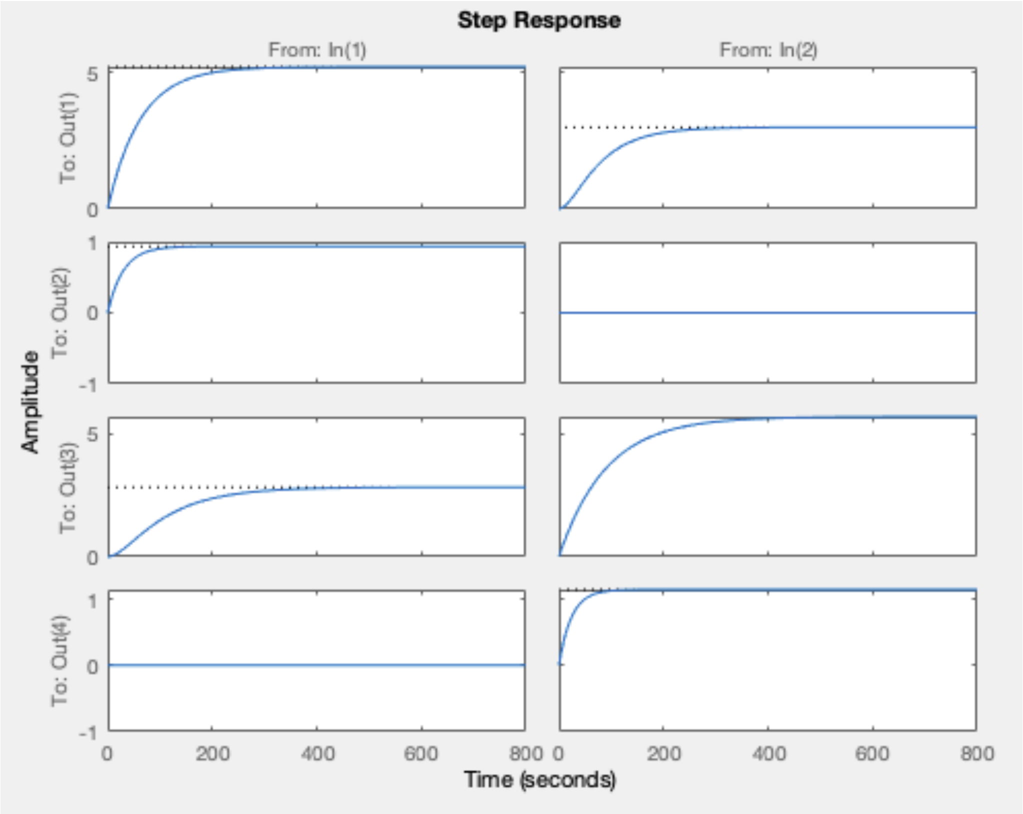
```
Ttransient = 5/rho_CT
```

```
Ts = 1
```



System Discretization

Discretize the system with a sampling time Ts=1 s and with the zero-order holder (ZOH).

Done through the MATLAB command $d\_system \;=\; c2d(system, 1, 'zoh')$

Discrete Time:

$$\mathrm{x}_{k+1} = Fx_k + Gu_k$$
$$y_k = Hx_k$$

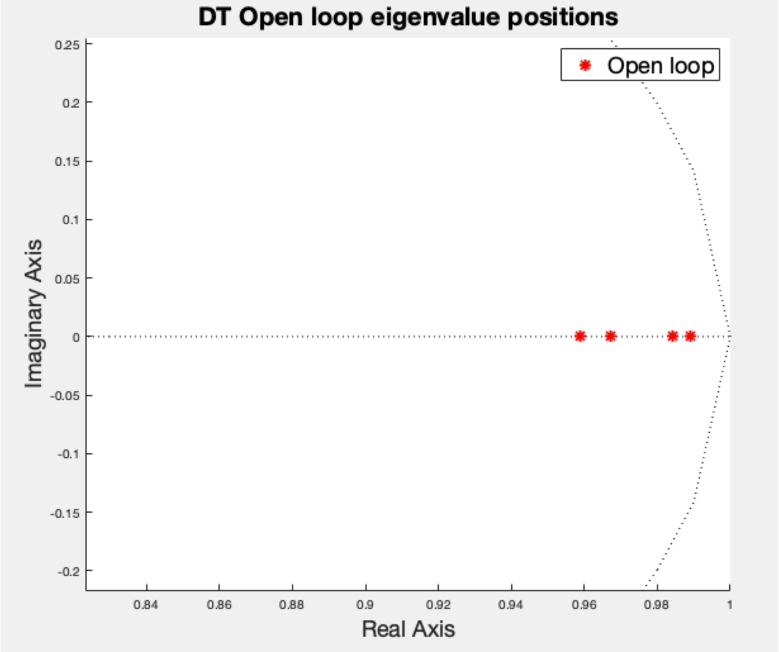
```
Ftot =

    0.9842         0         0    0.0407
         0    0.9672         0         0
         0    0.0326    0.9890         0
         0         0         0    0.9590
```

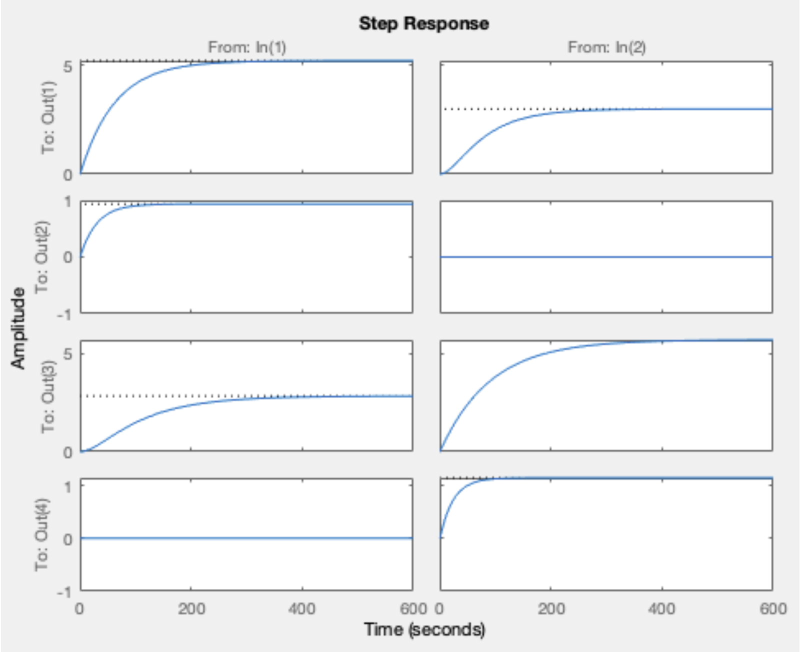
```
H =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
G =

    0.0826    0.0010
    0.0307         0
    0.0005    0.0625
         0    0.0469
```

Open Loop System Analysis

Spectral radius= 0.9890 ⇨ All the eigenvalues of the matrix F are strictly inside the unitary circle, so the open-loop system is **asymptotically stable**.





Controllability and Observability

Controllability

$$rank[\lambda_i - A \quad B] = 4 \qquad \forall\, \lambda_i \ \in eig(A)$$

$$\Downarrow$$

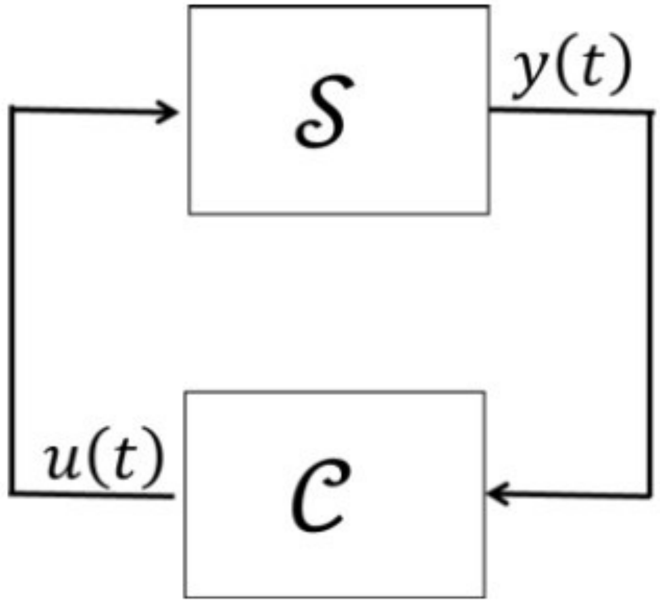
The system is controllable since matrix Co has full rank both in discrete time and contunuos time for every eigenvalue

Observability

$$rank\begin{bmatrix}\lambda_i - A \\ C\end{bmatrix} = 4 \qquad \forall\, \lambda_i \ \in eig(A)$$

$$\Downarrow$$

The system is observable since matrix Ob has full rank both in discrete time and contunuos time for every eigenvalue



CENTRALIZED CONTROL

Centralized Control

The idea of the centralized control is to compute a unique regulator that controls the system through a static state-feedback control law because the regulator can access to all the measurements of the system outputs.

The resulting closed loop system will be then $\dot{x} = (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}_{\boldsymbol{x}}\boldsymbol{C})x$

$$ContStruc = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$K_c = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix}$$

GOAL

Compute K_x that makes the closed loop stable by optimizing some LMI

In the centralized case K_x is a full 2x4 matrix since all the channels have access to the measurements of every other channel.

Performances are improved by optimizing LMIs!

Centralized Control
Fixed modes and LMI for stability

In order to design a stabilizing gain, it is necessary to check the presence of **fixed modes**. It can be done through an iterative approach represented by the *di_fixed_modes* function.

In our system there are no continuous time nor discrete time centralized fixed modes.

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We can obtain a control law that stabilize the system by optimizing the LMI for stability

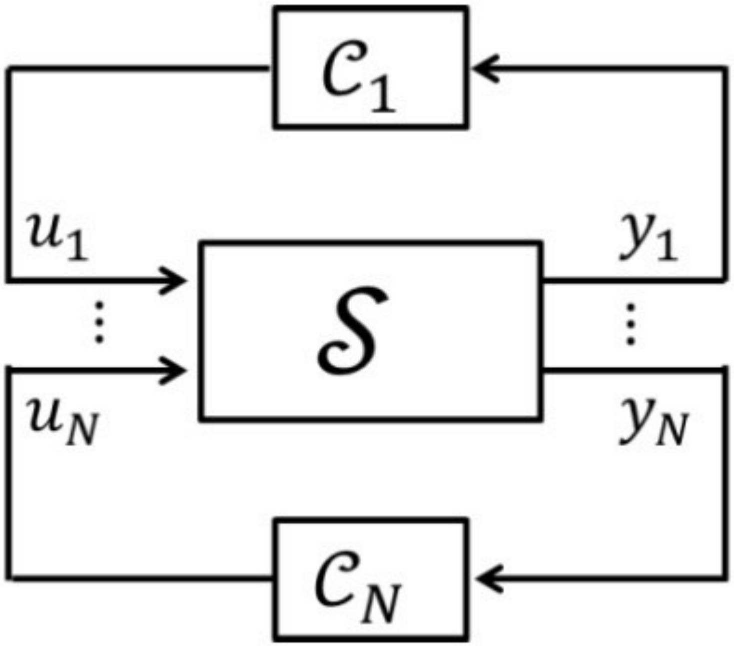
Theorem

$\exists K\colon\ A + BK$ is Hurwitz stable $\leftrightarrow\ \exists P = P^T > 0 : (A+BK)^T P + P(A+BK) < 0.$

Renaming $Y = P^{-1}$ and $L = KY$, we obtain:

LMI for stability

$$\begin{cases} Y > 0 \\ YA^T + AY + L^T B + BL < 0 \end{cases}$$



DECENTRALIZED CONTROL

Decentralized Control

The idea of a decentralized control is to compute multiple independent regulator, one for every channel i, where each of them controls the system only through y_i and u_i

The resulting closed loop system is, also in this case $\dot{x} = (\boldsymbol{A} + \boldsymbol{BK})x$.

$$ContStruc = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

GOAL

Compute K that makes the closed loop stable by optimizing some LMI

In the decentralized case K_x is a block-diagonal matrix, where on the diagonals there are full-matrices of dimension $n_{u_i} \times n_{y_i}$ for every channel i

$$K_c = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 \\ 0 & 0 & K_{23} & K_{24} \end{bmatrix}$$

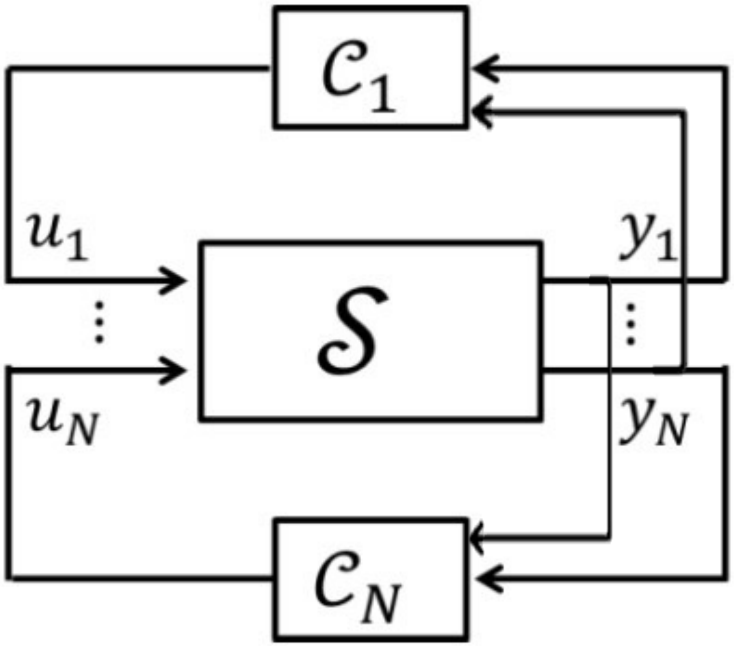
Decentralized Control

Continuous Stabilizing Control Law

The procedure followed is the same of the centralized case, but with a different structure of matrix K.

In our system there are no continuos time nor discrete time decentralized fixed modes.

The LMI for stabilization are the one used in the centralized case, with the difference that in the decentralized case is not possible to arbitrarly place the closed-loop eigenvalues with an output-feedback static control law, neither with a state-feedback static control law.



DISTRIBUTED CONTROL

Distributed Control

The idea is again to obtain a static static-feedback control law, resulting in the closed loop system $\dot{x} = (\boldsymbol{A} + \boldsymbol{BKC})x$

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GOAL
Compute K that makes the closed loop stable by optimizing some LMI

$$ContStruc = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$ContStruc = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Also in the distributed case is not possible to arbitrarly place the closed-loop eigenvalues with an output-feedback static control law, neither with a state-feedback static control law.

The structure of the matrix $K_{distrib}$ is sparse and changes according to the configuration considered. In particular $K_{distrib}$ will have a zero matrix corresponding to the block (i,j) when output j is not available to controller Ci.

Design of Static Control Laws using LMIs

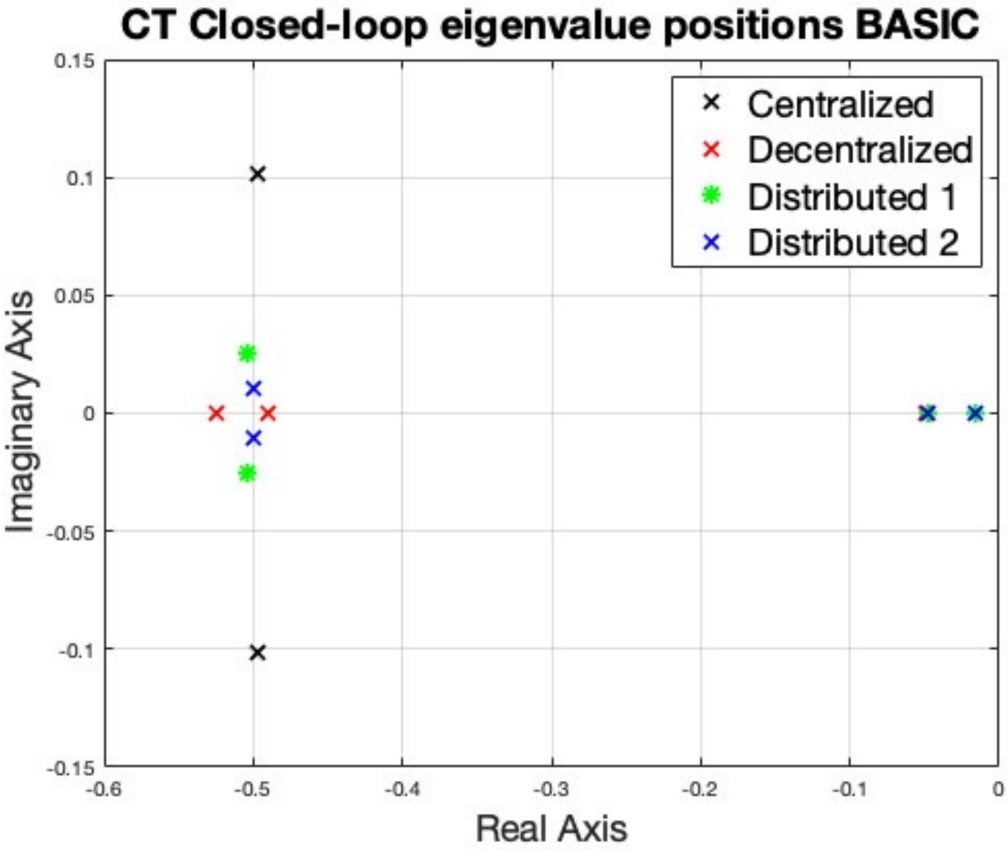
We considered different LMIs problems for both continuous time and discrite time systems:

- Basic Controller for Asymptotic Stability
- Reduction of the Control Effort
- Limitation on the Spectral Abscissa of $A + BK_x$ $(Continuous - Time)$
- Limitation on the Spectral Radius of $A + BK_x$ $(Discrete - Time)$
- Eigenvalues of $A + BK_x$ in a disk $(Continuous - Time)$

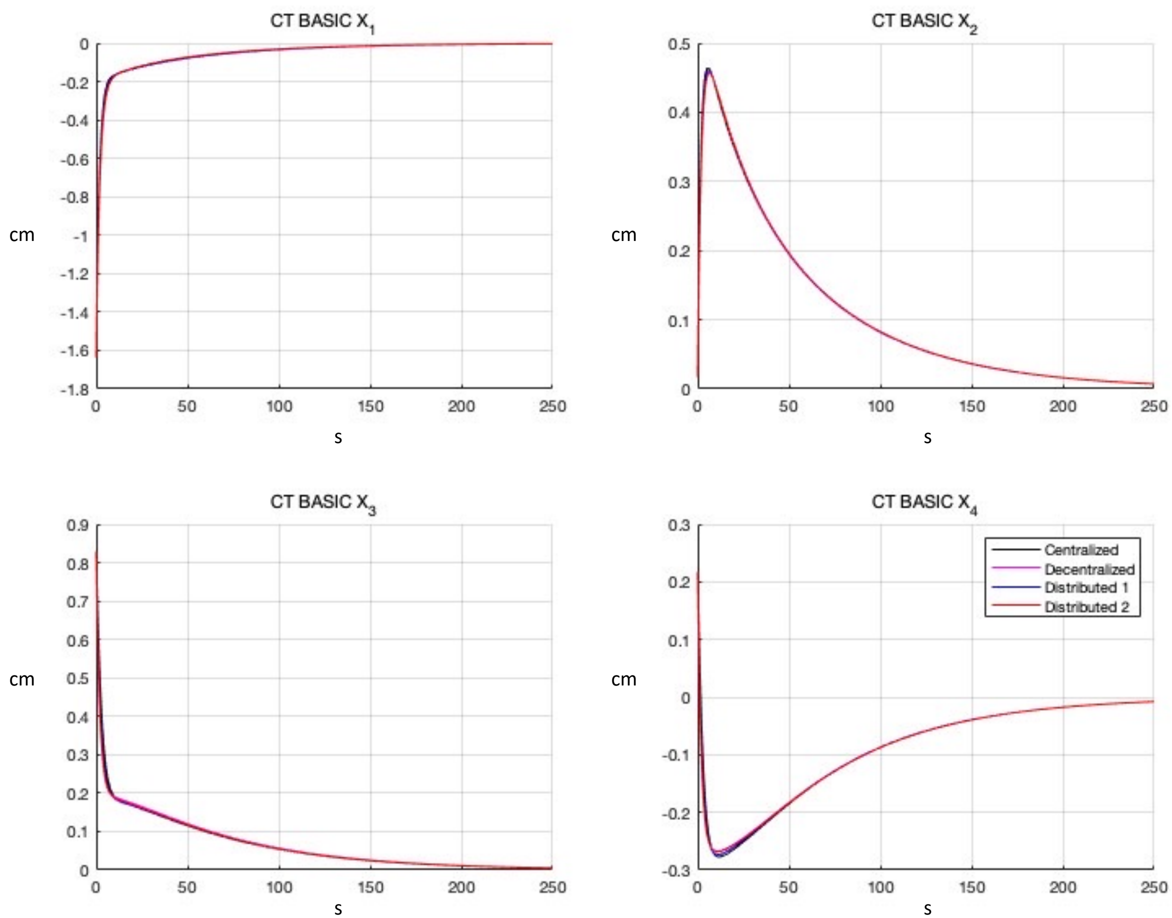
Basic Controller for Asymptotic Stability

- Continuous-Time

$$YA^T + AY + L^T B^T + BL < 0$$
$$Y > 0$$



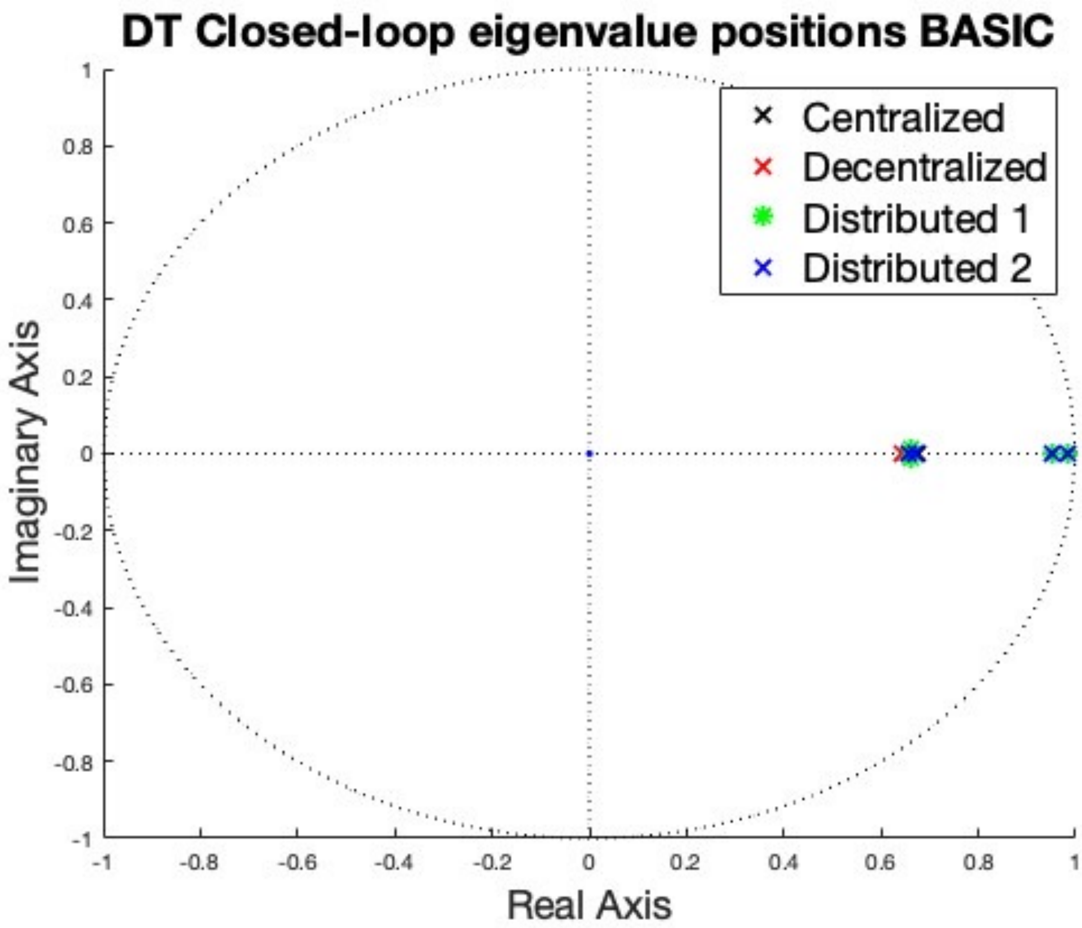
Basic Controller for Asymptotic Stability



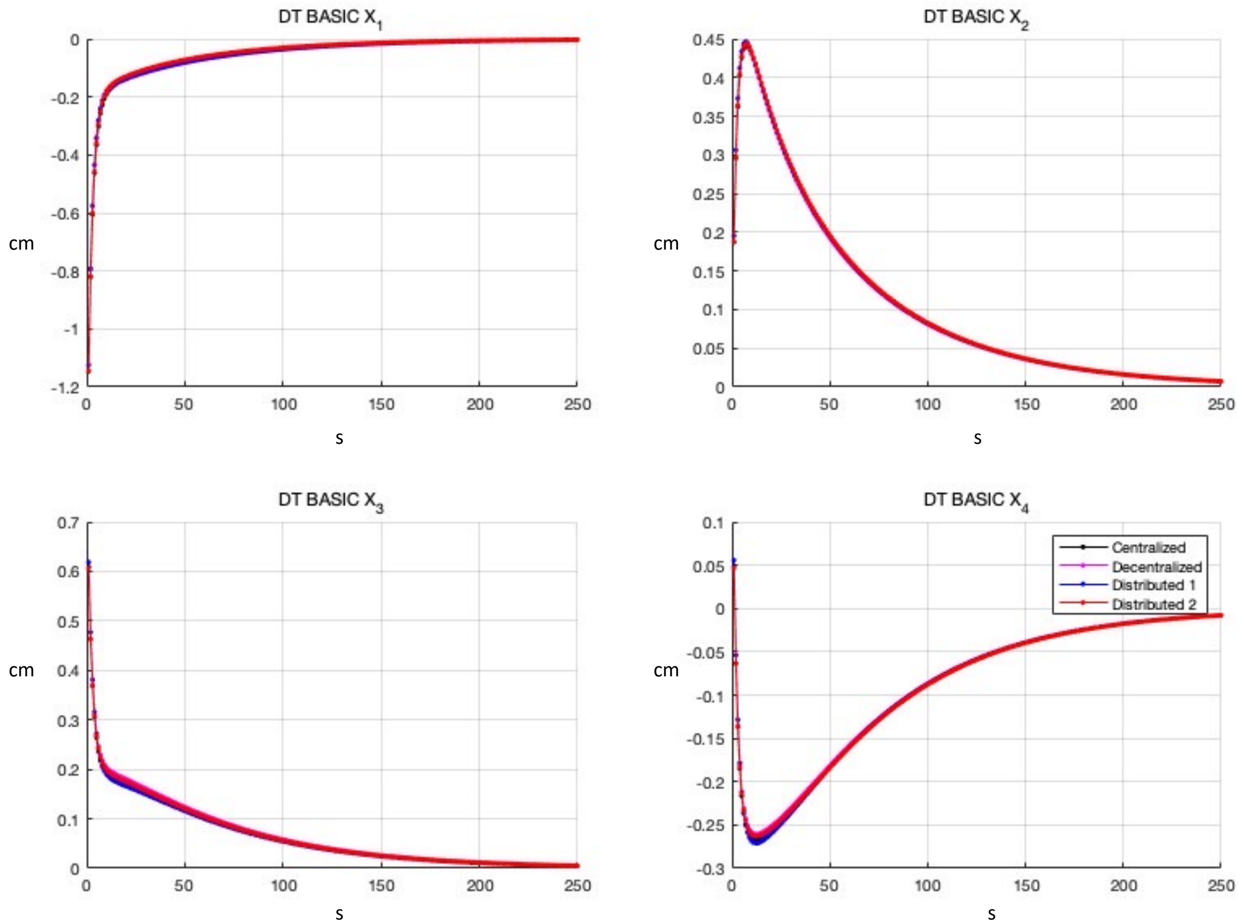
Basic Controller for Asymptotic Stability

- Discrete-Time

$$\begin{bmatrix} P - FPF^T - FL^TG^T - GLF^T & GL \\ L^TG^T & P \end{bmatrix} > 0$$



Basic Controller for Asymptotic Stability



Reduction of the Control Effort

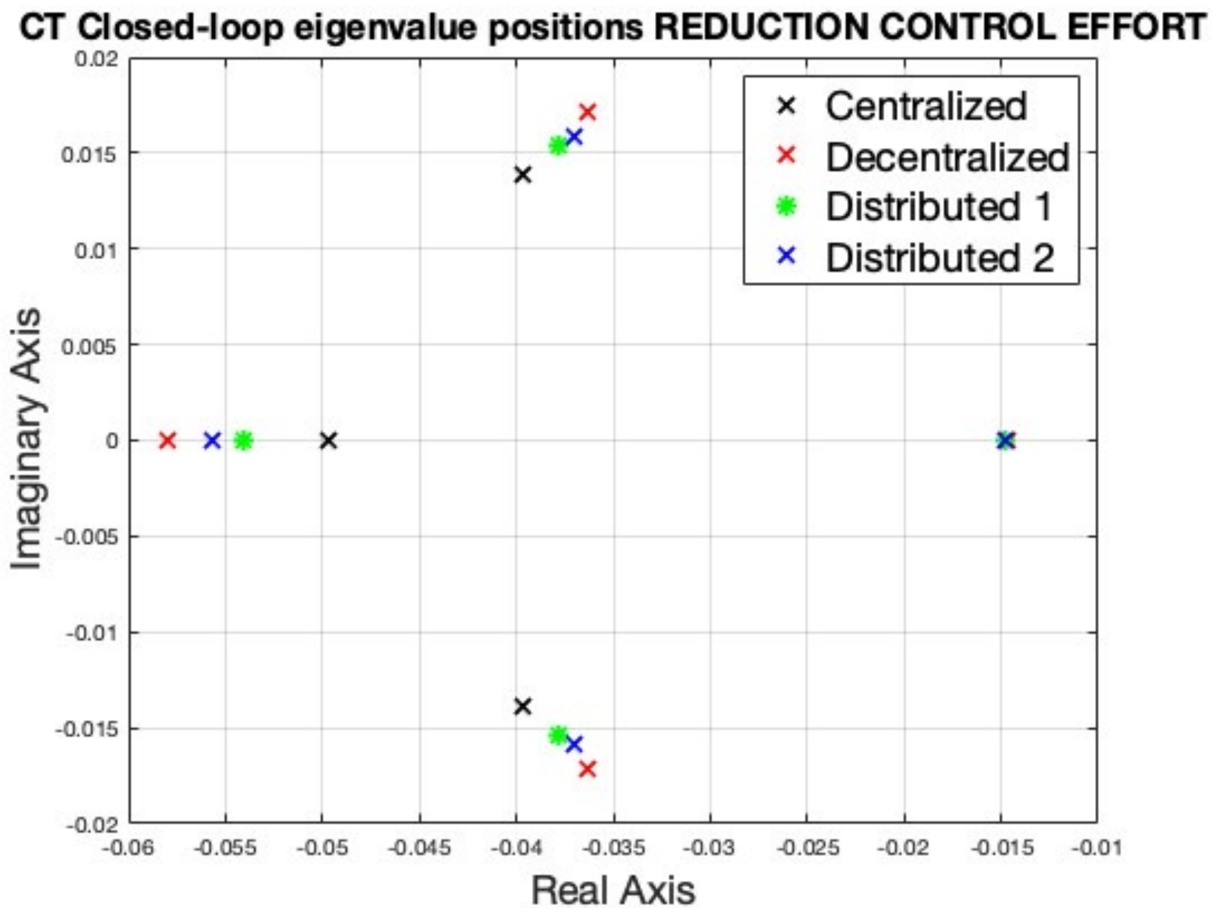
- Continuous-Time

- **L minimization**

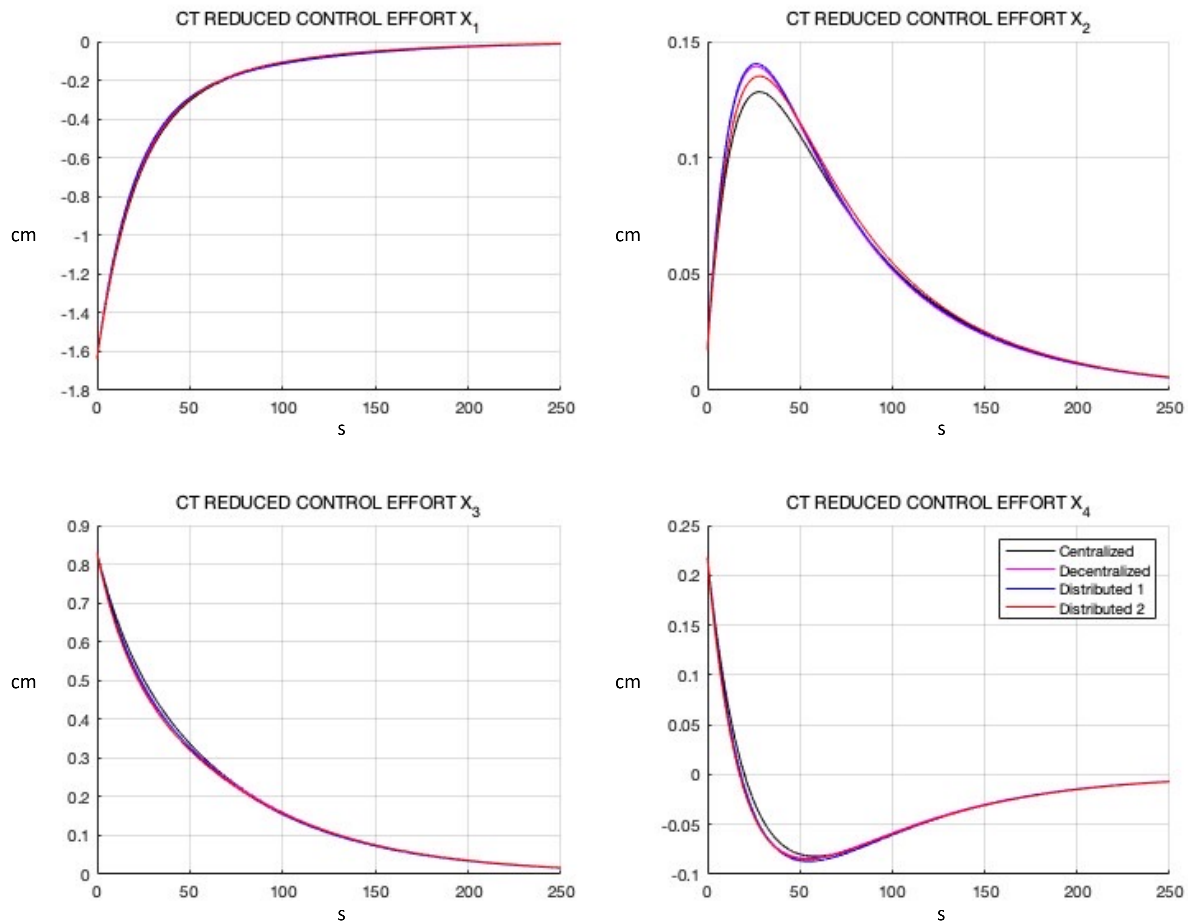
$$\begin{bmatrix} \kappa_L I & L^T \\ L & I \end{bmatrix} \geq 0$$

- **Y minimization**

$$\begin{bmatrix} \kappa_Y I & I \\ I & Y \end{bmatrix} \geq 0$$

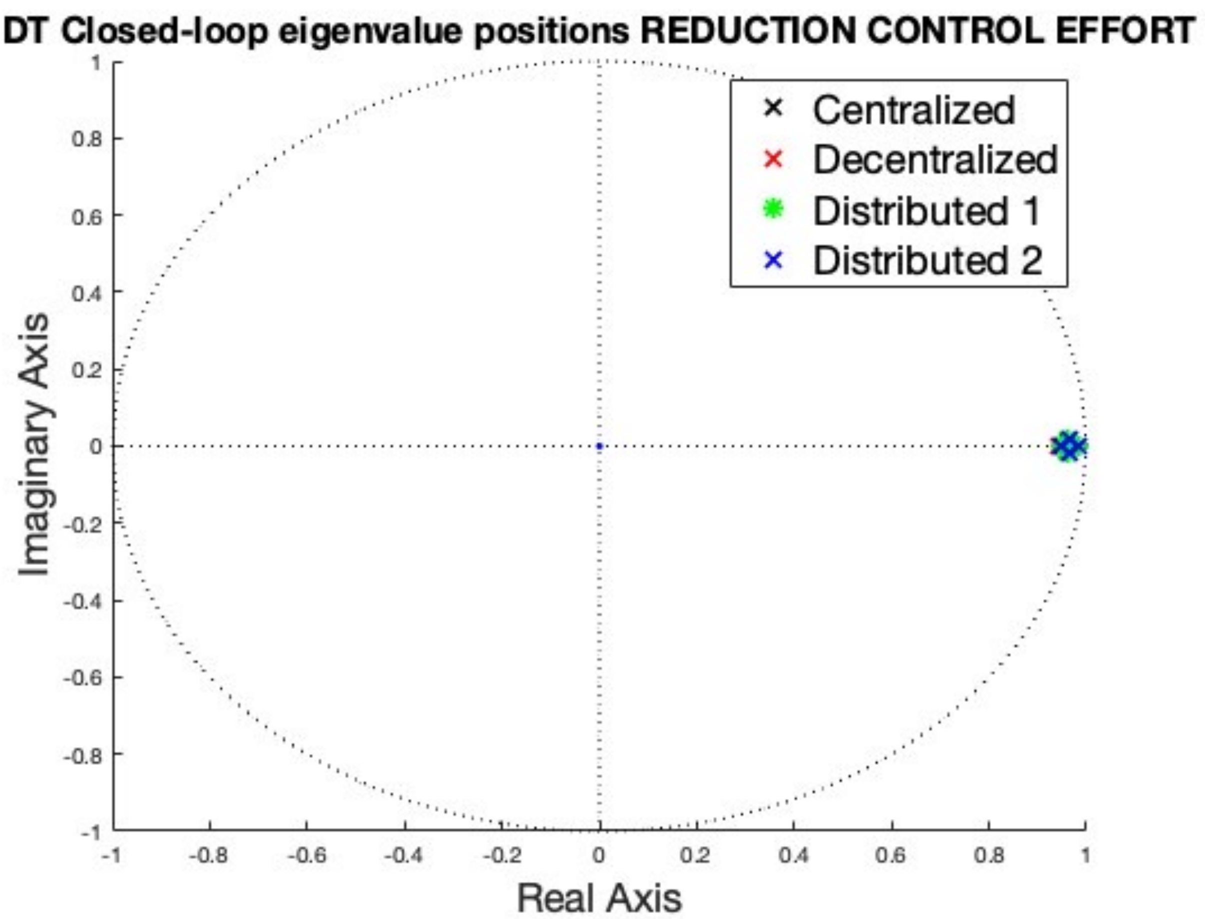


Reduction of the Control Effort

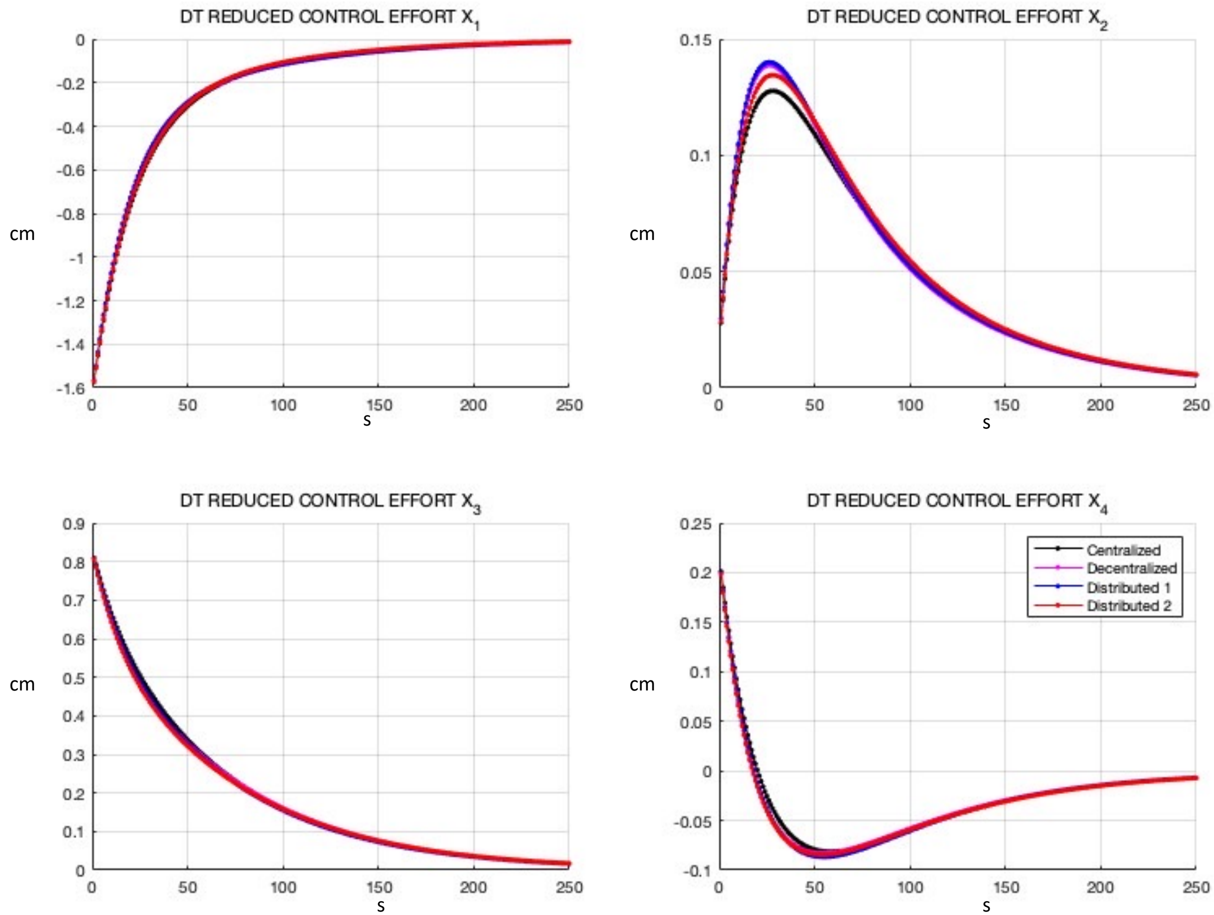


Reduction of the Control Effort

- Discrete-Time



Reduction of the Control Effort

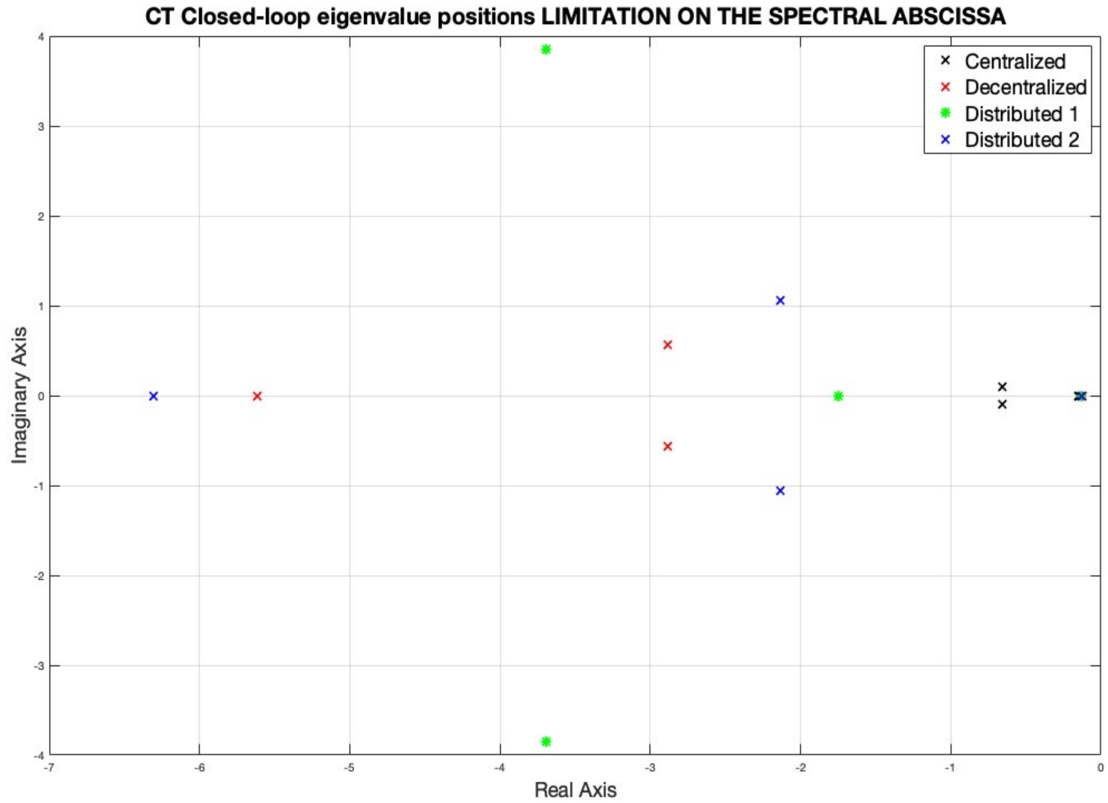


Limitation on the Spectral Abscissa of $A + BK_x$

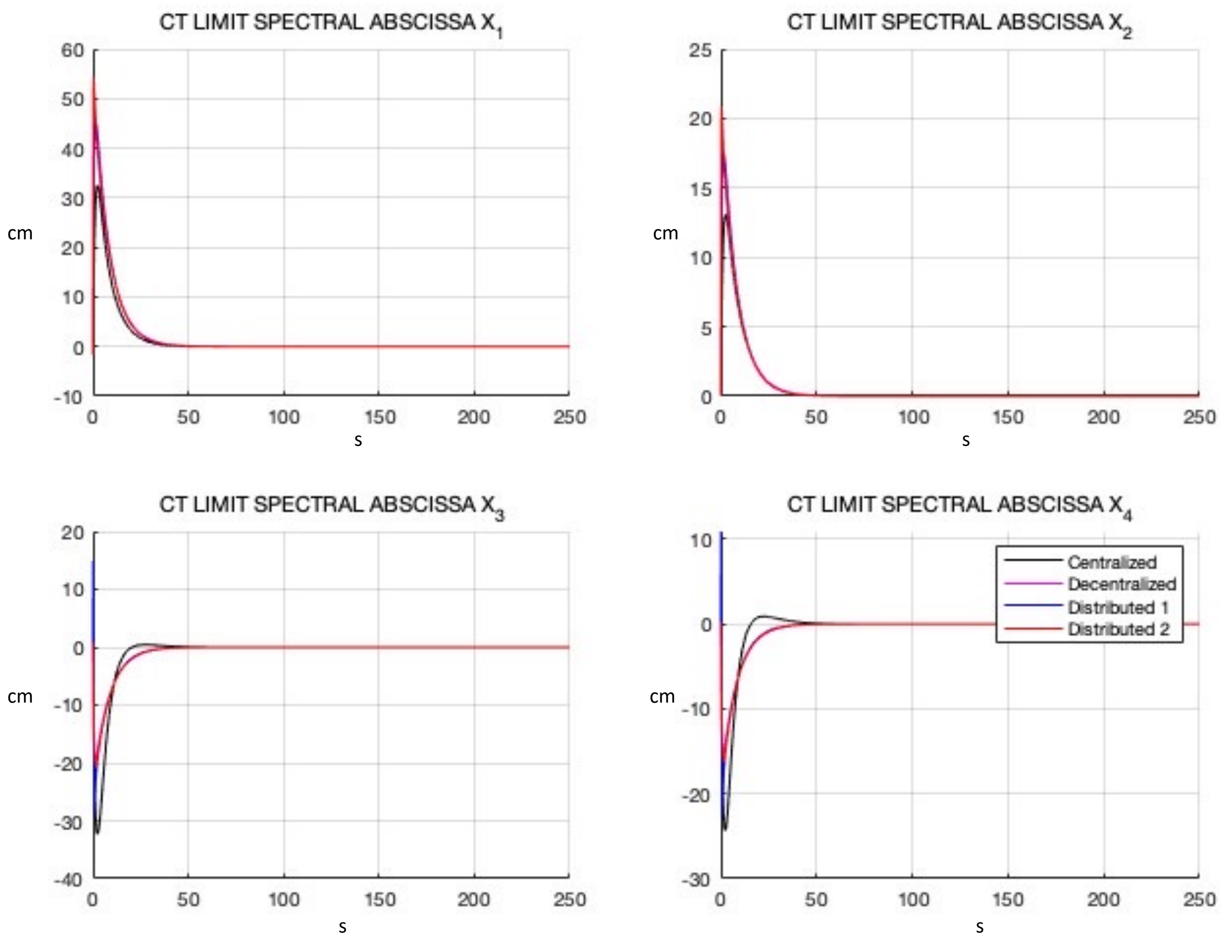
- Continuous-Time

$$YA^T + AY + L^T B^T + BL + 2\alpha Y < 0$$

- Parameter: $\alpha = 0.1$



Limitation on the Spectral Abscissa of $A + BK_x$

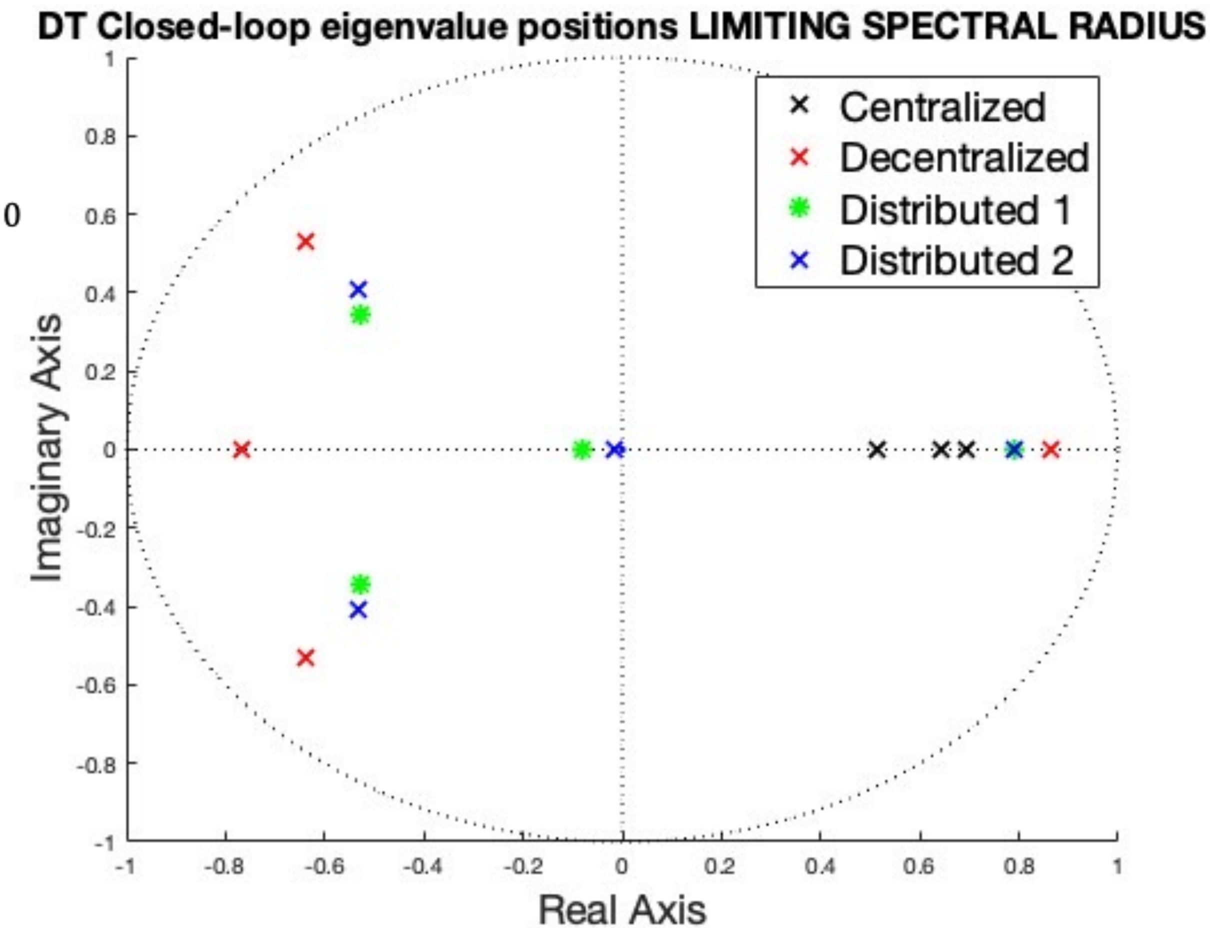


Limitation on the Spectral Radius of $A + BK_x$

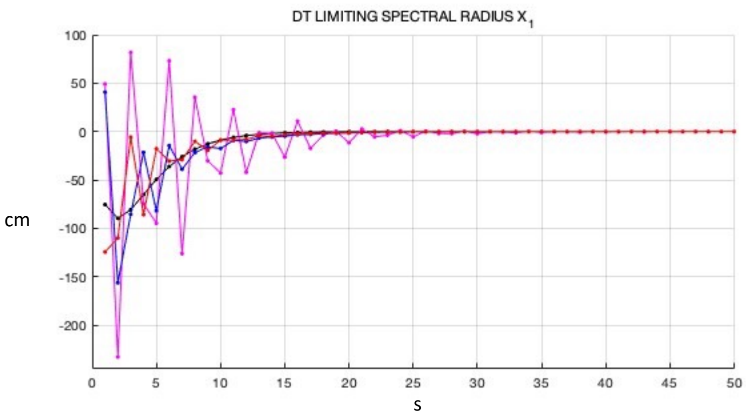
- Discrete-Time

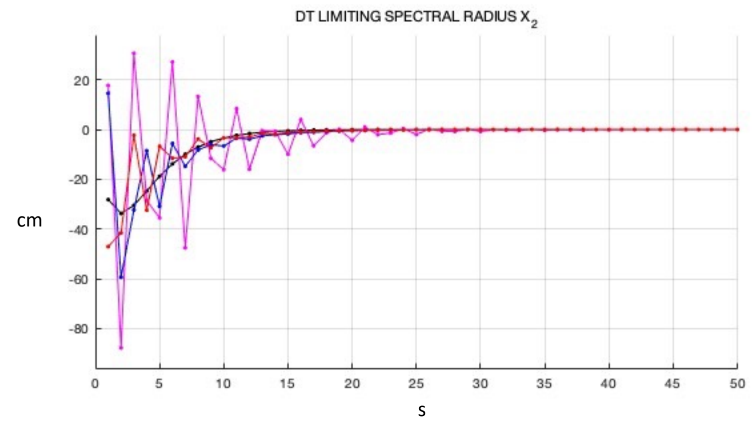
$$\begin{bmatrix} \rho^2 P - APA^T - AL^T B^T - BLA^T & BL \\ L^T B^T & P \end{bmatrix} > 0$$

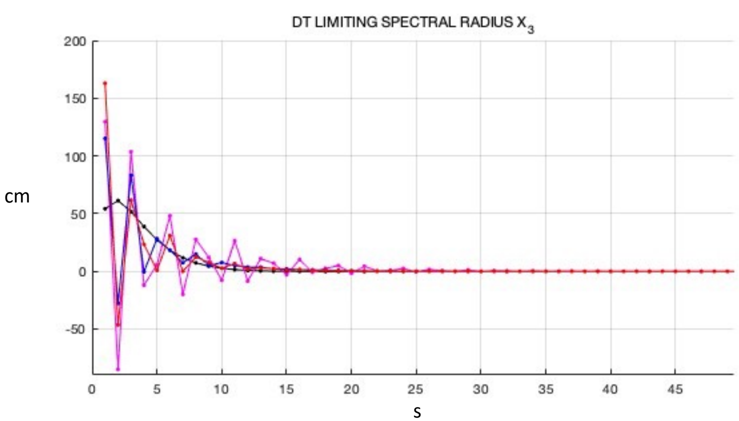
- Parameter: $\rho = 0.8$

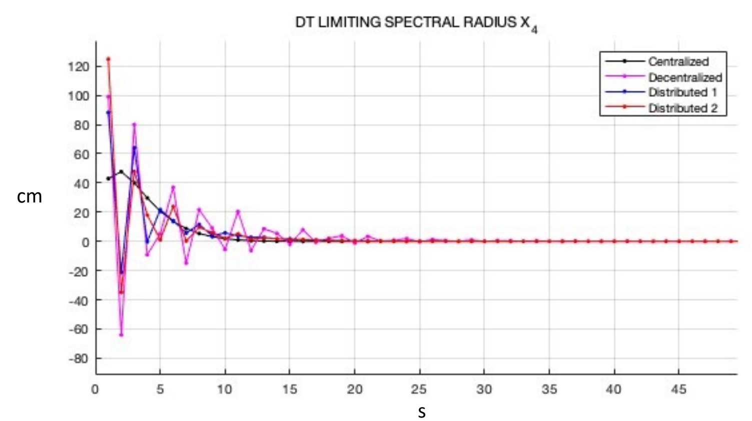


Limitation on the Spectral Radius of $A + BK_x$









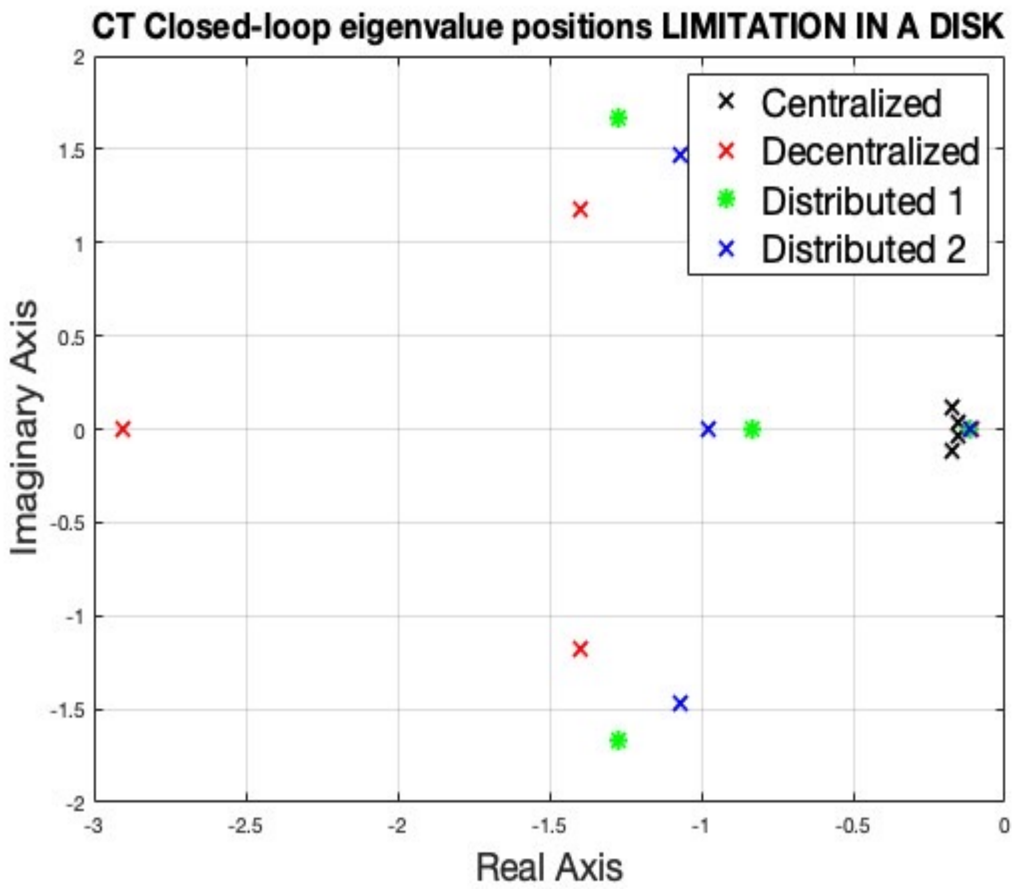
Eigenvalues of $A + BK_x$ in a disk

- Continuous-Time

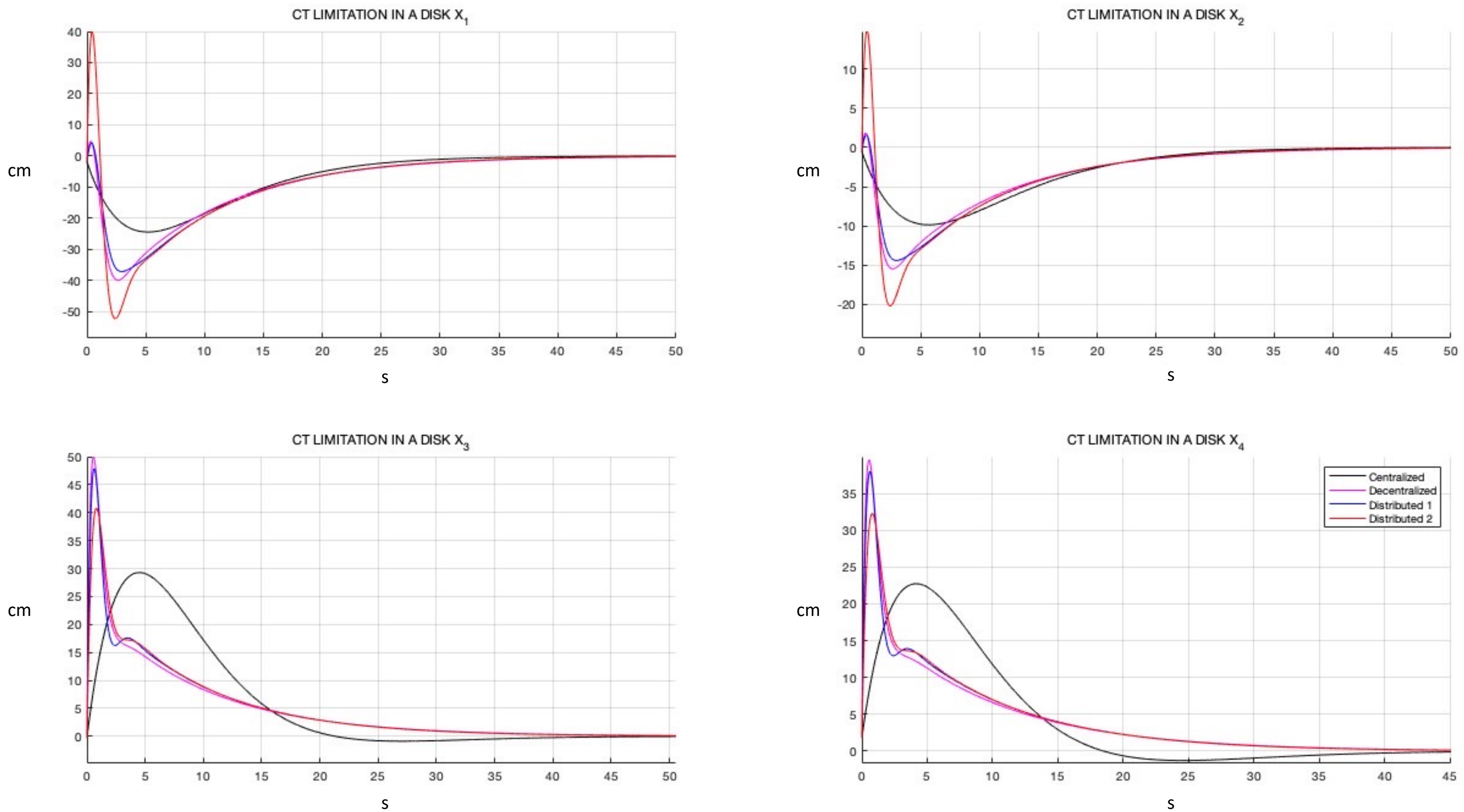
$$\begin{bmatrix} (\rho^2 - \alpha^2)P - APA^T - AL^TB^T - BLA^T - \alpha(PA^T + AP + L^TB^T + BL) & BL \\ L^TB^T & P \end{bmatrix} > 0$$

- Parameter: $\alpha = 6$

  $\rho = 0.8$



Eigenvalues of $A + BK_x$ in a disk



Summary

```
Results STABLE CONTROLLER (Continuous-time):
Scegli visualizzazione barra laterale .ty=0, rho=0.50759, FM=.
-  Decentralized: Feasibility=0, rho=0.52429, FM=.
-  Distributed1 (u2-x1): Feasibility=0, rho=0.50443, FM=.
-  Distributed2 (u1-x2): Feasibility=0, rho=0.49984, FM=.
Results STABLE CONTROLLER (Discrete-time):
-  Centralized: Feasibility=0, rho=0.98396, FM=.
-  Decentralized: Feasibility=0, rho=0.98401, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=0.98397, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=0.98398, FM=.
Results REDUCTION OF THE CONTROL EFFORT (Continuous-time):
-  Centralized: Feasibility=0, rho=-0.014738, FM=.
-  Decentralized: Feasibility=0, rho=-0.014721, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=-0.014789, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=-0.014736, FM=.
Results REDUCTION OF THE CONTROL EFFORT (Discrete-time):
-  Centralized: Feasibility=0, rho=0.98538, FM=.
-  Decentralized: Feasibility=0, rho=0.9854, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=0.98532, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=0.98539, FM=.
```

Summary

```
Results LIMITATION ON THE SPECTRAL ABSCISSA (Continuous-time):
-  Centralized: Feasibility=0, rho=-0.14129, FM=.
-  Decentralized: Feasibility=0, rho=-0.12053, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=-0.12937, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=-0.1273, FM=.
Results LIMITATION ON THE SPECTRAL RADIUS (Discrete-time):
-  Centralized: Feasibility=0, rho=0.69379, FM=.
-  Decentralized: Feasibility=0, rho=0.86396, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=0.79106, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=0.78894, FM=.
Results EIGENVALUES IN A DISK (Continuous-time):
-  Centralized: Feasibility=0, rho=-0.15009, FM=.
-  Decentralized: Feasibility=0, rho=-0.10686, FM=.
-  Distributed (u2-x1): Feasibility=0, rho=-0.11179, FM=.
-  Distributed (u1-x2): Feasibility=0, rho=-0.11301, FM=.
```